Characterization of optical discs

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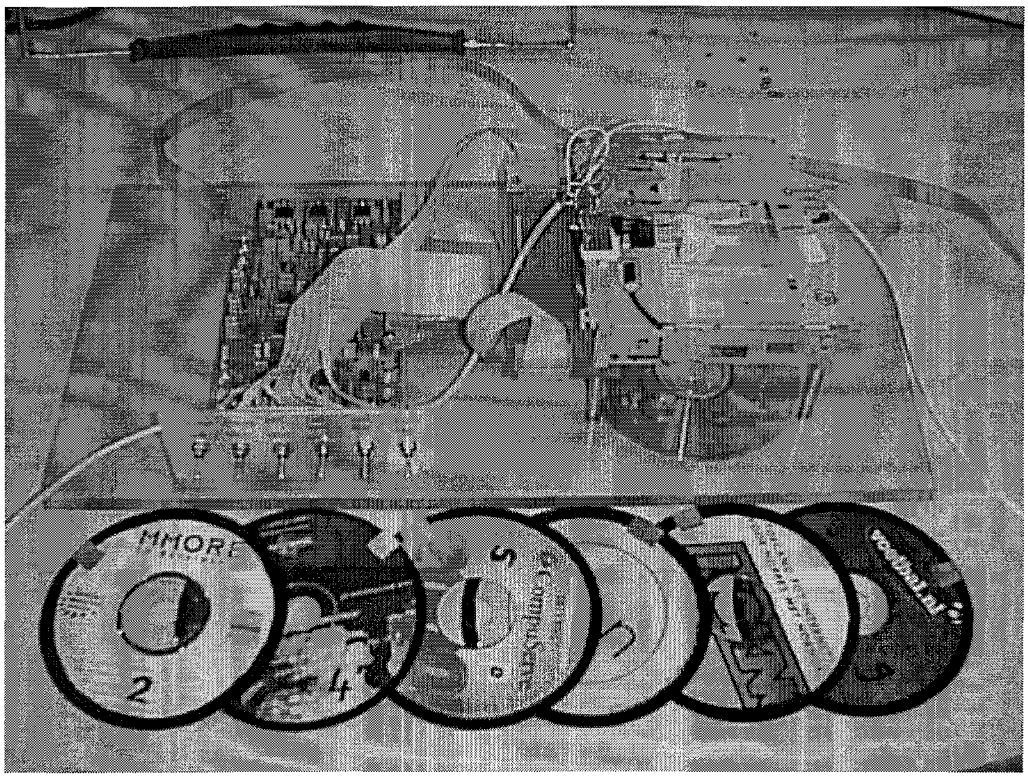
DCT report nr.:2001.14



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Ir. P.W.J.M. Nuij

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1 Introduction

In this paper optical discs are investigated. Examples of these are audio compact discs or CD-ROM's. The CD-player used is a car loader with a radial rotating arm as displayed in figure 1.1

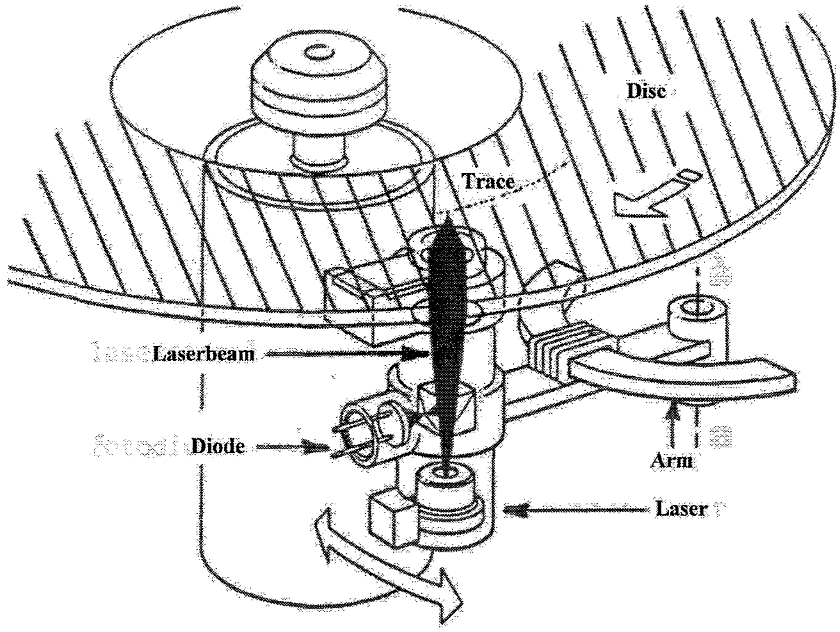


Figure 1.1 CD-player mechanism

Nowadays usually linear two-stage-mechanisms (with a long stroke sledge and a short stroke second actuator) are used in CD-players, but the this does not affect the topic of interest of this report.

For an outsider all optical discs look the same. Once the CD-player will not play the disc, the CD-player will be returned to the store with the complaint that it does not work like it is supposed to work. Of course this can be the reason, be it can also be the problem of the disc. Although the machine, which produces the discs, is an expensive and accurate tool, none of the discs it produces is perfect.

A detailed view of a disc is displayed in figure 1.2. In this figure five tracks are visible, with their pits and dams.

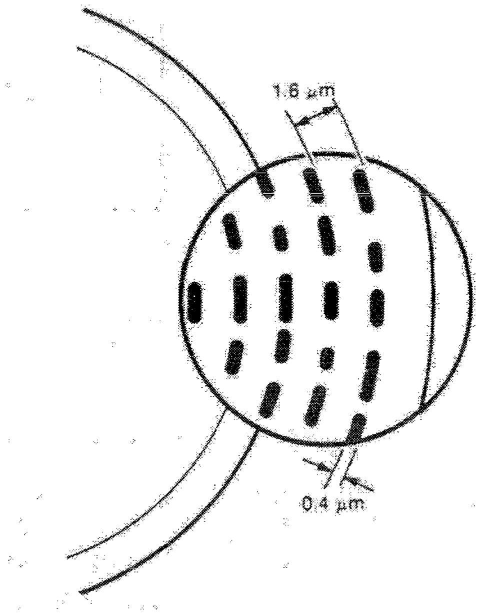


Figure 1.2 Detailed view of the disc

There has to be told that the tracks in this figure are ideal tracks. This means that these tracks follow an exact spiral, instead of the "wobble" tracks in practice. The intention of the research done in this paper is to develop an algorithm that can filter the disturbances out of the error (which has been measured). When this algorithm is known different discs can be compared with each other. Also conclusions can be made about the contribution of the higher harmonics of the rotational frequency to these disturbances. With this information different discs can be characterized.

In a CD-player two errors occur which are interesting with respect to the characterization of optical discs. These are the focus error of the laser (focus loop) and the radial error of the rotating arm (radial loop). In this paper only the radial loop has been investigated, because the radial error contains the most information about the specific disc that is in the CD-player.

Some irregularities are present in the disc after they are produced. These are unroundness of the disc, eccentricity of the hole and unroundness of the track (which has to be followed by the laser spot). In the Hard Disc Drive (HDD) industry these last irregularities are called 'Repeatable Run Outs' (RRO). It is of course possible to produce an almost perfect disc at a specific machine, but this will cost the manufacturer too much money, which results in less profit. For this reason requirements for the CD-player and tolerances for the disc are introduced. The requirement is a maximum radial error of 0.1 μm of the laser spot and a maximum disturbance of the disc, resulting from the irregularities mentioned, from 100 μm. This requirement is graphically illustrated in figure 1.3 (left figure). The two right graphics of figure 1.3 show the maximum tolerances of the disc.

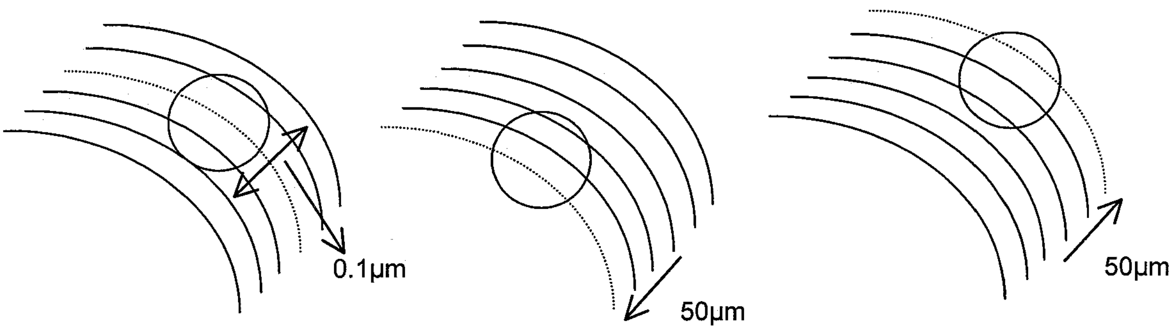


Figure 1.3 requirement and tolerances

As can be noticed from this figure, the laser spot follows the track, which oscillates with an amplitude of ± 50 μm, with a maximum radial error of 0.1 μm.

For the characterization process, first a model for the CD-player has to be developed. This experimental approach will be outlined in chapter 2. The next step is to develop the filter algorithm for the disturbance, which will be explained in chapter 3. In chapter 4 this algorithm will be used to compare different discs with each other. Conclusions and recommendations are made in chapter 5.

2 Model Estimation

2.1 Theory

When a model for the CD-player has to be obtained, injecting a signal into the CD-player and measure its response on every frequency of interest, is not the right way to do this, because this system is located in the closed loop of the block diagram (figure 2.1).

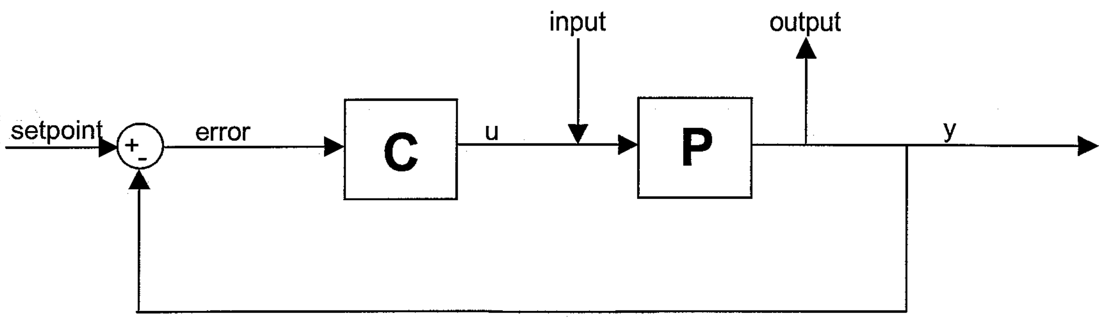


Figure 2.1 block diagram

In this diagram C represents the controller, P the plant, *u* the controlled variable and *y* the output. When a signal is injected in the closed loop at the point called *input* in the figure above and the response is measured at the point called *output*, not the plant P is measured, but P/(1+PC), which is called the process sensitivity. And from this transfer function the plant P cannot be calculated easily. Another problem is the fact that a signal cannot be injected and read out anywhere in the loop. In the plant, in this case the CD-player, there are several in advance specified points where signals can be injected an read out. These points are depicted in the figure below (figure 2.2).

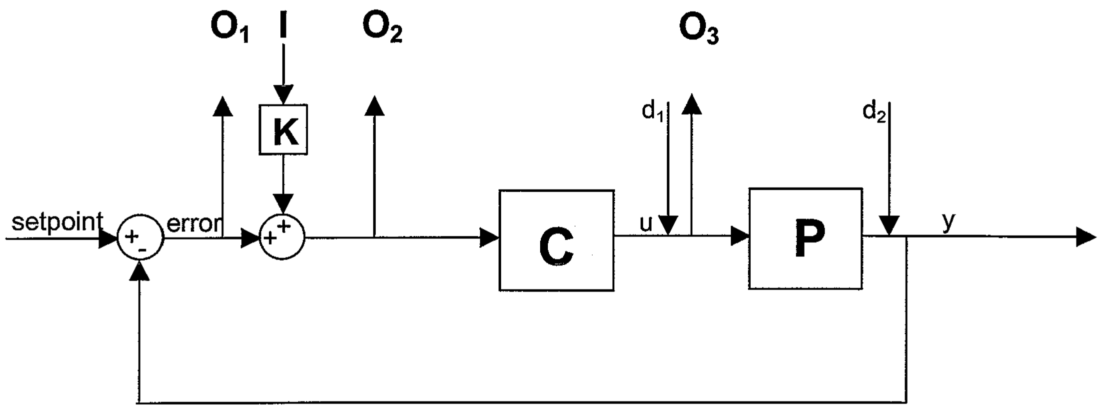


Figure 2.2 input/output points

These points are also depicted in figure 2.2.1 where the PCB (printed circuit board) is displayed.

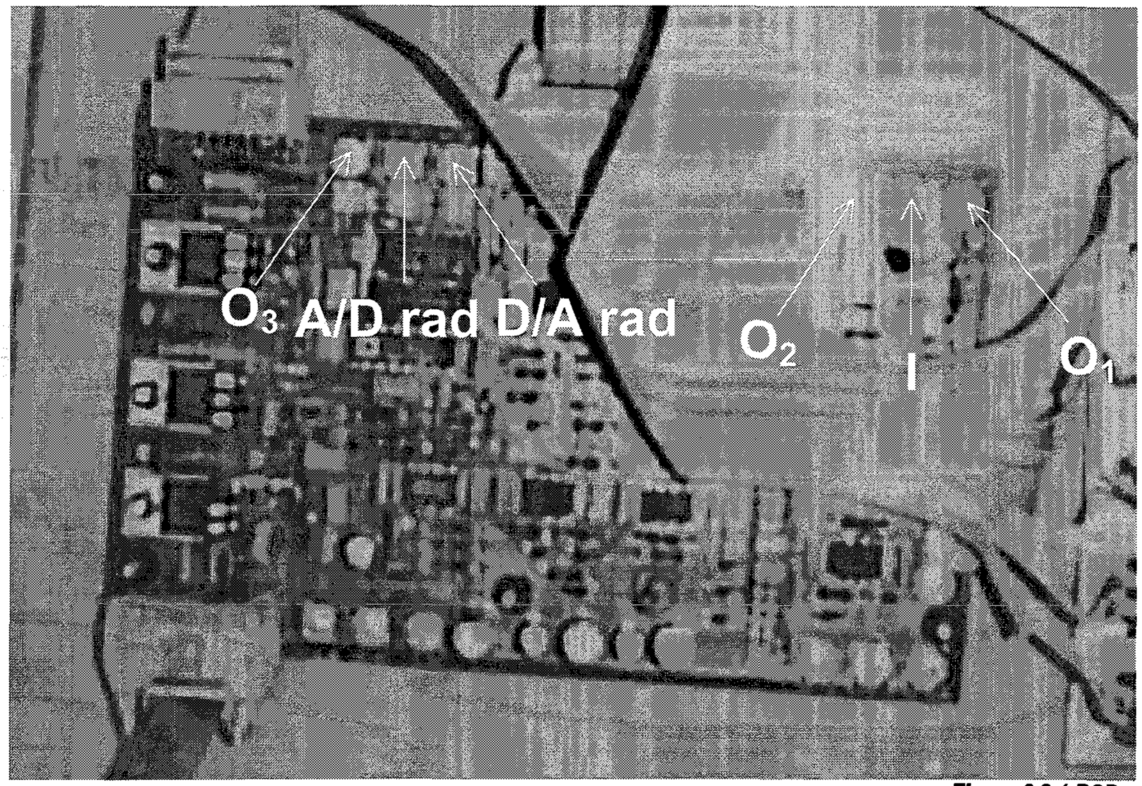


Figure 2.2.1 PCB

In these figures **I** represents a point in the loop where a signal can be injected, $\mathbf{O_1}$, $\mathbf{O_2}$ and $\mathbf{O_3}$ readout-points, **K** the gain of the sumpoint and d_1 & d_2 disturbances. With these available points two measurements are possible: the closed loop measurement and the sensitivity measurement. At the closed loop measurement a signal is injected at point **I** and read out at point $\mathbf{O_1}$. For the sensitivity measurement these points are **I** and $\mathbf{O_2}$. The closed loop relations for the two measurements now become as follows:

Closed loop measurement:

$$O_1 = \frac{P(s)C(s)K(s)}{1+P(s)C(s)} \cdot I + \frac{P(s)}{1+P(s)C(s)} \cdot d_1 + \frac{1}{1+P(s)C(s)} \cdot d_2$$

Sensitivity measurement:

$$O_2 = \frac{K(s)}{1+P(s)C(s)} \cdot I + \frac{P(s)}{1+P(s)C(s)} \cdot d_1 + \frac{1}{1+P(s)C(s)} \cdot d_2$$

In both measurements disturbances occur. These disturbances will also appear in the measured output. Since the closed loop response has a decreasing magnitude for high frequencies, disturbance d_2 becomes large compared to the influence of the injected signal. For the sensitivity measurement disturbance d_1 is the restraining factor. At low frequencies this disturbance is relatively large. This can be discounted for by a right choice of the signal-noise ratio. Disturbance d_2 can be compensated for by the choice of the injected signal. When white noise is chosen to be the signal to be injected into the system, the cross spectrum between this signal and d_2 becomes zero (since these two signals are uncorrelated). After this measurement an estimation of the sensitivity frequency response function is available. The first step to take is to compensate for the gain **K** of the sumpoint. When the transfer of the sumpoint has been measured, the gain **K** of this sumpoint is known. At this point the sensitivity frequency response function can be multiplied with this gain. This measurement is done by injecting a signal at point **I** and measuring at point $\mathbf{O_2}$. In order to get just the gain of the sumpoint the loop has to be cut open. This is done by turning off the laser.

At this time, the open loop transfer function can be calculated via:

$$S(s) = \frac{1}{1 + P(s)C(s)} \quad \Rightarrow \quad P(s)C(s) = S(s)^{-1} - 1$$

The next step now is to measure the transfer function of the controller C,which is necessary to retrieve a model for the plant P. This can be done in two ways. The first is through a measurement. This is done by injecting a signal at point **I** and readout at point $\mathbf{O_3}$. Since this is a closed loop system, the laser and the focus have to be turned off (opening the loop) to get only the controller out of this measurement. The second method is a more mathematically method. Since the electrical scheme of the the CD-player is known (Appendix E), the transfer function of the controller can be calculated with existing electrical engineering rules. When the open loop frequency response function is divided by the resulting model of the controller, the plant P is obtained. In order to get a usable plant for further calculations this result has to be fitted. This is done with the *frfit* routine in Matlab. With this algorithm the order of the numerator, the order of the denumerator and the number of integrators can be specified. During the fitting procedure zoom intervals and weighting functions can be added. The inputs of this algorithm are the frequency response data and the frequency vector. The output is a state-space description of the fitted frequency response function. In the next subsection, the previous explained procedure will be executed in practice.

2.2 Experimental

The sensitivity measurement has been done with *SigLab*. There has been chosen to use the *VNA* (virtual network analyser) method for the measurements because with this method the user immediately sees on the screen how the application is performing and it is very quick in comparison with for example *VSS* (virtual swept sine). With this last method a better coherence for low frequencies can be obtaind, but since fitting will be done later on, this will not cause any trouble. For a detailed description of the *VNA* measurement see Appendix A. The used adjustments were a measurement bandwidth of 10 kHz, a record length of 8192, no triggering, a *Hanning* window and 256 averages. The injected signal has been pink noise with a Volt peak of 0.4 (the difference between white noise and pink noise is that white noise has constant power when frequency is plotted on a linear scale, and pink noise has constant power when frequency is plotted on a log scale). The best sensitivity measurement that has been obtained, is displayed in figure 2.3.

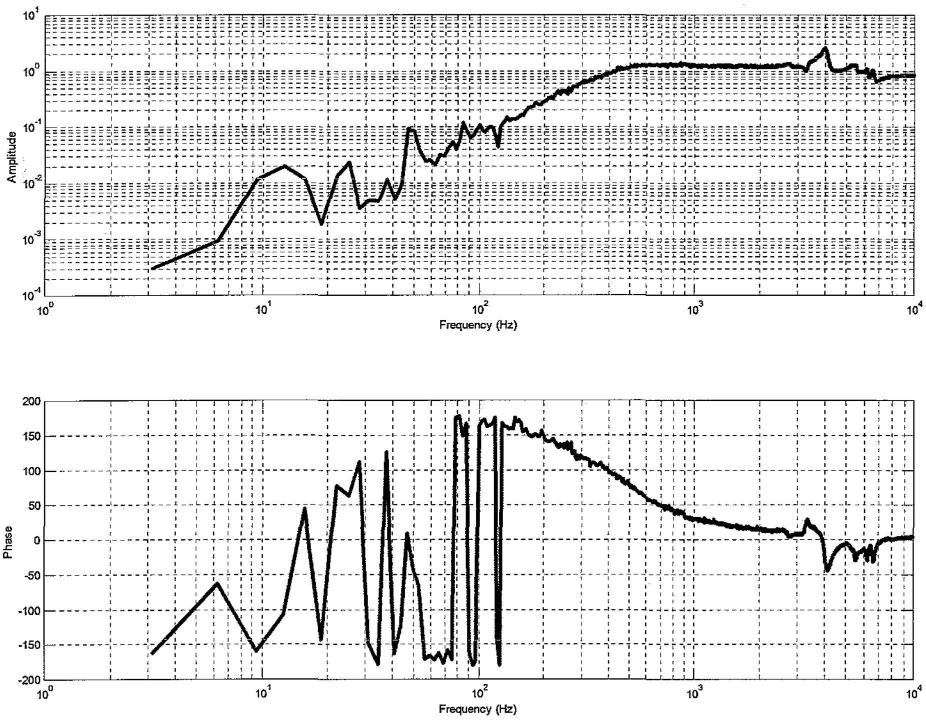


Figure 2.3 sensitivity function

From this figure the following remarks can be made:

- for low frequencies the sensitivity has a slope of +2 and a phase of +180 degrees
- the high frequent slope and phase are both zero
- some resonance peaks are visible at higher frequencies

The next step would be a measurement of the internal controller of the CD-player. As explained before, there are two ways to come to a transfer function of the controller. In figure 2.4 both results are displayed. The result of the impedance calculation causes the following transfer function of the controller (for a complete calculation see Appendix B):

$$H(s) = -\frac{0.3346212 \cdot s^2 + 541.196 \cdot s + 1 \cdot 10^5}{0.1652332 \cdot s^2 + 5173.196 \cdot s + 1 \cdot 10^5}$$

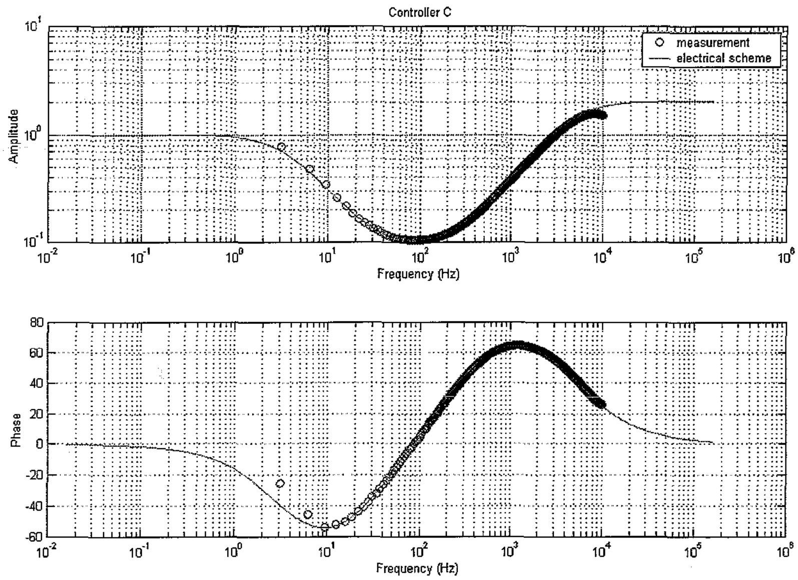


Figure 2.4 controller

As one can see they don't differ in amplitude and in phase. From the transfer function the location of the zeros and the poles can be calculated (and can also be verified in figure 2.4):

- Poles: -3.08 Hz and $-4.98 \cdot 10^3$ Hz
- Zeros: $-2.24 \cdot 10^2$ Hz and $-3.39 \cdot 10^1$ Hz

At this point the plant can be extracted from the obtained results, as already has been explained in the previous subsection. The result of this extraction is visible in figure 2.5 (the non-smooth line).

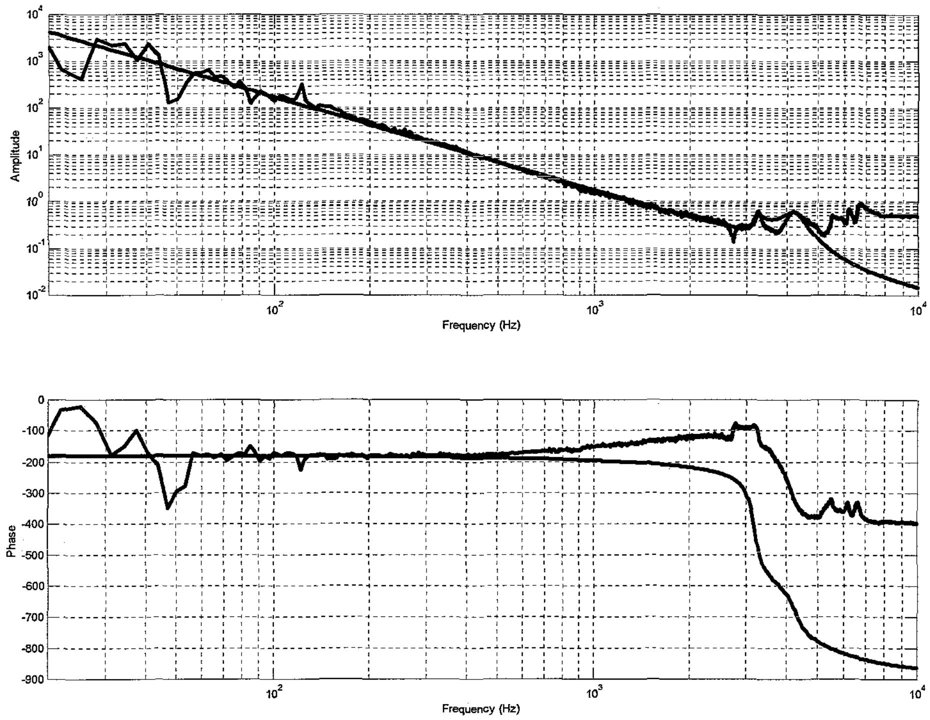


Figure 2.5 experimental extraction of the plant and the fit

This result is what one might expect; a -2 slope with some highfrequent resonance peaks and a phase of minus 180 degrees. The fact that the phase is not exactly –180 degrees at frequencies above 500 Hz could lie in the presence of electrical cross talk. This means mutual effect of two wires next to each other. The result after the fitting procedure is shown in figure 2.5 with the smooth line. There has been decided to fit two peaks of the transfer function. The reason of this is to limit the complexitiy of the model.

3 Disturbance modelling

3.1 Theory

The next step in the characterization process will be the modelling of the different disturbances of the different discs. The disturbances that affect the radial error are the following:

- eccentricity of the disc
- track unroundness
- mechanical vibration and shocks

The major disturbances of these are the eccentricity of the disc and the track unroundness. When looking at the spectrum of these disturbances, a peak will occur at the rotational frequency of the disc. Also the higher harmonics of this rotational frequency will be visible. Since in the experiments measurements will be done on a test set-up in a laboratorium the contribution of vibrations and shocks will be low. The question that now may rise is how to extract the disturbance from measurements. When a block diagram is made with the disturbance assumed to be a setpoint (figure 3.1), the following relations can be formed.

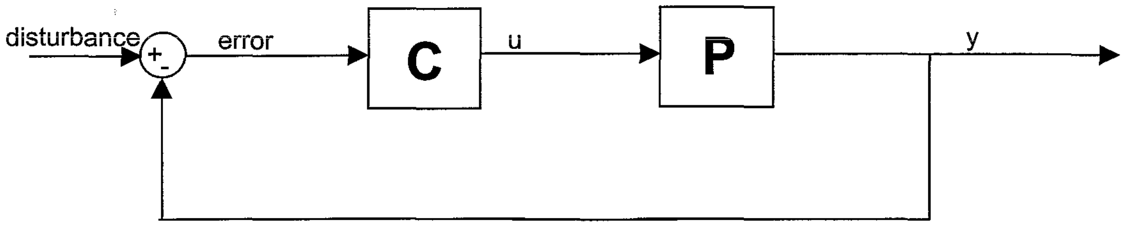


Figure 3.1 block diagram

The transfer between the disturbance and the setpoint:

$$error = \frac{1}{1 + P(s) \cdot C(s)} \cdot disturbance = S(s) \cdot disturbance$$

When now the error will be measured, the disturbance can be estimated by:

$$disturbance = S(s)^{-1} \cdot error = (1 + P(s) \cdot C(s)) \cdot error$$

This is of course only true when one assumes that only these disturbance are acting on the system, which will be the case when resonances and vibrations do not play a role of interest. This will be explained in practice later on in this report. In the next subsection this procedure will executed in practice.

3.2 Experiments

The measurement of the radial error has been done by letting the CD-player rotate the disc, using the internal controller and reading out the error at point **O_1** in figure 2.2. This measurement has been done for 15 different plates with a record length of 3.2 seconds. For simplicity only the results of one of the measurements are displayed here. Since the measurements are done with *SigLab* the units on the Y-axis are in *V(olts)*. The precise conversion rate to *μm* or *m* (micrometers or meters) is not known, but this will not be a problem in analyzing the differences between different discs. This because the conversion rate is of course for every plate exactly the same. This conversion rate has to be known precisely when one wants to say something about the size of the error. In the used CD-players an error of 0.1 *μm* and an eccentricity of 100 *μm* is acceptable. The measured error for one disc is displayed in figure 3.2.

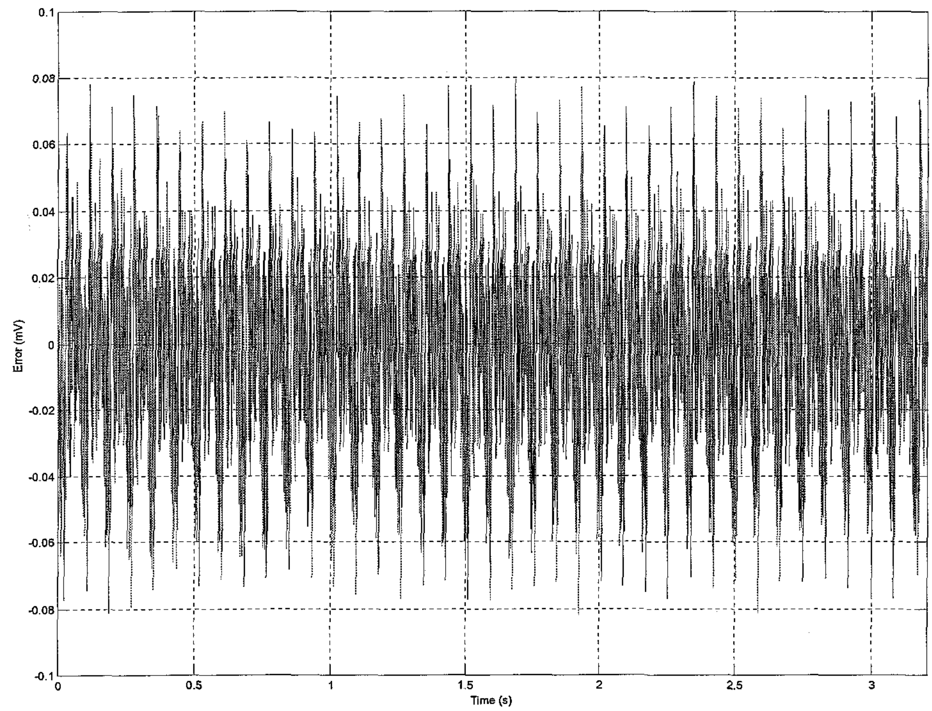


Figure 3.2 radial error

This error now has to be inversely filtered with the sensitivity function in order to obtain the disturbance. The procedure for desiging a good filter for this activity is described in section 3.2.1.

3.2.1 Filter design

At this point it is not very difficult to calculate the inverse of the sensitivity function. Since the fitted plant and the calculated controller are available, the open loop frequency response function can be calculated, which results in (figure 3.3):

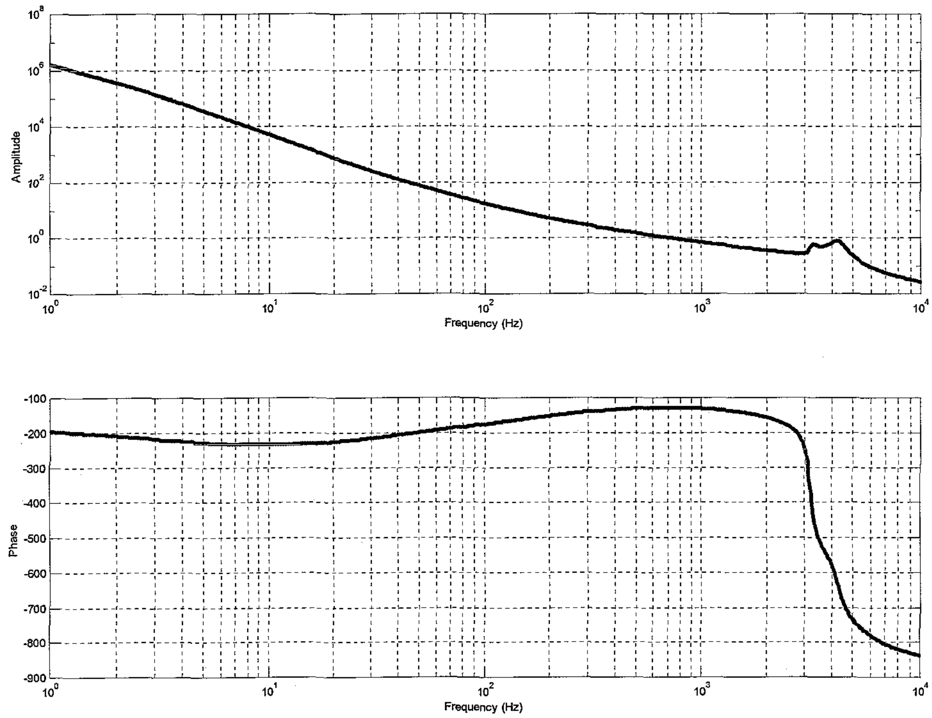


Figure 3.3 open loop transfer function

In this frequency response function a phase lift is visible around the bandwidth (800 Hz).

When the open loop is known, the inverse sensitivity can be calculated, according to:

$$(1+P(s)\cdot C(s))$$

The bode diagram of this inverse sensitivity function is displayed in figure 3.4.

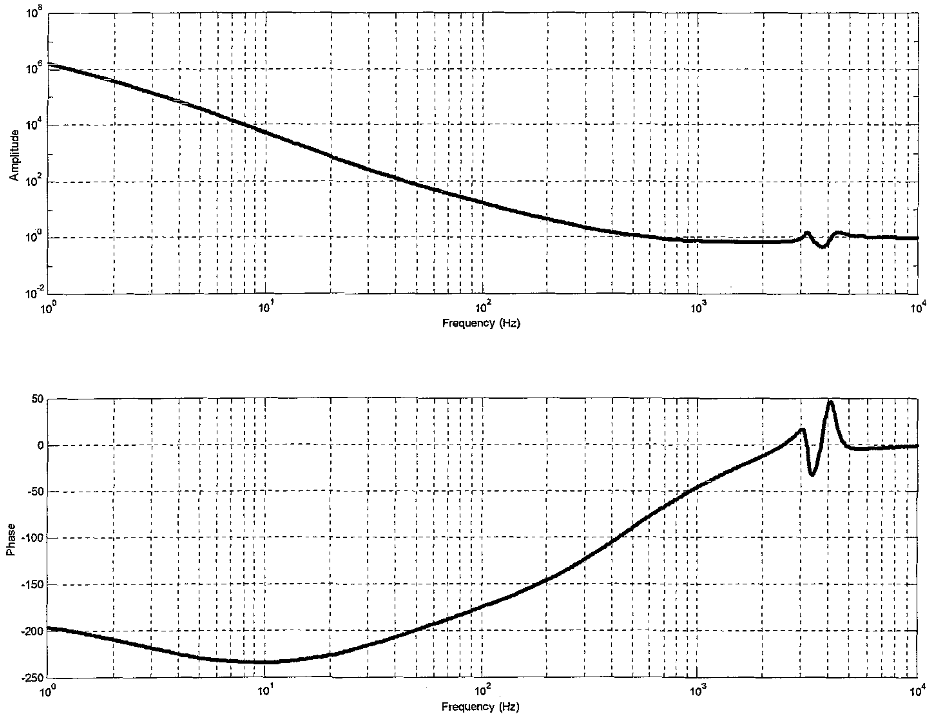


Figure 3.4 inverse sensitivity

The problem that rises now, when the error will be filtered with this inverse sensitivity, is the fact that for low frequencies this inverse sensitivity goes to infinity. This means that the low frequent part of the error will be increased tremendously. Also the high frequent part of the error is not the topic of interest when the eccentricity of a disc is investigated.

For this reason the inverse sensitivity function has to be filtered through a bandpass filter. Since the rotation frequency lies around 12 Hz, the passband has been chosen between 10 Hz and 1000 Hz. The filter order was chosen to be 8, resulting in a +4 slope for low frequencies and –4 slope for high frequencies. The reason for this choice is to get a lower weight for frequencies below the 10 Hz and also for frequencies above the 1000 Hz. The bandpass filter that has to perform this task has the following bode diagram (figure 3.5):

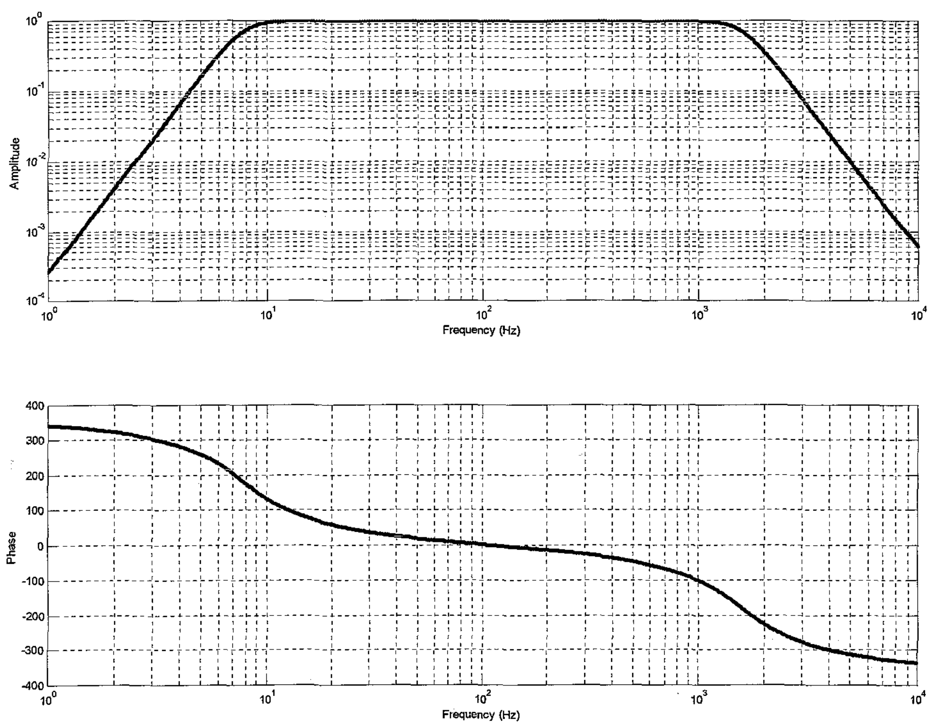


Figure 3.5 bandpass filter

When the inverse sensitivity function is filtered through this bandpass filter, the result, which will be used to filter the error, is displayed in figure 3.6. As can be seen, for low and high frequencies the weight is decreased and for the frequency range between 10 Hz and 1000 Hz, the resulting filter is the same as the inverse sensitivity.

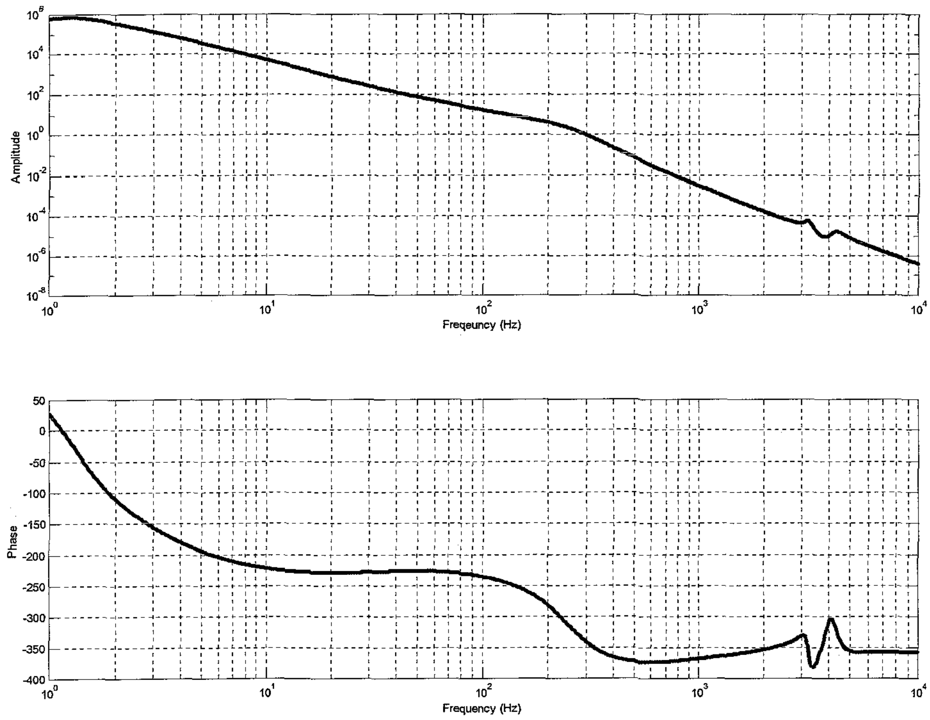


Figure 3.6 resulting filter

3.2.2 Disturbance extraction

At this point the measured error can be filtered through the obtained filter, which will result in the disturbance caused by the eccentricity of the disc and the track unroundness. The filtering has been done with *simulink* (the scheme used is displayed in Appendix D). The resulting disturbance which results after this filtering is displayed in figure 3.7.

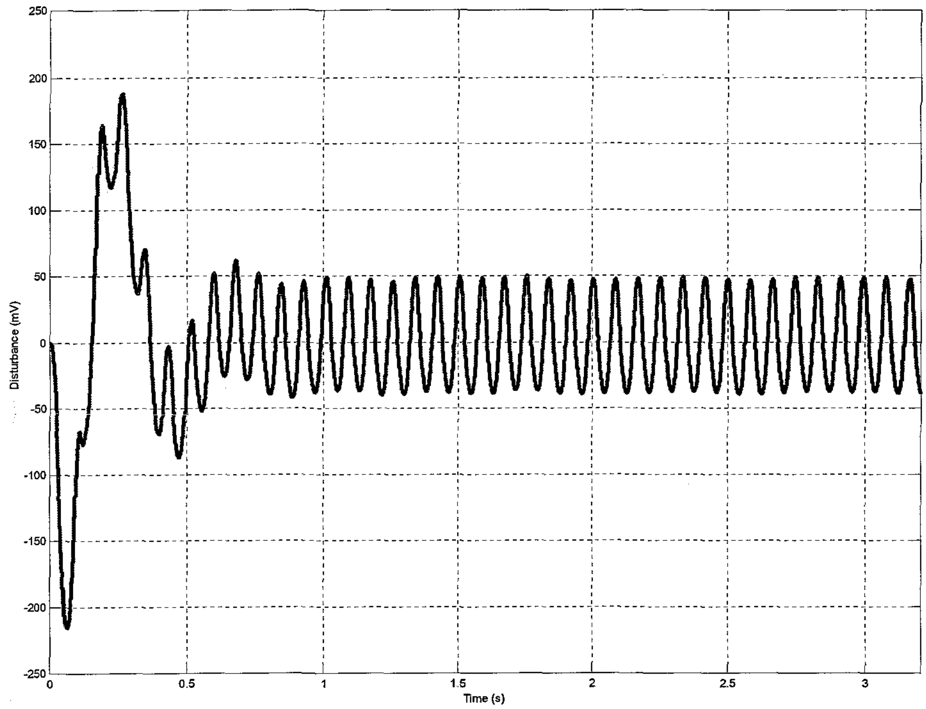


Figure 3.7 disturbance

The heavy oscillations in the beginning are the result of the transient response of the filter. As can be noticed the allowable factor of 1000 between the error and the eccentricity is reached for this particular disc. The magnitude of the eccentricity of the disc becomes more obvious when one looks at the spectrum of this disturbance. For this purpose, the corresponding spectrum is displayed in figure 3.8.

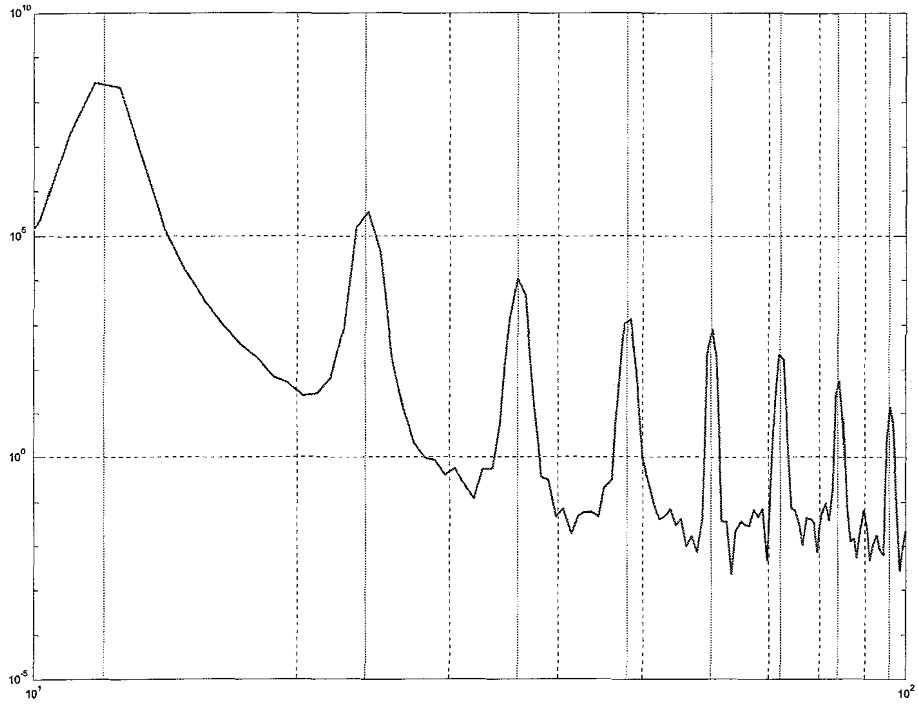


Figure 3.8 spectrum of the disturbance

In this figure, the vertical lines denote the first harmonic (12 Hz) and all the higher harmonics of this frequency. As can be seen, all the harmonics contribute to the eccentricity. The next step now would be to look at the eccentricity of different plates, which will be done in the next chapter.

4 Comparison of different discs

4.2 Introduction

At this point there can be thought about comparing the eccentricity of different discs. An example for instance is to standarize one disc and compare the rest with respect to this one. In order to get a good result, the mean of different measurements has to be taken. There has been decided to measure the error of each plate 6 times. Between each measurement the disc has been taken out of the CD-player and put in again. For one arbitrary disc these resulting errors are displayed in figure 4.1.

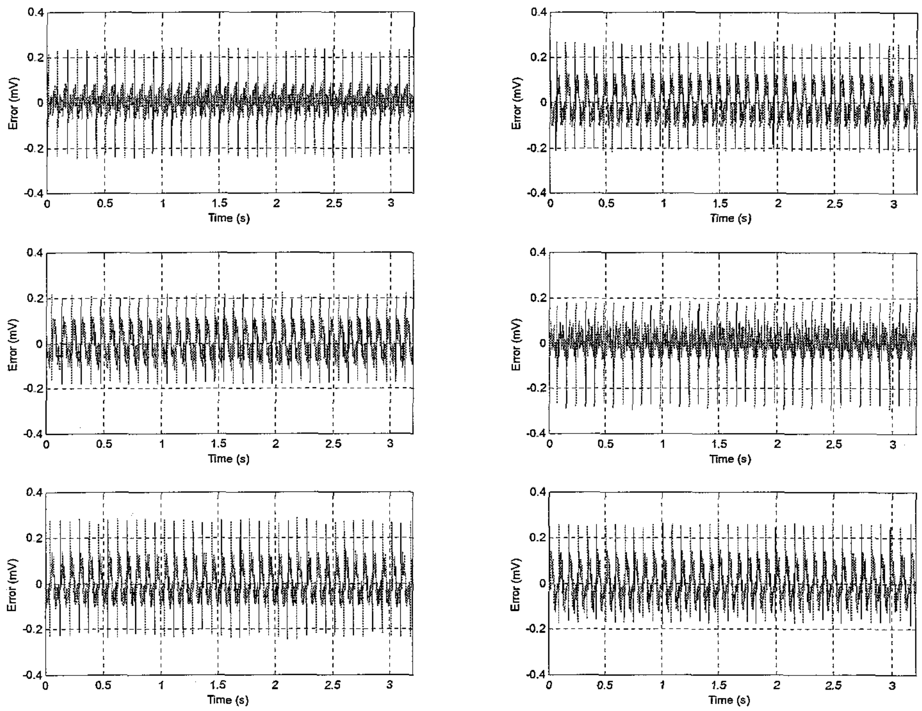


Figure 4.1 errors with replacing the disc

As can be noticed the errors are not all six the same. This can be the result of different support at each measurement, but also a different trace can be measured. This could result in different eccentricities and for this reason a comparison can be difficult. Since with the used line-up it is not possible to measure each time the same trace, but the disc can be taken out of the CD-player, the second prblem can be solved. This same disc has also been measured without taking out the disc between the different measurements. In this way the different ways the disc lies at the rotor would be eliminated. The resulting errors of this second measurement are visible in figure 4.2.

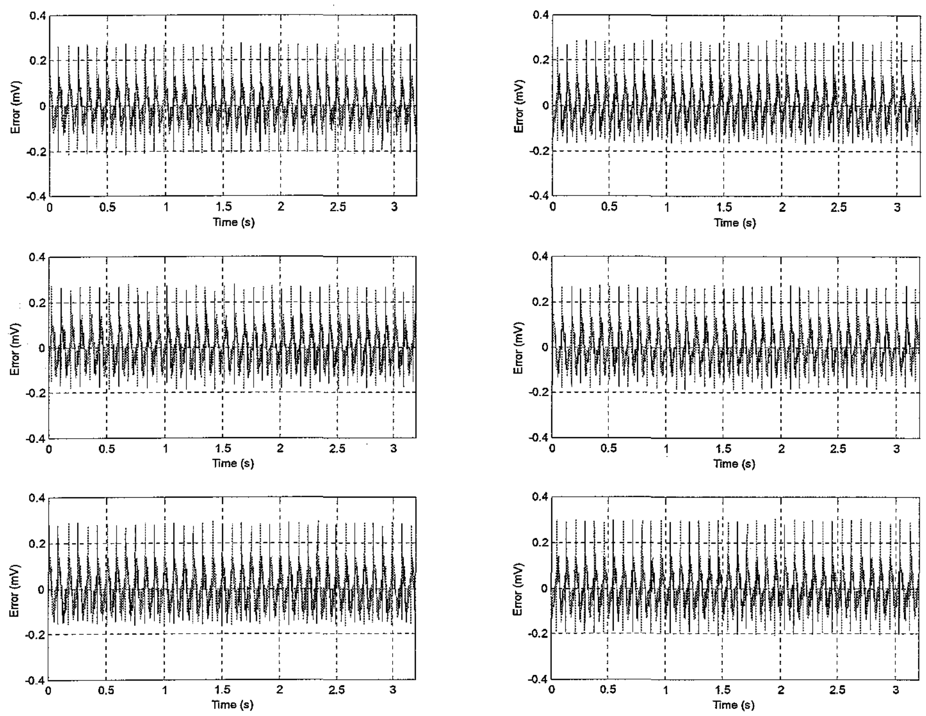


Figure 4.2 errors without replacing the disc

An interesting phenomenon to look at, is the difference between the spectra of the different eccentricities of these measurement techniques. In figure 4.3, the 12 obtained spectra are displayed. The measurements without displacement are plotted with the solid red lines and the one with replacement with dashed black lines. In the upper figure all the measurements are visible, where in the lower figure the means of each six have been calculated.

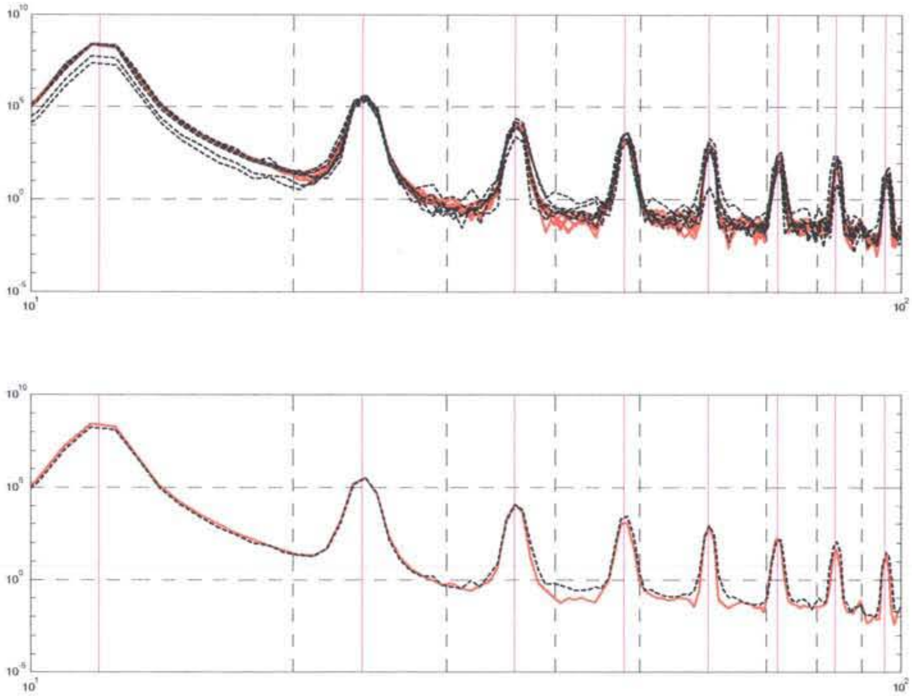


Figure 4.3 spectra of the disturbances

Clearly visible in this figure is the difference in deviation between the two methods in the upper part of the figure. When looking at the means of the two methods (lower part) they come closer to each other (but the scale of the Y-axis has to be taken into account). An other way to analyze these results is to normalize the first harmonic at 1 and than look at the contributions of the higher harmonics to this first one. When is this is done for the figure above the following spectra arise (figure 4.4).

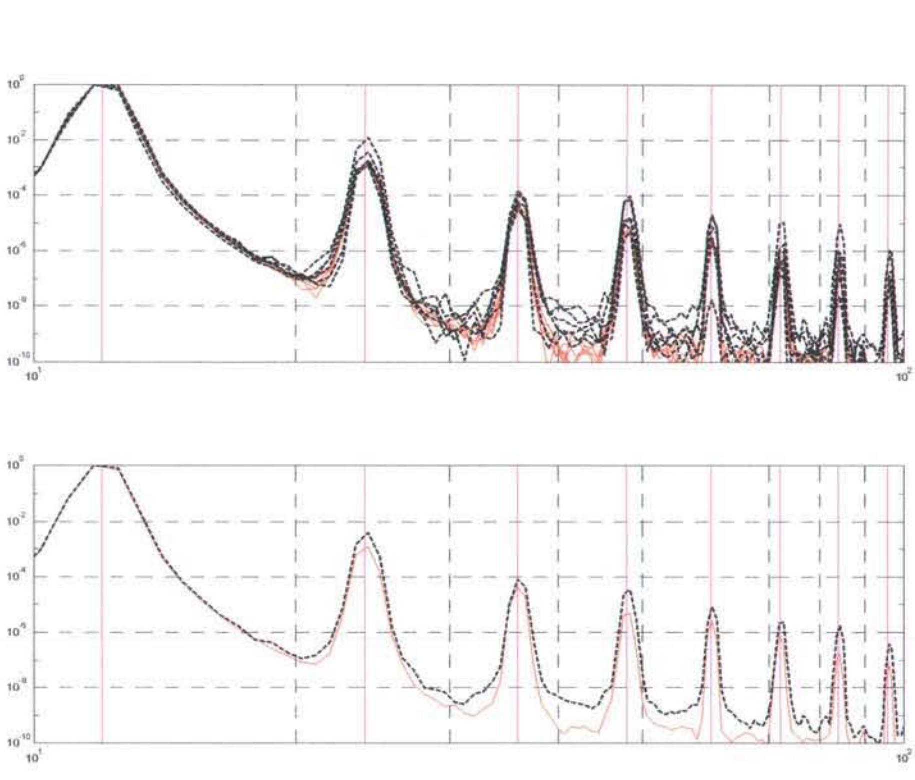


Figure 4.4 normalized spectrum

Also in this figure are in the upper window all 12 measurements and in the lower window the means. From this figure (especially from the means) it becomes very clear that when one normalizes the first harmonics of the spectra, the differences between the methods, when looking at the contribution of the higher harmonics, is not all to large. There has been decided after these results, that for comparing eccentricities of different discs the method can be chosen on free will. The replacement method has my own preference, because with this method a coincidental perfect support of the rotor of a high eccentric plate does not result in a good performance according to the spectra analysis.

4.2 Results

Measurements have been done on 10 discs. These consist out of 3 discs from 3 series and 1 disc where the hole has been treated with a file to enlarge the eccentricity. The radial error from each disc has been measured 3 times. In the upper window of figure 4.5 all the spectra of all the measurements are displayed. Every serie has been plotted in a different colour (blue black and red) and the disc which was treated with the file is the magenta one.

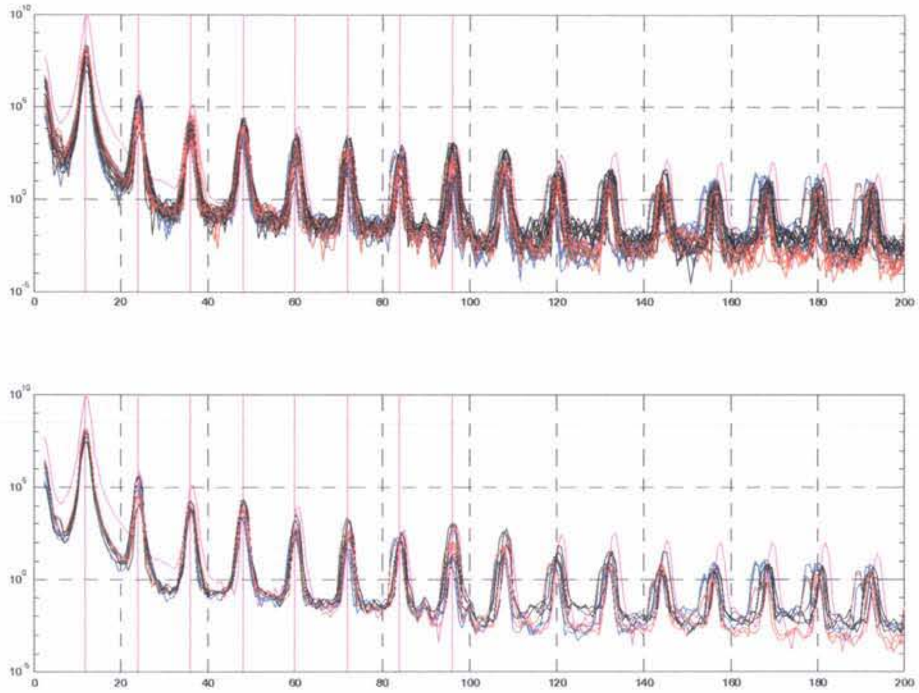


Figure 4.5 spectra of the disturbances

As can be noticed from this figure, one red peak moves along the frequency axis. This means that this is not exactly a multiple of the rotational frequency. To be able to say something about the higher harmonics in relation to the first one, again the normalized spectra were formed. This result is displayed in figure 4.6.

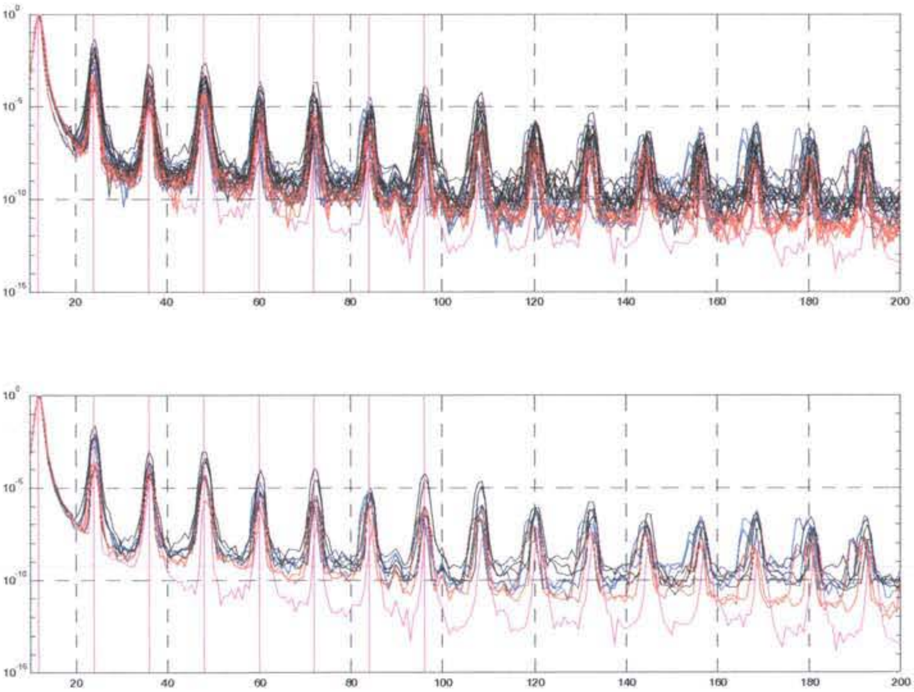


Figure 4.6 Normalized spectra

In this figure the same colours are used as in the previous one. When looking at the magenta coloured line, there can be concluded that when the disturbance (eccentricity) increases, the contribution of the higher harmonics to the first one decreases.

5. Conclusions and recommendations

In this report an algorithm has been developed which filters the eccentricity of the hole of the disc and the track unroundness, out of the measured radial error. In order to obtain this filter a sensitivity measurement has been done. With this estimation of the sensitivity frequency response function and a calculation of the controller, an estimated frequency response function of the plant is calculated. A fit of this plant frequency response function and the controller model are the basis of the filter to be designed.

A disadvantage of this filter is that the obtained result (the extracted disturbance) contains both the eccentricity of the hole as well as the track unroundness. An other problem is the experimental set-up. Not every measurement of the radial error can be done at the same track. In this manner the experiment is not quite reproducible.

Some conclusions that can be made according to the obtained results are:

- According as the eccentricity and the track unroundness increase, the contributions of higher harmonics to the first one decrease.
- Measurements with replacing the disc between each measurement show more deviations than measurements without replacing the disc.

Recommendations for further research would be:

- Use an experimental set-up that allows a reproducible measurement (same trace each measurement).
- Design an algorithm that can extract the disturbance by just letting the rotating arm rest against the assessment. In this way a sensitivity measurement would not be necessary.
- Try to develop experiments that can split the eccentricity of the disc and the track unroundness.

References

[1] M. Steinbuch, M.L. Norg. Advanced motion control: An industrial perspective. European Journal of Control (1998)4:278-293

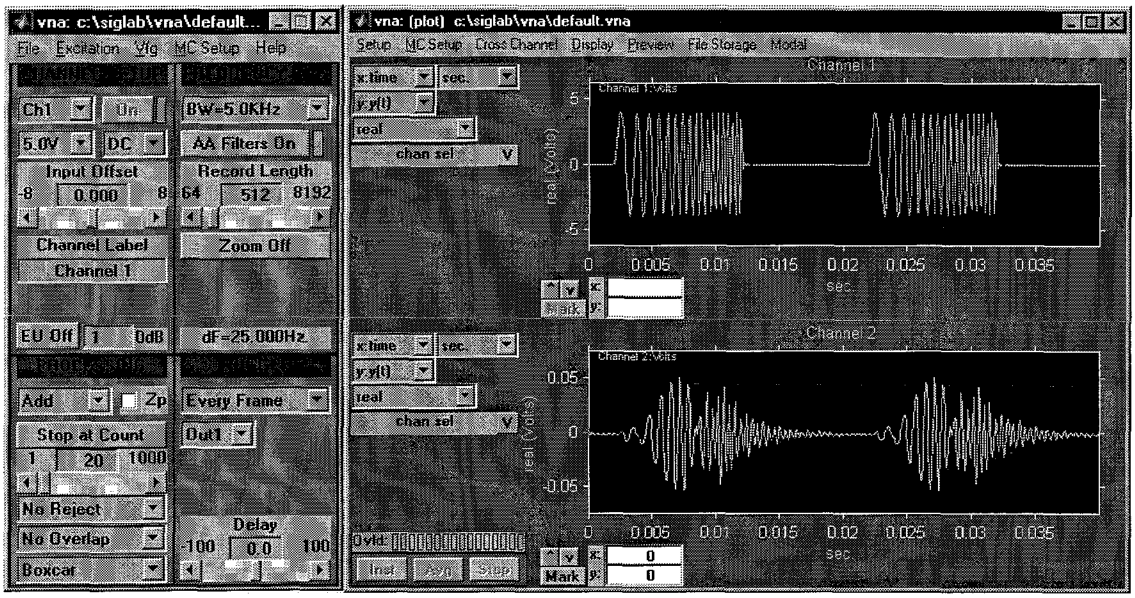
[2] M. Jansens, L. Greefs. Modellering van het radiale arm systeem van de Philips Compact Disc. Katholieke Universiteit Leuven.

Appendix A

Explanation of the VNA method for a senstivity measurement

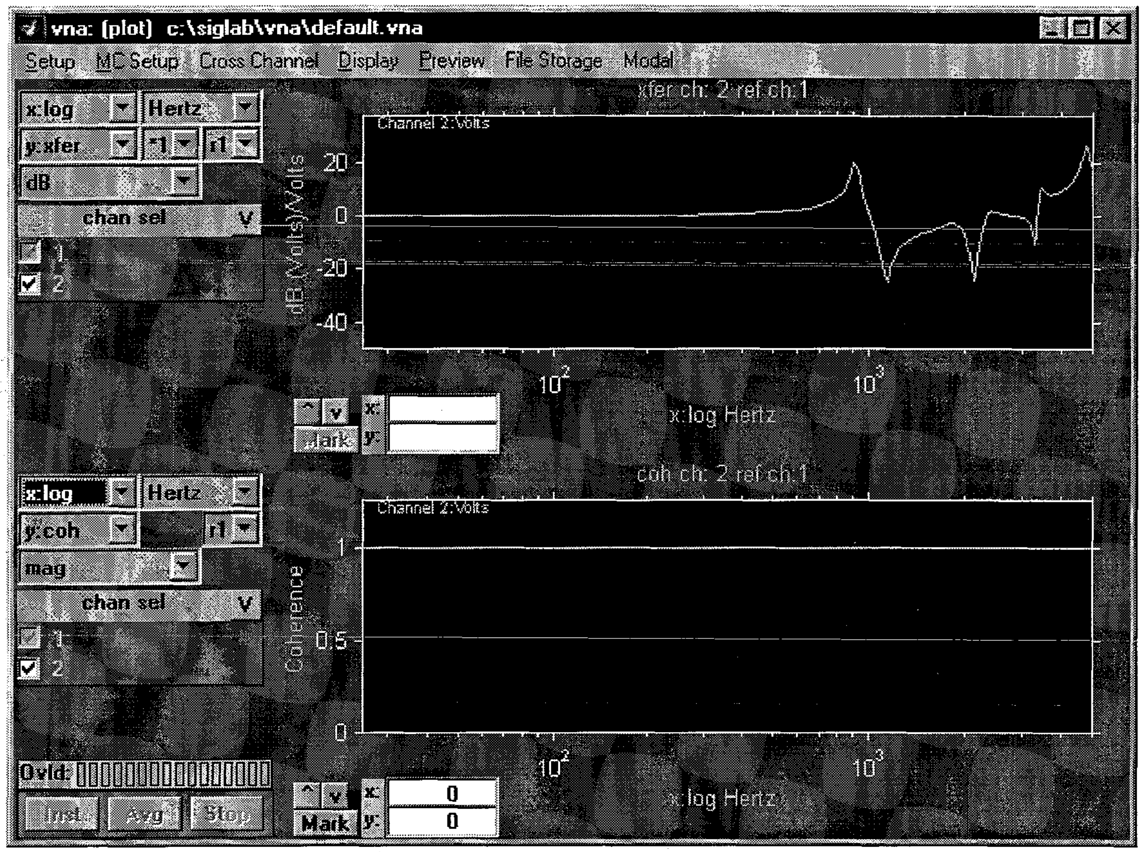
For the measurements of the sensitivity function and the controller the *VNA* (virtual network analyzer) method was used. Here follows a brief explanation of how to use this.

Step 1:
Start *SigLab*. Select *VNA* from the *network* window.The following windows appear:

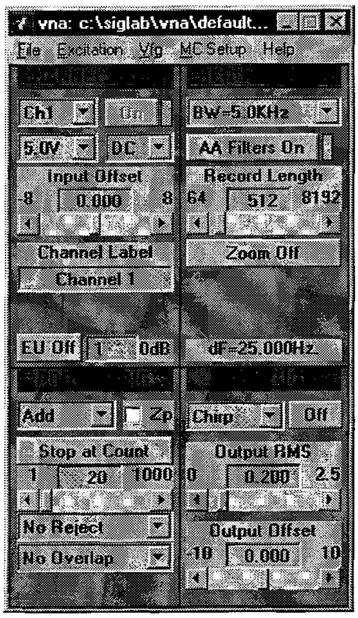


Step 2: (the right window)
In the right window two plots from *SigLab* are visible. These appear every time *SigLab* starts up. At the left from the plots one can select what has to be displayed in the plot. For the senstivity measurement it is wise to select channel 1 and 2 in both plots. This can be done by selecting both channels in the "*chan sel*" drop down list. In the upper window the transfer function now can be displayed by selecting "*y:xfer*" in the "*y:y(t)*" drop down window. In the "*x:time*" window the logscale has to be selected through "*x:log*". In the lower plot the same adjustments have to be made, but instead of "*y:xfer*", "*y:coh*" has to be selected. The results of these actions are that during the measurement the transfer function from channel 2 over channel 1 is visible in the upper plot and in the lower plot the coherence of this measurement is visible.

The right window should now look like:



Step 3: (The left window)
In the left window four menus are visible. For a sensitivity measurement triggering isn't necessary and can be turned *off* in the "*trigger*" menu. Now select "*linked to measurement*" in the excitation tab of the main toolbar. The left window should now look like:

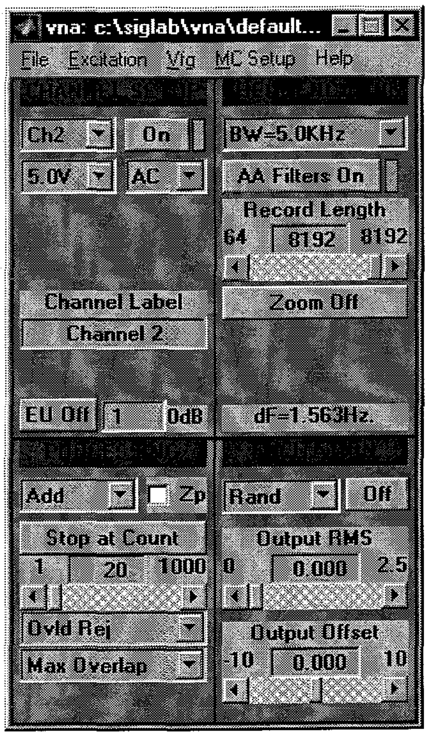


Channel setup:
If the wires between *SigLab* and the system are connected well, (one wire from *SigLab*'s output 1 to *SigLab*'s input 1, one wire from *SigLab*'s output 1 to the injection point at the system and one wire from the readout point at the system to *SigLab*'s input 2, only channel 1 and channel 2 have to be on. For a good measurement the current has to be AC (alternating current) for both channels. The range has to be selected in such a way that the signal in the appropriate channel doesn't leave this range.

Excitation:
In this drop out window the user can choose the kind of signal to inject in the system. The options are *Chirp* and *Rand*. The *Chirp* signal a sawtooth signal and *Rand* represents white noise. When selecting one, first put the *Output RMS* level to zero (otherwise one may destroy the system).

Processing:
Choose the *ADD* option. The number of counts to stop can be taken arbitrary, but at least 50 is recommended (this number represents the number of averages made to come to an good result). In the *No Reject* window choose *ovld reject*.
When *chirp* is selected:
-No Overlap
When *Rand* is selected:
-Max Overlap

Frequency RNG:
Choose the bandwidth of the measurement. Make sure that this bandwidth is at least twice the frequency of the highest frequency one wants to measure (with respect to the nyquist frequency). Leave the AA (anti-aliasing) filters on. Select the number of steps *SigLab* has to take over the frequency range. When the bandwidth is 5 kHz, a record length of 64 means that the frequency steps made are 200 Hz each. A record length of 8192 results in a frequency step of 1.563 Hz. The choice at this point is between a quick,inaccurate measurement on one side or a long,accurate measurement on the other side. Choose the accurate measurement, with a bandwidth of 5 kHz. Leave *zoom* off. The whole left window should now look something like this:



At this point one can start with the measurement. The first step to take now is to start the system. Next turn on the excitation by switching the *Off* button to *On* in the excitation menu in the left window. Now slowly increase the *Output RMS* till the injected signal can be heard (not too hard of course). Finally the sensitivity measurement can be started in the right window by pushing the *AVG* button. If there occurs an overload one of the two green lights (left bottom of the right window) will turn red and the range of the appropriate channel has to be adjusted. This has to be done by first *stop* the measurement, then adjust the range and finally average again. When one is satisfied with the measurement, the data can be saved using the left window (file...save as...sensitivity.vna).

Exporting data to Matlab

The measured data can also be used in *Matlab*. Therefore one has to import this data and unpack it. The best way to do this is to load the data and rename it.

Load measured data

load sensitivity.vna –mat

Rename data

Time vector:
time=SLm.tdxvec;

Frequency vector:
hz=SLm.fdxvec;

Time data:
timedata=SLm.scmeas(1,2).tdmeas;

Frequency response data:
frfdata=SLm.xcmeas(1,2).xfer;

Coherence:
coherence=SLm.xcmeas(1,2).coh;

At this point one can use the measurement in the *Matlab* environment.

Appendix B

Calculation of the transfer function of the controller

On the electrical scheme of the CD-player one can find the radial controller and calculate the transfer function of this controller for further use in programs such as *Matlab*. The elements of which the controller is built are displayed in figure B.1.

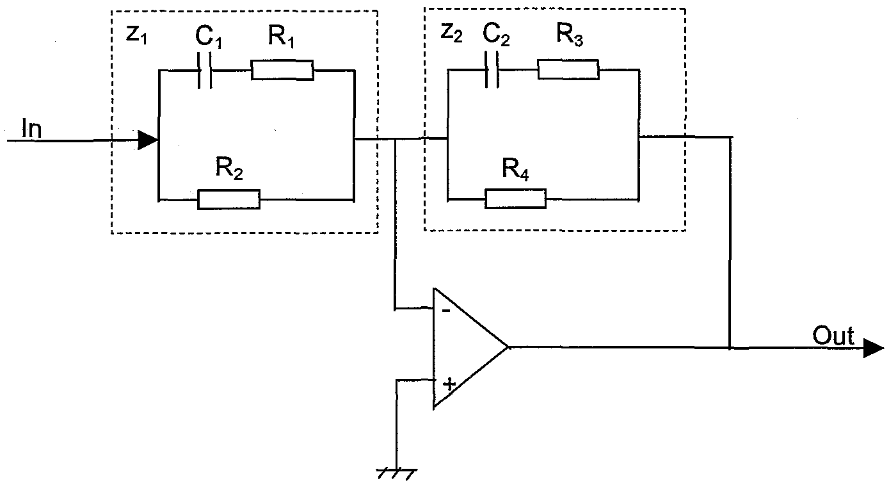


Figure B.1 Controller scheme

In this scheme *C* represents a condensator and *R* a resistance. Z_1 and Z_2 represent the impedances of the 2 combinations of condensators and resistances. *In/Out* refer to the voltage levels. The numerical values of the condensators and resistances are the following:

$C_1 = 6.8 \cdot 10^{-9}$ F = 6.8 nF
C_2 = 470 nF
$R_1 = 4.7 \cdot 10^{3}\ \Omega = 4.7\ k\Omega$
$R_2 = 100\ k\Omega$
$R_3 = 10\ k\Omega$
$R_4 = 100\ k\Omega$

In order to make the calculation easier one can make an equivalent scheme of that in figure B.1 which is displayed in figure B.2. In this figure also the current and its direction are displayed.

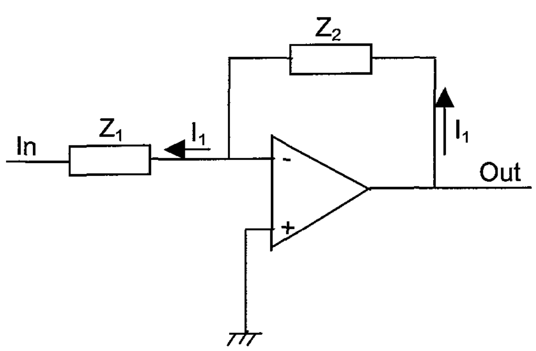


Figure B.2 equivalent controller scheme

One can easily find the the current I_1 by using Ohm's law for the two separate circuits. For the first circuit follows the following equation:

$$I_1 = \frac{V_{out} - V_-}{z_2}$$

with V_- the voltage at the negative side of the amplifier, which in this equation is equal to that on the positive side. Since the positive side is connected to earth, which has zero voltage, V_- will also be taken zero afterwards. The second equation becomes:

$$I_1 = \frac{V_- - V_{in}}{z_1}$$

Eliminating I_1 from these equations produces:

$$\frac{V_{out} - V_-}{z_2} = \frac{V_- - V_{in}}{z_1}$$

which, with $V_- = 0$, becomes:

$$\frac{V_{out}}{z_2} = -\frac{V_{in}}{z_1} \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{z_2}{z_1}$$

The resulting equation is the complex transfer function of the controller. The minus sign indicates how the controller is defined in the scheme. For analyzing and plotting purposes this minus sign has to be removed (otherwise the phase has turned 180 degrees).

The first step is to calculate the impedances z_1 and z_2. Since for two parallel resistances the following rule holds:

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 \cdot R_2}$$

the next result can be obtained for z_1:

$$z_1 = \frac{\left(\frac{1}{j\omega C_1} + R_1\right) \cdot R_2}{\left(\frac{1}{j\omega C_1} + R_1\right) + R_2} = \frac{j\omega C_1}{j\omega C_1} \cdot \frac{\left(\frac{1}{j\omega C_1} + R_1\right) \cdot R_2}{\left(\frac{1}{j\omega C_1} + R_1\right) + R_2} = \frac{(1 + R_1 j\omega C_1) \cdot R_2}{1 + j\omega C1 \cdot (R_1 + R_2)} = \frac{R_2 + R_1 R_2 j\omega C_1}{1 + j\omega C_1 \cdot (R_1 + R_2)}$$

In the same way z_2 can be calculated, resulting in:

$$z_2 = \frac{R_4 + R_3 R_4 j\omega C_2}{1 + j\omega C_2 \cdot (R_3 + R_4)}$$

The transfer function now becomes:

$$H(j\omega) = -z_2 \cdot z_1^{-1} = -\frac{R_4 + R_3 R_4 j\omega C_2}{1 + j\omega C_2 \cdot (R_3 + R_4)} \cdot \frac{1 + j\omega C_1 \cdot (R_1 + R_2)}{R_2 + R_1 R_2 j\omega C_1}$$

With the earlier given numerical values of the condensators and resistances, this transfer function becomes:

$$H(j\omega) = -\frac{(1 \cdot 10^5 + 4.7 \cdot 10^2 \cdot j\omega) \cdot (1 + 7.1196 \cdot 10^{-4} \cdot j\omega)}{(1 + 5.17 \cdot 10^{-2} \cdot j\omega) \cdot (1 \cdot 10^5 + 3.196 \cdot j\omega)}$$

Computing this transfer function and replacing $j\omega$ with *s* gives the final transfer function *H(s)*:

$$H(s) = -\frac{0.3346212 \cdot s^2 + 541.196 \cdot s + 1 \cdot 10^5}{0.1652332 \cdot s^2 + 5173.196 \cdot s + 1 \cdot 10^5}$$

This function has two poles and two zeros at the following frequencies:
Poles:
-19.342 rad/s and $-3.1289 \cdot 10^4$ rad/s
Zeros:
$-1.4046 \cdot 10^3$ rad/s and $-2.1277 \cdot 10^2$ rad/s

In Hertz this becomes:

- Poles: -3.08 Hz and $-4.98 \cdot 10^3$ Hz
- Zeros: $-2.24 \cdot 10^2$ Hz and $-3.39 \cdot 10^1$ Hz

Appendix C

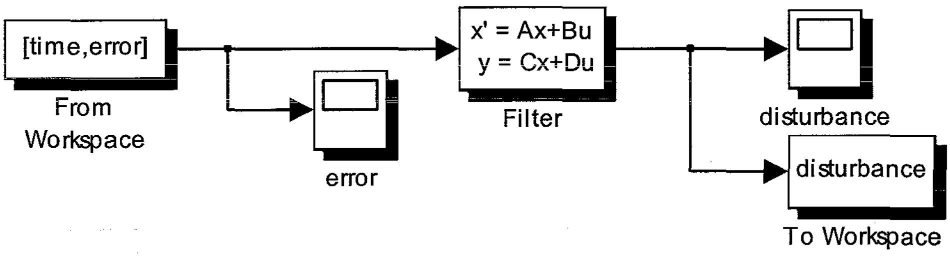
Disturbance_filt.m

```
clear all
close all

%sample time
Fs=2.56e4;
Ts=1/Fs;

%controller
num=[0.3346212 541.196 1e5];
den=[0.1652332 5173.196 1e5];
cont=tf(num,den);

%plant
load m:\vakken\stage2\metingen\plantdata3
p=ss(a,b,c,d);

%loading errordata
load m:\vakken\stage2\error\cd_hema_02\meting01.vna -mat
error1=SLm.scmeas(1,2).tdmeas;

Time=SLm.tdxvec;
Time=Time';
Freqvec=SLm.fdxvec;

%calculation band-pass filter
[num,den]=butter(8,[10/2*pi 1000/2*pi],'s');
boter=tf(num,den);

%calculation openloop
syspc=series(p,cont);

%calculation inverse sensitivity
sinv=1+syspc;

%calculation inverse sensitivity and band-pass filter
s=series(sinv,boter);

%simulinkdata
[a,b,c,d]=ssdata(s);

%simulation
sim('sim_disturbance_new')

%spectra calculation
[Px1,Fd1] = spectrum(disturbance1(20000:81910),8192,4096,hanning(8192),Fs);

%deleting the first part
Px1=Px1((4:size(Px1)),1);
Fd1=Fd1(4:size(Fd1));

%plotting spectra
figure(1)
subplot(211)
loglog(Fd7,Px1(:,1)),grid
hold on
plot(12*ones(1000,1),logspace(-30,10,1000),'m')
```

```
plot(24*ones(1000,1),logspace(-30,10,1000),'m')
plot(36*ones(1000,1),logspace(-30,10,1000),'m')
plot(48*ones(1000,1),logspace(-30,10,1000),'m')
plot(60*ones(1000,1),logspace(-30,10,1000),'m')
plot(72*ones(1000,1),logspace(-30,10,1000),'m')
plot(84*ones(1000,1),logspace(-30,10,1000),'m')
plot(96*ones(1000,1),logspace(-30,10,1000),'m')
subplot(212)
loglog(Fd1,P1gem1),grid

%normalization factor calculation
normfactor1=max(Px1);

%normalization
Px1=Px1./normfactor1;

%plotting normalization
figure(2)
subplot(211)
loglog(Fd7,Px1),grid
hold on
plot(12*ones(1000,1),logspace(-30,10,1000),'m')
plot(24*ones(1000,1),logspace(-30,10,1000),'m')
plot(36*ones(1000,1),logspace(-30,10,1000),'m')
plot(48*ones(1000,1),logspace(-30,10,1000),'m')
plot(60*ones(1000,1),logspace(-30,10,1000),'m')
plot(72*ones(1000,1),logspace(-30,10,1000),'m')
plot(84*ones(1000,1),logspace(-30,10,1000),'m')
plot(96*ones(1000,1),logspace(-30,10,1000),'m')
subplot(212)
loglog(Fd1,P1gem),grid
```

Appendix D

Simulink scheme



Appendix E

Electrical scheme