Effective output and availability of windturbines for household loads

EFFECTIVE OUTPUT AND AVAILABILITY OF WINDTURBINES FOR HOUSEHOLD LOADS

by

P. FERON

E.H. LYSEN

August 1981 R 491 D

University of Technology Eindhoven
Department of Physics
Wind Energy Group

CONTENTS Page

ABSTRACT

A theoretical model of a small-scale wind energy system producing electricity for a group of housholds connected to the grid is presented.

By means of (1) a Weibull distribution of the wind velocity, (2) a linear output model of the wind turbine and (3) a measured load distribution function of a group of Dutch households, the average values of the electricity surplus and electricity deficit are calculated for the wind regime in Den Helder, the Netherlands.

Two parameters proved to be useful to describe the system: the effective energy output and the power availability.

1. INTRODUCTION

Wind energy systems for generating electricity can roughly be divided into small-scale (< 100 kW) and large-scale systems (> 100 kW).
In the Netherlands the interest for the application of small-scale wind turbines is increasing rapidly. Wind turbines are being installed at farms, near small factories etc.

The objective of this study is to give a theoretical description of a small-scale wind energy system consisting of a wind turbine, a limited number of households and the grid.
One can distinguish two different operating conditions:

1. The power produced by the wind turbine exceeds the power demand of the households: surplus power is delivered to the grid.
2. The power produced by the wind turbine is not sufficient to fulfil the power demand of the households: the deficit power has to be supplied by the grid.

This is illustrated in figure 1.

In the following the energy flows in the system will be calculated by means of a probability analysis, after a description of the different ingredients for the model.

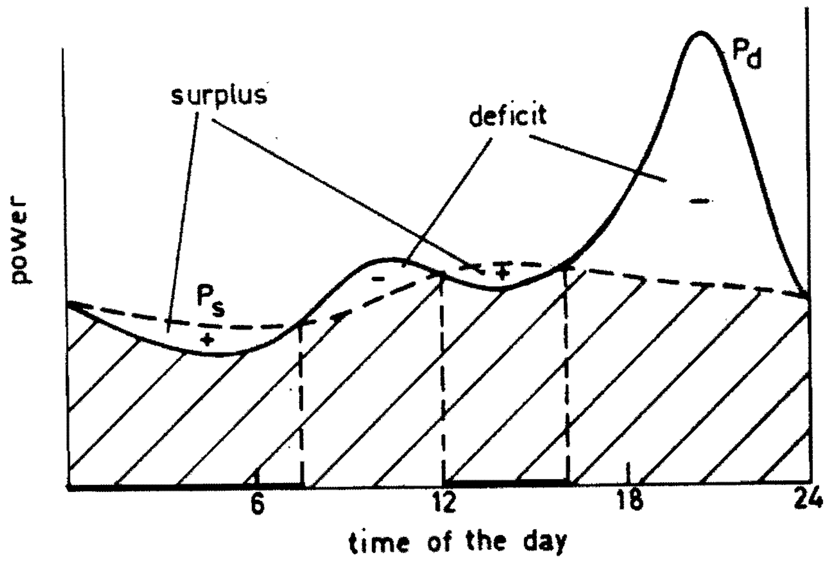

Figure 1

Typical example of the power supply of a wind turbine and the power demand of a number of households during an arbitrary day.

P_s : Power supply by the wind turbine.

P_d : Power demand of the households.

▨ : Total wind energy used by the households.

—— : Time during which the system is self-supporting, i.e. $P_s > P_d$.

2. INGREDIENTS FOR THE MODEL

The system consists of three components - wind turbine, households and grid - of which the grid is regarded as an infinite source of electricity, always available. As far as the first and second components are concerned we will derive probability functions for the output of the wind turbine and the load of the households.

2.1 Wind turbine output

The amount of electricity produced by the wind turbine is determined by the characteristics of the wind turbine and the wind regime of the site.

We assume a linear power output - wind velocity characteristic (figure 2). The wind velocities are hourly averages.
Mathematically:

$$\begin{aligned}
&\text{If} \quad 0<V<V_c && \text{then} \quad P_s = 0 \\
&\text{If} \quad V_c<V<V_r && \text{then} \quad P_s = \frac{V-V_c}{V_r-V_c} \cdot P_r \\
&\text{If} \quad V_r<V<V_f && \text{then} \quad P_s = P_r \\
&\text{If} \quad V>V_f && \text{then} \quad P_s = 0
\end{aligned} \tag{2.1}$$

The probability functions of the wind velocity are described by means of a Weibull function [1].
In the case of a Weibull shape factor k = 2 (valid for Den Helder, the Netherlands) the probability distribution function is (figure 3):

$$F(V) = 1 - \exp\left(-\frac{\pi}{4}\left(\frac{V}{\bar{V}}\right)^2\right) \tag{2.2}$$

($\bar{V}$ is the average wind velocity) and its first derivative, the probability density function, is:

$$f(V) = \frac{\pi}{2}\frac{V}{\bar{V}^2} \exp\left(-\frac{\pi}{4}\left(\frac{V}{\bar{V}}\right)^2\right) \tag{2.3}$$

Combination of formulas 2.1 and 2.3 and introduction of $x = \frac{V}{\bar{V}}$ leads to a power probability density function $f_s(P_s)$:

If $P_s = 0$ then $f_s(P_s) = [1+\exp(-\frac{\pi}{4} x_f^2) - \exp(-\frac{\pi}{4} x_c^2)] \cdot \delta(P_s)$

If $0<P_s<P_r$ then
$$f_s(P_s) = \frac{\pi}{2} \left(\frac{x_r-x_c}{P_r} \cdot P_s + x_c\right) \cdot \frac{x_r-x_c}{P_r} \cdot \exp\left[-\frac{\pi}{4} \cdot \left(\frac{x_r-x_c}{P_r} \cdot P_s + x_c\right)^2\right] \tag{2.4}$$

If $P_s = P_r$ then $f_s(P_s) = (\exp(-\frac{\pi}{4} x_r^2) - \exp(-\frac{\pi}{4} x_f^2))\ \delta(P_s-P_r)$

If $P_s > P_r$ then $f_s(P_s) = 0$

$\delta(P_s)$ is the Dirac-function:
$$\delta(P_s) = 0 \quad \text{if} \quad P_s \neq 0$$
$$\delta(P_s) = \infty \quad \text{if} \quad P_s = 0$$
$$\int_{-\infty}^{\infty} \delta(P_s)dP_s = 1$$

The probability distribution function $F_s(P_s)$ is found by integration of formula 2.4 (figure 4).

If $0 \leq P_s \leq P_r$ then
$$F_s(P_s) = 1 + \exp(-\frac{\pi}{4} x_f^2) - \exp\left(-\frac{\pi}{4}\left(\frac{x_r-x_c}{P_r} \cdot P_s + x_c\right)^2\right) \tag{2.5}$$

If $P_s > P_r$ then $F_s(P_s) = 1$.

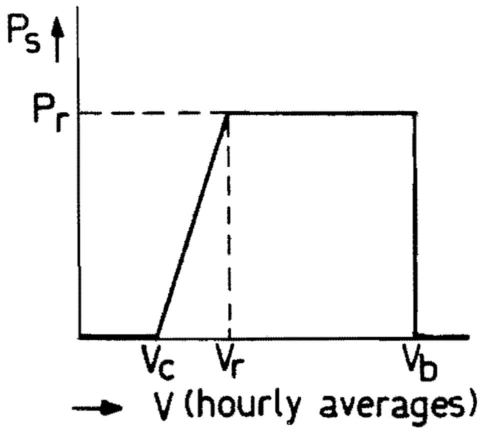

Figure 2

Power versus wind velocity

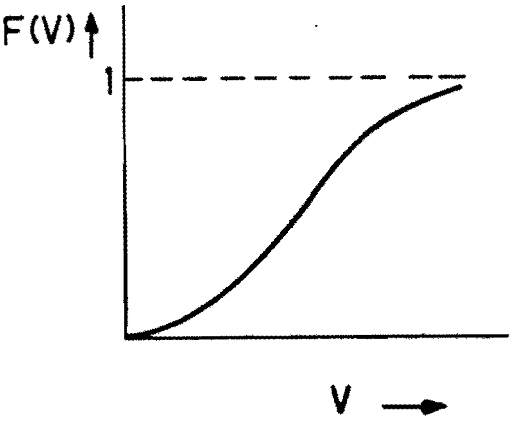

Figure 3

Probability distribution function of the wind velocity

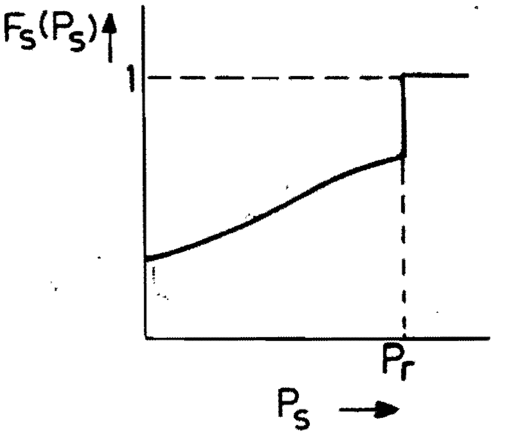

Figure 4

Probability distribution function of the power output

2.2 Household load

No theoretical description of the household load could be traced. For our purpose we used a study [2] which included the measurement of the daily pattern of the power demand of 180 households in Groningen (Netherlands). This was done in 1974/1975 for several periods equally spread out over the four seasons.
We deduced the probability distribution function from the given cumulative power frequency distribution on an annual basis.
The probability distribution function $F_d(P_d)$ (figure 5) and its first derivative, the probability density function $f_d(P_d)$, were both normalised on an average demanded power ($\overline{P}_d$).

$F_d(P_d)$	$f_d(P_d)$	Interval
0	0	if $0 \leq P_d < 0.36\ \overline{P}_d$
$0.300\ \frac{P_d}{\overline{P}_d} - 0.108$	$\frac{0.300}{\overline{P}_d}$	if $0.36\ \overline{P}_d \leq P_d < 0.54\ \overline{P}_d$
$1.336\ \frac{P_d}{\overline{P}_d} - 0.667$	$\frac{1.336}{\overline{P}_d}$	if $0.54\ \overline{P}_d \leq P_d < 0.72\ \overline{P}_d$
$1.522\ \frac{P_d}{\overline{P}_d} - 0.801$	$\frac{1.522}{\overline{P}_d}$	if $0.72\ \overline{P}_d \leq P_d < 0.90\ \overline{P}_d$
$0.737\ \frac{P_d}{\overline{P}_d} - 0.095$	$\frac{0.737}{\overline{P}_d}$	if $0.90\ \overline{P}_d \leq P_d < 1.08\ \overline{P}_d$
$0.460\ \frac{P_d}{\overline{P}_d} + 0.204$	$\frac{0.460}{\overline{P}_d}$	if $1.08\ \overline{P}_d \leq P_d < 1.27\ \overline{P}_d$
$0.190\ \frac{P_d}{\overline{P}_d} + 0.547$	$\frac{0.190}{\overline{P}_d}$	if $1.27\ \overline{P}_d \leq P_d < 2.17\ \overline{P}_d$
$0.160\ \frac{P_d}{\overline{P}_d} + 0.612$	$\frac{0.160}{\overline{P}_d}$	if $2.17\ \overline{P}_d \leq P_d < 2.35\ \overline{P}_d$
$0.032\ \frac{P_d}{\overline{P}_d} + 0.913$	$\frac{0.032}{\overline{P}_d}$	if $2.35\ \overline{P}_d \leq P_d < 2.72\ \overline{P}_d$
1	0	if $P_d \geq 2.72\ \overline{P}_d$

Table 1 The probability distribution and the probability density of the power demand P_d of a group of 180 households in Groningen, the Netherlands (1974/1975) [2]

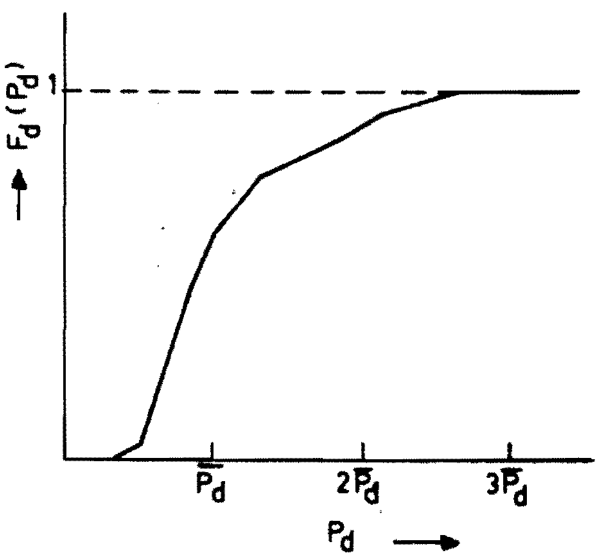

Figure 5

Probability distribution function of the demanded power.

3. THE MODEL

3.1 The integral expressions

Using the four probability functions of section 2 it is possible to derive a number of integral expressions (see appendix) for:

1. The electricity generated by the windturbine and used by the households in a period Δt:

$$\Delta t * \{\int_0^\infty P_d(1-F_s(P_d))f_d(P_d)dP_d + \int_0^\infty P_s(1-F_d(P_s))f_s(P_s)dP_s\} \tag{3.1}$$

2. The surplus electricity delivered to the grid in a period Δt:

$$\Delta t * \{\int_0^\infty P_s F_d(P_s)f_s(P_s)dP_s - \int_0^\infty P_d(1-F_s(P_d)f_d(P_d)dP_d\} \tag{3.2}$$

3. The electricity deficit supplied by the grid in a period Δt:

$$\Delta t * \{\int_0^\infty P_d F_s(P_d)f_d(P_d)dP_d - \int_0^\infty P_s(1-F_d(P_s))f_s(P_s)dP_s\} \tag{3.3}$$

This is illustrated in figure 1.
With these integral expressions we can describe the performance of the wind energy system with two parameters:

3.2 Effective output η

This is defined as the fraction of the mean total electricity production of the wind turbine which is used by the households.
From formula 3.1 we get:

$$\eta = \frac{\int_0^\infty P_d(1-F_s(P_d))f_d(P_d)dP_d + \int_0^\infty P_s(1-F_d(P_s))f_s(P_s)dP_s}{\int_0^\infty P_s f_s(P_s)dP_s} \tag{3.4}$$

3.3 Availability τ

This is defined as the mean fraction of time during which the wind turbine produces a surplus of electricity:

$$\tau = \int_0^\infty f_s(P_s)F_d(P_s)dP_s \qquad (3.5)$$

This is illustrated by the thick line in figure 1.

4. A PRACTICAL EXAMPLE

4.1 Actual calculation method

The theoretical model as presented in section 2 and 3 has been worked for Den Helder (Netherlands; $\overline{V}_{annual} = 7$ m/s) and a Dutch wind turbine "Klaver 10" (manufactured by Lagerwey/van de Loenhorst, Kootwijkerbroek). The method included the following:
- dividing the day in 12 two-hourly intervals
- dividing the year in 4 seasons.

This resulted in 48 time intervals and we assumed that the probability distribution of household load and wind turbine output in each interval are approximately independent. Furthermore we assumed the annual production of electricity by the wind turbine to be equal to the annual consumption of the households.

4.2 Description of the wind turbine and load

The "Klaver 10" wind turbine possesses three blades and the rotor has a diameter of 10 m. Its characteristics are: $P_r = 10$ kW; $V_c = 4$ m/s; $V_r = 9$ m/s; $V_f = 20$ m/s. Estimated lifetime is over 20 years.
The daily pattern of the power output of the wind turbine and the household load is shown in the figures 6 and 7 for each season (two-hourly averages).

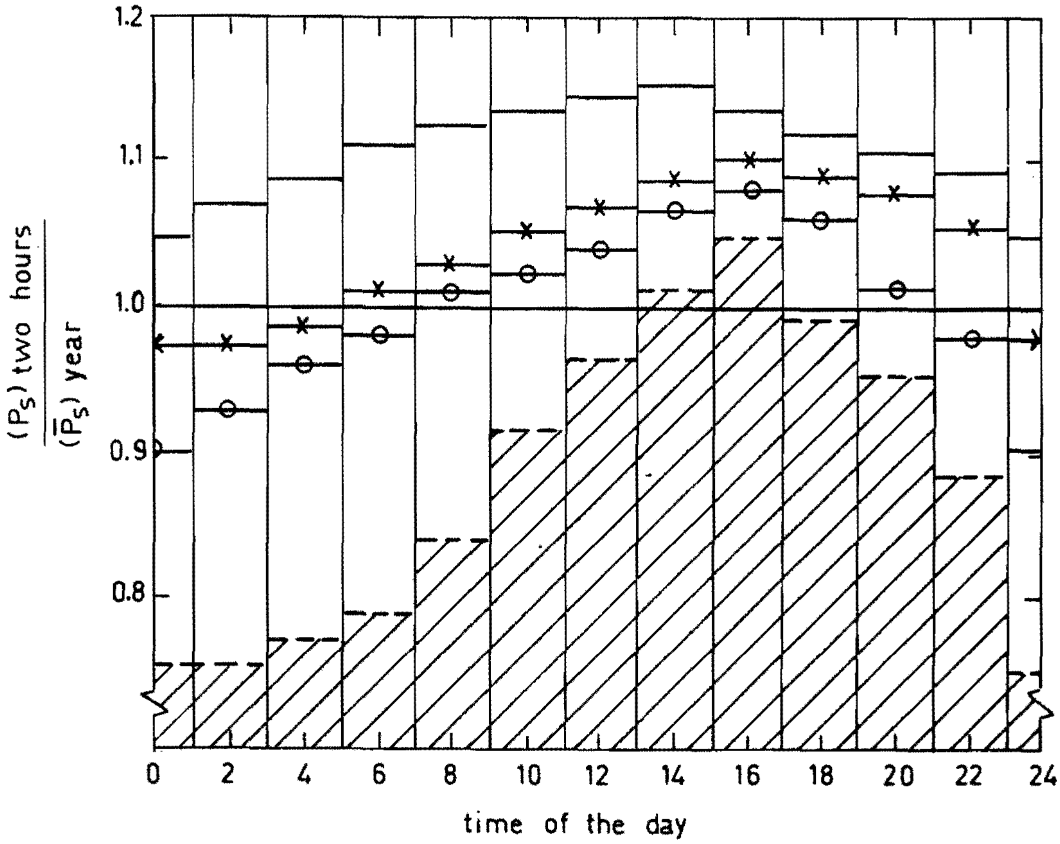

Figure 6

Daily pattern of the power supply of the wind turbine related to the annual power supply average in Den Helder.

——— Winter
—o— Spring
– – – Summer
—x— Autumn

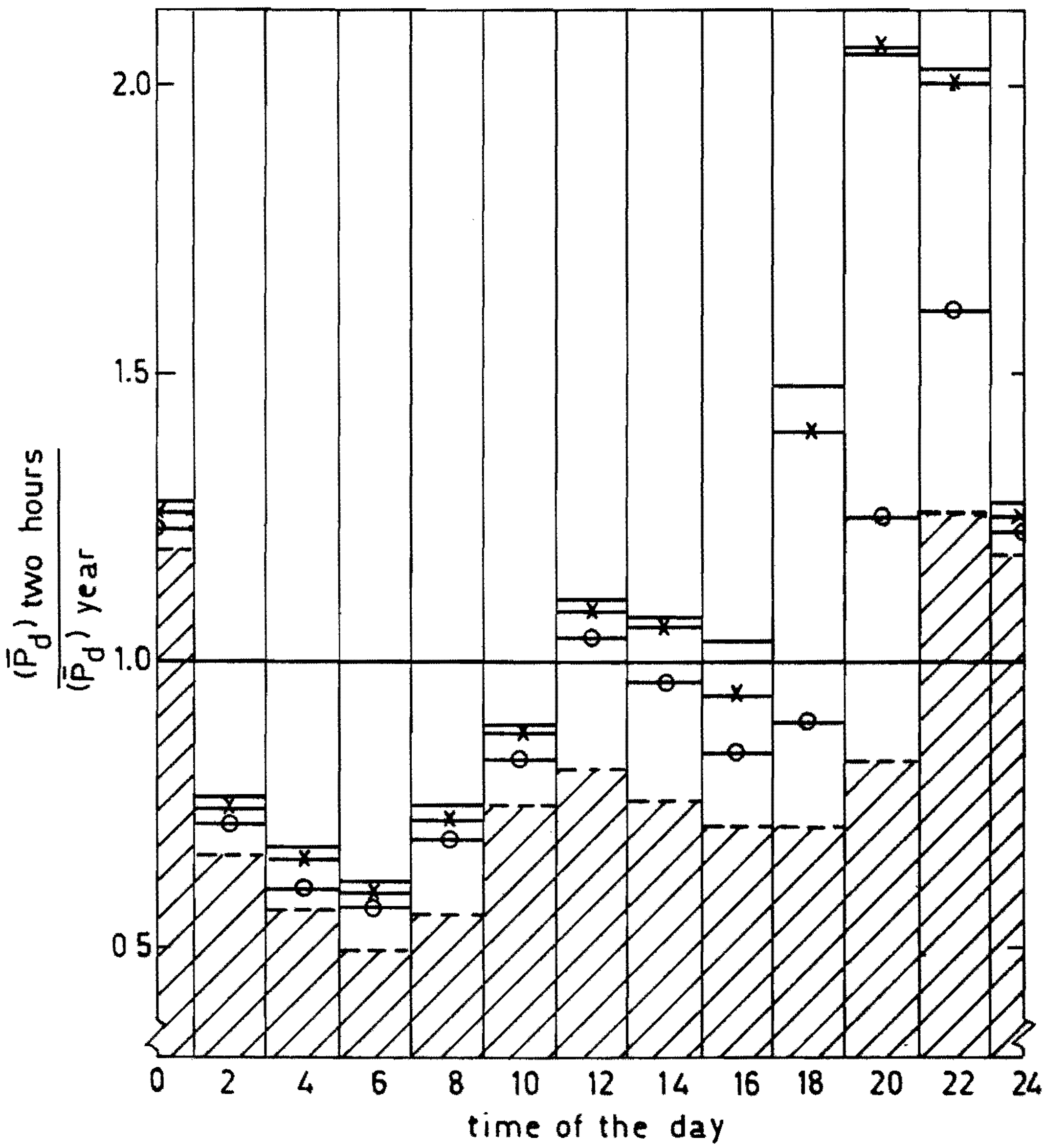

Figure 7

Daily pattern of the power demand of a group of households in Groningen related to the annual power demand average.

——— Winter

—o— Spring

– – – Summer

—x— Autumn

4.3 Results of the calculations

In table 2 the different amounts of electricity in the four seasons are shown: total power output, wind turbine output used by the households, output surplus, household deficit.

	windturbine output [kWh]	household electricity demand [kWh]	windturbine output used by the households [kWh]	electricity surplus [kWh]	electricity deficit [kWh]	effective output [%]
	A	B	C	A-C	B-C	C/A
Winter	12,081	12,498	7,179	4,902	5,319	59.4
Spring	10,648	10,097	5,693	4,955	4,404	53.5
Summer	9,533	8,399	4,601	4,932	3,798	48.3
Autumn	10,993	12,293	6,541	4,452	5,752	59.5
Annual	43,255	43,287	24,014	19,241	19,273	55.5

Table 2 The calculated amounts of electricity in a wind turbine household system, situated in Den Helder, the Netherlands.

4.3.1 Effective output

The daily pattern of the effective output per season is shown in figure 8. This figure reveals the following: the effective output η reaches a minimum at about 5.00 a.m. (demanded power small compared to the wind turbine output).
The η increases during the morning, parallel to the increasing power demand, reaching a maximum at about 10.00 a.m. Then it decreases slowly to reach a minimum at about 3.00 p.m. After this it starts to increase strongly to reach its largest peak in the evening hours (at about 9.00 p.m.) because of the large power demand at that time (lighting and television). During the beginning of the night the effective output decreases rapidly until it reaches the first mentioned minimum in the morning.

Note the effect of the early darkness during winter and autumn on the effective output (time-intervals 4.00 p.m. and 6.00 p.m.).
In table 1 the seasonal effective outputs are shown.
Because of the large power demand in winter and autumn compared to spring and summer the values of the effective output are much larger during the first mentioned seasons.

4.3.2 Availability

The daily pattern of the availability τ per season is shown in figure 9. This pattern is anti-parallel when compared to the pattern of the effective output η. This is clear because a high value of the effective output implicates that a large fraction of the wind turbine output is used, but this happens most often when the power demand exceeds the power supply by the wind turbine.

4.3.3 Economics

Pay-back times have been computed based on the discounted cash-flow method with the following assumptions:

1) Wind turbine output used by the households saves Dfl. 0.18/kWh and the annual rate of increase of the kWh price is 6%.
2) Surplus electricity is sold to the grid at a price of Dfl. 0.055/kWh, with an annual rate of increase of 8%.
3) Costs of the wind turbine: Dfl. 37,000.-; annual maintenance costs: Dfl. 500.- (annual rate of increase of the price: 5%); no subsidies. (all prices 1979).

With 3 different rates of interest (7, 9, 11%) the calculated pay-back times are respectively 8, 9 and 10 years.

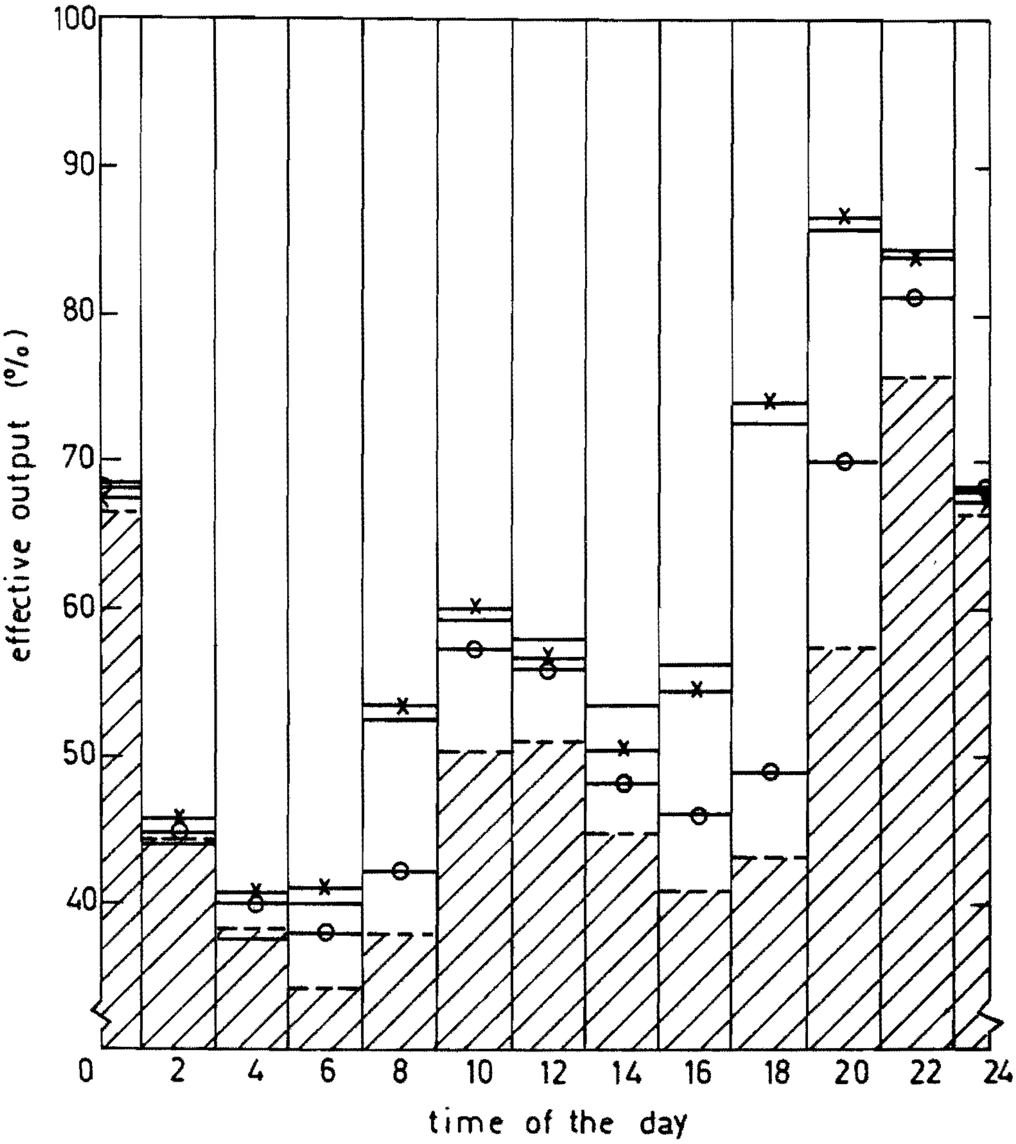

Figure 8

Daily pattern of the effective output η per season.

——— Winter

—o— Spring

– – – Summer

—x— Autumn

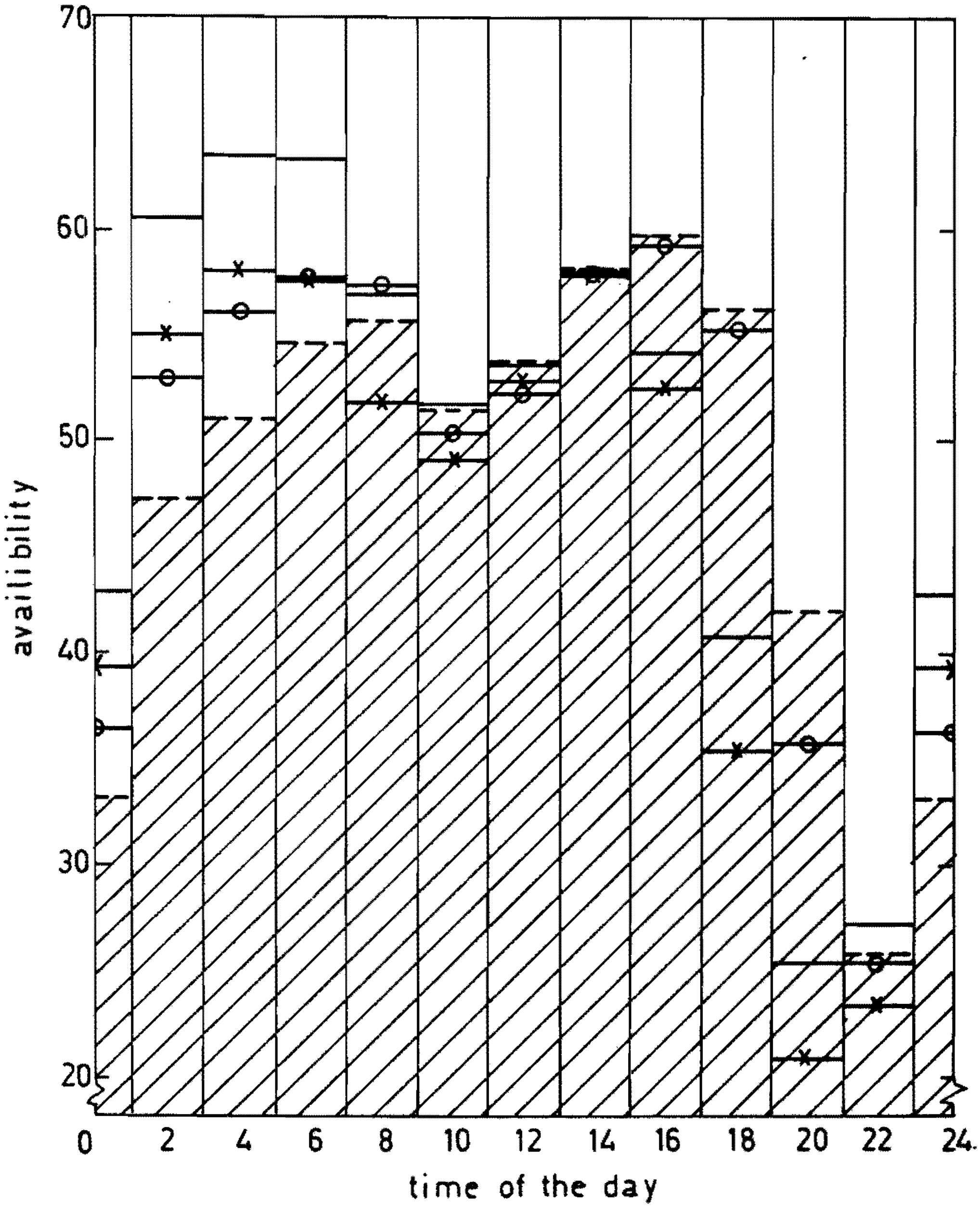

Figure 9

Daily pattern of the availability τ per season.

——— Winter

—o— Spring

- - - Summer

—x— Autumn

5. CONCLUSIONS AND RECOMMENDATIONS

When compared to the estimated life-time of the wind turbine the computed pay-back times do permit a positive conclusion about the feasibility of small wind turbines for households at coastal sites in the Netherlands.

From the daily pattern of the effective output we see that the variations in household power demand and not the wind variations influence the system's performance the most.

From the same graphs we learn that during the night large amounts of electricity are delivered to the grid. At night the grid has an over-capacity so storage in hot water boilers, instead of selling the electricity could be an economical alternative.

The model can easily be extended to other systems i.e. other wind turbines, other typical loads for instance schools, offices of factories, other ratios of annual windturbine output and annual electricity demand of the load.

The computed amounts of electricity etc. are average values. Extension of the model to a probability distribution for pay-back times, effective output and availability is useful.

6. LITERATURE

[1] The estimation of the parameters of the Weibull wind speed distribution for wind energy utilisation purposes.
M. Stevens, P.T. Smulders, Wind Engineering, Vol. 3, No. 2, 1979.

[2] An empirical study on the domestic use of electricity
G.J. van Helden, University of Groningen, 1975 (In Dutch).

APPENDIX: DERIVATION OF THE INTEGRAL EXPRESSIONS

The energy-flows in the wind energy system are governed by respectively the probability distribution functions $F_d(P_d)$, $F_s(P_s)$ and the probability density functions $f_d(P_d)$, $f_s(P_s)$, of the demanded power P_d and power P_s supplied by the wind turbine. $F_d(P)$ is the chance that the demanded power is equal or less than a value P, while $f_d(P)dP$ is the chance that the demanded power has a value in the interval [P, P+dP].

The average demanded amount of electricity in a period of time Δt is equal to $\Delta t * \int_0^\infty P_d f_d(P_d) dP_d$.

Similarly the average production of electricity in the period Δt is equal to $\Delta t * \int_0^\infty P_s f_s(P_s) dP_s$.

We assume that the demanded power P_d and the supplied power P_s are not correlated. For short periods like the two hours-period we have used this is a reasonable assumption.

The two possible operating conditions at a given time are: $P_s > P_d$ or $P_s < P_d$.

I. Power supply exceeds power demand ($P_s > P_d$):

The households use P_d and the surplus $P_s - P_d$ is delivered to the grid.

- The amount of electricity used by the households in a time interval Δt is equal to the chance $f_d(P_d)dP_d$ the power demand has a value in the interval $[P_d, P_d + dP_d]$, multiplied by the chance $f_s(P_s)dP_s$ that the power supply has a value in the interval $[P_s, P_s + dP_s]$, multiplied by the demanded power P_d and integrated over all possible values of P_s and P_d under the condition that $P_s > P_d$ and finally multiplied by Δt. This gives us:

$$\Delta t * \iint P_d f_s(P_s) f_d(P_d) dP_s dP_d \qquad \text{for } P_s > P_d \qquad (A.1)$$

We will work out this integral by means of figure A.
The integral is equal to the average value of P_d on the upper part of the first quadrant in figure A.

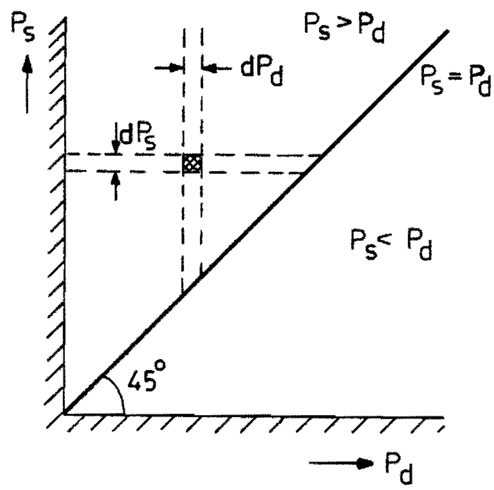

Figure A
Plane to illustrate the calculation of the integral expressions

The easiest way to compute the integral is to integrate P_s from P_d to ∞ first (see figure A: vertical direction) and then to integrate P_d from 0 to ∞. From formula A.1 we then arrive at a single integral:

$$\Delta t* \int_0^\infty P_d f_d(P_d) \{\int_{P_d}^\infty f_s(P_s) dP_s\} dP_d =$$

$$\Delta t* \int_0^\infty P_d(1-F_s(P_d))f_d(P_d)dP_d \qquad (A.2)$$

- The amount of electricity delivered to the grid is equal to the average value of $P_s - P_d$ on the upper part of the first quadrant in figure A:

$$\Delta t* \iint (P_s-P_d)f_s(P_s)f_d(P_d)dP_s dP_d \text{ for } \qquad \text{for } P_s > P_d \qquad (A.3)$$

This can be transformed into a single integral as well:

$$\Delta t* \Big[\int_0^\infty P_s f_s(P_s) \{\int_0^{P_s} f_d(P_d)dP_d\} \; dP_s \quad -$$

$$\int_0^\infty P_d f_d(P_d)\{ \int_{P_d}^\infty f_s(P_s)dP_s\} \; dP_d \Big] =$$

$$\Delta t * \left[\int_0^\infty P_s F_d(P_s) f_s(P_s) dP_s - \right.$$

$$\left. \int_0^\infty P_d (1 - F_s(P_d) f_d(P_d) dP_d \right] \qquad (A.4)$$

II. Power demand exceeds power supply ($P_s < P_d$):

The households use P_s and the deficit $P_d - P_s$ is supplied by the grid.

- The amount of electricity used by the households in a time interval Δt is equal to the average value of P_s in the part in figure A where $P_s < P_d$:

$$\Delta t * \iint P_s f_s(P_s) f_d(P_d) dP_s dP_d \qquad \text{for } P_s < P_d$$

(A.5)

This can be transformed into a single integral (see I):

$$\Delta t * \int_0^\infty P_s f_s(P_s) \left\{ \int_{P_s}^\infty f_d(P_d) dP_d \right\} dP_s \quad =$$

$$\Delta t * \int_0^\infty P_s \left\{1 - F_d(P_s)\right\} f_s(P_s) dP_s \qquad (A.6)$$

- The amount of electricity deficit in a time interval Δt is equal to the average value of $P_d - P_s$ in the part in figure A where $P_s < P_d$:

$$\Delta t * \iint (P_d - P_s) f_s(P_s) f_d(P_d) dP_s dP_d \qquad \text{for } P_s < P_d$$

(A.7)

Transformed into a single integral:

$$\Delta t * \Big[\int_0^\infty P_d f_d(P_d) \{ \int_0^{P_d} f_s(P_s) dP_s \} dP_d \quad -$$

$$\int_0^\infty P_s f_s(P_s) \Big\{ \int_{P_s}^\infty f_d(P_d) dP_d \Big\} dP_s \Big] \quad =$$

$$\Delta t * \Big[\int_0^\infty P_d F_s(P_d) f_d(P_d) dP_d \quad -$$

$$\int_0^\infty P_s (1-F_d(P_s)) f_s(P_s) dP_s \Big] \qquad \text{(A.8)}$$

Adding formula A.6 to formula A.2 we find the total amount of electricity used by the households in a time interval Δt.

The availability is defined as the time fraction during which the system is self-supporting. This is equal to the chance that P_d and P_s have values in the upper part of the first quadrant in figure A:

$$\tau = \iint f_s(P_s) f_d(P_d) dP_s dP_d \qquad \text{for } P_s > P_d \qquad \text{(A.9)}$$

$$= \int_0^\infty f_s(P_s) F_d(P_s) dP_s$$