The Likert attitude scale : theory and practice

THE LIKERT ATTITUDE SCALE: THEORY AND PRACTICE

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1 Introduction: the scaling theory

In this report, after the introduction, the main methodological and statistical aspects of the theory of attitude scales are briefly mentioned (sections 2 and 3). This is followed by a general description of a computer program that is based on this theory (section 4). The program enables the analysis of data in order to decide if a set of statements is sufficiently homogeneous to accept the sum score as a new variable. This is illustrated in section 5 with an example from the marketing area.

In the following, by attitude scale is meant the type of scale which is associated with the name of R.Likert and which is often denoted as a Likert scale [1]. This type of scale is widely used in the social and behavioral sciences, including interdisciplinary subject centered fields of interest, such as educational and organizational research, marketing and public opinion research. Many people are somehow familiar with being interviewed concerning their opinion or feelings with regard to some issue, reacting to statements by way of putting a cross in the column "strongly agree"; circling a 2 for "rather unimportant"; a 3 for "neither attractive nor unattractive"; and the like.
A common misunderstanding seems to be that a Likert scale is a single statement with a 5 points agree-disagree respons modality, instead of a theory about a set of such statements. This scaling theory and its application is the topic of this report.
The central idea of the scaling theory is that the unknown position of a person on a latent mental attribute (e.g.: a disposition, an attitude, an opinion, a notion, an impression, an intention, a view, a conception, a judgement), is estimated by his agreement or disagreement with statements that are relevant and valid for this latent attribute. It is thus assumed that each person has a fixed position on an underlying latent continuum and that his reaction to each statement is a repeated indication of this position. Each reaction is supposed to have a true score component, contributing to the location of an individual on the latent continuum, and a random error component.

Starting from this basic idea a statistical model can be developed, based on two important implications concerning the covariance matrix of the reactions to the statements: (1) the true score variances of the statements are equal to each other and (2) the true score component correlations are unity. This can be argued as follows.

If the reactions to a set of statements are the repeated measurements of an underlying latent attribute, then the systematic, true variance of each item is exclusively caused by this latent factor. All other variance in the items is random error variation. Now we introduce the following notation:

Let subject j's sum score on k items be: Y_j; j's score on item i: S_{ij}; j's true score component of item i: X_{ij} and j's error term component of item i: e_{ij}. For subject j the scaling model can now be written as:

$$Y_j = \sum_{i=1}^{k} S_{ij} = \sum_{i=1}^{k} (X_{ij} + e_{ij}) = \sum_{i=1}^{k} X_{ij} + \sum_{i=1}^{k} e_{ij} .$$

Because $E(\sum_{i=1}^{k} e_{ij}) = 0$ (the expectation of the sum of the random error components of k items is zero), $E(Y_j) = \sum_{i=1}^{k} X_{ij}$. In other words, $E(Y_j)$ is the true sum score of subject j on k items.

This is an important feature of the theory: without ever knowing true score components and error components of single items for individual respondents, we know that the expectation of the sum score of k items is a reliable estimator of subject j's position on the latent attribute. Furthermore, according to the theory, the true score component vectors $(X_{i1}, \ldots, X_{iN})$, $(X_{k1}, \ldots, X_{kN})$ are identical to each other for all i,k. As a consequence, the true score component variance $\frac{1}{N} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)^2$ of item i is independent of i and is, therefore, the same for all the items. Besides, the true score component covariance $\frac{1}{N} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)(X_{kj} - \bar{X}_k)$ of items i and k is the same for all i,k and is, moreover, equal to the constant variance. As a consequence, the true score correlation between items i and k is unity for all i,k.

The objective of the data analysis is to find the degree of similarity between the observed covariance matrix and the covariance matrix of true score components which is implied by the theory.

In the following, a set of observed variables (statements, items) will be called an instrument or a subscale, provided that requirements, yet to discuss, hold sufficiently.

The implications of the outlined scaling theory should be examined statistically, in order to know if in a research situation the requirements of the model hold sufficiently.
Two aspects are relevant in this context: homogeneity and internal consistency of the instrument.

Homogeneity is the extent to which single items measure the same attribute equally. Internal consistency is the extent to which, after splitting up an instrument randomly into parts, these parts are similar. Remaining methodological points of attention, such as the several forms of validity, are beyond the scope of this report.
With regard to homogeneity, a statistic is computed, called 'Cronbach's Alpha'. Background and deduction of Cronbach's Alpha are traced in section 2. Internal consistency is expressed by the discriminating power of each item; this aspect can be examined by computing a statistic called 'item rest correlation'. This is the subject matter of section 3.

2 Homogeneity: Cronbach's Alpha

The statistic Alpha is based on the concept of reliability in the classical theory of mental tests [2]. According to this theory, the reliability of an instrument consisting of a set of k items is the true variance of the instrument as a proportion of its total variance. The symbol for the coefficient of reliability of an instrument of k parallel statements is r_{kk}.
However, only the total variances and covariances of single items are known, not their component parts "true" and "error". Therefore, the purpose of the following steps is to express r_{kk} in the total variances and covariances of observed variables.

The starting point are definitions of the total, true and error variances of single items and of the instrument, and a general statistical rule. This rule is as follows: the variance of the sum score of k single scores equals the sum of the elements of the kxk covariance matrix of k single scores.

The symbols for the variances of single item i are:

s_{ik}^2 : total; s_{it}^2 : true; s_{ie}^2 : error variance of item i.

Definition of the relationship between these variances:

$$s_{ik}^2 = s_{it}^2 + s_{ie}^2 \qquad (1).$$

For all i, in the notation used earlier, this equation is equivalent to:

$\frac{1}{N} \sum_{j=1}^{N} (S_{ij} - \bar{S}_i)^2 = \frac{1}{N} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)^2 + \frac{1}{N} \sum_{j=1}^{N} (e_{ij} - \bar{e}_i)^2$ where: S_{ij} is the observed score of subject j on item i; X_{ij} is the true score component of S_{ij} and e_{ij} is the error component of S_{ij}. For reasons of simplicity, in the following the notation in equation (1) will be used.

Analogous to the single items, the symbols for an instrument are:

s_k^2 : total; s_t^2 : true; s_e^2 : error variance of an instrument

consisting of k parallel items.

Definition of the relationship between these variances:

$$s_k^2 = s_t^2 + s_e^2 \qquad (2).$$

In the notation used earlier, equation (2) is equivalent to

$$\frac{1}{N} \sum_{i=1}^{k} \sum_{j=1}^{N} (S_{ij} - \bar{S}_i)^2 = \frac{1}{N} \sum_{i=1}^{k} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)^2 + \frac{1}{N} \sum_{i=1}^{k} \sum_{j=1}^{N} (e_{ij} - \bar{e}_i)^2.$$

Definition (2) can be written as: $s_t^2 = s_k^2 - s_e^2$. From the concept of reliability it follows that r_{kk} now can be defined as:

$$r_{kk} = \frac{s_t^2}{s_k^2} = \frac{s_k^2 - s_e^2}{s_k^2} = 1 - \frac{s_e^2}{s_k^2} \qquad (3).$$

Definition (1) can be written as: $s_{ie}^2 = s_{ik}^2 - s_{it}^2$. Adding up over k items results in:

$$\sum_{i=1}^{k} s_{ie}^2 = \sum_{i=1}^{k} s_{ik}^2 - \sum_{i=1}^{k} s_{it}^2 .$$

Besides, $s_e^2 = \sum_{i=1}^{k} s_{ie}^2$: the error variance of the instrument is equal to the sum of the error variances of k single items. The reason is that the expectation of the covariances of the error components is zero: $E(\frac{1}{N} \sum_{j=1}^{N} (e_{ij} - \bar{e}_i)(e_{kj} - \bar{e}_k)) = 0$ for all i,k. In other words, the expectation of the off diagonal elements in the error components covariance matrix is zero. Therefore:

$$s_e^2 = \sum_{i=1}^{k} s_{ie}^2 = \sum_{i=1}^{k} s_{ik}^2 - \sum_{i=1}^{k} s_{it}^2 \qquad (4).$$

Substitution of (4) in (3) results in:

$$r_{kk} = 1 - \frac{\sum_{i=1}^{k} s_{ik}^2 - \sum_{i=1}^{k} s_{it}^2}{s_k^2} \qquad (5).$$

According to the scaling theory:

$$\sum_{i=1}^{k} s_{it}^2 = k\, s_{it}^2 \qquad (6)$$

because $\sum_{i=1}^{k} s_{it}^2 = \frac{1}{N} \sum_{i=1}^{k} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)^2 = k \frac{1}{N} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)^2$.

Substitution of (6) in (5) results in:

$$r_{kk} = 1 - \frac{\sum_{i=1}^{k} s_{ik}^2 - k\, s_{it}^2}{s_k^2} \qquad (7).$$

Besides:

$$s_t^2 = k^2\, s_{it}^2 \qquad (8)$$

as a consequence of the rule that the variance of the sum score of k items is equal to the sum of the elements of the kxk covariance matrix of the items and because, as was argued earlier, in the kxk covariance matrix of true score components, the variances and the covariance of items i and k are a constant for all i,k.

Substitution of (8) in (3) results in:

$$r_{kk} = \frac{k^2\, s_{it}^2}{s_k^2}$$; dividing both terms by k results in:

$$\frac{k\, s_{it}^2}{s_k^2} = \frac{r_{kk}}{k} \qquad (9).$$

Substitution of (9) in (7) results in:

$$r_{kk} = 1 - \frac{\sum_{i=1}^{k} s_{ik}^2}{s_k^2} + \frac{r_{kk}}{k} \qquad (10).$$

Rewriting of (10) results in:

$$r_{kk} = \frac{k}{k-1} \left(1 - \frac{\sum_{i=1}^{k} s_{ik}^2}{s_k^2} \right) \qquad (11).$$

In the covariance matrix of k items, $\sum_{i=1}^{k} s_{ik}^2$ represents the sum of the diagonal elements, whereas s_k^2 represents the total variance of the sum of k items, corresponding to the sum of the elements of the kxk covariance matrix of k items. In other words: in (11), r_{kk} is expressed exclusively in the observed total variances and covariances of k items.

L.J. Cronbach renamed r_{kk} in (11) as α [3]. Equation (11) is known as the definition of "Cronbach's Alpha". In conclusion, Cronbach's Alpha is the coefficient of homogeneity for a set of k parallel items. This can be illustrated in more detail by examining a concrete covariance matrix.

<u>Matrix 1.</u> Covariance matrix of items 1, 2, ..., k, ℓ, ..., m.

	1	2	...	k	ℓ	...	m
1	a_{11}			a_{1k}	$a_{1\ell}$		a_{1m}
2							
:							
:							
k	a_{k1}			a_{kk}	$a_{k\ell}$		a_{km}
ℓ	$a_{\ell 1}$			$a_{\ell k}$	$a_{\ell\ell}$		$a_{\ell m}$
:							
:							
m	a_{m1}			a_{mk}	$a_{m\ell}$		a_{mm}

In matrix 1 the definition of Cronbach's α for items 1, 2, ..., k is:

$$\alpha_{1-k} = \frac{k}{k-1} \left(1 - \frac{\sum_{i=1}^{k} a_{ii}}{\sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij}} \right) \qquad (12)$$

as $\sum_{i=1}^{k} s_{ik}^2$ in (11) equals $\sum_{i=1}^{k} a_{ii}$ in this matrix (the sum of the diagonal elements) and s_k^2 in (11) equals $\sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij}$ in the covariance matrix: the sum of the elements of the kxk covariance matrix.

As was argued earlier, if the requirements of the model hold perfectly, the variances and the covariances of the items are equal to a constant. It follows from (12) that Cronbach's Alpha is unity in this case:

$$\alpha_{1-k} = \frac{k}{k-1} \left(1 - \frac{k}{k^2} \right) .$$

Therefore, the statistic Alpha is a measure of similarity between the covariance matrix of what in section 2 was called S_{ij} (observed) scores and the covariance matrix of X_{ij} (true component) scores.

3 Internal consistency: item rest correlation

Alpha is a scale characteristic expressing the overall similarity of item distributions, but does not provide information on the contribution of each single component part. This function is performed by item rest correlations, which express the discriminating power of single items.
By the discriminating power of an item is meant: the extent to which the distribution of the scores on item i predict the distribution of the sum of the scores of all the other items correctly.
Item rest correlation can easily be defined in the covariance matrix that was introduced in section 2. Definition :

$$r_{i,1-k} = \frac{\sum_{j=1}^{k} a_{ij}}{(a_{ii} \sum_{j=1}^{k} \sum_{\ell=1}^{k} a_{j\ell})} \quad \text{where: } j \neq i, \ell \neq i \qquad (13).$$

This definition is equivalent to the definition of the product moment correlation of the score on item i and the sum of the scores on items 1 to k, minus i. In a famous article, G.W.Bohrnstedt showed that item rest correlations can be computed directly from the covariance matrix without actually adding up scores on single items and computing correlations in two dimensional tables [4]. (13) Is a measure of the discriminating power of item i.

Yet two other definitions in the covariance matrix are important. They were proposed by Bohrnstedt in the same article. First he showed that the correlation between two sets of sums of items (two subscales) can also be computed directly from the covariance matrix, without actually adding up sets of scores on single items.

Secondly, for two single items this was well known; Bohrnstedt's definition is not just equivalent to, but identical to the definition of product moment correlation.

The first of these definitions is:

$$r_{1-k,\ell-m} = \frac{\sum_{i=1}^{k} \sum_{j=\ell}^{m} a_{ij}}{\left(\sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij} \sum_{i=\ell}^{m} \sum_{j=\ell}^{m} a_{ij}\right)^{\frac{1}{2}}} \qquad (14).$$

Bohrnstedt proved that (14) is equivalent with the product moment correlation of the sum of scores on items 1, 2, ..., k and the sum of scores on items ℓ, ..., m.

The second definition is:

$$r_{ij} = \frac{a_{ij}}{(a_{ii} \quad a_{jj})^{\frac{1}{2}}} \qquad (15).$$

This is the well known definition of the product moment correlation of items i and j.

Item rest correlations, item subscale correlations and subscale intercorrelations are useful in the iterative construction of instruments. Items with negative item rest correlation should be reflected. This reflects the fact that these items have a semantically opposite meaning, compared to the rest of the set.

In the literature, $\alpha \geq .85$ and item rest correlations $\geq .40$ are considered as the minimum requirements for an instrument consisting of 10 or more items. In the case of less than 10 items, $\alpha \geq .75$ is considered acceptable.

4 The description of a program in Turbo Pascal

The input to the program is a data matrix of N strings of M items. The researcher can choose a level (e.g. 0.45) above which item intercorrelations are listed. After this items can be reflected. The options of printing correlations and reflection occur at all relevant places in the program.

The next option is the construction of an instrument (a subscale) out of a set of items, starting from a few strong item intercorrelations. On the screen are reported:

- item rest correlations by (14);
- Alpha by (12);
- a list of items which show correlations over a chosen level with the actual instrument by (14). This level has to be specified at the program start. On the basis of experience with the program, 0.30 is advised.

It can now be decided to leave out items from the actual subscale under construction and/or to add new items in a following run. The option of reflection is available again. When an instrument is completed, another can be constructed the same way. A single item can be defined as a scale as well.

During program execution, items which have been reflected are indicated by a minus sign. Missing data should have a score 9 in the data matrix. The results are stored in a file RESULTS that can be printed after the program has finished. This file contains:

- the most recent composition of the subscales, including reflection;
- for each subscale: item rest correlations and Alpha;
- a matrix of subscale intercorrelations.

5 An application

The sketched scaling model can be applied in many situations. In doing this, in general a hypothesis about the existence and identification of one or more latent underlying factors is examined. This will be illustrated with a marketing application.

In a project on the demand for logistic support, part of the questionnaire was devoted to opinions about consultants (A; 7 aspects); the likelihood of the use of logistic support in the near future (B; one single question); opinions about the character and function of logistics in the company (C; 8 statements). These parts of the questionnaire are reported first.

A. How important in the approach of consultants do you evaluate the following aspects?

1 problem approach;
2 expertise and knowledge;
3 communication between consultant and firm;
4 budgetary control;
5 time planning;
6 adchieved completion data;
7 application of knowledge and expertise in the firm.

Response modality: 5 points scales, ranging from "very important": 1, to "very unimportant": 5.

B. (item 8) How likely is it in your opinion, that your firm will use consultant support in the next 5 years in the area of logistics?

Response modality: 5 points scale, ranging from "very likely": 1, to "excluded": 5.

C. 8 Statements:

9 In the sector in which I work there is a great demand for consultant support in the logistic function.

10 The supply of logistic services in my sector is not known.

11 The control of the flow of goods in my business will become yet more important in the future.

12 In the near future logistics will be an important policy area in my business.

13 Logistics is just a marketing tool.

14 The importance of Electronic Data Interchange (EDI) in logistics will grow rapidly.

15 In my company there is scope for improvement in both information management and logistics.

16 Logistics is just a cost centre.

Response modality: 5 points scales, ranging from "strongly agree": 1, to "strongly disagree": 5.

The covariance matrix of these 16 items (N = 140) is shown in table 5.1.

The first step in the analysis is a listing of strong item intercorrelations in order to identify the heart of one or more subscales. The results are reported in table 5.2.

Table 5.2 Item intercorrelations ≧ .45 according to (15).

items		r
1	2	.48
1	7	.48
3	7	.46
4	5	.62
5	6	.46
11	12	.70
12	13	.51
13	14	.48

Next, in a number of runs we examine which subscales can be developed, starting from the previously identified central items. For each run is reported: items in the actual subscale with item rest correlations according to (14); Alpha according to (12); item subscale correlations ≧ .30 according to (14) under the broken line.

Table 5.1 Covariance matrix of 16 statements.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.71	.324	.211	.236	.314	.197	.356	.127	.193	.133	.073	.120	.116	.067	.041	-.027
2	.324	.63	.216	.239	.213	.127	.243	.050	.133	.156	.054	.040	.120	-.022	.088	.015
3	.211	.216	.63	.269	.161	.027	.326	-.019	.163	.023	.077	.062	.043	.022	.091	-.097
4	.236	.239	.269	.80	.497	.168	.108	-.060	.210	.054	-.027	.022	.104	.044	.038	.094
5	.314	.213	.161	.497	.80	.358	.141	-.096	.228	.147	-.067	-.029	.027	-.031	.021	.108
6	.197	.127	.027	.168	.358	.74	.078	.114	.210	.066	.003	.160	.037	.058	.119	.014
7	.356	.243	.326	.108	.141	.078	.78	.130	.128	.042	.180	.165	.179	.072	.119	-.089
8	.127	.050	-.019	-.060	-.096	.114	.130	1.26	.392	-.018	.330	.438	.288	.156	.374	-.123
9	.193	.133	.163	.210	.228	.210	.128	.392	.95	.017	.319	.489	.343	.245	.387	-.069
10	.133	.156	.023	.054	.147	.066	.042	-.018	.017	1.22	-.011	-.101	-.202	-.088	.001	.210
11	.073	.054	.077	-.027	-.067	.003	.180	.330	.319	-.011	1.04	.849	.331	.248	.314	-.177
12	.120	.040	.062	.022	-.029	.160	.165	.438	.489	-.101	.849	1.41	.656	.440	.479	-.339
13	.116	.120	.043	.104	.027	.037	.179	.288	.343	-.202	.331	.656	1.18	.525	.445	-.131
14	.067	-.022	.022	.044	-.031	.058	.072	.156	.245	-.088	.248	.440	.525	1.01	.336	-.124
15	.041	.088	.091	.038	.021	.119	.119	.374	.387	.001	.314	.479	.445	.336	.91	-.122
16	-.027	.015	-.097	.094	.108	.014	-.089	-.123	-.069	.210	-.177	-.339	-.131	-.124	-.122	1.28

Run 1 item	item rest and item subscale correlations	Alpha
4	.62	
5	.62	.77
- - -	- - -	- - -
1	.40	
2	.35	
3	.34	
6	.38	

Run 2 item	item rest and item subscale correlations	Alpha
1	.40	
4	.56	
5	.64	.71
- - -	- - -	- - -
2	.47	
3	.38	
6	.40	
7	.33	
9	.31	

Run 3 item	item rest and item subscale correlations	Alpha
1	.51	
2	.47	
4	.55	
5	.59	.74
- - -	- - -	- - -
3	.42	
6	.38	
7	.37	
9	.31	

The next 2 runs are skipped in this report.

Run 6

item	item rest and item subscale correlations	Alpha
1	.59	
2	.52	
3	.50	
4	.52	
5	.50	
7	.44	.77
- - -	- - -	- - -
6	.32	
9	.31	

Subscale 1 is completed now, since there are no item subscale correlations ≧ .40.

We continue with the first run of the next subscale (run 7).

Run 7

item	item rest and item subscale correlations	Alpha
11	.59	
12	.75	
13	.45	.76
- - -	- - -	- - -
8	.35	
9	.44	
14	.45	
15	.48	

In the next run, 4 items (8,9,14,15) are added at the same time.

Run 8 item	item rest and item subscale correlations	Alpha
8	.40	
9	.51	
11	.54	
12	.70	
13	.56	
14	.44	
15	.56	.80

As there are no items with item subscale correlations ≧ .40, subscale 2 is completed as well.

Finally, the results of the analysis are summarized in table 5.3.

The interpretation of the results doesn't cause much difficulty. We have found 2 subscales which are relatively mutually independent (r = .17). Subscale 1 expresses the importance in general of the knowledge and expertise of consultants, and of an effective communication. Aspect 6 (achieved completion data) differs sematically from the other aspects.

Table 5.3 Subscale and item intercorrelations according to (13), (14) and (15).

	subscale 1	subscale 2	item 6	item 10	item 16
subscale 1		.17	.32	.14	.00
subscale 2	.17		.16	-.07	-.19
item 6	.32	.16		.07	.01
item 10	.14	-.07	.07		.17
item 16	.00	-.19	.01	.17	

This is reflected in the fact that item 6 cannot be added to subscale 1. Subscale 2 expresses a general evaluation of logistic support. Here, the statements 10 and 16 drop out, which seems to have a semantic explanation as well.

The advantage of representing the data by the sum scores on these 2 subscales is twofold.

In the first place, the results are easily surveyable, compared with a detailed description of aspects with slightly different meanings. The differentiation between the respondents can now be shown by just 2 scores, each of them representing a general underlying factor.

Secondly, the most important implication of the model is that these sum scores on subscales have a known reliablility (Alpha). This is not true for each single item. Moreover, the more items a subscale contains, the more random error in individual reactions to items will fall away. As a consequence, the sum scores will approach the true scores.

Notice that in the presented application, reflecting items is not required, as a consequence of the formulation of the statements.

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