Plasmo-magnetoelastic waves in a semiconducting heterostructure

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Plasmo–magnetoelastic waves in a semiconducting heterostructure. I. Plasmo-elastic longitudinal modes. II. SH-Magnetoelastic modes.

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Plasmo–magnetoelastic waves in a semiconducting heterostructure. I. Plasmo–elastic longitudinal modes

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Wave propagation in semiconducting heterostructures is studied. Interactions of the electrical field with mechanical (elastic) and magnetic fields are considered. This first part deals with longitudinal (Rayleigh–type) plasmo–elastic waves in a thin layer on a half space (substrate); both layer and substrate are semiconducting. The waves show an exponential decay with depth in the substrate; the longer the waves the more of the wave energy is transmitted through the layer. Also purely plasmonic waves can exist. These waves are dissipative, and exist only for values of the wave number exceeding a certain critical value. In our general model an electrical relaxation term is included in the generalized Ohm's law; the influence of this relaxation aspect has received special attention.

INTRODUCTION

Semiconducting media are very rich in many physical phenomena occurring there simultaneously because of their comparable significance. Not only their electrical properties are important, but especially the interactions between electrical and mechanical, magnetic or, possibly, thermal fields give a new look on applications of semiconductors in practice. An example of such an interaction is the *elastodiffuse* effect: an interaction between electrical and elastic fields. In the example we have considered, it was found that the influence of the *plasmonic field* (i.e. the electronic charge) on the elastic one is small (in fact negligible) but, on the other hand, the elastic field generates extra plasmonic fields of technically relevant magnitude. All this is in correspondence with known physical observations on semiconductors (Maruszewski and Van de Ven (forthcoming)).

The aspects mentioned above become very evident in heterostructures built up of a thin layer on a half space (substrate). In this first part the propagation of longitudinal plasmo–elastic waves in such a heterostructure is considered, whereas the second part is more concerned with magneto–elastic shear waves in a semiconducting heterostructure.

Relaxation effects in semiconducting media is a relatively new aspect in the study of semiconductors. Here, we have included a relaxation time τ in the diffusion equation for the electronic charge. The influence of relaxation is investigated by comparing the results for $\tau \neq 0$ with those for $\tau = 0$. All numerical calculations have been performed for a heterostructure consisting of a *ZnSe*-layer on a *Ge*-substrate.

BASIC EQUATIONS

Consider an elastic semiconducting body, possibly built up of two or more semiconducting sub–bodies (heterostructure). The material is isotropic and, at least for each sub–body separately homogeneously. The semiconductor is doped (extrinsic semiconductor of n-type) and, therefore, hole field quantities may be neglected in comparison with the electronic field ones. The diffusion of impurities is neglected, but the relaxation nature of the charge field (*plasmonic field*) is taken into account. Such a situation mainly occurs in so–called *relaxation semiconductors.*

We are interested in the propagation of waves in such bodies. These waves show an interaction between the elastic and plasmonic fields, hence, they are called *plasmo–elastic waves.* The equations which govern these processes have been derived from an extended thermodynamical model (Maruszewski and Van de Ven (forthcoming), Maruszewski (1987a,b)). As shown there, these equations can be reduced to a set of two equations for the unknowns: the displacement $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ and the electric charge density per unit of mass $N = N(\mathbf{x}, t)$. These two equations read (index notation, including summation convention is used here)

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \lambda^n N_{,i} = \rho \ddot{u}_i \ ,$$
$$\tau \ddot{N} + (1 + \frac{\tau}{\tau^+}) \dot{N} + \frac{1}{\tau^+} N - D N_{,ii} \qquad (1)$$
$$+ \frac{\sigma \lambda^n}{\rho^2} u_{j,jii} = 0 \ .$$

Here, λ and μ are the Lamé parameters, λ^n is the elastodiffusive constant, ρ the density, τ and τ^+ are the relaxation time and the life time of the electrons, respectively, D is the diffusion coefficient and σ the electrical conductivity. A superimposed dot ($\dot{}$) means differentiation with respect to the time t, whereas $,i$ stands for $\partial/\partial x_i$. In the first equation, the equation of motion, the influence of the plasmonic field on the elastic field is represented by the elastodiffusive term: $\lambda^n N_{,i}$, whereas the opposite effect, i.e. the influence of the elastic field on the plasmonic field appears in the second equation, the diffusion equation, as the term preceded by $\sigma \lambda^n$.

The set (1) must be accompanied by the following jump conditions

$$[\![\mu (u_{i,j} + u_{j,i}) n_j + (\lambda u_{k,k} - \lambda^n N) n_i]\!] = 0 \ ,$$
$$[\![D N_{,i} n_i]\!] + [\![s (1 + \tau \frac{\partial}{\partial t}) N]\!] = 0 \ , \qquad (2)$$

where s is the coefficient of surface recombination. The first relation expresses the continuity of the stress vector. The outer surface is supposed to be free of stress. At material interfaces between sub–bodies still some more, physically trivial, jump conditions hold (i.e. continuity of displacements and electron charge).

As shown by Maruszewski and Van de Ven (forthcoming) by means of a dimension analysis, the influence of the plasmonic field on the elastic field, i.e. the terms preceded by λ^n in the first (mechanical) equations of (1) and (2), is negligibly small. On the other hand, the opposite effect, that is the influence of the elastic field on the electric charge field, represented by the λ^n-term in (1), is of physical relevance. All this is in correspondence with physical observations on semiconductors. Therefore, we shall neglect in $(1)^1$ and $(2)^1$ the elastodiffusive terms with λ^n. These equations then represent the purely elastic wave problem, which is assumed to be well–known (cf. Farnell (1978)).

LONGITUDINAL WAVES

We wish to apply the equations of the preceding section to a heterostructure consisting of a half space (*substrate*) with grown on it a thin epitaxial film (*layer*) of thickness h. In this layer longitudinal (Rayleigh–type) waves can propagate. We take a coordinate system $\{Ox_1x_2x_3\}$ with the Ox_1x_2-plane coinciding with the interface between half space and film, and the x_3-axis pointing into the half space. Hence, the upper (free) surface of the layer is given by $x_3 = -h$ and the interface by $x_3 = 0$, whereas in the half space $x_3 > 0$.

Let us assume that the propagation of the waves is along the x_1-axis. This implies that the resulting problem is two– dimensional, in the Ox_1x_3-plane, meaning that the variables u and N are functions of x_1, x_3 and t only and that $u_2 = 0$. In that case, the displacement components u_1 and u_3 can be expressed in potentials according to

$$u_1 = \varphi_{,1} - \psi_{,3} \ , \quad u_3 = \varphi_{,3} + \psi_{,1} \ . \qquad (3)$$

Then, (1) yields three equations for the three unknown variables $\varphi(x_1, x_3, t)$, $\psi(x_1, x_3, t)$ and $N(x_1, x_3, t)$. These equations reveal that the general solution can be assumed of the form

$$\{\varphi, \psi, N\} = \{\hat{\varphi}(\tilde{x}_3), \hat{\psi}(\tilde{x}_3), \hat{N}(\tilde{x}_3)\} e^{ik(x_1 - vt)} \ , \qquad (4)$$

where $\tilde{x}_3 = k x_3$, k is the (real) wave number and v is the wave velocity (which can be complex, with Re $v > 0$ and Im $v \leq 0$, but is real for purely elastic waves).

The solution of the purely elastic problem can be found, for instance, in Farnell (1978) Sect. 2.7.1. Introducing the longitudinal and transverse elastic wave velocities by

$$c_L = \left(\frac{\lambda + 2\mu}{\rho} \right)^{1/2} \ , \quad c_T = \left(\frac{\mu}{\rho} \right)^{1/2} \ , \qquad (4)$$

respectively, we state that if $c_{TI} < v < c_{TII} < c_{LI} < c_{LII}$ (the subindex indicates that the constant refers to the layer (I) or to the substrate (II)) there exist purely elastic longitudinal surface waves, and for these waves $\hat{\varphi}(\tilde{x}_3)$ takes the form (ψ is no longer relevant to us here) *for* $-kh < \tilde{x}_3 < 0$,

$$\hat{\varphi} = \hat{\varphi}^I(\tilde{x}_3) = \frac{1}{k^2} [A_1 \sinh \zeta_1 \tilde{x}_3 + A_2 \cosh \zeta_1 \tilde{x}_3] \ ,$$

for $\tilde{x}_3 > 0$,

$$\hat{\varphi} = \hat{\varphi}^{II}(\tilde{x}_3) = \frac{1}{k^2} A_3 e^{-\zeta_2 \tilde{x}_3} \ , \qquad (5)$$

where

$$\zeta_{1(2)} = \left[1 - \left(\frac{v}{c_{LI(II)}} \right)^2 \right]^{1/2} \ . \qquad (6)$$

The (real) wave velocity v follows from a dispersion relation of the form $v = V(k)$, for fixed h. We have calculated for a *ZnSe*- layer of thickness $h = 10^{-6} m$, and for a *Ge*-substrate, v and the associated normalized eigenvector (i.e. $A_1 \div A_3$). Hence, from now on the purely elastic

problem (that is $\hat{\varphi}(\tilde{x}_3)$) is considered known.

For $\hat{N}(\tilde{x}_3)$ there then remains the following problem (from $(1)^2$)
for $-kh < \tilde{x}_3 < 0$,

$$\frac{d^2\hat{N}^I}{d\tilde{x}_3^2} - \zeta_3^2\hat{N}^I = \Lambda_I k^2\Big(\frac{d^2}{d\tilde{x}_3^2} - 1\Big)^2\hat{\varphi}^I(\tilde{x}_3) =$$

$$= (1-\zeta_1^2)^2\Lambda_I[A_1\sinh\ \zeta_1\tilde{x}_3 + A_2\cosh\ \zeta_1\tilde{x}_3]\ , \quad (7)$$

for $\tilde{x}_3 > 0$ (analogously)

$$\frac{d^2\hat{N}^{II}}{d\tilde{x}_3^2} - \zeta_4^2\hat{N}^{II} = (1-\zeta_2^2)^2\Lambda_{II}A_3e^{-\zeta_2\tilde{x}_3}\ , \quad (8)$$

where

$$\zeta_{3(4)}^2 = \Big[1+\frac{1}{k^2D\tau^+} - \frac{\tau v^2}{D} - i\Big(1+\frac{\tau}{\tau^+}\Big)\frac{v}{kD}\Big]_{I(II)}\ , \quad (9)$$

and

$$\Lambda_{I(II)} = \Big(\frac{\sigma\lambda^n}{\rho^2D}\Big)_{I(II)}\ . \quad (10)$$

The boundary (at $x_3 = -h$) and jump (at $x_3 = 0$) conditions for N follow from $(2)^2$. At the upper surface of the film they yield (with(4))

$$\frac{d\hat{N}^I}{d\tilde{x}_3} + S\hat{N}^I = 0\ , \quad \text{at } \tilde{x}_3 = -kh\ , \quad (11)$$

where

$$S = \frac{s_I}{kD_I}(1 - ik\tau_I v)\ . \quad (12)$$

We assume that there exists no surface recombination of electrons on the interface $x_3 = 0$ and, moreover, we require that the electron charge is continuous across this plane. This yields

$$\frac{d\hat{N}^I}{d\tilde{x}_3} - D\ \frac{d\hat{N}^{II}}{d\tilde{x}_3} = 0\ , \quad (D = D_{II}/D_I)\ ,$$
$$\hat{N}^I - \hat{N}^{II} = 0\ , \quad \text{at } \tilde{x}_3 = 0\ . \quad (13)$$

The general solution of (7)-(8) reads
for $= kh < \tilde{x}_3 < 0$,

$$\hat{N} = \hat{N}^I(\tilde{x}_3) = B_1\sinh\ \zeta_3\tilde{x}_3 + B_2\cosh\ \zeta_3\tilde{x}_3$$
$$-\Gamma_1\sinh\ \zeta_1\tilde{x}_3 - \Gamma_2\cosh\ \zeta_1\tilde{x}_3\ , \quad (14)$$

for $\tilde{x}_3 > 0$,

$$\hat{N} = \hat{N}^{II}(\tilde{x}_3) = B_3e^{-\zeta_4\tilde{x}_3} - \Gamma_3e^{-\zeta_2\tilde{x}_3}\ , \quad (15)$$

where

$$\Gamma_{1(2)} = \frac{(1-\zeta_1^2)^2\Lambda_I}{(\zeta_3^2-\zeta_1^2)}\ A_{1(2)}\ ,$$
$$\Gamma_3 = \frac{(1-\zeta_2^2)^2\Lambda_{II}}{(\zeta_4^2-\zeta_2^2)}\ A_3\ . \quad (16)$$

Hence, Γ_1, Γ_2 and Γ_3 are known coefficients, representing the particular solution generated by the purely elastic wave. The coefficients B_1, B_2 and B_3 follow from the boundary conditions (12) and (13), which here result in

$$d_1B_1 - d_2B_2 = d_3\Gamma_1 - d_4\Gamma_2\ ,$$
$$\zeta_3B_1 + \zeta_4DB_3 = \zeta_1\Gamma_1 + \zeta_2D\Gamma_3\ , \quad (17)$$
$$B_2 - B_3 = \Gamma_2 - \Gamma_3\ ,$$

where $(\tilde{h} = kh)$

$$d_1 = \zeta_3\cosh\zeta_3\tilde{h} - S\sinh\zeta_3\tilde{h}\ ,$$
$$d_2 = \zeta_3\sinh\zeta_3\tilde{h} - S\cosh\zeta_3\tilde{h}\ ,$$
$$d_3 = \zeta_1\cosh\zeta_1\tilde{h} - S\sinh\zeta_1\tilde{h}\ , \quad (18)$$
$$d_4 = \zeta_1\sinh\zeta_1\tilde{h} - S\cosh\zeta_1\tilde{h}\ .$$

In case

$$\Delta := \zeta_3d_2 + \zeta_4Dd_1 \neq 0\ , \quad (19)$$

(17) admits the solution

$$B_1 = \{(\zeta_1d_2 + \zeta_4Dd_3)\Gamma_1 + \zeta_4D(d_2-d_4)\Gamma_2$$
$$-D(\zeta_4-\zeta_2)d_2\Gamma_3\}/\Delta\ ,$$
$$B_2 = \{(\zeta_1d_1 - \zeta_3d_3)\Gamma_1 + (\zeta_3d_4 + \zeta_4Dd_1)\Gamma_2$$
$$-D(\zeta_4-\zeta_2)d_1\Gamma_3\}/\Delta\ , \quad (20)$$
$$B_3 = \{(\zeta_1d_1 - \zeta_3d_3)\Gamma_1 + \zeta_3(d_4-d_2)\Gamma_2$$
$$+(D\zeta_2d_1 + \zeta_3d_2)\Gamma_3\}/\Delta\ .$$

With $B_1 \div B_3$ determined, the solution for the longitudinal plasmo-elastic waves (of Rayleigh-type) is known. We have calculated for a fixed value of h (i.e. $h = 10^{-6}(m)$) and for several values of $k \in [10^3, 10^6](m^{-1})$,

firstly the purely elastic waves (i.e. v and $A_1 \div A_3$) and, secondly, the plasmo–elastic waves ($B_1 \div B_3$). For a *ZnSe*-layer on a *Ge*-substrate the numerical values of the material coefficients can be found in Maruszewski and Van de Ven (forthcoming), Table 1. The relevant ones here are

$$\tau_I^+ = 10^{-8}(\text{sec}) \,, \quad \tau_{II}^+ = 10^{-5}(\text{sec}) \,, \quad D = 6.25 \,,$$

$$s_I = 10^3(\text{m/sec}) \,, \quad \Lambda_I = 60.76 \,, \quad \Lambda_{II} = 7.067 * 10^{-3} \,.$$

We have preformed these calculations for two, extreme, values of $\tau_P/\tau_P^+ (P = I, II)$, namely (i) $\tau_P/\tau_P^+ = 1$ (maximum relaxation) and (ii) $\tau_P/\tau_P^+ = 0$ (no relaxation). The main results are shown in Figs 1-4. In Figs 1 and 2 the amplitude $|\hat{N}(z)|$ of the plasmo–elastic waves is displayed as function of $z = x_3/h$ ($z \in [-1, 2]$) for $k = 10^6(m^{-1})$ and $k = 10^4(m^{-1})$ and for $\tau_P/\tau_P^+ = 1$ and 0, respectively.

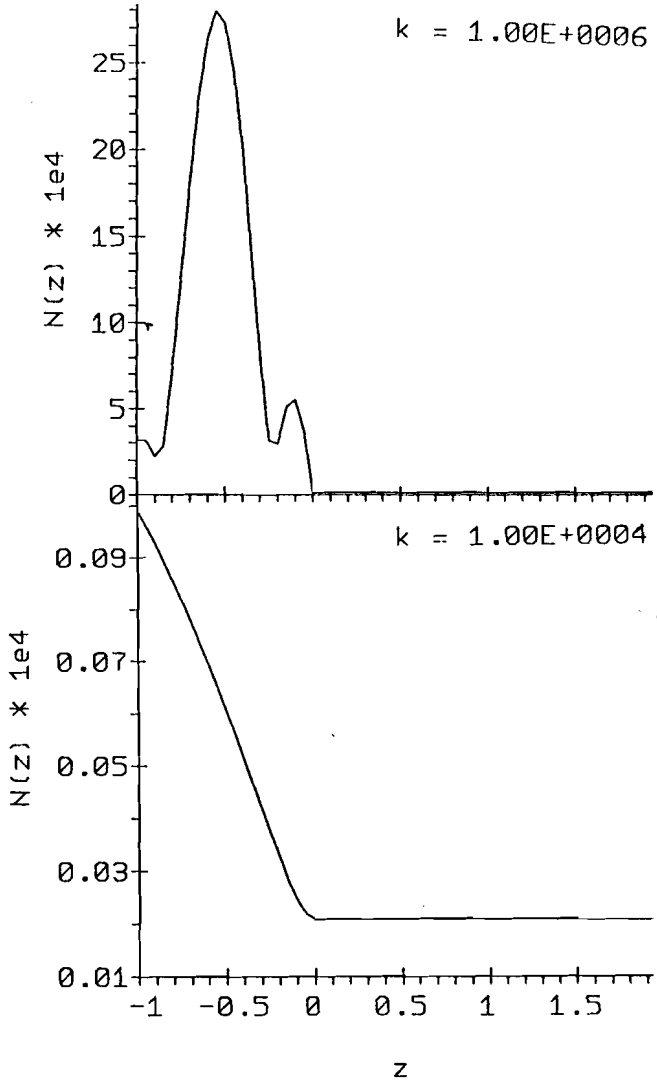

FIG 1. The amplitude $|\hat{N}(z)|$ of the plasmo–elastic wave for two values of k ($k = 10^6(m^{-1}) \Rightarrow v = 3015$(m/sec), and $k = 10^4(m^{-1}) \Rightarrow v = 3175$(m/sec)) and for $\tau/\tau^+ = 1$.

It is seen that the longer the waves are the better they penetrate the substrate, whereas the shorter the waves the more of the wave energy is transmitted along the surface. For very short waves ($k = 10^6(m^{-1})$) longitudinal plasmo–elastic waves practically exist only in the layer; we say then that "the wave escapes to the surface". However, this effect is much stronger for the relaxation semiconductor (Fig 1) than for the one without relaxation (Fig 2). From a comparison of the figures we see that, firstly, the amplitude of the waves increases with increasing k (shorter waves)–this will become even clearer in the next figure (Fig 3)–and, secondly, that this amplitude in the absence of relaxation ($\tau_P = 0$) is much larger (about a factor 10^2) than in the medium with (maximum) relaxation ($\tau_P = \tau_P^+$). This effect is also shown in Fig 3,

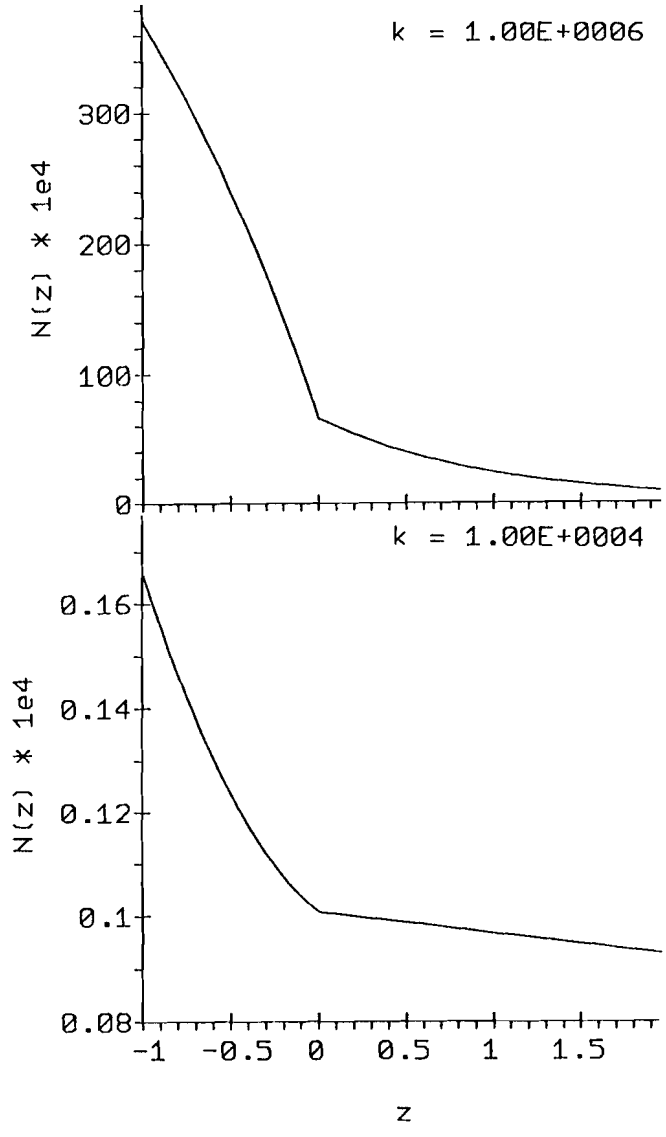

FIG 2. The amplitude $|\hat{N}(z)|$ of the plasmo–elastic wave for $h = 10^{-6}(m)$, for $k = 10^6$ or $10^4(m^{-1})$ and for $\tau/\tau^+ = 0$.

where the amplitude at the interface is plotted as a function of k ($k \in [1, 10^6](m^{-1})$) for the two cases $\tau_P/\tau_P^+ = 1$ and $\tau_P/\tau_P^+ = 0$. One sees that in the first case $|\hat{N}(0)|$ attains a maximum at $k = 4.1 * 10^5(m^{-1})$ (and a second, but smaller one, at $k = 4.8 * 10^5(m^{-1})$), whereas this maximum in the second case lies beyond the region $[1, 10^6]$. We note that in the neighbourhood of such a maximum, the interface plays the role of a frequency filter for the waves. Finally, we have investigated the behaviour of $|\hat{N}(0)|$ for $k = 10^6(m^{-1})$ as function of $T := \tau_P/\tau_P^+$ ($P = I, II$; we took the same value for T in layer and substrate). The result is depicted in Fig 4, showing that $|\hat{N}(0)|$ attains a maximum for $T = 1.1 * 10^{-4}$, and is monotoneously decreasing for larger values of T (on to a value of $|\hat{N}(0)| = 1.53 * 10^{-5}$ for $T = 1$).

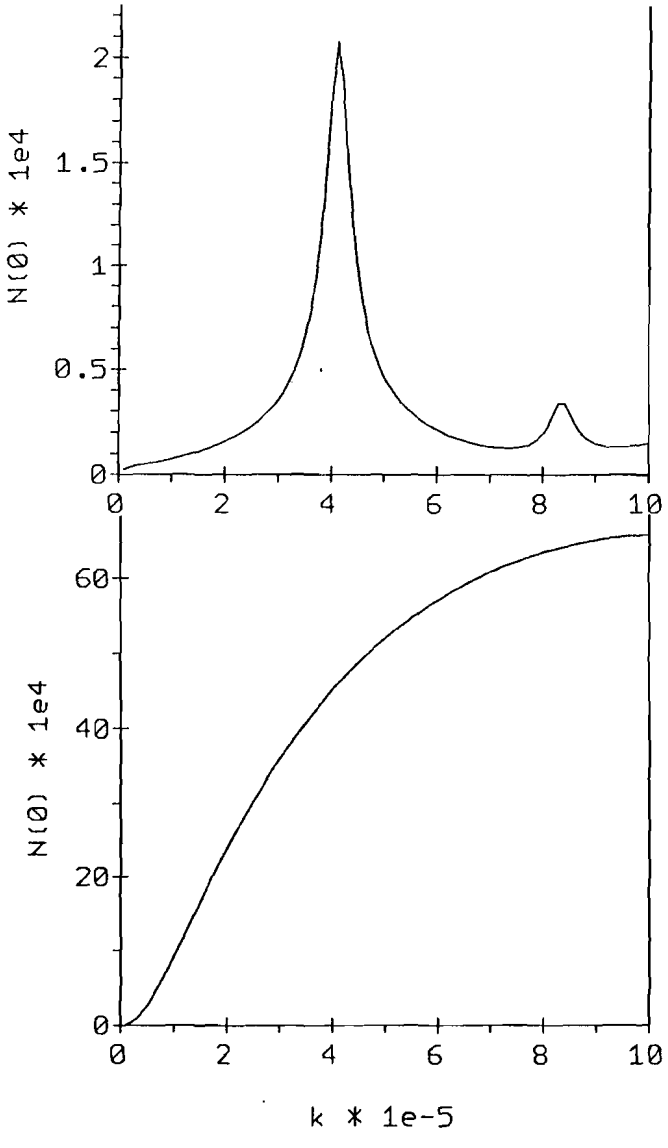

FIG 3. The amplitude $|\hat{N}(0)|$ at the interface $z = 0$ of the plasmo–elastic wave as function of $\hat{k} = k * 10^{-5} \in [0, 10]$, for $h = 10^{-6}(m)$ and for $\tau/\tau^+ = 1$ and $\tau/\tau^+ = 0$, respectively.

PURELY PLASOMONIC WAVES

In case (19) is not satisfied so that

$$\Delta = \zeta_3 d_2 + \zeta_4 D d_1 = 0 \,, \tag{21}$$

the system (17) with the right–hand sides taken equal to zero has a non–trivial solution. This implies the existence of purely plasmonic waves, (i.e. only $(B_1, B_2, B_3) \neq 0$). With (18) the dispersion relation (21) can be rewritten as

$$(\zeta_3^2 - \zeta_4 DS)\frac{\tanh \zeta_3 \tilde{h}}{D\zeta_3} - \frac{S}{D} + \zeta_4 = 0 \,. \tag{22}$$

Solution of this equation yields the wave velocity v as function of k for a fixed value of h. Since the plasmonic waves are dissipative, we must require $Im(v) < 0$, and, moreover, $Re\ v > 0$. It turned out that there exists a solution of (22), satisfying these restrictions, only for values of k exceeding a certain critical value $k_1 = k_1(h)$, for $h > 0$. Hence, for $0 < k < k_1$ no solution exists, whereas for $k > k_1$ we can obtain from (22) a complex wave velocity $v \in C$, with $Re\ v > 0$ and $Im\ v < 0$.

The critical value k_1 not only depends on h, but also on the ratio τ_P/τ_P^+. For $\tau_P = \tau_P^+$ the behaviour of k_1 as function of h (for $h \in [0, 10^{-6}](m)$) is plotted in FIG 5. It is seen that k_1 slightly decreases with increasing h. Our calculations revealed that the wave velocity $v(v \in \mathbb{R}$, for $k = k_1)$ was almost constant: its value changed from $v = 314$ (m/sec) for $h \to 0$ to $v = 317$ (m/sec) for $h = 10^{-6}(m)$. Hence, these values of v are much lower than those for the purely elastic waves.

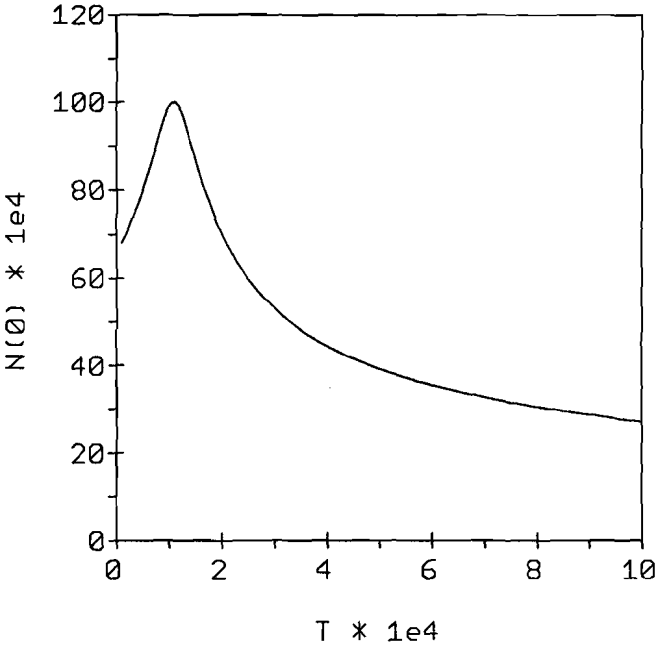

FIG 4. The amplitude $|\hat{N}(0)|$ as function of $T = \tau/\tau^+$, for $h = 10^{-6}(m)$ and $k = 10^6(m^{-1})$.

For $\tau_P/\tau_P^+ = 0$ no purely plasmonic waves exist; (22) then only has a solution with Re $v = 0$. To show how k_1 depends on the ratio τ_P/τ_P^+, we have calculated k_1 for $h = 10^{-6}$ (m) for several values of $T \in [0, 1]$. The results are shown in FIG 6. It turned out that there did not exist a solution for k_1 (with $Rev > 0$) for values of T between $0 \leq T < T_c = 0.103$, and that the value of k_1 strongly decreases for $T \downarrow T_c$. Finally, we note that our calculations of $v \in C$ for $k > k_1$ revealed that the dissipative character of the waves became stronger for smaller values of T (less relaxation).

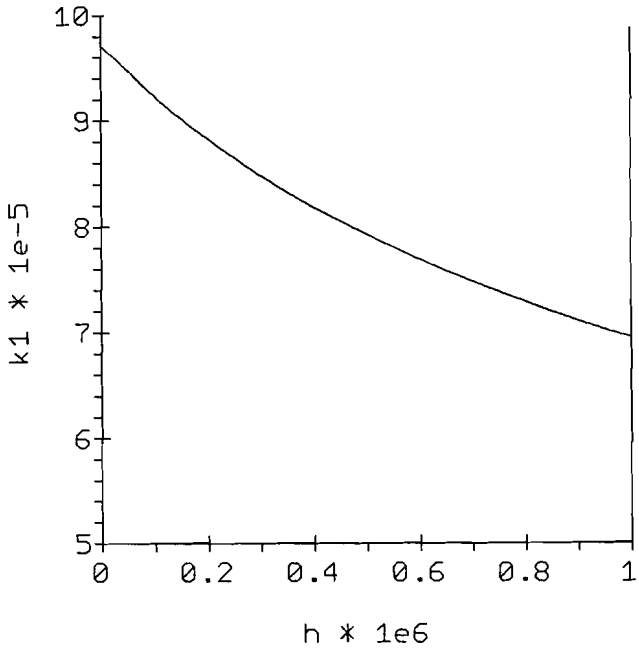

FIG 5. The critical wave number k_1 as function of h for $\tau_P/\tau_P^+ = 1$.

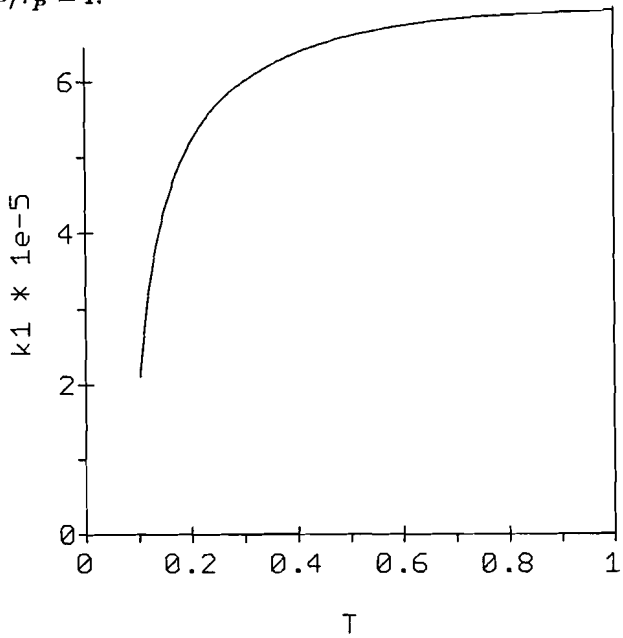

FIG 6. The critical wave number k_1 as function of T for $h = 10^{-6}$ (m).

CONCLUSIONS

Starting from a set of two equations and jump conditions, describing the displacement and the charge field (*plasmonic field*) in a relaxation semiconductor, we have derived a system for longitudinal plasmo–elastic waves in such a medium. We have applied this to a semiconducting heterostructure built up of a thin layer on a half space or substrate. Thus, we have calculated the wave velocity and the amplitudes of plane longitudinal plasmo–elastic surface waves in the structure. The effect of the plasmonic field on the elastic field is extremely small (in fact negligible), but the opposite effect of the elastic field on the plasmonic field (elastodiffusive effect) remains relevant. In fact, it was just the latter effect that caused the occurrence of plasmo–elastic waves, which were generated by purely elastic waves. Due to the dissipative character of the charge diffusion, the wave should be slightly dissipative, but this effect is so extremely small that it is neglected here.

The plasmo–elastic waves decay in the substrate. Characteristic for the presence of the layer was the tendency of the waves to escape to the layer; a tendency which was stronger for shorter waves. Also, this effect was stronger for relaxation semiconductors than for non–relaxation ones. On the other hand, the amplitudes for the plasmo–elastic waves were much larger in semiconductors without relaxation than in those with relaxation.

We have also considered purely plasmonic waves. It was shown that they only can exist for values of k larger than a certain critical value $k_1 = k_1(h)$, depending on the thickness h of the layer and on the relaxation ratio T. The influence of relaxation was twofold: firstly, the value of k_1 increases for increasing T from $T = 0.1$ on and, secondly, dissipation was smaller in a medium with relaxation then in one without relaxation.

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PLASMO-MAGNETOELASTIC WAVES IN A SEMICONDUCTING HETEROSTRUCTURE. II. SH-MAGNETOELASTIC MODES.

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The paper is devoted to the analysis of propagation of the *SH*-magnetoelastic (Love-type) waves in an epitaxial heterostructure collected of a magnetic semiconducting layer and nonmagnetic semiconducting substrate placed into an applied constant magnetic field perpendicular to the interface of the structure. Numerical results have been obtained for *ZnSe-Ge* heterostructure. The first order approximation of the dispersion of *SH*-magnetoelastic modes comparing to the dispersion of the purely elastic *SH* modes has been calculated.

INTRODUCTION

Contrary to the first part of this paper "van de Ven, Maruszewski: Plasmo-magnetoelastic waves in a semiconducting heterostructure", the second part is devoted to considerations concerning the transverse (*SH*) magnetoelastic modes in an epitaxial semiconducting heterostructure. This time the heterostructure is collected of two sub-bodies: the magnetic epitaxial layer and nonmagnetic substrate, both n-type homogeneous and isotropic semiconductors. Because of the magnetic properties of the structure just the transverse modes of the plasmo-magnetoelastic waves are the most intreresting.

Remark, that in this part we try to avoid repetition of notation explanation that has been done in the previous part.

SH-MAGNETOELASTIC WAVES

The problem concerns propagation of the *SH*-magnetoelastic waves in a semiconducting heterostructure collected of the epitaxial layer $-h < x_3 < 0$ and the substrate $x_3 > 0$ (h is the thickness of the layer). The propagation direction is x_1. The structure is placed into the applied magnetic field of induction $\mathbf{B}_0$ perpendicular to the interface $x_3 = 0$. The general wave equations and boundary conditions for the case when the elastic, electronic (plasmonic) and magnetic fields interact with each other have been derived in "Maruszewski, van de Ven: Plasmo-magnetoelastic waves in magnetic epitaxial film grown on a semiconductor". Particular analysis of them indicates that the wave propagation problem concerning the above geometry can be split into two separate ones: into the problem dealing with the longitudinal and the problem dealing with the transverse modes. The former case has been analyzed in the first part of this paper.

The final equations governing the latter one read

$$c_{TP}^2 u_{2,ii}^P - \ddot{u}_2^P + \frac{B_o}{\rho_P} h_{2,3}^P = 0 , \tag{1}$$

$$h_{2,ii}^P + \tau_P^n \dot{h}_{2,ii}^P - \mu_o \sigma_P (1+\chi_P) \dot{h}_2^P + B_o \sigma_P \dot{u}_{2,3}^P = 0 , \tag{2}$$

for $P = I, II$ (as previously, I indicates the layer and II - the substrate). We see that in this case only the u_2-component of the displacement and h_2-component of the magnetic field are coupled. Therefore we call their evolution in space (x_1, x_3) and time t the *SH*-magnetoelastic waves. In the vacuum space outside the structure we simply have $u_2 = h_2 = 0$. The form of the magnetic side of the problem is assumed as $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $|\mathbf{b}| << |\mathbf{B}_0|$, $\mathbf{B}_0 = B_o \mathbf{e}_3$, $\mathbf{b}^P = \chi_P \mathbf{h}^P$, $\chi_I = \chi$, $\chi_{II} = 0$ (χ - denotes the magnetic susceptibility).

The boundary conditions are as follows:

- at the free surface $x_3 = -h$:

$$h_2^I = 0 , \quad u_{2,3}^I = 0 , \tag{3}$$

- while at the interface $x_3 = 0$

$$h_2^I = h_2^{II} , \quad \rho_I c_{TI}^2 u_{2,3}^I = \rho_{II} c_{TII}^2 u_{2,3}^{II} ,$$

$$u_2^I = u_2^{II} , \quad \frac{1}{\sigma_I} h_{2,3}^I = \frac{1}{\sigma_{II}} h_{2,3}^{II} . \tag{4}$$

We assume now that the solutions of (1) and (2) in the form

$$\hat{u}_2^P = \hat{u}^P(x_3)\, e^{ik(x_1-vt)};$$

$$\hat{h}_2^P = \hat{h}^P(x_3)\, e^{ik(x_1-vt)}. \tag{5}$$

In this way we obtain two sets of equations

- in the layer ($-h<x_3<0$),

$$\hat{u}^I_{,33} + k^2\beta_1^2\hat{u}^I + A_1^I\hat{h}^I_{,3} = 0, \tag{6}$$

$$\hat{h}^I_{,33} - k^2\gamma_1^2\hat{h}^I - A_2^I\hat{u}^I_{,3} = 0, \tag{7}$$

- in the substrate ($x_3>0$)

$$\hat{u}^{II}_{,33} - k^2\beta_2^2\hat{u}^{II} + A_1^{II}\hat{h}^{II}_{,3} = 0, \tag{8}$$

$$\hat{h}^{II}_{,33} - k^2\gamma_2^2\hat{h}^{II} - A_2^{II}\hat{u}^{II}_{,3} = 0, \tag{9}$$

where

$$A_1^P = \frac{B_o}{\rho_P c_{TP}^2}, \tag{10}$$

$$A_2^P = \frac{ik\nu B_o\sigma_P}{1-ik\nu\tau_P^n}. \tag{11}$$

The general solutions of (6)-(9) read

$$\hat{u}^I = L_1\cos k\delta_1 x_3 + L_2\sin k\delta_1 x_3 + L_3\cos k\delta_3 x_3 + L_4\sin k\delta_3 x_3, \tag{12}$$

$$\hat{h}^I = \frac{A_2^I}{k}\left[\frac{\delta_1}{\delta_1^2+\gamma_1^2}(L_1\sin k\delta_1 x_3 - L_2\cos k\delta_1 x_3 + \frac{\delta_3}{\delta_3^2+\gamma_1^2}(L_3\sin k\delta_3 x_3 - L_4\cos k\delta_3 x_3)\right] \tag{13}$$

and

$$\hat{u}^{II} = L_5\, e^{-k\delta_2 x_3} + L_6\, e^{-k\delta_4 x_3}, \tag{14}$$

$$\hat{h}^{II} = \frac{A_2^{II}}{k}\left[\frac{\delta_2}{\gamma_2^2-\delta_2^2}L_5\, e^{-k\delta_2 x_3} + \frac{\delta_4}{\gamma_4^4-\delta_4^4}L_6\, e^{-k\delta_4 x_3}\right], \tag{15}$$

where

$$\delta_{1,3}^2 = w_1^2 \mp \left(w_1^4 + \beta_1^2\gamma_1^2\right)^{\frac{1}{2}}, \tag{16}$$

$$2w_1^2 = \beta_1^2 - \gamma_1^2 + \frac{1}{k^2}A_1^I A_2^I, \tag{17}$$

$$\delta_{2,4}^2 = w_2^2 \pm \left(w_2^4 - \beta_2^2\gamma_2^2\right)^{\frac{1}{2}}, \tag{18}$$

$$2w_2^2 = \beta_2^2 + \gamma_2^2 - \frac{1}{k^2}A_1^{II}A_2^{II}, \tag{19}$$

$$\beta_1^2 = \left(\frac{\nu}{c_{TI}}\right)^2 - 1, \tag{20}$$

$$\gamma_1^2 = 1 - \frac{i\nu\mu_o(1+\chi)\sigma_I}{k(1-ik\nu\tau_I^n)}, \tag{21}$$

$$\beta_2^2 = 1 - \left(\frac{\nu}{c_{TII}}\right)^2, \tag{22}$$

$$\gamma_2^2 = 1 - \frac{i\nu\mu_o(1+\chi)\sigma_{II}}{k(1-ik\nu\tau_{II}^n)}. \tag{23}$$

From the boundary conditions (3) and (4) we can determine the constants L_1 - L_6, which results in a characteristic equation of the form

$$Y_{\alpha\beta}L\beta = 0, \quad \alpha,\beta = 1,\ldots,6. \tag{24}$$

Nontrivial solutions of the above set of linear algebraic equations exists ony if

$$\det Y_{\alpha\beta} = 0. \tag{25}$$

The influence of the magnetic field B_o in (17) and (19) is represented by the last terms in the right-hand sides of these equations. In practice these (dimensionless) terms are very small compared to unity. This brings us to define

$$\frac{1}{k_2}A_1^I A_2^I =: i\,\varepsilon_1, \quad \frac{1}{k_2}A_1^{II}A_2^{II} =: -\varepsilon_2, \tag{26}$$

where $|\varepsilon_1|$, $|\varepsilon_2| \ll 1$.

The purely elastic solution (i.e. $B_o=0$) is obtained if $\varepsilon_1=\varepsilon_2=0$, and contains only the coefficients L_3, L_4 and L_6 ($L_1=L_2=L_5=0$, then; note that $(\delta_1^2+\gamma_1^2)=(\delta_2^2-\gamma_2^2)=0$ if $\varepsilon_1=\varepsilon_2=0$.

From the system (24) only the equations for $\alpha=1$, $\alpha=3$ and $\alpha=4$ remain, and they yield in the common way the following characteristic equation (cf. "Farnell: Types and properties of surface waves")

$$\tan\left[kh\left((\nu/c_1)^2-1\right)^{\frac{1}{2}}\right] = \frac{\rho_{II}c_2^2}{\rho_I c_1^2}\frac{\left(1-(\nu/c_1)^2\right)^{\frac{1}{2}}}{\left((\nu/c_1)^2-1\right)^{\frac{1}{2}}}, \tag{27}$$

for $c_I<\nu<c_2$ and with (throughout this section)

$$c_{1,2} = c_{TI,II}. \tag{28}$$

This equation is the dispersion relation for purely elastic *SH*-waves (Love waves). In case ε_1 and ε_2 are unequal to zero, this relation will be changed slightly. It is our purpose

to calculate in first order of ε the changes in the dispersion relation.

To this end, we substitute (26) into (17) and (19), put the thus obtained relations into (16) and (18), respectively, and develop these with respect to ε. The result reads, up to $O(\varepsilon^2)$,

$$\delta_1^2 = -\gamma_1^2 + i\frac{\gamma_1^2}{\beta_1^2+\gamma_1^2}\varepsilon_1\,, \tag{29}$$

$$\delta_3^2 = \beta_1^2 + i\frac{\beta_1^2}{\beta_1^2+\gamma_1^2}\varepsilon_1\;; \tag{30}$$

and

$$\begin{aligned}\delta_2^2 &= \gamma_2^2 + \frac{\gamma_2^2}{\beta_2^2-\gamma_2^2}\varepsilon_2\,,\\ \delta_4^2 &= \beta_2^2 - \frac{\beta_2^2}{\beta_2^2-\gamma_2^2}\varepsilon_2\,.\end{aligned} \tag{31}$$

In the derivation of the latter realtions we have assumed that $\mathrm{Re}\gamma_2^2 > \mathrm{Re}\beta_2^2$.

An immediate consequence of (30) and (31) is that $(\delta_1^2+\gamma_1^2)$ and $(\delta_2^2-\gamma_2^2)$ (appearing in the denominator in the formulas (13) and (15) for the amplitudses $\hat{h}^{I,II}$) become $O(\varepsilon^2)$. To get rid of this inconvenience we renormalize the coefficients L_1, L_2 and L_5, that is we replace them by

$$(\delta_1^2+\gamma_1^2)\,\hat{L}_1\,,\quad (\delta_1^2+\gamma_1^2)\,\hat{L}_2\,,\quad (\delta_2^2-\gamma_2^2)\,\hat{L}_5\,,$$

respectively. Furthermore, we change the numbering of the coefficients and the sequence of the equations (24) in such a way that we arrive at the reordered set

$$Z_{\alpha\beta}M_\beta = 0\,, \tag{32}$$

where

$$\begin{aligned}&M_1 = L_3\,,\quad M_2 = \delta_3 L_4\,,\quad M_3 = L_6\,,\\ &M_4 = \frac{L_1}{\delta_1^2+\gamma_1^2}\,,\; M_5 = \frac{\delta_1 L_1}{\delta_1^2+\gamma_1^2}\,,\; M_6 = \frac{L_5}{\delta_2^2-\gamma_2^2}\,.\end{aligned} \tag{33}$$

The matrix Z is of the form

$$Z = \begin{pmatrix} Z_{11} & Z_{12} & 0 & \varepsilon Z_{14} & \varepsilon Z_{15} & 0\\ 0 & Z_{22} & Z_{23} & 0 & \varepsilon Z_{25} & \varepsilon Z_{26}\\ Z_{31} & 0 & Z_{33} & \varepsilon Z_{34} & 0 & \varepsilon Z_{36}\\ Z_{41} & Z_{42} & 0 & Z_{44} & Z_{45} & 0\\ 0 & Z_{52} & Z_{53} & 0 & Z_{55} & Z_{56}\\ Z_{61} & 0 & Z_{63} & Z_{64} & 0 & Z_{66}\end{pmatrix} \tag{34}$$

where ε is defined as

$$\varepsilon = \frac{\gamma_1^2}{\beta_1^2+\gamma_1^2}\varepsilon_1 = \frac{1}{i}(\delta_1^2+\gamma_1^2)\,. \tag{35}$$

The elements in the 4°, 5° and 6° columns and the 1°, 2° and 3° rows of Z are preceded by a factor ε in order to indicate that these elements are $O(\varepsilon^2)$ with respect to the remaining ones. The part of Z containing the first three rows and columns describes the purely elastic SH-waves.

Assuming ε small, we can develop the determinant of Z with respect to ε. In doing so we neglect $O(\varepsilon^2)$-terms and, moreover, we use the fact that the determinant of the 3x3 submatrix $Z_{\alpha\beta}$, $\alpha,\beta \in (1,2,3)$, is also $O(\varepsilon^2)$ (in accordance with (27)). After some elementary calculations (the details of which are omitted here) we find that $\mathrm{Det}Z = 0$ is equivalent to the relation

$$\Gamma - \varepsilon\,\Lambda = 0\,, \tag{36}$$

where

$$\Gamma = \delta_3 \sin kh\delta_3 - \frac{\mu_{II}}{\mu_I}\delta_4 \cos kh\delta_3\,, \tag{37}$$

(it will turn out that this term is $O(\varepsilon^2)$) and

$$\begin{aligned}\Lambda = {}& \frac{-1}{\gamma_1(S_1+\frac{\sigma}{\tau}\frac{\gamma_1}{\gamma_2}C_1)}\,[\beta_1 n_3(s_3+\frac{\sigma}{\tau}\frac{\beta_1}{\gamma_2}C_1)\\ &-\mu\beta_2(C_1-c_3)n_3 + \sigma\beta_2 n_4\left(\frac{\beta_2}{\gamma_2}-1\right)C_1\,]\cdot\\ &\cdot\,[-i(\gamma_1 S_1+\mu\beta_2 c_3) + i\frac{\sigma\gamma_1^2}{\tau\gamma_2}(C_1-c_3)\\ &-\frac{r\mu}{\tau}\frac{\gamma_1^2}{\gamma_2^2}(\beta_2-\gamma_2)\,c_3]\\ &+i(C_1-c_3)\left[\frac{\sigma}{\tau}\frac{\beta_1^2}{\gamma_2}n_3 - \mu\beta_2 n_3 + \sigma\beta_2 n_4\left(\frac{\beta_2}{\gamma_2}-1\right)\right]\\ &-\frac{r\mu}{\tau\gamma_2^2}[\beta_1^2 n_3 + \tau\beta_2^2 n_4]\,(\beta_2-\gamma_2)c_3\,.\end{aligned} \tag{38}$$

In (38) we have used the following abbreviations (notice that in the evaluation of Λ we may neglect all ε-terms, which among others implies that we may use $\delta_1 = i\gamma_1$, $\delta_2 = \gamma_2$, $\delta_3 = \beta_1$, $\delta_4 = \beta_2$)

$$\begin{aligned}&S_1 = \sin kh\gamma_1\,,\quad C_1 = \cos kh\gamma_1\,,\\ &s_3 = \sin kh\beta_1\,,\quad c_3 = \cos kh\beta_1\,,\\ &n_3 = (\beta_1^2+\gamma_1^2)^{-1}\,,\quad n_4 = (\beta_2^2-\gamma_2^2)^{-1}\\ &\mu = \frac{\mu_{II}}{\mu_I}\,,\quad \tau = \frac{1-ik\nu\tau_I''}{1-ik\nu\tau_{II}''}\,,\quad \sigma = \frac{\sigma_{II}}{\sigma_I}\tau\,,\end{aligned} \tag{39}$$

$$r = \frac{1}{\varepsilon}(\delta_2^2 - \gamma_2^2) = \frac{\gamma_2^2(\beta_1^2+\gamma_1^2)}{\gamma_1^2(\gamma_2^2-\beta_2^2)}\frac{\varepsilon_2}{\varepsilon_1}.$$

In calculating $\gamma_{1,2}$ and $\beta_{1,2}$ for use in (38)-(39) we must take for v the velocity v_o of the purely elastic *SH* wave (following from (27)). This is consistent with the neglect of $O(\varepsilon^2)$-terms in the evaluation of Λ.

We proceed by developing the right-hand side of (37) with respect to ε. This has to be done in two steps: firstly, we substitute (30)-(31) for $\delta_1 \div \delta_4$, and, secondly, we must account for the fact that v is a first order perturbation (in ε) of v_o. In this way we obtain

i)

$$\Gamma = (\beta_1 \sin kh\beta_1 - \mu\beta_2 \cos kh\beta_1) + \varepsilon\Delta \tag{40}$$

with

$$\Delta = \frac{i}{2}\frac{\beta_1}{\gamma_1^2}(s_3 + kh\beta_1 c_3) - \frac{\mu}{2}\frac{\beta_2}{\gamma_2^2}(r c_3 + kh\beta_1 s_3). \tag{41}$$

In (40), Λ is preceded by a factor ε and, therefore, for the explicit calculation of Δ we may use $v = v_o$ in (41)

ii)

$$\beta_1 \sin kh\beta_1 - \mu\beta_2 \cos kh\beta_1 = V\frac{v_1}{c_1}\varepsilon, \tag{42}$$

with

$$V = \frac{v_o}{\beta_1 c_1}\left[s_3 + \beta_1 c_3 + \mu\beta_2 s_3 + \rho c_3\right], \tag{43}$$

and

$$\rho = \frac{\rho_{II}}{\rho_I}. \tag{44}$$

Oncemore, it is noted in the evaluation of V according to (43) one must use v_o for v, that means that in (43) (and in (41) and (38)) one must read for $\beta_{1,2}$

$$\beta_1 = \left[\left(\frac{v_o}{c_1}\right)^2 - 1\right]^{\frac{1}{2}}, \quad \beta_2 = \left[1 - \left(\frac{v_o}{c_2}\right)^2\right]^{\frac{1}{2}}. \tag{45}$$

The right-hand side of (42) is $O(\varepsilon^2)$ because the left-hand side of it is zero for $v = v_o$ in accordance with (27).

Recapitulating the results derived above we see that we have expressed $\mathrm{Det}Z = 0$ in the relation

$$V\frac{v_1}{c_1} + \Delta - \Lambda = 0, \tag{46}$$

from which the following expression for the first order perturbation (in ε) of the phase velocity v can be derived ($c_I = c_{TI}$)

$$\frac{v_1}{c_{TI}} = \frac{\Lambda - \Delta}{V}. \tag{47}$$

The complete relation for the phase velocity then reads

$$\frac{v}{c_{TI}} = \frac{v_o}{c_{TI}} + \frac{\Lambda-\Delta}{V}\varepsilon + O(\varepsilon^2). \tag{48}$$

This is the first order perturbation for small ε (i.e. $|\varepsilon| \ll 1$) of the dispersion we are looking for.

NUMERICAL RESULTS

On denoting

$$R(kh) = \frac{\Lambda-\Delta}{V}\varepsilon \tag{49}$$

we did calculations for $R(kh)$ (in its first order approximation) of dispersion of the *SH*-magnetoelastic modes in the case of *ZnSe-Ge* heterostructure with respect to various thickness of the epitaxial (*ZnSe*) layer h.

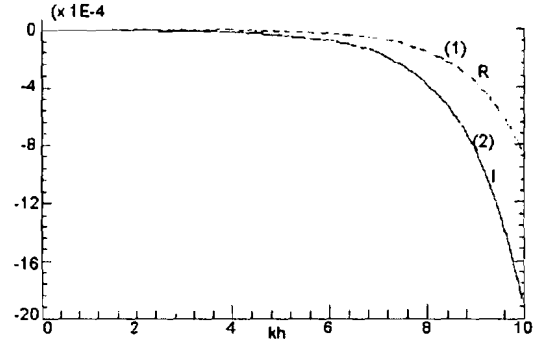

Fig. 1. Real(1) and imaginary(2) parts of $R(kh)$ for $h = 1\ m$.

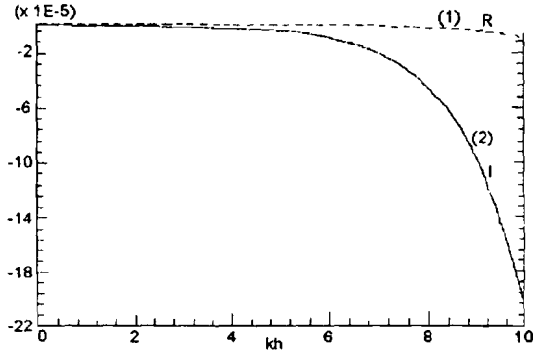

Fig. 2. Real(1) and imaginary(2) parts of $R(kh)$ for $h = 0.1\ m$.

It is easily seen from Figs. 1-4 that the *SH*-magnetoelastic modes are (with respect to their length) much more damped for the thick layer (long wave propagation) than for very thin one (short wave propagation) where damping effect practically vanishes. For the thickness of the layer between 0.1 m and 0.01 m (the real part of the phase velocity perturbation

is practically negligible), there is no influence of the plasmo-magnetic field on the purely elastic *SH* waves (27).

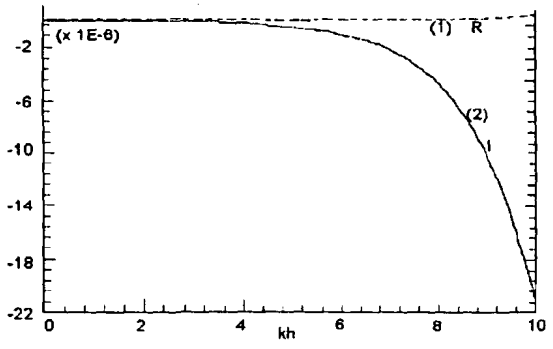

Fig. 3. Real(1) and imaginary(2) parts of $R(kh)$ for $h = 10^{-2}$ *m*.

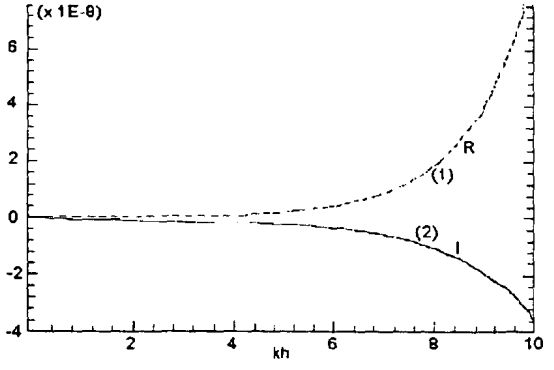

Fig. 4. Real(1) and imaginary(2) parts of $R(kh)$ for $h = 10^{-4}$ *m*.

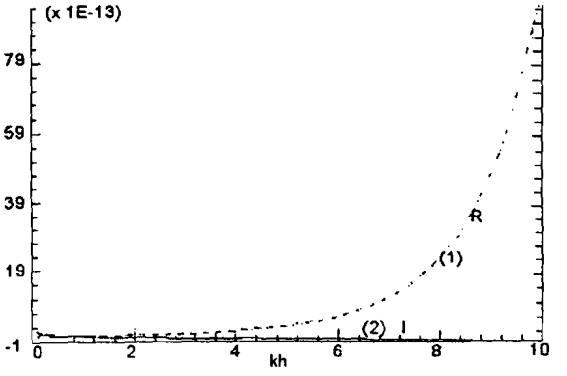

Fig. 5. Real(1) and imaginary(2) parts of $R(kh)$ for $h = 10^{-6}$ *m*.

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