On aggregation in production planning

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ON AGGREGATION IN PRODUCTION PLANNING

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There is a renewed interest in aggregate planning and hierarchical production planning, see for instance Axsäter [1] and Hax and Meal [4]. But there are not yet many results on the relationship between the characteristics of the production system and the right stratification of the production planning process. That relationship is the subject of this paper. Some (illustrated) ideas will be given and some suggestions for future research. In section 2 some simple deterministic cases with 2 or 3 products are considered. Section 3 discusses the role of deterministic models in a stochastic environment. In section 3 a purely stochastic case with N products and fixed production quantities is discussed.

1. INTRODUCTION

In production planning, as in all other kinds of planning, the planning horizon and the level of aggregation are important charateristics. Generally there are more planning levels and the planning horizon and the level of aggregation are related to each other. The lower level planning is detailed and has a short planning horizon. In the higher level planning the variables are more aggregated and the planning horizon is longer. The higher level planning determines restrictions(budgets) for the lower level planning. The structure of the complete planning process necessary to control an organization depends on one hand on the flexibility of the organization and on the other hand on the instability of the environment. A higher flexibility makes it easier to aggregate, to work with shorter planning horizons for the detailed planning. A higher degree of variability makes it necessary to work with detailed plans over a longer planning horizon.

In production planning we can distinguish four kinds of aggregation:

1. Aggregation over types of product.
2. Aggregation over production stages.
3. Aggregation over capacities.
4. Aggregation over time.

We will consider in particular aggregation over types of products and aggregation over (parallel) capacities.

There is a renewed interest in aggregate planning and hierarchical production planning, see for instance Axsäter [1] and Hax and Meal [4]. But there are not yet many results on the relationship between the characteristics of the production system and the right stratification of the production planning process. That relationship is the subject of this paper. Some (illustrated) ideas will be given and some suggestions for future research.

In the next section we will illustrate with some very simple examples that the right level of aggregation depends as well on the flexibility of the organization as on the instability of the environment. The possibility to switch production easily from one product to another makes it possible to aggregate over products. The mobility between capacities (substitutability of capacities) makes it possible to aggregate over capacities. But in both cases the variability of the demand restricts the possibility to aggregate.

The demand in the examples treated in section 2 is assumed to be deterministic. Of course deterministic models are frequently used in production planning, but mainly in the rolling plan context. Important characteristics of such a rolling plan are the structure of the deterministic model used each period (level of aggregation, planning horizon, etc.) and the kind of forecasting procedures. These characteristics being fixed the quality of the rolling plan depends on the instability and the inpredictability of the environment (demand, capacity, etc.). This is shortly discussed in section 3.

In section 4 we consider the case of one

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production unit with many products. Demand is assumed to be partly unpredictable. It is clear that in such cases there is always an aspect of stored capacity in the individual item inventories. This aspect is especially important in cases with a rigid production capacity. In such cases the spread of the individual inventories around the average inventory is rather stable and may be estimated rather well without being very specific about the precise production strategy. This estimate leads to an approximation of the inventory cost function for the whole system which only depends on the total inventory. So, one can split up the problem of the construction of a complete strategy in the construction of a strategy controlling total inventory and total capacity usage and a rule to distribute the total capacity usage over the different products. The best way to distribute the total capacity usage is in general according to shortest run-out time.

2. SIMPLE EXAMPLES OF AGGREGATION

In this section some simple examples of aggregation will be given, aggregation over products and aggregation over capacities (production units).

2.1 One production unit, two products

We consider the situation of two products made by the same production unit (see fig. 1).

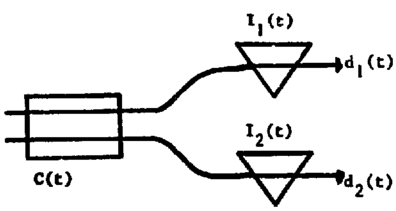

Figure 1.

Demand for both products is assumed to be known. Let $d_1(t)(d_2(t))$ be the demand for product 1 (2) in period t. The production rate for both products is the same. The total production capacity may vary from period to period and is denoted by C(t). There are two types of cost: inventory costs and production costs. Let $x_1(t)(x_2(t))$ be the production of product 1 (2) in period t. The production cost in period t depends only on $x_1(t) + x_2(t)$ and is given by $f(x_1(t) + x_2(t))$. Let $I_1(t)$ $(I_2(t))$ be the inventory of product 1 (2) at the end of period t. The inventory cost for product 1 (2) in period t is assumed to be $I_1(t)$ $|I_1(t)|(|I_2(t)|$. The starting inventories are assumed to be zero $(I_1(0) = I_2(0) = 0)$. The purpose is to minimize the total costs over the next T periods. This problem can be represented in the following way:

$$\text{Min} \sum_{t=1}^{T} \{|I_1(t)| + |I_2(t)| + f(x_1(t) + x_2(t))$$

such that

$$I_1(t+1) = I_1(t)+x_1(t+1)-d_1(t+1), \quad t=0,\ldots,T-$$

$$I_2(t+1) = I_2(t)+x_2(t+1)-d_2(t+1), \quad t=0,\ldots,T-$$

$$I_1(0) = I_2(0) = 0$$

$$x_1(t) \geq 0, \quad x_2(t) \geq 0, \quad t=1,\ldots,T$$

$$x_1(t) + x_2(t) \leq C(t) \qquad t=1,\ldots,T$$

The obvious aggregated version of this problem is

$$\text{Min} \sum_{t=1}^{T} \{|I(t)| + f(x(t)\}$$

such that

$$I(t+1)=I(t)+x(t+1)-d_1(t+1)-d_2(t+1),$$
$$t=0,\ldots,T-1$$

$$I(0) = 0$$

$$x(t) \geq 0, \quad t=1,\ldots,T$$

It is clear that the optimal cost for the ag gregated problem is less than or equal to th optimal cost of the detailed problem. Let $x^*(t)$ be the optimal (total) production derived from the aggregated model. If it is possible to construct a solution $x_1^0(t)$, $x_2^0(t)$ for the detailed problem with $x_1^0(t) + x_2^0(t) = x^*(t)$ and the corresponding inventories such that they never have opposite signs then $x_1^0(t)$, $x_2^0(t)$ has the same cost (in the detailed problem) as $x^*(t)$ in the aggregated problem and $x_1^0(t)$, $x_2^0(t)$ is optimal therefore. It is clear that if $x^*(t) \geq |d_1(t)-d_2(t)|$ for all t then one can construct a solution $x_1^0(t)$, $x_2^0(t)$ for the detailed problem such that $x_1^0(t)+x_2^0(t)=x^*(t)$ and the correspondin inventories are in all periods equal to eac other. So $x^*(t) \geq |d_1(t)-d_2(t)|$ is a sufficie condition for aggregation. The detailed T-period problem can be solved by first solvin the T-period aggregated problem and then dis tributing the total production such that the inventories remain equal. In this distribution step it is not necessary to look ahead further than 1 period. The total T-period

planning problem is split up in two levels, a T-period aggregated problem and a 1-period detailed problem. As mentioned in the introduction the higher level planning determines a budget (the total production) for the lower level planning.
It is a little unsatisfactory that in the condition for aggregation the optimal solution of the aggregated problem is used already. In case of a convex production cost function we know that $x^*(t))\geq\min_t\{d_1(t)+d_2(t)\}$.

So a sufficient condition for aggregation in that case is

$$\min_t\{d_1(t)+d_2(t)\}\geq|d_1(t)-d_2(t)| \quad \text{for all } t$$

It is clear that in case of variable demand this condition is not as easily satisfied as in case of a stable demand. The instability of the environment, mentioned already in the introduction, is indeed important.

It is not essential here that the inventory costs are linear. In case of a general inventory cost function one gets the same condition for aggregation. But the aggregated inventory cost function is no longer identical to the inventory cost functions for the individual products. Suppose $h(x)$ is the inventory cost function for the individual product. Then the aggregated inventory cost function should be $2h(\frac{x}{2})$. In case $h(x)=x^2$ the aggregated inventory cost function is $\frac{x^2}{2}$. If the production cost function is linear, so $f(x) = x$, then the only function of the inventory is that it stores capacity, it buffers the differences between total demand and capacity available. It does not matter in which product the capacity is stored as long as it is possible to prevent that the inventories get opposite signs. Also in cases where there is another kind of production function or where aggregation is not fully allowed there is always this aspect of stored capacity in the inventories of the individual products. We will come back to this in section 4.
The existence of set-up costs will complicate the problem. As long as the production quantities are so small that one may expect in most periods both products being produced the problem does not change much. But if that is not the case the problem gets more complicated indeed. This shows that aggregation over time is related to aggregation over products. If the periods in the planning are longer, problems of set-up costs are less severe.

2.2 Two production units, two products

We consider the situation of two products and two production units. Product 1 is made by production unit I, product 2 by production unit II (see figure 2).

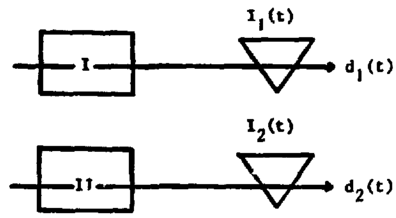

Figure 2.

The assumptions with respect to the products are the same as in case 1. The production capacity of the production units is restricted by the manpower capacity. Part of the people can only be deployed at either production unit I or production unit II. Another part of the people can be deployed at both production units. This leads to the following production restrictions

$$x_1(t) \leq p_1(t)+b(t)$$

$$x_2(t) \leq p_2(t)+b(t)$$

$$x_1(t)+x_2(t) \leq p_1(t)+p_2(t)+b(t)$$

Production costs are assumed to depend on the total production only, $f(x_1(t)+x_2(t))$. As in case 1 we want to minimize the costs over the first T periods. This leads to a problem which is almost equal to the problem in the previous case. The production restriction in that problem is replaced by the three restrictions given above. The aggregated problem is the same as in case 1 with $C(t)$ replaced by $p_1(t)+p_2(t)+b(t)$. In this case a sufficient condition for aggregation is

$$\min\{p_1(t)+b(t),\ p_2(t)+b(t),\ x^*(t)\}$$
$$\geq |d_1(t)-d_2(t)| \quad \text{for all } t$$

Disaggregation (distribution of the total production) is as in case 1. The part $b(t)$ is the mobility between the two production units.

2.3 Two production units, three products

In this case we will introduce another kind of mobility between production units. We consider the situation of three products and two production units. Product 1 has to be made by production unit I, product 2 has to be made by production unit II, but product 3 may either be made by production unit I or by production unit II (see fig. 3).

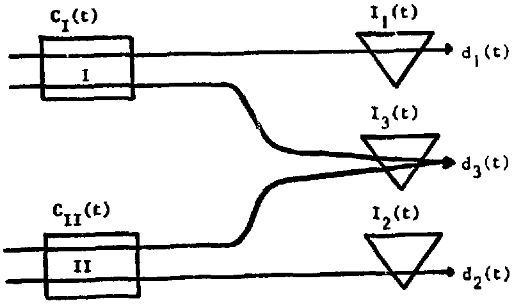

Figure 3.

Now we assume that all demand has to be delivered in time and that the only costs which can be influenced are the linear inventory costs. The purpose is to minimize the total inventory costs over the first T periods. All starting inventories are equal to zero. The problem may be presented in the following way:

$$\min \sum_{t=1}^{T} \{I_1(t)+I_2(t)+I_3(t)\}$$

such that

$$I_1(t+1)=I_1(t)+x_1(t+1)-d_1(t+1), \quad t=0,\dots,T-1$$

$$I_2(t+1)=I_2(t)+x_2(t+1)-d_2(t+1), \quad t=0,\dots,T-1$$

$$I_3(t+1)=I_3(t)+x_3(t+1)-d_3(t+1), \quad t=0,\dots,T-1$$

$$I_1(0) = I_2(0) = I_3(0) = 0$$

$$I_1(t) \geq 0,\ I_2(t) \geq 0,\ I_3(t) \geq 0, \quad t=1,\dots,T$$

$$x_1(t+1) \leq C_I(t),\ x_2(t+1) \leq C_{II}(t), \quad t=1,\dots,T$$

$$x_1(t+1)+x_2(t+1)+x_3(t+1) \leq C_I(t)+C_{II}(t), \quad t=1,\dots,T$$

$$x_1(t) \geq 0,\ x_2(t) \geq 0,\ x_3(t) \geq 0, \quad t=1,\dots,T$$

The aggregated model is

$$\min \sum_{t=1}^{I} I(t)$$

such that

$$I(t+1)=I(t)+x(t+1)-d_1(t+1)-d_2(t+1)-d_3(t+1), \quad t=0,\dots,T-1$$

$$0 \leq x(t) \leq C_I(t) + C_{II}(t), \quad t=1,\dots,T$$

$$I(0)=0,\ I(t) \geq 0, \quad t=1,\dots,T$$

A sufficient condition for aggregation is

$$d_1(t) \leq C_I(t),\ d_2(t) \leq C_{II}(t) \text{ for all } t.$$

If this condition is satisfied then for $x^*(t)$ the optimal solution of the aggregated problem, we have $x^*(t) \geq d_1(t)+d_2(t)$ for all t. One can construct the optimal solution of the detailed problem in the following way: Choose $x_1(t)=d_1(t)$, $x_2(t)=d_2(t)$ and $x_3(t)=x^*(t)$ + - $(d_1(t)+d_2(t))$. This is a feasible solution of the detailed problem with costs equal to the costs of the optimal solution of the aggregated problem. Hence, it is an optimal solution of the detailed problem.

This is a very trivial case of course, but nevertheless it shows something of the relationship between mobility and aggregation. The higher the demand of product 3 as fraction of the total demand the higher the mobility and the easier the conditions for aggregation are satisfied. In case of very unstable demand $d_1(t)$ and $d_2(t)$ the conditions for aggregation are easily violated. So the unstability of the environment is also important again.

3. THE QUALITY OF ROLLING PLANS

In all cases considered in section 2 we assumed the planning horizon to be given and the demand to be known over the whole planning horizon. This is a very severe assumption. Of course, deterministic models are frequently used in production planning, but mainly in the rolling plan context. In case a rolling plan is used, each period the following activities have to be executed:

1. The state of the system is observed.
2. Forecasts are made for the values of the exogeneous variables over the planning horizon.
3. A plan is made for the whole planning horizon. The models used in this step are deterministic in general, the forecasts are assumed to be perfect.
4. The first period decisions are implemented

In thinking about rolling plans it is important to be aware of the fact that rolling plans are used in situations where the forecasts are not perfect. The quality of a rolling plan is influenced by the choice of the forecasting procedure, the length of the horizon and the kind of model used in step 3. The quality will depend on the unpredictability of the environment. One may expect that a long planning horizon and a very detailed planning model will not contribute much to the quality of the planning in case of a high unpredictability.

Consider a system as in subsection 2.1, but with the capacity constant and with partly unknown demand instead of known demand. If one wants to evaluate the quality of a rolling plan one has to model this unpredictability of the demand.

One may assume for instance that the demand is generated in the following way:

$d_i(t+1)=q_i(t+1)+p_i(t+1)+c_i$

where c_i is some constant and

$q_i(t+1)=\rho q_i(t)+a_i(t)$

$p_i(t+1)=\rho p_i(t)+b_i(t)$

The $a_i(t)$ and $b_i(t)$ are independent normally distributed variables with mean 0 and standard deviations σ_i^a and σ_i^b. The realizations of the $b_i(t)$ are assumed to be known beforehand, while the realization of $a_i(t)$ becomes known during period t. The standard deviation σ_i^a is a measure of the unpredictability of the demand of product i. The forecasts for $d_i(t+1)$, $d_i(t+2),\dots$, needed at the beginning of period t+1 are equal to $\rho q_i(t)+p_i(t+1)$, $\rho^2 q_i(t)+p_i(t+2),\dots$. A reasonable measure of the quality of a rolling plan in such a stochastic environment is the average cost over an infinite horizon. The forecasting procedure being fixed, the only other possibilities to influence this average cost are the planning horizon and the planning model used in step 3.

The interesting point here of course is the influence of the level of aggregation of this planning model. If for all t the forecasts satisfy the conditions for aggregation given in the previous section the results for the aggregated model (plus a disaggregation step) are precisely identical to the results for the detailed model. But in case the demand is generated as described above the demand forecasts will not always satisfy the conditions for aggregation and it is important to know the difference in quality (average cost) between a rolling plan with a detailed planning model in step 3 and a rolling plan with an aggregated planning model in step 3.

The choice of the aggregate inventory cost function is also relevant. Suppose the individual inventory cost function is a convex function h(x). If it is possible to keep the inventories equal from period to period then the right aggregate inventory cost function is $2h(\frac{x}{2})$. In case this is not always possible one may get a better rolling plan by using in step 3 an aggregate planning model with a higher aggregate inventory cost function.

In case of quadratic production cost and inventory cost it is possible to calculate numerically the average cost for a rolling plan for this system. See Baker, Peterson [2] and Baker, Smits, Wijngaard [3] for details. That paper is concentrated on the influence of the planning horizon for different degrees of unpredictability. But the influence of the level of aggregation can be calculated in the same way. If the cost are non quadratic the calculation of the average cost under a rolling planning model is more difficult. Simulation is needed in such cases in general.

4. ONE PRODUCTION UNIT WITH SEVERAL PRODUCTS

In this section we will consider the case of one production unit and several products with (partly) unpredictable demand. In subsection 2.1 it was mentioned that in all such cases there is always an aspect of stored capacity in the individual product inventories. We will concentrate on that aspect here.
In cases with a high utilization rate one may expect that this stored capacity aspect is important. In such cases there is not much short term flexibility in the capacity usage and the best one can do is to use the available capacity to make the run-out times of the different products as equal as possible. The possibility to keep run-out times equal depends mainly on the unpredictable variability of the demand of the individual products, the production leadtime and the minimal production quantities. Hence, one may get a good estimate for the equality of the run-out times without using the precise form of the complete production strategy. This estimate may be used to construct an aggregate inventory cost function. In this way it is possible to find a good production strategy in a hierarchical way. In the first place one determines an aggregate production strategy in which the total production is given as function of the total inventory. Here the aggregate inventory cost function is used. In the second place the total production is distributed over the individual products according to run-out times, taking into account the minimal production quantities.
We will illustrate this with a specific example.

4.1 <u>Special case with N identical products</u>

Let there be N identical products. Customers arrive according to a Poisson process with intensity λ and order with probability 1/N one unit of product i. The fixed production quantity is q. The duration of a production run is d. At the end of each production run one has to decide to start a new production run or to wait until the inventories are lower. Suppose a new production run is started at time t. Then it is generally best to take the product with the lowest inventory position (= inventory on hand plus on order minus backorders).

Of course the pattern of replenishment depends on the complete production strategy. But we know the average intensity (λ/q) and we may expect that the spread of the individual inventories around the average inventory is insensitive for other characteristics of the replenishment process. Therefore we assume the replenishment process to be Poisson. But we may not assume of course that the probability of a replenishment for an arbitrary individual product is $1/N$. The products with the lowest inventory levels have the highest probability to get a replenishment. This causes in fact that the spread of the individual inventories around the average inventory tends to a steady state. The spread is the set of deviations of the individual product inventories from the average inventory. If there are many products the dependency between these inventory deviations is small in the steady state. If there were no dependency at all the steady spread would be characterized only by the steady state distribution of the individual product inventory deviation. We assume that this is the case indeed. The distribution function F of this steady deviation can be used to construct an aggregate inventory cost function. Let the individual inventory cost be given by the function $h(x)$. Then one may choose as aggregate inventory cost function

$$N \int_{-\infty}^{+\infty} h(\frac{x}{N} + y)dF(y)$$

Here x is the total inventory (and hence x/N the average inventory). A way to estimate the distribution function F is described in the next subsection.

4.2 Approximation of the distribution function of the steady state deviation

We assume first that $d=0$.

Suppose the inventories at a certain time are x_i, then the inventory deviations are

$$y_i := x_i - x/N \text{ where } x := \sum_{i=1}^{N} x_i.$$

If a customer for product 1 arrives the inventory deviations become

$$y_1 + \frac{1}{N} - 1, \quad y_2 + \frac{1}{N}, \ldots, y_N + \frac{1}{N}$$

If a replenishment for product 1 arrives the inventory deviations become

$$y_1 - \frac{q}{N} + q, \quad y_2 - \frac{q}{N}, \ldots, y_N - \frac{q}{N}$$

The calculation of F is iterative. Let F_0 be a first estimate. Suppose the inventory deviation of product i at time t is y_i. We assume that the other deviations are distributed according to F_0. That means that the possibility that the inventory of i is the smallest is $(1-F_0(y_i))^{N-1}$. So the probability that the inventory of product i is replenished between t and $t + \Delta t$ is $(\lambda/q)\Delta t.\ (1-F_0(y_i))^{N-1}$.

The behaviour of the inventory deviation of product i is described by a Markov process with the following properties:

From each state y transitions can occur to

$$y - \frac{q}{N}, \quad y + q - \frac{q}{N}, \quad y + \frac{1}{N}, \quad y - 1 + \frac{1}{N}.$$

The probabilities of these transitions to take place between t and $t + \Delta t$, given that the state at time t is y, are

$$(\lambda/q)\Delta t\{1-(1-F_0(y))^{N-1}\}, (\lambda/q)\Delta t(1-F_0(y))^{N-1},$$
$$\lambda\Delta t(1-\frac{1}{N}), \ \lambda\Delta t\frac{1}{N}$$

It is easy to determine the steady state distribution for this Markov process. The corresponding distribution function may be considered as the next approximation of F and is denoted by F_1.

It is possible to follow the same procedure with F_1 instead of F_0. In this way one can construct a sequence of approximations of F: $F_0, F_1, F_2, \ldots$. The iteration may be stopped as soon as F_{n+1} is close to F_n.

If $d > 0$ one has to distinguish between the inventory (= inventory on hand minus backorders) and the inventory position (= inventory on hand plus on order minus backorders). In this case we have to consider also the deviation of the inventory position of an individual product from the average inventory position. Now we assume that the events of production run starts follow a Poisson process and that in the steady state the individual product inventory position deviations are independent of each other. Let G be the distribution function of these deviations and let G_0 be a first approximation of G. The behaviour of the inventory position deviation is described by the same Markov Process as the behaviour of the inventory deviation in the case $d = 0$, with F_0 replaced by G_0. As in that case it is possible to construct a sequence of approximations of G: $G_0, G_1, \ldots$.

From the behaviour of the inventory position deviation follows easily the behaviour of the inventory deviation:

events due to a replenishment are delayed over a time d. That means that at each iteration step one can also construct an approximation of F.

4.3 Construction of a complete production storage

The construction of a complete production strategy consists of two steps. In the first step one constructs an aggregate production strategy. That means in this case that one has to develop a criterion on which the decision, when to start a new production run, can be based. In the second step one has to determine a criterion for which product to choose. This second step is trivial in this case, at least in case of a convex inventory cost function, the product with the lowest inventory has to have the highest priority. In the first step one has to consider an aggregated model. The above constructed aggregate inventory cost function has to be used here. The resulting aggregate production strategy will be a critical number strategy in this case: Start a new production run if and only if the aggregate inventory position is less than some number S.

4.4 Generalization and remarks

It is possible to generalize the example to a case with more stages of production. Assume that one has to decide whether or not to start a new production order at the first stage as soon as this stage becomes empty. A production order, once started, is finished in d periods. The above described approach works well for this case too. It is not essential in that approach that there is only one production order in process. Of course for larger d one will get a wider spread of the inventories and a higher aggregate inventory cost function.

The assumption of identical products is a very severe one. But it is possible to use the same approach in cases where the products are not identical. For instance in cases with production quantities and demand rates which vary from product to product. In such cases one has to concentrate on the run-out times instead of on the inventories. The construction of an aggregate inventory cost function is essentially the same.

The above described approach relies heavily on the insensitivity of the steady state spread of the inventories (or run-out times) for changes in the production strategy. The influence is via the replenishment process. The relationship between the production strategy and the characteristics of the replenishment process is very complex. It is difficult therefore to check the insensitivity of the spread for specific changes in the production strategy. But it is relatively easy to check the influence of the intensity of the replenishment process, the difference between a Poisson replenishment process and a more general replenishment process, the effect of autocorrelation in the replenishment process, the effect of correlation with the demand process, and so on. If the spread is insensitive for all these changes one may expect that it is also insensitive for changes in the production strategy. It may be useful therefore to execute some checks on the insensitivity of the spread for changes in the replenishment process.

On the other hand it may be useful to check how close the replenishment process under a single critical number strategy (see subsection 4.3) is to a Poisson process.

In case of an infinite capacity (many parallel production units available) the best production strategy is to start a production run of a certain product as soon as the inventory position of that product is below some level s. In that case the steady state inventory position deviation is homogeneously distributed on the set s+q-1, s+q-2,...,s . That shows that the insensitivity of the spread for the applied production strategy can only hold if the capacity is tight. In cases with a not very tight capacity constraint it may be possible to use a decomposition approach instead of an aggregation approach. In a decomposition approach the individual products are controlled independently; as soon as the inventory of a certain product comes below some critical level an order for q units is placed at the production unit. If the capacity is not infinite the order is not always delivered after d units of time since the capacity can be occupied by other orders. One needs an estimate of this delay. This delay corresponds to the waiting time for a queue with arrival rate λ/q and service time d. Of course the arrival process is very complex, but if there are many products one may get a good approximation of the waiting time by assuming that the arrival process is Poisson.

The waiting time distribution calculated in this way can be used in an individual product model to determine the reorder level. It would be interesting to compare the performance of the strategy based on aggregation and the strategy based on decomposition.

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