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LETTER TO THE EDITOR

THE CONSISTENT SOLUTION OF THE UNDAMPED CYCLOTRON-WAVE PROPAGATION PROBLEM

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In a recent publication[1]) the nonlinear dielectric behaviour of a cold plasma drifting along a static magnetic field (B_0) in a large amplitude E.M. wave ($\hat{E}$, ω, k) near and at cyclotron resonance has been treated. The permittivity, ε, of the plasma has been calculated from the reactive current; this current has been found by the summation over velocity space of the appropriate velocity component of the individual particles. To that end, the characteristic quantities, which define the individual motion of the particles such as the amplitude and the period of the energy oscillation have been expressed as functions of $\hat{E}$, B_0 and, close to the resonance, of the index of refraction, n. As a result, also the expressions for the reactive currents are functions of the permittivity, and close to the resonance:

$$n = \sqrt{\varepsilon_r} = n(\tilde{g}, \beta, \tilde{\alpha}, n) \tag{1}$$

(*cf.* eqs. (2) and (34) of ref. 1), where the symbols have the same meaning as in ref. 1:

$$\tilde{g} = \frac{e\hat{E}}{\gamma_0 m\omega c(1 - v_{z0}/v_p)} \doteq \frac{g}{\gamma_0(1 - v_{z0}/v_p)},$$

the E.M. field-strength parameter†,

$$\beta = \frac{eB_0}{\gamma_0 m\omega(1 - v_{z0}/v_p)} - 1,$$

the magnetic-field parameter,

$$\tilde{\alpha} = \frac{\omega_p^2}{\gamma_0\omega^2} = \frac{n_e e^2}{m\varepsilon_0\omega^2\gamma_0},$$

the electron-density parameter.

† For convenience a slightly different E.M. field parameter $\tilde{g}$ instead of g has been used: the parameter $\tilde{g}$ contains a small correction factor due to the zero-order drift velocity along the static magnetic field. As in ref. 1 the zero-order transverse velocity is supposed to be zero, so $\gamma_0 \equiv m(0)/m = 1/(1 - v_{z0}^2/c^2)^{\frac{1}{2}}$.

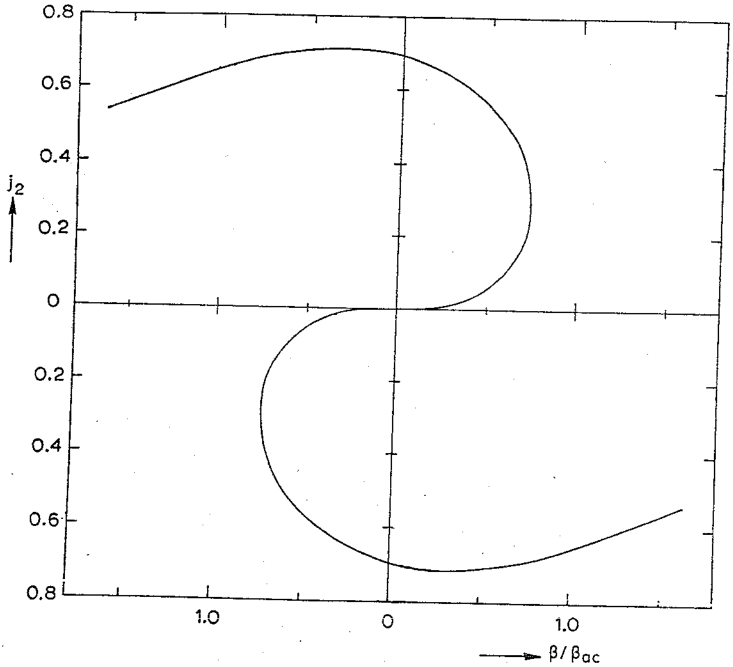

Fig. 1. The dielectric function j_2 as a function of the normalized magnetic-field parameter β/β_{ac}.

In ref. 1 an elaborate numerical procedure is described for the solution of eq. (1) in terms of $n = n(\beta)$ for a given set of values of $\tilde{g}$ and of $\tilde{\alpha}$ (*cf.* fig. 8 of ref. 1). Once this has been done, also the characteristic quantities of the motion can be calculated as functions of β. Though, in principle, the solution of eq. (1) can be obtained, one is forced to solve the problem again for any new set of values of the E.M. field strength and density, *i.e.* the parameters $\tilde{g}$ and $\tilde{\alpha}$. To avoid this cumbersome numerical procedure, an alternative, approximate, procedure is presented here which results in generalized expressions for all quantities.

It has been established, that under the condition that $\tilde{\alpha}' \gg$ the quantity $\tilde{g}^2\,[(1-\varepsilon)/\tilde{\alpha}]\,C$ is a function of β/β_{ac} only (*cf.* fig. 1):

$$[(1-\varepsilon)/\tilde{\alpha}]\,C = j_2(\beta/\beta_{ac}), \tag{2}$$

where

$$C = (\tilde{\alpha}\tilde{g}^2)^{\frac{1}{3}}, \qquad \text{and} \qquad \beta_{ac} = 0.73\,C; \tag{3}$$

β_{ac} is the width of the nonlinear resonance.

This is the consistent solution of $\varepsilon - 1$ as a function of β for any set of values of $\tilde{g}$ and $\tilde{\alpha}$. Now it is possible to substitute this solution in the expressions for the energy oscillation (*cf.* eqs. (18)–(23) of ref. 1). This enables

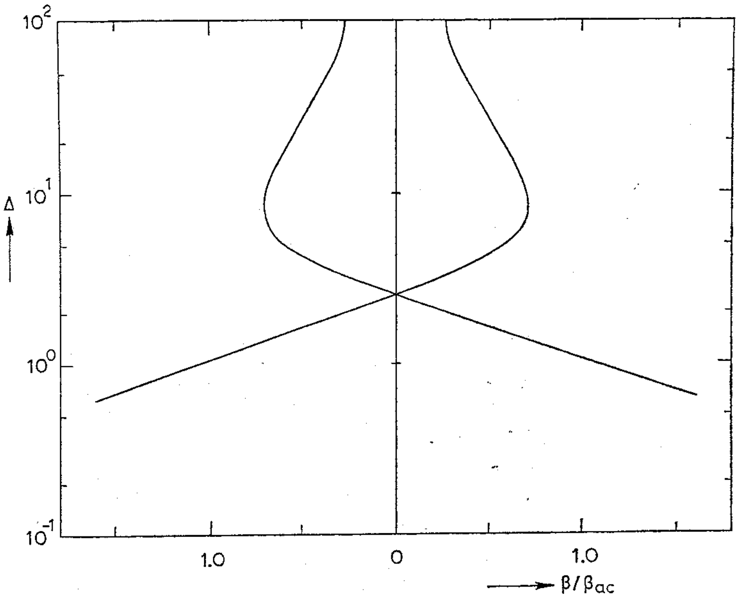

Fig. 2. The normalized energy amplitude as a function of the magnetic-field parameter β/β_{ac}.

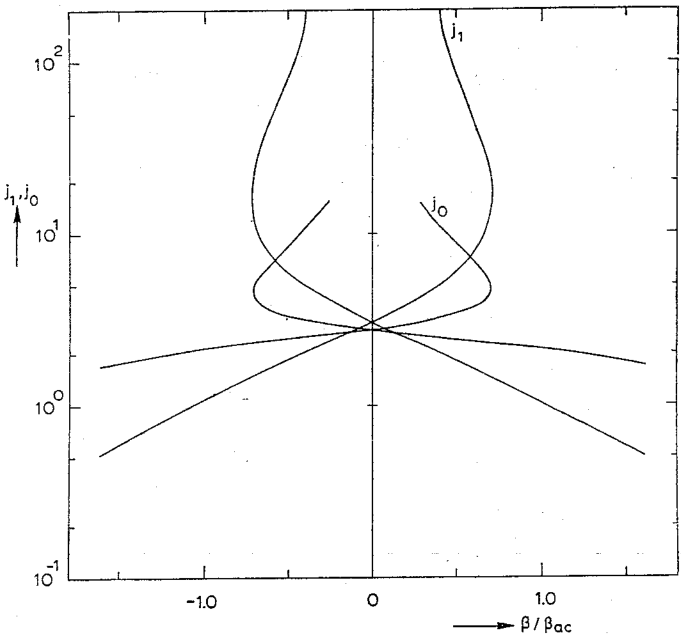

Fig. 3. Oscillation-time functions j_0 and j_1 as functions of magnetic-field parameter β/β_{ac}.

us to express the energy-amplitude oscillation time and length in new functions of Δ, j_0 and j_1 which depend only on β/β_{ac}. In terms of these functions which are shown in figs. 2 and 3 we find as a complete solution of the problem the following:

the energy amplitude:

$$\gamma_{extr} - \gamma_0 \approx \frac{\Delta(\beta/\beta_{ac})}{C} \gamma_0(1 - v_{z0}/v_p)\, \tilde{g}^2; \tag{4}$$

the oscillation time:

$$\omega t_{os} = \tau_{os} = \frac{2j_0}{C}\left[\frac{1}{1 - v_{z0}/v_p} + \frac{j_1}{j_0}\frac{C^2}{\tilde{\alpha}}\right]; \tag{5}$$

the oscillation length:

$$kz_{os} = \left\{\frac{2v_{z0}/v_p}{1 - v_{z0}/v_p}\frac{j_0}{C} + 2j_1\left[\frac{C}{\tilde{\alpha}} - j_2\right]\right\}. \tag{6}$$

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REFERENCE

1) Schram, D. C., Physica **40** (1968) 422.