Use of the method of progressive means in the analysis of errors in a line standard measurement

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USE OF THE METHOD OF PROGRESSIVE MEANS IN THE ANALYSIS OF ERRORS IN A LINE STANDARD MEASUREMENT

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samenvatting

This investigation deals with an analysis of errors that are present in a line standard which was constructed using a similar parent standard. Some of these errors have been estimated in a previous work refered to in the present report, and a special technique was employed to separate the systematic errors from the random errors. A modified curve drawn using the progressive means of points in the original graph of errors in the line standard was used as the basis, and the random errors of the dividing machine employed to engrave lines on the line standard was thus estimated.

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Part II

THE USE OF THE METHOD OF PROGRESSIVE MEANS IN THE ANALYSIS OF ERRORS IN A LINE STANDARD MEASUREMENT.

1. Introduction

1.1. In precise engineering metrological work lengths have to be measured very accurately and for this purpose very accurate measuring instruments have to be used. A class of these instruments depend solely on line standards for their manufacture and calibration. Thus it is obvious that the line standard itself must have a high degree of accuracy.

A line standard can be made by suitably engraving lines on a specially prepared steel surface. By calibrating this line standard using appropriate means and knowing the errors in each of the lines, it is possible to produce a second line standard with a greater accuracy.

This report deals with an analysis of the various errors which are present in such a line standard, produced from a parent standard.

The construction of the line standard in question and the measurements on it had been performed at the metrological laboratories of the Technische Hogeschool Eindhoven, and reference is made to the report of P.H.J. Schellekens and E.A. Khokar - 'Analysis of Errors in a Line Standard Measurement' - (WT - Rapport No. 0226)., in this respect.

1.2. The primary objective of the present investigation was to estimate the magnitude of the error imparted to the line standard by the engraving system of the dividing machine, during its construction.

A knowledge of this magnitude is important and also useful for the later construction of more accurate line standards.

2. Brief description of the procedure

2.1. The parent line standard was measured by a laser interferometer (for details, refer the above mentioned report). A set of measurements consisting of a forward movement and a backward movement of the scale, was performed on each line, the measurement on any line giving the distance of this line from the zero line of the scale. Thus two values were obtained for each line. The arithmetic mean of these values was calculated for each line, and the error in each of the lines, i.e. the difference between the observed mean value and the nominal mm reading, was obtained.

2.2. These errors were made use of to make a correcting tape for the dividing machine, which engraved lines on the new standard.

2.3. Principle of the dividing machine - Fig.1 gives a schematic diagram of the dividing machine.

2.4. Construction of the new line standard. By means of the photoelectric-microscope and the servo-system, the parent line standard is adjusted for each line on the scale. Each time the scale moves through the distance between two consecutive lines on it, a motion is imparted to the new line standard to be made and the engraving system of the dividing machine engraves a line on it. Each line on the new scale is thus drawn after the systematic error in the corresponding line of the parent standard has been corrected for.

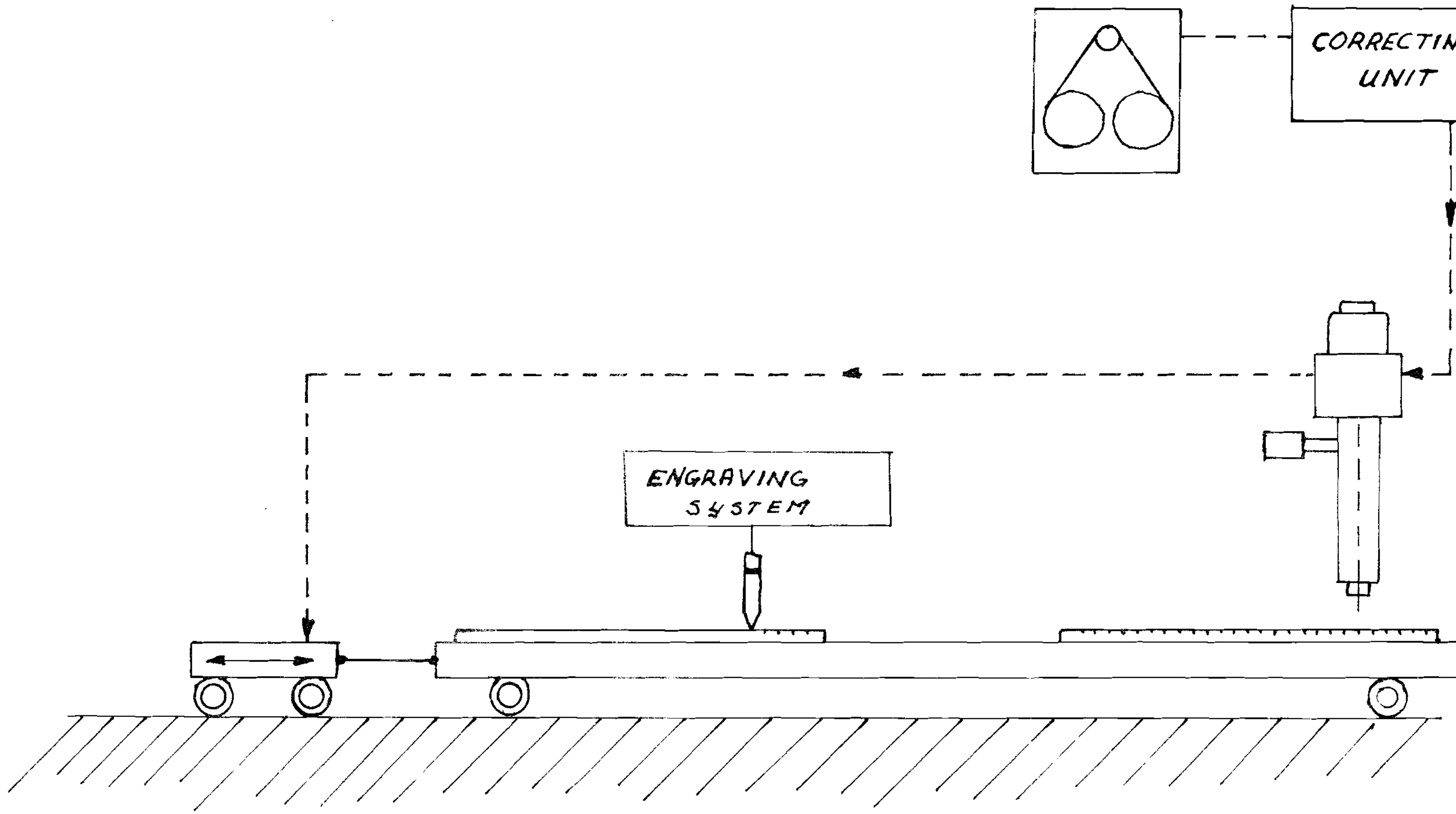
2.5. The new line standard so constructed was measured using the laser interferometer, as described in the report mentioned in section 1. Two sets of observations each consisting of a measurement in a forward direction and in a backward direction have been made on each line. Thus, in all 4 values were obtained for each line. The mean of these 4 values was used to calculate the error in each of the lines of the new standard.

These errors were plotted on a graph (cf. Computer Programme RA-2980-24). A portion of this graph is shown in Appendix 1

2.6. Some of the errors built in this new line standard have been estimated in the above-mentioned report. The present investigation took into account these estimates and went a step further in analysing some of the remaining errors. The primary objective of estimating the imprecision of the engraving system of the dividing machine, has been achieved, in the present work.

2.7. Approach - A plot of the errors in the new line standard (Graph I, Appendix 1, page 9) consists of a very large number of sharp mavericks. For all practical purposes, it was suggested that a somewhat modified curve drawn from this graph, but yet showing the same features as those found in it, could be used in its place, the criterion of the modified curve being that it should depict, more or less, the same features of the original graph.

2.7.1. To get a reasonably good modified curve, four graphs were drawn tentatively by making use of the progressive means. (This method is described in Appendix 2


CORRECTING UNIT
ENGRAVING SYSTEM

The graphs were drawn by taking the progressive means of 7, 11, 21 and 51 points respectively of the original graph (cf. Computer Programme

Portions of these graphs a, b, c, d are shown in Appendix 3, page 11

The graphs (a) and (b) drawn by taking the progressive means of 7 and 11 points respectively, show sufficiently well, the features in the original graph.

The graphs (c) and (d) do not show these features to the same extent.

2.7.2. At this stage another improvement was effected in the case of the graphs (a) and (b). Two new graphs (a') and (b') were generated by taking the progressive means of 3 and 5 points respectively from graphs (a) and (b). Portions of these graphs are shown in Appendix 4, page 12. These graphs (a'), (b') resembled their original graphs (a), (b) and at the same time were smoother than the original ones. So it was intended to choose the more preferable graph out of (a') and (b') to proceed with the investigation.

2.7.3. To see how closely these two graphs resembled the original graph giving errors, the differences in the ordinates were calculated as indicated below and two new graphs (e) and (f) were plotted. (cf. Computer Programme RA-3495) (APPENDIX 5 page 13)

For both (e) and (f) the difference z_i is given by:

$$z_i = y_i - y'_i ,$$

where y_i = ordinate of error at point i of original graph I and
y'_i = ordinate at same point i of graphs (a') or (b').

2.7.4. Evaluation. An examination of graphs (e) and (f) reveal that the differences z_i are of a random nature. The variances in the two cases were calculated (cf. Computer Programme RA-3899) and found to agree very closely.

A number of points taken from each of the graphs when plotted on probability paper gave straight lines, showing that the points belong to a Gaussian distribution (Appendix 6, page 14)

This analysis eliminates, in effect, the influence of the systematic errors present. Thus, it is possible to calculate the total imprecision of the measurement due to random errors only.

2.7.4.1. The random errors that are known to influence the measurement are those present in:

(i) the correcting tape;

(ii) the microscope and servo-system;

(iii) measurements of the new line standard;

(iv) the engraving system of the dividing machine.

A knowledge of the three imprecisions due to (i), (ii) and (iii) makes it possible to estimate the imprecision of the engraving system.

3. Summary

The technique of using progressive means is seen to provide a convenient means of extracting information from a complicated graph as the one dealt with in this report (Graph I). In this particular case, the influence of the systematic errors could be eliminated by this method and it was possible to compute the random error of the dividing machine. This value has been estimated in Section 5.

4. Investigation and Analysis of Data

4.1. Sources of Error

4.1.1. Random errors in the dividing procedure

(i) Random errors of the correcting tape, with a standard deviation $\sigma_{c.t}$ (these include the random errors present in the measurements of the parent line standard).

A value of 81 nm has been obtained for σ_M in the earlier work. Taking into account the pair of measurements for each line, the $\sigma_{c.t}$ for the mean of the two readings was calculated as $\frac{81}{\sqrt{2}}$ nm.

(ii) Random errors of the microscope and servo-system, with a standard deviation σ_m.

This value has been estimated from the relation

$$\sigma_M^2 = \sigma_m^2 + \sigma_{o.s}^2$$

where σ_m = standard deviation of random errors of microscope and servo-system.

$\sigma_{o.s}$ = standard deviation of random errors of the optical system.

A value of 60 nm has been obtained for σ_m previously.

(iii) Random errors of the dividing system, with a standard deviation σ_d.

4.1.2. Errors in the measurement

(i) Random errors in measurement of the new line standard, with a standard deviation $\sigma_{n.l.s}$.

This quantity has been estimated in the earlier work and the value is 40.5 nm. (2.5)

4.2. Systematic errors - Systematic errors were eliminated from the measurements as far as possible by carefully controlling the conditions of the experiment. But, however, it is surmised that the shape of the surface of the line standard could have influenced the measurements systematically, in particular, as this factor was uncontrollable (see concluding remarks).

5. Estimation of the results

The standard deviation of the differences z_i gives the total random variation σ_{Tot}.

As two nearly equal standard deviations were obtained for the differences in the two cases (graphs (a'),(b')), the best estimate of the variances could be arrived at by pooling these variances

$$\sigma^2_{Tot} = \frac{\sigma_1^2(n_1-1) + \sigma_2^2(n_2-1)}{n_1 + n_2 - 2}$$

$$\therefore \quad \sigma_{Tot} = 109 \text{ nm.}$$

Using the above notation, since the σ_s' are all independent, σ_{Tot} can be expressed as:

$$\sigma^2_{Tot} = \sigma^2_{c.t} + \sigma^2_m + \sigma^2_{n.l.s} + \sigma^2_d$$

$$\therefore \quad \sigma^2_d = \sigma^2_{Tot} - (\sigma^2_{c.t} + \sigma^2_m + \sigma^2_{n.l.s})$$

$$= 109^2 - \left| \left(\frac{81}{\sqrt{2}}\right)^2 + 60^2 + (40.5)^2 \right|$$

$$\therefore \quad \sigma_d = 58 \text{ nm}$$

CONCLUDING REMARKS

The analysis give a value of 58 nm for the imprecision of the engraving system of the dividing machine.

The investigation into the influence of the systematic effects was started , but could not be finished for want of time. In this respect altitude measurements of surface of the line standard were made in order to see a possible correlation between the shape of the surface and the systematic errors in the original graph.

The modified curve (b') was chosen to represent the errors best, and with this as the starting point another curve was obtained for the differences z_i as follows:

$$z_i = (x_i - \bar{x}) - r(y_i - \bar{y})$$

where $r = \dfrac{\sum |x_i - \bar{x}|}{\sum |y_i - \bar{y}|}$, a scaling factor.

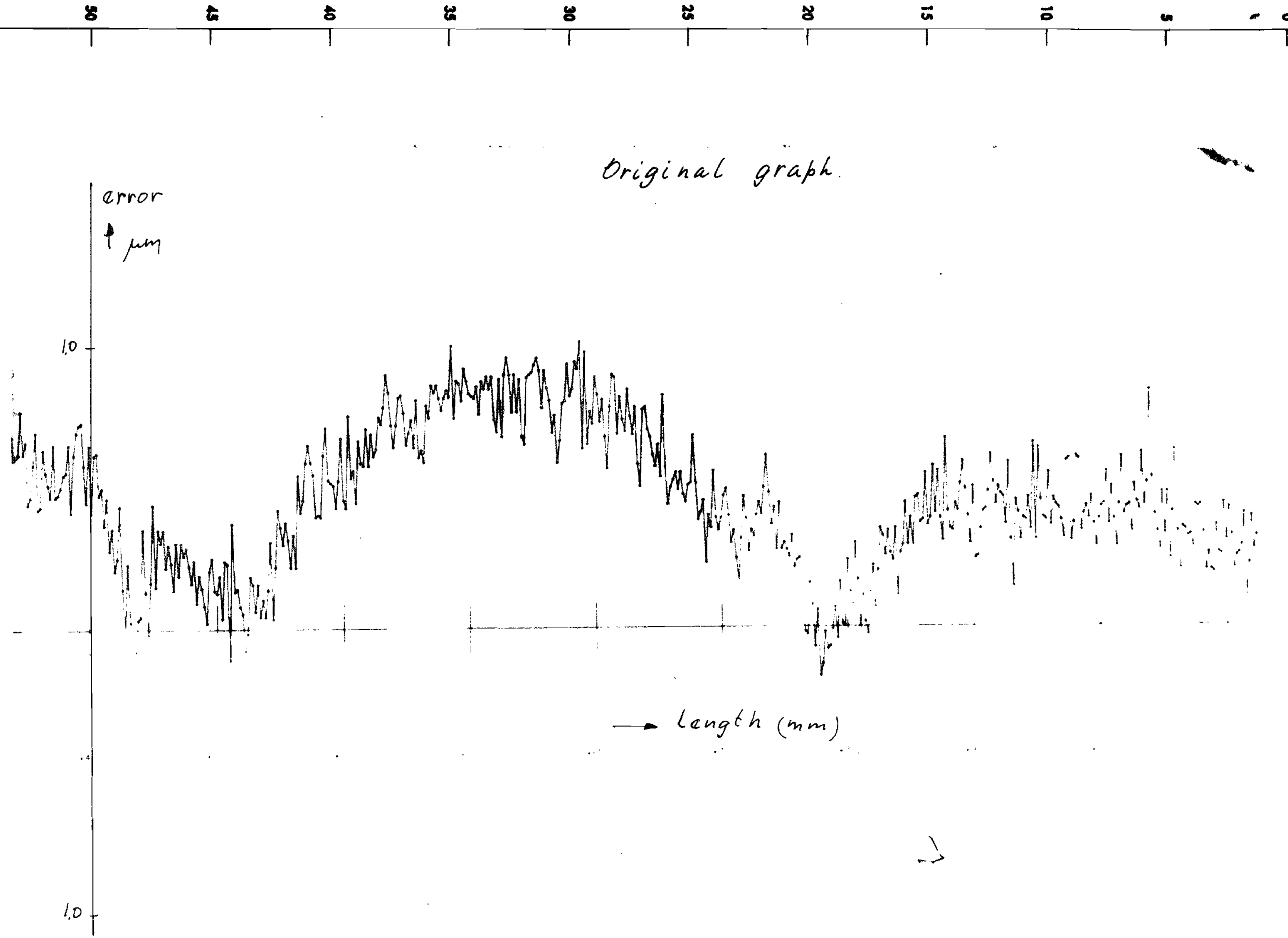
In these, x's refer to the ordinates of the modified curve (b') and
y's refer to the ordinates of the shape of the surface curve.

The curves x_i, y_i, z_i drawn to the same scale are shown in Appendix 7 (page 15)

It is intended to perform Fourier Analysis on the differences z_i curve and it is expected that this treatment will lead to the detection and consequent estimation of any systematic influence of the shape of the surface over the measurements of the line standard.

BIBLIOGRAPHY

SCHELLEKENS, P.H.J. and KHOKAR, E.A. - Analysis of Errors in a Line Standard Measurement - (WT Rapport No. 0226).


Original graph
error
↑ µm
1,0
1,0
→ length (mm)
50
45
40
35
30
25
20
15
10
5
0

APPENDIX - 1

APPENDIX - I

A-1. Calculation of progressive means.

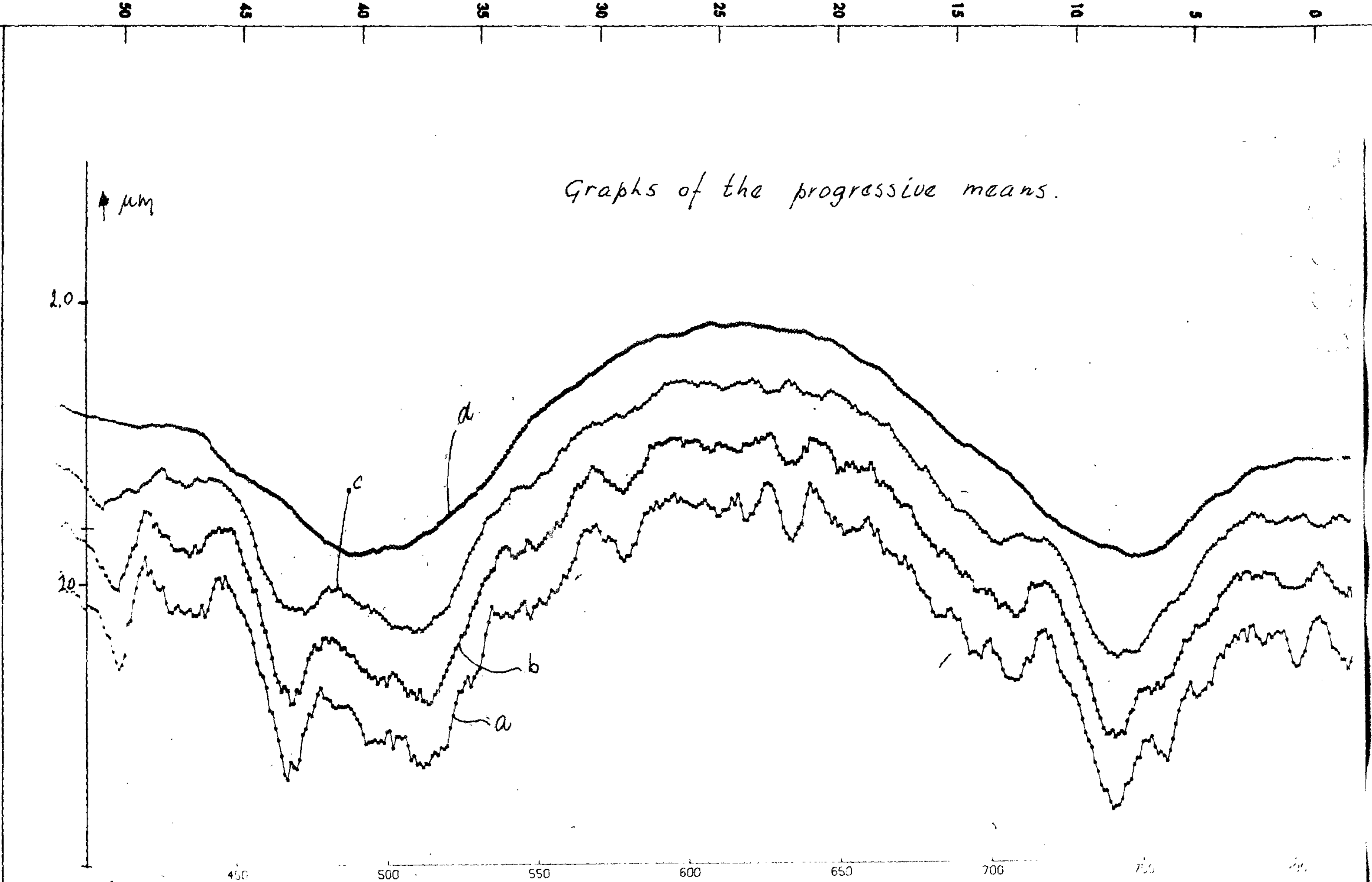
(a) The progressive means of 7 points were calculated as follows: Let the first 7 points of the line standard denoted by $i = 0, 1, \ldots 6$ have errors $x_0, x_1, \ldots x_6$ etc.

Then the 1st mean $\bar{x}_1 = \dfrac{x_0 + x_1 + \ldots + x_6}{7}$, the mean occurring at the line $n = 3$.

The 2nd mean for the 7 pts $n = 1, 2, \ldots 7$ and the second mean $\bar{x}_2 = \dfrac{x_1 + x_2 + \ldots + x_7}{7}$, the mean occurring at the line $n = 4$.

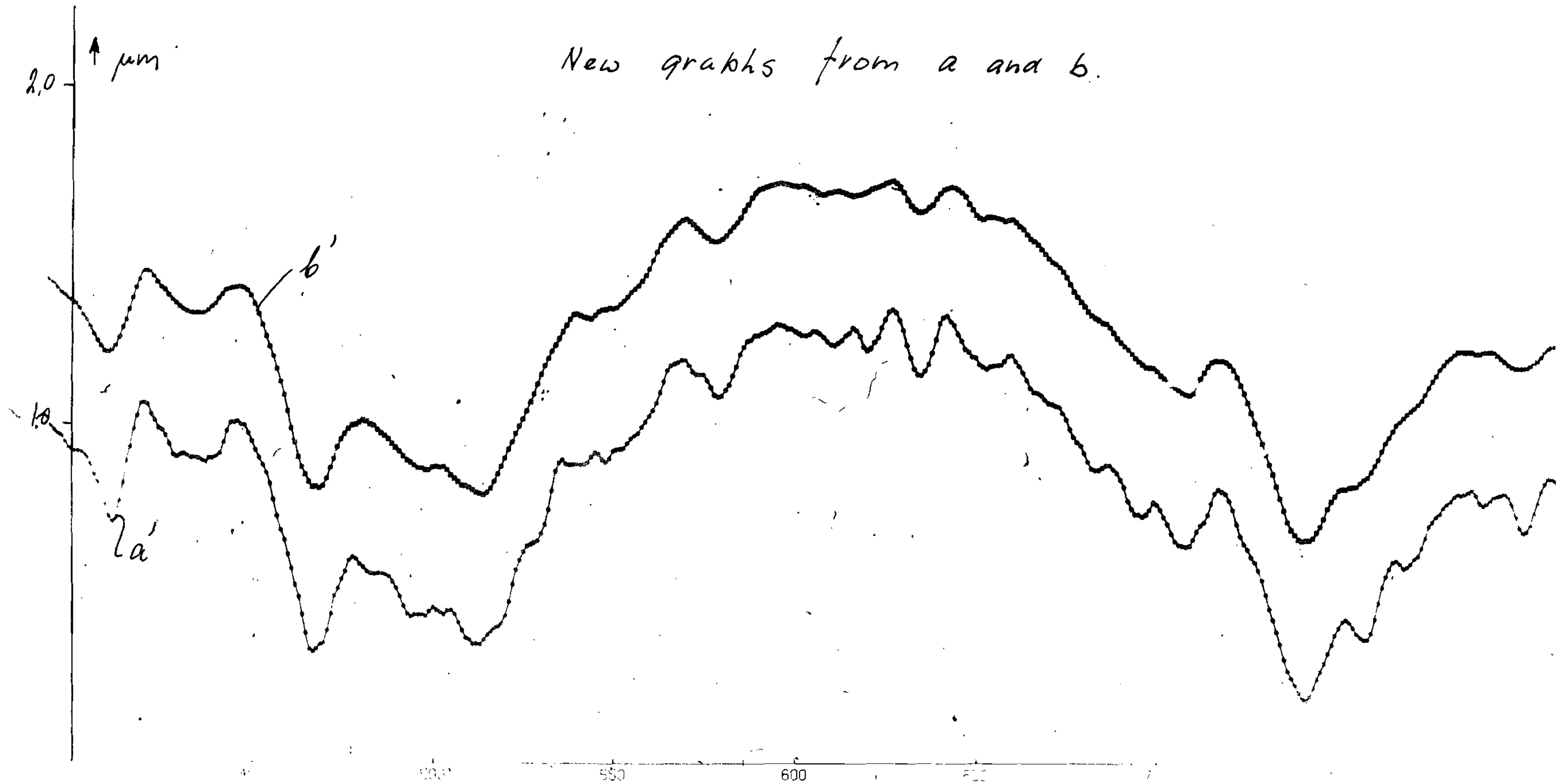
The curve of progressive means was drawn using the values $\bar{x}_1, \bar{x}_2, \ldots$ etc. The same method was used for drawing the other three graphs.

Graphs of the progressive means.



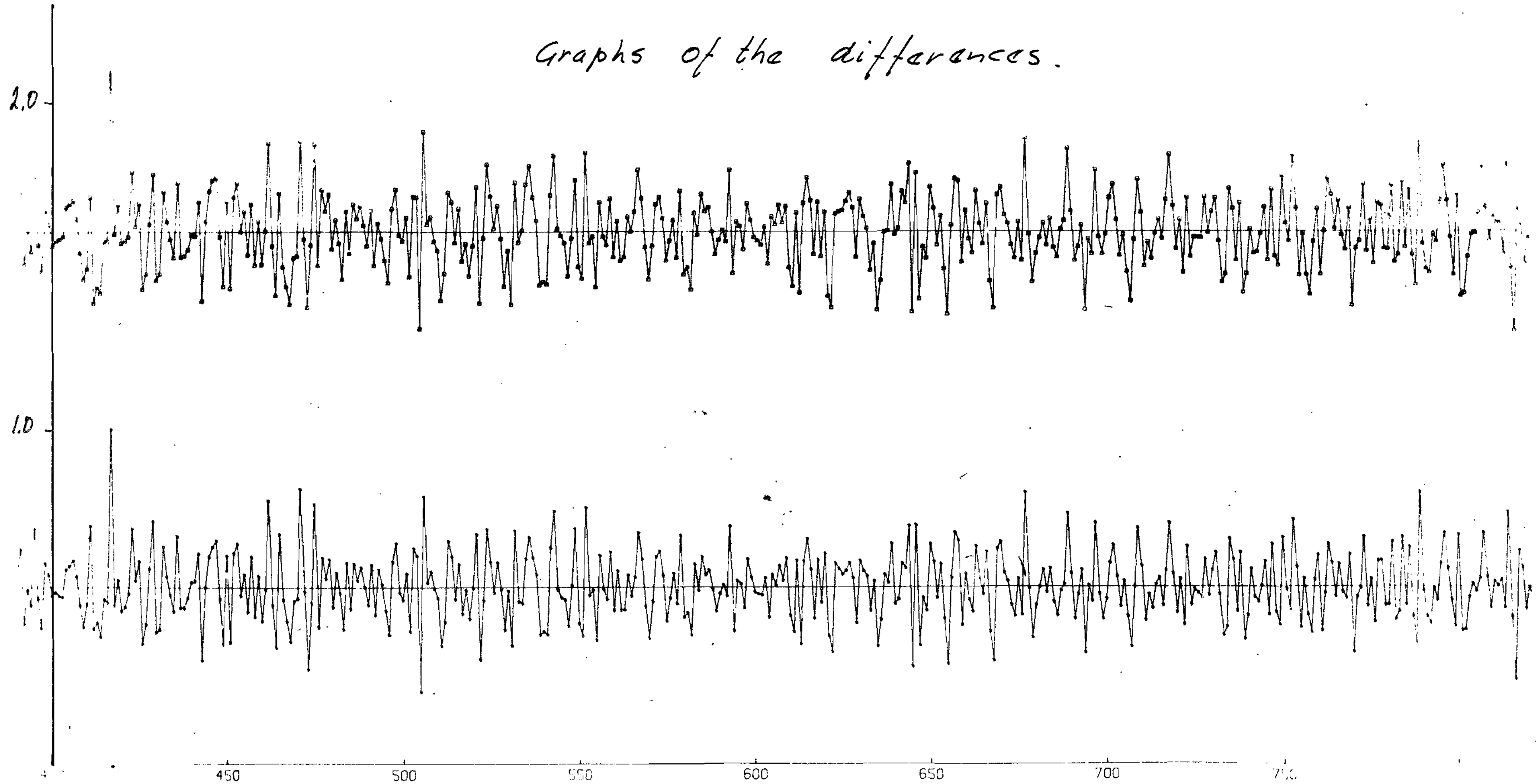
APPENDIX - 3

New graphs from a and b.

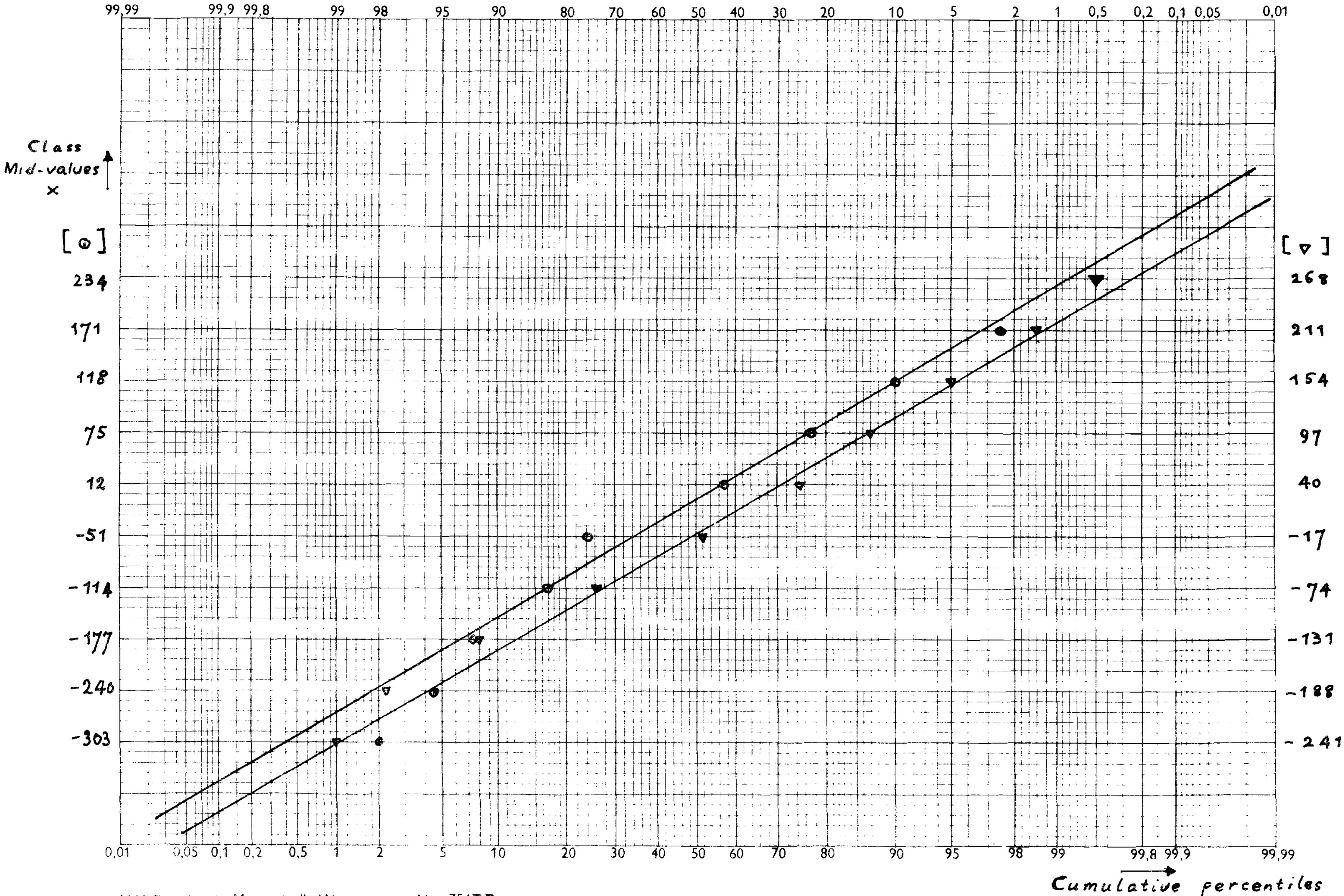


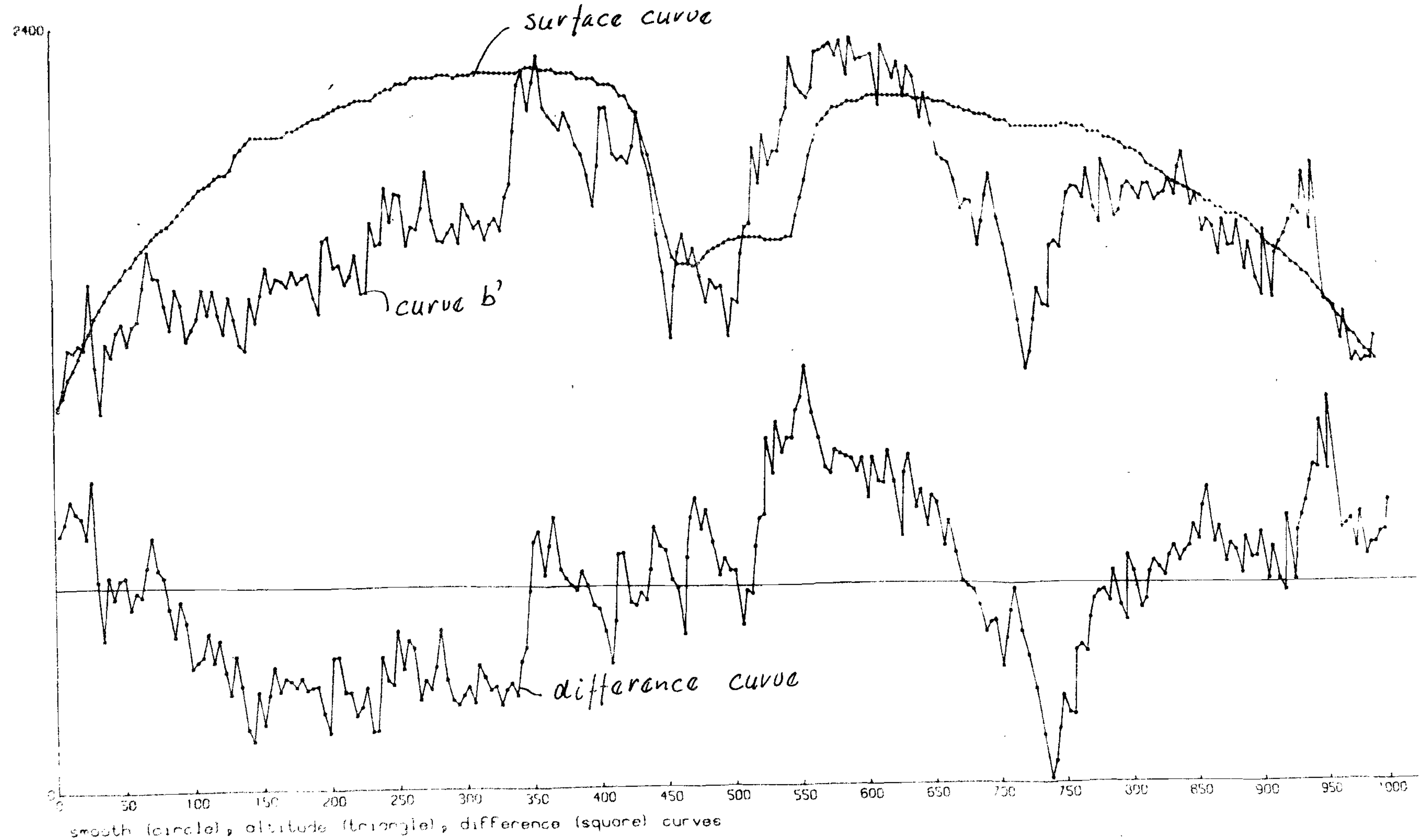
APPENDIX - 4

Graphs of the differances.



APPENDIX - 5


99,99 99,9 99,8 99 98 95 90 80 70 60 50 40 30 20 10 5 2 1 0,5 0,2 0,1 0,05 0,01
Class Mid-values x
[⊙]
234
171
118
75
12
-51
-114
-177
-240
-303
[▽]
268
211
154
97
40
-17
-74
-131
-188
-241
0,01 0,05 0,1 0,2 0,5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99,8 99,9 99,99
Cumulative percentiles


surface curve
curve b'
difference curve
2400
0
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
800
850
900
950
1000
smooth (circle), altitude (triangle), difference (square) curves

APPENDIX - 7