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Adaptive mutual synchronization of a CFT robot system

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Traineeship report

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Abstract

In this report an adaptation mechanism for linear friction parameters is developed in order to achieve mutual synchronization for one direction of the CFT robot system. In particular, a mutual synchronization scheme is extended with an adaptation mechanism for the linear friction parameters in order to improve the performance. A stability analysis is made to develop the adaptation mechanism. During simulations the behaviour and performance of the adaptive mutual synchronization scheme is assessed. Finally the adaptive mutual synchronization scheme is tested on the CFT robot system in the DCT laboratory at the Eindhoven University of Technology. The experiments illustrate that it is possible to achieve adaptive mutual synchronization, but a part of the interconnections between the robots are lost and restrictions are given to the weighting factor in the synchronization error. However it is seen that the performance of the scheme has made an enormous improvement.

Contents

Chapter 1

Introduction

1.1   Scope of the assignment

In actual production processes such as manufacturing, automotive applications and teleoperation systems there is a high requirement on flexibility and manoeuvrability of the involved systems. The tasks, which have to be performed, are sometimes so complex that they cannot be executed by a single system. So individual systems have to work together to execute a certain task. For that purpose there exist several synchronization schemes.

This report will discuss adaptive mutual synchronization of a two identical CFT robot system. The system to validate the developed controller consists of two identical CFT robots. The CFT robot is a Cartesian pick and place robot of four degrees of freedom, developed by Philips Centre of Manufacturing Technology.

At the Eindhoven University of Technology, Alejandro Rodriguez (2002) carried out a four years Ph.D. program on the analytical investigation of synchronization of multi-composed systems with only position measurements. The CFT robots were used to experimentally validate the developed controllers. In this program a model based controller was used. First, a model of the CFT robot was derived and the model parameters, such as the friction coefficients, were estimated using a Kalman filter and least squares techniques. Because some model parameters, especially the friction parameters, change in time and place, it is evident that the performance of the CFT robot system can be improved using an adaptive controller.

During another trainingship Nico Rademakers (2003) achieved leader-follower synchronization with an adaptive controller for the linear friction parameters. The goal of this study is to achieve mutual synchronization with an adaptive controller for the linear friction parameters. The difference between the two trainingships is the synchronization scheme that is used. In the leader-follower scheme there is one leader and the other robots follow the leader. Using the mutual scheme all the robots are equal with respect to each other and the synchronization goal is achieved by defining a specific reference trajectory for each robot.

To accomplish this goal a stability analysis will be made to develop an adaptive controller for the mutual synchronized system. Thereafter the controller is tested in a computer simulation. In addition the controller gains are tuned during these simulations. Finally the controller is implemented in the CFT robot systems and experimentally validated. Because each direction contains for each robot 5 gains, only the forward and backward movement of the robot is used for adaptive mutual synchronization.

1.1   Outline

The experiments are carried out on the CFT robot system. In chapter 2 the CFT robot system is discussed. In addition a full mathematical description of the CFT robot system is presented, which includes a kinematic and dynamic model and a set of physical parameters. In chapter 3 adaptive control and mutual synchronization are discussed. A stability analysis is made in order to develop an adaptation mechanism for the linear friction parameters. The developed adaptive mutual synchronization scheme is tested in a simulation environment. These simulations are presented in chapter 4. In particular, attention is paid to the influence of the weighting gain in the synchronization error on the performance of the scheme. In addition it is assessed if it is useful to use an adaptation mechanism. Finally, the developed adaptive mutual synchronization scheme is tested on the CFT robot system. The experimental obtained results are presented in chapter 5. Besides the experimental obtained results, the influence of the measurement noise and how to handle the measurement noise is discussed in chapter 5. In chapter 6 some conclusions are drawn and recommendations are given for future work.

Chapter 2

The CFT robot system

In this section a short description about the kinematics and dynamics of the CFT robot system is given. The CFT robot system is used to validate the designed adaptive controller on simulation level and experimentally. To do some computer simulations a full mathematical description of the robot is needed, which includes a kinematic and dynamic model and a set of physical parameters. In this chapter only a summary of the models and relations are given. For further details the reader is referred to Rodriguez-Angeles et al. (2002), Rodriguez-Angeles (2002) or Nijmeijer and Rodriguez-Angeles (2003).

2.1   The CFT robot

The CFT robot is an industrial pick and place robot, designed and built by the Philips Centre of Manufacturing Technology. The CFT robot is a Cartesian robot with a basic elbow configuration. The arm consists of two joints and is placed on a rotating base. The rotating base can move forward and backward. So the robot has four degrees of freedom in Cartesian space: rotation, up and down movement of the arm, forward and backward movement of the arm and forward and backward movement of the whole robot. The tool at the end of the outer link is passively actuated and designed to keep a horizontal plane all the time. Four DC brushless servomotors actuate the robot. Attached to the shaft of the motors are encoders with a resolution of 2000 RPR. This results in an accuracy of $\pm$ 0.5 mm in all motion directions. The servomotor link pair is to be considered as a rigid joint.

To evaluate the developed controller the system is equipped with a DS1005 dSPACE system. The dSPACE system takes care of the implementation of the developed controller and the communication with the robots. During the experiments the sampling frequency is set at 2 kHz.
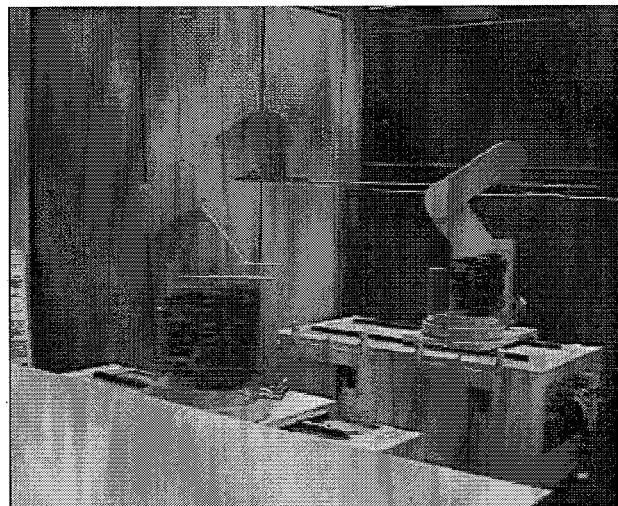
Figure 2.1 Two CFT robot systems at the DCT laboratory at the Eindhoven University of Technology.

2.2.1   Kinematics in Cartesian space

The CFT robot can be described by four Cartesian coordinates (x_{c1}, x_{c2}, x_{c3} and x_{c4}) which describe respectively the up and down movement of the arm, forward and backward movement of the arm, rotation of the arm, forward and backward of the whole robot (Figure 2.2). Table 2.1 describes the dimensions of the robot.

Dimension	Value [m]	Dimension	Value [m]
L_2, d_{1_2}	·0.25	L_8	0.48
L_4	0.05	D_4, d_5	0.0
L_5	0.35	D_6	0.04
L_6	0.30	d_s	0.185
L_7	0.08	$D_{2\_0'}$	0.0916

Table 2.1 Dimensions of the CFT robot.
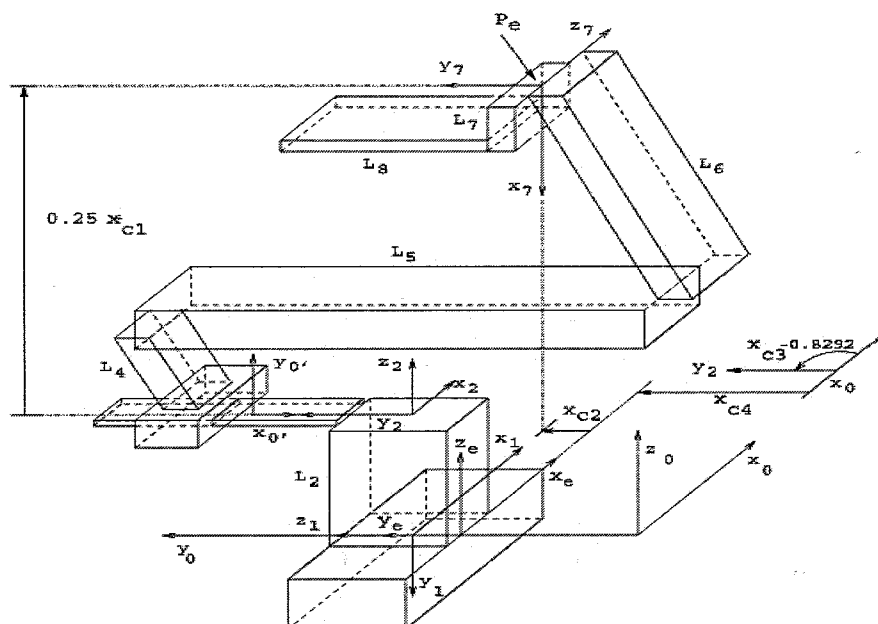
Figure 2.1 CFT robot, Cartesian coordinates

2.2.2 Kinematics in joint space

To describe the kinematics in joint space seven coordinates are needed. To obtain the kinematics in the joint space, Denavit-Hartenberg parameters are used. They are listed in Table 2.2. The coordinates q_1 and q_3 are the translations along the z_1, z_3 axis respectively. On the other hand the q_2, q_4, q_5, q_6 and q_7 joint coordinates are the rotations around the z_2, z_4, z_5, z_6 and z_7 axis respectively.

Because the robot is kinematically constrained, four coordinates can describe the kinematics in the joint space. The constraints are:

$$q_3 = L_4\left(\cos\left(-q_4 - q_5 + \frac{\pi}{2}\right) + \cos\left(-q_4 + \frac{\pi}{2}\right)\right) + d_{2\_0'}$$

$$q_6 = -q_5$$

$$q_7 = \pi - q_4$$

(2.1)

The four reduced coordinates, which describe the kinematics in joint space, are: q_1 translation of the whole robot, q_2 rotation of the whole robot, q_4 rotation of link z_4 and q_5 rotation of link z_5.

i	a_i	α_i	d_i	q_i
0	0	$-\pi/2$	-	-
1	0	$\pi/2$	q_1	0
2	0	$-\pi/2$	L_2	q_2
3	0	$-\pi/2$	q_3	$-\pi/2$
4	L_4	0	0	q_4
5	L_5	0	0	q_5
6	L_6	0	d_6	q_6
7	L_7	0	d_7	q_7
8	L_8	0	0	$\pi/2$
9	-	-	0	0

Table 2.2 Denavit-Hartenberg parameters for the CFT robot.
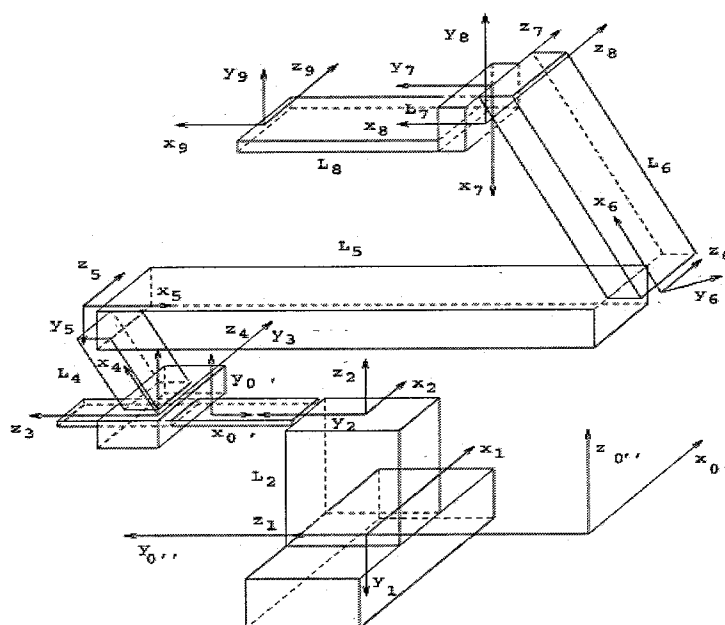
Figure 2.2 Joint frames for the CFT robot

2.3 The CFT robot dynamics

The dynamics of the robot can be derived by the Euler Lagrange approach. In particular, the Euler Lagrange approach is straightforward to compute and from control viewpoint results in a very convenient set of equations. The dynamics of robot 1 (plate number 669358) and robot 2 (plate number 677528) are of the form (Rodriguez-Angeles et al., 2002) (Rodriguez-Angeles, 2002) (Nijmeijer and Rodriguez-Angeles, 2003):

$$M(q_i,\vartheta)\ddot{q}_i + C(q_i,\dot{q}_i,\vartheta)\dot{q}_i + G(q_i,\vartheta) + F(\dot{q}_i,\vartheta) = \tau_{if} \qquad \text{i=1,2} \quad (2.2)$$

Here $M(q_i,\vartheta)$ is the symmetric, positive definite inertia matrix, $C(q_i,\dot{q}_i,\vartheta)$ denotes the coriolis and centrifugal forces, $G(q_i,\vartheta)$ represents the gravity forces and $F(\dot{q}_i,\vartheta)$ are the friction forces.

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The friction force can be modelled by (Rodriguez-Angeles et al., 2002) (Rodriguez-Angeles, 2002) (Nijmeijer and Rodriguez-Angeles, 2003):

$$F(\dot{q}_i, \vartheta) = B_{v,i} \cdot \dot{q}_i + B_{f1,i}\left(1 - \frac{2}{1 + e^{2\omega_{1,i}\dot{q}_i}}\right) + B_{f2,i}\left(1 - \frac{2}{1 + e^{2\omega_{2,i}\dot{q}_i}}\right) \qquad (2.3)$$

Here $B_{v,i}$ represents the diagonal viscous friction coefficient matrix and the other terms approximate the Coulomb and Stribeck effects. To tackle the problem of the discontinuity that the Coulomb friction represents at zero velocity, an approximation based on an exponential function is used. Note that the parameters B_v, B_{f1} and B_{f2} appear in a linear way in the model (2.3).

The vector ϑ (Table 3.2) is a vector with estimated physical parameters of the individual robot. The physical parameters ϑ_p, p=13,..., 32 are estimated by using an extended Kalman filter, whereas the remaining parameters ϑ_p, p=1,..., 12 are identified by considering the linear least square method. The entries of the matrices and the estimated physical parameters for robot 1 as well as robot 2 are listed in appendix A and appendix B.

Parameter	Description
ϑ_1	$m_1 + m_2$
ϑ_2	$m_2 l_{xc2}$
ϑ_3	$M_2 l_{yc2}$
ϑ_4	$m_2(l_{xc2}^2 + l_{yc2}^2) + m_3(l_{yc3}^2 + l_{zc3}^2) + m_4 l_{zc4}^2 + m_5 l^2 - z_{c5} + m_6 l_{zc6}^2 + m_7(l_{yc7}^2 + l_{zc7}^2) + m_8(l_{xc8}^2 + l_{zc8}^2) + I_{xx2} + I_{yy2} + I_{yy3} + I_{zz3} + I_{zz4} + I_{zz5} + I_{zz6} + I_{yy7} + I_{zz7} + I_{xx8} + I_{zz8}$
ϑ_5	$m_4 l_{xc4}$
ϑ_6	$m_4 l_{yc4}$
ϑ_7	$m_6 l_{xc6}$
ϑ_8	$m_6 l_{yc6}$
ϑ_9	$m_5 l_{xc5}$
ϑ_{10}	$m_5 l_{yc5}$
ϑ_{11}	M_6
ϑ_{12}	$m_7 + m_8$

Parameter	Description
ϑ_{13}	B_{v1}
ϑ_{14}	B_{v2}
ϑ_{15}	B_{v4}
ϑ_{16}	B_{v5}
ϑ_{17}	$B_{f1,1}$
ϑ_{18}	$B_{f1,2}$
ϑ_{19}	$B_{f1,4}$
ϑ_{20}	$B_{f1,5}$
ϑ_{21}	$B_{f2,1}$
ϑ_{22}	$B_{f2,2}$
ϑ_{23}	$B_{f2,4}$
ϑ_{24}	$B_{f2,5}$
ϑ_{25}	$w_{1,1}$
ϑ_{26}	$w_{1,2}$
ϑ_{27}	$w_{1,4}$
ϑ_{28}	$w_{1,5}$
ϑ_{29}	$w_{2,1}$
ϑ_{30}	$w_{2,2}$
ϑ_{31}	$w_{2,4}$
ϑ_{32}	$w_{2,5}$

Table 2.3 The vector ϑ with the physical parameters.

Chapter 3

Adaptive mutual synchronization

In this section a short description about synchronisation and adaptive control is given. For adaptive mutual synchronization it is first necessary to understand what synchronization is and what the goal of adaptive control is. Therefore a general introduction in synchronization is given and the goal and set-up of adaptive control is described. Subsequently a controller is developed and stability analysis of an adaptive mutual synchronized system is given.

3.1 Synchronization schemes

There are some different types of synchronization. In case of disconnected systems that present synchronization, this is referred to as natural synchronization. When synchronization is achieved by particular system interconnections, without any artificially introduced action, this is called as self-synchronized system. One refers to a controlled-synchronized system if there exist external actions and/or artificial interconnections. From a control viewpoint controlled-synchronization is the most interesting one. There are basically two different synchronization schemes for controlled-synchronization, namely external synchronization and internal (mutual) synchronization.

The first scheme is a leader-follower synchronization scheme. The goal of leader-follower synchronization is for the leader to track a predefined or arbitrary trajectory, whereas the goal for the follower is to follow a trajectory based on the actual states of the leader robot. So there is only communication from the leader to the follower. This means that the leader determines the behaviour of the follower. Therefore this scheme is also called a coordinated scheme. Since this scheme is not considered in this report, no further attention to this scheme will be paid.

On the other hand there is a cooperative scheme, which is often called a mutual synchronization scheme. Here the robots are equal with respect to each other. The goal of this synchronization scheme is to minimize the error between the robots and the desired trajectory and the error between the relative states of the robots at the same time. There are interconnections between all robots, such that all robots have influence on the combined dynamics. This is an advantage of mutual synchronization. Since, as an error occurs in the robot, which in leader-follower synchronization scheme is the follower, this will, in contrast to leader-follower synchronization, influence the dynamics of the whole system.

For mutual synchronization it is necessary to design a controller and interconnections to guarantee that the states of one robot are synchronized with respect to the desired trajectory and the states of the other robots. For further details about synchronization the reader is referred to Rodriguez-Angeles (2002) or Nijmeijer and Rodriguez-Angeles (2003)

3.2 Adaptive control

In general the starting point to design a feedback controller is the following: There is a model of a system available and it is assumed that the structure of the model is correct. The parameters are assumed to be constant with respect to place and time. But mostly this is not correct. The model is never correct due to unmodelled dynamics, The parameters are slowly varying with respect to time and sometimes fast varying with respect to the state.

To handle these uncertainties an adaptive controller can be used. The idea of an adaptive controller is to estimate the unknown or uncertain parameters in real time and use them in the control law, in order to achieve satisfactory behaviour in the presence of uncertainties or varying parameters of the system. Therefore, the velocity of the robot and the synchronization velocity error are used in an adaptation mechanism to fit the uncertain and unknown parameters in such a way that the tracking error is minimized.

3.3 Adaptive mutual synchronization

In section 2.3 it is shown that the dynamics of the CFT robots can be modelled by:

$$M(q_i,\vartheta)\ddot{q}_i + C(q_i,\dot{q}_i,\vartheta)\dot{q}_i + G(q_i,\vartheta) + F(\dot{q}_i,\vartheta) = \tau_{if} \quad i=1,2 \qquad (3.1)$$

To create interconnections between the two robots and to achieve mutual synchronization, for each robot i the reference trajectory is defined as the desired trajectory minus the errors between the relative states of robot i and the other robot j:

$$q_{ri} = q_d - k(q_i - q_j) \qquad (3.2.a)$$

$$\dot{q}_{ri} = \dot{q}_d - k(\dot{q}_i - \dot{q}_j) \qquad (3.2.b)$$

$$\ddot{q}_{ri} = \ddot{q}_d - k(\ddot{q}_i - \ddot{q}_j) \qquad (3.2.c)$$

The synchronization error s_i are defined for each robot i as the error between robot i and the desired trajectory plus the sum of the errors between the relative states of robot i and the other robot j:

$$s_i = q_i - q_{ri} = e_{ii} + k \cdot e_{ij} \qquad (3.3.a)$$

$$\dot{s}_i = \dot{q}_i - \dot{q}_{ri} = \dot{e}_{ii} + k \cdot \dot{e}_{ij} \qquad (3.3.b)$$

$$\ddot{s}_i = \ddot{q}_i - \ddot{q}_{ri} = \ddot{e}_{ii} + k \cdot \ddot{e}_{ij} \qquad (3.3.c)$$

where, the error between robot i and the desired trajectory is defined as:

$$e_{ii} = q_i - q_d \qquad (3.4.a)$$

$$\dot{e}_{ii} = \dot{q}_i - \dot{q}_d \qquad (3.4.b)$$

$$\ddot{e}_{ii} = \ddot{q}_i - \ddot{q}_d \qquad (3.4.c)$$

The error between the robot i and the other robot j is defined as:

$$e_{ij} = q_i - q_j \qquad (3.5.a)$$

$$\dot{e}_{ij} = \dot{q}_i - \dot{q}_j \qquad (3.5.b)$$

$$\ddot{e}_{ij} = \ddot{q}_i - \ddot{q}_j \qquad (3.5.c)$$

The controller for the i-th robot is given by:

$$\tau_{if} = M(q_i)\ddot{q}_{ri} + C(q_i,\dot{q}_i)\dot{q}_{ri} + G(q_i) + \hat{F}(\dot{q}_i,\hat{\theta}) - K_d\dot{s}_i - K_p s_i \qquad (3.6)$$

where $\hat{F}(\dot{q}_i,\hat{\theta})$ represents the friction term with the estimated linear friction parameters. It is assumed that the inertia matrix, Coriolis and centrifugal forces and the gravity forces are exact. The closed loop error dynamics for robot i becomes now:

$$M(q_i)\ddot{s}_i + C(q_i,\dot{q}_i)\dot{s}_i + K_d\dot{s}_i + K_p s_i = \hat{F}(\dot{q}_i) - F(\dot{q}_i) \qquad (3.7.a)$$

$$M(q_i)\ddot{s}_i + C(q_i,\dot{q}_i)\dot{s}_i + K_d\dot{s}_i + K_p s_i = We_\theta \qquad (3.7.b)$$

Because the uncertain friction parameters are linear (3.7.a), this can be written in the following form:

$$\hat{F}(\dot{q}_i) - F(\dot{q}_i) = W_i e_{\theta i} \tag{3.8}$$

with $e_{\theta i}$ defined as:

$$e_{\theta i} = \hat{\theta}_i - \theta_i \tag{3.9}$$

where θ_i is the vector with the exact linear friction parameters for robot i, equivalent to the physical parameters for p=13,..., 24 in the vector ϑ (Table 2.3) , while $\hat{\theta}_i$ is the vector with the estimated linear friction parameters for robot i.

$$\theta_i = \begin{pmatrix} B_{v1} & B_{v2} & B_{v4} & B_{v5} & B_{f1,1} & B_{f1,2} & B_{f1,4} & B_{f1,5} & B_{f2,1} & B_{f2,2} & B_{f2,4} & B_{f2,5} \end{pmatrix}^T \tag{3.10}$$

$$W_i = \begin{pmatrix}
\dot{q}_1 & 0 & 0 & 0 \\
0 & \dot{q}_2 & 0 & 0 \\
0 & 0 & \dot{q}_3 & 0 \\
0 & 0 & 0 & \dot{q}_4 \\
\left(1 - \dfrac{2}{1+e^{2\omega_{1,1}\dot{q}_1}}\right) & 0 & 0 & 0 \\
0 & \left(1 - \dfrac{2}{1+e^{2\omega_{1,2}\dot{q}_1}}\right) & 0 & 0 \\
0 & 0 & \left(1 - \dfrac{2}{1+e^{2\omega_{1,4}\dot{q}_1}}\right) & 0 \\
0 & 0 & 0 & \left(1 - \dfrac{2}{1+e^{2\omega_{1,5}\dot{q}_1}}\right) \\
\left(1 - \dfrac{2}{1+e^{2\omega_{2,1}\dot{q}_1}}\right) & 0 & 0 & 0 \\
0 & \left(1 - \dfrac{2}{1+e^{2\omega_{2,2}\dot{q}_1}}\right) & 0 & 0 \\
0 & 0 & \left(1 - \dfrac{2}{1+e^{2\omega_{2,4}\dot{q}_1}}\right) & 0 \\
0 & 0 & 0 & \left(1 - \dfrac{2}{1+e^{2\omega_{2,5}\dot{q}_1}}\right)
\end{pmatrix}^T \tag{3.11}$$

The goal of the adaptation mechanism is to make an estimate of the linear friction parameters in such a way that s_i -> 0 for $t \rightarrow \infty$. Therefore we need Lyapunov's theorem and a candidate Lyapunov function is defined as:

$$V(s_i, \dot{s}_i, e_{\theta i}) = \frac{1}{2} \dot{s}_i^T M(q_i) \dot{s}_i + \frac{1}{2} s_i^T K_p s_i + \frac{1}{2} e_{\theta i}^T \Gamma e_{\theta i} \tag{3.12}$$

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$V(s_i,\dot{s}_i,e_{\theta i})$ is positive definite for K_p and $\Gamma > 0$, $s_i,\dot{s}_i,e_{\theta i}$ and $V(s_i,\dot{s}_i,e_{\theta i}) = 0$ only if $s_i = 0, \dot{s}_i = 0$ and $e_{\theta i} = 0$

$$\dot{V}(s_i,\dot{s}_i,e_{\theta i}) = \dot{s}_i^T (M(q_i)\ddot{s}_i + \frac{1}{2}\dot{M}(q_i) \cdot \dot{s}_i + K_p s_i) + e_{\theta i}^T \Gamma \dot{e}_{\theta i} \qquad (3.13)$$

$$\dot{V}(s_i,\dot{s}_i,e_{\theta i}) = \dot{s}_i^T \left(\frac{1}{2}\dot{M}(q_i) - C(q_i,\dot{q}_i) \right) \dot{s}_i - \dot{s}_i^T K_d \dot{s}_i + e_{\theta i}^T \left(\Gamma \dot{e}_{\theta i} + W^T \dot{s}_i \right) \qquad (3.14)$$

$$= 0 \qquad\qquad \leq 0$$

To guarantee that $\dot{V}(s_i,\dot{s}_i,e_{\theta i}) \leq 0$, $e_{\theta i}^T \left(\Gamma \dot{e}_{\theta i} + W^T \dot{s}_i \right)$ is chosen equals zero, so

$$\dot{e}_{\theta i} = -\Gamma^{-1} W \dot{s}_i \qquad (3.15)$$

which results in

$$\dot{V}(s_i,\dot{s}_i,e_{\theta i}) = -\dot{s}_i^T K_d \dot{s}_i \leq 0 \qquad (3.16)$$

$\dot{V}(s_i,\dot{s}_i,e_{\theta i})$ is semi-negative definite if $K_d \geq 0$. The synchronization errors $s_i,\dot{s}_i$ and the parameter errors $e_{\theta i}$ are stable, but not necessarily asymptotically stable. To prove asymptotic stability, Barbalat's lemma is used.

Because $s_i,\dot{s}_i,e_{\theta i}$ are stable,

$$\lim_{t\to\infty} \int_0^t \dot{V}(s_i,\dot{s}_i,e_\theta)dt = \lim_{t\to\infty} -\int_0^t \dot{s}_i^T K_d \dot{s}_i dt = \frac{1}{2}\dot{s}_i^T M(q_i)\dot{s}_i + \frac{1}{2}s_i^T K_p s_i + \frac{1}{2}e_{\theta i}^T \Gamma e_{\theta i} \Big|_0^T = a \quad (3.17)$$

with a a constant.

from $M(q_i)\ddot{s}_i = -C(q_i,\dot{q}_i)\dot{s}_i - K_d\dot{s}_i - K_p s_i + We_{\theta i}$ and $s_i,\dot{s}_i,e_{\theta i}$ are stable it follows that:

$$\ddot{V}(s_i,\dot{s}_i,e_{\theta i}) = -2\dot{s}_i^T K_d \ddot{s}_i \leq M \text{ so } \dot{V}(s_i,\dot{s}_i,e_{\theta i}) \text{ is uniform continuous.}$$

From uniform continuity, (3.15) and Barbalat's lemma it follows that:

$$\lim_{t\to\infty} \dot{V}(s_i,\dot{s}_i,e_{\theta i}) = 0 \text{ so } \dot{s}_i(t) = 0 \text{ for } t \to \infty \text{ and also } \ddot{s}_i(t) = 0 \text{ for } t \to \infty \qquad (3.18)$$

Using the result of (3.17) in (3.7.b) it follows that:

$$K_p s_i = We_{\theta i} \qquad (3.19)$$

$We_{\theta i}$ is constant and $e_{\theta i}$ is constant, so $\dot{W}e_{\theta i} = 0$. If $\dot{W}$ is not equal to zero than $e_{\theta i} = 0$ and $s_i = 0$.

Now it is proved that s_i en $\dot{s}_i$ are asymptotically stable. However it is not yet proved that asymptotical stability also means global asymptotic synchronization of the robots. This proof is given in Rodriguez-Angeles (2002) or Nijmeijer and Rondriguez-Angeles (2003).

Chapter 4

Adaptive mutual synchronization on simulation level

The controller designed in chapter 3 will be tested in a simulation environment and thereafter in an experiment. The problems with the coupled dynamics will be avoided, so first only the q_1 direction will be considered. A model feedback controller controls the q_2 direction, where are assumed to be exact. A PD controller controls the q_4 and q_5 directions. This is because otherwise there are too many calculations for an experiment on the CFT robot system. In the future programming the controller in C instead of programming the controller in Matlab Simulink can perhaps solve this problem.

4.1   Simulation adaptive mutual control of the q_1 direction, varying k

For mutual synchronisation for each direction a desired trajectory is needed. Instead of following each other, the system should also follow a desired trajectory. In this case the desired trajectory is a sine wave with a frequency of 0.2 rad/s for x_{c1}, x_{c2}, x_{c3} and x_{c4} with amplitudes and initial position as given in Table 4.1. The initial velocity is equal to zero in all directions.

direction	amplitude	initial position
x_{c1}	0.08	-0.1343
x_{c2}	0.1	0.2766
x_{c3}	0.9420	2.4
x_{c4}	0.1	-0.265

Table 4.1 Initial positions and amplitudes of the desired trajectory.

Remark that:

$$x_{c1} = 0.25 - (L_4 + L_6)(\cos(q_4 + q_5) + \cos(q_4))$$

$$x_{c2} = d_{2\_0'} - d_s - L_6(\sin(q_4 + q_5) + \sin(q_4))$$

$$x_{c3} = q_2 + 2.4$$ (4.1)

$$x_{c4} = q_1 - d_s$$

So the q_1 direction is equal to the x_{c4} direction, which is the translation of the whole robot. The velocities and accelerations are obtained by taken the time derivative of the measured position signal. Using the desired trajectory and its derivatives, the reference trajectory for each direction can be calculated:

$$q_{ri} = q_d - k(q_i - q_j)$$ (4.2.a)

$$\dot{q}_{ri} = \dot{q}_d - k(\dot{q}_i - \dot{q}_j)$$ (4.2.b)

$$\ddot{q}_{ri} = \ddot{q}_d - k(\ddot{q}_i - \ddot{q}_j)$$ (4.2.c)

During the simulation the linear friction parameters in the q_1 direction, B_{v1}, $B_{f1,1}$ and $B_{f2,1}$, contain an initial error of 50 percent of the true linear friction parameter in the simulation model. The nonlinear friction parameters are assumed to be faultless. Even the linear and nonlinear friction parameters in the q_2 direction are assumed to be exact.

Hereafter the gains k, K_p, K_d and Γ are tuned. The gains K_p and K_d are chosen not too high, because otherwise it takes too much time to get a constant error s. On account of the initial error the Γ gains cannot be chosen arbitrarily high, because if these gains are chosen too high numerical problems occur and the system becomes unstable. If the feedback gains, K_p and K_d, are chosen higher, the adaptation gain Γ should be chosen higher as well. If the adaptation gain Γ cannot be chosen higher, it will take more time to get a constant error s.

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The price to be paid, if the feedback gains are less powerful, is that the error s is bigger than with stronger feedback gains. The feedback gains are chosen smaller, because for this study not the size of the error s is interesting, but convergence and stability of the system. The gains are nevertheless chosen with respect to the gain tuning procedure described in Rodriguez-Angeles (2002) and Nijmeijer and Rodriguez-Angeles (2003).

Finally the matrices K_p and K_d are chosen as:

$$K_p = \begin{pmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 12000 & 0 \\ 0 & 0 & 0 & 12000 \end{pmatrix} \tag{4.3}$$

$$K_d = \begin{pmatrix} 60 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 \\ 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & 300 \end{pmatrix} \tag{4.4}$$

The Γ matrices are tuned by trial and error:

$$\Gamma_{robot1} = diag\begin{pmatrix} 2.5 \cdot 10^{-4} \\ 4 \cdot 10^{-3} \\ 4 \cdot 10^{-3} \end{pmatrix} \quad \Gamma_{robot2} = diag\begin{pmatrix} 1.67 \cdot 10^{-4} \\ 4 \cdot 10^{-3} \\ 4 \cdot 10^{-3} \end{pmatrix} \tag{4.5}$$

The weight between e_{ii} and e_{ij} is described by the weighting factor k. The e_{ii} is the error between the desired trajectory and the robot. The e_{ij} is the error between the robot i and the other robot j. The sum of these errors forms the error s_i:

$$s_i = e_{ii} + \sum k \cdot e_{ij} \tag{4.6}$$

During the simulation each 50 seconds till 250 seconds the weighting factor k will increase with 0.2. Thereafter the weighting factor k is kept constant at value 1. Figure 4.1 shows the weighting factor k during the simulation.
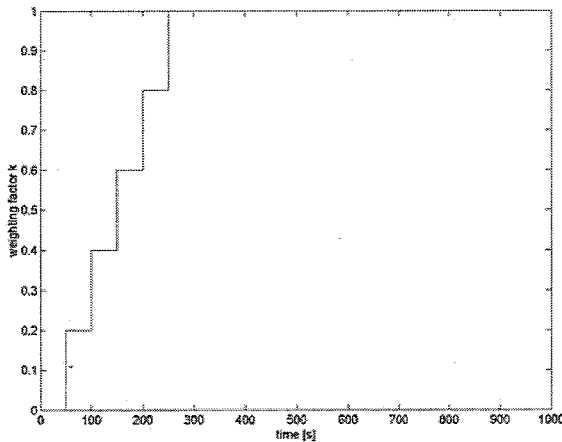

Figure 4.1 The weight factor k during simulation.

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If simulated with these settings, the following results are obtained:
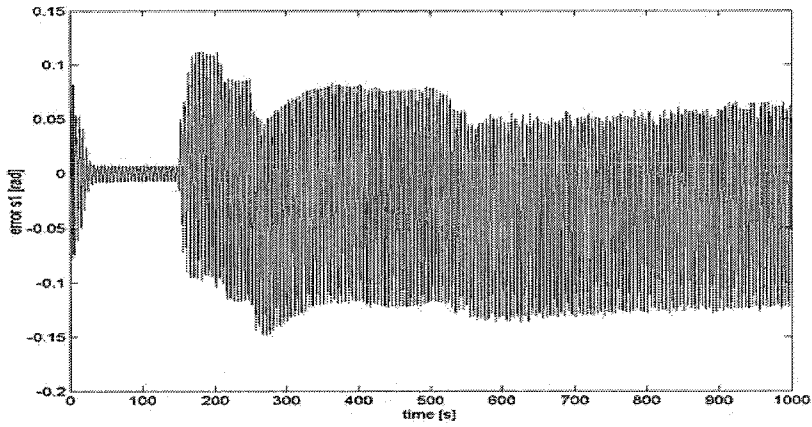

Figure 4.2a error s_1 robot 1 during simulation.
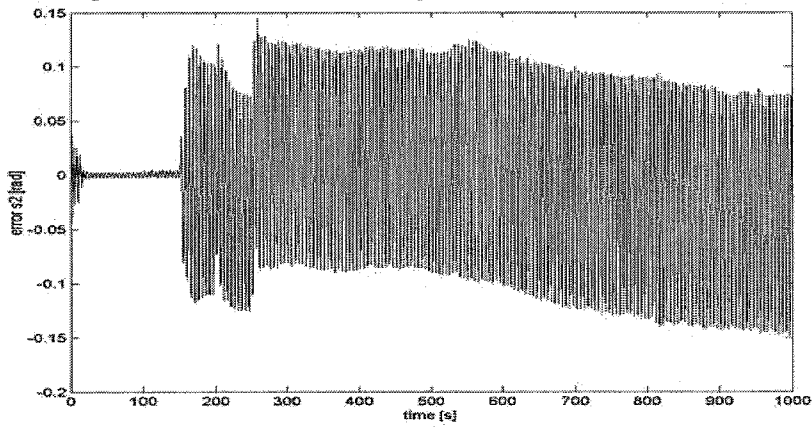

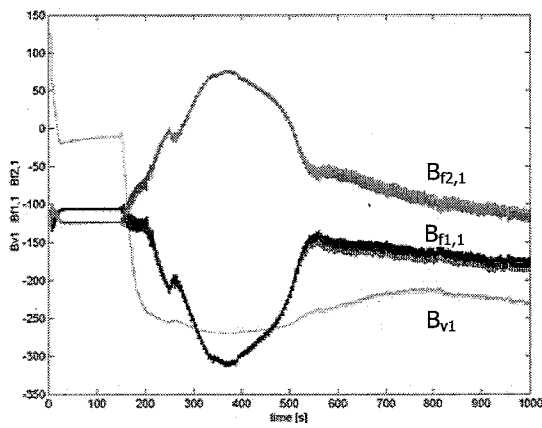Figure 4.2b error s_2 robot 2 during simulation

Figure 4.3a linear friction parameters robot 1 during simulation.
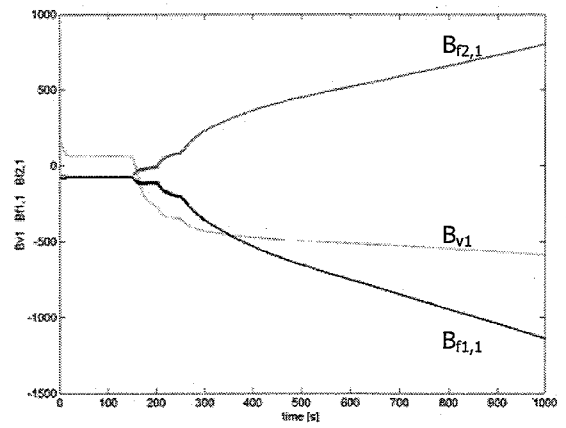

Figure 4.3b linearfriction parameters robot 2 during simulation.

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Clearly it is seen that after 150 seconds, so k=0.6 or higher, the controller does not work properly anymore. The error s exploded and the friction parameters drift away. Of course it is not necessary that the friction parameters are constant, but they have to behave in an acceptable range. Otherwise this can lead to unreasonable control actions, which can result in instability of the system. The same behaviour is also obtained for simulations where the weighting factor k is applied with other, smooth, functions. This means that the behaviour of the system is independent on how the weighting factor is applied to the system.

It can be concluded that the weighting factor k cannot be chosen arbitrarily high. It is not known how the weighting factor k should be chosen. The only way to choose a weighting factor k is by trial and error. For adaptive control it is even not logical to start with a high weighting factor. This is because of the following reasons:

If a robot should reckon with the other robots, firstly it is necessary to know its own behaviour. This is necessary to react correctly on the behaviour of the other robots. If a robot is incapable to predict its own behaviour, it will react, based on the incorrect information, on the behaviour of the other robots and create errors. So instead of minimize the error it will create an error. Therefore first each robot should estimate its own uncertain linear friction parameters, so the robot can react in a correct way.

The other reason, why it is logical, is the following one. Adaptive mutual synchronization where the own system is not well known is solving the reversed problem. If still one robot creates an error than due to synchronization the error of one robot becomes the problem of all the other robots. All the other robots have to solve, in cooperation with the robot with the error, the problem of one robot. This while the other robots know nothing about the behaviour of this robot. So they can only solve the error for this robot to react by themselves. All robots have to reckon with the robot, which flatly refuses to do his task. The consequence is that these robots are forced to make errors with respect to the desired trajectory q_d. The result is that, in total, it becomes something like: Minimize the collective errors, whereas the task (following q_d) is not done.

Mutual synchronization means as well that the estimated linear friction parameters are not only based on the error between the robot and the desired trajectory, but are also based on the errors between the relative states of the robots. So the estimated linear friction parameters represent not anymore the linear friction parameters of the robot, but are chosen in such way that the synchronization error s is minimized.

During mutual synchronization one robot is accusing another robot of errors committed by itself. Therefore it would be preferable to first solve the own problem and thereafter solve the problems with respect to each other. In an ideal situation where it is possible to force the errors between the robots and desired trajectory to zero, the errors between the relative states of the robots are even zero. In this ideal situation synchronization seems to be redundant.

This is slightly pessimistic with respect to synchronization. Of course synchronization has advantages. If a robot tracks a desired trajectory, measurement noise and other external disturbances disturb the tracking. In case of a synchronised system, the system can deal with these disturbances. For example: two robots should lift a pipe horizontal. The load to each robot is not equal to each other. This causes that one robot lifts faster than the other robot. In case of a synchronised system the pipe remains horizontal, as a consequence of the interconnections between the robots. On the other hand if there are no interconnections one robot lifts faster than the other, which results in a falling pipe. So if the robots are not able to track its desired trajectory without errors, synchronization is useful.

4.2   Simulation adaptive mutual control of the q_1 direction, k=0.1

To compare the behaviour of a controller with adaptation mechanism and without adaptation mechanism, a simulation is done where the first 200 seconds the weight factor k equals to 0 and thereafter equals to 0.1. The other gains are set as in section 4.1.
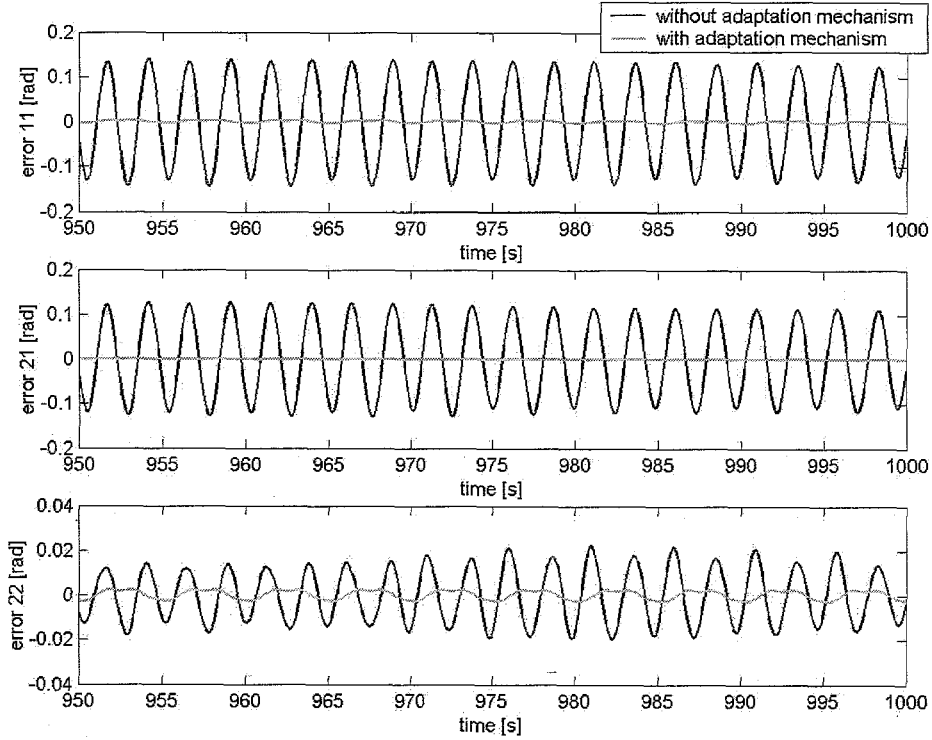
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Figure 4.4 Errors during simulation with and without adaptation mechanism.

In order to express the performance of the adaptive synchronization scheme, an error index is defined (Manssouri, 2002). Since only the position of the robots can be measured, the error index is defined by the synchronization position error s (4.6) or the position error e (3.5).

$$error\ index = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} e(\tau)^T e(\tau) d\tau \tag{4.7}$$

$$performance \sim [error\ index]^{-1} \tag{4.8}$$

error	error index
without adaptation mechanism	
e_{11}	$6.0800\ 10^{-3}$
e_{22}	$2.8403\ 10^{-4}$
e_{21}	$3.8347\ 10^{-3}$
with adaptation mechanism	
e_{11}	$4.2190\ 10^{-6}$
e_{22}	$4.4512\ 10^{-6}$
e_{21}	$5.9168\ 10^{-8}$

Table 4.1 Error indices during simulation with and without adaptation mechanism.

In Table 4.1 the error indices of the errors e_{11}, e_{22} and e_{21} are calculated during 50 seconds. The calculation starts at 950 seconds and stops at 1000 seconds. From Table 4.1 it is obvious

that it is useful to use an adaptation mechanism during mutual synchronization of the CFT robot system. At 1000 seconds the adaptation mechanism is still active. So an improvement of the results can be expected. This trend can also be been seen in Figure 4.5.
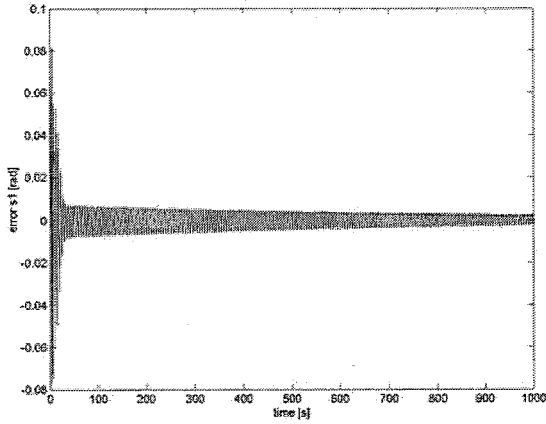

Figure 4.5a Error s_1 during simulation.   Figure 4.5b Error s_2 during simulation.
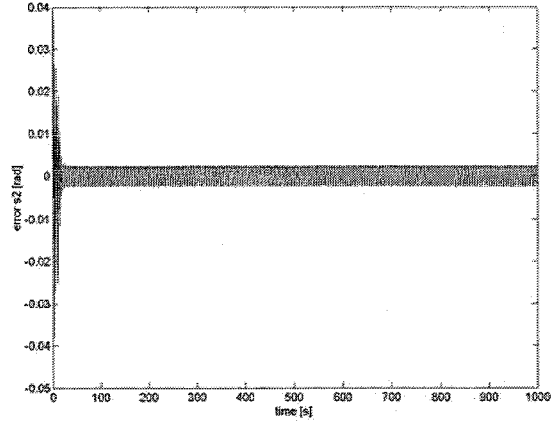
error	error index
from t=200 s until t=250 s	
s_1	$2.2657 10^{-5}$
s_2	$3.2640 10^{-8}$
from t=950 s until t=1000 s	
s_1	$4.1866 10^{-6}$
s_2	$4.5118 10^{-6}$

Table 4.2 Error indices during simulation calculated at t=200 and t=1000.

It is remarkable, when looking to Figure 4.5a and Figure 4.5b, that only the synchronization error s_1 becomes smaller. In Table 4.2 error indices are calculated at two different points in time. Looking to the error indices in Table 4.2, it is seen that the error s_1 and error s_2 are of the same order after 1000 seconds. It is even remarkable that the error s_2 is slightly increased during the simulation. To say more about the behaviour between the errors s for robot 1 and 2 it is necessary to make a longer simulation. If the error s becomes smaller, the parameter adaptation becomes slower. So if the error becomes small, it is necessary to increase the diagonal terms of the Γ matrices for a faster adaptation of the linear friction parameters in order to increase the performance of the system.

Chapter 5

Adaptive mutual synchronization: experiments

To validate the developed controller, experiments are necessary. In the simulations a dynamic model of the real system is used. There is no measurement noise, the physical parameters are constant in time and place and there are no other environmental disturbances. So in the simulations a very "clean" environment is used. Therefore it is necessary to validate the designed controller on a real system. To handle with measurement noise there are some possibilities. These will be discussed in section 5.1. The different experiments will be discussed in section 5.2 and section 5.3.

5.1   How to handle the measurement noise

Measurement noise will occur during the experiments. Because only the position is measured, the velocities and accelerations are obtained by differentiating the position measurements. The measurement noise causes a lot of trouble on the differentiated signals.

The controller needs the following signals:

$$\tau_{if} = M(q_i)\ddot{q}_{ri} + C(q_i,\dot{q}_i)\dot{q}_{ri} + G(q_i) + \hat{F}(\dot{q}_i,\hat{\vartheta}) - K_d\dot{s}_i - K_p s_i \tag{5.1}$$

$$\dot{e}_{\vartheta i} = -\Gamma^{-1}W\dot{s}_i \tag{5.2}$$

$$\dot{q}_{ri} = \dot{q}_d - \sum k(\dot{q}_i - \dot{q}_j) \tag{5.3.a}$$

$$\ddot{q}_{ri} = \ddot{q}_d - \sum k(\ddot{q}_i - \ddot{q}_j) \tag{5.3.b}$$

$$s_i = q_i - q_{ri} \tag{5.4.a}$$

$$\dot{s}_i = \dot{q}_i - \dot{q}_{ri} \tag{5.4.b}$$

Because the viscous friction coefficient consists of multiplication of the robot velocity with the synchronization velocity error $\dot{s}_i$, the differentiated robot velocity appears quadratic in the adaptation and control law. Because of the measurement noise on the measured position, the derivative of this signal contains also measurement noise. The squared measurement noise can disrupt the parameter adaptation. The derived controller should be modified in such a way, that it becomes robust for measurement noise. A controller, which can handle with measurement noise, is presented in De Jager (2003). In this controller the $\dot{q}^2$ term is removed. This is done by using the reference velocity $\dot{q}_r$ instead of the derivative of the measured position, $\dot{q}$, in the regressor W of the adaptation mechanism. Thereafter the $\dot{s}$ is replaced by a combination of $\dot{s}$ and s . The advantage of this is that the adaptation not only depends on the error in velocity but also depends on the position error. This results in a more consistent estimation of the friction parameters.

$$\dot{z}_i = \dot{s}_i + \Lambda s \tag{5.5}$$

The controller becomes then:

$$\tau_{if} = M(q_i)\ddot{q}_{ri} + C(q_i,\dot{q}_{ri})\dot{q}_{ri} + G(q_i) + \hat{F}(\dot{q}_{ri},\hat{\vartheta}) - K_d\dot{s}_i - K_p s_i \tag{5.6}$$

$$\dot{e}_{\vartheta i} = -\Gamma^{-1}W(\dot{q}_{ri})\dot{z}_i \tag{5.7}$$

The desired trajectories, position, velocity and acceleration respectively, are numerically generated. This implies that they do not contain any measurement noise and can be considered as smooth signals without noise. The reference velocity depends on the desired velocity and the derivative of the measured position. However the derivative of the measured position contains measurement noise. So the measurement noise still appears quadratic in the adaptation and control law. The only way to avoid this is to replace $\dot{q}_r$ by $\dot{q}_d$. If $\dot{q}_r$ is replaced by $\dot{q}_d$ then the control law becomes:

$$\tau_{if} = M(q_i)\ddot{q}_{ri} + C(q_i,\dot{q}_d)\dot{q}_d + G(q_i) + \hat{F}(\dot{q}_d,\hat{\vartheta}) - K_d \dot{s}_i - K_p s_i \qquad (5.8)$$

From (5.8) and (3.1) the closed loop error dynamics

$$M(q_i)\ddot{s}_i + C(q_i,\dot{q}_d)\dot{e}_{ii} + K_d \dot{s}_i + K_p s_i = \hat{F}(\dot{q}_d) - F(\dot{q}_i) \qquad (5.9)$$

In (5.9) it is seen that a part of the interconnections between the robots are removed, a velocity error $\dot{e}_{ii}$ instead of a velocity synchronization error $\dot{s}_i$ in the closed loop error dynamics, which is not desirable. To keep the interconnections the factor k is kept so small that the signal with the measurement noise becomes small with respect to the signals without measurement noise in the reference trajectory.

The second derivative of the measured position contains a lot of noise. This is the result of differentiating a not smooth signal, in this case the position with measurement noise. The disturbances are so large that they disturb the whole control law. The only option is to replace $\ddot{q}_{ri}$ by $\ddot{q}_d$. The disadvantage of this is that a part of the interconnections between the robots is removed. The controller becomes now:

$$\tau_{if} = M(q_i)\ddot{q}_d + C(q_i,\dot{q}_{ri})\dot{q}_{ri} + G(q_i) + \hat{F}(\dot{q}_{ri},\hat{\vartheta}) - K_d \dot{s}_i - K_p s_i \qquad (5.10)$$

$$\dot{e}_{\vartheta i} = -\Gamma^{-1} W(\dot{q}_{ri}) \dot{z}_i \qquad (5.11)$$

Because of the changes in the control law and adaptation mechanism a new stability analysis is necessary. Because this is very difficult and assumptions with respect to the measurement noise are needed, this is not done.

5.2   Mutual synchronization

To evaluate the controller with adaptation mechanism, first the performance of a controller without adaptation mechanism will be considered. The friction parameters are set as listed in appendix B. These are the best estimates available at the moment. The gains are set as follows.

$$K_p = \begin{pmatrix} 2000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 12000 & 0 \\ 0 & 0 & 0 & 12000 \end{pmatrix} \qquad (5.12)$$

$$K_d = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 \\ 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & 300 \end{pmatrix}$$

(5.13)

First of all the influence of synchronization on the different errors is assessed. Therefore an experiment is done, where the first 50 seconds the weighting gain k is equal to zero, after 50 seconds the weighting gain k switches to k=0.1. The results of this experiment are presented in Figure 5.1 and Table 5.1. In Table 5.1 the error indices of the errors e_{11}, e_{22} and e_{21} during the experiment are presented. The error indices are for both situations calculated during 50 seconds steady state behaviour.
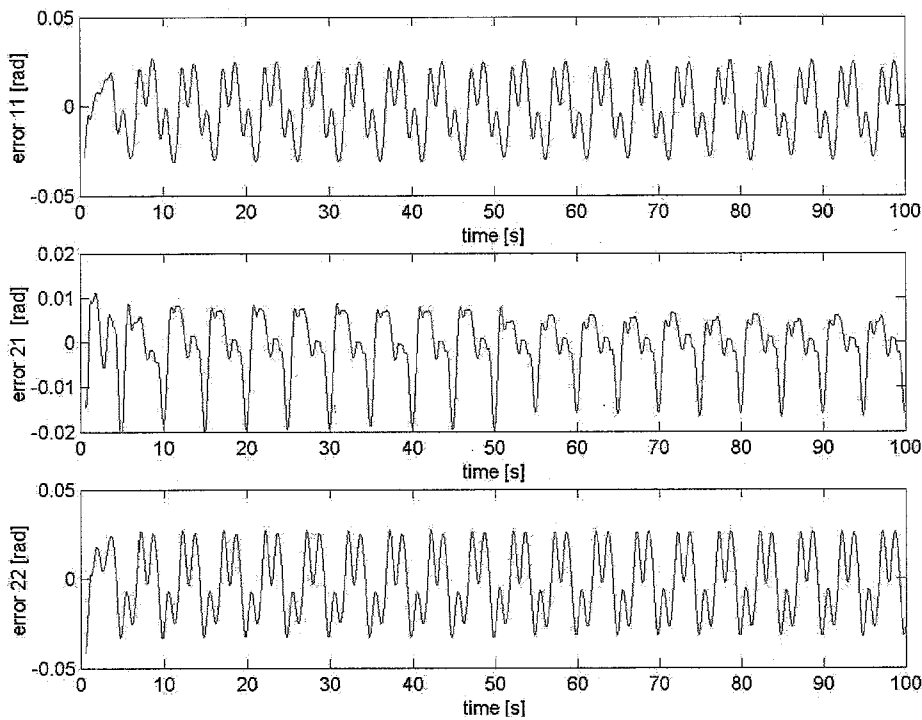
Figure 5.1 Errors during experiment with and without mutual synchronization.

error	error index
without mutual synchronization	
e_{11}	$2.8488 \ 10^{-4}$
e_{22}	$3.3966 \ 10^{-4}$
e_{21}	$5.8151 \ 10^{-4}$
s_1	$2.8579 \ 10^{-4}$
s_2	$3.5154 \ 10^{-4}$
with mutual synchronization	
e_{11}	$2.7499 \ 10^{-4}$
e_{22}	$3.3998 \ 10^{-4}$
e_{21}	$3.5737 \ 10^{-5}$
s_1	$2.7242 \ 10^{-4}$
s_2	$3.5041 \ 10^{-4}$

Table 5.1 Error indices during experiment with and without mutual synchronization.

In Figure 5.1 and Table 5.1 it is seen that synchronization had a positive effect on the cooperation of the two robots, because after 50 seconds, when the mutual synchronization is started, the error between the robots, e_{21}, becomes smaller. The error between the robot and the desired trajectory increase slightly for robot 2, but decreases for robot 1. Because the error e_{22} increases and e_{11} and e_{21} decreases it is difficult to say something about the effect on the total performance, tracking the desired trajectory as well as cooperation of the robots, of the system. Therefore for both situations the synchronization error s, $s_i = e_{ii} + k \cdot e_{ij}$ with k= 0.1, is calculated. In Table 5.1 it is seen that for robot 1 the performance with mutual synchronization is improved with 4.6 percent, while for robot 2 the performance is still improved slightly with 0.3 percent. However, for both robots the total performance with mutual synchronization is improved.

5.3 Adaptive Mutual synchronisation

During this experiment the friction parameters are initially set as listed in appendix B, which are the best estimates available at this moment. The intention is to get a better tracking with respect to each other as well as with respect to the desired trajectory with the developed adaptation mechanism (5.9). The gains K_p and K_d are set as in 5.2.1. The adaptation gain Γ is set as in the simulations done in chapter 4.

$$\Gamma_{robot1} = diag\begin{pmatrix} 2.5 \cdot 10^{-4} \\ 4 \cdot 10^{-3} \\ 4 \cdot 10^{-3} \end{pmatrix} \qquad (5.14)$$

$$\Gamma_{robot2} = diag\begin{pmatrix} 1.67 \cdot 10^{-4} \\ 4 \cdot 10^{-3} \\ 4 \cdot 10^{-3} \end{pmatrix} \qquad (5.15)$$

The Λ gain in $\dot{z}_i = \dot{s}_i + \Lambda s$ is set for both robots different. This is done by trial and error, to require that the friction parameters contain in an acceptable range.

$$\Lambda_{robot1} = 5$$

$$\Lambda_{robot2} = 2 \qquad (5.16)$$

In this experiment the first 600 seconds adaptive control without mutual synchronization is used. This means that the weighting gain k is equal to 0 during this period. After 600 seconds the weighting gain k is changed to 0.1. Figure 5.3 shows the experimental obtained errors e_{11}, e_{22} and e_{21} during this experiment.
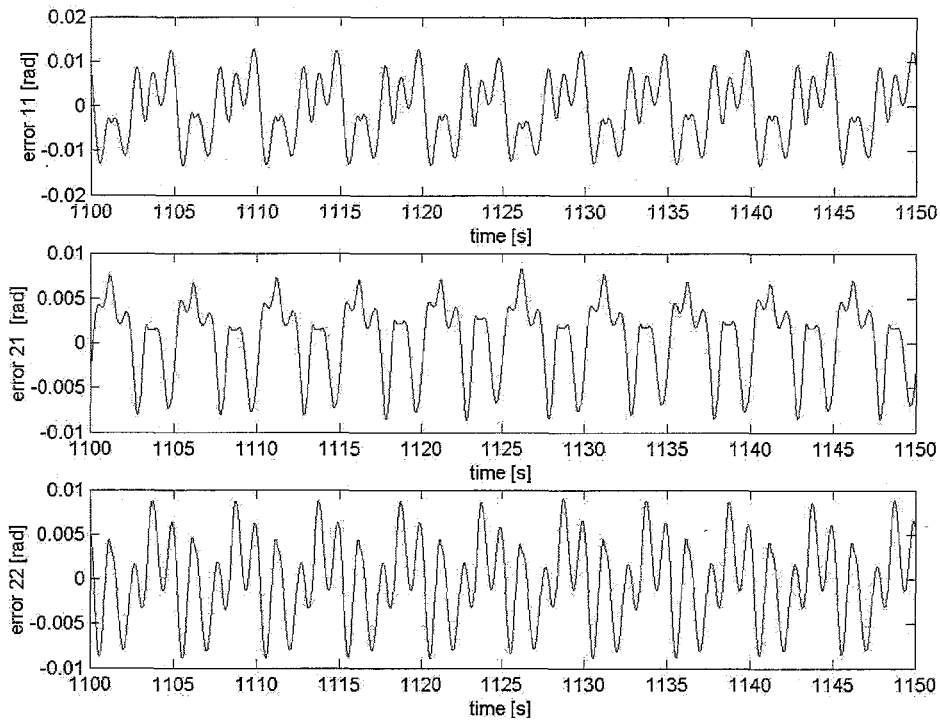
Figure 5.2 Mutual synchronization with adaptation mechanism, experiment.

error	error index
without adaptation mechanism	
s_1	$2.7242\ 10^{-4}$
s_2	$3.5041\ 10^{-4}$
with adaptation mechanism	
s_1	$5.2710\ 10^{-5}$
s_2	$2.0111\ 10^{-5}$

Table 5.2 Error indices during experiment with and without adaptation mechanism.

In Table 5.2 the error indices during the steady state behaviour of the experiment without adaptation mechanism and the experiment with adaptation mechanism are calculated during 50 seconds. The synchronization error s_1 is reduced with approximately 80 percent and the synchronization error s_2 with approximately 94 percent using an adaptation mechanism during mutual synchronization. This means that using this adaptation mechanism during mutual synchronization is very useful.

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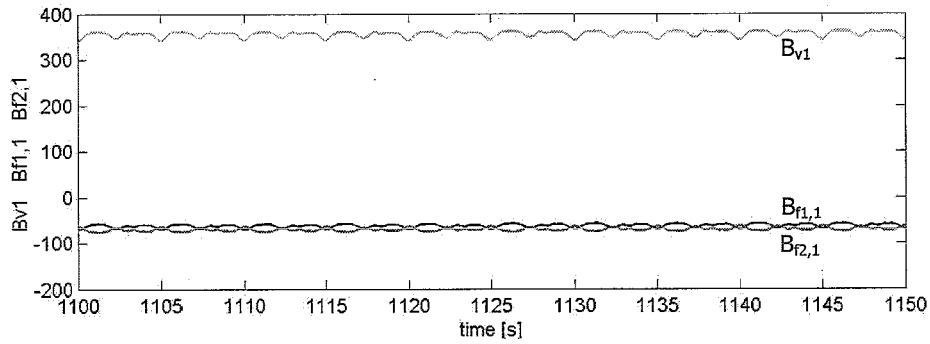
Figure 5.3a Friction parameters robot 1 during experiment.
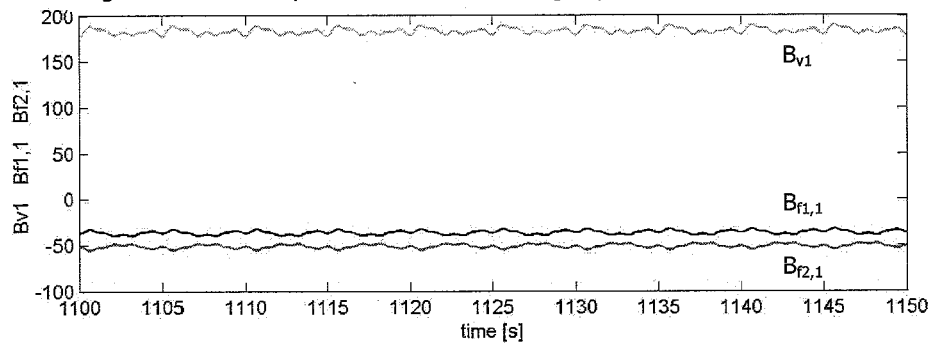
Figure 5.3b Friction parameters robot 2 during experiment.

In Figure 5.4a and Figure 5.4b the linear friction parameters of robot 1 and robot 2 are shown. It is seen that the linear friction parameters are more or less constant, which is desirable during steady state behaviour. If the linear friction parameters exhibit drift like the friction parameters in Figure 4.3a and Figure 4.3b, unreasonable control actions can be expected that can lead to instability of the system.

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Chapter 6

Conclusions and recommendations

6.1   Conclusions

An adaptive controller for the linear friction parameters is developed to achieve mutual synchronization of the CFT robot system in one direction. Only the q_1 direction is considered, which is the forward and backward movement of the whole robot. The influence of the coupled dynamics is constant, because only the friction parameters in the q_1 direction are estimated. It is seen that mutual synchronization improves the performance of the system. The performance of the system with mutual synchronization can be improved using an adaptation mechanism for the linear friction parameters with approximately 80 percent for robot 1 and with approximately 94 percent for robot 2. So it is clear that an adaptation mechanism is very useful in comparison to a controller without an adaptation mechanism.

The weighting factor k, which weights the error between the desired trajectory and robot i and the error between the relative states of robot i and robot j in the synchronization error s, $s_i = e_{ii} + k \cdot e_{ij}$, cannot be chosen arbitrarily large. How this gain has to be chosen is unknown and it is also unknown what the optimal choice for this weighting factor is. For some weighting factors the controller does not work properly anymore. The estimated parameters exhibit drift, which in the course of time can lead to unreasonable control actions that can lead to instability of the system. The weighting factor used during the simulations and experiments is therefore chosen by trial and error.

For adaptive mutual synchronization it is not even logical to start with a high weighting gain. Therefore are two reasons. The first reason is that if a robot should reckon with another robot, it is important to know first its own behaviour very well in order to react in a correct way on the behaviour of the other robots. Therefore first each robot should estimate its own uncertain linear friction parameters, so the robot can react in a correct way.
The second reason is that starting with a high weighting gain is the reversed problem. One robot is accusing another robot of errors committed by itself. It will be logical to first solve the own errors and thereafter solve the errors with respect to each other. In the most extreme situation all robots track the desired trajectory faultless, which means that the errors between the robots are equal to 0 as well.

The estimated parameters depend not only on the error between the robot and the desired trajectory, but also on the error between the relative states of the robots. This means that the estimated linear friction parameters represents not anymore the linear friction parameters of the robot, but are chosen in such way that the synchronization error s is minimized.

Because the velocities and accelerations of the robots are obtained by differentiating the position measurements, there is a lot of noise on these signals in the experiment. It is very difficult to cope with the measurement noise, because in mutual synchronization also the reference trajectory, $q_{ri} = q_d - k(q_i - q_j)$, depends on measured position. This means that the velocity and acceleration of the reference trajectory depends also on the differentiated velocity and acceleration of the different robots. This means that the measurement noise still appears quadratic in the adaptation and control law if $\dot{q}_i$ is replaced with $\dot{q}_r$. The only signal that contains no measurement noise is q_d, but a part of the interconnections between the robots are lost if $\dot{q}_r$ and $\ddot{q}_r$ are replaced by $\dot{q}_d$ and $\ddot{q}_d$. The only way to handle the measurement noise is to keep the weighting factor k small, so that in the reference trajectory the signal with the measurement noise becomes small with respect to the signals without measurement noise. But still in $\ddot{q}_r$ contains so much noise that the

control law is disturbed. Therefore $\ddot{q}_r$ is replaced by $\ddot{q}_d$, which implies that a part of the interconnections between the robots are lost.

6.2   Recommendations

In this report only adaptive mutual synchronization is achieved for one direction. This should be extended to adaptive mutual synchronization of the four directions of the robot system. One of the possible problems to deal with is the coupled dynamics of the system. If problems with the adaptation of the parameters occur, it might be a solution to use the adaptation mechanism for each direction separately, i.e. do only adaptive control in one direction while the parameters in the other directions are fixed to the values earlier obtained with the adaptation mechanism. After some time switch to another direction and do adaptive control in this direction, while the parameters in the other directions are fixed to the values earlier obtained with the adaptation mechanism.

In this report the gains are chosen by trial and error. This means that maybe the performance of the robot system can be improved by tuning the different gains. Since the CFT robot system is nonlinear and coupled, the only way to improve the performance of the system is by online tuning, i.e. tuning while the system is running. This will be a very time-consuming task.

The velocities and accelerations of the robots are obtained by differentiating the measured position signal. To handle with the measurement noise in the system, a part of the interconnections between the robots are lost and restrictions are given to the weight factor k. Therefore it will be desirable to obtain the velocities and accelerations of the robots in another way. Options are to use an observer, a low-pass filter or the use of two different position measurements with each mean zero and then take the average of these position signals.

Only an adaptation mechanism for the linear friction parameters is used. Because also the nonlinear friction parameters are dependent on external circumstances, the performance of the scheme can be improved if these parameters are again determined. This can be done by using a Kalman filter for the nonlinear friction parameters, while the linear friction parameters are fixed to the values obtained with the adaptation mechanism.

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Appendix A Dynamic model of the CFT robot

Entries of the inertia matrix $M(q_i)$

The entries of the inertia matrix $M(q_i) \in \bullet^{4\times4}$, as function of the generalized coordinates $q_i = [q_{i,1} \quad q_{i,2} \quad q_{i,4} \quad q_{i,5}]^T$ are given by

$$M_{1,1} = \theta_1 + \theta_{11} + \theta_{12}$$

$$
\begin{aligned}
M_{1,2} =\ & (-\theta_{12}d_{2\_0'} - \theta_{11}d_{2\_0'} - \theta_3)\sin(q_2) + (\theta_2 + d_6\theta_{11})\cos(q_2) \\
&+ \frac{1}{2}((L_4 - L_5)(\theta_{12} + \theta_{11}) - \theta_9)(\cos(q_5 + q_2 + q_4) - \cos(q_5 - q_2 + q_4)) \\
&+ \frac{1}{2}(\theta_7 + \theta_5 + \theta_{12}L_6)(\cos(-q_2 + q_4) - \cos(q_2 + q_4)) \\
&+ \frac{1}{2}(\theta_8 + \theta_6)(\sin(q_2 + q_4) - \sin(-q_2 + q_4)) \\
&+ \frac{1}{2}(-\sin(q_5 - q_2 + q_4) + \sin(q_5 + q_2 + q_4))\theta_{10}
\end{aligned}
$$

$$
\begin{aligned}
M_{1,3} =\ & \frac{1}{2}(-\theta_5 - \theta_7 - \theta_{12}L_6)(\cos(q_2 + q_4) + \cos(-q_2 + q_4)) \\
&+ \frac{1}{2}((L_4 - L_5)(\theta_{12} + \theta_{11}) - \theta_9)(\cos(q_5 - q_2 + q_4) + \cos(q_5 + q_2 + q_4)) \\
&+ \frac{1}{2}(\theta_8 + \theta_6)(\sin(q_2 + q_4) + \sin(-q_2 + q_4)) \\
&+ \frac{1}{2}(\sin(q_5 + q_2 + q_4) + \sin(q_5 - q_2 + q_4))\theta_{10}
\end{aligned}
$$

$$
\begin{aligned}
M_{1,4} =\ & \frac{1}{2}((L_4 - L_5)(\theta_{11} + \theta_{12}) - \theta_9)(\cos(q_5 + q_2 + q_4) + \cos(q_5 - q_2 + q_4)) \\
&+ \frac{1}{2}(\sin(q_5 + q_2 + q_4) + \sin(q_5 - q_2 + q_4))\theta_{10}
\end{aligned}
$$

$$
\begin{aligned}
M_{2,2} =\ & ((L_5 - L_4)(\sin(q_5) + \sin(q_5 + 2q_4)) - 2\cos(q_4)d_{2\_0'})\theta_8 + \theta_4 \\
&+ (-2d_{2\_0'}\cos(q_4 + q_5) - L_4\sin(2q_5 + 2q_4))\theta_{10} + \theta_{12}d_{2\_0'}^2 \\
&+ ((\frac{1}{2} - \frac{1}{2}\cos(2q_5 + 2q_4))(L_5^2 + L_4^2) + 2L_4d_{2\_0'}\sin(q_4 + q_5) + d_6^2 \\
&+ d_{2\_0'}^2 + ((\cos(2q_5 + 2q_4) - 1)L_4 - 2d_{2\_0'}\sin(q_4 + q_5))L_5)\theta_{11} \\
&+ ((\cos(2q_4) - \cos(q_5) - 1 + \cos(q_5 + 2q_4))L_4 \\
&- 2d_{2\_0'}\sin(q_4))\theta_5 - 2(\sin(q_4 + q_5)L_5 + \sin(q_4)L_6)d_{2\_0'}\theta_{12} \\
&+ ((\cos(q_5 + 2q_4) - \cos(q_5))(L_4 - L_5) - 2d_{2\_0'}\sin(q_4))\theta_7 \\
&- \frac{1}{2}\theta_{12}(\cos(2q_4) - 1)L_6^2 - \frac{1}{2}(\cos(2q_5 + 2q_4) - 1)(L_5^2 + L_4^2)\theta_{12} \\
&+ (-L_4(\sin(2q_4) + \sin(q_5 + 2q_4) + \sin(q_5)) - 2\cos(q_4)d_{2\_0'})\theta_6 \\
&+ ((\cos(2q_5 + 2q_4) - 1)L_4 + (\cos(q_5) - \cos(q_5 + 2q_4))L_6)L_5\theta_{12} \\
&+ ((\cos(2q_5 + 2q_4) - 1)L_4 - 2d_{2\_0'}\sin(q_4 + q_5))\theta_9 \\
&+ (2\sin(q_4 + q_5)d_{2\_0'} - (\cos(q_5) + \cos(q_5 + 2q_4))L_6)L_4\theta_{12}
\end{aligned}
$$

$$M_{2,3} = -\theta_7 d_6 \cos(q_4) + \theta_8 d_6 \sin(q_4) + \theta_{11} d_6 (L_4 - L_5) \cos(q_4 + q_5)$$

$$M_{2,4} = \theta_{11} d_6 (L_4 - L_5) \cos(q_4 + q_5)$$

$$
\begin{aligned}
M_{3,3} =\ & (((L_5 - L_6)L_4 + 2L_5 L_6)\theta_{12} + L_5 L_4 \theta_{11} + (\theta_9 - \theta_5 - \theta_7)L_4 \\
& + 2\theta_7 L_5) \cos(q_5) + L_4(\theta_6 + \theta_8) \sin(2q_4) \\
& - L_4((\tfrac{1}{2}L_4 + L_6)\theta_{12} + \tfrac{1}{2}L_4 \theta_{11} + \theta_5 + \theta_7) \cos(2q_4) \\
& - L_4((L_6 + L_5)\theta_{12} + L_5 \theta_{11} + \theta_9 + \theta_7 + \theta_5) \cos(q_5 + 2q_4) \\
& + (\theta_8 + \theta_{10} + \theta_6)L_4 \sin(q_5 + 2q_4) + ((2L_5 - L_4)\theta_8 \\
& - (\theta_6 + \theta_{10})L_4) \sin(q_5) + (L_4^2 + (L_6 - L_5)L_4 + L_5^2 + L_6^2)\theta_{12} \\
& + ((\tfrac{1}{2}L_4 - L_5)(\theta_{12} + \theta_{11}) - \theta_9)L_4 \cos(2q_5 + 2q_4) \\
& + (L_5^2 - L_5 L_4 + L_4^2)\theta_{11} + (\theta_7 - \theta_9 + \sin(2q_5 + 2q_4)\theta_{10} - \theta_5)L_4
\end{aligned}
$$

$$
\begin{aligned}
M_{3,4} =\ & \tfrac{1}{2}(\sin(q_5 + 2q_4) - \sin(q_5))L_4 \theta_6 + \tfrac{1}{2}\theta_{12}(\cos(2q_5 + 2q_4) + 1)L_4^2 \\
& + (\tfrac{1}{2}\cos(q_5) - \cos(2q_5 + 2q_4) - \tfrac{1}{2}\cos(q_5 + 2q_4) - 1)L_4 \theta_9 \\
& + (\tfrac{1}{2}\sin(q_5 + 2q_4) + \sin(2q_5 + 2q_4) - \tfrac{1}{2}\sin(q_5))L_4 \theta_{10} \\
& + (L_5 \cos(q_5) - \tfrac{1}{2}(\cos(q_5) + \cos(q_5 + 2q_4))L_4)\theta_7 \\
& + ((L_5 - \tfrac{1}{2}L_4)\sin(q_5) + \tfrac{1}{2}L_4 \sin(q_5 + 2q_4))\theta_8 + \theta_{12}L_5^2 \\
& + (L_5^2 + \tfrac{1}{2}(\cos(q_5) - \cos(q_5 + 2q_4))L_4 L_5 + \tfrac{1}{2}(1 + \cos(2q_5 + 2q_4)) \\
& \times (L_4^2 - 2L_4 L_5))\theta_{11} - \tfrac{1}{2}(\cos(q_5) + \cos(q_5 + 2q_4))L_4 \theta_5 \\
& + (\cos(q_5)L_6 - \tfrac{1}{2}(2 + 2\cos(2q_5 + 2q_4) + \cos(q_5 + 2q_4) \\
& - \cos(q_5))L_4)L_5 \theta_{12} - \tfrac{1}{2}\theta_{12}(\cos(q_5) + \cos(q_5 + 2q_4))L_6 L_4
\end{aligned}
$$

$$
\begin{aligned}
M_{4,4} =\ & ((\tfrac{1}{2}L_4^2 - L_5 L_4)(\theta_{12} + \theta_{11}) - L_4 \theta_9) \cos(2q_5 + 2q_4) \\
& + (\tfrac{1}{2}L_4^2 - L_5 L_4 + L_5^2)(\theta_{12} + \theta_{11}) + (\sin(2q_5 + 2q_4)\theta_{10} - \theta_9)L_4
\end{aligned}
$$

Entries of the Coriolis matrix $C(q_i, \dot{q}_i)$

The entries of the Coriolis matrix $C(q_i, \dot{q}_i) \in \bullet^{4\times4}$ as function of the generalized coordinates $q_i = [q_{i,1} \quad q_{i,2} \quad q_{i,4} \quad q_{i,5}]^T$ are given by

$$C_{1,1} = C_{2,1} = C_{3,1} = C_{4,1} = 0$$

$$
\begin{aligned}
C_{1,2} =\ & \tfrac{1}{2}(\theta_7 + \theta_5 + L_6 \theta_{12})((\dot{q}_2 - \dot{q}_4)\sin(q_4 - q_2) + (\dot{q}_2 + \dot{q}_4) \\
& \times \sin(q_2 + q_4)) - ((\theta_{12} + \theta_{11})d_{2\_0'} + \theta_3)\dot{q}_2 \cos(q_2) \\
& + \tfrac{1}{2}(\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11}))((\dot{q}_4 + \dot{q}_2 + \dot{q}_5) \\
& \times \sin(q_5 + q_2 + q_4) + (\dot{q}_2 - \dot{q}_4 - \dot{q}_5)\sin(q_5 - q_2 + q_4)) \\
& - (\theta_2 + d_6 \theta_{11})\dot{q}_2 \sin(q_2) + \tfrac{1}{2}(\theta_8 + \theta_6)((\dot{q}_2 - \dot{q}_4)\cos(q_4 - q_2) \\
& + (\dot{q}_2 + \dot{q}_4)\cos(q_2 + q_4)) + \tfrac{1}{2}((\dot{q}_2 - \dot{q}_4 - \dot{q}_5) \\
& \times \cos(q_5 - q_2 + q_4) + (\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\cos(q_5 + q_2 + q_4))\theta_{10}
\end{aligned}
$$

$$C_{1,3} = \frac{1}{2}(\theta_8 + \theta_6)((\dot{q}_4 - \dot{q}_2)\cos(q_4 - q_2) + (\dot{q}_2 + \dot{q}_4)\cos(q_2 + q_4))$$
$$+ \frac{1}{2}(\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11}))((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)$$
$$\times \sin(q_5 + q_2 + q_4) + (\dot{q}_4 - \dot{q}_2 + \dot{q}_5)\sin(q_5 - q_2 + q_4))$$
$$+ \frac{1}{2}(\theta_5 + \theta_7 + L_6\theta_{12})((\dot{q}_4 - \dot{q}_2)\sin(q_4 - q_2) + (\dot{q}_2 + \dot{q}_4)$$
$$\times \sin(q_2 + q_4)) + \frac{1}{2}((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\cos(q_5 + q_2 + q_4)$$
$$+ (\dot{q}_4 + \dot{q}_5 - \dot{q}_2)\cos(q_5 - q_2 + q_4))\theta_{10}$$

$$C_{1,4} = \frac{1}{2}((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\cos(q_5 + q_2 + q_4) + (\dot{q}_4 - \dot{q}_2 + \dot{q}_5)$$
$$\times \cos(q_5 - q_2 + q_4))\theta_{10} + \frac{1}{2}(\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11}))$$
$$\times ((\dot{q}_4 + \dot{q}_2 + \dot{q}_5)\sin(q_5 + q_2 + q_4)$$
$$+ (\dot{q}_4 + \dot{q}_5 - \dot{q}_2)\sin(q_5 - q_2 + q_4))$$

$$C_{2,2} = -\frac{1}{2}(L_4\theta_6 - L_5\theta_8 + \theta_8 L_4)((2\dot{q}_4 + \dot{q}_5)\cos(2q_4 + q_5)$$
$$+ \dot{q}_5\cos(q_5)) - \dot{q}_4 d_{2\_0'}(\theta_5 + \theta_7 + L_6\theta_{12})\cos(q_4)$$
$$- (\dot{q}_4 + \dot{q}_5)(\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11}))d_{2\_0'}\cos(q_4 + q_5)$$
$$- \frac{1}{2}(\dot{q}_4 + \dot{q}_5)(2L_4\theta_9 - (L_5 - L_4)^2(\theta_{12} + \theta_{11}))\sin(2q_5 + 2q_4)$$
$$- \frac{1}{2}\dot{q}_4(2L_4\theta_5 - L_6^2\theta_{12})\sin(2q_4) - \dot{q}_4 L_4\theta_6\cos(2q_4)$$
$$+ \frac{1}{2}(L_4(\theta_5 + \theta_7) + L_6\theta_{12}(L_4 - L_5) - L_5\theta_7)(\dot{q}_5\sin(q_5)$$
$$- (2\dot{q}_4 + \dot{q}_5)\sin(2q_4 + q_5)) + (\dot{q}_4 + \dot{q}_5)\sin(q_4 + q_5)d_{2\_0'}\theta_{10}$$
$$- (\dot{q}_4 + \dot{q}_5)\cos(2q_5 + 2q_4)L_4\theta_{10} + d_{2\_0'}\dot{q}_4(\theta_8 + \theta_6)\sin(q_4)$$

$$C_{2,3} = \frac{1}{2}\dot{q}_2(L_6^2\theta_{12} - 2L_4\theta_5)\sin(2q_4) - \dot{q}_2\cos(2q_5 + 2q_4)L_4\theta_{10}$$
$$+ \frac{1}{2}\dot{q}_2((L_5 - L_4)^2(\theta_{12} + \theta_{11}) - 2L_4\theta_9)\sin(2q_5 + 2q_4)$$
$$+ (\dot{q}_2 d_{2\_0'}\theta_{10} + d_6\theta_{11}(\dot{q}_4 + \dot{q}_5)(L_5 - L_4))\sin(q_4 + q_5)$$
$$- (((L_5 - L_4)(\theta_{12} + \theta_{11}) + \theta_9)d_{2\_0'})\dot{q}_2\cos(q_4 + q_5)$$
$$+ \dot{q}_2((L_5 - L_4)(L_6\theta_{12} + \theta_7) - L_4\theta_5)\sin(2q_4 + q_5)$$
$$+ (\dot{q}_4 d_6\theta_8 - \dot{q}_2(L_6\theta_{12} + \theta_7 + \theta_5)d_{2\_0'})\cos(q_4)$$
$$+ (\dot{q}_2(\theta_8 + \theta_6)d_{2\_0'} + \dot{q}_4 d_6\theta_7)\sin(q_4)$$
$$+ \dot{q}_2((L_5 - L_4)\theta_8 - L_4\theta_6)\cos(2q_4 + q_5) - \dot{q}_2 L_4\theta_6\cos(2q_4)$$

$$C_{2,4} = ((\dot{q}_4 + \dot{q}_5)(L_5 - L_4)d_6\theta_{11} + \dot{q}_2 d_{2\_0'}\theta_{10})\sin(q_4 + q_5)$$
$$+ \frac{1}{2}\dot{q}_2((L_5 - L_4)^2(\theta_{12} + \theta_{11}) - 2L_4\theta_9)\sin(2q_5 + 2q_4)$$
$$- ((L_5 - L_4)(\theta_{12} + \theta_{11}) + \theta_9)d_{2\_0'}\dot{q}_2\cos(q_4 + q_5)$$
$$- \dot{q}_2 L_4\theta_{10}\cos(2q_5 + 2q_4)$$
$$- \frac{1}{2}\dot{q}_2(L_4\theta_6 + \theta_8(L_4 - L_5))(\cos(q_5) + \cos(2q_4 + q_5))$$
$$+ \frac{1}{2}\dot{q}_2((L_5 - L_4)(L_6\theta_{12} + \theta_7) - L_4\theta_5)(\sin(2q_4 + q_5) - \sin(q_5))$$

$$C_{3,2} = \dot{q}_2 L_4 \theta_{10} \cos(2q_5 + 2q_4) + \dot{q}_2 L_4 \theta_6 \cos(2q_4)$$
$$+ \dot{q}_2 d_{2\_0'}(\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11})) \cos(q_4 + q_5)$$
$$- \dot{q}_2 d_{2\_0'} \sin(q_4 + q_5)\theta_{10} + \frac{1}{2}\dot{q}_2(2L_4\theta_5 - L_6^2\theta_{12})\sin(2q_4)$$
$$+ \dot{q}_2(L_4\theta_5 - (L_5 - L_4)(L_6\theta_{12} + \theta_7))\sin(2q_4 + q_5)$$
$$+ \frac{1}{2}\dot{q}_2(-(L_5 - L_4)^2(\theta_{12} + \theta_{11}) + 2L_4\theta_9)\sin(2q_5 + 2q_4)$$
$$+ \dot{q}_2 d_{2\_0'}(\theta_5 + \theta_7 + L_6\theta_{12})\cos(q_4) - \dot{q}_2 d_{2\_0'}(\theta_8 + \theta_6)\sin(q_4)$$
$$+ \dot{q}_2(L_4\theta_6 + \theta_8(L_4 - L_5))\cos(2q_4 + q_5)$$

$$C_{3,3} = \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\theta_{12} + \theta_{11}) + 2\theta_9)\sin(2q_5 + 2q_4)$$
$$- \frac{1}{2}\dot{q}_5((2L_5L_6 - L_6L_4 + L_5L_4)\theta_{12} + (2L_5 - L_4)\theta_7 + \theta_{11}L_5L_4$$
$$+ L_4(\theta_9 - \theta_5))\sin(q_5) + L_4(\dot{q}_4 + \dot{q}_5)\theta_{10}\cos(2q_5 + 2q_4)$$
$$+ \frac{1}{2}L_4(2\dot{q}_4 + \dot{q}_5)((L_5 + L_6)\theta_{12} + \theta_7 + L_5\theta_{11} + \theta_5 + \theta_9)$$
$$\times \sin(2q_4 + q_5) + L_4\dot{q}_4(\theta_8 + \theta_6)\cos(2q_4)$$
$$+ \frac{1}{2}L_4\dot{q}_4((L_4 + 2L_6)\theta_{12} + 2\theta_7 + L_4\theta_{11} + 2\theta_5)\sin(2q_4)$$
$$+ \frac{1}{2}L_4(2\dot{q}_4 + \dot{q}_5)(\theta_8 + \theta_6 + \theta_{10})\cos(2q_4 + q_5)$$
$$- \frac{1}{2}\dot{q}_5(L_4(\theta_6 + \theta_{10}) + \theta_8(L_4 - 2L_5))\cos(q_5)$$

$$C_{3,4} = (\frac{1}{2}(\theta_8 + \theta_{10} + \theta_6)\cos(2q_4 + q_5) + \theta_{10}\cos(2q_5 + 2q_4))L_4$$
$$\times (\dot{q}_4 + \dot{q}_5) - \frac{1}{2}(\dot{q}_4 + \dot{q}_5)(L_5L_4\theta_{12} + (2L_5 - L_4)(\theta_7 + L_6\theta_{12})$$
$$+ \theta_{11}L_5L_4 + L_4(\theta_9 - \theta_5))\sin(q_5) + \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((L_5 + L_6)\theta_{12}$$
$$+ \theta_7 + L_5\theta_{11} + \theta_5 + \theta_9)\sin(2q_4 + q_5)$$
$$+ \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\theta_{12} + \theta_{11}) + 2\theta_9)\sin(2q_5 + 2q_4)$$
$$+ \frac{1}{2}(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)\theta_8 - L_4(\theta_{10} + \theta_6))\cos(q_5)$$

$$C_{4,2} = \dot{q}_2 d_{2\_0'}((L_5 - L_4)(\theta_{12} + \theta_{11}) + \theta_9)\cos(q_4 + q_5)$$
$$+ \frac{1}{2}\dot{q}_2((L_5 - L_4)(L_6\theta_{12} + \theta_7) - L_4\theta_5)(\sin(q_5) - \sin(2q_4 + q_5))$$
$$+ \frac{1}{2}\dot{q}_2(2L_4\theta_9 - (\theta_{12} + \theta_{11})(L_5 - L_4)^2)\sin(2q_5 + 2q_4)$$
$$+ \dot{q}_2 L_4 \theta_{10}\cos(2q_5 + 2q_4) - \dot{q}_2 d_{2\_0'}\theta_{10}\sin(q_4 + q_5)$$
$$+ \frac{1}{2}\dot{q}_2(L_4\theta_6 - (L_5 - L_4)\theta_8)(\cos(q_5) + \cos(2q_4 + q_5))$$

$$C_{4,3} = \frac{1}{2}\dot{q}_4(L_4(\theta_9 - \theta_5 + \theta_{11}L_5) + (2L_5L_6 - L_6L_4 + L_5L_4)\theta_{12}$$
$$+(2L_5 - L_4)\theta_7)\sin(q_5) + L_4(\dot{q}_4 + \dot{q}_5)\theta_{10}\cos(2q_5 + 2q_4)$$
$$+\frac{1}{2}\dot{q}_4 L_4((L_5 + L_6)\theta_{12} + \theta_7 + L_5\theta_{11} + \theta_5 + \theta_9)\sin(2q_4 + q_5)$$
$$+\frac{1}{2}\dot{q}_4 L_4(\theta_8 + \theta_6 + \theta_{10})\cos(2q_4 + q_5)$$
$$+\frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\theta_{12} + \theta_{11}) + 2\theta_9)\sin(2q_5 + 2q_4)$$
$$+\frac{1}{2}\dot{q}_4(L_4(\theta_6 + \theta_{10}) + \theta_8(L_4 - 2L_5))\cos(q_5)$$

$$C_{4,4} = \frac{1}{2}L_4(\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\theta_{12} + \theta_{11}) + 2\theta_9)\sin(2q_5 + 2q_4)$$
$$+L_4(\dot{q}_4 + \dot{q}_5)\theta_{10}\cos(2q_5 + 2q_4)$$

Entries of the gravity vector $g(q_i)$

The entries of the gravity vector $g(q_i) \in \bullet^4$, as function of the generalized joint coordinates $q_i = [q_{i,1} \quad q_{i,2} \quad q_{i,4} \quad q_{i,5}]^T$ and the acceleration due to gravity $g = 9.81 m/s^2$ are given by

$$g_1 = g_2 = 0$$

$$g_3 = -g(\theta_9 + \theta_{12}L_5 + L_5\theta_{11})\sin(q_4 + q_5) - g(\theta_6 + \theta_8)\cos(q_4)$$
$$-g(\theta_5 + \theta_{12}(L_6 + L_4) + L_4\theta_{11} + \theta_7)\sin(q_4) - g\theta_{10}\cos(q_4 + q_5)$$

$$g_4 = -g(\theta_9 + \theta_{12}L_5 + L_5\theta_{11})\sin(q_4 + q_5) - g\theta_{10}\cos(q_4 + q_5)$$

Entries of the vector of friction forces $f(\dot{q}_i)$

The friction forces $f(\dot{q}_i) \in \bullet^4$ in the transposer robot are modeled by (2.5), such that the entries of $f(\dot{q}_i)$ can be written as function of the generalized velocities $\dot{q}_i = [\dot{q}_{i,1} \quad \dot{q}_{i,2} \quad \dot{q}_{i,4} \quad \dot{q}_{i,5}]^T$ as follows

$$f_1(\dot{q}_1) = \theta_{13}\dot{q}_1 + \theta_{17}\left(1 - \frac{2}{1 + e^{2\theta_{25}\dot{q}_1}}\right) + \theta_{21}\left(1 - \frac{2}{1 + e^{2\theta_{29}\dot{q}_1}}\right)$$

$$f_2(\dot{q}_2) = \theta_{14}\dot{q}_2 + \theta_{18}\left(1 - \frac{2}{1 + e^{2\theta_{26}\dot{q}_2}}\right) + \theta_{22}\left(1 - \frac{2}{1 + e^{2\theta_{30}\dot{q}_2}}\right)$$

$$f_3(\dot{q}_4) = \theta_{15}\dot{q}_4 + \theta_{19}\left(1 - \frac{2}{1 + e^{2\theta_{27}\dot{q}_4}}\right) + \theta_{23}\left(1 - \frac{2}{1 + e^{2\theta_{31}\dot{q}_4}}\right)$$

$$f_4(\dot{q}_5) = \theta_{16}\dot{q}_5 + \theta_{20}\left(1 - \frac{2}{1 + e^{2\theta_{28}\dot{q}_5}}\right) + \theta_{24}\left(1 - \frac{2}{1 + e^{2\theta_{32}\dot{q}_5}}\right)$$

Appendix B Estimated parameters for robot 1 and robot 2

Parameter	description	Robot 1 (i=1)	Robot 2 (i=2)
ϑ_1	m_1+m_2	147.0161	121.3049
ϑ_2	$m_2 l_{xc2}$	2.1448	0.3107
ϑ_3	$M_2 l_{yc2}$	-0.6363	4.1955
ϑ_4	$m_2(l_{xc2}^2+l_{yc2}^2)+m_3(l_{yc3}^2+l_{zc3}^2)+$ $m_4 l_{zc4}^2+m_5 l^2 -z_{c5}+$ $m_6 l_{zc6}^2+m_7(l_{yc7}^2+l_{zc7}^2)+$ $m_8(l_{xc8}^2+l_{zc8}^2)+l_{xx2}+l_{yy2}+$ $l_{yy3}+l_{zz3}+l_{zz4}+l_{zz5}+$ $l_{zz6}+l_{yy7}+l_{zz7}+l_{xx8}+l_{zz8}$	0.5931	1.7453
ϑ_5	$m_4 l_{xc4}$	0.1701	0.8316
ϑ_6	$m_4 l_{yc4}$	-0.0561	0.8687
ϑ_7	$m_6 l_{xc6}$	0.8392	0.8105
ϑ_8	$m_6 l_{yc6}$	1.9397	1.6721
ϑ_9	$m_5 l_{xc5}$	-0.1428	-0.1879
ϑ_{10}	$m_5 l_{yc5}$	1.7807	1.7850
ϑ_{11}	m_6	0.1498	0.8759
ϑ_{12}	m_7+m_8	4.4844	4.1328
ϑ_{13}	B_{v1}	83.2946	97.2600
ϑ_{14}	B_{v2}	11.2104	9.0999
ϑ_{15}	B_{v4}	16.6527	11.6257
ϑ_{16}	B_{v5}	13.6684	9.6257
ϑ_{17}	$B_{f1,1}$	-72.6918	-54.9912
ϑ_{18}	$B_{f1,2}$	-32.9333	18.4710
ϑ_{19}	$B_{f1,4}$	5.2337	-3.5232
ϑ_{20}	$B_{f1,5}$	-3.0435	-5.8564
ϑ_{21}	$B_{f2,1}$	-85.4138	-46.5915
ϑ_{22}	$B_{f2,2}$	-42.4819	11.1605
ϑ_{23}	$B_{f2,4}$	-4.3254	2.2684
ϑ_{24}	$B_{f2,5}$	5.5640	8.2304
ϑ_{25}	$w_{1,1}$	149.9624	150.3190
ϑ_{26}	$w_{1,2}$	142.7894	136.8945
ϑ_{27}	$w_{1,4}$	8.6392	-35.3699
ϑ_{28}	$w_{1,5}$	27.6979	36.0641
ϑ_{29}	$w_{2,1}$	-100.2648	-98.9881
ϑ_{30}	$w_{2,2}$	-142.2786	-170.4702
ϑ_{31}	$w_{2,4}$	-1.8278	-89.3236
ϑ_{32}	$w_{2,5}$	12.0224	16.2942

Table B.1 Estimated parameters for the CFT robots.