# Rotables stocking and repair : a Markov approach

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**Rotables Stocking and Repair:**

**A Markov Approach**

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# ABSTRACT

In rotable maintenance and repair situations, the relation between the initial stock and service levels is, in general, calculated assuming fixed repair throughput rates. In this paper it is suggested that, in practice, people adjust there throughput rates in order to react to actual stock positions. The consequences of this procedure are investigated for an elementary maintenance and repair situation. For this elementary example it is shown that the procedure has favorable effects on the initial stock and service levels.

# 1. Introduction.

Of late decennia, much attention is called to production control problems. Systems have been developed to support management in taking control decisions, e.g. MRP 1 and 2, OPT etc. Such systems commonly require stable and predictable short-term demand conditions. In practice, short-term demand can be unpredictable, for instance in some maintenance situations. In a subset of those maintenance situations, we come across short-term demand uncertainty for both capacity **and** material: *The maintenance of repairable parts, the so-called rotables*, e.g. train equipment, engines, printed circuit boards etc. The cost-effective control of such maintenance situations is the subject of our research. The research is in progress since february 1991.

Rotables are in use "the so-called installed base" or hold in reserve, "the so-called turnaround". They can adopt two states; the failed and the repaired. Only rotables in use are prone to failure: Rotables in reserve are waiting for repair, under repair or repaired. During maintenance, failed rotables are exchanged by their repaired counterparts, if available. The failed rotables enter the turnaround for repair. Before maintenance can be activated, a number of rotables must be procured. The number to procure is referred to as "the initial stocking problem". When all equipment is in operation and assuming that rotables are never disposed off during repair, the initial stock and the turnaround are equal. The initial stock serves to increase the uptime or "service level" of the installed base. The greater the service level demanded, the greater the initial stock needed. The initial stock is a function of three interacting processes; the failure process, the repair process and the inventory holding process.

## The failure process.

The recoverable parts in use are prone to fail. The superposition of all part failures is called "the failure process". The failure process is influenced by the maintenance activation measure. Gits (1984) distinguishes three activation measures: failure based, use based and condition based. In case of a condition or a failure based maintenance activation measure, failures occur according to a (compound) Poisson process. The failure process will be unpredictable and varying on the short term. So will be demand.

## The repair process.

The repair process changes the state of the rotable: From the failed state to the repaired, repair contains the activities dis(assembly), inspection, part exchange and testing. The complexity of the repair process is dependent of the repairable part structure. The more levels the part structure is composed of, the more dis(assembly) is required.

**The inventory holding process.**

Repaired rotables are kept in inventory. To guarantee a quick exchange, the inventory must be situated near by the installed base. In case the installed base is widespread, the inventory must have a multi-stage structure. In general, parts tend to fail rarely. As a result many recoverable parts are "slowly moving".

Much literature is directed to the initial stocking problem. In practice, we come across a great variety of initial stocking problem types. However, the vast majority of literature addresses single or multi item initial stocking problems with the following features:

- a (compound) Poisson failure process,
- a single level repair process,
- a single or two-stage inventory holding process,

To reduce mathematical complexity many authors further assume:

- stationary demand,
- statistically independent failures,
- a fixed number of rotables,
- no lateral resupply between inventories,
- no batching of failed rotables,
- no subcontracting,
- ample capacity (until recently).

An overview is presented by Nahmias (1981). In the early literature, solution methods for the initial stocking problem have been published for a variety of situations under the collective noun METRIC. METRIC solves the problem assuming ample capacity. The most important representatives of METRIC are Sherbrooke (1968, 1986), Muckstadt (1973) and Slay (1984). More recently the ample capacity restriction is relaxed. The initial stocking problem is solved by means of closed queuing network theory. The most important representatives of this theory are Gross (1982, 1983), Balana et al. (1989) and Ebeling (1991). Queuing theory is more accurate than METRIC in solving the problem. However queuing theory is more intricate to solve complex initial stocking problems than METRIC. Therefore, lately, attention has been called to METRIC again. The gap between both approaches has recently been closed by means of approximations, Ahmed et al. (1992). None of these authors however pay attention to production control decisions. By applying control decisions, e.g. sequencing, the initial stocking problem is affected. The use of sequencing rules is proposed by Schneeweiß et al. (1992). These authors show that it is cost-effective to schedule expensive rotables into repair first.

In the literature so far, the service level is strictly increased by augmenting the number of rotables, not by considering the repair capacity on the short term. Short-term capacity is assumed to be fixed. However, the repair capacity affects the repair throughput rate, which in turn influences the service level. If the key capacity resources are constituted by human beings not by machines, the assumption of a fixed repair throughput rate is not likely to be true. In practice people are flexible. They may, for example, temporarily work harder to avoid stock out occurrences. On the other hand, people may slow down when repaired stock levels do. Such human behavior affects the service level. The change in service level is dependent of the measure of human flexibility and the irregularity of the failure process. In practice, it is not plausible that people can adjust their own effort beyond any limit. However, flexibility can be reinforced by regulating the number of working hours on the short-term.

This paper deals with an example of an elementary rotable maintenance situation e.g. a simple tool monitoring and repair situation, see figure 1. Upon failure a tool is exchanged by its repaired counterpart. We assume:

- a single tool type,

- zero exchange and transport times,

- a limited repair capacity resource consisting of one capacity resource,

- a repair capacity utilization rate smaller than one,

- exponentially distributed failure and repair throughput times,

- no scrap, so that the total number of rotables will be fixed.



f   : failure rate
r   : repair rate
Q1: failed queue
Q2: repaired queue
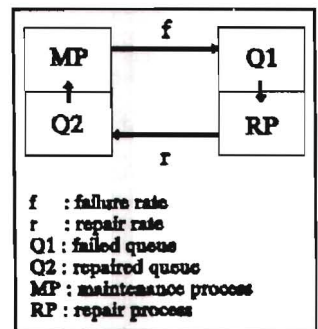MP : maintenance process
RP : repair process

Figure 1: Queuing network.

In section 2, we first assume a fixed repair throughput rate and calculate the relation between the initial stock and the service level for this example. Then the assumption of a fixed throughput rate is relaxed and the relation is calculated again. The example shows that throughput rate adjustment, if applied well, can be effective. Throughput rate adjustment can be reinforced by means of short-term capacity planning. Section 3 introduces a short-term capacity control function. The function is embedded in a hierarchical control structure. In section 4, the conclusions are drawn and further research to short-term capacity planning is advocated.

## 2. Throughput rates.

Consider figure 1. The example represents an elementary rotable maintenance situation. However simplified, the example contains aspects which are encountered in practice. It serves to gain understanding

in the impacts of these aspects. In the example, we show the relation between the service level and the initial spares problem. The service level is defined as the uptime of the installed base. Within the context of the assumptions, the network behaves as Markov process e.g. Kleinrock (1975), see figure 2.
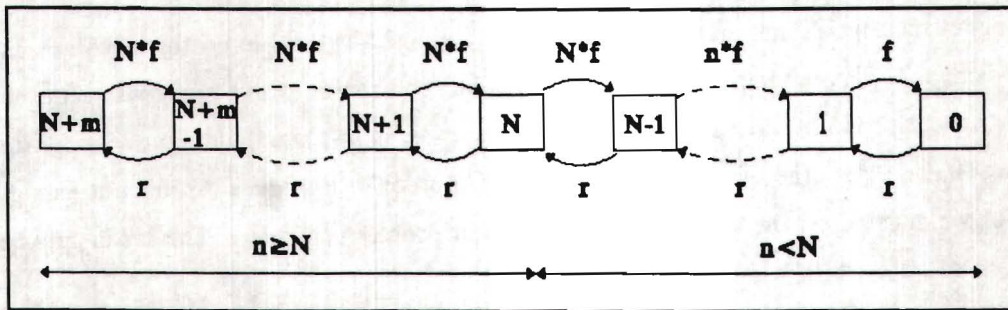


**Figure 2:** Markov process.

i : the state indicator; $0 \leq i \leq N+m$,

N : the number of installed, $N \geq 1$,

n : the number in operation, $0 \leq n \leq N$,

m : the initial stock, $m \geq 0$,

f : exponential failure rate per rotable,

r : repair throughput rate; 1/ repair throughput time,

$P_i$ : the steady state probability of state i; $P_{N+m}$: the probability that all tools are repaired; $P_0$: the probability that no tools are repaired.

With the help of Markov theory, we calculate all probabilities $P_i$. The cumulation of all states "$n \geq N$" yields the service level. Assuming steady state, equation (1) holds

$$P_{N+m-1} r = P_{N+m} N f \tag{1}$$

Therefore,

$$
\begin{aligned}
P_{N+m-1} &= P_{N+m} \, N(f/r) \\
P_{N+m-2} &= P_{N+m} \, N^2 (f/r)^2 \\
&\vdots \\
P_{N+1} &= P_{N+m} \, N^{m-1} (f/r)^{m-1} \\
P_N &= P_{N+m} \, N^m (f/r)^m \\
P_{N-1} &= P_{N+m} \, N^{m+1} (f/r)^{m+1} \\
&\vdots \\
P_0 &= P_{N+m} \, (1)(2)..(N-1) N^{m+1} (f/r)^{m+N}
\end{aligned}
\tag{2}
$$

and,

$$\Sigma_i \, P_i = 1 \tag{3}$$

Consider for example a two machine problem (N=2) with failure rate (f=0,2 1/weeks) and repair throughput time (1/r=2 weeks). If we choose the initial stock (m=4), we find $P_6 = 0{,}26$, $P_5 = 0{,}21$, $P_4 = 0{,}17$,

$P_3=0,13$, $P_2=0,11$, $P_1=0,09$ and $P_0=0,03$. So the service level, formally $100\Sigma_{i=2..6}P_i$, equals 88%. If we choose the initial stock (m=5), we find a service level of 91%.

In many maintenance situations, capacity restrictions are imposed by people not by machines. Lets, for example, assume that people temporarily work harder when confronted with a stockout occurrence and, on the other hand, slow down when stock levels do. In that case, people manipulate the repair throughput rate. As a result, we expect the service level to increase. In order to introduce throughput manipulation in our model, we must adjust the repair rate in the Markov process of figure 2. The repair rate becomes some function of state i "r(i)" e.g. Regterschot (1987).

In our example we choose a simple two-stage throughput function, see figure 3. People work on a low pace (r(-)) if confronted with the states N+m till N+m-x and on a high pace (r(+)) else. The equations (2) change as follows

$$
\begin{array}{ll}
P'_{N+m-1} = P'_{N+m} \; N(f/r(-)) \\
P'_{N+m-2} = P'_{N+m} \; N^2(f/r(-))^2 \\
\quad\quad\vdots \\
P'_{N+m-x} = P'_{N+m} \; N^2(f/r(-))^x \\
\quad\quad\vdots \\
P'_{N+1} = P'_{N+m} \; N^{m-1}(f/r(-))^x(f/r(+))^{m-1-x} \\
P'_{N} = P'_{N+m} \; N^m(f/r(-))^x(f/r(+))^{m-x} \\
P'_{N-1} = P'_{N+m} \; N^{m+1}(f/r(-))^x(f/r(+))^{m+1-x} \\
\quad\quad\vdots \\
P'_0 = P'_{N+m} \; (1)(2)..(N-1)N^{m+1}(f/r(-))^x(f/r(+))^{m+N-x}
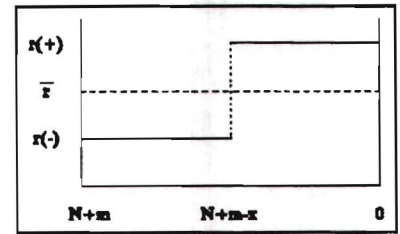\end{array}
\tag{4}
$$



**Figure 3**: 2-stage throughput

and again,

$$
\Sigma_i \; P_i' = 1
$$

For the Markov processes (2) and (4) to be comparable, we require the average throughput rates of both processes formally to be equal, i.e.

$$
\Sigma_i r(i)P_i' = \Sigma_i rP_i = r
\tag{5}
$$

Consider again the example where the initial stock equals four (N=2, m=4). In that example, the workload is (a) zero in 26% of the time, (b) amount to a maximum of two in roughly 37% of the time and (c) to a minimum of three in the remaining 37% of the time. Lets assume that people are able to increase their throughput by 10% in case (c), and to decrease their throughput by 10% in case (b), then we satisfy equation (5). Further, r(0), r(1), r(2) and r(3) equal 0.55 and r(4) and r(5) equal 0.45. Now we find a service level of 91%, equal to the case with initial stock five. Assuming the flexibility mentioned, we could have removed one tool and still obtain the same service level in this example.

## 3. Capacity Planning.

The example of section 2 shows that human flexibility can have a favorable effect on the stock and service levels. The flexibility better tunes the failure and breakdown rates. The gain in service level is dependent of the measure of flexibility and the irregularity of the failure process. In practice, it is not plausible that people can adjust their own effort beyond any limit. However, the service level increasing effect can be reinforced by regulating the daily working hours on the short-term. In practice this means that people on a daily or weekly base work an irregular number of hours, e.g. nine hours daily when confronted with stockout occurrences and seven hours when stocklevels do. Distinct from throughput rate adjustment as a result of human behavior, adjustment as a result of short-term capacity planning requires active control. At present, in rotable maintenance situations, control concepts are not yet equiped with a short-term capacity feature. The development of such a concept is the aim of our research.

A theory dealing with the design of control concepts for production situations, and useful in our research, is developed by Bertrand et al. (1990). The authors emphasize both material and capacity aspects: Both are essential in rotable maintenance situations. The theory is briefly explained.

In general, goodsflow and decision structures are too complex to design altogether. To reduce complexity, the authors introduce a technique called "decomposition". Among others they decompose goodsflow from production unit (PU) decisions, and aggregate from detailed decisions. On a goodsflow level both aggregate and detailed decisions are taken: On a PU level only detailed decisions are taken. Decisions concerning the capacity volume, a key topic in our research, are of an aggregate nature and consequently are taken on a goodsflow level.

The goodsflow control structure, designed by Bertrand et al. coordinates the workorder release to PUs. The control structure is composed of two basic functions, material coordination and workload control. Material coordination determines the release priorities: Workload control determines the aggregate release patterns. Unlike production situations, in rotable maintenance situations also the initial stocking problem must be solved on a goodsflow level. Further we add a short-term capacity planning function. The adjusted goodsflow structure is presented in figure 4. In the context of this paper we elaborate on the upper three functions.

On the most upper level an aggregate repair plan is drafted. The plan contains per rotable the number of repairs (in man-hours) to be expected in a given period. the plan is fed with, the number of installed, failure rates, the maintenance activation, historical data and the $\alpha$-service level. The aggregate repair plan

ties up the decision space for the next lower level functions. On a next lower level the aggregate capacity volume flexibility and the initial stocking problem must be solved. The capacity flexibility determines the average capacity level necessary to satisfy the repair plan. The function further determines the maximum deviation from the average capacity and the additional costs. The capacity flexibility function is affected by the capacity control function. In this paper we deal with a two-stage capacity control function alone. The initial stocking problem is affected by the flexible capacity function. The more flexible the capacity the smaller the initial stock. The measure of flexibility to use is dependent of the costs of this flexibility and the gain in initial stock reduction. More formally,
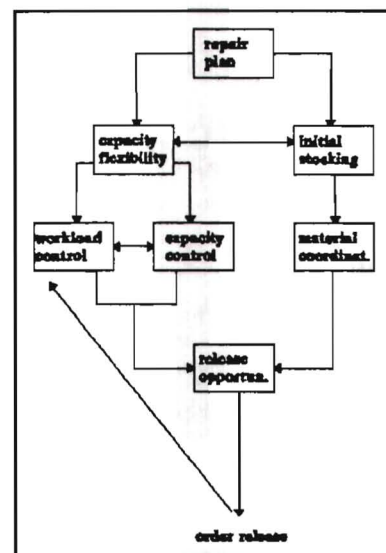


Figure 4: Goodsflow structure

$S(\ell)$    : The initial stock; Some function of the service level $\ell$.

$L(q)$    : The service level; Some function of the average failed queue length q.

$Q(r)$    : The average failed queue length; Some function of the average repair throughput rate r.

$r_{min}$    : The minimum repair throughput rate.

$r_{max}$    : The maximum repair throughput rate.

$$r = \Sigma_{i=0,..,N+m}P_i{'}r(-) + \Sigma_{i=0,..,N+m-x}P_i{'}(r(+)-r(-)) \tag{6}$$
$$\text{and } \Sigma_i r(i)P{'}_i = \Sigma_i rP_i = r; \; r_{min} \leq r(-) \leq r \; ; \; r \leq r(+) \leq r_{max}$$

The costs functions:

Cs    : The costs of initial stock; $s(\ell) * cs$.

Cr    : The costs of flexible capacity; $cr1\Sigma_{i=0,..,N+m}P_i{'}r(-) + cr2\Sigma_{k=0,..,N+m-x}P_i{'}(r(+)-r(-))$.

Ct    : Total costs; $Cs+Cr$.

When the function $S(\ell)$, $L(q)$ and $Q(r)$ and cost functions are known in some rotable situation, the optimal capacity flexibility can for example be determined with the following procedure. In the procedure we assume only one optimum.

```
Step 1  : r(-):=r(+):=r; For all i Calculate Pi; Ct:=Cs+Cr; y:=Ct.
Step 2  : If r(-)>rmin Then r(-):=r(-)-1; r(+):=r(+)+1;
          For all i Calculate Pi'; Ct:=Cs+Cr Else Stop.
Step 3  : If Ct<y Then y:=Ct; Step 2 Else Stop.
```

Determination of the functions $S(\ell)$, $L(q)$ and $Q(r)$ and their interaction in distinct rotable situations requires further research.

## 4. Conclusions.

The introduction shows that, from a mathematical point of view, already much literature has been published on initial stocking of rotables. However valuable, still many assumptions must be relaxed before complex real life problems can be solved. Therefore further research in the field must be stimulated.

In case of an irregular failure process, the example of section 2 shows that human behavior can have a favorable effect on the stock and service levels because failure and repair rates are better tuned. In practice, it is not plausible that people can adjust their own effort beyond any limit. However, the service level increasing effect can be reinforced by regulating the working hours on the short-term. Distinct from human behavior, the latter requires active control. In section 3 a control structure is presented composed which is composed of hierarchical function. We have elaborated on the interaction of the upper level functions. Further research is necessary to both the individual functions and their interaction.

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