

The polarization losses of offset antennas

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by

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The polarization losses of offset paraboloid antennas

by Jakob Dijk, Christiaan T.W. van Diepenbeek, Eduard J. Maanders
and Lambert F.G. Thurlings

Abstract

The electric field in the aperture of offset front-fed paraboloid antennas and open cassegrain antennas, excited by an electric dipole or Huygens source in the focus, is compared with the fields of front-fed circularly symmetrical paraboloid reflector antennas and classical cassegrain antennas. The aperture field forms the basis of expressions to calculate the polarization efficiency of all four types of antenna. Computed results are given, showing that offset antennas can compete with front-fed paraboloids if they are excited by an electric dipole; the classical cassegrain antenna, however, shows better results. If offset antennas are excited by a Huygens source, they are very unfavourable compared with the symmetrical antennas which show no cross-polarization.

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1. Introduction

It has been known for several years that, if a paraboloid reflector antenna is fed by a linearly polarized electrical dipole, the antenna system will radiate not only energy in the main polarization, but also a fair amount in an unwanted polarization, mostly called cross polarization or depolarization.

Condon [1] was one of the first to give a detailed analysis of this phenomenon. It appears that cross-polarized lobes, also called Condon lobes, are formed, having a maximum in planes at 45° to the principal plane.

Silver [2, p. 423] also mentions this cross-polarization, mainly as an abstract of Condon's work.

Cutler [3] gives a physical explanation as to the relation between aperture electric field lines and the polarization of the dipole feed, and explains the very unfavourable situation which occurs if the focus of the paraboloid falls between the aperture and apex of the paraboloid. This work has been continued by Jones [4], who investigates the radiation characteristics of paraboloid reflector antennas excited in their foci by a short electrical dipole feed, a short magnetic dipole feed, and a plane wave source, being a combination of an electric and a magnetic dipole. If this dipole pair is represented by dipole fields of equal intensity, commonly known as a Huygens source, it has been proved that the cross-polarized component of the aperture illumination disappears [5].

Kofman [5] has extended this work by considering other conical sections of revolutions as well as the paraboloid. The cross-polarized pattern of the reflector excited by any arbitrary feed system may be calculated, using the methods of Afifi [6] while Potter [7] has found an analytical expression for the polarization loss or polarization efficiency.

It is the latter expression which will also be reviewed in this paper. Potter [8] has also found a similar expression for cassegrain antennas which will be included in the present study.

Not much is known so far about off-set paraboloids and open cassegrain antennas. Hanfling [9] has shown a stereographic mapping method which contains the aperture field lines of an offset antenna, excited by several field sources, but without further details, while

Graham [10] describes the polarization of offset antennas and states that an offset cassegrain antenna can be designed to have very low cross-polarization losses, however, also without further explanations or calculations.

Since plans now exist for frequency re-use above 10 GHz by polarization diversity the interest in cross-polarization problems has recently increased considerably. Ludwig [11] has published a paper on the definition of cross polarization, and Kimber and Tischenko [12] calculate the current distribution of various reflector antennas with different illumination, but unfortunately without giving any numerical results.

Chu and Turrin [13] have discussed the beamshift of offset antennas with circular polarization and have calculated the level of cross-polarization sidelobes, predicting the poor polarization performance of the open cassegrain antenna.

It is the purpose of the present paper to obtain a more detailed insight into the cross-polarization losses of offset antennas. For this purpose we shall compare the front-fed paraboloid, the true cassegrain antenna, the offset front-fed paraboloid, and the open cassegrain antenna. In all the cases we shall use a short electrical linearly polarized dipole, a magnetic dipole and a Huygens source as a primary radiator. We will first compare the aperture electric fields, define afterwards a polarization factor and calculate this for different configuration. Finally, we show a practical example.

2. Aperture fields of reflector antennas illuminated by an electric dipole

2.1 Aperture field of a front-fed paraboloid

Let us consider a short electric dipole of length l (2, p.92), lying along the x axis of a cartesian coordinate system (Fig.1), with a current I flowing in the direction of the positive x-axis.

Expressed in ρ, ψ, ξ coordinates (Fig.2), the far zone components of the complex electric field are

$$\begin{aligned} \bar{E} &= E_{\psi} \bar{a}_{\psi} + E_{\xi} \bar{a}_{\xi} \quad \text{or} \\ \bar{E} &= \frac{j\eta I l e^{-jk\rho}}{2\lambda\rho} (-\bar{a}_{\psi} \cos\psi \cos\xi + \bar{a}_{\xi} \sin\xi), \end{aligned} \quad (1)$$

where $\eta = 120 \pi$ Ohms, \bar{a}_{ψ} and \bar{a}_{ξ} are unit vectors along the ψ and ξ axes, respectively, and k the wave number.

In x,y,z coordinates Eq. 1 becomes

$$\bar{E} = -E_0 [\bar{x}(\cos^2\psi \cos^2\xi + \sin^2\xi) - \bar{y}(\frac{1}{2} \sin^2\psi \sin 2\xi) + \bar{z} \cdot \frac{1}{2} \sin\psi \cos\xi], \quad (2)$$

where

$$E_0 = \frac{j\eta I l e^{-jk\rho}}{2\lambda\rho}.$$

If the dipole is oriented along the positive y-axis, it is readily seen that the electric field becomes

$$\bar{E} = (-\bar{a}_{\psi} \cos\psi \sin\xi - \bar{a}_{\xi} \cos\xi) \quad (3)$$

or

$$\bar{E} = -E_0 [-\bar{x} \frac{1}{2} \sin^2\psi \sin 2\xi + \bar{y}(\cos^2\psi \sin^2\xi + \cos^2\xi) + \bar{z} \frac{1}{2} \sin 2\psi \sin\xi]. \quad (4)$$

The surface currents induced in any arbitrary reflector by these fields can be computed straight forwardly using geometrical optical techniques.

Using the method employed before by Jones [4], the aperture field may now be found by first calculating the physic surface-current density on the reflector $\bar{K} = 2(\bar{n} \times \bar{H}_i)$, \bar{H}_i being the incident field and \bar{n} the unit vector normal to the surface at the point of incidence.

A simpler way to find the aperture field may be followed by investigating what happens with the fields $E_{\psi} \bar{a}_{\psi}$ and $E_{\psi} \bar{a}_{\psi}$ at the point of incidence. From Fig. 3 it is readily seen that the vector $E_{\psi} \bar{a}_{\psi}$ is perpendicular to the plane comprising the z-axis, radius ρ from focus to the surface of the reflector, the reflected ray, and the vector \bar{n} at the point of incidence (plane FGH). After reflection this vector remains perpendicular to the surface, but its direction reverses. Therefore,

$$\bar{E}_{\psi}^r = - E_{\psi} \bar{a}_{\psi} \quad (5)$$

the index r indicating reflection.

The vector $E_{\psi} \bar{a}_{\psi}$ lies in plane FGH and is perpendicular to the radius. To find out what happens with $E_{\psi} \bar{a}_{\psi}$ we will use Fig. 3 and define the indices n and τ as the directions normal and tangential to the paraboloid surface at the point of incidence. We now resolve E_{ψ} in $E_{\psi,n}$ and $E_{\psi,\tau}$ resulting in

$$\begin{aligned} E_{\psi,n} &= E_{\psi} \sin \frac{1}{2} \psi \\ E_{\psi,\tau} &= E_{\psi} \cos \frac{1}{2} \psi \end{aligned} \quad (6)$$

After reflection, $E_{\psi,n}$ is continuous and $E_{\psi,\tau}$ retains its sign. Therefore,

$$\begin{aligned} E_{\psi,n}^r &= E_{\psi} \sin \frac{1}{2} \psi \\ E_{\psi,\tau}^r &= - E_{\psi} \cos \frac{1}{2} \psi \end{aligned} \quad (7)$$

By means of the vectors \bar{a}_{\perp} and $\bar{a}_{//}$ [Fig. 3] and by resolving $E_{\psi,n}$ and $E_{\psi,\tau}$ along these vectors it is readily found, using Eqs. 7, that

$$E_{\perp}^r = - E_{\psi,n}^r \sin \frac{1}{2} \psi + E_{\psi,\tau}^r \cos \frac{1}{2} \psi = - E_{\psi} \quad (8)$$

$$E_{//}^r = E_{\psi,n}^r \cos \frac{1}{2} \psi + E_{\psi,\tau}^r \sin \frac{1}{2} \psi = 0 \quad (9)$$

The unit vector \bar{a}_{\perp} may be written

$$\bar{a}_{\perp} = \bar{z} \cos \frac{1}{2} \psi + \bar{y} \sin \frac{1}{2} \psi \quad (10)$$

If we use an electric dipole oriented along the positive x-axis, the reflected field \vec{E}_ψ^r follows from Eqs. 1, 8 and 10 resulting in

$$\vec{E}_\psi^r = E_0 \cos \psi \cos \xi (\cos \xi, \sin \xi, 0), \quad (11)$$

and Eq. 5 becomes $\vec{E}_\psi^r = -E_0 \sin \xi \vec{a}_\xi$

By means of Eqs. 1, 5 and 11 the aperture field E_A yields

$$\vec{E}_A = E^r(x, y, z) = E_0 \cos \psi \cos \xi (\cos \xi, \sin \xi, 0) - E_0 \sin \xi (-\sin \xi, \cos \xi, 0)$$

or

$$\vec{E}_A = E_0 \left[\left\{ 1 - \cos^2 \xi (1 - \cos \psi) \right\} \vec{x} - \frac{1}{2} \sin 2\xi (1 - \cos \psi) \vec{y} \right] \quad (12)$$

where

$$E_0 = \frac{j \eta I l e^{-jk(F+z_0)}}{2 \lambda r}$$

Using the same technique as described above, we find for the aperture field, if the dipole is oriented along the positive y-axis

$$\vec{E}_A = E_0 \left[-\frac{1}{2} \sin 2\xi (1 - \cos \psi) \vec{x} + \left\{ 1 - \sin^2 \xi (1 - \cos \psi) \right\} \vec{y} \right] \quad (13)$$

Apparently, equivalent results are obtained, because the paraboloid is circularly symmetrical and each linear polarization may be reduced to a polarization along the x-axis by means of rotation. However, if the dipole is oriented along the positive z-axis, the far field pattern is represented by

$$\vec{E} = -E_0 \sin \psi \vec{a}_\psi$$

and the aperture field by

$$\vec{E}_A = E_0 (\sin \psi \cos \xi \vec{x} + \sin \psi \sin \xi \vec{y}) \quad (14)$$

Although the results of a paraboloid with a dipole located along the z-axis are not of great practical value, they are mentioned here for the sake of completeness and because of the fact that the results are used below.

2.2. Aperture field of an offset paraboloid

Let us consider an offset paraboloid as illustrated in Fig. 4. The electric dipole will be located in the focus of the paraboloid oriented along the positive x'-axis of a x', y', z' coordinate system.

If this dipole has a dipole moment \bar{p}_0 [2, p.93], we may resolve \bar{p}_0 into two vectors, one along the x-axis and one along the z-axis resulting in

$$\bar{p}_0 = |\bar{p}_0| \cos \psi_0 \bar{x} + |\bar{p}_0| \sin \psi_0 \bar{z} \quad (15)$$

Therefore, we may replace the dipole oriented along the x' axis by a combination of a dipole along the x-axis with dipole moment $|\bar{p}_0| \cos \psi_0$ and a dipole along the z-axis with dipole moment $|\bar{p}_0| \sin \psi_0$.

The aperture field is now a superposition of the aperture fields according to Eq. 12 replacing E_0 by $E_0 \cos \psi_0$ and to Eq. 14 replacing E_0 by $E_0 \sin \psi_0$. We then obtain for the aperture field:

$$\begin{aligned} \bar{E}_A = & [E_0 \cos \psi_0 \{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \} + E_0 \sin \psi_0 \sin \psi \cos \frac{\psi}{2}] \bar{x} \\ & + [E_0 \cos \psi_0 \{ -\frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \} + E_0 \sin \psi_0 \sin \psi \sin \frac{\psi}{2}] \bar{y} \quad (16) \end{aligned}$$

If the dipole is oriented along the y' axis, Eq. 13 may be used again since the y'-axis is identical with the y-axis.

Although it is not of great value in practice, the aperture field may also be found if the dipole is oriented along the z' axis. The same procedure is now followed as before. The dipole moment \bar{p}_0 is divided along the x-axis and z-axis yielding

$$\bar{p}_0 = -|\bar{p}_0| \sin \psi_0 \bar{x} + |\bar{p}_0| \cos \psi_0 \bar{z} \quad (17)$$

This results in an aperture field

$$\begin{aligned} \bar{E}_A = & [- E_0 \sin \psi_0 \{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \} + E_0 \cos \psi_0 \sin \psi \cos \frac{\psi}{2}] \bar{x} \\ & + [- E_0 \sin \psi_0 \{ -\frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \} + E_0 \cos \psi_0 \sin \psi \sin \frac{\psi}{2}] \bar{y} \quad (18) \end{aligned}$$

2.3. Aperture field of a classical cassegrain antenna

The same technique as used in Sec. 2.1 may be employed to calculate the fields in the aperture of a cassegrain antenna. However, there are some fundamental differences because the dipole field is reflected twice before it arrives at the main reflector aperture. Therefore, the components \bar{E}_f and \bar{E}_ψ are to be known after this double reflection in order to calculate this aperture field. Let the electric dipole be located in focus F_1 and oriented along the positive x-axis [Fig.5]; after reflection by the curved surface of the hyperboloid the component E_f of Eq. 1 becomes

$$E_f^r \bar{a}_f = - E_0' \sin \frac{1}{2} \bar{a}_f \quad (19)$$

and after reflection by the paraboloid

$$E_f^{rr} \bar{a}_f = E_0' \sin \frac{1}{2} \bar{a}_f \quad (20)$$

To find the component E_ψ^{rr} we introduce the vectors \bar{n} , $\bar{\tau}$, $\bar{a}_{//h}$, $\bar{a}_{\perp h}$, $\bar{a}_{//p}$ and $\bar{a}_{\perp p}$, defined in Fig. 6 and all lying in plane $F_1 F_2 GH$. Resolving E_ψ into components along \bar{n} and $\bar{\tau}$ yields

$$\begin{aligned} E_{\psi,n} &= - E_\psi \sin \alpha \\ E_{\psi,\tau} &= - E_\psi \cos \alpha \end{aligned} \quad (21)$$

After reflection by the hyperboloid, Eq. 21 becomes

$$\begin{aligned} E_{\psi,n}^r &= - E_\psi \sin \alpha \\ E_{\psi,\tau}^r &= + E_\psi \cos \alpha \end{aligned} \quad (22)$$

These components are resolved along $\bar{a}_{//h}$ and $\bar{a}_{\perp h}$ resulting in

$$\begin{aligned} E_{//h}^r &= E_{\psi,n}^r \cos \alpha + E_{\psi,\tau}^r \sin \alpha \\ &= - E_\psi \sin \alpha \cos \alpha + E_\psi \sin \alpha \cos \alpha = 0 \end{aligned} \quad (23)$$

$$\begin{aligned} E_{\perp h}^r &= - E_{\psi,n}^r \sin \alpha + E_{\psi,\tau}^r \cos \alpha \\ &= E_\psi \sin^2 \alpha + E_\psi \cos^2 \alpha = E_\psi \end{aligned} \quad (24)$$

Upon reflection by the paraboloid as explained in Sec. 2.1, the reflected vector of E_{ψ}^r becomes

$$\bar{E}_{\psi}^{rr} = -E_{\psi} \bar{a}_{\perp, p} \quad (25)$$

Since the vectors $\bar{a}_{\perp, p} = \cos \frac{\psi}{2} \bar{x} + \sin \frac{\psi}{2} \bar{y}$ (Fig. 6), $\bar{a}_{\parallel} = -\sin \frac{\psi}{2} \bar{x} + \cos \frac{\psi}{2} \bar{y}$ and $\bar{E}_{\psi} = -\cos \psi \cos \frac{\psi}{2} \bar{a}_{\psi}$, we obtain by combining Eqs. 20 and 25

$$\bar{E}_{\psi}^{rr}(x, y, z) = \bar{E}_A = E_0' [\sin \frac{\psi}{2} (-\sin \frac{\psi}{2}, \cos \frac{\psi}{2}, 0)_{\bar{x}} + \cos \psi \cos \frac{\psi}{2} (\cos \frac{\psi}{2}, \sin \frac{\psi}{2}, 0)] \quad (26)$$

or

$$\bar{E}_A = -E_0' \left[\left\{ 1 - \cos^2 \frac{\psi}{2} (1 + \cos \psi) \right\} \bar{x} - \frac{1}{2} \sin 2 \frac{\psi}{2} (1 + \cos \psi) \bar{y} \right] \quad (27)$$

Defining $\psi_1 = 180^\circ - \psi$, Eq. 27 becomes

$$\bar{E}_A = -E_0' \left[\left\{ 1 - \cos^2 \frac{\psi_1}{2} (1 - \cos \psi_1) \right\} \bar{x} - \frac{1}{2} \sin 2 \frac{\psi_1}{2} (1 - \cos \psi_1) \bar{y} \right] \quad (28)$$

In the equations discussed in this section

$$E_0' = \frac{j \eta I l e^{-jkr_0}}{2 \lambda \rho'} \quad , \text{ where } \rho' \text{ is the distance between the primary focus and the surface of the main reflector } | \quad .$$

It is readily found that $r_0 = f/e + F + Z_0$ and $\rho' = \frac{2F}{1 + \cos \psi_1} + f/e$, where f is the distance between the two hyperboloid foci, e the hyperboloid eccentricity and Z_0 the depth of the paraboloid. If the dipole is oriented along the positive y-axis we find in a similar way

$$\bar{E}_A = -E_0' \left[\left\{ -\frac{1}{2} \sin 2 \frac{\psi_1}{2} (1 - \cos \psi_1) \right\} \bar{x} + \left\{ 1 - \sin^2 \frac{\psi_1}{2} (1 - \cos \psi_1) \right\} \bar{y} \right] \quad (29)$$

and if the dipole is oriented along the z-axis

$$\bar{E}_A = E_0' (\sin \psi_1 \cos \frac{\psi_1}{2} \bar{x} + \sin \psi_1 \sin \frac{\psi_1}{2} \bar{y}) \quad (30)$$

2.4. The aperture field of an open cassegrain antenna

The calculation of the aperture field of an open cassegrain antenna is much more complicated than the previous ones. The geometry is presented in Fig. 7. In general, the planes KGH (with the z-axis) and F₁KG (with the z'-axis) will not coincide. Therefore, the ray from the primary focus F₁ to the subreflector and the ray reflected from the paraboloid (GH) will generally not be located in the same plane. However, we may obtain a solution by considering the electric dipole to be located in F₁ and oriented along the positive x''-axis [Fig.8]. In the coordinate system x'', y'', z'' concentrated around F₁, the far field of this electric dipole is given by

$$\vec{E} = E_{\xi''} \vec{a}_{\xi''} + E_{\psi''} \vec{a}_{\psi''} = E_0 \sin \xi'' \vec{a}_{\xi''} - E_0 \cos \psi'' \cos \xi'' \vec{a}_{\psi''} \quad (31)$$

After reflection by the hyperboloid we obtain

$$E_{\xi''}^r \vec{a}_{\xi''} = - E_0 \sin \xi'' \vec{a}_{\xi''} (x'', y'', z'') \quad (32)$$

and

$$E_{\psi''}^r \vec{a}_{\psi''} = - E_0 \cos \psi'' \cos \xi'' \vec{a}_{\psi''} (x'', y'', z'') \quad (33)$$

where

$$\vec{a}_{\xi''} (x'', y'', z'') = \vec{a}_{\xi'} (x', y', z') = -\sin \xi' \vec{x}' + \cos \xi' \vec{y}' \quad (34)$$

and

$$\vec{a}_{\psi''} (x'', y'', z'') = \cos \psi_2 \cos \xi' \vec{x}' + \cos \psi_2 \sin \xi' \vec{y}' + \sin \psi_2 \vec{z}' \quad (35)$$

In total, the field reflected by the subreflector in x', y', z' coordinates is

$$\vec{E}^r (x', y', z') = -E_0 \sin \xi' (-\sin \xi', \cos \xi', 0) - E_0 \cos \psi'' \cos \xi' (\cos \psi_2 \cos \xi', \cos \psi_2 \sin \xi', \sin \psi_2) \quad (36)$$

Dipole fields originated from the secondary focus F₂ result in well-known aperture fields at the main reflector. Therefore, we will try to find such a field and identify it with Eq. 36.

In the x', y', z' coordinate system the electric dipole oriented along the positive x' -axis, located in F_2 has a far field of

$$\begin{aligned} E^{(1)}(x', y', z') &= E_0^{(1)} \sin \xi' (-\sin \xi', \cos \xi', 0) \\ &\quad - E_0^{(1)} \cos \psi_2 \cos \xi' (\cos \psi_2 \cos \xi', \cos \psi_2 \sin \xi', \sin \psi_2') \end{aligned} \quad (37)$$

and if the dipole is oriented along the z' -axis

$$E^{(2)}(x', y', z') = -E_0^{(2)} \sin \psi_2 (\cos \psi_2 \cos \xi', \cos \psi_2 \sin \xi', \sin \psi_2) \quad (38)$$

Added together, the total field originated from F_2 is

$$\begin{aligned} E_0^{(1)} \sin \xi' (-\sin \xi', \cos \xi', 0) + (-E_0^{(1)} \cos \psi_2 \cos \xi' - E_0^{(2)} \sin \psi_2) \cdot \\ \cdot (\cos \psi_2 \cos \xi', \cos \psi_2 \sin \xi', \sin \psi_2) \end{aligned} \quad (39)$$

If we wish the Eqs. 36 and 39 to be identical, the requirements are:

$$(1) E_0^{(1)} \equiv -E_0 \quad (40)$$

$$(2) E_0^{(2)} \sin \psi_2 + E_0^{(1)} \cos \psi_2 \cos \xi' \equiv E_0 \cos \psi'' \cos \xi' \quad (41)$$

$$\text{or } E_0^{(2)} \equiv E_0 (\cos \psi_2 + \cos \psi'') \cdot \frac{\cos \xi'}{\sin \psi_2} \quad (42)$$

In this way we may consider the reflected field from the hyperboloid to be the primary field for the paraboloid main reflector with an electric dipole located in F_2 , having a dipole moment $E_0^{(1)}$ oriented along the x' -axis and a dipole moment $E_0^{(2)}$ along the z' -axis.

Resolving these dipoles along x - and z -axes by means of the transformation

$$\begin{aligned} \bar{x} &= \cos \psi_0 \bar{x}' + \sin \psi_0 \bar{z}' \\ \bar{y} &= \bar{y}' \\ \bar{z} &= -\sin \psi_0 \bar{x}' + \cos \psi_0 \bar{z}' \end{aligned} \quad (43)$$

we obtain along the x -axis a dipole moment of

$$E_0^{(x)} = E_0^{(1)} \cos \psi_0 - E_0^{(2)} \sin \psi_0 \quad (44)$$

and along the z -axis

$$E_0^{(z)} = E_0^{(1)} \sin \psi_0 + E_0^{(2)} \cos \psi_0 \quad (45)$$

By means of Eq. 12 we obtain for dipole $E_0^{(x)}$ an aperture field of

$$\bar{E}^r = E_0^{(x)} \left[\left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} \bar{x} - \frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \right] \bar{y} \quad (46)$$

In accordance with Eq. 14 we find for dipole $E_0^{(z)}$ an aperture field of the paraboloid

$$\bar{E}^r = E_0^{(z)} \left(\sin \psi \cos \frac{\psi}{2} \bar{x} + \sin \psi \sin \frac{\psi}{2} \bar{y} \right) \quad (47)$$

From the above it is readily seen that the complete solution for the aperture field is

$$\begin{aligned} \bar{E}_A = & \left[E_0^{(x)} \left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(z)} \sin \psi \cos \frac{\psi}{2} \right] \bar{x} + \\ & + \left[- \left\{ E_0^{(x)} \frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(z)} \sin \psi \sin \frac{\psi}{2} \right] \bar{y} \quad , \quad (48) \end{aligned}$$

where $E_0^{(1)}$, $E_0^{(2)}$, $E_0^{(x)}$ and $E_0^{(z)}$ have the values as defined in Eqs. 40, 42, 44 and 45 and where

$$\begin{aligned} E_0 &= \frac{j \eta I l e^{-jk r_a}}{2 \lambda \rho'} \\ r_a &= z_0 + F + \frac{f}{e} \\ \tan \frac{1}{2} (100 - \psi'') &= \frac{e-1}{e+1} \tan \frac{1}{2} \psi_2 \\ \rho' &= \frac{f}{e} + \frac{2F}{(1 + \cos \psi)} \end{aligned}$$

and as explained in Appendix A

$$\begin{aligned} \sin \psi_2 &= \left[(\cos \psi_0 \sin \psi \cos \frac{\psi}{2} - \sin \psi_0 \cos \psi)^2 + (\sin \psi \sin \frac{\psi}{2})^2 \right]^{\frac{1}{2}} = \sqrt{A} \\ \cos \psi_2 &= \sin \psi_0 \sin \psi \cos \frac{\psi}{2} + \cos \psi_0 \cos \psi \\ \cos \frac{\psi}{2}' &= \frac{\cos \psi_0 \sin \psi \cos \frac{\psi}{2} - \sin \psi_0 \cos \psi}{\sqrt{A}} \\ \sin \frac{\psi}{2}' &= \frac{\sin \psi \sin \frac{\psi}{2}}{\sqrt{A}} \end{aligned}$$

If the electric dipole is oriented along the + y'' axis, the same procedure may be followed as above. We mention only the results

$$\begin{aligned} \bar{E}_A = & \left[E_0^{(x)} \left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(y)} \left\{ -\frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(z)} \sin \psi \cos \frac{\psi}{2} \right] \bar{x} \\ & + \left[E_0^{(x)} \left\{ -\frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(y)} \left\{ 1 - \sin^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(z)} \sin \psi \sin \frac{\psi}{2} \right] \bar{y} \end{aligned}$$

where

$$\begin{aligned}
 E_o^{(x)} &= -E_o^{(z)} \sin \psi_o \\
 E_o^{(y)} &= E_o^{(y)} \\
 E_o^{(z)} &= E_o^{(z)} \cos \psi_o \\
 E_o^{(x')} &= -E_o \\
 E_o^{(y')} &= E_o \left[\cos \psi_2 + \cos \psi'' \right] \frac{\sin \frac{\psi'}{2}}{\sin \psi_2} \\
 E_o &= \frac{j\eta I l e^{-jkra}}{2\lambda p'} \\
 r_o &= z_o + F + f/e \\
 p' &= f/e + \frac{2F}{1 + \cos \psi}
 \end{aligned}$$

and if finally the electric dipole is oriented along the z'' -axis, we obtain for the aperture field

$$\begin{aligned}
 \vec{E}_A &= \left[E_o^{(x)} \left\{ 1 - \cos^2 \frac{\psi'}{2} (1 - \cos \psi) \right\} + E_o^{(z)} \sin \psi \cos \frac{\psi'}{2} \right] \vec{x} + \\
 &+ \left[E_o^{(x)} \left\{ -\frac{1}{2} \sin 2 \frac{\psi'}{2} (1 - \cos \psi) \right\} + E_o^{(z)} \sin \psi \sin \frac{\psi'}{2} \right] \vec{y} ,
 \end{aligned} \tag{50}$$

where

$$\begin{aligned}
 E_o^{(x)} &= -E_o^{(z)} \sin \psi_o \\
 E_o^{(z)} &= E_o^{(z)} \cos \psi_o \\
 E_o^{(y')} &= E_o \frac{\sin \psi''}{\sin \psi_2} \\
 E_o &= \frac{j\eta I l e^{-jkra}}{2\lambda p'} \\
 r_o &= z_o + F + \frac{f}{e} \\
 p' &= f/e + \frac{2F}{1 + \cos \psi}
 \end{aligned}$$

3. Aperture fields of reflector antennas illuminated by a magnetic dipole

3.1. The far fields of a magnetic dipole

The far field of a magnetic dipole has already been considered by Silver [1,p.95]. We will mention here only the results in respect of the direction of the dipole moment in the x, y, z coordinate system. If the dipole moment is oriented along the positive x-axis, the far field in the ρ, ψ, ξ coordinates is

$$\vec{E} = E_0' [\cos \psi \cos \xi \vec{a}_\xi + \sin \xi \vec{a}_\psi] \quad (51)$$

If the dipole moment is oriented along the positive y-axis, we obtain

$$\vec{E} = E_0' [\cos \psi \sin \xi \vec{a}_\xi - \cos \xi \vec{a}_\psi] \quad (52)$$

and, finally, if the dipole is oriented along the +z-axis,

$$\vec{E} = E_0' \sin \psi \vec{a}_\xi \quad (53)$$

In all these equations we define E_0' (2,p.95) as

$$E_0' = m_0 \frac{k^2}{4\pi} \eta \cdot \frac{1}{\rho} \quad (54)$$

where m_0 is the magnetic moment.

3.2. The aperture field of a front-fed paraboloid

The aperture field may now be determined in a similar way as done in Sec. 2.1. If the dipole moment is oriented along the x-axis, the electric field after reflection becomes:

$$\vec{E}^r = -E_0' \cos \psi \cos \xi (-\sin \xi, \cos \xi, 0) - E_0' \sin \xi (\cos \xi, \sin \xi, 0)$$

or

$$\vec{E}_A = -E_0' \left[\left\{ \frac{1}{2} \sin 2\xi (1 - \cos \psi) \right\} \vec{x} + \left\{ 1 - \cos^2 \xi (1 - \cos \psi) \right\} \vec{y} \right] \quad (55)$$

If the dipole moment is oriented along the y -axis, we obtain, as shown before (4),

$$\bar{E}_A = E_0' \left[\left\{ 1 - \sin^2 \frac{\psi}{2} (1 - \cos \psi) \right\} \bar{x} + \frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \bar{y} \right] \quad (56)$$

and if the dipole moment is oriented along the z -axis

$$E_A = E_0' \left\{ \sin \psi \sin \frac{\psi}{2} \bar{x} - \sin \psi \cos \frac{\psi}{2} \bar{y} \right\} \quad (57)$$

3.3. The aperture field of an offset paraboloid

The aperture field of an offset paraboloid excited by a magnetic dipole follows from the results in Sec. 3.2.

If the dipole moment is oriented along the x' -axis, we find

$$\begin{aligned} \bar{E}_A = & \left[E_0' \cos \psi_0 \left\{ -\frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0' \sin \psi_0 \sin \psi \sin \frac{\psi}{2} \right] \bar{x} + \\ & + \left[-E_0' \cos \psi_0 \left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0' \sin \psi_0 (-\sin \psi \cos \frac{\psi}{2}) \right] \bar{y} \end{aligned} \quad (58)$$

If the dipole is oriented along the y' -axis, we may use Eq. 56 since $y' = y$ and if the dipole moment points in the z -direction.

$$\begin{aligned} E_A = & \left[E_0' \sin \psi_0 \left\{ \frac{1}{2} \sin 2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0' \cos \psi_0 \sin \psi \sin \frac{\psi}{2} \right] \bar{x} \\ & + \left[E_0' \sin \psi_0 \left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0' \cos \psi_0 (-\sin \psi \cos \frac{\psi}{2}) \right] \bar{y} \end{aligned} \quad (59)$$

3.4. The aperture fields of classical and open cassegrain systems

Following the same procedure as for classical cassegrain systems illuminated by an electric dipole, the aperture fields of a classical cassegrain antenna illuminated by a magnetic dipole are as follows

(a) The dipole moment is oriented along the positive x-axis:

$$\vec{E}_A(x, y, z) = -E_0' \left[\frac{1}{2} \sin 2\xi (1 - \cos \psi_1) \vec{x} + \{1 - \cos^2 \xi (1 - \cos \psi_1)\} \vec{y} \right], \quad (60)$$

(b) The dipole moment is oriented along the positive y-axis:

$$\vec{E}_A(x, y, z) = E_0' \left[\{1 - \sin^2 \xi (1 - \cos \psi_1)\} \vec{x} + \frac{1}{2} \sin 2\xi (1 - \cos \psi_1) \vec{y} \right] \quad (61)$$

(c) The dipole moment is oriented along the positive z-axis:

$$\vec{E}_A(x, y, z) = E_0' \left\{ -\sin \psi_1 \sin \xi \vec{x} + \sin \psi_1 \cos \xi \vec{y} \right\}, \quad (62)$$

The results for the open cassegrain antenna are collected in the survey below.

Magnetic dipole moment oriented along positive x''-axis

$$\vec{E}_A = \left[-E_0^{(x)} \frac{1}{2} \sin 2\xi (1 - \cos \psi) + E_0^{(z)} \sin \psi \sin \xi \right] \vec{x} + \left[-E_0^{(x)} \{1 - \cos^2 \xi (1 - \cos \psi)\} - E_0^{(z)} \sin \psi \cos \xi \right] \vec{y} \quad (63)$$

where

$$E_0^{(x)} = E_0^{(1)} \cos \psi_0 - E_0^{(2)} \sin \psi_0$$

$$E_0^{(z)} = E_0^{(1)} \sin \psi_0 + E_0^{(2)} \cos \psi_0$$

$$E_0^{(1)} = E_0'$$

$$E_0^{(2)} = -E_0' (\cos \psi'' + \cos \psi_2) \cdot \frac{\cos \xi'}{\sin \psi_2}$$

$$E_0' = \frac{k^2}{4\pi} \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{m_0}{\rho'} \sin \theta \exp(-jk r_a)$$

$$r_a = z_0 + F + f/e$$

$$\rho' = f/e + \frac{2F}{1 + \cos \psi}$$

Magnetic dipole moment oriented along positive y''-axis

$$\vec{E}_A = \left[-E_0^{(x)} \left\{ \frac{1}{2} \sin 2\frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(y)} \left\{ 1 - \sin^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(z)} \sin \psi \sin \frac{\psi}{2} \right] \vec{x} \\ + \left[-E_0^{(x)} \left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(y)} \left\{ \frac{1}{2} \sin 2\frac{\psi}{2} (1 - \cos \psi) \right\} - E_0^{(z)} \sin \psi \cos \frac{\psi}{2} \right] \vec{y}$$

where

(64)

$$E_0^{(x)} = -E_0^{(z)} \sin \psi_0$$

$$E_0^{(y)} = E_0^{(y)}$$

$$E_0^{(z)} = E_0^{(z)} \cos \psi_0$$

$$E_0^{(1)} = E_0'$$

$$E_0^{(2)} = -E_0' (\cos \psi'' + \cos \psi_2) \frac{\sin \frac{\psi}{2}}{\sin \psi_2}$$

Magnetic dipole moment oriented along positive z''-axis

$$\vec{E}_A = \left[-E_0^{(x)} \left\{ \frac{1}{2} \sin 2\frac{\psi}{2} (1 - \cos \psi) \right\} + E_0^{(z)} \sin \psi \sin \frac{\psi}{2} \right] \vec{x} + \\ + \left[-E_0^{(x)} \left\{ 1 - \cos^2 \frac{\psi}{2} (1 - \cos \psi) \right\} - E_0^{(z)} \sin \psi \cos \frac{\psi}{2} \right] \vec{y}$$

where

(65)

$$E_0^{(x)} = -E_0^{(1)} \sin \psi_0$$

$$E_0^{(z)} = E_0^{(1)} \cos \psi_0$$

$$E_0^{(1)} = -E_0' \frac{\sin \psi''}{\sin \psi_2}$$

In all three configurations

$$E_0' = \frac{k^2}{4\pi} \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{m_0}{\rho'} \sin \theta e^{-jk r_a}$$

$$r_a = z_0 + F + f/e$$

$$\rho' = f/e + \frac{2F}{1 + \cos \psi}$$

4. The aperture fields of reflector antennas illuminated by a Huygens source

A combination of an electric dipole and a magnetic dipole of equal intensity and crossly oriented is often called a Huygens source [4]. If this source is located in the focus of a paraboloid antenna in such a way that the electric dipole orients along the positive x-axis and the magnetic dipole along the positive y-axis, it is readily seen from Eqs. 1 and 52 that the far field of the Huygens source may be written as

$$\vec{E} = E_0 (1 + \cos \psi) (-\cos \frac{\psi}{2} \vec{a}_\psi + \sin \frac{\psi}{2} \vec{a}_\xi) \quad (66)$$

and if the magnetic dipole points in the -x direction and the electric dipole in the +y direction from Eqs. 3 and 51

$$\vec{E} = -E_0 (1 + \cos \psi) (\sin \frac{\psi}{2} \vec{a}_\psi + \cos \frac{\psi}{2} \vec{a}_\xi) \quad (67)$$

where $E_0 = E'_0$

The aperture fields of a front fed paraboloid illuminated by a Huygens source referred to Eq. 66 are readily found by superposition of Eqs. 12 and 56.

In accordance with Jones [4], we find

$$\vec{E}_A = E_0 (1 + \cos \psi) \vec{x} \quad (68)$$

In the same way if the electric dipole is oriented along the +y axis and the magnetic dipole along the -x axis

$$\vec{E}_A = E_0 (1 + \cos \psi) \vec{y} \quad (69)$$

It appears that in both cases the cross-polarization component disappears. The Huygens source used for Eq. 67 is the same as that used for Eq. 66, but rotated over an angle of 90 degrees.

A classical cassegrain antenna shows similar results. From Eqs. 28 and Eqs. 61 we find

$$\vec{E}_A = -E_0 (1 + \cos \psi_i) \vec{x} \quad (70)$$

and from Eqs. 29 and 60

$$\vec{E}_A = -E_0 (1 + \cos \psi_i) \vec{y} \quad (71)$$

The aperture field of an offset antenna illuminated by a Huygens source may be found by combining the aperture fields originated by an electric dipole oriented along the positive x' - axis and a magnetic dipole oriented along the positive y' - axis. The resultant aperture field is found from Eqs. 16 and 56 by letting $E_0 = E'_0$. If the electric dipole is oriented along the positive y' - axis and the magnetic dipole along the negative x' - axis, we find the aperture field similarly as above by superposition of the aperture fields from Eqs. 13 and 58. As will be noticed, the cross polarization component does not disappear.

One is easily tempted to simplify the cross-polarization component by letting $E'_0 = E_0 \cos \Psi_0$. In this way the superposition of Eqs 16 and 56 leads to:

$$E_A = \{ E'_0 \cos \Psi_0 (1 + \cos \psi) + E'_0 \sin \Psi_0 \sin \psi \cos \frac{\psi}{2} \} \bar{x} + E'_0 \sin \Psi_0 \sin \psi \sin \frac{\psi}{2} \bar{y} \quad (72)$$

and the superposition of Eqs. 13 and 58 to:

$$E_A = - E'_0 \sin \Psi_0 \sin \psi \sin \frac{\psi}{2} \bar{x} + \{ E'_0 \cos \Psi_0 (1 + \cos \psi) + E'_0 \sin \Psi_0 \sin \psi \cos \frac{\psi}{2} \} \bar{y} \quad (73)$$

If we try to find the aperture fields of an open cassegrain antenna by combining Eqs. 63 and 49 or Eqs. 64 and 48, it appears also that no simplification takes place. Therefore, it is of little value to rewrite here the equations found before. As will be noticed, the cross-polarization component in the aperture does not disappear either.

5. The polarization efficiency

In accordance with Potter (7), the polarization efficiency of an antenna is defined by the ratio of antenna gain including the effects of cross polarization, to antenna gain if the cross polarized energy were zero everywhere.

Thus

$$\eta_p = \frac{\left| \int_0^{2\pi} \int_0^{\psi} E_{mp}(\psi, \xi) \rho^2 \sin \psi \, d\psi d\xi \right|^2}{\left| \int_0^{2\pi} \int_0^{\psi} [E_{mp}^2(\psi, \xi) + E_{cp}^2(\psi, \xi)]^{\frac{1}{2}} \rho^2 \sin \psi \, d\psi d\xi \right|^2} \quad (74)$$

where $E_{mp}(\psi, \xi)$ represents the electric field in the aperture with principal polarization and $E_{cp}(\psi, \xi)$ that of the cross polarization. By means of Eq.74 and the equations for the electric field in the aperture found in the previous paragraphs it is now possible to calculate the polarization efficiency.

In the case that a front-fed paraboloid is investigated, the distance ρ between paraboloid and focus is $\rho = 2F/(1+\cos\psi)$, and because all the fields involved are proportional to $\exp(-jk(F+z_0))/\rho$, Eq.74 may be replaced by

$$\eta_p = \frac{\left| \int_0^{2\pi} \int_0^{\psi} \frac{E_{mp}(\psi, \xi)}{E_0} \tan \frac{1}{2} \psi \, d\psi d\xi \right|^2}{\left| \int_0^{2\pi} \int_0^{\psi} \left[\left(\frac{E_{mp}(\psi, \xi)}{E_0} \right)^2 + \left(\frac{E_{cp}(\psi, \xi)}{E_0} \right)^2 \right]^{\frac{1}{2}} \tan \frac{1}{2} \psi \, d\psi d\xi \right|^2} \quad (75)$$

If the paraboloid is illuminated by an electric dipole oriented along the +x-axis, the aperture fields to be used are

$$E_{mp} = E_0 [1 - \cos^2 \xi (1 - \cos \psi)] \quad (76)$$

$$E_{cp} = -\frac{1}{2} E_0 \sin 2\xi (1 - \cos \psi) \quad (77)$$

where

$$E_0 = \frac{j \eta I \mathcal{L} e^{-jk(F+z_0)}}{2\lambda \rho}$$

It is possible to simplify Eq. 75 by substituting Eq. 76 and Eq. 77, but this does not increase the insight into the problem. An approximation of this equation as carried out by Potter (7), has the drawback that it gives only reliable results for very shallow paraboloid reflectors with subtending angles of less than 60 degrees. The results of Eq. 75, computed without any approximation, applied to front-fed paraboloid reflector antennas are presented in Fig. 9.

In the case of a classical cassegrain antenna (Figs. 5 and 6), the integration is carried out over the angles ξ and ψ_2 . The fields are now proportional to the factors

$$E'_0 = \frac{j\eta I L}{2\lambda} \frac{e^{-jk(z_0 + F + f/e)}}{\rho'}$$

where

$$\rho' = \frac{2F}{1 + \cos \psi_2} + f/e$$

The aperture fields are expressed in the angles ξ and ψ_1 which have to be transformed to ξ, ψ_2 by

$$\tan \frac{1}{2} \psi_1 = \frac{e-1}{e+1} \tan \frac{1}{2} \psi_2 \quad (78)$$

We can now replace Eq. 74 by

$$\eta_p = \frac{\left| \int_0^{2\pi} \int_0^{\Psi} \frac{E_{mp}(\xi, \psi_2)}{E_0} \cdot \tan\left(\frac{1}{2} \psi_2\right) d\xi d\psi_2 \right|^2}{\left| \int_0^{2\pi} \int_0^{\Psi} \left[\left(\frac{E_{mp}(\xi, \psi_2)}{E_0} \right) + \left(\frac{E_{cp}(\xi, \psi_2)}{E_0} \right) \right]^{\frac{1}{2}} \tan\left(\frac{1}{2} \psi_2\right) d\xi d\psi_2 \right|^2}$$

If the subreflector is illuminated by an electric dipole, oriented along the positive x-axis, the aperture fields to be used are

$$\frac{E_{mp}}{E'_0} = -1 + \cos^2 \xi \left[1 - \cos \xi \cdot 2 \operatorname{arc} \tan \left\{ \frac{e-1}{e+1} \tan\left(\frac{1}{2} \psi_2\right) \right\} \right] \quad (79)$$

$$\frac{E_{cp}}{E'_0} = \frac{1}{2} \sin 2\xi \left[1 - \cos \xi \cdot 2 \operatorname{arc} \tan \left\{ \frac{e-1}{e+1} \tan\left(\frac{1}{2} \psi_2\right) \right\} \right]. \quad (80)$$

Fig. 10 shows the computed results. Where the polarization efficiency is given in relation to the subtended angle of the main reflector with the magnification ratio $M = \frac{e+1}{e-1}$ as a parameter.

When an offset paraboloid antenna is investigated Eq. 75 may still be used, however, the integration limits will differ. As explained in Appendix B, ψ will have to be integrated between $\psi_0 - \psi$ and $\psi_0 + \psi$. The integration limits of ξ , ξ_L and ξ_R are

$$\xi_L = - \arccos \left(\frac{\cos \psi - \cos \psi_0 \cos \psi}{\sin \psi_0 \sin \psi} \right) \quad (81)$$

$$\xi_R = + \arccos \left(\frac{\cos \psi - \cos \psi_0 \cos \psi}{\sin \psi_0 \sin \psi} \right) \quad (82)$$

where ψ_0 is the offset angle and ψ the angular aperture of the mainreflector. (In the open cassegrain antenna ψ is called ψ_2).

Eq. 75 is then written as

$$\eta_p = \frac{\left| \int_{\psi_0 - \psi}^{\psi_0 + \psi} \int_{\xi_L}^{\xi_R} \frac{E_{mp}(\xi, \psi)}{E_0} \tan \frac{1}{2} \psi d\psi d\xi \right|^2}{\left| \int_{\psi_0 - \psi}^{\psi_0 + \psi} \int_{\xi_L}^{\xi_R} \frac{\sqrt{E_{mp}^2 + E_{cp}^2}}{E_0} \tan \frac{1}{2} \psi d\psi d\xi \right|^2} \quad (83)$$

where the main and cross polarized fields for the offset paraboloid and open cassegrain antenna have been discussed in the previous sections for various illuminations.

In the case of an open cassegrain antenna the efficiency factor becomes a little more complicated. It is readily shown that the factor E_0 in the aperture fields is equal to that of the classical cassegrain antenna and that the integration limits are the same as for the offset antenna.

Figs. 11 A and 11 B show the polarization efficiency of an offset illuminated by an electric dipole oriented along the positive x' - axis and positive y - axis respectively.

No calculations have been carried out on antenna structures illuminated by a magnetic dipole, since this is of only academic interest.

The polarization efficiency of an offset antenna illuminated by a Huygens source, in accordance with Eqs. 16 and 56 or Eqs. 13 and 58, is shown in Fig. 12. A separate figure of offset paraboloid antennas illuminated by a "modified" Huygens source as explained by Eqs. 72 and 73 is not published because the results are very similar to those presented in Fig. 12 .

The results obtained with an open cassegrain antenna are given in Figs. 13A, 13B and 14 . In Figs 13A and 13B the polarization efficiency has been calculated for the case that the electric dipole is oriented along the positive x'' - axis and positive y'' - axis respectively. The results for illumination by a Huygens source are given in Fig. 14. The eccentricity of the hyperboloid subreflector was 1.5. For both offset paraboloids and open cassegrain antennas the offset angle served as parameter.

6. A practical example

In the previous section a Huygens source was presented with equal intensities of a magnetic and an electric dipole. However, many feed patterns may be divided in electric and magnetic dipoles with unequal intensities. In this section we work out a practical example.

A very popular feed system used to illuminate a reflector surface is the open waveguide excited with the TE_{10} mode described by Silver [2, p.343] and Jones (4). The field components of a rectangular waveguide excited in the TE_{10} mode and the electric field vector oriented along the x-axis, is, in accordance with Silver, represented by

$$\begin{aligned} \bar{E}_\psi(\psi, \xi) &= C \frac{\cos \xi}{\rho} \left[1 + \frac{\beta_{10}}{k} \cos \psi \right] F(\psi, \xi) e^{-jk\rho} \bar{a}_\psi \\ \bar{E}_\xi(\psi, \xi) &= -C \frac{\sin \xi}{\rho} \left[\frac{\beta_{10}}{k} + \cos \psi \right] F(\psi, \xi) e^{-jk\rho} \bar{a}_\xi \end{aligned} \quad (84)$$

where

$$F(\psi, \xi) = \frac{\cos[(\pi a/\lambda) \sin \psi \cos \xi]}{[(\pi a/\lambda) \sin \psi \cos \xi]^2 - (\pi/2)^2} \cdot \frac{\sin[(\pi b/\lambda) \sin \psi \sin \xi]}{(\pi b/\lambda) \sin \psi \sin \xi}$$

In this equation it is assumed that the reflection coefficient at the opening of the waveguide is zero. The symbols a and b are waveguide dimensions and C is a coefficient depending upon the wavelength and dimensions [2, p.343]. Further, β_{10} stands for the phase constant for the TE_{10} mode, and k the propagation constant, equal to $\frac{2\pi}{\lambda}$.

The polarization vector is

$$\bar{a}_i = \frac{\cos \xi}{\rho} \left(1 + \frac{\beta_{10}}{k} \cos \psi \right) \bar{a}_\psi - \frac{\sin \xi}{\rho} \left(\cos \psi + \frac{\beta_{10}}{k} \right) \bar{a}_\xi \quad (85)$$

If the dimensions of the waveguide are such that $\beta_{10}/k = 1$ the polarization vector reduces to

$$\bar{a}_i = \cos \xi \bar{a}_\psi - \sin \xi \bar{a}_\xi \quad (86)$$

which is equal to that of a Huygens source.

However, in practice this cannot be realized as normally

$$\frac{\beta_{10}}{k} = \frac{\lambda}{\lambda_{g10}} \quad (87)$$

where λ_{g10} is the wavelength in the guide [2, p.205]

$$\lambda_{g10} = \frac{\lambda}{[1 - (\lambda/2a)^2]^{1/2}} \quad (88)$$

for the TE₁₀ mode. Therefore $\beta_{10} = k$ only for $\lambda \ll a$.

Nevertheless, this polarization vector is very popular and used by several authors such as Afifi [6], Carter [14] and Tartakovski [15], as it simplifies the complicated mathematical work considerably.

If we want to study the cross-polarization properties of antennas illuminated by this feed, we must know the waveguide dimensions, frequency range and cut-off frequency.

If we study a rectangular horn in the X-Band (8200 - 12400 MHz) the dimensions a and b are 0.900 x 0.400 inches and the cut-off frequency is 6560 MHz. The proportions of the lowest and highest frequencies to the cut-off frequency are 1.25 and 1.90, a relationship which is also found for waveguides in other frequency bands. Let λ_1 be the longer wavelength = 3.66 cm and λ_2 the shorter = 2.42 cm. The wavelength in the waveguide is then for λ_1

$$\lambda_{g10} = \frac{3.66}{\sqrt{1 - \left(\frac{3.66}{4.57}\right)^2}} = 6.11 \text{ cm,}$$

and for λ_2

$$\lambda_{g10} = \frac{2.42}{\sqrt{1 - \left(\frac{2.42}{4.57}\right)^2}} = 2.85 \text{ cm.}$$

From Eq. 87 we then obtain for

$$\beta_{10}/k = \lambda_1/\lambda_{g10} = 3.66/6.11 = 0.60$$

and

$$\beta_{10}/k = \lambda_2/\lambda_{g10} = 2.42/2.85 = 0.85$$

The polarization vector is now for λ_1

$$\bar{a}_i = \frac{\cos \frac{\xi}{2}}{\rho} (1 + 0.60 \cos \psi) \bar{a}_\psi - \frac{\sin \frac{\xi}{2}}{\rho} (0.60 + \cos \psi) \bar{a}_\xi \quad (89)$$

and for λ_2

$$\bar{a}_i = \frac{\cos \frac{\xi}{2}}{\rho} (1 + 0.85 \cos \psi) \bar{a}_\psi - \frac{\sin \frac{\xi}{2}}{\rho} (0.85 + \cos \psi) \bar{a}_\xi \quad (90)$$

The polarization properties apparently depend on the frequency.

If such a feed is used to illuminate a front-fed paraboloid antenna, it is readily found by means of the theory developed in Sec. 2.1. that the aperture field

$$\bar{E}_A = E_0 \frac{\cos \frac{\xi}{2}}{1 + \cos \psi} (1 + m \cos \psi) (\cos \frac{\xi}{2}, \sin \frac{\xi}{2}, 0)_{x_0} - E_0 \frac{\sin \frac{\xi}{2}}{1 + \cos \psi} (m + \cos \psi) (-\sin \frac{\xi}{2}, \cos \frac{\xi}{2}, 0)$$

or

$$\bar{E}_A = \frac{E_0}{1 + \cos \psi} \left[\left\{ (1 + m \cos \psi) \cos^2 \frac{\xi}{2} + (m + \cos \psi) \sin^2 \frac{\xi}{2} \right\} \bar{x} + \left\{ (1 + m \cos \psi) \cos \frac{\xi}{2} \sin \frac{\xi}{2} - (m + \cos \psi) \cos \frac{\xi}{2} \sin \frac{\xi}{2} \right\} \bar{y} \right] \quad (91)$$

where m is any value between 0.60 and 0.85.

E_0 is the amplitude factor of the feed system and is in accordance with Silver (2, p.343)

$$E_0 = \frac{\cos[(\pi a/\lambda) \sin \psi \cos \frac{\xi}{2}]}{[(\pi a/\lambda) \sin \psi \cos \frac{\xi}{2}]^2 - \frac{1}{2} \pi} \times \frac{\sin[(\pi b/\lambda) \sin \psi \sin \frac{\xi}{2}]}{(\pi b/\lambda) \sin \psi \sin \frac{\xi}{2}} \quad (92)$$

The results for three different values of m are given in Fig. 15.

7. Conclusions

It has been demonstrated in the foregoing that by calculating the aperture electric fields of antennas with a paraboloid (main) reflector, expressions may be derived for the polarization efficiency or polarization loss. These expressions are found not only for front-fed paraboloids, but also for classical cassegrain antennas, front-fed offset paraboloids, and open cassegrain antennas. Both electric dipole excitation and excitation by a Huygens source are investigated as they give a good insight into the problems and facilitate comparative studies. Moreover, there are a number of realistic feeds, such as a rectangular horn excited in the TE 01 mode, having polarization properties close to the Huygens source. An example of this kind has been worked out, showing that the polarization losses decrease considerably if the polarization vector approach that of a Huygens source. If investigations are required for feeds with polarization properties different from those as discussed here, the same techniques may be used.

After the electric aperture field has become known, an expression may be found for the polarization efficiency η_p . Carrying out the computation, it is readily seen that the front-fed paraboloid has very bad polarization properties, becoming worse for deep paraboloids. In the case that the focus falls within the aperture plane ($\psi_2 = 90^\circ$), the polarization efficiency falls to 89 % (Fig. 9). On the other hand, the true cassegrain antenna has much better properties, which not only depend upon the subtending angle of the main reflector, but also on the magnification ratio $M = \frac{e+1}{e-1}$, which has been introduced as a parameter (Fig. 10). The result becomes worse for low M values and deep main paraboloids; however, for $M = 2$ and $\psi_2 = 90$ degrees, the true cassegrain antenna still retains a polarization efficiency of 99 %, being considerably more than in the case of front-fed paraboloids with equal ψ_2 . Offset paraboloid antennas show an increase in the losses at increasing subtending angle and increasing offset angle. If we compare the front-fed paraboloid with the offset paraboloid, it appears that the former shows better results at equal subtending angles than the offset antenna with an electric dipole polarized along the x'-axis; e.g. a front-fed paraboloid with a subtending angle of 60 degrees has a polarization efficiency of 98,5 %, while an offset paraboloid with subtending and offset angles of 60 degrees shows an efficiency of only 91 % (Fig. 11A).

If the dipole is polarized along the y' -axis, the efficiency even drops to 89 % (Fig. 11B).

If we study the results obtained with an open cassegrain antenna illuminated by an electric dipole, it appears that not much difference is noticed if the dipole is oriented along the x'' -axis or y'' -axis. At offset angles and subtending angles of about 60 degrees it appears that the efficiency drops to 90 % which is of the same order as for offset front-fed paraboloids (Figs. 13A and 13B). The results obtained by illumination by a Huygens source are, both for offset antennas and open cassegrain antennas, similar to those obtained by illumination by an electric dipole. The results clearly depend on the offset and subtending angles rather more than on the polarization of the feed. At offset angles and main reflector subtending angles of ca. 60 degrees an efficiency of ca. 90 % is noticed again (Figs. 12 and 14).

If we try to improve the polarization properties of offset paraboloids by an "improved" Huygens feed as presented by Eqs. 72 and 73, it appears that no remarkable differences are found compared with an illumination by a true Huygens source. We also investigated the losses of open cassegrain antennas in relation to the eccentricity of the hyperboloid subreflector. Using eccentricities of 2.0 and 2.5, the results are very similar to those with eccentricities of 1.5.

Compared with the symmetrical front-fed paraboloid antenna and the classical cassegrain antenna, offset antennas are very unfavourable when illuminated by a Huygens source. The Huygens source gives zero polarization losses for symmetrical paraboloid reflector antennas, but the losses of offset antennas are of the same order as those calculated for offset antennas illuminated by an electric dipole. This conclusion is supported by the fact that for eccentricities differing from $e = 1.5$ similar results are obtained.

More study is required to find out whether feeds may be designed having polarization properties which may improve the polarization losses of offset antennas. However, the present study makes the use of offset antennas for purposes where a polarization discrimination of more than 30 dB is required, very questionable.

Appendix A

The relationship between (ξ, ψ) and (ξ', ψ')

In the x, y, z coordinate system a fieldpoint P is given by

$$P(x, y, z) = (\rho \sin \psi \cos \xi, \rho \sin \psi \sin \xi, -\rho \cos \psi) \quad (A_1)$$

and in the x', y', z' coordinate system

$$P(x', y', z') = (\rho \sin \psi_2 \cos \xi', \rho \sin \psi_2 \sin \xi', -\rho \cos \psi_2) \quad (A_2)$$

The relationship between the two coordinate systems follows from

$$\begin{aligned} \bar{x}' &= \cos \psi_0 \bar{x} + \sin \psi_0 \bar{z} \\ \bar{y}' &= \bar{y} \\ \bar{z}' &= -\sin \psi_0 \bar{x} + \cos \psi_0 \bar{z} \end{aligned} \quad (A_3)$$

If we transform $P(x', y', z')$ into the x, y, z system, we obtain

$$P \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \psi_0 & 0 & \sin \psi_0 \\ 0 & 1 & 0 \\ -\sin \psi_0 & 0 & \cos \psi_0 \end{pmatrix} \begin{pmatrix} \sin \psi \cos \xi \\ \sin \psi \sin \xi \\ -\cos \psi \end{pmatrix} = \begin{pmatrix} \cos \psi_0 \sin \psi \cos \xi - \sin \psi_0 \cos \psi \\ \sin \psi \sin \xi \\ -\sin \psi_0 \sin \psi \cos \xi - \cos \psi_0 \cos \psi \end{pmatrix} \quad (A_4)$$

resulting in

$$\begin{aligned} \sin \psi_2 \cos \xi' &= \cos \psi_0 \sin \psi \cos \xi - \sin \psi_0 \cos \psi \\ \sin \psi_2 \sin \xi' &= \sin \psi \sin \xi \\ -\cos \psi_2 &= -\sin \psi_0 \sin \psi \cos \xi - \cos \psi_0 \cos \psi \end{aligned} \quad (A_5)$$

From Eqs. A₅ expressions are found for ψ_2 and ξ_2 , viz.:

$$\sin \psi_2 = \left[(\cos \psi_0 \sin \psi \cos \xi - \sin \psi_0 \cos \psi)^2 + (\sin \psi \sin \xi)^2 \right]^{\frac{1}{2}} = \sqrt{A}$$

$$\cos \psi_2 = \sin \psi_0 \sin \psi \cos \xi + \cos \psi_0 \cos \psi$$

$$\cos \xi' = \frac{\cos \psi_0 \sin \psi \cos \xi - \sin \psi_0 \cos \psi}{\sqrt{A}}$$

$$\sin \xi' = \frac{\sin \psi \sin \xi}{\sqrt{A}}$$

Appendix B

The integration limits of offset antennas

The boundary of an offset paraboloid is determined by the intersection of a paraboloid with a cone where the axes of cone and paraboloid intersect in the focus of the paraboloid. The angle between the two axes is called the offset angle Ψ_0 . From Fig. 16 it may be seen that if the cone has a vertex angle Ψ , the limits of the integration variable ψ are $\Psi_0 - \Psi$ and $\Psi_0 + \Psi$. It is also seen that the integration limits of the variable ξ , ξ_L and ξ_R are dependent on ψ . To find the relationship of ξ_L and ξ_R to ψ , we will use the Eqs. A₁ and A₂ from Appendix A and express P(x,y,z) in P(x',y',z') or

$$P \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \Psi_0 & 0 & -\sin \Psi_0 \\ 0 & 1 & 0 \\ \sin \Psi_0 & 0 & \cos \Psi_0 \end{pmatrix} \begin{pmatrix} \sin \psi' \cos \xi' \\ \sin \psi' \sin \xi' \\ -\cos \psi' \end{pmatrix} = \begin{pmatrix} \cos \Psi_0 \sin \psi' \cos \xi' + \sin \Psi_0 \cos \psi' \\ \sin \psi' \sin \xi' \\ \sin \Psi_0 \sin \psi' \cos \xi' - \cos \Psi_0 \cos \psi' \end{pmatrix}$$

This expression should be equal to Eq. A₁, therefore,

$$\sin \psi \cos \xi = \cos \Psi_0 \sin \psi' \cos \xi' + \sin \Psi_0 \cos \psi' \quad (B_2)$$

$$\sin \psi \sin \xi = \sin \psi' \sin \xi' \quad (B_3)$$

$$-\cos \psi = \sin \Psi_0 \sin \psi' \cos \xi' - \cos \Psi_0 \cos \psi' \quad (B_4)$$

If we eliminate ξ' from the Eqs. B₂ and B₄, we find

$$\sin \Psi_0 \sin \psi \cos \xi = \cos \psi' - \cos \Psi_0 \cos \psi$$

For $\psi' = \psi$, which is obtained for ξ_L and ξ_R , and $\psi_0 \neq 0 = \psi$ we find

$$\cos \xi_{L,R} = \frac{\cos \psi - \cos \psi_0 \cos \psi}{\sin \psi_0 \sin \psi} \quad (B_5)$$

If $\psi = \psi_0 \pm \psi$, $\cos \xi_{L,R} = +1$ and $\xi_{L,R} = 0$, which follows from Fig. B₁.
From Eq. B₅ follows

$$\xi_L = - \arccos \left[\frac{\cos \psi - \cos \psi_0 \cos \psi}{\sin \psi_0 \sin \psi} \right] \quad (B_6)$$

$$\xi_R = + \arccos \left[\frac{\cos \psi - \cos \psi_0 \cos \psi}{\sin \psi_0 \sin \psi} \right] \quad (B_7)$$

Relations B₆ and B₇ are only valid for

$\psi > 0$ and $\psi_0 - \psi > 0$.

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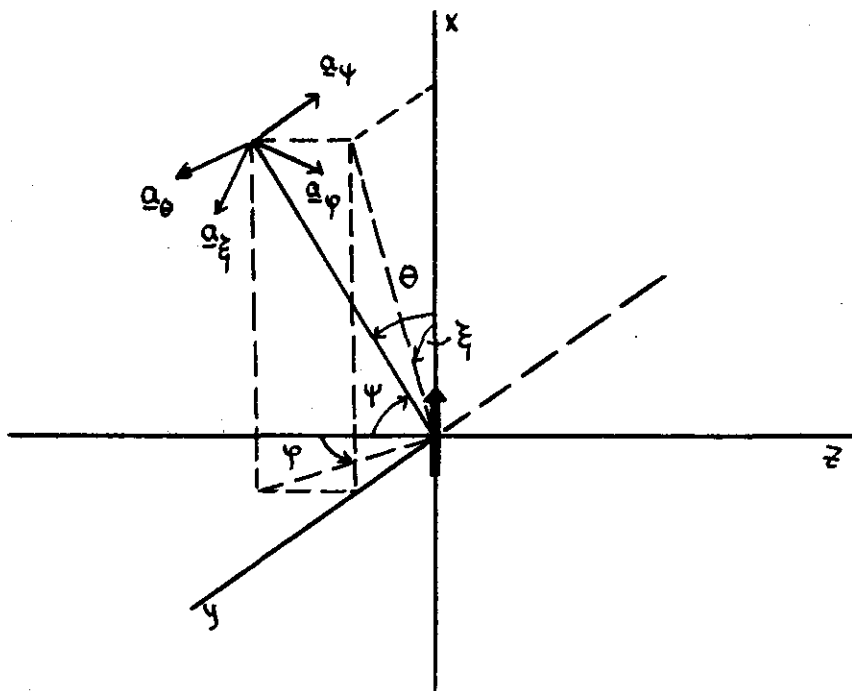


Fig. 1 Electric dipole oriented along the positive x -axis of a cartesian coordinate system.

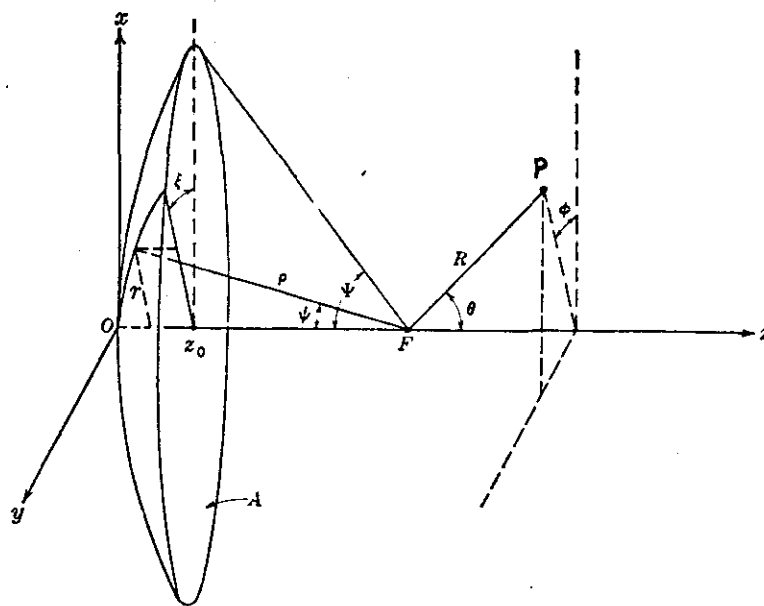


Fig. 2 Geometry of the parabolic reflector with ρ , ψ , ξ coordinates.

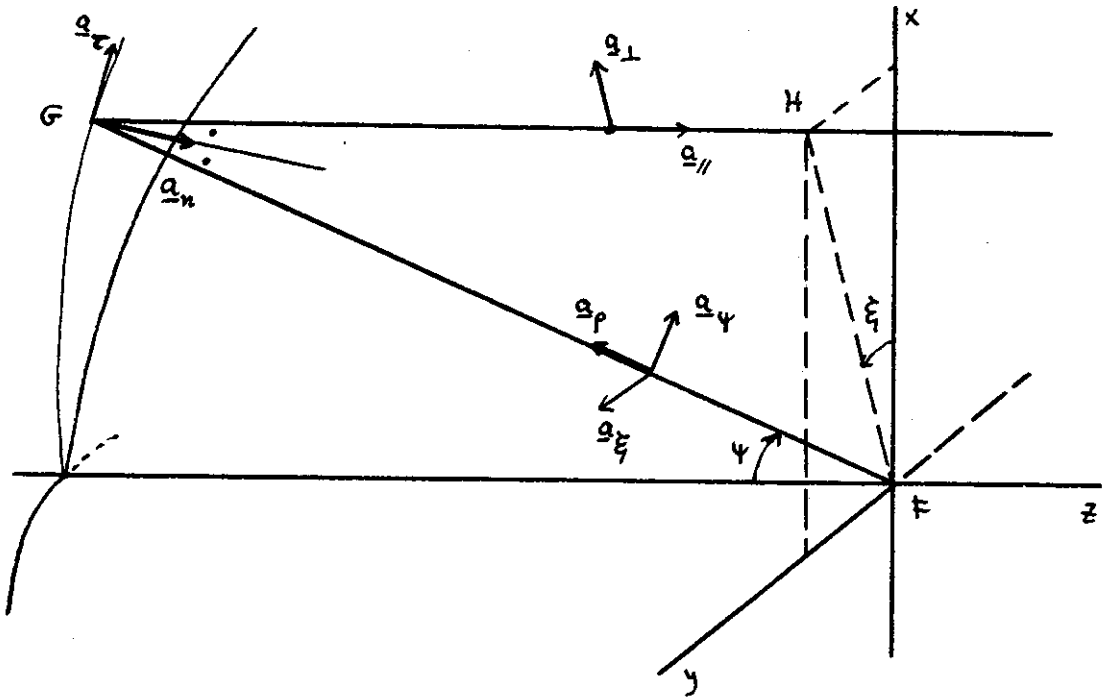


Fig. 3 Geometry of the parabolic reflector with incident and reflected rays and vectors.

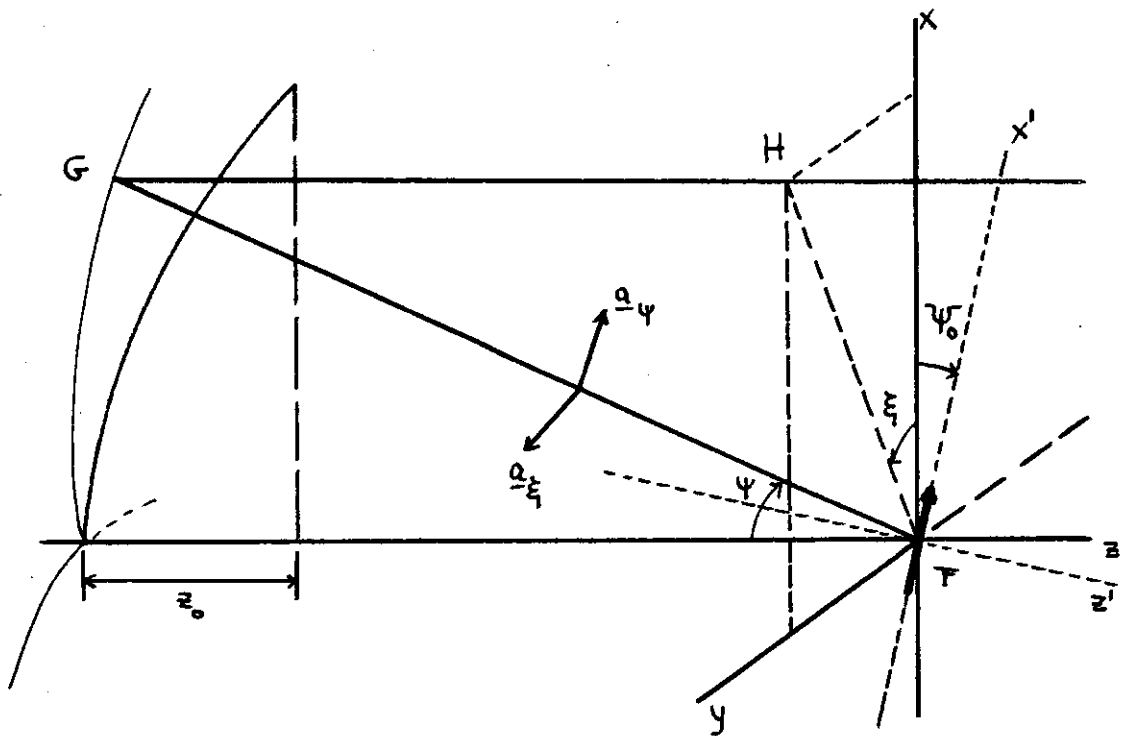


Fig. 4 Offset paraboloidal reflector.

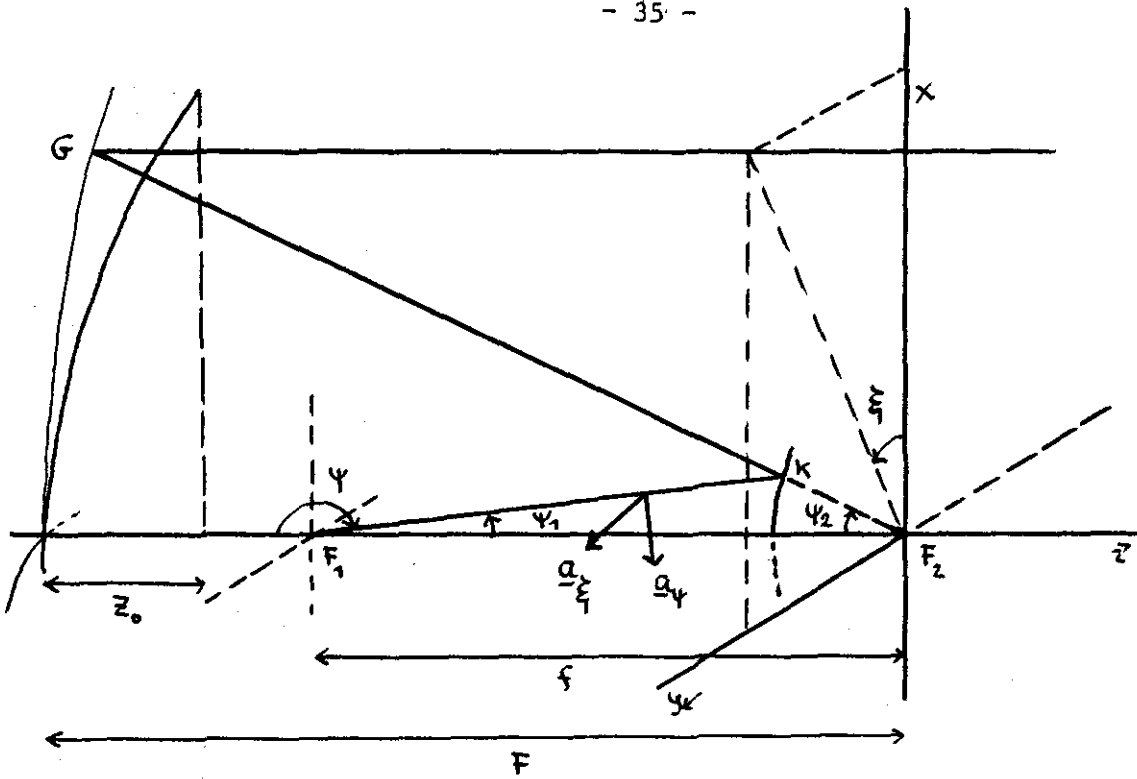


Fig. 5 Geometry of the classical cassegrain antenna.

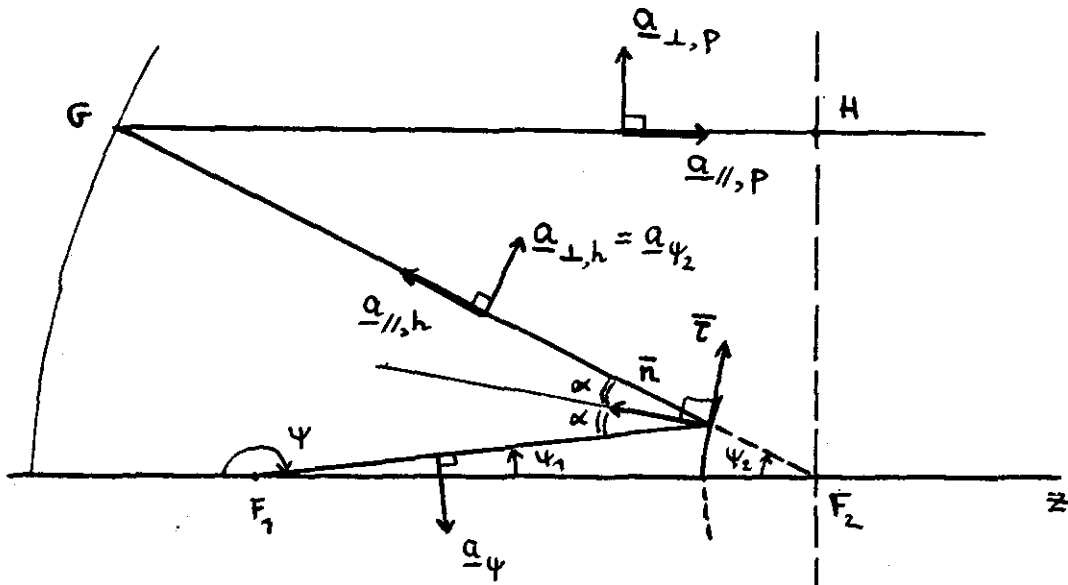


Fig. 6 Vectors in the classical cassegrain antenna.

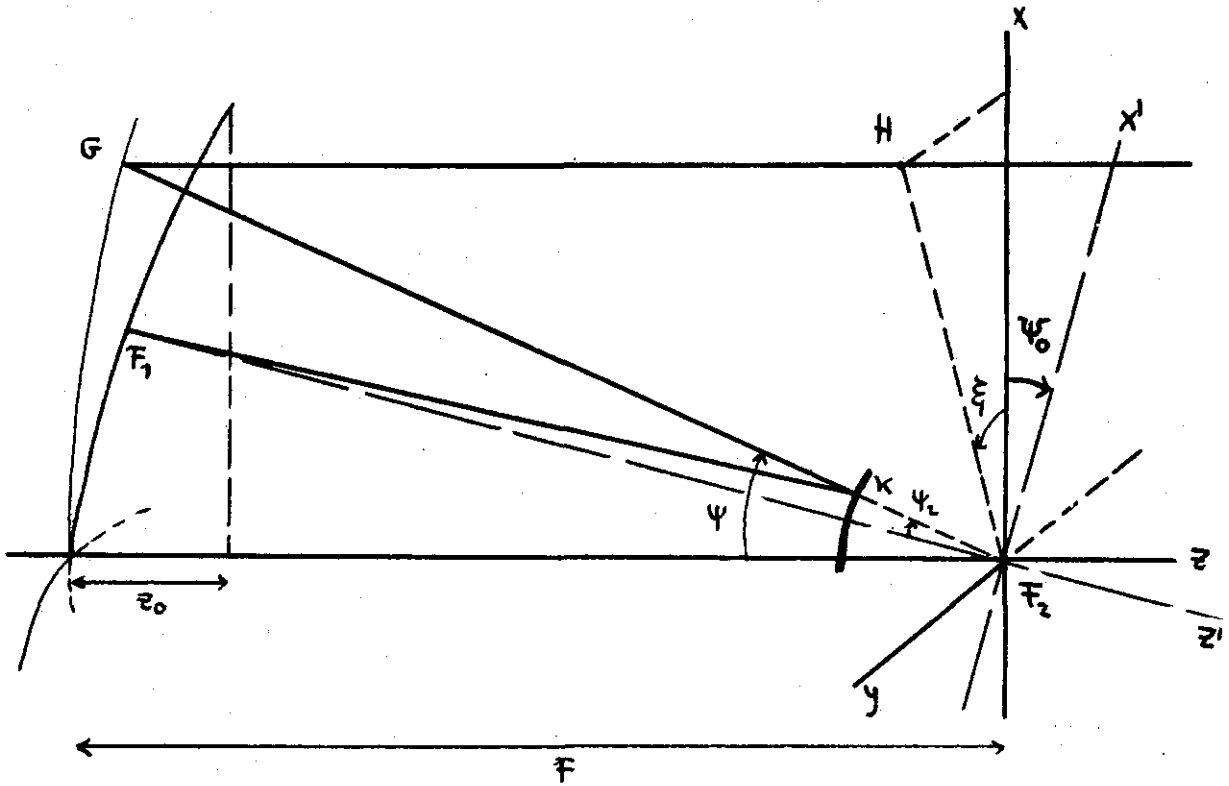


Fig. 7 Geometry of the open cassegrain antenna.

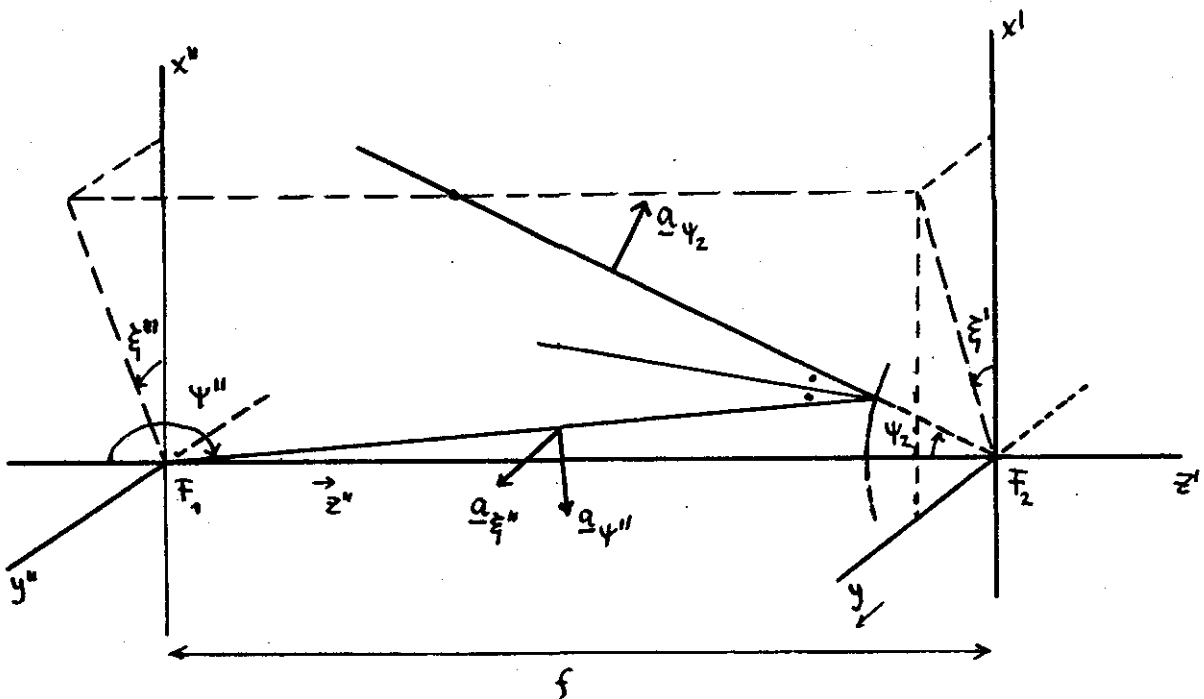


Fig. 8 Vectors in the open cassegrain antenna.

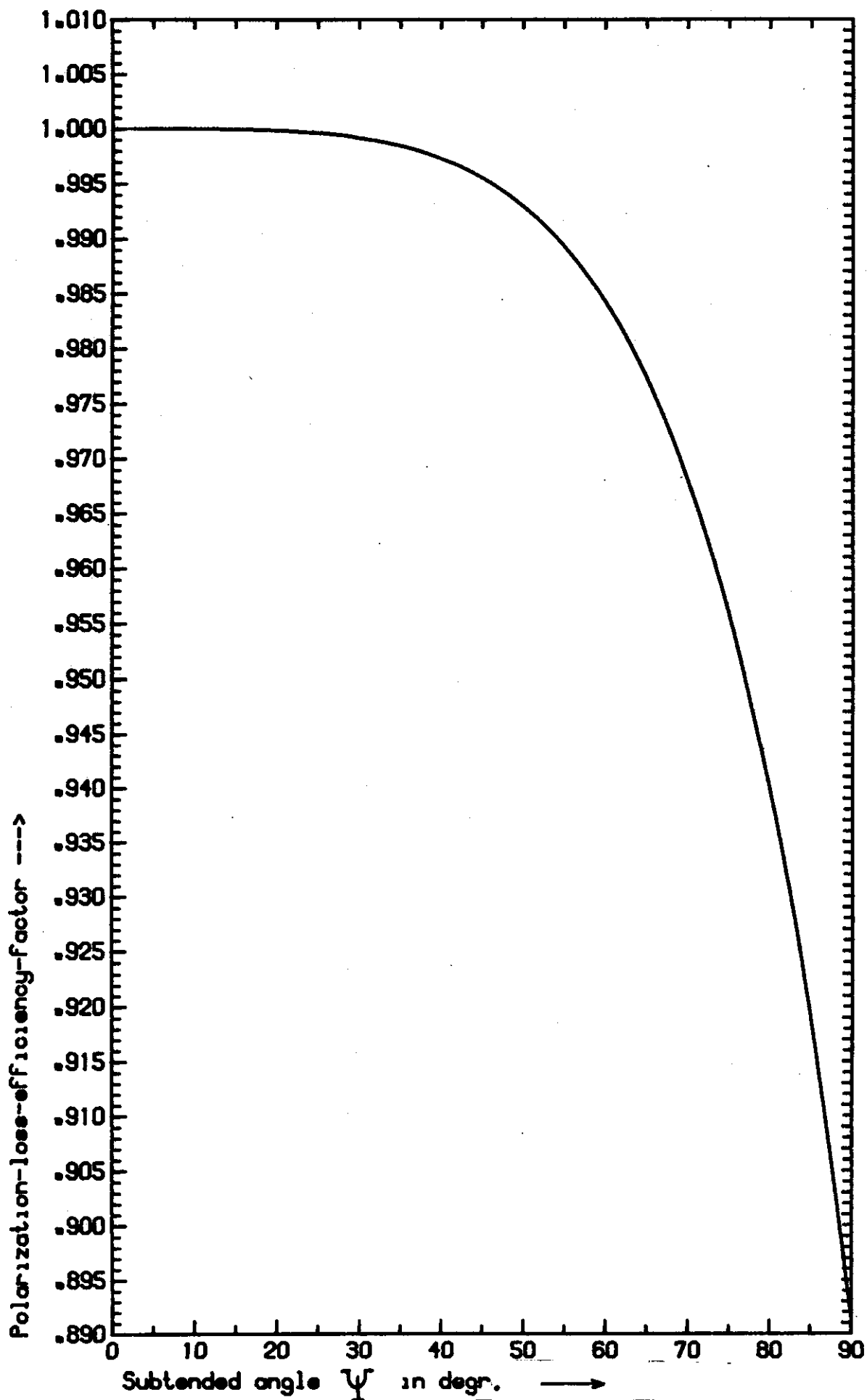


Fig. 9 Polarization loss efficiency factor of a parabolic reflector.

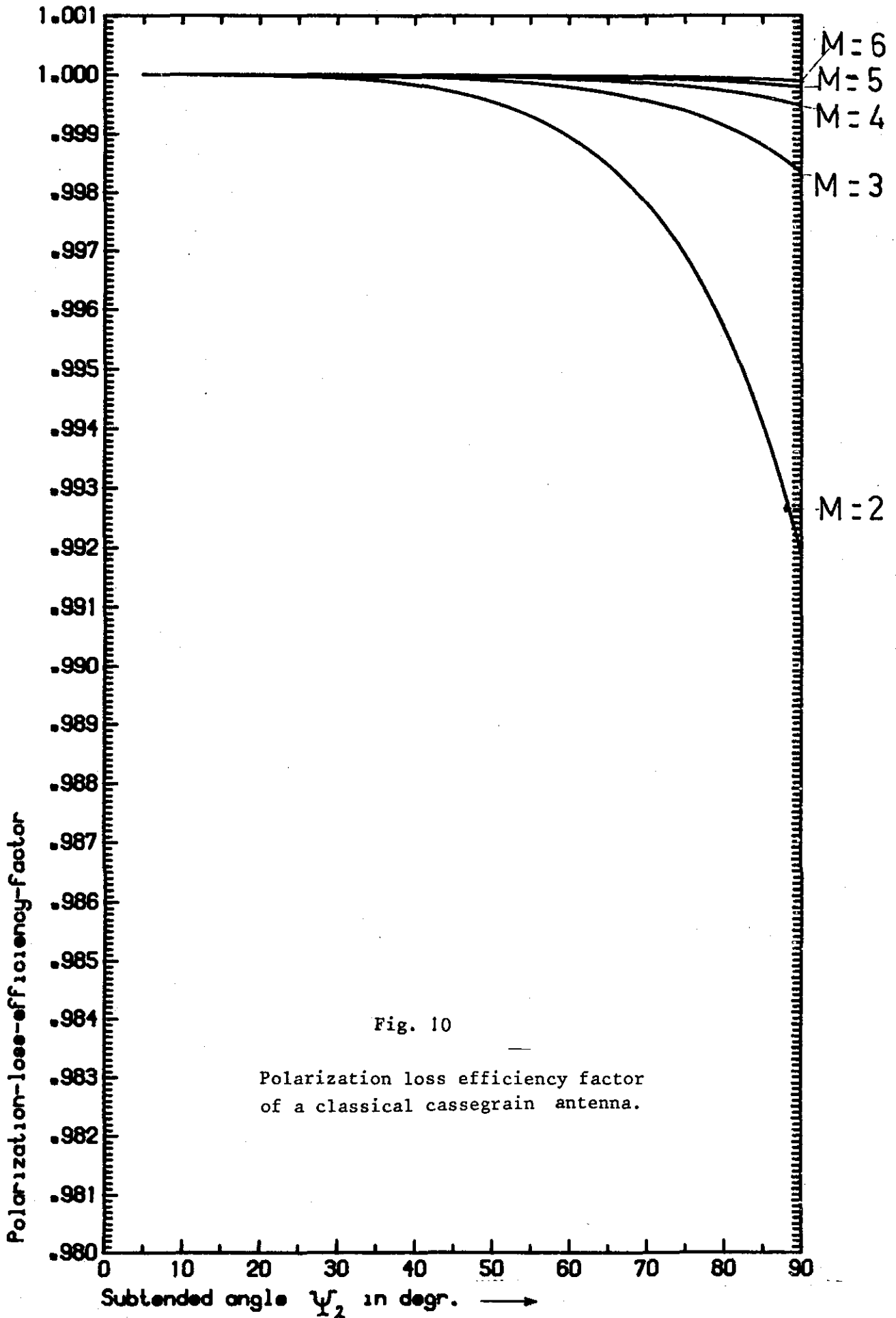
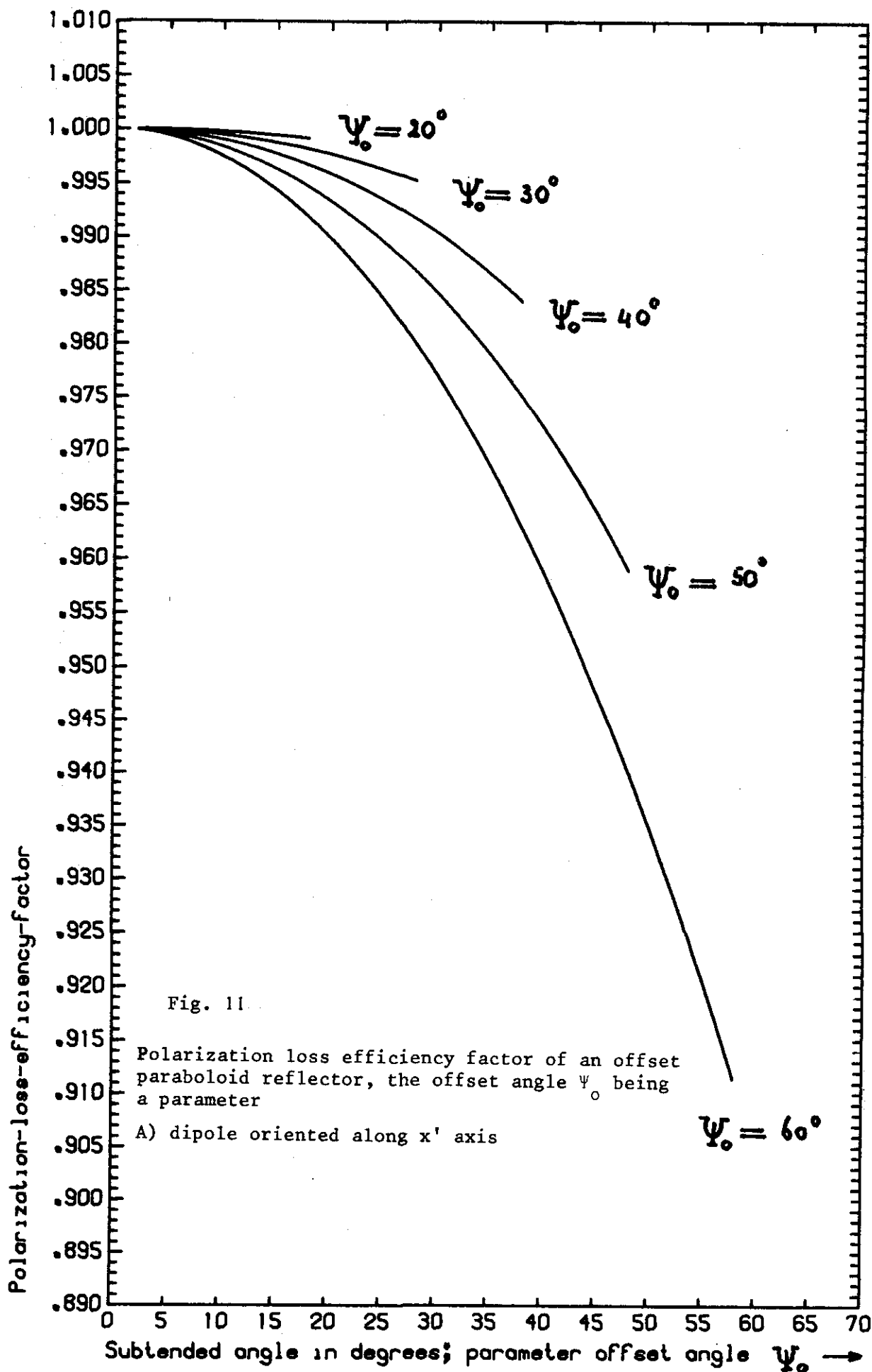
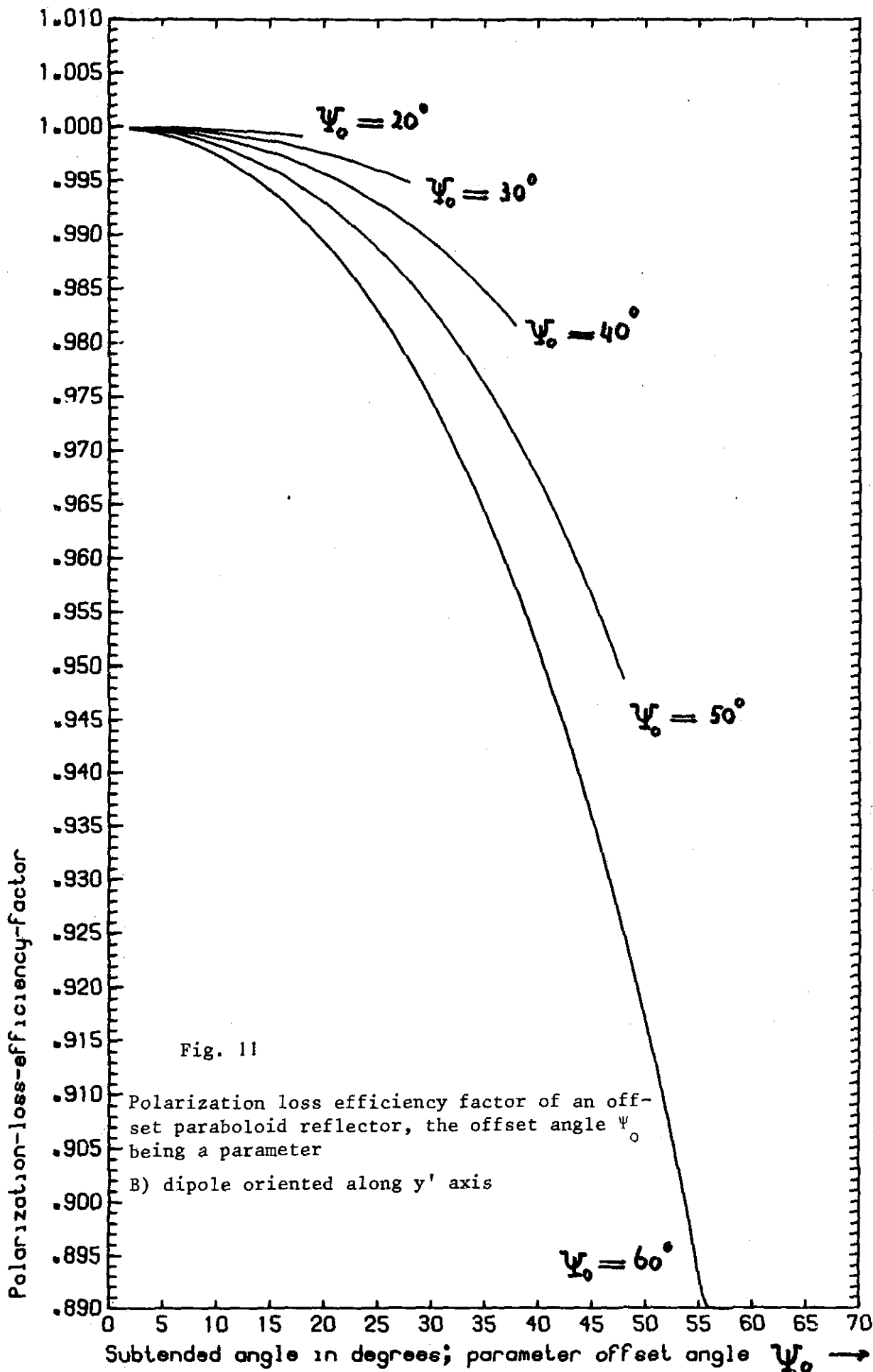


Fig. 10

Polarization loss efficiency factor
of a classical cassegrain antenna.





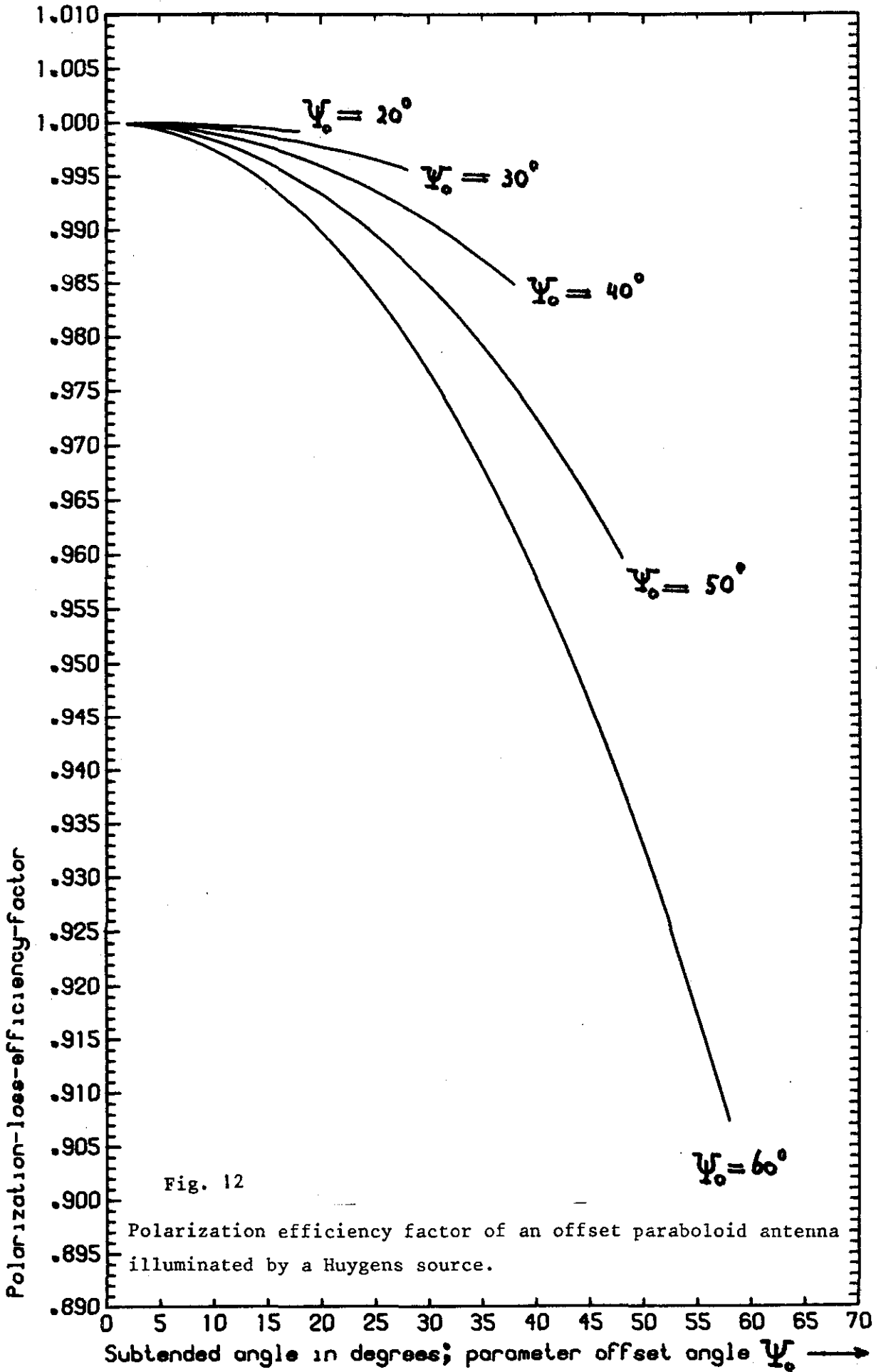
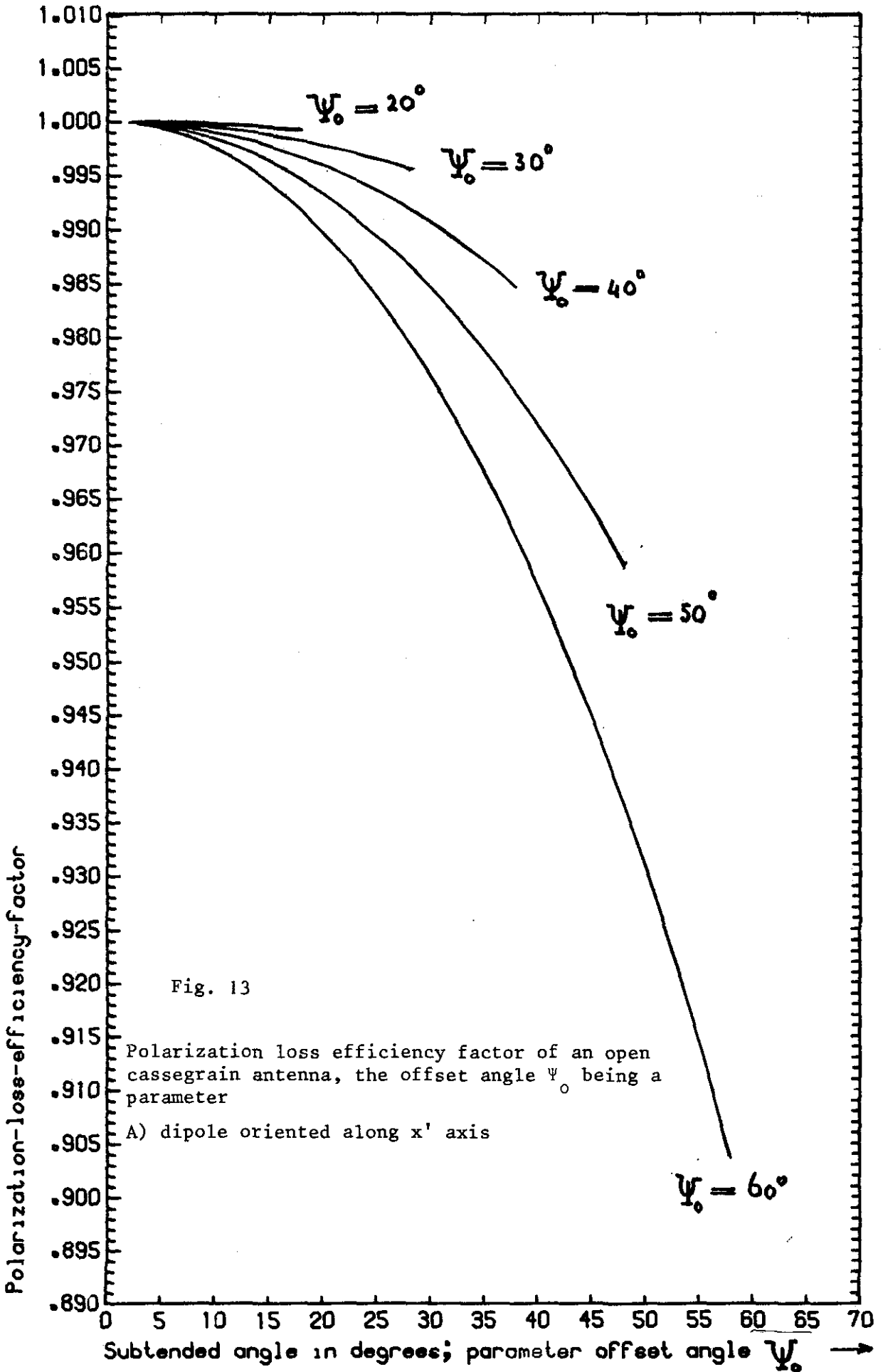
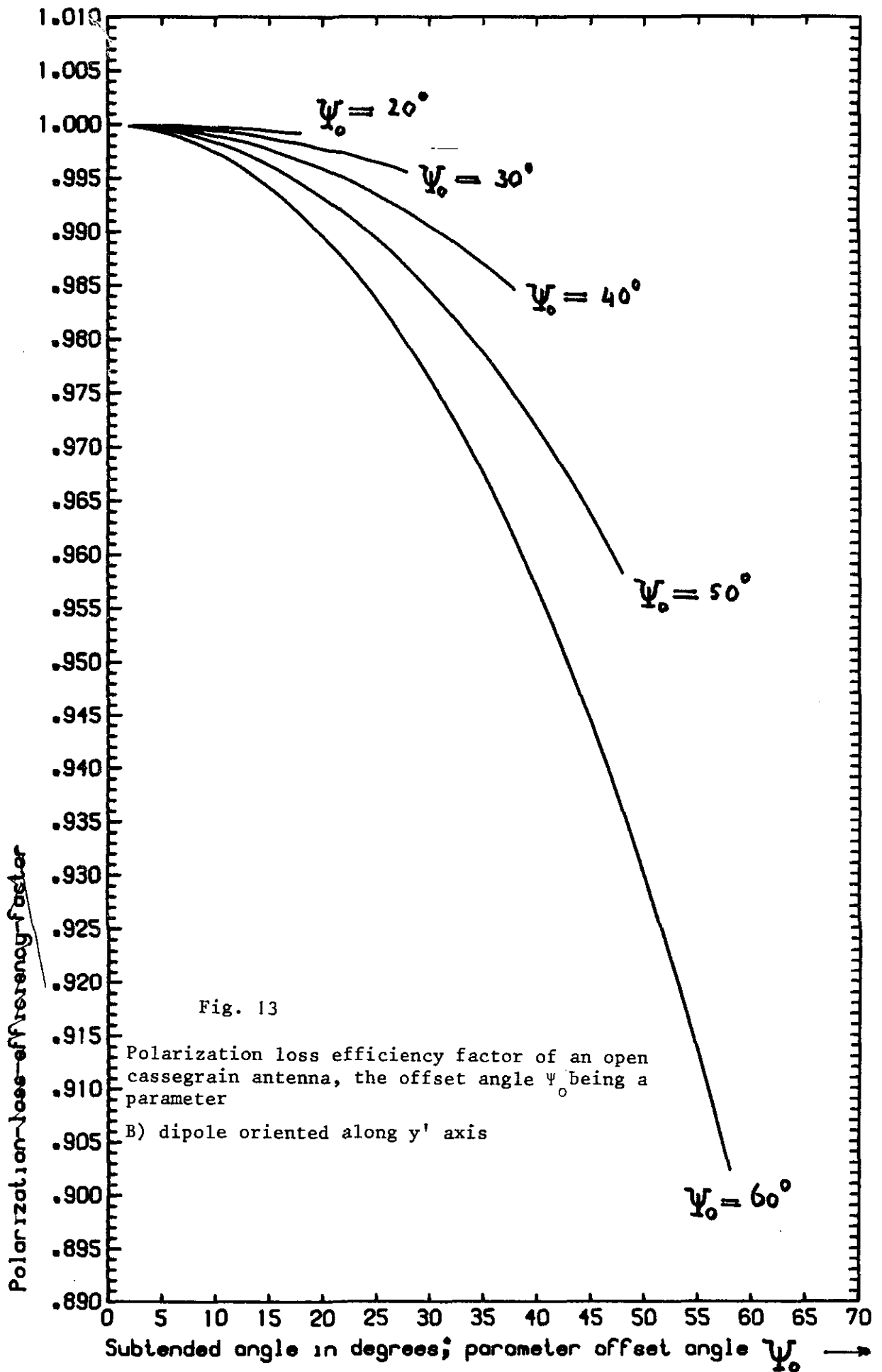


Fig. 12

Polarization efficiency factor of an offset paraboloid antenna illuminated by a Huygens source.





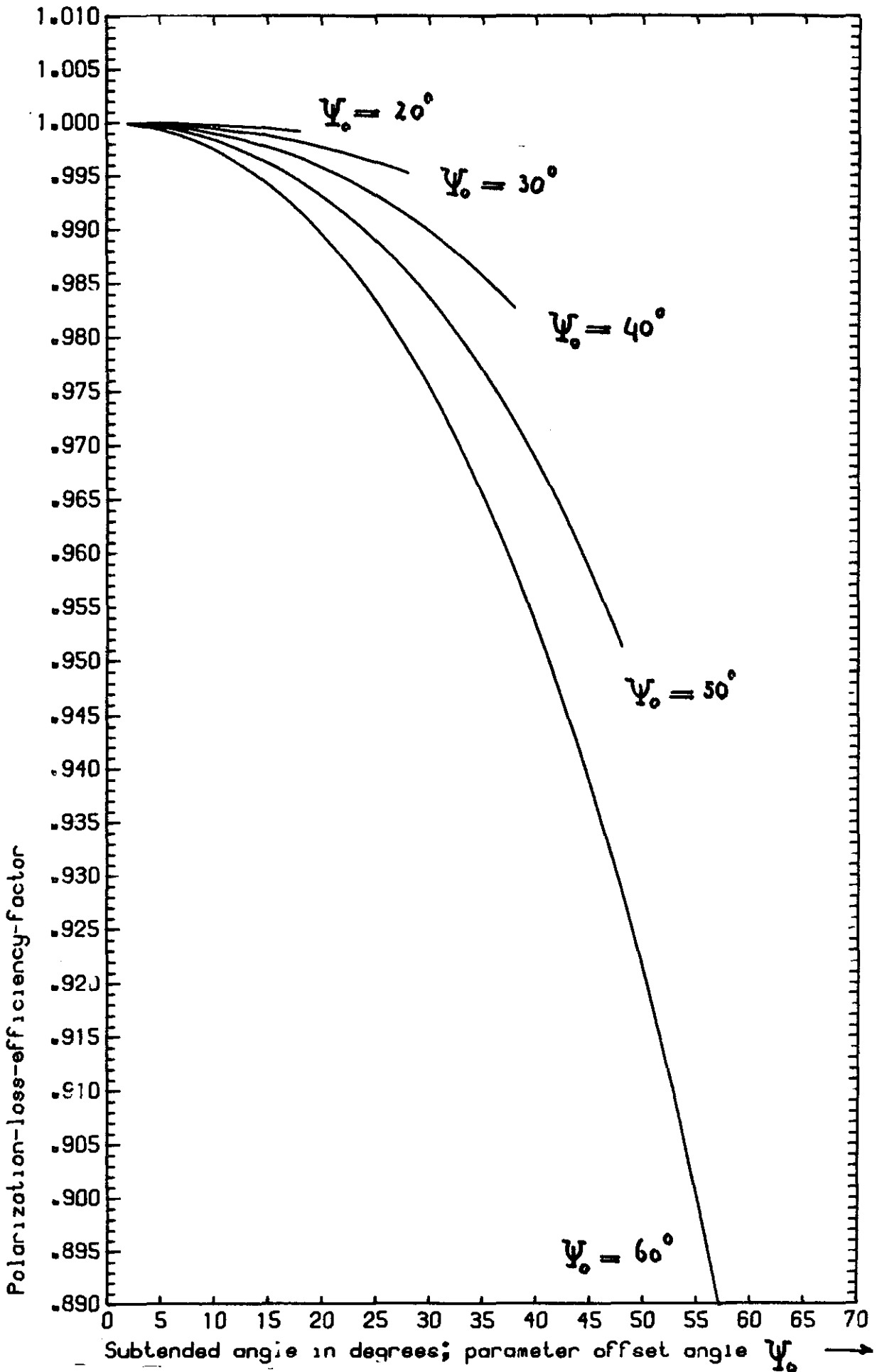


Fig. 14 Polarization loss efficiency factor of an open cassegrain antenna illuminated by a Huygens source.

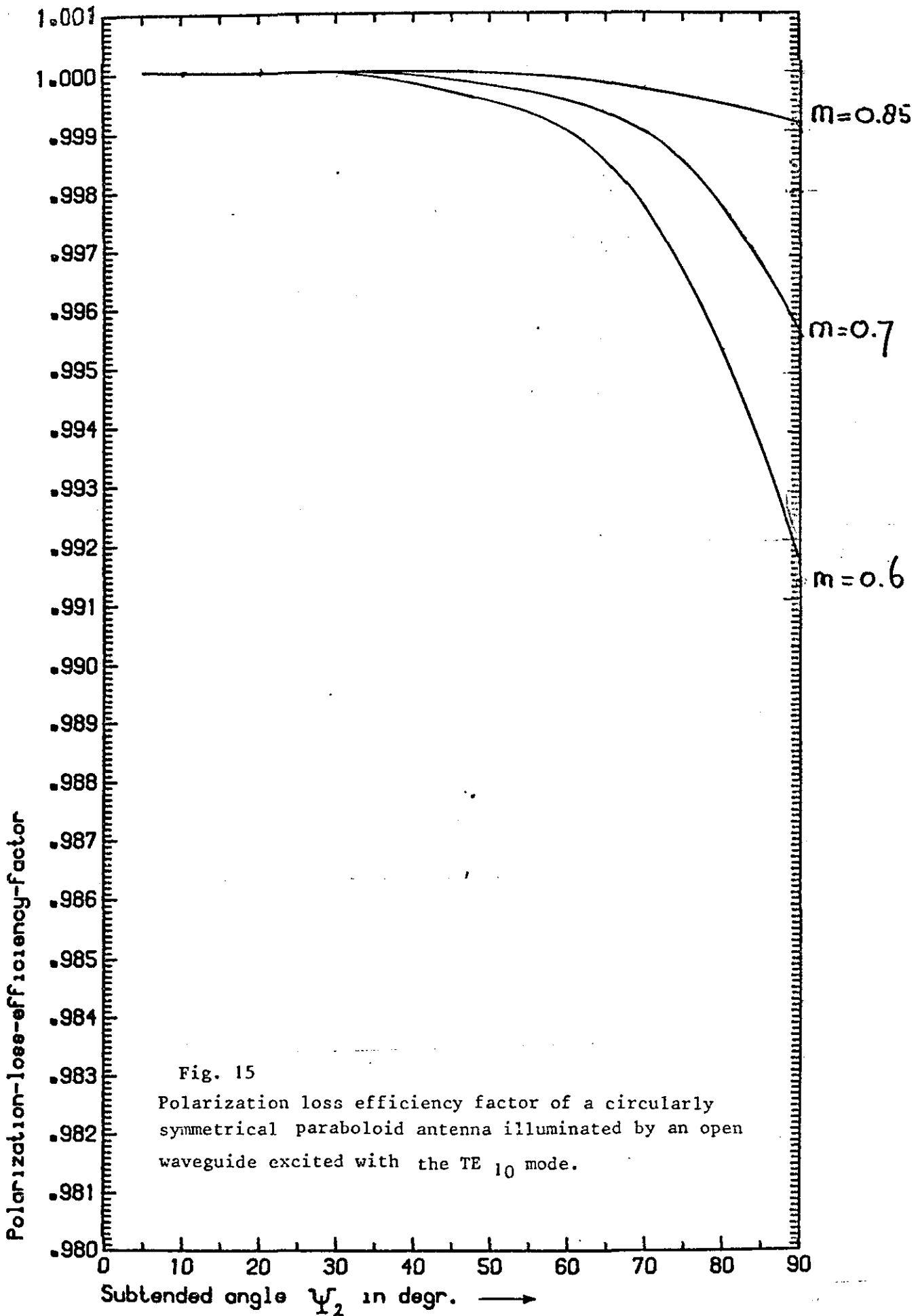


Fig. 15
Polarization loss efficiency factor of a circularly symmetrical paraboloid antenna illuminated by an open waveguide excited with the TE₁₀ mode.

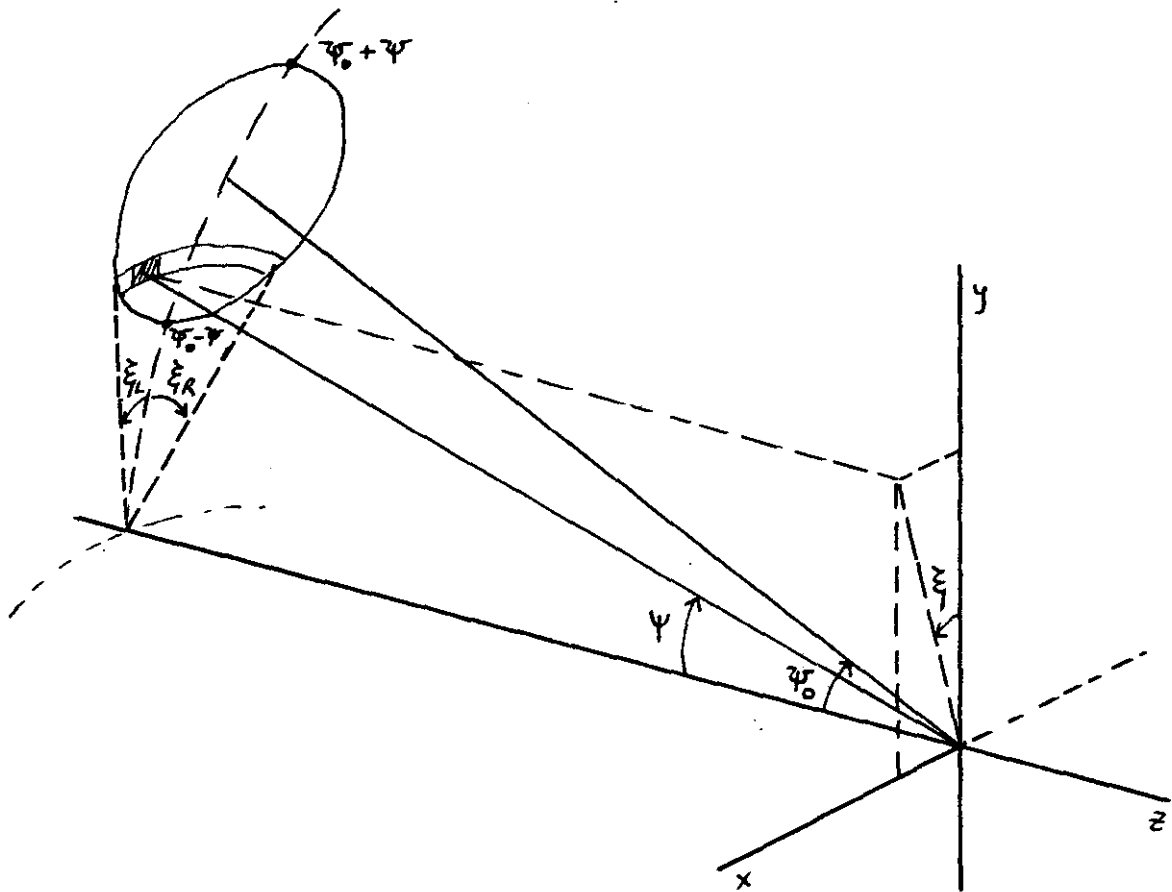


Fig. 16 The integration limits of offset antennas.

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