Approximation of structural optimization problems by means of designed numerical experiments

APPROXIMATIONS OF STRUCTURAL OPTIMIZATION PROBLEMS BY MEANS OF DESIGNED NUMERICAL EXPERIMENTS

A.J.G.Schoofs, D.H. van Campen
Department of Mechanical Engineering
Eindhoven University of Technology
P.O.Box 513, 5600 MB Eindhoven, The Netherlands

Abstract

In this paper the Experimental Design Theory (EDT), [1], is used as a tool in building approximate objective and constraint functions to be applied in structural optimization problems, in order to reduce the number of full finite element (FEM) analyses, [2]. FEM analyses can be regarded as numerical experiments, where the design variables are treated as settings, and the weight, displacements, stresses, etc. can be regarded as responses of the numerical experiment. The approximating functions will be derived for these responses by using optimal experimental designs and regression techniques.The proposed method is illustrated with an example.

1. Experimental Design Theory

1.1 Regression model

We use the following notations: $\underset{\sim}{x}$ is a column matrix, $\underline{x}$ is a stochastic variable and $\hat{x}$ is an estimated variable.

EDT consists of two main parts. The first part concerns the planning of experiments and ends up with a list of experiments to be carried out. This list is called the experimental design (ED). In the second part the experimental results are analyzed and fitted to some mathematical relationship: the regression model.

When a structure is determined by n design variables, denoted by the column, $\underset{\sim}{x}$, we may search for t functions describing the responses

$$y_j = y_j(\underset{\sim}{x}) \qquad j = 1,\dots,t \tag{1}$$

in a certain limited area according to the bounds of the design variables given by

$$x_i^l \le x_i \le x_i^u \qquad i = 1,\dots,n \tag{2}$$

In the sequel we will consider only one response quantity, y_j, and for brevity we omit the index j. To find the relation $y = y(\underset{\sim}{x})$ we assume a mathematical model. Mostly a linear model will apply of the form

$$\underline{y} = \underset{\sim}{f}^T(\underset{\sim}{x})\underset{\sim}{\beta} + \underline{e} = \beta_1 f_1(\underset{\sim}{x}) + \dots \beta_k f_k(\underset{\sim}{x}) + \underline{e} \tag{3}$$

where the components β_1 ,..., β_k of the column $\underset{\sim}{\beta}$ are unknown parameters; the model is linear in the components of $\underset{\sim}{\beta}$. The functions

$f_1(\underset{\sim}{x}), \ldots, f_k(\underset{\sim}{x})$ are the components of the column $\underset{\sim}{f}(\underset{\sim}{x})$. We can choose both linear and non-linear functions for them; in most cases for Eq.(3) a polynomial is chosen. The variable $\underline{e}$ accounts for the stochastic or deterministic model error that is inherent in every model assumption.

1.2. Parameter and response estimation

A feasible point in the design variable space is characterized by specific values of all design variables within the bounds given by Ineq. (2). The formulation of an ED implies the choice of a certain number, say N, of such points. For a proper estimation of β_i, i = 1,...,k, see Eq. (3), the number N should exceed the number k.

For this moment we assume that somehow an ED has been determined consisting of N points, represented by the sets of design variables $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_N$. If we analyze the structure at these points yielding the column of response quantities $\underset{\sim}{y} = [y_1 \; y_2 \; \ldots. \; y_N]^T$, then by using a least-squares technique the unknown parameters $\underset{\sim}{\beta}$ can be estimated from

$$\hat{\underset{\sim}{\beta}} = (X^T X)^{-1} X^T \underset{\sim}{y} \tag{4}$$

where X is the (N*k) "design matrix", which is given by

$$X = [\underset{\sim}{f}(\underset{\sim}{x}_1) \; \underset{\sim}{f}(\underset{\sim}{x}_2) \; \ldots. \; \underset{\sim}{f}(\underset{\sim}{x}_N)]^T \tag{5}$$

Subsequently, for an arbitrary design point, $\underset{\sim}{x}$, within the bounds (2) the response variable can be estimated from the explicit approximation

$$\hat{y}(\underset{\sim}{x}) = \underset{\sim}{f}^T(\underset{\sim}{x})\hat{\underset{\sim}{\beta}} \tag{6}$$

It is our purpose to use regression models of the type of Eq.(6) to formulate and solve optimization problems.

1.3. Use of sensitivities

Differentiation of the mathematical model Eq.(3) with respect to the design variable x_i gives

$$\frac{\partial y}{\partial x_i} = \beta_1 \frac{\partial f_1}{\partial x_i} + \ldots + \beta_k \frac{\partial f_k}{\partial x_i} + \frac{\partial e}{\partial x_i} \qquad i=1,\ldots,n \tag{7}$$

In FEM-formulations such sensitivities of y can efficiently be computed and thus Eq.(7) can be used with advantage, together with Eq.(3), to estimate the parameters $\underset{\sim}{\beta}$. Furthermore, the accuracy of partial derivatives of the resulting regression models then will be increased, which is advantageous for use of the regression models in optimization algorithms.

1.4. Accuracy of the estimates

A measure for the accuracy of $\hat{\beta}$ is the variance-co-variance matrix $V(\hat{\beta})$, which is defined as

$$V(\hat{\beta}) = E\left[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T\right] = (X^TX)^{-1}\sigma^2 \tag{8}$$

where E is the expected value operator, and σ^2 is the variance of the response variable y. For the response estimator $\hat{y}(x)$ the variance $V(\hat{y}(x))$ is used as a measure for its accuracy. From (6) and (8) follows

$$V(\hat{y}(x)) = f^T(x)(X^TX)^{-1}f(x)\sigma^2 \tag{9}$$

1.5. Planning of the experiments

The first part of EDT concerns the determination of the list of experiments to be carried out, the experimental design (ED), in such a way that model parameters and responses can accurately be estimated. For this purpose several methods are available. We will use a relatively recently developed method: the optimal experimental design theory, [1].

1.6. Optimal experimental design

The formulation of an ED implies the choice of a certain number, N, of points in the design variable space limited by the bounds given by Ineq.(2). The objective in optimal experimental design is to determine these N points in such a way from, in general, much a larger set of so-called candidate points, that the variances of the estimated parameters, or the variance of the estimated response quantity, are minimized.

1.7. Discrete levels of design variables

On principle all real design variable values within the bounds are allowed for a candidate point. For the purpose of efficiency, however, we only allow a very limited number of discrete values, called levels, of every design variable. The choice of the number of levels for a certain design variable depends on the order of the variable in the assumed mathematical model, see Eq.(3). A linear effect can be estimated by means of, at least, two levels. A quadratic effect needs at least three levels, and so on. For function types others than polynomial terms, for example trigonometric functions, similar considerations can be applied.

2. Model building

The building of an accurate regression model for a given system or structure is an iterative process. Initially the following questions have to be resolved to some degree:

- which variables play a role and what is their range of interest,
- which form of functions $f_i(\underset{\sim}{x})$, see Eq.(3), may be suitable to describe the searched relationship.

A good strategy is to begin with moderate model demands thus reducing the initial computing costs. The iterative model building process is able to enhance models in a cost efficient way.

At the start of each iteration step a model assumption of the type of Eq.(3) must be available. The iteration step then involves generation of an ED, collection of data, followed by estimation of the parameters from the collected data, and the evaluation of the model. Evaluation implies answering questions like:

- Is the model valid?
- Are the estimated parameters accurate enough?
- Are the response predictions accurate enough for all relevant values of x?

If the results of the testing require further model improvement, it is necessary to perform another model building cycle consisting of experimental design, data collection, parameter estimation, and retesting.

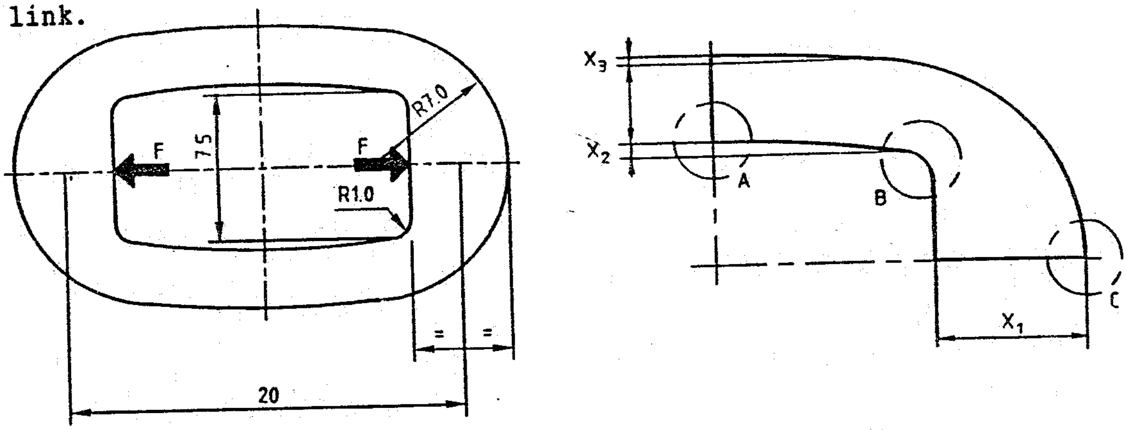
3. Computer program for model building and optimization

We developed an interactive computer program called CADE,[2], which stands for "Computer Aided Design of Experiments". Besides for experimental design, also facilities for the analysis of experiments have been implemented. For the experimental design part, the core of the program ACED, Welch [3], has been used. In CADE the optimality criteria and algorithms of ACED have been generalized to the case of simultaneous observation of several response quantities.

CADE has been coded in Fortran 77, and runs on Apollo D3000 work-stations, Vax systems and a Alliant FX40 computer. The program originally consisted of three main modules, being model input, design of experiments and parameter estimation.Recently we added a fourth module to CADE. Using this module, several developed regression models can be composed to formulate an objective function and constraint functions defining a (structural) optimization problem. Subsequently, the optimization problem is solved using CADE by means of an Sequential Quadratric Programming (SQP) algorithm.

4. Application

Van Campen et al.[4], applied the method on a stress concentration problem in a link of a chain of a continues variable transmission system, see Fig.1a. The loading force F is 268 N. Each section of the chain contains a number of links of about 0.5 mm in thickness. The pins which transmit the driving force to conical discs are locked up by the links in subsequent sections of the chain. Only a symmetric loading case was considered, allowing us to use only one quarter of the link in the FEM model. Fig.1b shows the design variables x_1, x_2 and x_3 which were used to vary the geometry of the link.



(a) (b)

Fig. 1 a. View of the link; b. Definition of the design variables

In Fig.1b three areas, A, B, and C are indicated with potentially high tensile stresses along the contour of the link. The maximum tensile stresses are denoted by σ_A, σ_B, and σ_C respectively and the objective was to derive regression models for these quantities. The level of the stresses can be influenced by variation of the geometry parameters x_1, x_2, and x_3. Hence these parameters were used as design variables.
The design variables are subject to the constructive constraints

$$4.5 \le x_1 \le 6.0 \ , \qquad 0.0 \le x_2 \le 0.6 \ , \qquad 0.0 \le x_3 \le 0.6 \tag{13}$$

Each design variable was varied on four levels. For the set of candidate points from which the experimental design had to be selected all possible combinations of the levels were used resulting in 4x4x4 = 64 candidate points.For each stress area a mathematical model was assumed containing 11 unknown parameters. One FEM analysis provides 4 observations, namely one value of the stress and three values of its partial derivatives:

$$\sigma_i \ , \quad \frac{\partial \sigma_i}{\partial x_1} \ , \quad \frac{\partial \sigma_i}{\partial x_2} \ , \quad \frac{\partial \sigma_i}{\partial x_3} \ , \qquad i = A, \ ,B \ ,C \tag{14}$$

Hence, a minimum of 3 (~ 11/4) FEM runs was required. The number of design points, N, was chosen 5. These 5 points were selected from the 64 candidate points using the optimal experimental design module of CADE.

The model fitting process resulted in the following regression models for the three stress quantities

$$\sigma_A = 540.3 - 110.1x_1 + 7.7x_1^2 + 201.6x_2 + 10.5x_2^2 - 76.3x_3^2 - \\ - 17.1x_1x_2 - 3.0x_1x_2x_3 + 12.6x_1x_3 \, , \qquad (15a)$$

$$\sigma_B = 870.9 - 199.3x_1 + 15.2x_1^2 - 263.6x_2 - 46.1x_2^2 + \\ + 41.2x_1x_3 + 3.1x_1^2x_3 - 7.0x_1^2x_3 \, , \qquad (15b)$$

$$\sigma_C = 1311.6 - 335.9x_1 + 23.7x_1^2 + 44.2x_2 + 7.7x_2^2 - \\ - 39.2x_3^2 - 0.9x_1^2x_2 + 6.0x_1x_3^2 \, , \qquad (15c)$$

In order to test the capability of the procedure one hundred test points were chosen in the design space at random. The FEM observations in these points were compared with the predictions of the models (15). Fig. 2 shows the distributions of the residuals.

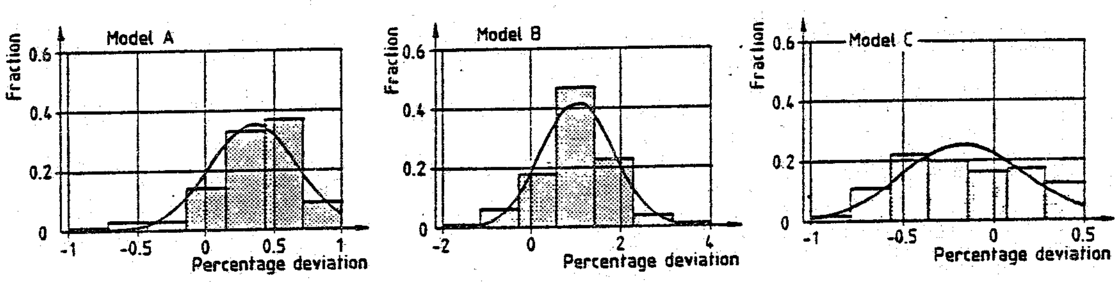


Fig. 2 Distribution of residuals of 100 random test points for the approximations in the areas A, B, and C.

We may conclude that, based on as little as five FEM analyses (and using partial derivatives), regression models of good overall fit could be derived.

References

[1] Fedorov, V.V. Theory of optimal experiments, Academic Press, New York, 1972.

[2] Schoofs, A.J.G. Experimental design and structural optimization, Ph.D. thesis, Eindhoven University of Technology, The Netherlands, 1987.

[3] Welch, W.J. Algorithm for the construction of experimental designs. User's manual ACED, version 1.6.1, Univ. of British Columbia, Vancouver, Canada, 1985. Press, Eindhoven, 1991.

[4] Campen, D.H. van, Nagtegaal, R, and Schoofs, A.J.G. Approximation methods in structural optimization using experimental designs for multiple responses, Chapter 5.5, Multicriteria design optimzation, Procedures and applications,(Eds. Eschenauer, H., Koski, J., and Osyczka, A.), Springer-Verlag, Berlin, Heidelberg, 1990.