# Composite computed torque control of robots with elastic motor transmission

# COMPOSITE COMPUTED TORQUE CONTROL OF ROBOTS WITH ELASTIC MOTOR TRANSMISSIONS

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Abstract. The main problem in *the control of robots with elastic transmissions between the actuators and the rigid links* is caused by the number of control inputs being less than the number of degrees of freedom. This problem has been faced by a composite control law consisting of the conventional 'rigid' computed torque controller for link-based trajectory tracking and a 'flexible' computed torque part multiplicated with the inverse of the stiffness matrix for stabilization of the elastic deflections. The resultant control system resembles the socalled 'two-time scale sliding control' technique of Slotine and Hong (1987), but in our approach the stiffnesses of the elastic motor transmissions do not have to be relatively large neither is there the restriction that there have to be as many motor inputs as elastic transmissions. The goal of the composite controller is that the individual link trajectories will follow the desired trajectories while the elastic-transmission forces/torques, which are not directly constrained by the output specifications, remain on a certain 'manifold' due to *the natural flexibility behavior of the system.* The key concept is illustrated with simulation results of a translation-rotation robot with one torsional-elastic motor transmission.

Keywords. Control theory; industrial robots; flexible robots; computed torque control; Lyapunov methods; nonlinear systems; control system design.

## INTRODUCTION

### Flexible Manipulator Control

Today, industrial robots are used for various purposes. Because of hardware limitations in on-line applications, until now, robot control has been studied extensively under the assumption that the actuator transmissions are stiff and that the links can be modelled as rigid bodies. Therefore, most of today's robots have a very stiff (and thus heavy) construction in order to avoid deformations and vibrations. For *higher operating speeds*, industrial robots should be *lightweight* constructions to reduce the driving force/torque requirements and to enable the robot arm to respond faster. However, a lightweight manipulator may have flexibility in the link structure and elasticity in the transmissions between actuators and links. For most manipulators, elasticity of the motor transmissions has a greater significance for the design of the controller than the deformation of the flexible links. Furthermore, link flexibility can be approximately modeled by a chain of rigid sublinks interconnected by elastic joints. Hence, more accurate models involving *elastic transmissions* should be taken into account to pursue better dynamic performance of industrial robots. The application of more complex control algorithms is possible now due to the availability of advanced multiprocessor equipment for real-time manipulator control.

### Computed Torque Control

A well known approach to improve the behavior of manipulators is the *computed torque control* method, sometimes called inverse dynamics control or static nonlinear state feedback control. Here, the control law is designed explicitely on the basis of a model, in order to compensate for robot nonlinearities and to guarantee a desired closed-loop behavior. In its original version, this control method appears to be applicable only to rigid manipulators. If flexibilities play an important role, it often results in an *instable* system behavior. Therefore, the control system must deal with control of the elastic vibrations as well as trajectory tracking. However, it is not possible to find a control input for a flexible manipulator which will accomplish perfect tracking of any desired trajectory in space while totally damping the undesired elastic deflections. It is more realistic to search for a control law achieving both a *reasonable trajectory tracking and a certain stabilization of acceptable vibrations.* A composite control technique shall be developed to extend the familiar computed torque control scheme for rigid manipulators to a control scheme for manipulators with elastic motor transmissions.

## A MANIPULATOR WITH ELASTIC MOTOR TRANSMISSIONS

### Introduction

In this paper, we consider manipulators that can be modeled as an open chain of n rigid links interconnected by cilindrical, revolute or prismatic joints with one degree of freedom per joint. One end of the chain is fixed to the ground and the other end (the gripper) has to follow a specified trajectory in space. It is assumed that from this specification the desired trajectory of each link can be determined.

Since each joint allows one relative motion of the connected link, n generalized coordinates are necessary and sufficient to describe the kinematics of the *links.* These coordinates are the components of a vector $\underline{q}_l \in \mathbb{R}^n$. The desired path of $\underline{q}_l$ in time is denoted by $\underline{q}_{ld} = \underline{q}_{ld}(t)$.

Each joint has its own actuator and its own transmission between the actuator and the driven link. The motor torques (used in a generalized sense, i.e. denoting both torques and forces) acting on the transmissions are the robot control inputs. In this paper, we consider the case in which *some or all transmissions are elastically deformable.* Then, for each elastic deformation it is necessary to introduce an extra coordinate to describe the rotation of the rotor of the motor. These extra coordinates are the components of a vector $\underline{q}_m \in \mathrm{R}^e$ with $e \leq n$.
For the sequel it is advantageous to regroup the coordinates $\underline{q}_l$ of the links in two vectors $\underline{q}_s \in \mathrm{R}^{n-e}$ and $\underline{q}_e \in \mathrm{R}^e$, where $\underline{q}_s$ and $\underline{q}_e$ contain the coordinates of the *direct driven links* (i.e. driven by the stiff transmissions), respectively the coordinates of the *elastically driven links* (i.e. driven by the elastic transmissions). See Fig. 1.

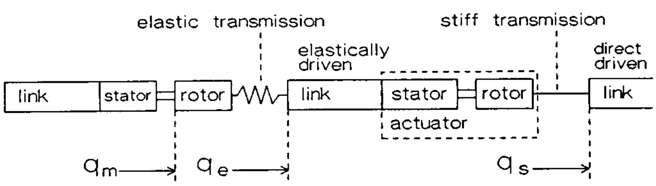



Fig. 1: Elastically driven links – direct driven links.

This completes the introduction of the total vector of generalized coordinates $\underline{q} \in \mathrm{R}^{n+e}$:

$$\underline{q} = \begin{bmatrix} \underline{q}_s \\ \underline{q}_e \\ \underline{q}_m \end{bmatrix} . \tag{1}$$

The components of the vector $\underline{\epsilon}$ defined by

$$\underline{\epsilon} = \underline{q}_m - \underline{q}_e , \tag{2}$$

characterize *the deformations of the elastic motor transmissions.* Hence, if these transmissions are modeled as massless linear springs, *the elastic-transmission torques* being the components of a vector $\underline{z}_e \in \mathrm{R}^e$ are related to $\underline{\epsilon}$ by

$$\underline{z}_e = K\underline{\epsilon} , \tag{3}$$

where $K \in \mathrm{R}^{e\times e}$ is the positive definite diagonal stiffness matrix.

## The Partitioned Dynamic Manipulator Model

Using a Lagrangian approach, the dynamic model of the manipulator can be written in the standard form

$$M(\underline{q})\ddot{\underline{q}} + \underline{n}(\underline{q}, \dot{\underline{q}}, t) = H\underline{u} , \tag{4}$$

where $\underline{q} \in \mathrm{R}^{n+e}$ is the vector of generalized coordinates,
$\underline{u} \in \mathrm{R}^n$ is the vector of actuator input torques,
$\underline{n} \in \mathrm{R}^{n+e}$ is the vector containing all other torques, for instance due to Coriolis and centrifugal accelerations, friction, gravity, etc.,
$M \in \mathrm{R}^{(n+e)\times(n+e)}$ is the symmetrical, positive definite inertia matrix,
$H \in \mathrm{R}^{(n+e)\times n}$ is the distribution matrix.

The actuator torques, i.e. the components of $\underline{u}$, are also regrouped in two subvectors $\underline{u}_s \in \mathrm{R}^{n-e}$ and $\underline{u}_e \in \mathrm{R}^e$ where $\underline{u}_s$ and $\underline{u}_e$ contain the torques of the actuators with a stiff, respectively an elastic transmission. If M and $\underline{n}$ are partitioned in accordance to the partitioning of $\underline{q}$, the dynamic model can be given in partitioned form by:

$$M_{ss}\ddot{\underline{q}}_s + M_{se}\ddot{\underline{q}}_e + \underline{n}_s = \underline{u}_s , \tag{5}$$
$$M_{es}\ddot{\underline{q}}_s + M_{ee}\ddot{\underline{q}}_e + \underline{n}_e = \underline{z}_e , \tag{6}$$
$$M_{mm}\ddot{\underline{q}}_m + \underline{n}_m = \underline{u}_e - \underline{z}_e . \tag{7}$$

The equations of motion of the direct driven links and the motor rotors connected to the stiff transmissions are formulated in eq. (5), while eq. (6) represents the motions of the elastically driven links manipulated by the elastic-transmission torques $\underline{z}_e$ The dynamics of the motor rotors connected to the same elastic transmissions are given in eq. (7).

With eq. (3) it is possible to eliminate $\underline{q}_m$. Rearranging the equations of motion then yields a set in $\underline{q}_l$ and $\underline{z}_e$, which is more *suitable for control design*:

$$M_l\ddot{\underline{q}}_l \;+\; M_zF\ddot{\underline{z}}_e + \underline{n}_l = \underline{u} , \tag{8}$$
$$M_e\ddot{\underline{q}}_l \;+\; \underline{n}_e = \underline{z}_e , \tag{9}$$

$$\text{where } \underline{q}_l = \begin{bmatrix} \underline{q}_s \\ \underline{q}_e \end{bmatrix} , \quad \underline{u} = \begin{bmatrix} \underline{u}_s \\ \underline{u}_e \end{bmatrix} , \quad \underline{n}_l = \begin{bmatrix} \underline{n}_s \\ \underline{n}_e + \underline{n}_m \end{bmatrix} ,$$
$$M_l = \begin{bmatrix} M_{ss} & M_{se} \\ M_{es} & M_{ee} + M_{mm} \end{bmatrix} ,$$
$$M_z = \begin{bmatrix} 0 \\ M_{mm} \end{bmatrix} , \quad M_e = [\ M_{es} \quad M_{ee}\ ] , \tag{10}$$

$F \in \mathrm{R}^{e\times e}$ is the positive definite diagonal *flexibility matrix*:

$$F = K^{-1} \tag{11}$$

If all actuator transmissions are very stiff (i.e., the largest flexibility element in $F$ is very small), eq. (8) simplifies to the equations of motion of *the equivalent rigid-transmission manipulator*:

$$M_l(\underline{q}_l)\ddot{\underline{q}}_l + \underline{n}_l(\underline{q}_l, \dot{\underline{q}}_l, t) = \underline{u} , \tag{12}$$

# COMPOSITE COMPUTED TORQUE CONTROL

## Introduction

The first problem in controlling a rigid-link manipulator with elastic motor transmissions, is that only the desired link coordinates $\underline{q}_{ld} = \underline{q}_{ld}(t)$ can be determined directly from the known desired gripper path, while there is no indication for a certain desired trajectory of $\underline{z}_e(t)$.
To obtain a smooth robot performance in space, we define a reference trajectory $\underline{q}_{lr} = \underline{q}_{lr}(t)$ for the link variables, which will converge to $\underline{q}_{ld}$ after progression in time. Further, the idea is to formulate a *'reference manifold'* $\underline{z}_{er}(t)$ on which the controller tries to keep the elastic-transmission torques $\underline{z}_e(t)$, instead of trying to suppress them totally.

The second problem is that there are more degrees of freedom than control inputs.
The goal of the *composite controller* developed in this paper is to track the reference trajectory of the links, while stabilizing the elastic vibrations around the specified reference manifold.

## The Computed Torque Controller

As argued at the beginning of this paper, it is appealing to try to find the analogue for flexible manipulators of the socalled computed torque control method for rigid robots. However, an elastic-transmission robot does not allow a nonlinear feedback control as for rigid manipulators, since there are less control inputs than degrees of freedom. For the n-th order model (8) it is possible to choose the next *computed torque control law*:

$$\underline{u} = M_l(\underline{q}_l)(\ddot{\underline{q}}_{lr} + K_l\dot{\underline{e}}_{lr}) \;+\; \underline{n}_l(\underline{q}_l, \dot{\underline{q}}_l, t) + M_zF(\ddot{\underline{z}}_{er} + K_z\dot{\underline{e}}_{zr}) \;, \tag{13}$$
$$\dot{\underline{q}}_{lr} = \dot{\underline{q}}_{ld} + \Lambda\underline{e}_l \;, \quad \forall t \geq t_0; \quad \underline{q}_{lr}(t_0) = \underline{q}_{ld}(t_0) \tag{14}$$
$$\underline{z}_{er} = M_e(\underline{q}_l)(\ddot{\underline{q}}_{lr} + K_l\dot{\underline{e}}_{lr}) \;+\; \underline{n}_e(\underline{q}_l, \dot{\underline{q}}_l, t) , \tag{15}$$

where $\underline{q}_{l_r}$ is the chosen *reference trajectory* of the link positions; in this case is $\underline{s}_r = \underline{\dot{e}}_{lr}$ a sliding surface for $\underline{q}_l$ according to Asada and Slotine (1986),

$\underline{z}_{er}$ is the chosen *reference manifold*, in this case the vector of elastic-transmission torques necessary to manipulate the links along the reference trajectory,

$\underline{e}_l = \underline{q}_{ld} - \underline{q}_l$ is the *tracking error*,

$\underline{e}_{lr} = \underline{q}_{l_r} - \underline{q}_l$ is the *reference error*,

$\underline{e}_{zr} = \underline{z}_{er} - \underline{z}_e$ is the *reference manifold error*,

$K_l \in \mathrm{R}^{n \times n}$ and $K_z \in \mathrm{R}^{e \times e}$ are both diagonal, positive definite gain matrices.

According to definition (15), $\underline{z}_{er}$ can be written as a function of $\underline{q}_l$, $\underline{\dot{q}}_l$ and t. From the definition of the control input signals (13), we know that the synthesis of $\underline{u}$ involves the first and second time-derivatives of $\underline{z}_{er}$. In the simulations of this paper this is achieved by simply differentiating twice the computed torques $\underline{z}_{er}$. But since M and $\underline{n}$ are smooth functions of their arguments, in future it seems to be better just to *make use of the dynamic robot model*. In this way, $\underline{\dot{z}}_{er}$ and $\underline{\ddot{z}}_{er}$ are explicit functions of $\underline{q}_l$, $\underline{\dot{q}}_l$, $\underline{z}_e$, $\underline{\dot{z}}_{er}$ and t, which implies that the composite controller can be implemented if *the generalized coordinates and velocities are measurable at all time t*. The second condition is that the desired trajectory $\underline{q}_{ld}(t)$ and all its time-derivatives up to the fourth order are known and uniformily bounded.

## The Composite Controller

In the adopted control strategy (13), we recognize the next *composite control* form:

$$\underline{u} = \underline{u}_r + \underline{u}_f \, , \tag{16}$$

where

$\underline{u}_r$ is the computed torque controller designed as it would be done for the equivalent *rigid-transmission manipulator* (12) and ment directly for tracking the desired link-based trajectory in space, $\underline{q}_{ld}(t)$,

$\underline{u}_f$ is the corrective control term to compensate the effects of *transmission-flexibilities* by stabilizing the elastic torque vector $\underline{z}_e(t)$ around its reference manifold $\underline{z}_{er}(t)$ (which obviously indirectly depends on the link-based trajectory of interest, $\underline{q}_{ld}(t)$, too).

Notice, that if there are no elastic deformations in the actuator transmissions (i.e., $\| F \|_2 \rightarrow 0$), from eq. (13) just remains the 'rigid' computed torque controller we all know. The addition of the 'flexible' computed torque part with the flexibility matrix F expresses the extra effort needed to stabilize the occuring elastic vibrations.

## Stability of the Closed-Loop System

Stability is an extremely important factor for control design, especially for the kind of flexible robot systems as considered in this paper. Lyapunov's stability theorems make possible a method of synthesizing control laws which guarantee stability of the closed-loop system.

In *the second stability approach of Lyapunov*, the first step is the derivation of the equivalent error equations of the closed-loop system.
The three individual parts of the model describing the closed-loop error dynamics of
- (5) all motor rotors with links interconnected by the stiff transmissions,
- (6) the links manipulated by the elastic transmissions and
- (7) the motor rotors connected to the elastic transmissions are resp.:

$$\begin{aligned}
[\, M_{ss} \quad M_{se} \,]\,(\underline{\ddot{e}}_{lr} + K_l \underline{\dot{e}}_{lr}) &= \underline{0} \, , && (17)\\
[\, M_{es} \quad M_{ee} \,]\,(\underline{\ddot{e}}_{lr} + K_l \underline{\dot{e}}_{lr}) &= \underline{e}_{zr} \, , && (18)\\
[\, 0 \quad M_{mm} \,]\,(\underline{\ddot{e}}_{lr} + K_l \underline{\dot{e}}_{lr}) \; + & \\
M_{mm} F(\underline{\ddot{e}}_{zr} + K_z \underline{\dot{e}}_{zr}) + \underline{e}_{zr} &= \underline{0} \, . && (19)
\end{aligned}$$

Then, in order to obtain a short notation of the error equations of the overall control system, the next *total reference error* vector $\underline{e}_{lz} \in \mathrm{R}^{n+e}$ shall be defined:

$$\underline{e}_{lz} = \begin{bmatrix} \underline{e}_{lr} \\ \underline{e}_{zr} \end{bmatrix} . \tag{20}$$

Now, the n *equivalent error equations* of the closed-loop system are (see also the combination of control law (13) with the system dynamics (8)):

$$M_{lz}(\underline{\ddot{e}}_{lz} + K_{lz} \underline{\dot{e}}_{lz}) = \underline{0} \, , \tag{21}$$

In the second step, a positive-definite *Lyapunov function candidate* V(t) of the total error vector $\underline{e}_{lz}$is chosen:

$$V = \frac{1}{2} \underline{\dot{e}}_{lz}^T P \underline{\dot{e}}_{lz} \, , \tag{22}$$

where $P \in \mathrm{R}^{(n+e) \times (n+e)}$ is a constant positive definite matrix.
Then, a sufficient condition for *uniform asymptotic stability* of the system is that the time derivative of this Lyapunov function V is negative definite (by using error equation (21)):

$$\dot{V} = \underline{\dot{e}}_{lz}^T P \underline{\ddot{e}}_{lz} = -\underline{\dot{e}}_{lz}^T P K_{lz} \underline{\dot{e}}_{lz} \, . \tag{23}$$

Finally, since $\dot{V} < 0$ if $\underline{\dot{e}}_{lz}(t) \neq \underline{0}$, the system will be stable in the sense that the total error goes to zero or at least remains bounded in time.

# AN EXAMPLE

## A TR-Robot with one Elastic Transmission

In this simulation example, we consider a three-degrees-of-freedom translation-rotation (TR-) robot with one elastic motor transmission. A schematic drawing of it is given in Fig. 2. The actuator at the prismatic joint translates a carriage in horizontal direction via a stiff transmission. At the revolute joint on the carriage is fixed an inverted pendulum, which is driven by the elastic transmission. This elastic motor transmission is modeled as a linear-elastic, massless and torional spring.

The desired trajectories for link-motion control $[q_{sd}(t), q_{ed}(t)]$ are derived from a certain trajectory of the payload at the end of the

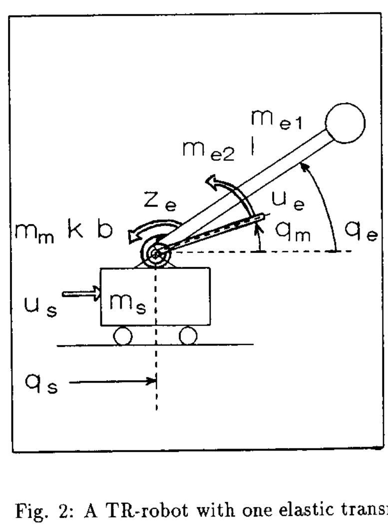



Fig. 2: A TR-robot with one elastic transmission

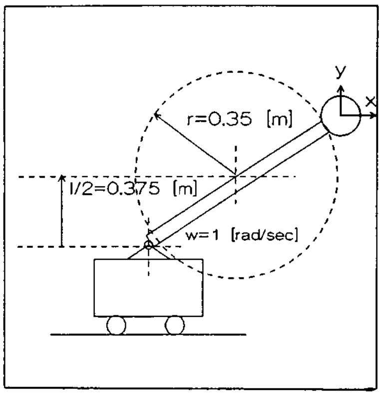



Fig. 3: The desired trajectory of the payload

robot arm, $[x_d(t), y_d(t)]$. This gripper trajectory is specified to be a constant circulation in two-dimensional space under the assumption that there are no elasticities in the system (see Fig. 3).

The next system variables and parameters will be used:

- $q_s$ is the horizontal translation of the carriage,
- $q_e$ is the rotation of the arm,
- $q_m$ is the rotation of the motor rotor acting on the rotating arm via the elastic transmission,
- $u_s$ is the force acting directly on the translating carriage via a stiff transmission,
- $z_e$ is the elastic-transmission torque acting on the rotating arm,
- $u_e$ is the moment of the motor rotor acting on the torsional-elastic transmission,

$m_s = 10kg$ is the mass of the carriage,
$m_{e1} = 2kg$ is the mass of the payload at the end of the arm,
$m_{e2} = 3kg$ is the mass of the arm,
$m_m = 5kgm^2$ is the inertia of the motor rotor connected to the elastic transmission,
$l = 0.75m$ is the length of the arm,
$k = 2Nm/rad$ is the stiffness of the elastic motor transmission,
$b = 0.5Nm$ is the friction constant of the motor rotor.

The dynamic model of this robot is described in the general form of equation (24):

$$M(\underline{q})\underline{\ddot{q}} + \underline{n}(\underline{q}, \underline{\dot{q}}, t) = H\underline{u} \ ,$$

where

$$\begin{aligned}
\underline{q} &= \begin{bmatrix} q_s \\ q_e \\ q_m \end{bmatrix} , \ \underline{u} = \begin{bmatrix} u_s \\ u_e \end{bmatrix} , \ H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} , \\
\underline{n} &= \begin{bmatrix} n_s \\ n_e - z_e \\ n_m + z_e \end{bmatrix} = \\
&= \begin{bmatrix} -(m_{e1} + \frac{m_{e2}}{2})\cos(q_e)\dot{q}_e^2 \\ -z_e \\ b\dot{q}_m + z_e \end{bmatrix} , \\
M &= \begin{bmatrix} M_{ss} & M_{se} & 0 \\ M_{es} & M_{ee} & 0 \\ 0 & 0 & M_{mm} \end{bmatrix} =, \\
&= \begin{bmatrix} m_s + m_{e1} + m_{e2} & -(m_{e1} + \frac{m_{e2}}{2})l\sin(q_e) & 0 \\ * & (m_{e1} + \frac{m_{e2}}{3})l^2 & 0 \\ * & * & m_m \end{bmatrix} , \\
z_e &= k(q_m - q_e) \ .
\end{aligned}$$

With this notation, we immediately obtain the particular system dynamics structure as depicted in equations (5)- (6)- (7).

Control Simulation Results

Here, some simulation results obtained with the composite controller are presented. The tracking output errors of the payload ($e_x = x_d - x$, $e_y = y_d - y$) and their time-derivatives are relatively small, while the control input signals are quite normal for motors (see Fig. 4: $k = 2[Nm/rad], K_l = diag[10], K_z = 10$). If the initial link coordinates $q_s(t_0)$ and $q_e(t_0)$ and their time-derivatives are not according to the desired trajectory, the tests still show a very acceptable system performance, except that the absolute magnitudes of the force $u_s$ and the torque $u_e$ become very large just after $t = t_0$.

In the simulation tests, indeed it appears that the elastic-transmission torque $z_e$ plays an important role in the flexible control system, in that it depends from the system dynamics and from the actual trajectory of the links (eq. (6)), but not from the magnitude of the stiffness k of the torsional-elastic motor transmission (which is compensated by $\epsilon$ in definition (2); see the same Fig. 4). This explains the remarkable effect that the magnitude of k does not influence the ranges in between which the output errors $e_x$, $e_y$ and their time-derivatives fluctuate: wether we try $k = 0.2$, $k = 2$ or $k = 20[Nm/rad]$, the output (velocity) errors always remain in between $\pm 0.01[m(/sec)]$. Thus, there seems to be no restriction to the magnitude of the stiffness k, this in contradiction to the control strategy of Slotine and Hong (1987), where k has to be 'very large'. As can be expected, these output (velocity) error ranges change by alternating the control gain $K_l$: for example, they become $\pm 0.001[m(/sec)]$ as $K_l$ is chosen to be $K_l = diag[100]$ instead of $K_l = diag[10]$.

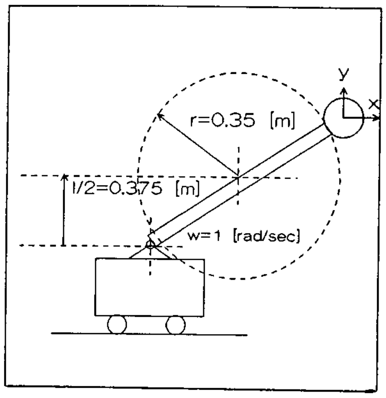
## CONCLUSIONS

Using the idea of computed torque control, in this paper a control law is presented which incorporates the effects of motor-transmission flexibilities on the system. The goal of this *composite controller* is to follow a specified link-coordinate based trajectory in space while stabilizing the elastic deflections around a certain reference manifold due to the natural flexibility behavior of the system. Simulations of a translation-rotation robot with one elastic transmission between the motor and the rotating arm have shown the effectiveness of this proposed composite control approach.

One attractive feature of this composite strategy is that the control term $\underline{u}_r(t)$ can be designed on the basis of well-established *control schemes for rigid manipulators*, such as the well-known computed torque method utilized in this controller design or such as the adaptive sliding control approach of Slotine and Li (1986).

Another advantage of this approach is that the mechanical stiffnesses of the linear-elastic transmissions do not have to be assumed large in formulating the dynamic model of the robot system. This, in contradiction to the '*singular perturbation*' approach of Slotine and Hong (1987) for manipulators only with relatively stiff actuator transmissions. A crucial issue of their method is the assumption that the smallest stiffness is sufficiently large so as to preserve a certain time scale separation. In case the elastic transmissions are 'not stiff enough', they propose the use of socalled integral manifolds to obtain a more accurate control system which accounts for the effect of flexibility up to a certain order of the largest flexibility parameter.

A third advantage of this control strategy is that *not all* motor transmissions have to be elastic; so the number of degrees of freedom is less than or equal to two times the number of motors: $(n + e) \leq 2n$.

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## Control Simulation Results

Here, some simulation results obtained with the composite controller are presented. The tracking output errors of the payload ($e_x = x_d - x$, $e_y = y_d - y$) and their time-derivatives are relatively small, while the control input signals are quite normal for motors (see Fig. 4: $k = 2[Nm/rad], K_l = diag[10], K_z = 10$). If the initial link coordinates $q_s(t_0)$ and $q_e(t_0)$ and their time-derivatives are not according to the desired trajectory, the tests still show a very acceptable system performance, except that the absolute magnitudes of the force $u_s$ and the torque $u_e$ become very large just after $t = t_0$.

In the simulation tests, indeed it appears that the elastic-transmission torque $z_e$ plays an important role in the flexible control system, in that it depends from the system dynamics and from the actual trajectory of the links (eq. (6)), but not from the magnitude of the stiffness k of the torsional-elastic motor transmission (which is compensated by $\epsilon$ in definition (2); see the same Fig. 4). This explains the remarkable effect that the magnitude of k does not influence the ranges in between which the output errors $e_x$, $e_y$ and their time-derivatives fluctuate: wether we try $k = 0.2$, $k = 2$ or $k = 20[Nm/rad]$, the output (velocity) errors always remain in between $\pm 0.01[m(/sec)]$. Thus, there seems to be no restriction to the magnitude of the stiffness k, this in contradiction to the control strategy of Slotine and Hong (1987), where k has to be 'very large'. As can be expected, these output (velocity) error ranges change by alternating the control gain $K_l$: for example, they become $\pm 0.001[m(/sec)]$ as $K_l$ is chosen to be $K_l = diag[100]$ instead of $K_l = diag[10]$.

## CONCLUSIONS

Using the idea of computed torque control, in this paper a control law is presented which incorporates the effects of motor-transmission flexibilities on the system. The goal of this *composite controller* is to follow a specified link-coordinate based trajectory in space while stabilizing the elastic deflections around a certain reference manifold due to the natural flexibility behavior of the system. Simulations of a translation-rotation robot with one elastic transmission between the motor and the rotating arm have shown the effectiveness of this proposed composite control approach.

One attractive feature of this composite strategy is that the control term $\underline{u}_r(t)$ can be designed on the basis of well-established *control schemes for rigid manipulators*, such as the well-known computed torque method utilized in this controller design or such as the adaptive sliding control approach of Slotine and Li (1986).

Another advantage of this approach is that the mechanical stiffnesses of the linear-elastic transmissions do not have to be assumed large in formulating the dynamic model of the robot system. This, in contradiction to the *'singular perturbation'* approach of Slotine and Hong (1987) for manipulators only with relatively stiff actuator transmissions. A crucial issue of their method is the assumption that the smallest stiffness is sufficiently large so as to preserve a certain time scale separation. In case the elastic transmissions are 'not stiff enough', they propose the use of socalled integral manifolds to obtain a more accurate control system which accounts for the effect of flexibility up to a certain order of the largest flexibility parameter.

A third advantage of this control strategy is that *not all* motor transmissions have to be elastic; so the number of degrees of freedom is less than or equal to two times the number of motors: $(n + e) \leq 2n$.

However, last but not least must be mentioned that *all system state variables and their first time-derivatives have to be available* by measurement or on-line estimation.

## FUTURE RESEARCH

There are several interesting research issues that arise at this point:

- In this article are considered flexible manipulators with elasticity concentrated at the actuator transmissions. In future, this approach will be extended to the case of elasticity distributed along the manipulator's structure (i.e., consideration of *flexible arms* modeled as elastic joints).
- Further has to be adressed the problem if there is *lack of full state availability* (as for example the flexible variables cannot be measured) and if there have to be designed an output-trajectory controller to realize tracking of a prespecified path in space of the gripper at the end of an *redundant* robot arm (i.e., we have to deal with *end-effector* control instead of link-based control).
- Unfortunately, the computed torque control method also relies heavily on an accurate prior knowledge of the robot system dynamics and, therefore, the approach presented in this paper has to be expanded further to an *adaptive control* technique in which the unknown or time-varying system parameters will be adjusted on-line (for example basically according to the method of Slotine and Li (1987)). The global asymptotic stability of the overall system shall then be guaranteed through the hyperstability theorem of Popov (1969).
- Finally,*sliding control* according to Asada and Slotine (1986) can be practized in order to robustify the system against parametric uncertainties and (environmental) disturbances.

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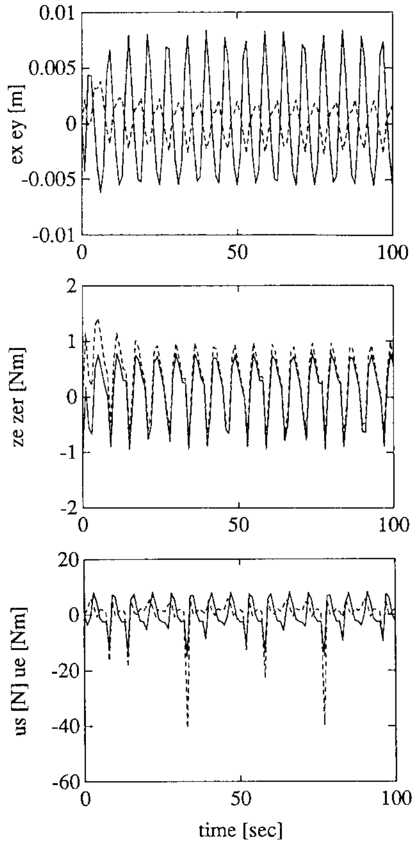


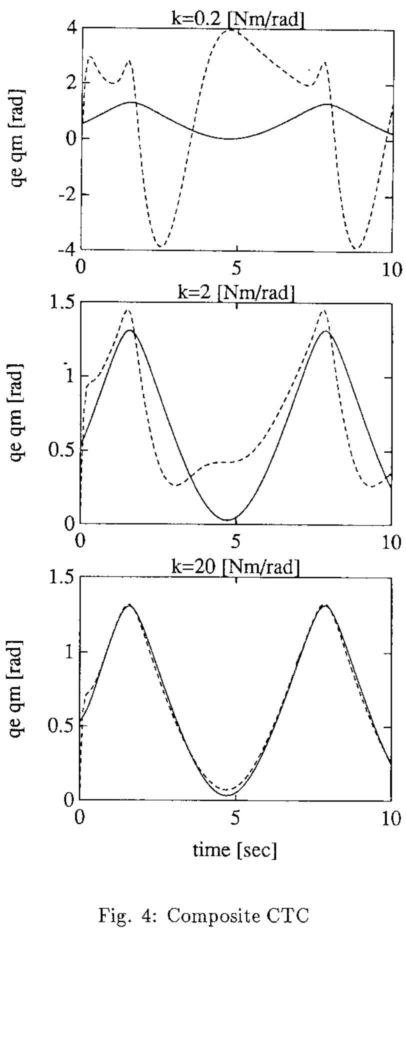


Fig. 4: Composite CTC