

# Bayesian identification of LPV-BJ models : a multidimensional kernel approach

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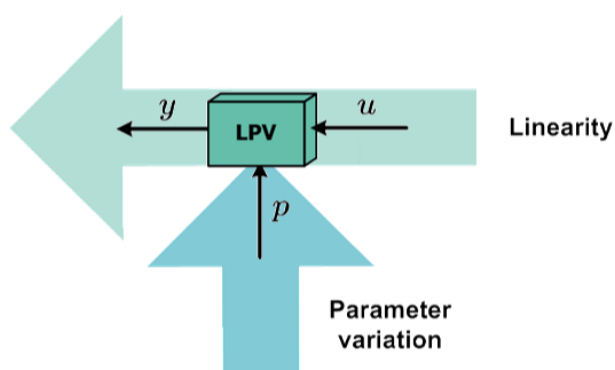
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# Bayesian Identification of LPV-BJ Models: A Multidimensional Kernel Approach

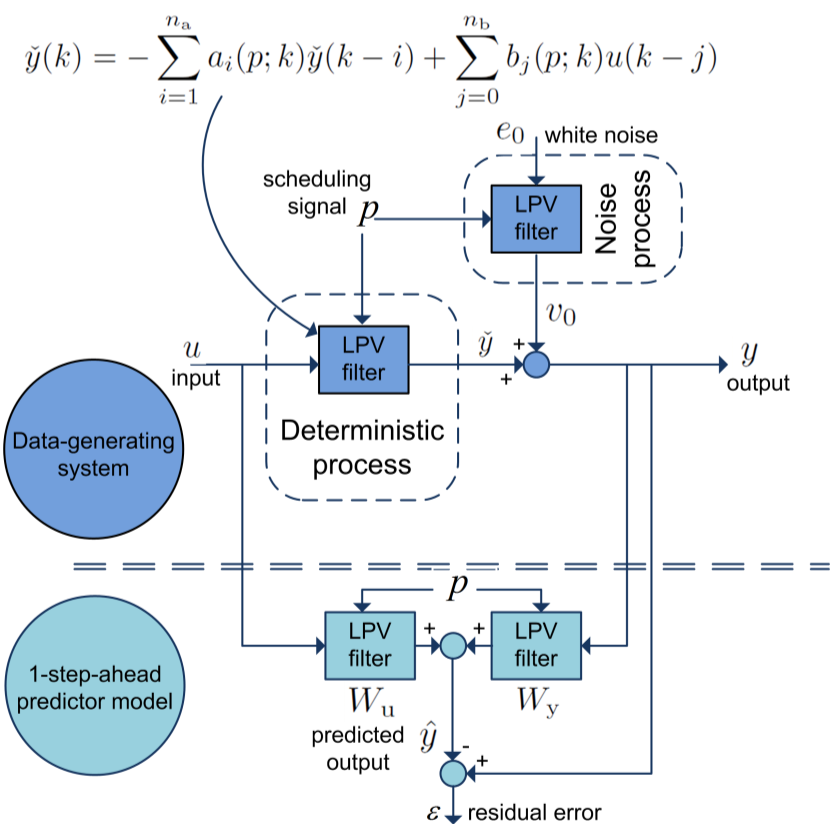
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## Motivation

Many industrial control systems exhibit nonlinear behaviour, where linear modeling is becoming insufficient to support model based control design such that the increasing performance specifications can be fulfilled. However, *Linear Parameter-Varying* (LPV) systems offer a powerful framework to deal with such situations while preserving the linear relationship between the input and the output signals.



In this work, we introduce a nonparametric approach in a Bayesian setting to efficiently estimate, both in the stochastic and computational sense, LPV Box-Jenkins (LPV-BJ) models.



## Bayesian identification for LPV-BJ systems

The one-step-ahead predictor (IIRs model form) can be written as

$$y(k) = \underbrace{\sum_{i=1}^{\infty} f_{y_i}(p, k, i) q^{-i} y(k)}_{W_y} + \underbrace{\sum_{j=0}^{\infty} f_{u_j}(p, k, j) q^{-j} u(k)}_{W_u} + e_0(k), \quad (1)$$

$f = \hat{y}(k|k-1)$

which can be considered as a standard Gaussian process regression model

$$y(k) = f(x^{(k)}) + e_0(k),$$

where  $x^{(k)} = \{u^{(k)}, p^{(k)}, y^{(k-1)}\}$  denotes the past measurements till time  $k$ , e.g.,  $u^{(k)} = \{u(\tau)\}_{\tau \leq k}$ ,  $f \sim \mathcal{GP}(0, K)$  and it can be viewed as a sum of two independent zero mean Gaussian random fields  $f^u, f^y$

$$f = \sum_{l=1}^{\infty} f_l^y + \sum_{l=1}^{\infty} f_l^u$$

The covariance of  $f$  should express

- coefficient functions dependency on  $p$ .
- Stability of the predictor.

$$K(x^{(k)}, x^{(k')}) = \sum_{l=1}^{\infty} K_l^u(x^{(k)}, x^{(k')}) + \sum_{l=1}^{\infty} K_l^y(x^{(k)}, x^{(k')}),$$

where

$$K_l^y(x^{(k)}, x^{(k')}) = \beta_l^y y(k-l) \exp\left(-\frac{\|p^{(k,l)} - p^{(k',l)}\|_2^2}{\sigma_y^2}\right) y(k'-l),$$

$$K_l^u(x^{(k)}, x^{(k')}) = \beta_l^u u(k-l) \exp\left(-\frac{\|p^{(k,l)} - p^{(k',l)}\|_2^2}{\sigma_u^2}\right) u(k'-l),$$

where  $\beta_l^u = \lambda_1 e^{-l\lambda_2}$  and  $\beta_l^y = \lambda_3 e^{-l\lambda_4}$  are the so-called decay terms. The hyperparameters are  $\sigma_u, \sigma_y, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}^+$ . These unknown hyperparameters are tuned using marginal likelihood maximization.

## Simulation example

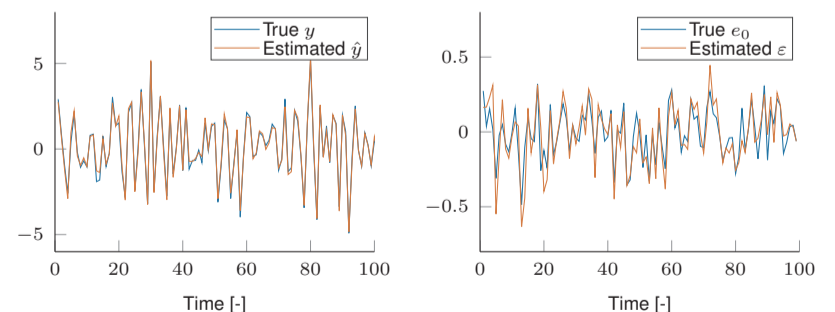
The coefficient functions of the process part of the considered example are

$$\begin{aligned} a_1(\cdot) &= 0.1p^2(k-1), & a_2(\cdot) &= \tan^{-1}(p(k-1)) \cos(p(k-2)), \\ b_0(\cdot) &= -\exp(-p(k)), & b_1(\cdot) &= 1 - 0.5p^2(k) + p(k-1), \\ b_2(\cdot) &= \tan^{-1}(p(k-2)). \end{aligned}$$

The noise process  $v(k)$  is a colored noise generated by an LPV-ARMAX filter with coefficient functions

$$\begin{aligned} c_1(\cdot) &= 0.8p^2(k-1), & c_2(\cdot) &= 0.5 \tan^{-1}(p(k-2)), \\ d_1(\cdot) &= 0.2p^3(k-1), & d_2(\cdot) &= 0.5 \sin(p(k-2)). \end{aligned}$$

The data set consists of 1000 data points with SNR = 20dB.



(a) System output.

(b) Residuals.

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