# Optimal trajectory control of a linear robotarm by a state space method

# Optimal Trajectory Control of a Linear Robotarm by a State Space Method

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With a linear robotarm - driven by a DC motor, with a maximum velocity of 1 m/s, an acceleration of 5 $m/s^2$, an accuracy of 0.01 mm and for loads up to 50 kg - a trajectory control is performed.
The desired trajectory is firstly carried out by the robot by movement of the endeffector. A force sensor delivers herewith the inputsignals for the necessary motoraction. So the movement of the endeffector of this robotarm represents a certain trajectory - a sequence of positions in time - and subsequently by identification a sequence of motorinputsignals. Then the desired trajectory is again performed eventually with varying parameters, in which the known motor control signals are updated by the new control algorithm.
This control law is based on a description of the robotarm by a state space model (including non linearities) in discrete form. The optimal control algorithm is based on the matrix-ricatti equation by minimization of a performance criterion-function and so the control signals are derived. An investigation is made of the static state feedback, which is implemented in a microprocessor. Attention is paid to the influence of the variation of the so-called weighing factors.
Results of trajectory performances are compared with those obtained by conventional P.I.D. controllers.

KEYWORDS: Actuator, state space, optimal algorithm, matrix ricatti, microprocessor.

## 1. INTRODUCTION

In order to obtain experience in robot design and to test advanced control systems as well a modular robot system has been developed, which will consist of linear and rotary actuators. One of each type has been developed in the mean time. The first module is a linear actuator and this paper deals with the development of an advanced control system for trajectory control of the linear arm.

The trajectory control of robots is rather complicated -inherent to the construction of robots- caused by specific properties, which will be mentioned here.

* In generally robots are more dimensional and have a number degrees of freedom. Links and joints are built together. The final movement of the endeffector is the result of rotations/ translations of the individual joints. In control space the dimension is even higher than the mentioned D.O.F. because position and velocity in the same direction may both form coordinates in the control space.
* Depending upon the application one may deal with high accelerations, velocities and a high position accuracy.
* Friction and the fact that the different degrees of freedom may influence each other, mean that these systems can often only be described by non-linear differential equations.
* Different loads but also inherent system parameter-values may vary much during the performance of a trajectory. Moments of inertia can easily vary by a factor 4 [5]. This makes the right adjustment without adaptation of the controller parameters also difficult.
* The phenomena of elasticity of the links makes positioning without vision difficult. In some cases feedback can be applied to reduce this problem. [9].

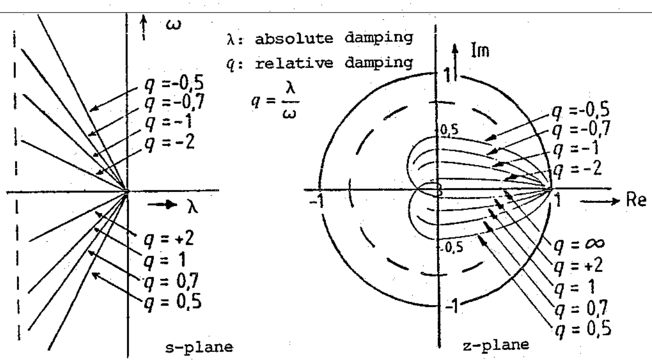



Fig. 1 Absolute and relative damping in s- and z- plane.

With attention to the items mentioned above the robot controllers are developed, and the next categorisation can be made:
- the individual joint P.I.D.-control.
- the computed torque method.
- the robust controller method.

At robust controller design the goal is to construct a controller with a minimal performance sensitivity to model uncertainty, including both parametric uncertainty as well as high frequency unmodeled dynamics.

Methods dealing with frequency domain techniques are in principle only valid for systems described by linear differential equations because they are based on the Laplace transform i.e. s-transform for continuous and z-transform for discrete systems.

Response terms as overshoot, indication and settling time (2%, 5%) are due to the pole-zero configuration of the controlled system in relation to the absolute and relative damping lines in the s-plane and the z-plane as shown in Fig. 1.

For the design of P.I.D.-controllers for single input-single output (SISO) systems the Ziegler-Nichols rules can be used and this is based on the experience of the controller-expert. However for coupled multi input - multi output (MIMO) systems this is difficult and it is even worse whenever parameter variations are involved and adaptive control should be applied.

From the considerations given above, it may be stated that the control of a multi-dimensional robot is difficult. Specially this becomes clearly when a predetermined trajectory is performed. One is faced with problems of:
- coupling and multidimensionality (according to control space).
- non linearity.
- parameter variation and adaptation.

In this paper a method is described, which is in principle able to tackle - to a certain extent - the stated problems. The experiments have been carried out with a linear robot-arm and the applied method is not limited.

The desired trajectory is known or firstly carried out by the robot by movement of the end effector. The positions of the endeffector represent that desired trajectory and by identification a related sequence of motor input signals is obtained. Then the desired trajectory is again performed eventually with varying parameters, in which the already known motor control signals are updated - by state feedback - by the new control algorithm. This control law is based on a description of the robotarm by a state-space model, according to:

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{u}(t) \qquad \underline{y}(t) = C\,\underline{x}(t) \qquad (1)$$

The description is kept in the time domain, so no transformation is carried out. This means that certain non-linearities of the additive type still can be handled here.

The optimal algorithm of the controller is based on the matrix-ricatti equation by minimization of a performance criterion-function and so the control signals are derived.
The performance integral may contain e.g. contributions of the deviations in trajectory positions and velocities but also the control efforts like the motor control signals. Even the boundaries for the control signals may be taken into account. The updating of the parameters in the algorithm for adaptive control depends on the time to solve the matrix-ricatti equation and this is strongly dependent on the number of dimensions. While with the conventional P.I.D.-action the control quality is mostly described in global response sense, gives the state space method a numerical measure for performance quality and the control effort.

## 2. DESCRIPTION OF THE SYSTEM

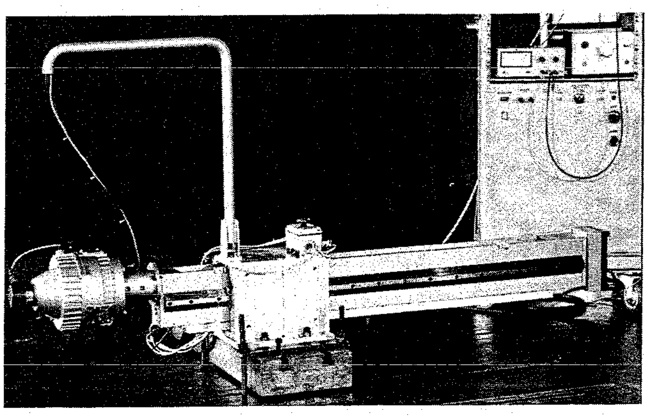


Fig. 2 Photograph of the linear robotarm.

## 2.1. The construction of the linear robotarm.

The linear actuator is the first part of a modular robot system, and the major specifications are shown in Table 1.

| | |
|---|---|
| maximum velocity | 1 m/s |
| maximum acceleration | 5 $m/s^2$ |
| maximum load | 50 kg |
| stroke | 1 m |
| position accuracy | 0.01 mm |
| position measuring system | Heidenhain LS 513 |
| power source | DC-motor: Axem MC 19 PR 26 |
| control system | µC PID - or state controller |

Table 1. Design specifications of the linear robotarm.

The mechanical construction is fairly stiff due to the hollow frame construction, while at the same time the masses are kept minimal.
The arm has been constructed with side guide-ways, which enables the preloading of the spindle and the application of roller bearings for the main bearing system.

The rotation of the motor into a translation of the actuator is converted by a spindle with a ballscrew nut. An advantage of this combination is that the back-lash can be eliminated by preloading the nut.
A disadvantage is that the rotary speed is limited because of the critical speed of the spindle.

The applied ballscrew nut transmission SKF 25 x 25 R has a pitch of 25 mm, which is rather large. Although vibration problems may arise - due to the slenderness of the spindle - it can be concluded from calculations and measurements that the lowest critical speed is 54 rad/s, while the maximum speed is 40 rad/s.

The DC motor is of the disc-armature type with the following characteristics:
- a very small mechanical time constant. With a load of 50 kg this becomes 26 ms.
- a continuously adjustable speed
- the largest torque at low speed.

The disadvantage however is the poor resistance against overheating. This is important because the criterium for design was very determined by the 100% duty cycle. Coupled to the motorshaft is also a tachogenerator and a rotational encoder.

For direct position measurement along the arm an optical linear digital incremental encoder has been mounted, type Heidenhain LS513 with a length of 1020 mm and an accuracy of 0.01 mm. The necessary frequency range of the encoder is determined by the speed of the arm and the accuracy of the lineal.

The free end of the linear robotarm is extended with a one dimensional force sensor, based on the bending principle and measured by strain gauges. The force sensor is used in the TEACH mode.

## 2.2. The control system hardware

An advanced control system is needed, because of the desired high performance of the linear robotarm. (Fig. 3)

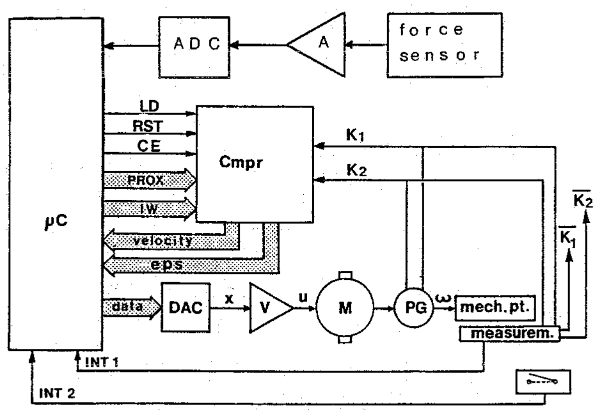



Fig.3 The control system hardware

This gives the opportunity to test several control algorithms and also to apply adaptive control i.e. to change the controller parameters during operation when the characteristics of the system are changing. As an option, the linear or rotational encoder can be chosen for direct or indirect position measurement. For the implementation of the controller is used a singleboard computer of Intel iSBC 86/05 with a clockfrequency of 8 MHz, with a programmable interuptcontroller and - timer and some parallel input and output gates. The signals of the force sensor are read by a 8-bits DAC. The servo-amplifier is fed by a 12-bits or a 8-bits DAC. The last one is sufficient to avoid noise sensitivities. For comparing the actual and the desired position c/q velocity a comparator of 16 bits is applied. With an accuracy of 0,01 mm on a total length of 655 mm the 16-bits comparator is sufficient. The monitor communication and the loading of the control program from the Intel development system is done via a serial gate. The control program can also be loaded as a ROM into the control computer.

## 2.3. Dynamic analysis of the linear actuator.

An analysis of the dynamic behaviour is necessary to optimize the control of the linear actuator. The P.I.D.-control system for this one-dimensional case is given in Fig. 4.

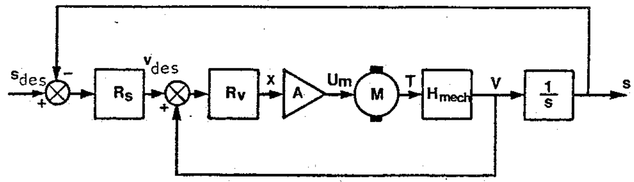



Fig. 4 Scheme of the control system

The signal flow is from the desired position $s_{des}$ through all the control elements (the position-controller $R_s$, the velocity-controller $R_v$, the amplifier A, the motor M and the mechanical system $H_{mech}$) to the actual position s of the arm.
The elements - except the mechanical system $H_{mech}$ - have known characteristics or can be established ($R_s$, $R_v$) as a control algorithm. In the case of a controller based on the state space method it is very necessary to start with a valid model, because this controller is strongly dependent on the procesmodel as shown in Ch 3.

For the estimation of the lowest eigenfrequency a single- D.O.F. system is considered. A moment of inertia $J_i$ is transferred to a mass $m_i = J_i\ (2\pi/h_{sp})^2$. The masses can be transferred and summarized to the end of the chain of masses and the springs, applied in series, can be reduced to one spring constant by (2).

$$^1/_k = \sum_{i=1}^{n} {^1/_{k_i}} \qquad m_{T,j} = m_j \left(\frac{\sum_{i=1}^{j} {^1/_{k_i}}}{^1/_k}\right)^2 \tag{2}$$

With the respectively data the lowest eigenfrequency fo becomes:

$$m = 125\ kg \qquad k = 1,3\ 10^7\ N/m \qquad fo = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

| Parameter | Value |
|---|---|
| $J_1$ Moment of inertia of the motor and half of the loaded spindle | $J_1=12,4.10^{-4}+1/2(\frac{1402-1}{1402})^2 . .2,51.10^{-4} kgm^2$ |
| $J_2$ Moment of inertia of the remaining part of the spindle | $J_2=1/2(\frac{1402+1}{1402}).2,51.10^{-4} kgm^2$ |
| $m_1$ Mass of a part of the spindle | $m_1=1/2(\frac{1402+1}{1402}).4,16$ kg |
| $m_2$ Mass of the motor, the arm frame, the remaining part of the spindle and the load ($m_L$) | $m_2=77 +1/2(\frac{1402-1}{1402}).4,16+m_L$.kg |
| $k_1$ Torsional stiffness (spindle) | $k_1=9,85.10^2+0,688.1$ Nm/rad |
| $k_2$ Stiffness of the housing | $k_2=1,25.10^8$ N/m |
| $k_3$ Axial stiffness of the spindle | $k_3=1,8.10^7+5,2.1.10^4$ N/m |
| $d_1$ Damping factor of the motor | $d_1=4,3.10^{-3}$ Nm.s/rad |
| $d_2$ Damping factor of the nut | $d_2=1,6.10^{-2}$ Nm.s/rad |
| $d_3$ Damping of the main bearing | $d_3=120$ N.s/m |
| $d_4$ Torsional material damping (spindle) | $d_4=8.10^{-5}$ Nm.s/rad |
| $d_5$ Axial material damping (housing) | $d_5=0,05$ N.s/m |
| $d_6$ Axial material damping (spindle) | $d_6=0,05$ N.s/m |

Table 2. Model parameters.

For the maximum acceleration $a_{max} = 5\ m/s^2$ the estimated maximum amplitude of the vibration is $x_{max} = \frac{k}{m}\ a_{max} = 5.10^{-5}$ m.
A dynamic analysis is needed when a maximum frequency above 51 Hz and an accuracy better than 0.05 mm is required.
For a higher order analysis the lumped mass model of Fig. 5 is used [3].

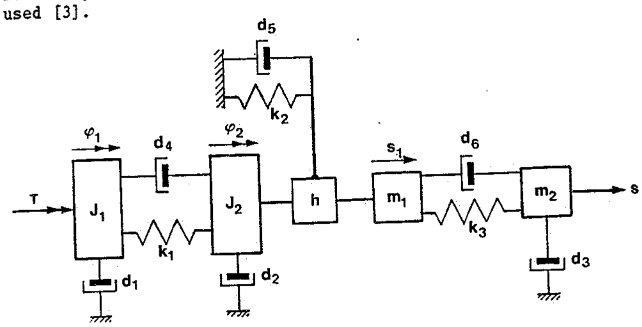



Fig. 5 Dynamic lumped mass model.

It is useful to have insight in the dynamic behaviour of the actuator, although the developed first version of the state space controller - because of matrix dimensions and calculation time - is not based on this sophisticated model.
The input signal is the motortorque (T) acting on its disc-armature ($J_1$) and the motorshaft connected to the spindle by a spring ($k_1$). The moment of inertia of the spindle is distributed over its length. Therefore the moment of inertia of the spindle is added partly to that one of the motor ($J_1$) and the remaining part is represented by ($J_2$). The ballscrew-nut, described by (3) is fixed to the frame by the spring ($k_2$)

$$s_1 = \phi_2\ h_{sp}/2\pi \tag{3}$$

The mass of the spindle ($m_1$) is linked to the end of the arm by the axial stiffness of the spindle ($k_3$). At the end of the arm a mass ($m_2$) is located, representing the mass of the armframe, the motor and the external load.

If the degrees of freedom are represented by the vector $\underline{q}$, the external moments by $\underline{Q}$ and with M is mass-matrix, D is damping-matrix, K is stiffness-matrix in which:

$$\underline{q}^T = (\phi 1,\ \phi 2,\ s) \quad \text{and} \quad \underline{Q}^T = (T,\ 0,\ 0) \tag{4}$$

then the robotarm is described by the differential equation in matrix notation:

$$M\ \ddot{\underline{q}} \ +\ D\ \dot{\underline{q}}\ +\ K\ \underline{q}\ =\ \underline{Q} \tag{5}$$

The dynamic model of the linear robotarm has been measured with a Fourier analyzer (HP 5423) and calculated with a special programme with the values for the parameters of Table 2. The eigenfrequencies vary with the varying arm position. At each arm position 3 modes are found, given in Table 3, within the rows respectively left and right the calculated and measured values.

| Arm position l (mm) | mode f1 (Hz) calc. | meas | mode f2 (Hz) calc. | meas. | mode f3 (Hz) calc. | meas |
|---|---|---|---|---|---|---|
| 180 | 82 | - | 123 | 103 | 708 | 384 |
| 355 | 90 | 73 | 132 | 124 | 703 | 389 |
| 510 | 95 | 59 | 141 | 119 | 694 | 375 |
| 755 | 101 | 92 | 153 | 145 | 685 | 389 |
| 1005 | 105 | 113 | 166 | 163 | 679 | 406 |

Table 3. Calculated and measured eigenfrequencies.

## 3. OPTIMAL TRAJECTORY CONTROL.

### 3.1. The state space description method.

Systems may be represented in the state variable form by a number of differential equations and described as a vector-matrix differential equation:

$$\dot{\underline{x}}\ (t) = f\ [\ \underline{x}\ (t),\ \underline{u}\ (t),\ t\ ] \tag{6}$$

where $\underline{x}$ (t) = n-dimensional state vector
$\underline{u}$ (t) = r-dimensional control vector.

These equations are in general non-linear with time varying parameters. In the linear case general expressions for the system are:

$$\dot{\underline{x}}\ (t) = A\ (t)\ \underline{x}\ (t) + B\ (t)\ \underline{u}\ (t) \tag{7}$$
$$\underline{y}\ (t) = C\ (t)\ \underline{x}\ (t). \qquad \underline{x}\ (t_o) = \underline{x}_o$$

in which: $\underline{x}$ (t) = state vector    A (t) = system matrix
$\underline{u}$ (t) = control vector    B (t) = control matrix
$\underline{y}$ (t) = output vector    C (t) = output matrix.

For time-invariant systems the matrices A, B en C are constant. The dynamic behaviour (stability, transientphenomena) is determined by the eigenvalues of matrix A. They are the roots of the characteristic equation and the location of these poles in the left half of the s-plane (Fig 1) determine the response time and the possible overshoot.

For improving the dynamic-$^c/q$ transient behaviour the location of these poles may be changed by applying state feedback as a linear control law:

$$\underline{u}\ (t) = -\ L\ \underline{x}\ (t)\ +\ \underline{r}\ (t) \tag{8}$$

The new input vector $\underline{r}$ (t) is the new reference signal, which should be followed (servo-problem) or kept constant (regulator-problem) by the system. This makes however essentially no difference. For the feedback system of Fig.6 the next equation can be derived:

$$\dot{\underline{x}}\ (t) = [A\ -\ B\ L]\ \underline{x}\ (t)\ +\ B\ \underline{r}\ (t). \tag{9}$$

The dynamic behaviour of the modified system is now determined by the eigenvalues of matrix [A - BL].

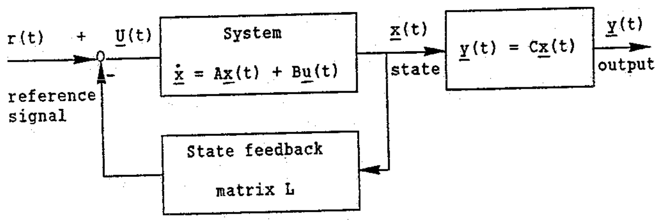



Fig. 6 System with linear state feedback.

State feedback makes it possible that a fully controllable system performs any desired dynamics. In this sense an original slow system can be made faster by locating the new poles deep in the left half of the s-plane. Big eigenvalues however mean a matrix L with big components and subsequently big values for the inputsignals $\underline{u}$ (t).

The choice of the state feedback makes a balance between the result-output $\underline{x}$ (t) and the effort-input $\underline{u}$ (t). The weighing is one object of the optimisation philosophy (Ch 3.2).

### 3.2. The optimal linear control law.

In this section an optimal control strategy will be discussed, in which the balance between control effort and outputresult is an optimum. Assuming the system of (7) with initial condition:

$$\dot{\underline{x}}\ (t) = A\ (t)\ \underline{x}\ (t) + B\ (t)\ \underline{u}\ (t)\ ; \qquad \underline{x}\ (t_o) = \underline{x}_o$$

and controlled by the linear state-feedback

$$\underline{u}\ (t) = -\ L\ (t)\ \underline{x}\ (t) \tag{10}$$

Then the closed sysem is described by:

$$\dot{\underline{x}}\ (t) = [A\ (t)\ -\ B\ (t)\ L\ (t)\ ]\ \underline{x}\ (t)\ ;\ \underline{x}(t_0) = \underline{x}(0) \tag{11}$$

The feedbackmatrix L (t) must be chosen such that a defined performance criterion J (t) over a certain time interval $[t_o,\ t_e]$, with $t \in [t_o,\ t_e]$ becomes a minimum.

$$J(t)=\int_t^{t_e} [\underline{x}^T(\tau)Q(\tau)\underline{x}(\tau)\ +\ \underline{u}^T(\tau)R(\tau)\underline{u}(\tau)]d\tau\ +\ \underline{x}^T(t_e)P(t_e)\ x(t_e) \tag{12}$$

The terms in this performance criterion function J represent respectively the contributions of successively the output, the control input and the output at final time with their corresponding weighing matrices Q, R and $P_e$, which have to be symmetrical and positive definite. Within the class of linear control laws L(t), an optimal control law $L^o(t)$ has to be found which forces J to be a minimum. From $L^o(t)$ the optimal control strategy $\underline{u}^o(t)$ then can be derived:

$$\underline{u}^o(t) = -\ L^o(t)\ \underline{x}(t). \tag{13}$$

From (12) it is found that J(t) can be considered as additional "performance cost" over the interval $[t,\ t_e]$, which obeys the following equation:

$$J(t) = \underline{x}^T(t)\ P(t)\ \underline{x}(t) \tag{14}$$

with the next begin- and end values:

$$J(t_o) = \underline{x}^T(t_o)\ P(t_o)\ \underline{x}(t_o) = J \tag{15}$$
$$J(t_e) = \underline{x}^T(t_e)\ P(t_e)\ \underline{x}(t_e).$$

So matrix P(t) plays an important role in the derivation of the optimal control law and obeys a matrix differential equation with the systemmatrices A(t) en B(t) and the weighing matrices Q(t) and R(t) which is called a matrix ricatti equation.

The optimal $P^o(t)$ is found by solving the next matrix-ricatti equation:

$$-\dot{P}^o(t)=A^T(t)P^o(t)+P^o(t)A(t)+Q(t)-P^o(t)B(t)R^{-1}(t)B^T(t)P^o(t) \qquad (16)$$

$$P^o(t_e)=P_e$$

which gives the solution for the optimal control law:

$$L^o(t) = R^{-1}(t)B^T(t)P^o(t) \qquad (17)$$

$$\text{with } J_{min} = \underline{x}^T(t_o)P^o(t_o)\underline{x}(t_o)$$

Equation (16) is solved by defining a Hamilton-matrix F as:

$$F = \begin{bmatrix} A^T & Q \\ BR^{-1}B^T & -A \end{bmatrix} \qquad (18)$$

The eigenvalues of F are divided in those of the right and those of the left half plane of the s-plane and the last are identical to the eigenvalues of the closed system i.e. of: $| A - BL |$.

For time invariant systems the matrices A, B, Q, R are constant and for the stationary control situation also $P(t)$ and $L(t)$ become constant respectively $\bar{P}^o$ and $\bar{L}^o$. This yields:

$$\bar{L}^o = R^{-1}B^T\bar{P}^o \qquad (19)$$

The calculation of $P^o(t)$ from the matrix-ricatti equation (16) is very suitable to be done by a digital computer - by special programs like PC.Matlab, Matrix.X - because many standard operations like inversion and multiplication of matrices, determination of eigenvalues and eigenvectors are involved.

As said above for the time-invariant case $L^o$ is a constant and the calculation can be done off-line and only once like a fixed P.I.D. controller. However if the system has time (or trajectory) dependent parameters then $A(t)$, $B(t)$ have to be known or identified and $L^o(t)$ has to be calculated many times during the trajectory. This might be time critical.

## 4. IMPLEMENTATION.

### 4.1. Description of the model.

The dynamic analysis of the linear robotarm of Ch.2 with a number of eigenfrequencies is necessary to get insight in the dynamic behaviour and for estimation of the parametervalues but it is too extensive for control purposes. With this first attempt of developing a controller with the state space method the model is described by:

$$[\frac{J}{h}+(M_o+M_L)h]\ddot{s} +[\frac{K_e^2}{R_ah} +b]\dot{s}+[T_{wc}+\frac{K_e(U_b-U_{off})}{R_a}+Fh] = \frac{K_eK_v}{R_a}U_i \qquad (20)$$

$s$ = position of the endeffector
$J/h$ = equivalent mass of the translating moment of inertia
$M_o$ = translating mass of the actuator
$M_L$ = translating mass of an external load
$F$ = external force to the endeffector
$h$ = pitch of the spindle
$K_e$ = motorconstant
$R_a$ = motor-resistance
$b$ = viscous friction coëfficient
$T_{wc}$ = torque due to the coulombfriction
$U_b$ = voltage loss over the motorbrushes
$U_{off}$ = offset of the amplifier
$K_v$ = gain of the amplifier
$U_i$ = imputvoltage to the amplifier

$T_{wc}$ is dependent of the sign of $\dot{s}$, and $U_b$, $U_{off}$ of the sign of $U_i$. Substitution of the relevant data delivers the next differential equation:

$$(0.395+1.44\ 10^{-3}M_L)\ \ddot{s} + 11.85\ \dot{s}+(c+1.44\ 10^{-3}F) = U_i \qquad (21)$$

$$\ddot{s} + \alpha \dot{s} + \beta = \gamma U_i \qquad (22)$$

For the additive nonlinearity c - dependent on the sign of $\dot{s}$ and $U_i$ according to the given matrix - a simple correction is made in the control software and therefore the factor c and subsequently the value of $\beta$ is not essential.

| c | $\dot{s}>o$ | $\dot{s}<o$ |
|---|---|---|
| $U_i>o$ | 0.9 | 0.58 |
| $U_i<o$ | 0.24 | -0.08 |

Defining a state vector: $\underline{x}(t)= \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} \int_o^t s(\tau)\,d\tau \\ s(t) \\ \dot{s}(t) \end{bmatrix}$

a control vector $\underline{u}(t) = U_i$ and a outputvector $\underline{y}(t) = s(t)$ and according to $\dot{\underline{x}}(t) = A\ \underline{x}(t) + B\ \underline{u}(t)$ and $\underline{y}(t) = C\ \underline{x}(t)$

this gives the next equations:

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\alpha \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix} \underline{u}(t)\ ;\ \underline{y}(t) = [0\ 1\ 0]\ \underline{x}(t) \qquad (23)$$

### 4.2 The teach and replay mode.

The desired trajectory is firstly carried out by the robot by movement of the endeffector (Fig. 7). A force sensor delivers herewith the inputsignals for the necessary motoraction (proportional to the desired velocity). This is called the TEACH-mode and a nominal trajectory $s_n(t)$ is registred. So the movement of the endeffector of this robotarm represents a certain trajectory - a sequence of positions in time - related to a series of nominal motorinputsignals $U_n(t)$. For the identification the signals $s_n(t)$, $\dot{s}_n(t)$ and $\ddot{s}_n(t)$ are discretized (with sampletime $t_s$=1,2 ms) in the usual way.

From these data and the original assumed model an optimal control law is derived. This may be done off-line for the stationary case, but if adaptive control is required, this must be done repetitive one-line during the trajectory performance.

In the REPLAY-mode the desired trajectory is again carried out, eventually with varying parameters, in which the nominal motorinputsignals are updated via corrections by the new control algorithm.

Deviations of the nominal values of state and input along the trajectory are called perturbations:

$$\tilde{\underline{x}}(t) = \underline{x}(t) - \underline{x}_n(t) \quad ; \quad \tilde{\underline{u}}(t) = \underline{u}(t) - \underline{u}_n(t) \qquad (24)$$

Then the system can be described by the linearized model:

$$\dot{\tilde{\underline{x}}}(t) = A\ \tilde{\underline{x}}(t) + B\ \tilde{\underline{u}}(t) \quad ; \quad \tilde{\underline{y}}(t) = C\ \tilde{\underline{x}}(t) \qquad (25)$$

in which A, B, C are the Jacobians given in (23).

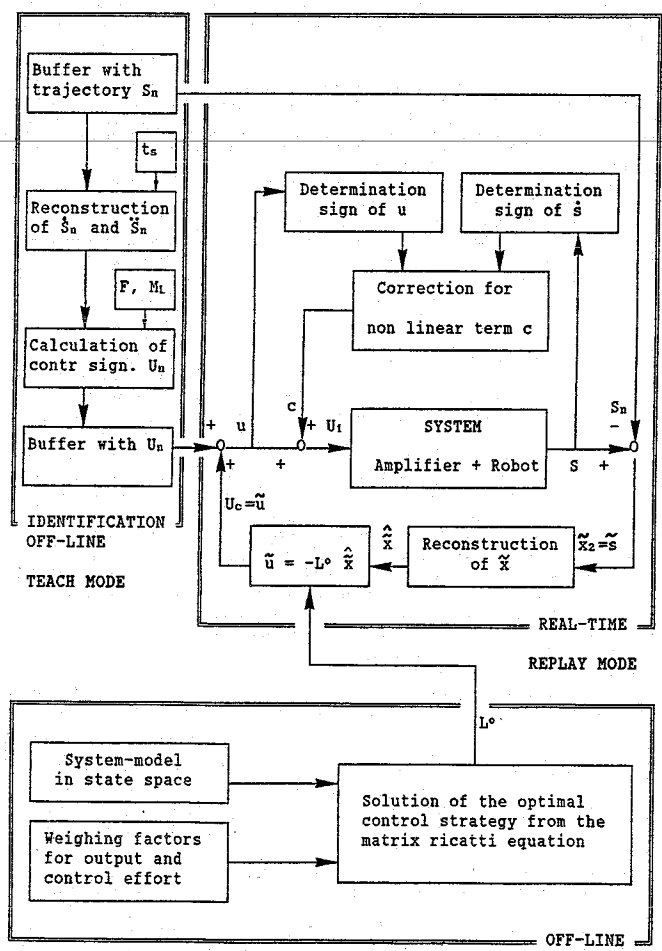



Fig. 7 Teach and Replay of the controlled system.

### 4.3 The applied optimal linear control law.

The controllability of the system, with matrices A, B is firstly investigated with the next controllability matrix:

$$[B \;\; AB \;\; A^2B] = \begin{bmatrix} 0 & 0 & j \\ 0 & \gamma & -\alpha\gamma \\ \gamma & -\alpha\gamma & \alpha^2\gamma \end{bmatrix} \qquad (26)$$

Rank = 3, which implies full controllability. According to the statements made in Ch 3.2 about the performance criterionfunction J and the weighing matrices Q en R (with normalisation) and without weighing the endvalue $x(t_e)$ the expression for J becomes:

$$J = \int_{t_0}^{t_e} \{ q_1 \tilde{x}_1^2(t) + q_2 \tilde{x}_2^2(t) + q_3 \tilde{x}_3^2(t) + u^2(t) \} \, dt \qquad (27)$$

If only position - and velocity feedback is applied ( the $\int s \, dt$ is not fed back) then the state vector $\underline{x}(t)$ and the matrices A, B, Q and R are reduced to:

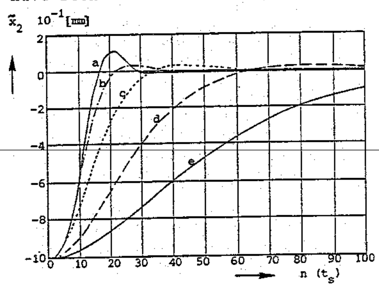
$$\underline{x}(t) = \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} ; \; A = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix} ; \; B = \begin{bmatrix} 0 \\ \gamma \end{bmatrix} ; \; Q = \begin{bmatrix} q_2 & 0 \\ 0 & q_3 \end{bmatrix} ; \; R = 1 \qquad (28)$$

According ch 3.2 and the eqs. (16) to (19) the optimal stationary control matrix $\bar{L}^o$ is calculated by solving $\bar{P}^o$ from the stationary matrix-ricatti equation. With the data from (28) the Hamilton matrix F (18) can be constructed. The negative eigenvalues of F equal the eigenvalues of [A-BL]. For the simple case of (28) these calculations can be done by hand and the result is:

$$\bar{L}^o = [\, l_1 \;\; l_2 \;\; l_3 ] = [\, o \;\; \sqrt{q_2} \;\; (-\alpha + \sqrt{\alpha^2 + \gamma^2 q_3 + 2\gamma\sqrt{q_2}})/\gamma ] \qquad (29)$$

Realizing that the factors $l_1$, $l_2$ and $l_3$ are related respectively to $\int s \, dt$, s and $\dot{s}$, so integral - , proportional - and differential feedback, these optimal "P.I.D.-values" in $\bar{L}^o$ are determined by the weighing factors of Q and R ($q_1$, $q_2$, $q_3$) and the components of matrices A en B ($\alpha$ and $\gamma$).

During the experiments trajectory control over different distances, also the whole range of 655 mm, - and some registered on a videotape [2] - have been performed with remarkable good results. Here some comparable responses are given in Fig. 8 about the dynamic behaviour if the start position deviation $x_2(o)$ = -1 mm with a sampletime $t_s$ = 1.2 ms. The calculations have been carried out with the software package PC-Matlab.

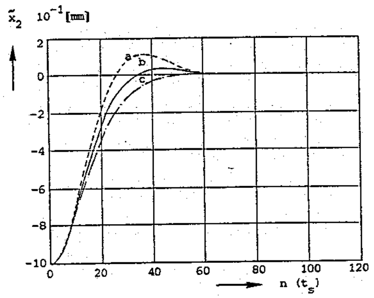



$\tilde{x}_2(t_0)$= -1[mm], $t_s$=1.2[ms]

$q_1$=$q_3$=0 , $q_2$=variabel

| | $q_2$ | L | | |
|---|---|---|---|---|
| a) | $10^5$ | [0 | 312 | 8] |
| b) | $10^6$ | [0 | 972 | 18] |
| c) | $10^7$ | [0 | 2978 | 38] |
| d) | $10^8$ | [0 | 8885 | 73] |
| e) | $10^9$ | [0 | 25255 | 131] |

Fig. 8A Optimal responses for variation of $q_2$



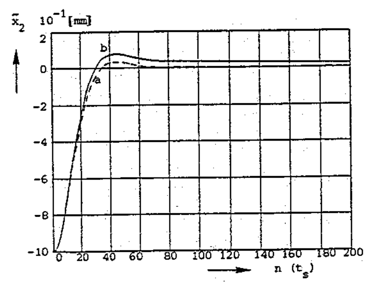

$\tilde{x}_2(t_0)$= -1[mm], $t_s$=1.2[ms]

$q_1$=$q_3$=0 , $q_2$ = $10^7$

$l_1$, $l_2$ constant, $l_3$=variabel

| | L | | |
|---|---|---|---|
| a) | [0 | 2978 | 28] |
| b) | [0 | 2978 | 38] |
| c) | [0 | 2978 | 48] |

Fig.8B Responses for variation of $l_3$(response b is optimal)

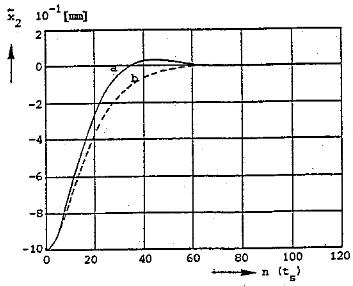



$\tilde{x}_2(t_0)$= -1[mm], $t_s$=1.2[ms]

$q_2$=$10^7$, $q_3$=0, $q_1$=variabel

| | $q_1$ | L | | |
|---|---|---|---|---|
| a) | 0 | [0 | 2978 | 38] |
| b) | $10^8$ | [9346 | 3130 | 40] |

Fig. 8C Optimal responses for variation of $q_1$




$\tilde{x}_2(t_0)$= -1[mm], $t_s$=1.2[ms]

$q_2$=$10^7$, $q_1$=0, $q_3$=variabel

| | $q_3$ | L | | |
|---|---|---|---|---|
| a) | 0 | [0 | 2978 | 38] |
| b) | 2000 | [0 | 3130 | 40] |

Fig.8D Optimal responses for variation of $q_3$

Fig. 8 Responses with the (optimal) state space controller.

In Fig 8A optimal responses with a variation of $q_2$(weighing factor of position deviation) with $q_1$ = $q_3$ = o are shown. From this the optimal value of $q_2$ can be studied.
In Fig. 8B the different responses on variations of $l_3$ - around the optimal value (response b) - are shown.
In Fig. 8C two optimal responses with the variation of $q_1$ (weighing factor of the integral position deviation) with $q_3$ = o are shown. This illustrates the effect of the $q_1$ value.
In Fig. 8D two optimal responses with a variation of $q_3$ (weighing factor of the velocity deviation) with $q_1$ = o are shown. This illustrates the effect of the $q_3$ value.

### CONCLUSIONS.

The trajectory control of robots is rather difficult inherent to the specific properties - more dimensionality, non linearity, coupling of dimensions, variations of parameters, elasticity - of robots.
A controller is developed to handle some of these items. With a state space controller no transformation to the freqency-domain has to be made, so e.g. additive non-linearities can be treated.
An optimal control algorithm with minimizing a performance criterion function based on the matrix-ricatti equation is specially suitable for digital computers, because of the ability to perform matrix operations. In this sense also the components of more dimensions can be handled. The optimal control strategy for the static case - with constant paramters for the traject - can easily be calculated.
The dynamic case - with updating of the control strategy on line - needs still much faster computers because the matrix-ricatti equation has to be solved frequently during the trajectory performance.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