Concepts and trade-offs in supply chain finance

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Kasper van der Vliet

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To Henk and Jehan.

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Kasper van der Vliet
Eindhoven, April 2015.

Contents

Chapter 1
Introduction

This thesis concerns supply chain finance. A supply chain is a network of companies involved in producing and moving a product or service to a customer. The companies involved use several resources. One of these resources is capital, i.e., money provided by investors or lenders to buy other resources, such as equipment, materials, or knowledge. Capital is a special resource. Apart from providing a means to buy other resources, it is used as a basis for the valuation of a company. A company's worth is the amount of capital equivalent to the net value of its present-day assets and its future cash flows. A company can therefore use its future cash flows to obtain capital. It can do so by selling claims on its future cash flows to those who own capital. The latter, however, require a compensation for the risks associated with buying these claims; a premium. Besides the risk of the cash flow itself, this premium depends on the quality of information and the amount of control offered to capital owners. Generally, the less transparency and/or the less control, the higher the premium.

Thus, the premium offered to a company depends also on its supply chain structure, i.e., the arrangements, processes and technologies that allow it to offer better information and control to capital providers. Companies in a supply chain can *pro-actively* lower each other's premiums by informing or committing to capital providers. For instance, a company can inform a capital provider of its intention to make a payment to another company in the future. Subsequently, the capital provider can make a custom credit offer to this company for the corresponding cash flow. When companies facilitate financing to each other in such a way, trade-offs arise between the respective companies. How should companies reconfigure their financial arrangements? Or, should they reconfigure their operational arrangements? This dissertation aims to answer such questions;

it deals with concepts and trade-offs on alternative capital flows between companies within a supply chain.

The goal of this chapter is to provide the reader a general context for our research and offer some guidance on the remainder of this thesis. Therefore, in Section 1.1, we first position the research conducted for this thesis in a wider *theoretical* context. Subsequently, in Section 1.2, we present an outline of the remainder of this thesis.

1.1 The Capital Market, Information Asymmetry, and Reverse Factoring

Finance is inextricably linked with operations management. Indeed, any activity that is concerned with the management of resources with the goal to produce a product or deliver a service more effectively affects the cash flows of those who own or employ the respective resources. The decision when to order materials, how to prioritize jobs, or where to locate workers: they eventually affect a company's cash flows one way or the other. We generally evaluate these decisions based on the expected cost, profit, or the net present value of the cash flows. The cash required for implementing a decision eventually comes from one source: the capital market. The purpose of the capital market is to match the supply and demand for cash. It allows for trading of the timing and risk of cash flows.

The capital market is considered to be a major catalyzer of job creation, economic growth, and innovation. It allows people to save money, e.g., for their retirement, to buy a home, or even start a business. For existing businesses, it supports the funding of day-to-day operations, R&D initiatives, or even the acquisition of other businesses. The actors of the capital market can be divided into three categories: (i) users of capital, (ii) providers of capital, and (iii) intermediaries of capital, i.e., organizations that primarily mediate funds (between users and providers) and provide associated services. Users of capital generally want to raise capital at the lowest possible cost. Providers of capital, on the other hand, want the maximum return for their risk, or equivalently, the minimum risk for their return. Intermediaries of capital want to mediate between providers and users with the maximum possible return.

While the capital market has long been a topic of study in economics and finance, it is not until about half a century ago that a very influential idea was formed on how firms' financial choices are linked to the functioning of the capital market. Modigliani and Miller (1958) proved that when the capital market is perfect, the financial policy of a firm is irrelevant to its value. Thus, a firm cannot create value from decisions that concern how much money to borrow, whether to issue (more) equity, or when to pay out dividends to its shareholders[1]. A perfect capital market is a market without frictions, i.e., all of its actors are rational and have perfect information on each other, there are no transaction costs, regulations, and taxes, all assets are perfectly divisible, and there is perfect competition (i.e., no participant is large enough to set the price).

While it is generally known that real capital markets are not frictionless, the Modigliani-Miller theorem provides a useful benchmark to explore the determinants of firms' financial choices. Therefore, the theorem also acted as a 'detonator' for the theory of corporate finance (Tirole, 2006). Correspondingly, corporate finance theory is also referred as the 'study of financing frictions' (Hennessy and Whited, 2007). The theorem resulted in a commonly accepted view in research that the consideration of phenomena of the real capital market, such as a credit constraint or a credit arbitrage (i.e., the ability to benefit from a price difference), in financial decision-making should be motivated based on a relaxation of the assumptions of a perfect capital market. This view is evident from the corporate finance literature (see, e.g., Dotan and Ravid, 1985) as well as the recently emerging operations-finance interface research stream (see, e.g., Birge, 2014). The latter addresses situations where firms can create value or improve risk management by jointly considering operations management and finance decisions.

Information asymmetry, the absence of complete information on the assets and actions of capital users to capital providers, is a friction that is particulary relevant to the studies conducted for this thesis. Indeed, providers of capital generally cannot perfectly verify or monitor the state of the assets that are being financed from their capital, at least not without incurring a significant cost. As a result of information incompleteness and the cost associated with verification and monitoring, capital providers increase the cost of capital, condition or constrain the provision of capital, or they may even completely deny it (Akerlof, 1970; Stiglitz and Weiss, 1981; Myers and Majluf, 1984; Hubbard, 1997).

The significance of information asymmetry (and the inherent cost of mitigating

[1]Miller (1991) explains the intuition for this result with the following analogy: *"Think of the firm as a gigantic tub of whole milk. The farmer can sell the whole milk as it is. Or he can separate out the cream, and sell it at a considerably higher price than the whole milk would bring...The Modigliani-Miller proposition says that if there were no costs of separation, (and, of course, no government dairy support program), the cream plus the skim milk would bring the same price as the whole milk."* Thus, if a firm sells its safe cash flows (cream) to lenders, it is eventually left with more risky and thus lower valued equity (skimmed milk). Consequently, the firm's shareholders would adjust their price (i.e, expected return) of their equity such that the total value of the firm remains the same after the debt transaction.

it), however, can vary significantly across different types of firms. For instance, publicly listed firms generally have easier (cheaper) access to capital markets than privately owned (less transparent) firms. As also shown by the 2008 crisis, small- and medium-sized enterprises (SMEs) typically face the greatest challenges to obtaining (affordable) credit during periods in which credit is rationed (Berger and Udell, 2006; Kraemer-Eis et al., 2010). To the extent that some firms have easier access to capital than others, firms have an incentive to exploit their financial strength to pass on credit to other supply chain members (Schwartz, 1974). In our studies, we mostly consider a basic supply chain setting in which a supplier sells to one or more buyers that are financially stronger than him.

Recently, *supply chain finance* has emerged as a concept in which firms take an integral perspective to financing in their supply chain (Pfohl and Gomm, 2009). The transparency offered to capital providers on supply chain processes is one of the areas in which firms can add value from an integral perspective. Based on increased transparency, capital providers can reconcile their decisions concerning the price and/or availability of credit in the supply chain. A financial arrangement called 'reverse factoring' relies on this principle and provides a common basis for the studies in this thesis. In this arrangement, a creditworthy buyer confirms its future payment obligations to its suppliers to a financial institution. Based on the knowledge that the risk of offering financing against the corresponding entitlements (i.e., receivables) is low, the financial institution can facilitate early financing options cheaply to the respective suppliers. Financial institutions (e.g., Citi, HSBC, and Deutsche Bank) have shown increased interest in reverse factoring the last decade. Despite the lower interest rates collected than in conventional financing, their interest is, amongst other things, driven by the lower internal capital requirements[2] of providing financing that is backed by a creditworthy entity in the supply chain, and the potential gains in efficiency from expanding financing services through one entity instead of dealing with multiple entities in the supply chain (Hurtrez and Salvadori, 2010).

In this dissertation, we present four studies in which we develop concepts and investigate trade-offs concerning supply chain finance. In the first study, we develop a framework for supply chain finance. In the second and third study, we consider trade-offs concerning reverse factoring implementation. Specifically,

[2]Banks or other financial institutions have to hold an amount of capital that is required by its financial regulator. These requirements are put into place to ensure that they do not take excess risk on becoming insolvent. The requirements generally also account for the risk of the type of financing transactions that banks facilitate. Banks need to hold less capital for less risky financing transactions, such as reverse factoring. Note that the lower rate collected in a reverse factoring (vs. conventional) financing transaction does thus not necessarily imply that the transaction is less profitable for a bank.

we explore a supplier's trade-off concerning the extension of payment terms, and subsequently, a buyer's trade-off concerning the improvement of service levels. Our final study considers how reverse factoring imposes a friction on suppliers, and how this friction can be mitigated through horizontal collaboration. Specifically, we consider how pooling receivables between suppliers mitigates the indivisibility of financing transactions. We explore how the value of this pooling initiative interacts with pooling on the operational level.

One of the main contributions of this thesis is that it shows how operations-finance interactions can have a significant impact on the value on the arrangements made between firms. For instance, one of the trade-offs that we consider in this thesis deals with estimating the cost of extending payment terms to a customer. In the literature, this cost is conventionally assessed purely based on the expected amount of capital that is tied up in outstanding receivables. We find, however, that this approach tends to underestimate the true cost of a payment term extension as it neglects the impact of the latter on a firm's cash volatility. In addition, we show that a buyer offering cheaper finance to a supplier does not naturally encourage the supplier to give preferential treatment or better service. Thus, managers should consider contractually requiring a better service if they expect this from a supply chain finance arrangement. We also show that supply chain finance can benefit firms in horizonal collaboration settings. Specifically, we show that receivables pooling and investment pooling can interact with each other such that the combined benefit of the pooling initiatives can be greater than the sum of the benefits of the individual initiatives.

Our contribution to research on the interface of operations and finance are threefold. First, we provide a framework that allows researchers to focus on key trade-offs in supply chain finance. Secondly, we identify complex interactions between inventory, receivables, and cash management in a multi-period setting. Thirdly, we demonstrate how simulation-based optimization is a powerful research method for analyzing complex interactions. The models that we consider in thesis are difficult, perhaps even impossible to solve mathematically. However, we can still explain most our findings based on existing theory and concepts.

1.2 *Overview of the Thesis*

The core chapters of this thesis, Chapters 2-5, include one conceptual study (Chapter 2) and three modeling studies (Chapters 3-5). In Chapter 6, we summarize the main results of the preceding chapters of this thesis, present some managerial implications, and discuss future research directions. In the next paragraphs, we present the main research question, methodology, and the contribution of each of the core chapters of this thesis.

Ch. 2 A Framework To Advance Research on Supply Chain Finance

In this chapter, we present a framework that allows us to position supply chain finance practises and focus research efforts on related trade-offs. The framework is developed based on emerging trends in the area of supply chain finance and a case study of two manufacturers. Practises and concepts are ultimately typified based on two dimensions: strategic objectives and implementation tactics. We highlight the main trade-offs on these dimensions. Subsequently, we highlight our contributions by showing the positions of the three studies of the remainder of this thesis in the framework.

Chapter 3 The Price of Reverse Factoring: Financing Rates vs. Payment Delays

In this chapter, we explore a trade-off that a supplier faces in a reverse factoring arrangement. The study is inspired from cases in which investment-grade buyers use reverse factoring to induce their suppliers to grant them more lenient payment terms. Specifically, we explore the following main research question:

What extensions of payment terms allow the supplier to benefit from reverse factoring?

We develop a periodic review inventory model of a supplier which can use conventional sources of financing and/or reverse factoring. The firm's objective is the minimization of average cost per period, consisting of both inventory and financing cost. We find that an extension of payment terms induces a non-linear financing cost for the supplier, beyond the opportunity cost of carrying receivables. Furthermore, we find that the maximum size of the payment term extension that a supplier can accommodate in reverse factoring depends on demand uncertainty and the supplier's cost structure. Overall, this chapter illustrates that the financial implications of an extension of payment terms needs careful assessment in stochastic settings.

This chapter also appeared in print as van der Vliet et al. (2015).

Chapter 4 Reverse Factoring and Service Levels: Let it happen or make it work?

In this chapter, we investigate whether a buyer can expect to be served better from offering cheaper financing to its supplier by means of reverse factoring. Subsequently, we explore how much extra service the buyer can contractually agree with the supplier. We explore thus the following question:

What is the maximum service level improvement that a buyer can require, given the terms of his reverse factoring offer?

We develop a periodic review inventory model of a supplier that serves demands of two buyers with a minimum fill rate constraint. One of the buyers (A) facilitates early payment to the supplier through reverse factoring; the other (B) pays the supplier with a fixed payment delay. We introduce a rationing policy that allows the supplier to differentiate the fill rate to the two buyers. The supplier's objective is the minimization of average cost per period while satisfying the fill rate constraints of both buyers. We find that the optimal base stock decreases as a function of the reverse factoring rate, hence, a supplier does not naturally offer a better service level for reverse factoring, but rather collects the maximum financial savings. However, we find that buyer A can contractually require a significant fill rate improvement for reverse factoring. Furthermore, we find that maximum fill rate improvement depends on the relative size of the mean demand of buyer A, the demand uncertainty, and the deviation between lead time and payment term. Our work thus yields managerial insights about how much buyers can operationally benefit from reverse factoring.

Chapter 5 Pooling Receivables and its Interaction with Pooling Investment

A particular feature of reverse factoring forms a motivation of the study in this chapter: the indivisibility of an account receivable. Indeed, while reverse factoring allows a firm to obtain cheaper financing from selling its receivable, it typically must sell the *whole* receivable when it chooses to sell it. However, pooling receivables with other firms can mitigate the inherent cost of indivisibility. We consider a setting in which firms can pool both their investments and/or receivables and answer the following question:

If firms can pool receivables, pool investment, or pool both, is the benefit from pooling both super- or sub-additive?

We develop a stylized single period model of two identical firms operating in a make-to-order setting. We develop the model in such a way that we can analyze the benefit of each pooling scenario. We find that the optimal investment level of the firms can increase or decrease as a result of receivables pooling. Considering that receivables yield cheaper finance, the finding that receivables pooling can decrease optimal investment is surprising initially. It becomes intuitive, however, if we consider that the financial savings from a lower investment level may exceed the corresponding loss in profit when firms pool receivables. Furthermore, we find that the benefit from engaging in both types of pooling (as opposed to only one) can be sub- or super-additive. When the technology maturity level of a firm's assets is low (high), and the firm can thus considerably (only marginally) decrease unit production cost through investment, the benefit from pooling both is super-additive (sub-additive). Overall, our results suggest that simultaneous evaluation of the pooling concepts improves managerial decision making.

While, from this outline, it is clear that Chapters 3 and 5 are especially relevant for a supplier, and Chapter 4 for a buyer in the supply chain, the framework in Chapter 2 will allow us to see how the studies are linked together. Indeed, by positioning the models of each chapter in the framework, we highlight how we contributed to exploring different elements of supply chain finance.

Chapter 2

A Framework To Advance Research on Supply Chain Finance

2.1 Introduction

Supply Chain Finance (SCF) is the inter-company optimization of financing as well as the integration of financing processes with customers, suppliers, and service providers in order to increase the value of all participating companies (Pfohl and Gomm, 2009). Interest for SCF has grown significantly since the financial crisis of 2008 (Milne, 2009; Shang et al., 2009; Pezza, 2011; Seifert and Seifert, 2011; Wuttke et al., 2013). According to executives surveyed by the Aberdeen Group, the impact of demand volatility on available cash is a key factor behind these developments (Pezza, 2011). As increased demand volatility calls on the one hand for more investment in safety stocks but on the other hand induces a desire to hold more precautionary cash, balancing operational performance and financial resilience may become a challenge. This is especially true for SMEs as these generally have a harder time obtaining credit in crises due to intrinsic opacity on their creditworthiness (Kraemer-Eis et al., 2010). By providing tools and concepts in which firms take an integral perspective to financing in addition to operations in the supply chain, SCF may provide a remedy to such problems. Surprisingly, however, the supply chain management discipline itself is in practice often surprisingly little involved in SCF initiatives[1].

[1]We use the term "SCF arrangement" generally to denote a specific instance of SCF between firms. Nonetheless, we may refer to "SCF initiatives" when the emphasis is on firms' intentions or actions to realize an SCF arrangement. There is otherwise no substantive difference between the two referents.

The Aberdeen Group indicates that the supply chain management discipline is not involved in almost 50% of the SCF initiatives (Pezza, 2011). An executive interviewed by the Financial Times even says: "there is nothing very supply chain about finance right now" (Milne, 2009).

An SCF initiative that has become popular recently is 'reverse factoring' (Milne, 2009; Wuttke et al., 2013). Prominent manufacturing firms such as Volvo, Vodafone, Nestle and major retailers, such as Sainsbury, WalMart, and Metro, use it (Milne, 2009; Mason, 2012; O'Connel, 2009). Reverse factoring is an arrangement where a buyer facilitates early financing options for its suppliers by confirming future payment obligations to a factor, i.e., a financial intermediary (Klapper, 2006). Based on the confirmations provided by the buyer, the factor can fully reconcile the risk and pricing of any collateral transaction, and thus offer credit to the supplier at the same price it would to the buyer. Investment grade firms can thus use reverse factoring to reduce the cost of capital of their suppliers. Many firms use reverse factoring as a means to reduce their own working capital costs: by offering competitively priced early payment options, they induce suppliers to offer longer payment terms (see, e.g., Wuttke et al., 2013). Some firms, however, use reverse factoring to relax financial constraints that have a negative impact on supplier's operational performance (Aeppel, 2010; Mason, 2012; Boeing, 2012). We see thus that while some SCF initiatives are oriented towards financial benefits, others are orientated towards supply chain benefits.

Research on SCF fits with the growing body of literature at the interface of operations and finance (see, e.g., Birge, 2014; Zhao and Huchzermeier, 2015, for a review). Works in this area address situations where firms generally can create value or improve risk management by jointly considering operations management and financial management. The impact of operations-finance interactions are studied both in the context of a single the firm (see, e.g., Buzacott and Zhang, 2004) as well as in a supply chain setting (see, e.g., Lai et al., 2009; Kouvelis and Zhao, 2011). Research that explicitly considers financial arrangements in the supply chain generally focuses on trade credit, i.e., an arrangement in which firms allow a delayed payment to customers (Gupta and Wang, 2009; Yang and Birge, 2011). Some empirical works have analyzed the use of reverse factoring and propose general managerial advices to buyers concerning its implementation (see, e.g., Seifert and Seifert, 2011; Wuttke et al., 2013).

Trade-offs that result from the ability to adjust financing conditions within the supply chain remain largely unexplored in the literature. As one of the few, Pfohl and Gomm (2009) consider the trade-off between the the cost of information transfer to members within the supply chain vs. capital cost savings that result from it. In their setting, members of a supply chain are able to offer better financing conditions to each other due to the inherent information asymmetry between firms and financial intermediaries. Inspired by examples in which buyers subsidize financially distressed suppliers to mitigate supply disruptions, Babich (2010) analyzes the interactions between the decision to reserve capacity and the decision to grant financial aid to a supplier. We see thus in the literature that SCF initiatives can also be oriented towards financial benefits or supply chain benefits. Noting the diversity of approaches to SCF in the literature and in practise, we see the need for a framework to organize our future research efforts on SCF. We will do so in this chapter.

By identifying key trends from the literature and conducting a small case study on the SCF initiatives of two manufacturers, we develop a framework that positions SCF initiatives based on two dimensions. On the first dimension, we distinguish transaction-oriented from competence-oriented SCF initiatives. While the improvement of a competence, such as agility or any other operational measure, is the primary the objective in the latter, maximizing the transactional benefit that the SCF arrangement offers is the primary goal in the first. On the second dimension, we distinguish uniform from customized SCF initiatives. While in an uniform SCF initiative one type of SCF arrangement is implemented for all supply chain relationships, in a customized initiative SCF arrangements are tailored such that they fit the nature of the respective relationship. Having developed this framework, we use our framework to position the three quantitative studies that we have conducted as part of this thesis.

The remainder of this chapter is organized as follows. In Section 2.2, we offer a detailed consideration of three emerging trends in the intermediation of credit within supply chains. In Section 2.3, we report the corroborative findings from a case study on reverse factoring. In Section 2.4, we synthesize our observations and present a view of research opportunities by means of a conceptual framework. In Section 2.5, we summarize our findings and conclude by positioning the three studies of the remaining chapters of this dissertation in the framework.

2.2 *Trends changing the SCF landscape*

At the core of SCF we find the concept of 'intra-supply chain credit', i.e., credit intermediation between supply chain members, in contrast to credit granted only by specialized financial institutions. As noted earlier, intra-supply chain credit through trade credit is a well-established and well-researched phenomenon in the literature (see Seifert et al., 2013, for an extensive review). Trade credit is a form of short-term financing; depending on the industry, the repayment takes place within weeks or months (Ng et al., 1999). Some firms even offer long-term financing to customers through so-called captive financing vehicles (Brennan et al., 1988). Such practices are popular with manufacturers selling capital-intensive goods, such as producers of cars or agriculture equipment (Brennan et al., 1988; Looker, 1998). Scientific literature indicates the following four main motives for intra-supply chain credit (Mian and Smith, 1992; Petersen and Rajan, 1997):

(Motive 1) better information/control than intermediaries;

(Motive 2) price discrimination;

(Motive 3) transactional savings;

(Motive 4) quality control and assurance.

Nonetheless, recent developments in industry and findings in research on intra-supply chain credit suggest an evolution concerning both the motives and approaches on intra-supply chain credit. In the following paragraphs, we summarize three emerging trends.

Trend 1: *greater involvement of financial and/or technological intermediaries in intra-supply chain credit.* Close to 85% of global trade is facilitated by trade credit, yet the Society for Worldwide Interbank Financial Telecommunication believes that a significant share of it will migrate towards bank-intermediated services in the coming years (SWIFT, 2010). This migration suggests that the ability of firms to exploit relative advantages over financial intermediaries (motive 1) is stagnating. Indeed, banks and technology providers have significantly invested in platforms to mitigate informational asymmetries and automate the process of making funds available, based on events in the supply chain (Hurtrez and Salvadori, 2010; Casterman, 2013). Most of the current services can be classified as post-shipment solutions: financing is offered after completion of the physical transaction. Some pre-shipment services are also available, such as purchase-order financing or inventory financing, and the introduction of industry standards to better facilitate pre-shipment financing promises growth in this area (Casterman, 2013). Despite the investments, discriminatory practices (motive 2) sometimes remain because marginal costs for intermediaries are too high. For instance, smaller suppliers have been excluded from reverse factoring arrangements because their inclusion was thought not to offer significant benefits (Milne, 2009; Wuttke et al., 2013). Increasing technological maturity will yield more opportunities for value creation, i.e., beyond purely transactional savings (motive 3), and thus more involvement of intermediaries.

Trend 2: *supply chain risk management as a motive for intra-supply chain credit.* Industry practices are emerging in which risk management is the motive for offering credit in the supply chain. This development may be seen as an extension of motive 4. For instance, Caterpillar introduced reverse factoring as part of its program to 'gear' its supply base after the credit crisis of 2008 (Aeppel, 2010). Similarly, Rolls Royce introduced reverse factoring to strengthen the operational resilience of its supply chain (Gustin, 2014b). Boeing implemented reverse factoring as part of a larger campaign to help SME suppliers sustain highly skilled export-related jobs (Boeing, 2012). Levi Strauss & Co introduced reverse factoring to provide a financial incentive to suppliers in the Far East to meet environmental, labor, and safety standards (Donnan, 2014). In addition, recent research reveals that intra-supply chain credit creates value through mitigation and/or control of supply chain risk. Yang and Birge (2011) show how trade credit serves as a risk-sharing mechanism in the supply chain. Kim and Shin (2012) show how trade credit mitigates incentive problems in the supply chain by building inter-firm credit relationships. While the use of intra-supply chain credit for mitigation of supply chain risk may entail costs and/or risks for the facilitating company, these examples suggest that the net value of the engagement can be positive.

Trend 3: *multi-echelon perspective on the configuration of intra-supply chain credit.* Traditionally, intra-supply chain credit, such as trade credit, is arranged between two parties. A supply chain thus consists of multiple financial arrangements that are in principle independent. Theoretical and empirical evidence suggests that the two trends already identified - greater involvement of intermediaries and supply chain risk management motives - may be further developed by the implementation of more integrated perspectives on the financial dimensions of the supply chain. Industry practises are emerging in which these multi-echelon perspective are present. For instance, Unilever provides financial aid to suppliers in multiple tiers of the tea supply chain, in order to mitigate uncooperative working capital practices among them (Reason, 2005). Hewlett Packard makes use of a so-called 'Buy/Sell'-scheme that they also offer as a service to other firms (Hoffman et al., 2010). This scheme allows firms that have outsourced their manufacturing operations the opportunity to still retain control of their supply chain by buying items from second-tier suppliers, and subsequently selling them to first-tier suppliers. In research, multi-echelon perspectives are also appearing. For instance, some studies analyze the potential working capital savings from adjustments in payment schemes across multiple tiers of the supply chain (Hofmann and Kotzab, 2010; Randall and Farris, 2009). Song and Tong (2012) provide a new accounting framework that allows for evaluating key financial metrics under alternative payment schemes in serial supply chains. Luo and Shang (2013) illustrate the value of cash pooling in serial supply chains.

The three trends suggest that a firm initiating an SCF arrangement may face strategic considerations, i.e., those that concern the *objective(s)* of the respective arrangement, but also tactical considerations on *how* the arrangement can best contribute to achieving the desired objectives. A strategic consideration is for example: to what extent should a firm strive to mitigate operational risk rather than collect pure financial benefits in SCF arrangement? Subsequently, a corresponding tactical consideration would be: should the initiating firm involve service providers or intermediaries to make operational risk more transparent?

2.3 *SCF in practice: divergent approaches to reverse factoring*

In order to further investigate the relevance of the trends to SCF, we conducted a case study in which examined the implementation of reverse factoring at two European corporations, i.e., firm A and B. The study can be considered to be an instrumental case study Stake and Savolainen (1995). In an instrumental case study, one studies a particular case of a more general phenomenon to gain insight and/or develop theory. In our case, the phenomenon is the different approaches of firms in SCF initiatives.

Both firms are publicly listed multinational firms and operate in the technology sector. Due to inherent complexity of their end-products they rely on large, complex, global supply networks consisting of thousands of firms. At the time of our case study, firm A had operated reverse factoring arrangements for over a year, while firm B had just begun to offer them to its suppliers. We conducted semi-structured interviews with executives involved in the implementation process. We were particulary interested to place the firms' choices in light of the three trends that are discussed above. The main findings are displayed in Table 2.1. To give further context of the firms' initiatives, Figure 2.1 gives a schematic representation of a reverse factoring process. While the two firms contracted different banking partners (i.e., factors) to implement reverse factoring, the process was found to be identical across the firms.

Trend	Firm A	Firm B
1. Role of financial intermediary	Provides strategic guidance and supports implementation by providing credit assessments; suppliers segmented based on their transactional volume and capital cost.	Has a passive role in supplier offering; scheme is offered to all suppliers on the same terms.
2. Risk management	No clear focus on risk management; decrease working capital through extension of payment terms and/or reduce costs of goods sold through reduction of prices and standardization of payment terms. New terms contractually agreed in amendments.	Clear focus on risk management; increase the ability of the supply network to cope with demand volatility and support the realization of its growth ambitions concerning the customer base. No terms changed to the existing contracts.
3. Multi-level perspective	Promote reselling of working capital benefits to firms upstream in supply chain.	Enable financial flexibility for firms upstream in the supply chain.

Table 2.1 *Two firms with different approaches towards reverse factoring*

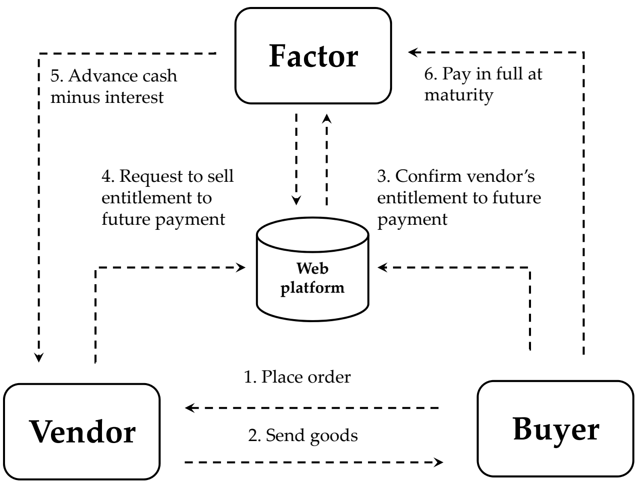


Figure 2.1 *Successive actions in reverse factoring*

The perspective of risk management (trend 2) provides the main contrast between each firm's approach to reverse factoring. Firm A is not concerned with risk management in this context, and sees reverse factoring primarily as a means to improve its working capital position and to lower its cost of procurement. In contrast, firm B has an explicit risk-related goal: improve suppliers' ability to respond to demand variability. More specifically, firm A uses reverse factoring to make standardized extensions to its payment terms, while still allowing suppliers to realize a reduction in their financing cost (with respect to the situation before the use of reverse factoring). Longer payment terms entail that firm A realizes a reduction in working capital, and standardization of payment terms reduces transaction costs. Firm B is not motivated by potential savings in working capital or savings in transaction costs from standardization of payment terms. The responsiveness of its suppliers - of which many are critical - is a significant factor for revenues, market share, and profit. Reverse factoring improves suppliers' liquidity and lowers their short-term financing costs, at least partially removing financial obstacles to operational agility. For firm B, the value of improved supply chain performance - better matching of supply with demand - appeared unquestionably greater than the foregone savings from a decrease of working capital.

The different objectives of the two firms are further reflected in the role of the financial intermediary and the tactics used in the reverse factoring implementation (trend 1). At both firms the intermediary provides the technology and funding and explains the scheme to suppliers, but firm A allows the intermediary a deeper role in determining how financing is offered. The intermediary here is a tactical partner in the supply chain, providing estimations of each supplier's cost of capital and working capital position. Firm A consequently sets a threshold level on the annual value of transactions, in order to determine which suppliers are eligible for implementation of reverse factoring. The return from implementations at a lower volume is thought not to justify the direct costs. Firm A thus first targets suppliers with the highest transactional volume and/or cost of capital, as these provide the greatest potential return in reduction of working capital and other costs. The terms offered to each supplier are tailored to the size of the potential benefits, and all agreements are ultimately contractual. Firm B employs no such criteria and makes the scheme available to all suppliers. Small but critical suppliers can have big impact on the ability of firm B to meet final market demand, so there is no reason to discriminate based on size. In contrast to firm A, firm B does not amend the terms (e.g., with respect to logistics performance) of its contracts with suppliers, which suggests that firm B sees interests to be naturally aligned in its supply chain.

Concerning the relevance of reverse factoring for their extended supply chain (trend 3), the firms again provide different but cogent views. Firm A notes that some suppliers - in particular, larger firms already have relatively good access to capital - initially saw little benefit from participating in the reverse factoring program. The possibility that such suppliers could in turn rearrange payment terms with their suppliers nevertheless became a decisive argument for participation. In contrast, firm B promotes propagation of cheap liquidity upstream in its supply chain, in order to stimulate further investment in supplier resilience.

2.4 A typology for future research on SCF

While the types of credit arrangements available to members in the supply chain are likely to become more diverse with time, the trends and the case study allows us to confirm basic dimensions as the basis for a typology: strategic objectives and implementation tactics. The dimensions are considered strategic and tactical because they concern decisions on the *aims* of SCF and on the *methods* used to reach those aims respectively. The dimensions are visualized in Figure 2.2. The figure contains other information than the dimensions, which will be explained later in this section. Next, we will further discuss the two dimensions and accompanying tradeoffs.

Strategic Objectives: Transaction-oriented vs. Competence-oriented SCF

Along the strategic dimension we distinguish transaction-oriented SCF at one extreme from competence-oriented SCF at the other. The contrast is validated by our case study: an SCF arrangement may offer transactional benefits through working capital benefits or savings in capital cost (see firm A), while it can also be employed to enhance a supply chain competence, such as supply base agility (see firm B). We consider the two orientations to be each other's opposites as realizing supply chain competencies generally requires firms to invest cash in resources. Collecting the transactional benefits from a SCF arrangement, by e.g. increasing payment terms or reducing prices of suppliers, eventually diminishes the initiating firm's ability to employ SCF as an enabler for investment. Indeed, if in reverse factoring suppliers' payment terms would be extended (or prices would be reduced) such that the resulting costs marginally equal the capital cost savings, investment is unlikely to result from the implementation.

The range of supply chain competencies that might be realized by SCF is as varied as the investment opportunities present in the supply chain. Resources

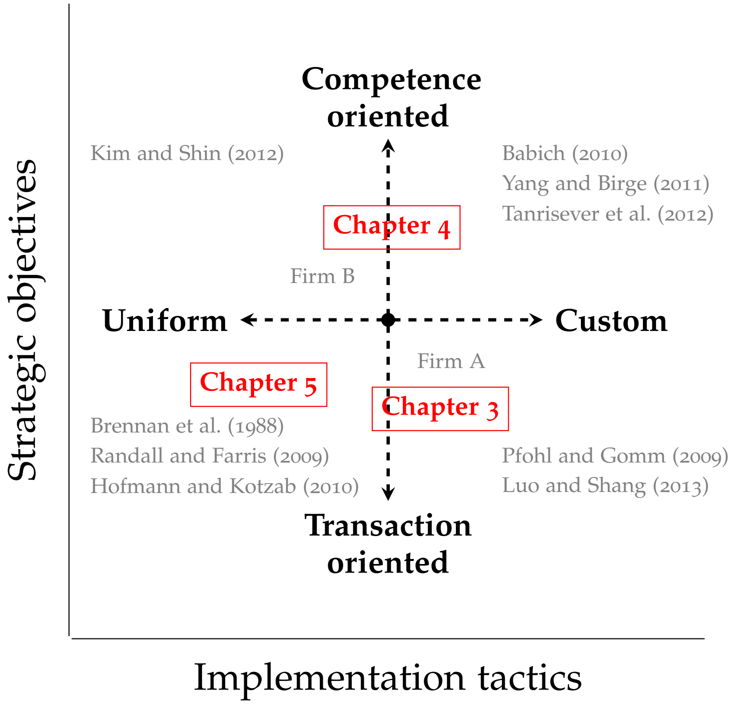


Figure 2.2 *A typology for SCF practices and SCF research efforts*

that potentially qualify as investment opportunities are inventory, production capacity, technology, and human resources. The investments made in these resources are greatly influenced by the cost of financing at the respective tier of the chain. As these investment decisions frequently impact the rest of the chain, potential adjustments through SCF can be of great value. As often appears in standard investment theory, however, an opportunity with high expected value may entail significant risk. For instance, a pre-shipment financing arrangement that helps vendors with raw materials purchases may expose the supply chain to higher obsolescence risks, which can lead to a great reduction in the risk-adjusted value. While the current literature on supply chain coordination of incentives through contracting is well developed (see, e.g., Cachon, 2003), the potential of SCF as a rectifying mechanism for investment problems in supply chains remains largely unexplored. Since the potential benefit and risk of a particular type SCF engagement is likely to change under different conditions, appropriate consideration should be given to trade-offs. For instance, Tanrisever et al. (2012) show that reverse factoring may promote operational improvement in the supply chain, but this improvement is highly sensitive to the terms proposed by the buyer.

Implementation Tactics: Uniform vs. Customized Implementations

Along the tactical dimension we distinguish uniform implementations from customized implementations. In a uniform implementation, the initiating firm follows more or less a single specification for SCF arrangements, while in a customized implementation the firms adjusts the type and terms the agreement to fit the nature their supply relationship(s). The contrast is again validated by our case study. Firm A employs a higher level of customization than firm B, since the terms offered to each supplier are tailored to the size of the transactional benefits that SCF yields for each supplier.

Customization measures may increase the total return of an SCF arrangement, but they will usually require greater investment in technology and/or administration. The marginal cost of customization thus has to be balanced with its marginal reward, generalizing the informational transfer setting of Pfohl and Gomm (2009). For instance, obtaining transparency on firm's financial or operational status is not always trivial, especially for SMEs. As shown by the case of firm A, customization may also entail a more prominent role for the relevant financial intermediary due to its specialization in financial analysis. For a competence-oriented SCF arrangement, a customization tactic may even bring about more complexities. Besides operational and financial risks, firms may also be exposed to agency or moral hazard problems. Funding from SCF may be diverted to personal interests or used to serve other investment than what has been agreed between the parties involved. Problems of moral hazard, e.g., between shareholders and lenders, or between managers and shareholders, are recognized in the literature of corporate finance and (contractual) remedies or incentive mechanisms are generally prescribed (Tirole, 2006). Literature on customization measures for ensuring effective supply chain investment from SCF arrangements is yet limited. In order to provide adequate answers, with appropriate consideration for the trade-offs and risks, multi-disciplinary and multi-method approaches may need to be applied or even new approaches developed (Sanders and Wagner, 2011).

The framework provides us a means for typifying SCF practises in industry (see approximate placements of the two firms in Figure 2.2), and formulate questions concerning trade-offs that are practically relevant. Are the choices of firm A and firm B (i.e., customized transaction-oriented and uniform competence-oriented SCF respectively) indeed optimal for them? Are there conditions under which they should reconsider their choices? Or, under what circumstances would uniform, transaction-oriented SCF or customized, competence-oriented SCF be advisable?

The resulting typology allows us also to position SCF research efforts, to have constructive dialogue on meaningful research directions, and formulate questions. To illustrate this, we have positioned some of the theoretical studies on intra-supply chain credit mentioned earlier in the typology. As financial intermediaries further proliferate their SCF services, and pre-shipment or multi-level financing arrangements gain more momentum, theoretical studies are needed to determine what competencies can be potentially enhanced by SCF and what type and degree of customization is advisable in each case.

2.5 *Conclusions and positioning of the studies in the remaining chapters of this thesis*

We develop a typology to position industry practises and theoretical studies on SCF. In this typology, we highlight that in SCF initiatives firms can reap direct benefits from the transaction, but they can also indirectly enhance the supply chain by improving its competence to mitigate supply disruptions or achieve operational efficiency. While there is empirical evidence of both approaches, conceptual and practical guidance on the conduct of competence-oriented SCF is scant, let alone how to customize competence-oriented SCF. Studies that consider the dynamics between physical and financial flows on the single firm-level show that measures to improve financial performance may eventually degrade operational performance (Gupta and Wang, 2009; Protopappa-Sieke and Seifert, 2010). The same principle seems to hold for inter-firm financing concepts (Tanrisever et al., 2012). Further study is needed. Explanatory research can help to identify the antecedents for a particular type of SCF arrangement. Normative research can help to identify the SCF configurations that do meet multi-objective criteria, i.e., financial efficiency and operational performance. Such research can entail significant opportunities for industrial users, financial intermediaries and providers to develop better SCF technologies.

In the remainder of this thesis, we present three modeling studies that consider issues in each of the four quadrants. See Figure 2.2 for the placements of Chapters 3, 4 and Chapter 5.

In Chapter 3, we explore explore the supplier's trade-off in a reverse factoring arrangement in which it is required to extend payment terms. We find that the trade-off depends on the demand uncertainty and on financial aspects that are generally not accounted for in practise when valuing the cost of a payment term extension, such as the firm's profit margin and its cost structure. The study illustrates that it makes sense to customize transaction-oriented SCF arrangements based on both operational and financial aspects of the firms involved, rather than only the financial aspects.

The study in Chapter 4 is inspired from our case study findings. As noted earlier, firm B initiates a competence-oriented SCF without contractually agreeing any terms on operational performance with its supplier. We explore whether firms, such as firm B, can expect to be served better than the other customers as a result of facilitating reverse factoring. We find that this not the case. We find, however, that such firms may in some case contractually agree significant service level improvements with their supplier, if they want to. Like in Chapter 3, we investigate how the size of the service level improvement depends on both operational as well financial aspects of the firms, and thus whether customization makes sense.

In Chapter 5, we explore a concept in which suppliers pool receivables to reduce a cost that is induced by a friction in reverse factoring. Specifically, the indivisibility of a receivable may give rise to still use a costlier, but flexible financing alternative. We consider how pooling receivables mitigates the adverse impact of this indivisibility and explore how the size of of the pooling benefit interacts with pooling on the operational level. Unlike the other studies, we consider do not consider how the financial or operational nature of the firms involved impact trade-offs. The firms are are identical, thus the arrangement can be considered to be uniform.

Chapter 3
The Price of Reverse Factoring: Financing Rates vs. Payment Delays

3.1 Introduction

Trade credit is a short-term loan between firms that is linked both in terms of timing and value to the exchange of goods between them (Ferris, 1981). Recent estimates suggest that around 80-90% of the world trade is facilitated by trade credit (Williams, 2008). In the manufacturing sector, accounts receivable make up 20-25% of the total assets of firms (Mian and Smith, 1992; Fewings, 1992). The role of trade credit in our economy is extensive, and it has consequently been the topic of investigation of many studies (Seifert et al., 2013). These studies link the provision of trade credit to information asymmetry, transaction costs, hedging, moral hazard, quality assurance, and many other motives and market phenomena.

Regardless of the motive, the provision of trade credit is considered to be an investment on the microeconomic level. Its terms should therefore account for the opportunity cost of tying up capital in an asset, i.e., the receivable. Financial management practices conventionally consider the provider's cost of capital as the basis for the opportunity cost (see, e.g., Brealey et al., 2011), but recent developments challenge this perspective. Indeed, revelation of information about the risk of a specific asset can be the basis for improved financing terms, which reflect the risk of the asset concerned as opposed to that of the firm in general. Pfohl and Gomm (2009) show how this results from an inter-company approach to financing, which they call 'Supply Chain Finance' (SCF). The term SCF is also increasingly used by financial institutions to denote payables financing or early payment services (Casterman, 2013). Among these, 'reverse factoring' is a prime example and has received considerable recent interest from the business and research community (Tanrisever et al., 2012; Wuttke et al., 2013). It is essentially a development of conventional factoring arrangements. In the latter, a firm independently sells one or more of its receivables to a financier - the factor - against a premium (Soufani, 2002a). In reverse factoring, the firm's client is also involved: the client makes an explicit guarantee to the factor that the payment obligation will be met (Klapper, 2006). This guarantee entails that the factor can offer financing at a rate as low as when the client itself would apply for funds. Investment grade firms can therefore use reverse factoring to realise a significant reduction in cost of credit for their suppliers.

According to Hurtrez and Salvadori (2010), recent technological advances allow reverse factoring to be offered efficiently, and challenging economic conditions have accelerated adoption. Specifically, the credit crisis increased the spread of short-term capital costs between large corporations and their SME suppliers; in some cases, the latter even saw their access to short-term capital cut. A study initiated by the Bank of England concludes that reverse factoring offers significant opportunities to rejuvenate lending to SME firms (Association of Corporate Treasurers, 2010). Nonetheless, many buyers also see reverse factoring as a means to reduce their own working capital costs: by offering competitively priced early payment options, they induce their suppliers to offer longer payment terms. From a survey among executives, Seifert and Seifert (2011) find that buyers managed to reduce net working capital by 13% on average through reverse factoring. While literature suggests that payment terms can be reconfigured in a collaborative spirit, the approach of some buyers appears to neglect this perspective. Some buyers indeed exploit their bargaining position to impose the payment term they want. For instance, Milne (2009) reports that a large corporation introduced reverse factoring as a 'sweetener' to an unpopular decision to move its payment terms to suppliers from 45 to 90 days. Wuttke et al. (2013) cite an executive of a major chemical firm: "We would say to our supplier, we will extend payment terms anyway. It is up to you to take our SCF[1] offer or leave it." In a survey among professionals, Aberdeen finds that 17% of the respondents experienced 'pressure' from trading partners to adopt supply chain finance (Pezza, 2011). These findings suggest that inter-firm power

[1] In trade publications, the general term Supply Chain Finance (SCF) is frequently used to refer to reverse factoring in particular.

can significantly impact the context of a reverse factoring implementation, and thus the benefit of the arrangement for suppliers may in some cases be open to question.

Even if there is a mutually agreed extension in *contractual* payment terms, the supplier can use reverse factoring to obtain early payment cheaply. Consequently, a trade-off between 'longer' and 'cheaper' arises. This provides the central motivation for our study. Adjusting payment terms on the basis of an adjustment in financing rates assumes that the net effect of these changes on the financing costs of a firm can be assessed. The assessment is often made by considering the cost of capital in conjunction with the average value of outstanding receivables and/or payables (Randall and Farris, 2009; Hofmann and Kotzab, 2010). This approach presumes that the configuration of trade credit can be made independently from operations; but some studies show that lot-size or inventory decisions can interact with the receipt and/or provision of trade credit (Gupta and Wang, 2009; Protopappa-Sieke and Seifert, 2010; Song and Tong, 2012). On account of the possibility for interaction between financial and operational terms, we explore the costs and benefits of payment term extension and reverse factoring respectively. While the potential impact of neglecting this interaction is especially a concern for the supplier, it may also have practical relevance to the buyer and even the factor. Indeed, if the supplier eventually does not collect the anticipated benefit of reverse factoring, he may eventually want to renegotiate with the buyer, not make use of the arrangement, or, exit it.

To explore the aforementioned trade-off, we take a discrete-time, infinite-horizon, base-stock inventory model of a supplier firm and incorporate financial dimensions in the state description. Initially, we assume that the supplier has only access to conventional short-term financing sources; we then extend this with the option to sell receivables through reverse factoring. Further, since a firm's opportunity cost rate for carrying receivables may influence its use of reverse factoring, we consider relevant alternative implementations for the latter: manual or auto discounting[2]. As the names suggest, with manual discounting the firm chooses when to discount its receivables, while with auto discounting the firm discounts receivables as soon as the factor receives the buyer's payment guarantee. The choice for manual discounting indicates that the firm rather holds (vs. discounts) its receivable until it needs cash. In all cases, the firm's objective is the minimisation of expected cost per period, defined as the sum of inventory and financing costs.

[2] Practitioners use the term "discounting" to describe the sale of a receivable by supplier in a reverse factoring system, presumably because the cash that the supplier receives is a only a fraction of the face value of the receivable.

Within the operations management literature, our work contributes first of all to the relatively young research line in the area of supply chain finance (Pfohl and Gomm, 2009; Randall and Farris, 2009; Wuttke et al., 2013). In particular, our work complements that of Tanrisever et al. (2012), who obtain analytical insights from a single period model of reverse factoring: we examine the conditions under which reverse factoring is economically viable in a multi-period setting. We find that manual and auto discounting are to be treated as different types of systems with different accompanying trade-offs. While auto discounting allows for making a trade-off independent of inventory operations, manual discounting involves a more complex trade-off which is conditioned on demand uncertainty and the supplier firm's cost structure. These parameters affect the expected volume of receivables being discounted, and thus the discounting cost, but also impact the expected volume of receivables and the associated opportunity cost. The overall impact of reverse factoring on the payment term decision may consequently be difficult to predict. Furthermore, we show that the ability to extend payment terms in an economically justified fashion with manual discounting may be restricted to settings where the opportunity cost rate for holding receivables is very low. In an extensive numerical study, we find a maximum opportunity cost rate of 0.5% per year for most of our settings. This rate corresponds to the rate of return of low-risk short-term investments, such as depositing money in a savings account or investment-grade corporate bond, which are unlikely to be the next best investment opportunity of SME's.

We contribute also to an emerging research area that considers interactions between inventory and financing in a multi-period stochastic setting (Maddah et al., 2004; Hu and Sobel, 2007; Gupta and Wang, 2009; Babich, 2010; Protopappa-Sieke and Seifert, 2010; Song and Tong, 2012; Luo and Shang, 2013). Our experiments suggest that while the cash retention level required to finance a base stock operation increases in the payment term, it approaches an asymptotic limit. The value of retained cash is therefore decreasing in the payment term. Viewing a payment term as lead time, this finding conforms with an intuition of Goldberg et al. (2012): when lead time is very long, the system is subject to so much randomness between an event and its consequence that 'being smarter' provides almost no benefit.

The remainder of this Chapter is structured as follows. In section 3.2 we discuss our research questions and literature relevant to our problem. In section 3.3 we describe the models we implement in our simulations. In section 3.4 we discuss the design of our experiments. In section 3.5 we present the results from the experiments. In section 3.6 we summarise our findings and draw final conclusions.

3.2 Research Questions and Literature

In this section, we motivate our research questions and discuss the literature relevant to our problem.

When a buyer induces its suppliers to grant a payment term extension in return for cheaper finance, the parties should know the cost of the extension. In financial management, the cost of granting an extension of payment terms to a customer is generally assessed based on the average value of outstanding receivables (Brealey et al., 2011). The average value of outstanding receivables is in turn the product of average daily volume of credit sales and average number of days until payment. The cost is then calculated by multiplying the average receivables value with the firm's required rate of return for the time period that the cash flows are delayed. The firm's weighted average cost of capital (WACC) is often used as a benchmark. (see, e.g., Randall and Farris, 2009). With this approach, the cost of trade credit is a linear function of payment terms and thus independent of variability in demand. We hypothesize that variability in demand will influence the amount of financing necessitated by an extension of payment terms. It is well known from stochastic inventory theory that longer replenishment lead times require higher levels of safety stock, in order to hedge against intervening demand uncertainty (Zipkin, 2000). Viewing a payment term as lead time, we expect that a firm's financial position is exposed to more variability when extending payment terms. Additional delay in payment entails the possibility of incurring more cash outlays and receipts between the moment of selling goods and collecting payment. As cash flow uncertainty is associated with the need to borrow money and/or hold more cash (Opler et al., 1999), financing cost may be a non-linear function of the payment term, regardless of the opportunity cost rate used for receivables. We thus formulate a first research question:

Research question 3.1. *What impact does extending payment terms have on the cost of managing a stochastic inventory operation?*

The presumption that the financing needed to support physical flows is a linear function of payment terms also suggests that differences in the cost of short-term credit can be exploited in a straightforward fashion. Firms would benefit as long as the multiplier of the initial payment term is no greater than the inverse of the multiplier of the initial financing rate. If the initial financing rate were halved, for instance, the initial payment term could be doubled. Several studies analyse the potential savings from adjusting payment terms this way (Randall and Farris, 2009; Hofmann and Kotzab, 2010; Wuttke et al., 2013). In contrast, by considering explicitly the effect that demand uncertainty has on financial flows in a single period model, Tanrisever et al. (2012) find that this inverse-proportional relationship will generally underestimate the cost of an extension of payment terms. We explore this finding in a multi-period inventory setting, where firms conduct transactions in an ongoing manner.

A further qualification of the benefits of reverse factoring results from a firm's opportunity cost rate for holding receivables. The pertinence of this consideration is evident when we consider that there are in practice two distinct ways in which firms use reverse factoring. Many choose an auto discounting policy, but some choose for manual discounting[3]. That both approaches exist and are applied is evident from trade literature (cf. Dunn, 2011; Gustin, 2014a), and both are included in a US Patent application for reverse factoring (Kramer, 2009). With auto discounting the firm discounts each of his receivables at the earliest possible moment, i.e., as soon as the factor offers the transaction. With manual discounting the firm manually chooses when to discount his receivables. Thus, the firm leaves receivables to be paid at the maturity date, unless it encounters a situation in which it would rather use the capital tied up in its receivable before this date. A bank overdraft embodies the same principle: it allows the firm to borrow money only when it needs it and pay interest only on the amount it borrows.

The choice for auto or manual discounting can also be explained based on how the respective firm assesses the magnitude of the opportunity cost rate for holding receivables relative to the discount rate of reverse factoring: lower, equal or higher. When the firm's opportunity cost rate is equal to the discount rate, the firm should be indifferent between the two options. Indeed, the expected gain from investing the cash that is otherwise tied up in receivables is then equal to the cost of discounting. With manual discounting the firm assesses the opportunity cost rate lower than the discount rate, while with auto discounting the firm assesses the opportunity cost rate higher than the discount rate. As the firm holds no receivables with auto discounting and thus incurs no opportunity cost anymore, the discount rate is the maximum opportunity cost rate with reverse factoring[4]. We thus formulate the second research question:

[3]To our knowledge there is no published study on the relative prevalence of each type of discounting. From conversations with firm managers and trade finance professionals we have the impression that auto discounting is more widely applied.

[4]While, before reverse factoring, the opportunity cost rate can indeed be greater than the discount rate, they are equal, however, as soon as reverse factoring is offered. It does not make sense for the firm to grant a payment term extension based on the return from investing the cash that otherwise is tied up in receivables. Indeed, this return would already have been made by the firm when the payment term is zero.

Research question 3.2. *What payment term extension would allow a supplier to benefit from reverse factoring? Specifically, what is the maximum payment term extension when the receivables holding cost is:*

(a) zero;
(b) equal to the cost of factoring;
(c) positive but lower than the cost of factoring?

Looking further to relevant literature, we note first the relation of our study to the growing body of research on the interface of operations and finance. Work in this area generally aims to identify conditions under which a tighter integration of the two disciplines creates value or allows improved risk management (Birge et al., 2007). Imperfections in capital markets are often assumed, since an interaction between investment and financing decisions is only then possible (Modigliani and Miller, 1958). In our case, reverse factoring mitigates information asymmetry between financial intermediaries and firms, yielding an option to exploit cheaper credit. Specifically, the payment guarantee from buyer to factor entails that the supplier can discount receivables at a cheaper rate than would otherwise be possible.

Three other research topics are particularly relevant to our study: inventory incorporating payment schemes, trade credit policy, and cash management. For inventory theorists, even in deterministic settings, payment schemes undermine the conventional assumption that capital needs are related to average inventory levels. Beranek (1967) was among the first to study the implications of alternative payment practices on the economic lot size decision. Haley and Higgins (1973); Goyal (1985) and Rachamadugu (1989) further enrich this stream. Kim and Chung (1990) propose a model to combine the lot-size decision and the discount offered to customers for early payment. Schiff and Lieber (1974) use control theory to study the relationship between inventory and accounts receivable policy. More recently, scholars have explored the significance of payment schemes by means of stochastic inventory models. For instance, Maddah et al. (2004) investigate the effect of receiving trade credit in a periodic review (s, S) inventory model. Gupta and Wang (2009) show that a base stock inventory policy continues to be optimal when a supplier gives trade credit, but requires adaptation of the base stock parameter.

Most research on trade credit itself is to be found in the economics literature. Given the existence of financial intermediaries, scholars have been interested to explain the role of trade credit (see Seifert et al., 2013, for an extensive review). In addition to this economic perspective, there is normative literature that explores the optimal credit policy to customers. Most of these studies consider the trade-off between lost sales when the policy is too tight and credit losses when policy is too easy. Davis (1966) is among the first to analyse trade credit in terms of marginal revenue and cost. Bierman and Hausman (1970) and Mehta (1970) formulate the credit decision respectively in a finite and infinite horizon framework. Fewings (1992) obtains closed-form solution for the value of granting credit and an upper bound on the acceptable default risk. Another series illustrates the nuances of correctly evaluating a credit policy (Oh, 1976; Atkins and Kim, 1977; Dyl, 1977; Walia, 1977; Kim and Atkins, 1978; Weston and Tuan, 1980). Nonetheless, this literature invariably assumes that inventory and/or procurement polices do not affect the trade credit decision.

Turning to the question of cash management, it is again the economics literature that describes the basic motives for holding cash. Principally, these are: (1) transaction costs, (2) precaution to adverse shocks and/or costly access to capital markets, (3) taxes, and (4) agency conflicts, such as the problem that entrenched managers would rather retain cash than increase payouts to shareholders (Bates et al., 2009). Many models for optimal cash management exist, of which the ones by Baumol (1952) and Miller and Orr (1966) are seminal. Both models propose cash control policies to balance liquidity with the opportunity cost of holding cash. Recently, scholars have explored the significance of linking liquidity and/or cash management with inventory theory. For instance, Hu and Sobel (2007) show that an echelon base stock policy is sub-optimal in a serial supply chain with liquidity constraints. Luo and Shang (2013) show the value of centralized cash retention in a two-stage supply chain.

Few studies explicitly incorporate all aspects mentioned above: inventory control, trade credit, and cash management. Exceptions include Protopappa-Sieke and Seifert (2010), who optimize order quantities for a finite horizon model that includes working capital restrictions and payment delays. Song and Tong (2012) propose new accounting metrics that allow correction of classical inventory prescriptions for the influence of payment schemes and possible borrowing to cover cash shortages. Luo and Shang (2014) consider a model in which a firm can both receive and grant payment delays; they show that a working capital dependent base stock policy is optimal. All these studies generally confirm that payment delays can have a significant impact on the cash dynamics of an inventory system, and consequently also on its funding requirements. Hence, accounting for the financing cost of only holding inventory when analyzing an inventory system, as is conventional in inventory theory, may imply a misjudgement of the true cost of the ordering policy proposed to control it. These aforementioned studies, therefore, include the financing requirements that result from mismatches between the incoming and outgoing cash flows of an inventory system. Our work complements the aforementioned studies by adding a further element to a firm's decision problem: the choice between conventional sources of capital and reverse factoring, given that the latter changes financing rates as well as payment delays.

3.3 Models

We develop a periodic review inventory model of a supplier that sells to a creditworthy buyer who offers reverse factoring. Periods are indexed by the variable t and one period represents $l \in \mathbb{R}^+$ years. At the end of each period, the supplier firm will receive a stochastic demand $D_t \geq 0$ from the buyer. In order to have its products ready before demand is revealed, the supplier orders stock at the start of each period. We assume that inventory is controlled by a base stock policy with base stock level I units. The supplier pays price c per unit for stock and sells at price $p > c$ per unit to the buyer. The ordered items are delivered immediately prior to the end of the period and the supplier pays for them upon delivery. Once demand is revealed, if it cannot be fully met from inventory, the unmet portion is back-ordered until the next period. For each backlogged unit, the buyer charges the supplier a penalty cost b. For each unsold unit, the supplier incurs a storage cost $h < b$. The supplier grants the buyer a payment term of $k \in \mathbb{N}^+$ periods. The payment term starts to count from the moment that a demanded unit is met from inventory. Once revenue from prior demands are collected and costs are paid, the supplier may at the end of each period release cash to shareholders.

In the initial version of our model, the supplier meets periodic expenses with cash retained from previous periods or by borrowing from a bank. Borrowing only occurs to the extent that retained cash is insufficient. The annualized interest charge for borrowing is β per monetary unit. Cash retention is governed by a constant threshold policy: cash is released whenever it exceeds a threshold level $T \geq 0$, but only to the extent that the cash level is returned to T. As shareholders could have invested retained cash elsewhere, an annualized opportunity cost rate of α is assessed on each monetary unit retained. In a perfect capital market we should expect $\alpha = \beta$, but we assume that capital market frictions may entail $\alpha < \beta$ or $\alpha > \beta$ (Myers and Majluf, 1984)

While we do not theoretically demonstrate a constant threshold policy to be the optimal form of cash management for our model, analogy to a base stock policy appears warranted by the opposition between α and β, which constitute respectively holding and shortage costs for cash. (Cf. discussion of Song and Tong, 2012, ,where a "base cash" policy is also used.) Cash management policies used in practice may be more more complex - e.g., the dual threshold model of Stone (1972) - but contextual factors that motivate these policies, such as net balance requirements of banking agreements, are absent from our model. We assume unlimited borrowing capacity. The model can impose a credit limit, but this forces much of our focus to lay on default events instead of purely on the change in financing needs that results from a payment period extension.

In every period the supplier in our model receives the money from the sales realized k periods ago. The supplier's total periodic payment includes a fixed cost f, variable expenses for the replenishment of its stock, inventory (holding and shortage) costs, and interest for debt outstanding during the period t. Furthermore, analogous to the opportunity cost rate for holding cash, an opportunity cost rate of η per year is assessed on each monetary unit of accounts receivable that result from the payment term. We assume $\eta < \alpha$ as the risk of investing in an account receivable is lower than the risk of investing in the firm itself. Indeed, while settlement of the account receivable is due after a known delay, the timing of cash dividends from the firm depends on demand and realized profits, and is consequently uncertain.

The state of the supplier at the start of period t is $S_t = (x_t, m_t, \boldsymbol{r}_t)$. The scalar x_t represents the inventory position and the scalar m_t represents the cash position. A tangible cash balance or a bank overdraft is represented by $m_t > 0$ or $m_t < 0$ respectively. The k-dimensional vector $\boldsymbol{r}_t$ with components $r_{t,i}$ for $i = 1, 2, \ldots, k$ represents the outstanding accounts receivable, i.e., the payments to be collected at the end of periods $t, t+1, \ldots, t+k-1$. The vector S_t conveys all information needed to implement the ordering and cash retention policies in period t: how many units needed to reach base stock, the associated cash payment and the amount of cash that will be received.

Figure 3.1 summarises the sequence of events in a period. At the start of period, the supplier observes its state S_t (1) and places an order at its raw materials supplier (2). At the end of the period, the supplier collects the cash from its oldest accounts receivable and the position of the other receivables is decremented (3). If needed, the supplier borrows money (4). Subsequently, the supplier collects the units that it has ordered (5) and makes a cash payment that includes its variable and fixed costs (6). If its policy allows, the supplier releases cash to its shareholders (7). Finally, the suppliers receives the demand from its buyer and meets it to the extent that its inventory allows. This creates a new account receivable (9).

Since the initial version of our model includes only a conventional source of short-term financing, bank borrowing, we henceforth refer to this as the

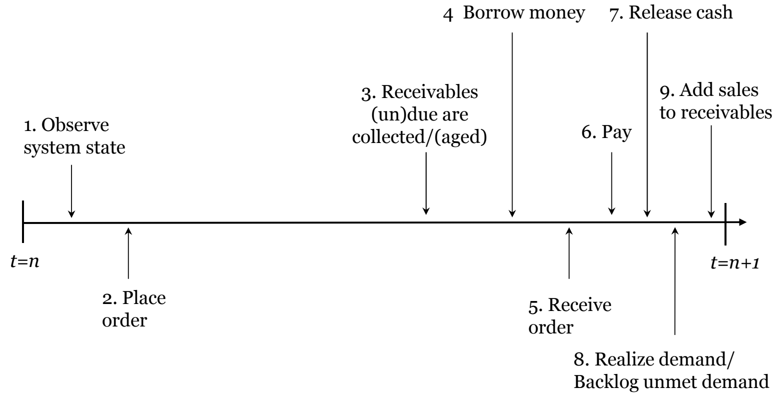


Figure 3.1 *The sequence of events within a single period.*

'conventional financing' model (CF). The mathematical formulation of this model is given in Section 3.3.1 below. In Section 3.3.2 we describe extensions to CF that model the application of reverse factoring. These extensions represent, respectively, the 'manual discounting' model (MD), where discounting only occurs when cash deficits arise (Section 3.3.2), and the 'auto discounting' model (AD), where discounting is always applied as soon as possible (Section 3.3.2).

3.3.1 *Conventional financing model (CF)*

When the firm finances its operations solely from internal cash and conventionally borrowing, the transition equations for inventory, cash, and receivables are as follows:

$$x_{t+1} = I - D_t \tag{3.1}$$

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + \min\{I, D_t\})p & i = k \\ r_{t,i+1} & i = 1, \dots, k-1 \end{cases} \tag{3.2}$$

$$m_{t+1} = \min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\} \tag{3.3}$$

where

$$P_t(I, x_t, m_t) = f + (I - x_t)c + h(x_t)^+ + b(-x_t)^+ + \beta(-m_t)^+,$$

and $(a)^+ = \max\{0, a\}$. Equation (3.1) specifies the inventory position at the start of period $t+1$ to be the base stock level minus the demand from period t. Equation (3.2) describes the payments to be collected in the next k periods. For

$i = k$ payment consists of revenue from demand that was in backlog at the start of period t, plus revenue from demand that occurs and is satisfied in period t. For $i \leq k-1$, record-keeping for payments due from demands prior to period t is updated. Equation (3.3) specifies the cash position at the start of period $t+1$ to be the cash position at the start of period t, plus the payment collected in period t, minus the periodical expenses $P_t(I, x_t, m_t)$. The period expenses $P_t(I, x_t, m_t)$ includes a fixed cost f, variable replenishment cost $(I - x_t)c$, inventory cost and incurred interest. If the cash position at the end of period t exceeds T, the firm releases exactly the amount of cash needed to bring the firm's cash position down to T.

For a specific joint base stock and cash management policy $Z = (I, T)$, we define $G_{CF}(Z)$ to be the long-run average cost per period.

$$\mathrm{G}_{CF}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} [h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - r_{t,1})^+ + \alpha(\min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\})^+ + \eta \sum_{i=1}^{i=k} r_{t,i}]. \tag{3.4}$$

The definition includes direct costs for inventory and borrowing and opportunity costs assessed on cash management and receivables. We wish to find the policy $Z^* = (I^*, T^*)$ that minimizes $G_{CF}(Z)$.

Note that the cash management cost is linked to uncertainty in the match between incoming and outgoing cash flows. If demand were constant, the firm would always be able to match these flows and would not need to borrow money and/or retain cash. Furthermore, there is an interaction between the base stock level I and the cash retention level T. The replenishment cost in period t depends on I and D_{t-1}, the demand of the preceding period. The cash available to meet the replenishment cost depends on T and the size of the demand met in period $t-k$. Even when the payment term is only one period a deficit can arise, since backlogged demand is included in the immediate replenishment cost but revenue is delayed.

3.3.2 *Reverse factoring model extensions*

Like factoring, reverse factoring allows a firm to discount a receivable, i.e., receive cash now instead of waiting until the agreed payment delay has elapsed. We define the scalar $\gamma \in (0, 1)$ to be the annualized fraction of the receivable's face value that the firm must pay to discount a receivable. For example, if $\gamma = 4\%$, then, with linear interest, reverse factoring gives the firm the opportunity to receive immediately 99% of the face value of a receivable that would otherwise result in cash payment in three months ($100 - 4 \times 3/12 = 99$). The remaining 1% constitutes the financial cost of the transaction (and revenue for the factor).

For the reverse factoring model we set $\gamma < \beta$, so cash from discounting receivables is preferred over cash from borrowing. This lower financing rate is a key characteristic of reverse factoring. Credit risk in the transaction is low, since there is an explicit guarantee from the firm's customer to the factor that payment will be made on the account receivable, and the customer is typically a large, creditworthy corporation. In conventional factoring, the customer is not necessarily credit-worthy and gives no such payment guarantee. So the transaction is risker for the factor and γ is typically higher than the cost of a bank loan for a firm (Soufani, 2002b). A further key characteristic of reverse factoring is the possible extension of the agreed payment delay. Even though the supplier can discount receivables at a lower rate, the cost of a discount transaction is increasing in the agreed payment delay. The trade-off between discount rate and agreed payment delay, as identified in our second research question, is consequently critical[5].

The static k-dimensional vector $\boldsymbol{\gamma}$ with components γ_j represents the rates applicable for discounting receivables that are otherwise due j periods from the beginning of the current period . We set $\gamma_1 = 0$ since receivable $r_{t,1}$ is due anyway at the end of period t. For $j > 1$ we set $\gamma_j = \gamma_{j-1} + l\gamma$ (recall l is the length of one period in years). The discount γ_j is applied at the moment the receivable is discounted. The discount increases in the due date of the receivable, so the firm discounts receivables in order of decreasing age, i.e., first the ones are due soonest. If, after discounting all of its receivables, the firm still needs more cash, it borrows money. The sequence of events with reverse factoring is the same as in Figure 3.1, except that at (4) the firm discounts receivables as needed and available, before resorting to borrowing.

Next we discuss further model extensions that accommodate the two ways of applying reverse factoring: manual discounting or auto discounting.

Manual discounting (MD)

In this case the supplier prefers to discount receivables rather than borrow to cover a cash deficit, but does not discount receivables if enjoying a cash surplus.

[5]Our model could just as well represent a conventional factoring transaction, but as typically $\gamma \geq \beta$ in this case, borrowing would clearly always be preferable. The existence of the market for factoring services in practice (, despite $\gamma \geq \beta$,) is partly due that our baseline assumption of unlimited access to loans does not apply.

This entails changes to the transition equations for receivables (3.2) and cash (3.3). For receivables we have

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + \min\{I, D_t\})p & i = k \\ (1 - \varphi_{t,i}(r_{t,1},..,r_{t,i+1}))r_{t,i+1} & i = 1,\ldots,k-1 \end{cases} \tag{3.5}$$

where

$$\varphi_{t,i}(r_{t,1},..,r_{t,i+1}) = \frac{\min\{(1-\gamma_n)r_{t,i+1},(P_t(I,x_t,m_t) - m_t - \sum_{n=1}^{n=i}(1-\gamma_n)r_{t,n})^+\}}{(1-\gamma_n)r_{t,i+1}}.$$

Here, $\varphi_{t,i}(r_{t,1},..,r_{t,i+1})$ is the fraction of $r_{t,i+1}$ that needs to be discounted in period t to exactly meet the firm's cash need in that respective period. This fraction is dependent on the cash from the receivable that matures in period t and the cash from potentially discounting all the receivables up to $r_{t,i+1}$, i.e., $r_{t,1},..,r_{t,i+1}$. There are three possible outcomes concerning $\varphi_{t,i}(r_{t,1},..,r_{t,i+1})$:

(a) $\varphi_{t,i}(r_{t,1},..,r_{t,i+1}) = 0$: The firm's cash need can be met from the receivable that matures in period t and the cash from potentially discounting all the receivables older than $r_{t,i+1}$. No fraction of $r_{t,i+1}$ is discounted.
(b) $0 < \varphi_{t,i}(r_{t,1},..,r_{t,i+1}) \leq 1$: The firm's cash need can not be met from the receivable that matures in period t and the cash from discounting all the receivables older than the receivable $r_{t,i+1}$, but it can be met after partially discounting $r_{t,i+1}$. A fraction of $r_{t,i+1}$ is discounted.
(c) $\varphi_{t,i}(r_{t,1},..,r_{t,i+1}) = 1$: The firm's cash need can not be met from the receivable that matures in period t and the cash from discounting all the receivables older than the receivable $r_{t,i+1}$ and $r_{t,i+1}$ itself. The receivable $r_{t,i+1}$ is discounted completely.

For cash we have the following transition equation:

$$m_{t+1} = \begin{cases} \min\{m_t + r_{t,1} - P_t(I,x_t,m_t), T\} & m_t + r_{t,1} - P_t(I,x_t,m_t) \geq 0 \\ 0 & m_t + r_{t,1} - P_t(I,x_t,m_t) < 0 \ \wedge \\ & m_t + \sum_{n=1}^{n=k}(1-\gamma_n)r_{t,n} - \\ & P_t(I,x_t,m_t) \geq 0 \\ m_t + \sum_{n=1}^{n=k}(1-\gamma_n)r_{t,n} - P_t(I,x_t,m_t) & \text{otherwise.} \end{cases} \tag{3.6}$$

There are three possible outcomes for the cash position that are captured by the respective cases in (3.6):

(a) After paying P_t and without discounting any receivables, the firm's cash position is non-negative. If the firm's cash position less than or equal to T, no cash is released to shareholders; If it exceeds T, excess cash is released to shareholders and the cash position returns to T.
(b) The firm must discount some receivables to meet P_t. Receivables are discounted so that the cash position equals zero. No cash is released to shareholders.
(c) Even after discounting all receivables, the firm has insufficient cash to meet P_t. Borrowing occurs, so the cash position is negative. No cash is released to shareholders.

Again we wish to find the policy $Z^* = (I^*, T^*)$ that minimises the long run average cost per period, but the new objective function $G_{MD}(Z)$ includes factoring as well as conventional borrowing:

$$G_{MD}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} [h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - r_{t,1})^+$$
$$+ \alpha(\min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\})^+ + \sum_{i=1}^{k} (\gamma_i \varphi_{t,i}(.) + \eta(1 - \varphi_{t,i}))r_{t,i}]. \quad (3.7)$$

Auto discounting (AD)

In this case the supplier discounts the full value of any receivable as soon as it is possible to do so. Due to sequence of events (Figure 3.1), holding costs for receivables are still incurred for one period. With auto discounting, the transition equations for receivables(3.2) and cash (3.3) are changed as follows.

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + \min\{I, D_t\})p & i = k \\ 0 & i = i, .., k-1 \end{cases} \quad (3.8)$$

$$m_{t+1} = \min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\} \quad (3.9)$$

We aim to find the policy $Z^* = (I^*, T^*)$ that minimises the long run average cost per period, as defined by the objective function $G_{AD}(Z)$:

$$G_{AD}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} [h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - \gamma_k r_{t,k})^+ + \alpha(\min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\})^+ + (\eta + \gamma_k) r_{t,k}]. \quad (3.10)$$

Note that the transition equations and the objective function of the auto discounting model are relatively simpler than those of the manual discounting model. Indeed, regardless of its cash need in each period, the supplier discounts all of its receivables at the earliest possible moment.

3.4 *Algorithm and Experimental Design*

The objective function of a base stock policy in a pure inventory setting is convex (Porteus, 2002), but the inclusion of the cash position and the outstanding receivables in the state description of our model precludes a comparable analytic insight. Besides the increased size of the state space, the interaction between the base stock parameter and cash retention parameter complicates analysis. In an initial exploration of the solution space by means of simulation, we find the objective function to exhibit convexity in both decision variables for all system configurations (CF, MD and AD) and a range of parameter values. Specifically, in all cases we find a unique policy Z^* that yields globally minimal average cost, and no policy that yields a local extreme point. Based on this insight, we utilise a 3-stage algorithm in our subsequent simulation experiments, in order to find the globally optimal policy efficiently. In this section we describe first this algorithm. Afterward we describe in detail the design of our experiments. The results of the experiments are given in Section 3.5.

3.4.1 *Solution Algorithm*

Algorithm 1 describes the 3-stage algorithm that we use in our simulation experiments to find the globally optimal policy. In its first two stages, the algorithm determines a truncated search interval, $[I_l, I_u] \times [T_l, T_u]$, through iterative gradient estimations in each policy dimension. In the third stage, stochastic approximation is used to find the optimal policy. Stochastic approximation is an iterative scheme that attempts to find a zero of the gradient of the objective function. It has been widely studied since the pioneering works of Robbins and Monro (1951) and Kiefer and Wolfowitz (1952) (Fu, 2006; Broadie et al., 2009). We use a multidimensional version of the Kiefer-Wolfowitz algorithm, which was first introduced by Blum (1954). As algorithms of this type are prone to poor finite-time performance, we make two improvements to reach faster convergence, following the proposals of Broadie et al. (2009). First, we use different tuning sequences in each dimension, in order to adapt better to the different convexity characteristics of each. Second, to avoid long oscillatory periods, we check in each iteration whether the next policy would be located within the truncated search interval; if it goes outside, the tuning sequence is amended to ensure that the next policy policy lies again within the search interval.

3.4.2 *Experimental design and parameter settings*

We answer our research questions by means of four sets of simulation experiments: Experiment 1 and Experiment 2(a) - Experiment 2(c), corresponding to the numbering of the research questions in Section 3.2.

As we are mainly interested in the behavior of the firm's expected cost as a function of the model parameters rather than the value itself, we experimented with two, three, where necessary, multiple different settings for each parameter. We chose our values such that they closely represent the market conditions in which manufacturing companies from developed countries operate. For instance, payment terms are generally not longer than a couple of weeks or months in the manufacturing sector (see e.g. Klapper et al., 2012). The annual interest rates for (unsecured) revolving credit lines can go up to $10-15\%$. A net profit margin between $10-30\%$ and an operating leverage ratio[6] between $0-0.6$ is common in the manufacturing sector (see, e.g., Brealey et al., 2011).

In each experiment, we explore 3×3 basic settings: all combinations of three possible levels for the expected net profit margin, $\omega = (\mu_D(p-c) - f)/\mu_D p$, and three possible levels for operating leverage, $\psi = f/(\mu_D c + f)$. The specific values of p, c and f that underlie the nine basic settings are shown in Table 3.1.

We define k_0 to be the initial payment term of the supplier to the creditworthy buyer, i.e., the payment term before reverse factoring. Furthermore, we define $k_e \equiv \max k \text{ s.t. } G_{MD}(k) - G_{CF}(k_0) \leq 0$, the maximum payment term extension with manual discounting, to be the longest payment term such that the firm has no greater financing cost with manual discounting than it did with conventional financing and initial payment term k_0. The definition of the maximum payment term extension k_e with *auto discounting* is analogous to that of manual discounting, but with $G_{AD}(.)$ in place of $G_{MD}(.)$.

In all experiments we take demand to be log-normally distributed with mean

[6]Operating leverage is a measure of the relationship between fixed cost and total cost for a firm.

Algorithm 1: Stochastic approximation algorithm for determination of Z^*

Step 0: Choose algorithm parameters

- initial step sizes a_0^k for $k = 1, 2$; default values $a_0^1 = 1, a_0^2 = 2$;
- initial policy $Z_0 = (I_0, T_0)$; default value $Z_0 = (\mu_D, 0)$;
- stopping condition v; default value $v = 1 \times 10^{-6}$;

Step 1: Localise $[I_l, I_u]$, the search interval for the base stock parameter
Set $Z_n = (\mu_D + na_0^1, 0)$ for $n \in \mathbb{N}^+$. Evaluate iteratively the gradient estimation $\tilde{G}'(n) = (\tilde{G}(Z_{n+1}) - \tilde{G}(Z_n))/a_0^1$. In each iteration, increase the number of replications dynamically until the confidence interval of the estimation, $[\tilde{G}_{LB}, \tilde{G}_{UB}]$, indicates a statistically significant direction, i.e., $\tilde{G}'_{LB}(n) > 0$ or $\tilde{G}'_{UB}(n) < 0$. If $\tilde{G}'_{UB}(n) < 0$, then $\{n\} \leftarrow \{n+1\}$; if $\tilde{G}'_{LB}(n) > 0$, store values as indicated below and move to step 2.

- initial policy for step 2: $\{Z_0\} \leftarrow \{Z_n\}$;
- base stock search interval for step 3: $[I_l, I_u] = [I_n - a_0^1, I_n + a_0^1]$.

Step 2: Localise $[T_l, T_u]$, the search interval for the cash retention parameter
Starting at policy Z_0, evaluate iteratively the gradient $\tilde{G}'(n)$ with $Z_n = (I_0, 0 + na_0^2)$ until $\tilde{G}'_{LB}(n) > 0$ or $\tilde{G}'_{UB}(n) < 0$. If $\tilde{G}'_{UB}(n) < 0$, then $\{n\} \leftarrow \{n+1\}$; if $\tilde{G}'_{LB}(n) > 0$ store values as indicated below and move to step 3.

- the initial policy for step 3: $\{Z_0\} \leftarrow \{Z_n\}$;
- the cash retention search interval $[T_l, T_u] = [T_n - a_0^2, T_n + a_0^2]$.

Step 3: Determine the joint optimal policy: Z^*

- Set $\{a_0^k\} \leftarrow \{0.1a_0^k\}$, $\{\tau^k\} \leftarrow \{0.1a_0^k\}$, and $\{\lambda^k\} \leftarrow \{0\}$ for $k = 1, 2$.
- Evaluate $\tilde{G}' = (\tilde{G}(I_0 + a_0^1, T_0) - \tilde{G}(Z_0))/a_0^1$ and set $\{\theta^1\} \leftarrow \{1/|\tilde{G}'_1|\}$.
- Evaluate $\tilde{G}' = (\tilde{G}(I_0, T_0 + a_0^2) - \tilde{G}(Z_0))/a_0^2$ and set $\{\theta^2\} \leftarrow \{1/|\tilde{G}'_2|\}$.
  Use the following recursion to calculate Z_{n+1}:

$$Z_{n+1} = Z_n - \left(a_n^1 \frac{\tilde{G}(Z_n + c_n^1) - \tilde{G}(Z_n)}{c_n^1}, a_n^2 \frac{\tilde{G}(Z_n + c_n^2) - \tilde{G}(Z_n)}{c_n^2} \right),$$

where

- $\boldsymbol{c}_n$ with $c_n^k = \tau^k / n^{\frac{1}{4}}$ for $k = 1, 2$ is the sequence of finite difference widths,
- $\boldsymbol{a}_n$ with $a_n^k = \theta^k / (n + \lambda^k)$ for $k = 1, 2$ is the sequence of step sizes.

  In each iteration, check:

- if $|\tilde{G}(Z_{n+1}) - \tilde{G}(Z_n)| < v$, return Z_{n+1} and terminate search;
- if $I_{n+1} < I_l$ or $I_{n+1} > I_u$, adapt λ^1 such that $I_l \leq I_{n+1} \leq I_u$;
- if $T_{n+1} < T_l$ or $T_{n+1} > T_u$, adapt λ^2 such that $T_l \leq T_{n+1} \leq T_u$.

$\mu_D = 10$ and coefficient of variation $c.v. = \mu_D/\sigma_D$ equal to either 0.25 or 0.50. Full detail of demand and cost parameters for each experiment appears in Table 3.2. In all experiments, one period corresponds to one week. Table 3.2 shows annual percentage rates (APR), which are converted to weekly rates in the experiments.

$\psi \backslash \omega$	0.1	0.2	0.3
0	10, 9, 0	10, 8, 0	10, 7, 0
0.3	10, 6.3, 27	10, 5.6, 24	10, 4.9, 21
0.6	10, 3.6, 54	10, 3.2, 48	10, 2.8, 42

Table 3.1 *Values of unit selling price p, unit cost c, and total fixed cost f underlying the experimental settings of net profit margin ω and operating leverage ψ.*

Experiment	1	2(a)	2(b)	2(c)
Scenario	$CF_{\eta=0\%}$	$CF_{\eta=0\%}$ vs. $MD_{\eta=0\%}$	$CF_{\eta=8\%}$ vs. $AD_{\eta=\gamma\%}$	$CF_{\eta>0\%}$ vs. $MD_{\eta>0\%}$
μ_D	10	10	10	10
$c.v.$	0.25, 0.5	0.25, 0.5	0.25, 0.5	0.25, 0.5
h	0.02	0.02	0.02	0.02
b	(0.1) 0.2 (0.4)	0.2	0.2	0.2
α	(4%) 8% (12%)	8%	8%	8%
β	8%	8%	8%	8%
η	0%	0%	8%	0.5%,1%,2%,4%
k	1-10	n.a.	n.a.	n.a.
k_0	n.a.	2,4	2,4	2,4
k_e	n.a.	2-12,4-14	2-12,4-14	2-12,4-14

Table 3.2 *Demand and cost parameter settings.*

The next paragraphs describe explicitly our four experiments. Although we determine the optimal policy Z^* for every experimental instance, we are generally most interested to compare policies or the performance of the system across different payment terms. Consequently, in order to facilitate the presentation, we use $Z^*(k) = (I^*(k), T^*(k))$ to denote the optimal policy for payment term k, and we rewrite the objective functions as $G_{(.)}(k)$, suppressing the immediate dependence on Z^*.

Experiment 1: *The impact of payment terms with conventional financing and no opportunity cost rate for holding receivables.*

Here we explore how payment terms impact total financing cost for the supplier firm when the opportunity cost rate for receivables is neglected, i.e., $\eta = 0\%$. In addition to the experimental settings shown in Table 3.1, we test the sensitivity of our findings to changes in: (i) the relative magnitude of inventory holding cost h, inventory to backlog cost b and (ii) the relative magnitude of cash opportunity cost rate α to the borrowing cost rate β. Specifically, we change b (from 0.2) to 0.1 and 0.4 such that h/b becomes $1/5$ and $1/20$ (instead of $1/10$)respectively. Furthermore, we change α (from 4%) to 8% and 12% such that α/β becomes $1/2$ and $3/2$ (instead of $1/1$) respectively.

Experiment 2(a): *Maximum payment term extension with no opportunity cost rate for holding receivables.*

We explore the trade-off between cheaper credit and extended payment terms in reverse factoring when there is no opportunity cost rate for holding a receivable, i.e., $\eta = 0\%$. For each initial payment term k_0, we determine $Z^*(k_0)$ when only conventional financing at rate β is used. Then, with reverse factoring at rate $\gamma \leq \beta$ also available, we determine the maximum extended payment term k_e.

Experiment 2(b): *Maximum payment term extension with greatest opportunity cost rate for holding receivables.*

We set the opportunity cost rate for holding receivables equal to the cost rate for factoring, $\eta = \gamma$, but otherwise explore the same trade-off as in Experiment 2(a). Accordingly, we seek the maximum extended payment term k_e.

Experiment 2(c): *Maximum payment term extension with intermediate opportunity cost rate for holding receivables.*

Again we explore the same basic trade-off as in Experiment 2(a), but now the opportunity cost rate for holding receivables is less than reverse factoring rate, $0 < \eta < \gamma$. We determine the maximum extended payment term k_e.

In all experiments we let the system start with zero cash, zero inventory, and zero receivables. In our algorithm, we begin to gather the data for evaluating a policy after a warm-up of 500 periods, which is determined based on Welch's procedure (Welch, 1983; Law and Kelton, 2000). We calculate 95% confidence intervals from 30 independent replications, each with total run-length of 20,000 periods (including warmup). Relative error is approximately 0.5% (Law and Kelton, 2000). After some initial calibration, we were able to locate the optimal policy within three minutes on a personal computer (with 2.53Ghz CPU, 4GB RAM).

3.5 Numerical results

Here we present and discuss the results from each experiment. Section 3.5.1 covers Experiment 1, the impact of a payment term extension on the firm's financing cost, and the accompanying sensitivity analysis for changes in inventory and cash management cost parameters. Sections 3.5.2 - 3.5.4 cover respectively Experiments 2(a) - 2(c), the trade-off between payment term extension and reverse factoring for the three different scenarios of opportunity cost rate for holding receivables.

3.5.1 The impact of payment terms with conventional financing and no opportunity cost rate for holding receivables.

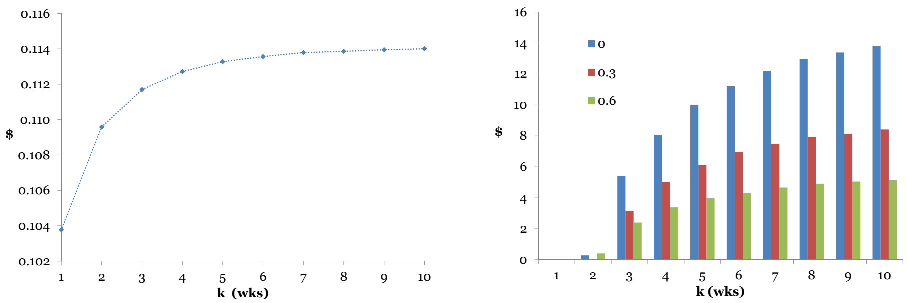
In all configurations of this experiment, we observe the following general relationship between financing cost and payment term:

The optimal cost $G_{CF}(k)$ increases asymptotically in the payment term k.

Figure 3.2a illustrates this finding. While the optimal cash retention level $T^*(k)$ increases asymptotically, the optimal base stock level $I^*(k)$ decreases slightly or remains constant. Changes in the base stock level occur because a backlog event delays the receipt of cash, which may entail a financing need. The relative impact of this is greater when the payment term is short, as the base stock then tends to be higher. Despite changes in the base stock level, changes in inventory cost appear statistically insignificant across the different payment term settings. The increase in cost from a payment term extension can thus be entirely attributed to greater variability in cash flow. As there is no opportunity cost rate for holding receivables in this experiment ($\eta = 0$), we conclude that a payment term extension entails greater financial costs than such opportunity costs alone.

The apparent concavity of the objective function implies that the relative cost of extending payment terms decreases with the pre-existing payment term. This makes intuitive sense. In a system with arbitrarily long payment terms, the incoming and outgoing cash flows become essentially independent. Additional delay in cash receipts resulting from a payment term extension should have negligible impact. The optimal cash retention level thus increases asymptotically. Being 'smarter' with cash provides little benefit when payment terms are very long. This same argument has already been used to explain the asymptotic optimality of constant-order policies for lost sales inventory models with large lead times (Goldberg et al., 2012).

Since we set $\alpha = \beta$ in this experiment, the cash retention level is the result of a trade-off that minimises the amount of capital needed for running the base stock operation. According to conventional finance literature, capital market frictions form the main motivation to retain and/or optimise cash (Myers and Majluf, 1984; Bates et al., 2009). Frictions can cause $\alpha \neq \beta$. In the last part of this section we therefore present a sensitivity study on the impact of these frictions.



(a) *Optimal expected total cost $G_{CF}(k)$ as function of payment term k. Fixed parameters: c.v. = 0.25, ω = 0.2 and $\psi = 0.3$.*

(b) *Optimal cash target $T^*(k)$ as function of payment term k for different levels of operating leverage ψ. Fixed parameters: c.v. = 0.25 and $\omega = 0.2$.*

Figure 3.2 *The impact of trade credit on system cost and cash retention*

Turning to the basic parameters that define our experimental scenarios, we examine their effect on the relative cost of a payment term extension. Specifically, if $G_{CF}(k)$ is the cost of an initial payment term k, then $\Delta G(k) \equiv (G_{CF}(k+1) - G_{CF}(k))/G_{CF}(k)$ is the relative cost of extending the payment term by one week. The results of Experiment 1 then support the following assertion:

The firm's relative cost of a payment term extension $\Delta G(k)$ is increasing in the coefficient of variation for demand, but decreasing in the initial payment term k, the net profit margin ω, and the operating leverage ψ.

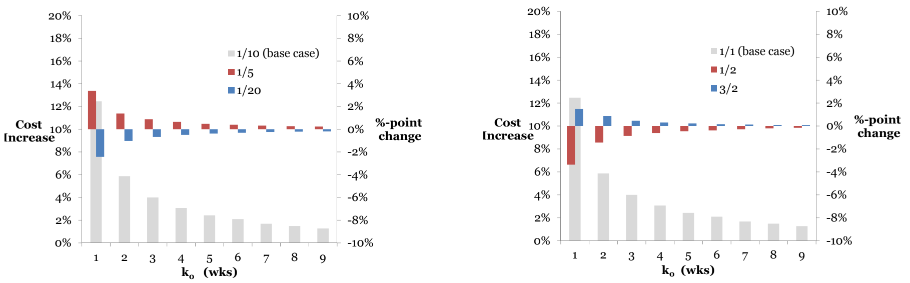
Sample paths show that a higher demand uncertainty causes higher uncertainty in the incoming and outgoing cash flows, exacerbating the impact of the payment term extension. The cash deficits or excesses accumulated will each tend to be greater in magnitude. A lower net profit margin or a lower operating leverage also increases the firm's sensitivity to an extension of payment terms. The effect of a higher net profit margin is intuitively reasonable, since it provides a greater buffer against the potential deficits that arise from a mismatch between incoming and outgoing cash flows. The effect of operating leverage is less obvious, since fixed cost are often considered to be burdensome. Firms with high operating leverage are even considered to more risky by investors (Brealey et al., 2011). While we indeed find that a higher operating leverage may imply a higher absolute cost, higher operating leverage makes the firm less sensitive to payment term extension. This appears to result from the relative stability of the outgoing cash flows for a firm with higher operating leverage. Moreover, the optimal cash retention level decreases with the operating leverage. Figure 3.2b illustrates this. Firms that rely more heavily on external purchases need to keep more cash to competitively sustain their payment terms to customers than firms that rely more on internal production with fixed costs. While a variable cost structure may be an attractive way to handle lower demand realisations in a market with non-stationary demand, it has negative implications for the ability to match incoming and outgoing cash flows when demand is stationary.

Sensitivity Study for Other Parameters

As the cost of a payment term extension is the basis of exploration in our subsequent experiments, we explore the sensitivity of our results to changes in the relative magnitude of inventory costs, h and b, and the relative magnitude of financing costs, α and β. These studies support the following assertion:

The relative cost of a payment term extension $\Delta G(k)$ is increasing in the ratio h/b and in the ratio α/β.

Changes in the inventory or financing cost ratios have a significant effect on the optimal base stock and cash retention level, but a minor effect on the the increase in financing cost that results from a payment term extension. Figure 3.3 provides illustration. Figure 3.3a shows the impact of varying the inventory cost ratio. As the value h/b increases, the sensitivity of costs to an increase in payment terms also increases. An increase in h/b entails a decrease in the optimal base stock level, which increases the probability of backlog. As explained earlier, backlog increases the probability of incurring a cash deficit, as it simultaneously delays cash receipts while additional cash is needed for stock replenishment. Figure 3.3b shows the impact of varying the financing cost ratio. As the value α/β increases, the relative cost of a payment term extension also increases. An increase in α/β means that the cost of holding cash becomes relatively expensive in comparison to borrowing, which limits the ability to use retained cash as a protection against cash flow uncertainty. If α/β is large, the firm may completely stop holding cash, i.e., $T^* = 0$ becomes the optimal cash retention threshold.



(a) *Percentage cost increase from a payment term extension of 1 week for base case $h/b = 1/10$ (left axis) and incremental change for $h/b = 1/5$ and $h/b = 1/20$ (right axis)*

(b) *Percentage cost increase from a payment term extension of 1 week for base case $\alpha/\beta = 1/1$ (left axis) and incremental change for $\alpha/\beta = 1/2$ and $\alpha/\beta = 3/2$ (right axis)*

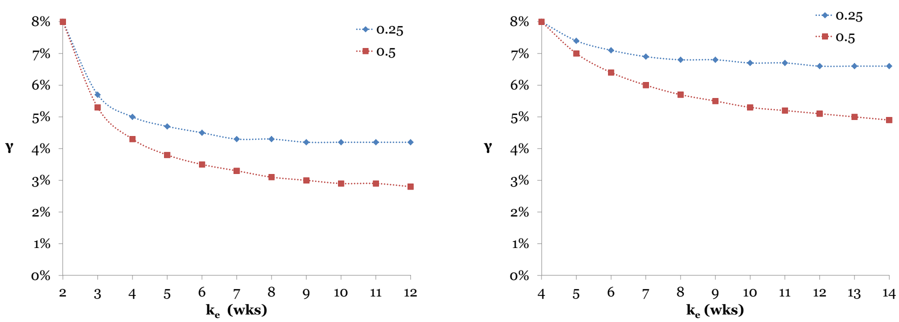
Figure 3.3 *Cost sensitivity to a payment term extension of 1 week for varying levels of inventory cost ratio h/b and financing cost ratio α/β. Fixed parameters c.v. $= 0.5, \omega = 0.1$, and $\psi = 0.5$.*

3.5.2 *Maximum payment term extension with no opportunity cost rate for holding receivables.*

Building on the insights provided by Experiment 1, we explore the maximum payment term extension that allows a firm to benefit from reverse factoring when the opportunity cost rate for holding receivables is negligible. Although the payment term extension increases the total value of outstanding receivables, the firm only considers the direct cost of financing its inventory operation. This experiment supports the following assertion:

When the opportunity cost rate for holding receivables is negligible and the firm uses manual discounting, the maximum payment term extension k_e for a given reverse factoring rate γ is decreasing in the coefficient of variation for demand, but increasing in the initial payment term k_0, the net profit margin ω, and the operating leverage ψ.

The significance of our main experimental parameters for the maximum payment term extension with reverse factoring appears consistent with their significance for the relative cost of a payment term extension in the conventional financing setting. Where previously we saw greater relative costs for an extension, here we see a smaller maximum possible extension. Figure 3.4 shows, for different values of initial payment term and demand uncertainty, the set of (γ, k_e) values for which the financing cost with reverse factoring and manual discounting case is equivalent to the initial cost with conventional financing. With reverse factoring and manual discounting, borrowing activity is generally reduced to negligible levels: factoring substitutes for borrowing. In some cases, most particularly when the experimental setting allows only a minimal payment term extension, borrowing may still occur. The optimal cash retention level may be positive in manual discounting ($T^* > 0$), but in contrast to the case with conventional financing, it decreases with the extended payment term. As payment terms get longer the firm acquires enough receivables to finance operations without borrowing or retaining cash.



(a) $k_0 = 2$ **(b)** $k_0 = 4$

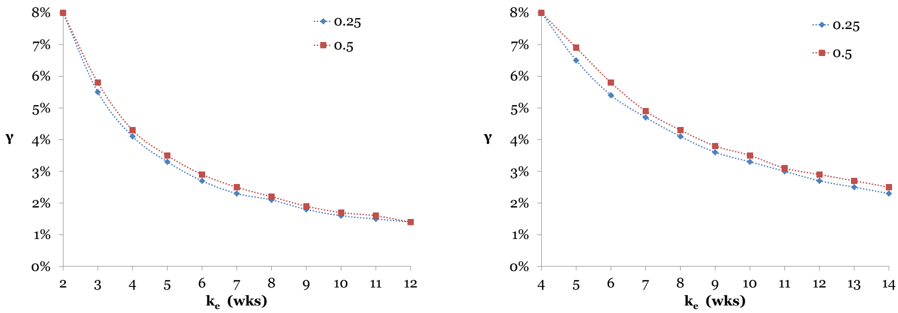
Figure 3.4 *Trade-off between factoring rate γ and maximum extended payment term k_e with manual discounting. c.v.* $= 0.25$, *or* 0.50. *Fixed parameters* $\eta = 0, \omega = 0.2$, $\psi = 0$.

3.5.3 *Maximum payment term extension with greatest opportunity cost rate for holding receivables.*

Here we explore the maximum payment term extension that allows a firm to benefit from reverse factoring when the opportunity cost rate for holding receivables η is the greatest, i.e., equal to γ, the cost of discounting them (see Table 3.1 and Table 3.2 for our experimental settings). Recall from Section 3.2 that when $\eta = \gamma$, the supplier is indifferent between manual and auto discounting. Although, *before* reverse factoring, a greater opportunity cost rate for receivables is in principle possible, the choice for auto discounting is constant at this point and beyond. Our experiments support the following assertion.

With reverse factoring and auto discounting, the maximum payment term extension k_e for a given reverse factoring rate γ is not affected by the coefficient of variation for demand, net profit margin ω, or operating leverage ψ.

Figure 3.5 shows, for different values of initial payment term and demand uncertainty, the set of (γ, k_e) values for which the financing cost with reverse factoring and auto discounting case is equivalent to the initial cost with conventional financing. Note that the maximum payment term extension and the reverse factoring rate are inverse-proportionally related. Since the firm discounts all of its receivables in every period, periodic expenses can almost always be met. Borrowing activity is negligible in this setting, even when operating leverage and demand uncertainty are both high. When the factoring rate is equal to (or less than) the opportunity cost rate for holding a receivable, a decision-maker can evaluate a proposed reverse factoring arrangement independently of the stochastic and economic aspects of inventory operations.



(a) $k_0 = 2$ **(b)** $k_0 = 4$

Figure 3.5 *Trade-off between factoring rate γ and maximum extended payment term k_e with auto discounting. c.v. $= 0.25$, or 0.50 Fixed parameters $\eta = \gamma$, $\omega = 0.2$, and $\psi = 0$.*

3.5.4 *Maximum payment term extension with intermediate opportunity cost rate for holding receivables.*

When the cost of receivables is higher than zero but below the cost of discounting, the cost of additional capital tied in receivables and the cost of additional capital required to fund cash deficits need to be accounted for in the trade-off. We wish to determine the maximum payment term extension that allows a supplier to benefit from reverse factoring. In this case the simulation results support the following assertion:

There exists an opportunity cost rate $\eta_{max} < \gamma$ such that no economically viable payment term extension is possible when $\eta_{max} < \eta < \gamma$. When $0 < \eta < \eta_{max}$, the maximum extended payment term k_e for reverse factoring is decreasing in η, decreasing in the net profit margin ω, and decreasing in the operating leverage ψ.

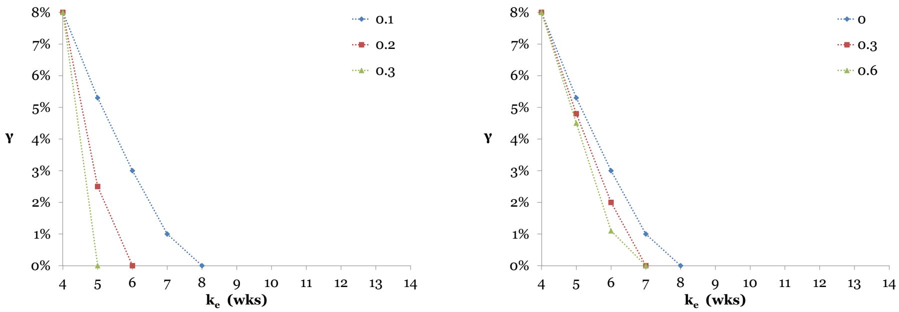
$\psi \backslash \omega$	0.1	0.2	0.3
0	1%	0.5%	0.5%
0.3	1%	0.5%	0.5%
0.6	1%	0.5%	0.5%

(a) $k_0 = 2, c.v. = 0.25$

$\psi \backslash \omega$	0.1	0.2	0.3
0	4%	1%	0.5%
0.3	2%	0.5%	0.5%
0.6	2%	0.5%	0.5%

(b) $k_0 = 4, c.v. = 0.25$

Table 3.3 *Maximum opportunity cost rate for holding receivables η_{max} such that a payment term extension $k_e > k_0$ is possible.*



(a) *Different levels of ω; $\psi = 0$ fixed.*

(b) *Different levels of ψ; $\omega = 0.1$ fixed.*

Figure 3.6 *Trade-off between γ and k_e with manual discounting. Fixed parameters $\eta = 0.005, k_0 = 4,$ and $c.v. = 0.25$.*

The maximum extended payment term appears to be highly sensitive to the opportunity cost rate for receivables. In this setting, the firm pays a dual premium for extended payment terms: the cost of carrying additional receivables and the cost of additional cash flow uncertainty. The possibility to extend payment terms and still realize a lower expected cost appears limited to settings where the opportunity cost rate for receivables is low. Table 3.3 shows the values of η_{max} that result from our basic experimental settings. In many cases, the opportunity cost rate for holding receivables must be below 0.5% if the firm is to extend payment terms and realize a reduction in financing cost. This rate corresponds to the rate of return of low-risk short-term investments, such as a savings account or an investment-grade corporate bond, which are unlikely to be the next best investment opportunity of SME's. The common presumption in industry that a firm assesses its opportunity cost rate for holding receivables at a rate equal to or higher than the reverse factoring rate can therefore be deceiving in terms of value creation.

Even though no cost-effective payment term extension is feasible when $\eta_{max} <$

$\eta < \gamma$, these cases may have practical relevance. As mentioned earlier, corporate customers sometimes unilaterally impose a payment term extension on their suppliers (e.g., Milne, 2009). Faced with such a situation, managers must still decide between auto discounting or manual discounting. Intuitively, since the cost of discounting any single receivable is greater than the opportunity cost, the selective approach of manual discounting should be preferred. Our experiments confirm this, showing that the relative advantage of manual discounting versus auto discounting increases in the opportunity cost of receivables and also in the size of the imposed payment term extension.

In contrast to the case of $\eta = 0$ examined in Experiment 2(a), the maximum extended payment term for reverse factoring appears to be *decreasing* in the net profit margin and operating leverage when $0 < \eta < \eta_{max}$. Figure 3.6 illustrates this. While a lower net profit margin or lower operating leverage make an extension of payment terms more costly in terms of cash flow variability, this same variability leads the firm to discount a greater proportion of its receivables. The average amount of outstanding receivables and the corresponding opportunity cost is thus ultimately lower. The latter effect tends to dominate when the opportunity cost rate for holding receivables increases toward η_{max}, so lower net profit margin and lower operating leverage then both facilitate longer payment terms. As $\eta \to 0$ the cost of cash flow variability becomes more important, and direction of significance for net profit margin and operating leverage tends to reverse. This contrast shows that a good estimation of the opportunity cost rate for holding receivables is essential if managers are to evaluate any payment term extension proposed in a reverse factoring arrangement.

3.6 Conclusions

Despite growing business interest for supply chain finance, little is yet scientifically known about the optimal management and benefits of such innovative financing arrangements. Our study focuses on reverse factoring, an arrangement that promises improvement of working capital financing for investment-grade buyers and their suppliers. The buyer facilitates cheaper short term financing for the supplier, and the latter in return may be asked to grant longer payment terms. We couple a periodic review, infinite horizon base stock inventory model with financing by conventional sources of capital and reverse factoring and explore the effect of a payment term extension.

The cost of a payment term is classically assessed purely based on the opportunity cost associated with tying up capital in average amount of receivables.

We show first that even without any opportunity cost for receivables, a payment term extension will generally entail greater financing costs. Additional capital is needed to cope with more variable cash flows. Introducing reverse factoring, we identify settings that allow a decision maker to make a payment term decision independently of inventory, and other settings where the maximum viable payment term extension depends on demand uncertainty, net profit margin, and operating leverage. We show that the significance of these parameters for financing costs may be complex and interrelated. Correspondingly, a decision about a payment term extension may be challenging.

Based on the data of a large provider of supply chain finance services, Klapper et al. (2012) finds that creditworthy buyers receive contracts with the longest maturities from their smallest, least creditworthy suppliers. While it is known that large buyers may use their strong bargaining position in extending trade credit terms with their suppliers (Wilson and Summers, 2002), this finding can also be seen as an indication of how emerging services impacts the trade credit landscape. Features of reverse factoring suggest that the facilitation of trade credit has become easier: financing rates are low and receivables can be discounted any time during the trade credit period. Our results suggest, however, that making payment term decisions based on the expected working capital changes will not account for the dynamics of stochastic inventory operations and their interaction with financing requirements.

There are several questions and issues left for future research. For instance, it is unknown what the optimal cash management (or even inventory management) policy is in our setting. Analytical studies, such that of Song and Tong (2012) show that obtaining properties is challenging, perhaps impossible. Based on our simulation, we find that similar to how a base stock policy propagates demand uncertainty to cash, a "base cash" policy eventually propagates demand uncertainty to shareholder's pay-outs. In practise, this could could entail a cost. It is unlikely, that the propagation can be mitigated with a threshold policy. Indeed, in this policy type demand uncertainty eventually increases the uncertainty of the timing/size of orders, and their corresponding cash flows. This suggests that our insights might be valid for a more general set of policies than the one that we consider. Another challenge is to extend our analysis for a firm with a borrowing constraint. Questions include: What should the firm do when it is approaches its borrowing constraint? (e.g, does it order less?) We still expect, however, that with a borrowing constraint financing cost still increase nonlinearly as a function payment terms. Indeed, the firm needs to either retain more cash or occasionally order less (and incur more backlog cost) to hedge itself against hitting the borrowing constraint.

Chapter 4

Reverse Factoring and Service Levels: Let it happen or make it work?

In Chapter 3 we have seen how reverse factoring allows a buyer to extend payment terms to its supplier on the premise of cheap finance. In fact, some literature even suggests that the scheme is developed in response to the growing need of firms to demand longer credit periods from their suppliers (Tyler, 2006; Hurtrez and Salvadori, 2010). For instance, consumer goods company P&G offers the scheme to suppliers in order to help them to deal with their extended payment terms (Serena, 2013). According to P&G, the extension helps them to free up $2 billion of cash to fund new investments and/or buy-backs of shares. Similarly, Dutch telecom provider KPN introduced reverse factoring as a 'sweetener' to its suppliers after having extended its payment terms from 45 to 90 days (Milne, 2009). The extension of payment terms in ongoing transactions increases a buyer's outstanding payables, hence, these initiatives could be characterized as initiatives in which buyers borrow more from suppliers.

Aside from the possibility to increase payables, reverse factoring seems to generate interest from investment-grade buyers based on motives that concern the reliability of their supply chain (see Chapter 2). For instance, Rolls Royce sees the introduction of reverse factoring as a way to mitigate the risk that suppliers cannot sufficiently invest in operations (Gustin, 2014b). It helps their suppliers to finance R&D and prototype building of tools, i.e. investments which are difficult to fund. In a recent Wall Street Journal article, an executive of Wal-Mart said that the company hopes that the introduction of reverse factoring will result in a more stable supplier base and more predictable supply of merchandise (O'Connel, 2009). In that same article, a consultant even suggests that reverse factoring may give Wal-Mart a competitive edge; he claims, *"Suppliers might be more inclined to give certain retailers 'preferential treatment' because of such programs, ..., the arrangement could influence how a supplier would look at Wal-Mart, especially if, ..., this program lets a supplier stay in business or make more money."* A statement of the Head of Trade Supply Chain Finance of Citi illustrates that such beliefs are not uncommon. He says that when there are shortages in the supply chain, suppliers *"typically will choose those who pay earliest. Consequently, implementing an SCF*[1] *programme can be a valuable means of securing supplies and suppliers"* (Ras, 2012).

The idea that firms favor a client that offers reverse factoring over other clients, i.e., clients that do not offer reverse factoring, makes intuitive sense. Indeed, while the firm needs to wait for its money until receivables mature for the other clients, receivables from a client offering reverse factoring can be converted into cash cheaply. Thus, satisfying the demands of the latter allows to repay debt or any other form of credit earlier. The facilitation of reverse factoring should therefore allow a buyer to encourage its supplier to give him more priority over his other buyers. Surprisingly, however, literature so far does not mention of any examples of buyers that contractually agree different service levels with suppliers in reverse factoring schemes. Indeed, most buyers seem to focus only on adjusting payment terms. Despite the intuition provided above, it is questionable whether suppliers 'naturally' provide better service to a buyer that offers reverse factoring. Indeed, it is also possible that a supplier will not invest in better service or will not give some type of preferential treatment; he just collects the maximum possible financial savings offered through the scheme. This may be so if the supplier hardly saves any money by giving preferential treatment to the respective buyer.

Our study is inspired from an interview with the treasurer of company B in the case study of Chapter 2. We will shortly summarise the case. Firm B is an OEM that had just launched a reverse factoring program to its supply base as part of a strategy to improve their supply base's resilience to demand uncertainty. Despite the significant transaction cost involved in implementing and running the program, the company did not propose any contractual changes to any supplier. That is, it changed neither its service level agreements nor its payment terms with suppliers. The treasurer explained that he believes that

[1] In trade publications, the general term Supply Chain Finance (SCF) is frequently used to refer to reverse factoring in particular.

the high cost of capital and liquidity shortfalls as a result of the 2008 financial crisis have impeded many of the company's suppliers from sustaining healthy levels of investment. While cost of capital and liquidity may indeed impede suppliers from making necessary investments to be resilient, the effectiveness of company's *B* tactic, i.e., not agreeing higher service levels with *any* supplier, is open to question. Indeed, it is unlikely that all of its suppliers experience tight credit constraints or experience tight credit constraints all the time.

The service level that a supplier provides to a client is dependent on many factors. For instance, Heide and Stump (1995) finds that while relationship continuity can improve the performance of a supplier, the dependency of a buyer, e.g., through specific investments, can deteriorate it. Aside from these aspects, it is however by no means clear that a retailer offering reverse factoring to a supplier may expect to be served better than other retailers based on simple economic grounds. Thus, we formulate the following research question:

Research question 4.1. *Is it in the supplier's benefit to provide a higher service level to a retailer that offers reverse factoring, or does this need to be contractually agreed by the retailer?*

If it is not in the supplier's benefit to provide the extra service, it may be worthwhile for the retailer to contractually agree the extra service. Stronger, it may even be worthwhile to contractually agree a higher service if the supplier already has the incentive to provide extra service. Indeed, the supplier may only commit to a service level improvement that is below the maximum level improvement that it can potentially offer to the retailer. We thus formulate the following research question

Research question 4.2. *What is the maximum service level improvement that a retailer can ask, given the terms of the reverse factoring offer?*

Our intuition on these questions are as follows. While serving a customer that offers reverse factoring yields cheap payment options, offering a higher service level to this customer, without comprising the service level of other retailers, should increase the supplier's operational cost. Consequently, a trade off between 'higher' and 'cheaper' should arise for the supplier. As both the *availability* and the cost of alternative financing can influence a the trade-off, we do not consider restrictions to alternative sources of capital in our study. We feel that including these restrictions forces the supplier to make two trade-offs at once, i.e., one based the need for financing and one based on the cost of financing. For a study based on the need for financing, we refer to the work of

Babich (2010), which explores how a buyer can control the supplier's financial state through direct financial injections.

To explore the retailer's trade-off in reverse factoring, we consider a discrete time, infinite horizon base stock model, with backorders, of a supplier serving two retailers with minimum fill rate constraints. We incorporate financial dimensions in the state description such that, next to cost of tying up capital in inventory, the cost of meeting the firm's demand for cash is measured. To meet its demand for cash, the firm has initially two options: borrowing and cash retained from previous periods. We extend these options with the option to discount receivables from one of the retailers. We use an inventory rationing policy introduced by Atkins and Katircioglu (1995), which allows us to raise the fill rate of the retailer offering reverse factoring. However, in order to ensure that the fill rate of the other retailer is not affected, the base stock level must be adapted. The firm's objective is to minimize the long-run expected inventory and financing cost while fulfilling the minimum fill rate requirements. Due to the complexity of our models, we use simulation-based optimization to solve them.

Our main contributions are as follows. First we find, in contrast to what trade literature suggests, that increased customer service is not a natural by-product of offering cheap payment options to a supplier. Namely, we find that the optimal base stock decreases as a function of the reverse factoring rate, as the supplier needs to hedge less against the financial cost of cash volatility. Thus, buyers should consider contractually agreeing their additional service desires as a result of reverse factoring introduction. Second, we show that the maximum service level that can be contracted in reverse factoring is a non-linear function of the reverse factoring rate and is dependent on the demand uncertainty, the difference between the lead-time and payment term, the ratio of the mean demands of the retailers, and the initial fill rate requirements. To the best of our knowledge, our study is the first one that quantitatively explores how much additional service can be asked from a supplier in return for offering reverse factoring. In light of the recent work of Murfin and Njoroge (2014), who empirically find that longer payment terms by investment-grade buyers are linked to cutbacks in capital expenditures by suppliers, our paper offers an alternative to how reverse factoring can be used by investment-grade buyers to stimulate capital expenditures to improve the supply chain. Depending on the setting, we find that a buyer may contractually agree a significant fill rate improvement, e.g., an improvement of a few percentage points on an initial fill rate of 90%.

The remainder of this chapter is structured as follows. In section 4.1 we discuss the literature relevant to our problem. In section 4.2, we describe the models we implement in our simulation. In section 4.3 we discuss the design of our experiments and our solution method. In section 4.4 we present the results from our experiments. In section 4.5 we draw final conclusions and discuss managerial insights.

4.1 Literature

Our work is related to two streams of literature: inventory models with financing, and inventory rationing. The first stream generally aims to determine the value of mutual consideration of financing and inventory decisions within firms, between firms, or between firms and investors and/or lenders. The models that have been developed so far can be categorized into three types: (1) single firm single-period, (2) single firm multi-period and (3) multi-echelon inventory models[2]. The models developed in this chapter belong to the third category. Therefore, we will consider some works of the first two categories briefly only, and discuss the related works in the latter category in more depth. Concerning category (1), many contributions have been already made (Buzacott and Zhang, 2004; Dada and Hu, 2008; Xu and Birge, 2008; Lai et al., 2009; Yang and Birge, 2011; Kouvelis and Zhao, 2011; Alan and Gaur, 2012). These studies generally consider an extended newsvendor setting to explore how financing decisions by the firm, its supplier, its customer, and/or financial intermediary, may impact the firm's ordering decision and the supply chain as a whole. One study that is particular relevant to our study is that of Tanrisever et al. (2012); they show how the value of reverse factoring is conditioned on the operating characteristics and the working capital policy of the supplier. In category (2), we find several studies. Babich and Sobel (2004) explore the coordination of financial and operational decisions to maximize the present value of proceeds from an Initial Public Offering (IPO). Xu and Birge (2006) present an integrated corporate planning model for making simultaneous production and financial decisions. Gupta and Wang (2009) show that a base stock inventory policy continues to be optimal when a supplier gives trade credit. Protopappa-Sieke and Seifert (2010) optimise order quantities for a finite horizon model that includes working capital restrictions and payment delays. Babich (2010) studies a manufacturer's joint inventory and financial subsidy decisions when facing a financially constrained supplier; he shows that an order-up-to policy and subsidize-up-to policy are optimal. Luo and Shang (2014) consider a model in which a firm can both receive and grant payment delays; they show that a

[2]Multi-echelon inventory models refers to models that study problems in which inventory can be located at multiple locations.

working capital dependent base stock policy is optimal. In category (3), not as many studies can be found as in category (1) or (2). The studies of Hu and Sobel (2007), Song and Tong (2012) and Luo and Shang (2013) are among the few in this category. The works show the complexity of obtaining analytical solutions due to interactions and the increased state space and provide insightful measures and perspectives to handle the complexity.

The models presented in this chapter can be considered to be extensions of those in Chapter 3. Specifically, the models accommodate a lead-time that is an arbitrary large number of periods instead of only one, and they include two customers instead of one. This latter gives rise to a rationing problem.

Inventory rationing is a relatively mature subfield of inventory management; it deals with distinguishing customer classes and giving each class a different a service level. The firm's wish to differentiate service can be motivated based on several grounds, such as the possibility to generate additional profit, or the differences in service requirements of different customers (e.g. critical vs. less critical). The problems related to serving multiple demand classes have been studied in both single-echelon and multi-echelon contexts.

Arslan et al. (2007) provide an extensive review of the research on rationing in single-product inventory models and categorizes them based on three dimensions: (1) the type of control policy, i.e., periodic or continuous review, (2) the treatment of demand shortages, i.e., lost sales or backorders, and (3) whether or not key developments are primarily focused on two vs. multiple demand classes. We refer to their paper for the complete review and consider only the works relevant to ours, i.e., works that consider periodic review models. Veinott (1965) was one of the first to analyse a periodic review single product inventory model with multiple demand classes and proposes a critical inventory level policy, i.e., a policy in which a customer (class) is not served anymore when inventory on hand falls below a certain critical level. Subsequently, (Topkis, 1968) studies this system and proposes a scheme that allows the calculation of the critical levels efficiently for the lost sales case only; he finds that the backorder case requires more computational efforts. Evans (1968) and Kaplan (1969) study similar periodic review systems for the lost-sales and back-ordering case respectively, but only with two customer classes. Nahmias and Demmy (1981) study a periodic review critical-level system with backorders and two demand classes. Cohen et al. (1988) consider a similar system and policy as Nahmias and Demmy (1981), but with lost sales. They develop a greedy heuristic which minimizes expected costs subject to a fill rate service constraint. However, the performance of this heuristic deteriorates as the fill rate requirement or the leadtime increases. Frank et al. (2003) study a periodic review system with two demand classes, one deterministic and the other stochastic, with lost sales. They show that the optimal ordering and rationing policy is state dependent and complex and present a simpler policy that works well compared to the optimal one. Except the studies of Frank et al. (2003) and (Cohen et al., 1988), all the studies discussed so far invariably assume that stock levels can be changed instantaneously. In the work of Frank et al. (2003) the leadtime is equal to one review period and in the model of Cohen et al. (1988) the leadtime is arbitrary large.

Rationing policies have also been widely studied in multi-echelon distribution systems, mostly two-echelon divergent systems (,i.e., one warehouse and multiple retailers). We refer to the work of Lagodimos and Koukoumialos (2008) for a review on the various rationing policies. Within this class of two-echelon systems, an important distinction is made based on whether inventory is carried at the warehouse (Federgruen, 1993). Eppen and Schrage (1981) were the first to study a system in which the warehouse does not hold inventory. They introduce a fair share allocation rule, which ensures that the stock-out probabilities at the retailers are equalized. Many works have extended this line of research (see Federgruen, 1993, for an overview). These works have mostly focused on allocation policies that have a simple (myopic) structure and can yield a cost that is close to the optimal. In addition to this line, there is also a line of research that focuses on allocation rules that aim to satisfy the service level constraints of different end customers (de Kok, 1990; Verrijdt and de Kok, 1995, 1996; Diks and de Kok, 1996). We refer to the work of Van der Heijden et al. (1997) for an extensive evaluation of such allocation rules in N-echelon distribution systems.

The work of Atkins and Katircioglu (1995) considers a setting that is most close to ours, i.e., a single-echelon, periodic review inventory setting with backorders, arbitrarily large leadtime and with non-identical demand streams. They study two model versions: a cost-oriented version in which the aim is to minimise the average inventory cost and service-oriented version in which the aim is to satisfy the service constraints of different customers (at minimum cost). Inspired by studies from Federgruen and Zipkin (1984) and of Federgruen (1993), they derive a lower bound for the optimal cost for a relaxed version of the cost-oriented model (i.e., one that allows negative inventory allocations). Subsequently, based on a correspondence of the relaxed cost-oriented model and the service-oriented model, they develop a heuristic allocation policy for the service-oriented model, which yields service levels that are close to target and an expected inventory cost that is at the lower bound. As their heuristic allows us to controllably vary the fill rate of different demand streams, we use it in our model.

4.2 The Models

Here we present the models we use to explore our research questions. First, in section 4.2.1 we introduce the general setting and a base case model of a supplier that serves two retailers and that uses conventional financing only. Subsequently, in section 4.2.2, we discuss an allocation policy that allows us to differentiate fill rates between the two retailers. In section 4.2.3, we discuss the model extensions for reverse factoring for one of the retailers.

4.2.1 Conventional Financing Model (C)

We consider a periodic review inventory model of a supplier serving two retailers from stock. Retailer A offer reverse factoring; retailer B pays after a fixed payment delay. The supplier reviews its inventory position at the start of each period. Periods are indexed by the variable t. At the end of each period, the supplier receives stochastic demands $D_{A,t}$ and $D_{B,t}$ with mean μ_A and μ_B and variance σ_A^2 and σ_B^2 from retailer A and B respectively. The supplier pays price c per ordered unit and sells demanded items at price $p > c$ per item to the retailers. Inventory is controlled according to a base stock policy, with base stock I units. At the start of each period, the supplier monitors its inventory position, and orders the items needed such that it is equal to the base stock. Orders are delivered after l periods with $l \in \mathbb{N}^+$; orders placed at the start of period $t - l - 1$ are delivered immediately prior to the end of the period t.

The demands of both retailers arrive at the end of the period, right after the supplier receives its delivery. If the retailers' demands cannot be met from the stock on hand the supplier rations the available stock among the two retailers and backorders their unmet demand portions. For each unsold item, the supplier incurs a variable inventory holding cost. Inventory holding cost typically includes two major components (Zipkin, 2000): a direct cost associated with holding inventory, including warehouse rental, insurance, and handling costs, and a financing cost as a result of having capital tied up in inventory. In inventory theory, these two cost components are often represented in a single parameter. Here, we consider only the second component as variable, which we denote as ρ per monetary unit tied up in inventory. The direct inventory cost, i.e., the warehouse rental, insurance, and handling costs, are assumed to be fixed and is represented by f monetary units per period.

The supplier meets its payment obligations either from internal cash, i.e., profits that are retained in previous periods, or by borrowing from a bank. Borrowing occurs to the extent that internal cash is insufficient, and is paid back to extent that it is possible again from profits in subsequent periods. This type credit is also referred to as revolving credit, or simply a 'revolver'. The annualized interest charge for borrowing is β per monetary unit; interest is charged at the end of each period based on the borrowing position at the start of a period. Cash retention is governed by a threshold policy that is analogous to a base stock policy: cash is released to shareholders in the form of dividends whenever it exceeds threshold T, but only to the extent that the cash level is returned to T. As with inventory, an annualized opportunity cost rate α is charged on each monetary unit retained in cash within the firm. In a perfect capital market, we should expect the shareholder's opportunity cost (required rate of return) for investing money in cash retention and/or inventory to be equal to the interest rate charged for borrowing money, thus $\beta = \alpha = \rho$. Later, in the sensitivity analysis (Section 4.3.2) we consider the case that capital market frictions may entail differences among these rates.

The supplier pays for its goods upon delivery. As in practise firms can pay their suppliers after a delay, this assumption may seem restrictive. However, it can be circumvented in our model, by subtracting the firm's payment period to its supplier from the firm's payment period to the retailers. We can thus account for all cases in which the firm's payment period to its supplier is shorter than that to the retailers. The supplier initially grants both retailers the same fixed payment term of $k \in \mathbb{N}^+$ periods. The payment term starts to count from the moment that a demanded unit is met from inventory. While this implies that shareholders incur a cost for capital tied up in receivables (see Chapter 3), we exclude this cost in our model. In a backorder inventory system with only conventional financing this should not affect the inventory and/or cash management; each monetary unit of revenue eventually undergoes the same payment delay regardless of the base stock I and/or cash target level T. When the firm uses reverse factoring this changes: as receivables are sold to finance inventory operations the expected amount of receivables that the supplier carries for retailer A becomes lower. Consequently, a firm may tend to adapt its policy such that the marginal cost and marginal value of discounting (vs. carrying) receivables are equalised[3]. In this model, we let the opportunity cost rate of carrying receivables to be zero. This implies that the supplier has no tendency to differentiate service between the retailers based on the amount of receivables that the supplier has to carry for each retailer. This allows us to focus purely on the trade-off between cost of offering service and the cost of financing inventory operations.

[3]The numerical results of Chapter 3 illustrate how the opportunity cost rate of receivables indeed impacts and complicates the trade-off on payment terms.

We use the following notation for the variables of the system:

- m_t: the firm's cash position; a tangible cash balance or a bank overdraft is represented by $m_t > 0$ or $m_t < 0$ respectively.
- $\boldsymbol{r}_{j,t} = (r_{j,t,1}, ..., r_{j,t,k})$: k-dimensional vector of outstanding receivables to be collected at the end of periods $t, t+1, ..., t+k-1$ for retailer j;
- $\boldsymbol{o}_t = (o_{t,1}, ..., o_{t,l-1})$: $(l-1)$-dimensional vector of outstanding orders to be collected at the end of periods $t, t+1, ..., t+l-2$;
- $o_t = \sum_{n=1}^{n=l-1} o_{t,n}$ sum of outstanding orders in period t (applicable only if $l \geq 2$);
- x_t: inventory on hand in period t;
- $b_{j,t}$: backorders of customer j in period t;
- $w_{j,t} = D_{j,t} + b_{j,t}$: demand of retailer j in period t plus its backorders at the start of the respective period;
- $a_{j,t}$: amount of stock allocated to retailer j in period t with $a_{j,t} \leq w_{j,t}$;
- D_t, b_t, w_t, a_t, $r_{t,i}$: summations of $D_{j,t}$, $b_{j,t}$, $w_{j,t}$, $a_{j,t}$, and $r_{j,t,i}$ over $j \in \{A, B\}$;
- $y_t = x_t - b_t + o_t$: inventory position in period t.

The system state at the start of period t is $S_t = (\boldsymbol{o}_t, x_t, b_{A,t}, b_{B,t}, \boldsymbol{r}_{A,t}, \boldsymbol{r}_{B,t}, m_t)$. Figure 4.1 summarizes the sequence of events in a period. At the start of the period, S_t is observed (1) and the order is placed (2). At the end of the period, cash is collected from the oldest accounts receivable and the position of others is decremented (3). Money is borrowed if needed (4), the units ordered are received (5) and the cash payment is made (6). If policy allows, cash is released from the firm to shareholders (7). Finally, demand is received and allocated to retailer A and B to the extent that inventory on hand allows. This creates new accounts receivable for the retailers (9). The transition equations for the outstanding orders, inventory on hand, backorders, cash position, and accounts receivable are as follows:

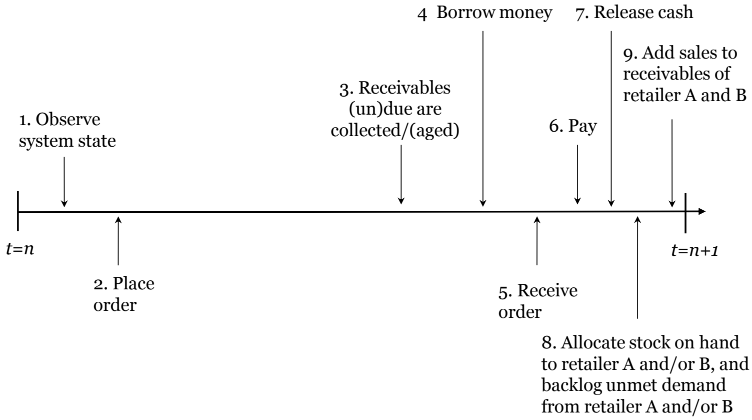


Figure 4.1 *The sequence of events within a single period.*

$$o_{t+1,i} = \begin{cases} I - y_t & i = l-1 \quad \wedge \quad l \geq 2 \\ o_{t,i+1} & i = 1,\dots,l-2 \quad \wedge \quad l \geq 3 \end{cases} \tag{4.1}$$

$$x_{t+1} = \begin{cases} (I - D_t)^+ & l = 1 \\ (x_t - b_t + o_{t,1} - D_t)^+ & l \geq 2 \end{cases} \tag{4.2}$$

$$b_{j,t+1} = (w_{j,t} - a_{j,t})^+ \quad j = A, B \tag{4.3}$$

$$r_{j,t+1,i} = \begin{cases} (a_{j,t})p & i = k \\ r_{j,t,i+1} & i = 1,\dots,k-1 \end{cases} \quad j = A, B \tag{4.4}$$

$$m_{t+1} = \min\{u_t, T\} \tag{4.5}$$

where

$$u_t = m_t + r_{t,1} - p_t,$$
$$p_t = f + vc + \beta(-m_t)^+,$$
$$v = \begin{cases} I - y_t & l = 1 \\ o_{t,1} & l \geq 2 \end{cases}$$
$$(a)^+ = \max\{0, a\}.$$

Equation (4.1) describes the transition of the orders to be collected in the next $l-1$ periods. This equation is only needed when $l \geq 2$; indeed, for $l = 1$ the orders placed at the start a period are collected immediately before the end of the same period. For $l \geq 2$, $o_{t,l-1}$ represents the order that is placed in the previous period; for $l \geq 3$, $o_{t,1}, ..., o_{t,l-2}$, represent the orders that are placed before the previous period and are updated each period.

Equation (4.2) specifies the changes in inventory on hand for $l = 1$ and $l \geq 2$. For $l = 1$, the inventory on hand at the start of period $t+1$ is the base stock minus demand from period t. For $l \geq 2$ the inventory on hand at the start of period $t+1$ is the inventory on hand, minus backorders and the order due, minus the total demand in period t, only if the latter exceeds zero. Otherwise, the inventory on hand is zero.

Equation (4.3) describes the backorders of each retailer j in period $t+1$ as the retailer's demand plus backorders in period t, minus the inventory allocation in respective period, only if the latter exceeds zero. Otherwise the retailer's backorders are zero.

Equation (4.4) describes the payments due in the next k periods. For $i = k$ the payment consists of the revenue from the allocation in period t. For payments that are due in $1 < i \leq k$ periods record-keeping is updated.

Equation (4.5) specifies the cash position at the start of period $t+1$ to be the cash position plus payments minus expenses made in period t. If, without dividend payments, the cash position exceeds the cash target T at the end of period t, the firm pays out dividends such that the firm's cash position is T again.

We define service level of retailer j, SL_j, to be the fill rate, i.e., the long-run average fraction of j's demand that is satisfied *directly* from stock (Zipkin, 2000):

$$SL_j = 1 - \lim_{t\to\infty} \sum_{t=1}^{\infty} \frac{D_{j,t} - (a_{j,t} - b_{j,t})^+}{D_{j,t}}. \tag{4.6}$$

As will become clear later on, the fill rate for retailer j, SL_j, depends on the supplier's base stock level I and the allocation policy used. For a specific joint base stock and cash management policy $Z = (I, T)$ we define $G_C(Z)$ to be the long-run average cost per period:

$$G_C(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} [\rho x_t + \alpha(m_t)^+ + \beta(-m_t)^+]. \tag{4.7}$$

The definition includes the inventory costs and the financing cost. The inventory cost are the cost incurred due to the capital tied up in inventory. The financing cost includes cash retention cost and borrowing cost, which are respectively the costs incurred due to the capital tied up in cash and the capital borrowed to finance operations. We assume that both retailers initially require the same minimum fill rate MSL. Given a stock allocation policy, we wish to find the policy $Z^* = (I^*, T^*)$ that satisfies the minimum fill rate for the retailers and minimizes the long-run average cost per period:

$$\begin{aligned} &\underset{Z}{\text{minimize}} && G_C(Z) \\ &\text{subject to} && SL_j(I) \geq MSL,\ j = A, B. \end{aligned} \tag{4.8}$$

4.2.2 *Allocation Policy*

Here, we introduce an allocation policy that allows the firm to vary the fill rates of the retailers. As we aim to explore the maximum service level improvement that the firm can grant in return for reverse factoring from retailer A, the allocation policy should allow us to increase fill rate of retailer A, SL_A, while keeping constant the fill rate of retailer B, SL_B. Naturally, the increase of SL_A should involve an adaptation of the base stock. To the best of our knowledge, there are no analytic results on the optimal rationing rule for our setting, even without considering the financial dimensions. Particularly, the possibility that backorders can be carried over consecutive review periods for each retailer complicates analysis.

Atkins and Katircioglu (1995) analyse a variant of our setting in which the firm's aim is to minimise average holding and backorder cost. They prove the structure of the optimal allocation and ordering policy for a relaxed version, specifically, a setting that allows negative stock allocations. Based on this policy, they propose a heuristic policy that gives service levels very close to the target service levels and a cost that is always at the lower bound. However, in their policy, the target service levels of retailers are expressed as 'backorder rates', i.e., the average amount of demand plus backorders minus the average allocation of a customer expressed as fraction of the mean demand of the respective customer, instead of the fill rates. By means of simulation, we find that that the backorder rate has a close relation with the fill rate. Indeed, the smaller the fraction of orders waiting to be satisfied for a particular retailer, the higher the fraction of orders that can be satisfied directly from stock for the respective retailer. Based on this relation, the heuristic can controllably differentiate the fill rates of two retailers in our model. We use the following modified version of their allocation policy:

$$a_{j,t} = (w_{j,t} - \Theta_t \zeta_j)^+ \quad j = A, B \tag{4.9}$$

where

$$\Theta_t = \begin{cases} 0 & w_t \leq q \\ \theta_{AB} & w_t > q \quad \wedge \quad a_{A,t}(\theta_{AB}) \geq 0 \quad \wedge \quad a_{B,t}(\theta_{AB}) \geq 0 \\ \theta_A & w_t > q \quad \wedge \quad a_{A,t}(\theta_A) \geq 0 \quad \wedge \quad a_{B,t}(\theta_A) \leq 0 \\ \theta_B & w_t > q \quad \wedge \quad a_{A,t}(\theta_B) \leq 0 \quad \wedge \quad a_{B,t}(\theta_B) > 0 \end{cases},$$

$$\zeta_A = \lambda \mu_A, \quad \zeta_B = \mu_B, \quad \zeta_{AB} = \zeta_A + \zeta_B,$$

$$\theta_i = \frac{w_{i,t} - \min\{w_t, q\}}{\zeta_i} \text{ with } i = A, B, AB, \text{ and}$$

$$q = \begin{cases} I & l = 1 \\ x_t - b_t + o_{t,1} & l \geq 2 \end{cases}.$$

Note from (4.9) that the retailer's allocation not only accounts for the current demand, but also the amount of backorders of a customer at the allocation moment. The tuning parameter λ with $\lambda \in (0, 1]$ allows us to increase the service level of retailer A. When $\lambda = 1$, the allocation policy ensures that retailer A and B receive the same service; when $\lambda < 1$, retailer A will have a better service than retailer B. Note that as $\lambda \to 0$, greater priority will be given to retailer A in the case of shortages. Our allocation policy differs from that of Atkins and Katircioglu (1995) in that the parameter Θ_t is a state and time-dependent parameter rather than a fixed parameter. This adaptation makes a search for a fixed Θ which satisfies the non-negative allocation constraints redundant. Later in this chapter, we will discuss an algorithm that allows to increase SL_A while keeping SL_B fixed.

4.2.3 *Reverse Factoring Model (R)*

In this section, we introduce reverse factoring for the demands from retailer A. Reverse factoring allows the firm to discount the receivables resulting from the demands from A. We assume that the firm uses a 'manual discounting' policy. This policy is explained in Section 3.2, page 27. It basically means that the firm discounts its receivables only when it needs them according to its internal operations, otherwise it leaves them to be paid at maturity. We define the scalar $\gamma \in (0, 1)$ to be the annualized fraction of the receivable's face value that the firm must pay to discount a receivable (see Section 3.3, page 31, for an example how this works). For the reverse factoring model, we set $\gamma < \beta$, so cash from discounting is preferred over cash from borrowing. The static k-dimensional vector γ with components γ_i represents the rates applicable for discounting receivables that are otherwise due i periods from the beginning of the current period. We set $\gamma_1 = 0$ since receivable $r_{t,1}$ is due anyway at the end of period t. For $i > 1$ we set $\gamma_i = \gamma_{i-1} + \tau\gamma$ (τ is the length of one period in years). The sequence of events with reverse factoring is the same as in Figure 4.1, except that at (4) the firm discounts receivables as needed and available, before resorting to borrowing.

The introduction of reverse factoring entails a change to the transition equations for receivables for retailer A (4.4) and the transition equation for cash (4.5). For the receivables from retailer A the transition equation (4.4) becomes:

$$r_{A,t+1,i} = \begin{cases} (a_{j,t})p & i = k \\ (1 - \varphi_{t,i}(r_{t,1}, .., r_{t,i+1}))r_{A,t,i+1} & i = 1, \ldots, k-1 \end{cases} \tag{4.10}$$

where

$$\varphi_{t,i}(r_{t,1}, .., r_{t,i+1}) = \frac{\min\{(1-\gamma_n)r_{A,t,i+1}, (u_t - \sum_{n=2}^{n=i}(1-\gamma_n)r_{A,t,n})^+\}}{(1-\gamma_n)r_{A,t,i+1}}$$

is the fraction of $r_{t,i+1}$ that needs to be discounted in period t to exactly meet the firm's cash need in the respective period. Cash is governed by the following transition equation:

$$m_{t+1} = \begin{cases} \min\{u_t, T\} & u_t \geq 0 \\ 0 & u_t < 0 \quad \wedge u_t + \sum_{n=2}^{n=k}(1-\gamma_n)r_{A,t,n} \geq 0 \\ u_t + \sum_{n=2}^{n=k}(1-\gamma_n)r_{A,t,n} & \text{otherwise.} \end{cases} \tag{4.11}$$

As explained in Section 3.3, there are three possible outcomes for the cash position captured by the respective cases in (4.11):

(a) After paying periodical expenses the firm's cash position is non-negative. No receivables need to be discounted.

(b) After paying periodical expenses the firm's cash position is negative. The firm must discount some receivables.

(c) Even after discounting all of its receivables, the firm has insufficient cash to meet its expenses, consequently it has to borrow.

The long run average cost per period of the firm becomes as follows:

$$G_R(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} [\rho x_t + \alpha(m_t)^+ + \beta(-m_t)^+ + \sum_{i=1}^{k} \gamma_i \varphi_{t,i}(.) r_{A,t,i}]. \tag{4.12}$$

Note that compared to firm's average cost in the conventional financing model (Equation 4.7) the firm's average cost now includes factoring cost. Again, we wish to find the policy $Z^* = (I^*, T^*)$ that satisfies the retailers' minimum fill rate constraint and minimizes the long-run average cost per period:

$$\begin{aligned} & \underset{Z}{\text{minimize}} && G_R(Z) \\ & \text{subject to} && SL_j(I) \geq MSL,\ j = A, B. \end{aligned} \tag{4.13}$$

4.3 Experimental Design and Solution Algorithm

Now that we have the model for both financing settings (C and R) and an allocation policy that allows to differentiate the retailers' service levels, we can answer the research questions by means of experimentation. In section 4.3.1, we describe the solution algorithm that we use for our exploration. Subsequently, in section 4.3.2, we describe in detail the design of the two experiments we conduct and the design of a sensitivity analysis.

4.3.1 Solution Algorithm

In an exploration of the solution space by means of simulation, we find the objective functions (4.7) and (4.12) to exhibit convexity in the cash target, T, but not in the base stock I. While the average inventory cost is a convex function of the base stock level, the average financing cost appears to be non-convex. This will be illustrated the next section. Nevertheless, as financing cost eventually increase asymptotically in the base stock, our simulation shows that, for any cash target level T, there is an unique base stock level I that yields a minimum total average cost. Thus for any given setting we can find a joint policy Z^* that yields a global minimum average cost. Based on this insight, we utilize the 3-stage algorithm that we have already developed in Chapter 3 (see Algorithm 1 on page 40). However, unlike the models in Chapter 3, the optimization problems are constrained; we aim to find the minimum average cost that satisfies a minimum fill rate constraint of A and B (see Equation 4.8 and 4.13). To ensure that we only consider solutions that satisfy the minimum fill rate constraint, we add another stage to the Algorithm 1. Specifically, we add Algorithm 2 (discussed next) to the three existing stages.

We define $\hat{I}(\lambda)$ to be the minimum base stock for allocation policy λ that satisfies the fill rate constraints of both retailers: $\hat{I}(\lambda) \equiv \min\{I(\lambda) : SL_j(I(\lambda)) \geq MSL, j = A, B\}$. Note that $\lambda \in (0, 1] \Rightarrow SL_A(I(\lambda)) \geq SL_B(I(\lambda))$, i.e., we only consider values for the allocation tuning parameter λ such that retailer A has a fill rate that is equal or greater than to that of B. Thus, $\hat{I}(\lambda) = \min\{I(\lambda) : SL_B(I(\lambda)) \geq MSL\}$. Given λ, we can find thus $\hat{I}(\lambda)$ by means of a bisection search algorithm that evaluates only the fill rate of retailer B (see Algorithm 2). Subsequently, in Step 0 of Algorithm 1 (see page 40), 'Choose algorithm parameters', we use $\hat{I}(\lambda)$ as the initial policy, thus $Z_0 \leftarrow (\hat{I}(\lambda), 0)$.

Algorithm 2: Algorithm for finding $\tilde{I}(\lambda)$

Choose $T = 0$
Choose $I_L = 0$ and $I_H = \delta l(\mu_A + \mu_B)$ with δ such that $\tilde{SL}_B(I_H) >> MSL$
Choose $\epsilon = .01$
while $(\tilde{SL}_B(\hat{I}(\lambda)) < MSL \vee \tilde{SL}_B(\hat{I}(\lambda)) > MSL + \epsilon)$ **do**
  $\hat{I} \leftarrow \frac{1}{2}(I_L + I_H)$;
  Evaluate $\tilde{SL}_B(\hat{I})$;
  If $\tilde{SL}_B(\hat{I}) > MSL$ **then** $I_H \leftarrow \hat{I}$ **else** $I_L \leftarrow \hat{I}$;
    return $\hat{I}$

4.3.2 *Experimental Design*

The next three paragraphs describe explicitly the two experiments and a sensitivity analysis. We are mostly interested in comparing the optimal policies and the corresponding cost of systems across different settings of the allocation policy λ. To facilitate presentation, let $Z^*_\nu(.) = (I^*_\nu(.), T^*_\nu(.))$ be the optimal policy with financing method ν with $\nu \in \{C, R\}$ as function of the input parameters $l, k, \lambda, \rho, \alpha, \beta$ and γ. Note that $\nu = C \Rightarrow \gamma = \emptyset$ (i.e., the factoring rate is not applicable in the case that firm uses conventional financing).

As in Chapter 3, we are interested in the behavior rather than absolute value of the firm's objective as a function of the model parameters in this study. Therefore, we experimented with multiple different settings for each parameter. The parameters are again chosen to be as realistic as possible for manufacturing firms in developed countries.

Experiment 1: *Supplier's inclination to offer a higher service in return for reverse factoring*

To explore whether the supplier benefits from providing a higher fill rate to retailer A, we investigate the change in the optimal base stock as a result of introducing reverse factoring for retailer A in an unconstrained setting:

$$\Delta I^* = I_R^*(.) - I_C^*(.) \quad \text{with } \gamma < \beta \wedge MSL = 0 \tag{4.14}$$

Here, $\Delta I^* > 0$ implies that the supplier is inclined to give a higher service as a consequence of receiving a reverse factoring offer. We conduct two experiments. First, we let $\mu_B = 0$ (and $\lambda = 1$) in both the conventional financing C and reverse factoring R model. Basically, this entails that we consider an inventory operation with retailer A only and thus no inventory is allocated to B in any period. Second, we explore ΔI^* when the demands from customer B are also considered. Specifically, we explore this for $\mu_B = \mu_A$ (and $\lambda = 1$). While the setting $\lambda = 1$ lets the supplier to treat the retailers equally in the allocation, it still allows us to explore whether giving extra service to retailer A is lucrative. Indeed, if $\Delta I^* > 0$ for $\mu_B > 0$, then λ can be tuned such that the extra service is only granted to retailer A.

The parameter settings for the experiments can be found in Table 4.1. In each experiment, we explore the average cost for a range of values of the base stock level I and the best cash target level for the respective base stock: $G_v(I, T^*)$. Thus, for each value of the base stock we search for the cash target level T^* that minimizes the average cost for that respective base stock level. We conduct 36 $(3 \times 2 \times 2 \times 3)$ experiments, in which we explore:

- 3 settings for the financing method: one with conventional financing C with $\alpha = \beta = 5\%$ and two with reverse factoring R with $\gamma = 3\%$ and 1% respectively.
- 2 settings for the mean demand of retailer A and retailer B: $(\mu_A, \mu_B) = (5, 0)$ and $(5, 5)$ respectively.
- 2 settings for the demand variation coefficient $c.v.$ with $c.v. = c.v._A = c.v._B$ of the retailers: $c.v. = 0.25$ and 0.5 respectively;
- 3 settings for the lead time, $l = 2, 4,$ and 6 (such that $l < k$, $l = k$, and $l > k$);

Experiment 2: *The maximum service level improvement that a retailer can require in return for the reverse factoring*

We define $SL^*_A(\gamma)$ as the maximum fill rate the supplier can offer to retailer A for reverse factoring with rate γ, such that:

- the supplier incurs no greater average cost than the optimal average cost with conventional financing sources and a non-discriminating allocation policy;
- after adopting reverse factoring, the initial minimum fill rate requirement for retailer B is still fulfilled.

To facilitate presentation, we rewrite the optimal average cost per period as function of the allocation policy λ as $G_\nu(\lambda)$ with $\nu \in \{C, R\}$, suppressing the immediate dependence on the optimal policy Z^*. The maximum fill rate $SL^*_A(\gamma)$ can then be formulated as follows:

$$\begin{aligned} SL^*_A(\gamma) \equiv \max_{\lambda} & \; SL_A(Z^*_R(\gamma,\lambda)) \qquad (4.15)\\ \text{subject to } & G_R(\gamma,\lambda) - G_C(\lambda_0) \leq 0,\\ & SL_B(I^*_R(\gamma,\lambda)) \geq MSL,\\ & SL_j(I^*_C(\lambda_0)) \geq MSL \;\; j = \{A, B\},\\ & \lambda_0 = 1,\; \lambda \in (0,1], \end{aligned}$$

all other parameters equal. For a given initial cost $Z^*_C(\lambda_0) : SL_j(I^*_C(\lambda_0)) \geq MSL, j = \{A, B\}$ and reverse factoring rate γ, the maximum fill rate $SL^*_A(\gamma)$ can be found by means of an iterative search that in each iteration lowers the tuning parameter λ (starting from λ_0), finds the corresponding optimal policy $Z^*_R(\gamma,\lambda) : SL_B(I^*_R(\gamma,\lambda)) \geq MSL$, and evaluates whether the difference $G_R(\gamma,\lambda) - G_C(\lambda_0)$ is still positive. The search stops when a point has been reached in which the difference cannot be reduced any further. Finally, it returns the fill rate of retailer A that belongs to the respective optimal policy: $SL_A(Z^*_R(\gamma,\lambda))$. By conducting this iterative search for a range of values for γ, we obtain a set of $(\gamma, SL^*_A(\gamma))$-values that show the maximum fill rate levels that retailer A can require for reverse factoring as function of the rate γ. This analysis is thus similar to the one conducted in Chapter 3, but instead of exploring the maximum *payment term* for a given reverse factoring offer γ we explore the maximum *fill rate*.

We conduct 17 experiments, in which we first explore $SL^*_A(\gamma)$ as function of γ for a base case setting and 16 (4×2) alternative settings for:

- demand variation coefficient of the retailers: *c.v.* $= 0.5$, and 1;

- the minimum fill rate: $MSL = 0.95$, and 0.97;
- the lead time: $l = 2$ and 6 (such that $l < k$ and $l > k$ respectively);
- the ratio of the mean demand of retailer A to retailer B, $\mu_A : \mu_B = 1 : 3$ and $3 : 1$.

The parameter settings that we use in our experiments can be found in Table 4.1.

Experiment	1		2	
Description	Exploration ΔI^*		Exploration $SL_A^*(\gamma)$	
parameter	Base Case	Variations	Base Case	Variations
(μ_A, μ_B)	(5,0)	(5,5)	(5,5)	(2.5,7.5),(7.5,2.5)
$(c.v._A, c.v._B)$	(0.25,0.25)	(0.5,0.5)	(0.25,0.25)	(0.5,0.5),(1,1)
p	10	-	10	-
c	9	-	9	-
ρ	5%	-	5%	-
α	5%	-	5%	-
β	5%	-	5%	-
γ	3%,1%	-	[5%,0%)\0.2%	-
k	4	-	4	-
l	4	2,6	4	2,6
I	0-50	-	n.a.	-
MSL	0	-	0,90	0.95,0.97
λ	1	-	(0,1]\0.02	-

Table 4.1 *Input parameter settings.*

Sensitivity Analysis

We explore the sensitivity of our results to the firm's expected net profit margin, the operating leverage and to changes in the financing cost rates.

While we set $(p, c, f) = (10, 9, 0)$ in the basic setting, we explore 9 (3×3) settings with three possible levels for the expected net profit margin, ω, and three possible levels for the operating leverage, ψ. Remember from Chapter 3 that $\omega = (\mu_T(p - c) - f)/\mu_T p$ and $\psi = f/(\mu_T c + f)$ with $\mu_T = \mu_A + \mu_B$. The alternative settings concerning unit selling price p, unit cost c, and fixed cost f are displayed in Table 4.2.

We expect that changes in the net profit margin impact the trade-off on the maximum fill rate $SL_A^*(\gamma)$ similarly to how they impact the trade-off on the maximum payment term k_e (See Section 3.5.2), i.e., the higher the expected net profit margin the lower the maximum fill rate for a given reverse factoring rate γ. In addition, we expect the same finding for changes in the operating leverage, i.e., the higher the operating leverage the higher the maximum fill rate for a given reverse factoring rate γ.

$\psi \backslash \omega$	0.1	0.2	0.3
0	10, 9, 0	10, 8, 0	10, 7, 0
0.3	10, 6.3, 27	10, 5.6, 24	10, 4.9, 21
0.6	10, 3.6, 54	10, 3.2, 48	10, 2.8, 42

Table 4.2 *Values of unit selling price p, unit cost c, and total fixed cost f underlying the experimental settings of net profit margin ω and operating leverage ψ.*

While we set $\beta = \alpha = \rho = 5\%$ in the basic setting, capital market frictions may entail differences among these rates. For example, as shareholders are residual claimants in bankruptcy, they might expect a higher return for the additional risk, and thus we might observe $\alpha > \beta$ and/or $\rho > \beta$ (Tirole, 2006). On the other hand, as lenders are generally less informed than shareholders on the quality of the firm's assets or the policies, we equally might observe $\beta > \alpha$ and/or $\beta > \rho$ as a result of the lenders' anticipation on these imperfections (Myers and Majluf, 1984). As far the relative size of the inventory holding cost rate to the cash retention cost rate goes, holding inventory is generally considered to be riskier than holding cash (Buzacott and Zhang, 2004). Indeed, while holding inventory exposes the shareholder to the risk of losing money from the inability to sell it at the purchase value in financial distress, holding cash generally does not imply such risk; cash can readily be used to satisfy potential claimants of the firm. Thus, we consider settings $\alpha > \beta$ or $\alpha < \beta$ but always require $\rho \geq \alpha$.

For each of the 9 different combinations for net profit and operating leverage in Table 4.2, we consider variations in the annual cost rate for:

- borrowing β: 2.5% and 10%,
- holding cash α: 1.25% and 2.5%,
- holding inventory ρ: 2.5% and 10%,

all else equal (remember that in the basic setting $\beta = \alpha = \rho = 5\%$). We expect that as the borrowing rate β increases, or opportunity cost rate for cash α increases, the maximum fill rate for a given reverse factoring rate γ increases too. On the other hand, as the opportunity cost rate for inventory ρ increases, the maximum fill rate for a given reverse factoring rate γ should decrease. Indeed, the higher the initial financial cost of an inventory operation, the higher the benefit from reverse factoring. In addition, the higher the cost of keeping inventory, and thus the cost investing in extra service, the lower the benefit from reverse factoring.

In all experiments, we take demand to be log-normally distributed. One period corresponds to one week. Table 4.1 shows annual percentage rates (APR) for financing, which are converted to weekly rates in the experiments. In each simulation run we let the system start with zero cash, zero inventory on hand, zero backorders, and zero receivables for the retailers. We begin to assess performance after a warm-up of 500 periods, which is determined based on Welch's procedure (Welch, 1983; Law and Kelton, 2000). We calculate 95% confidence intervals from 50 independent replications, each with total run-length of 20,000 periods (including warmup). Relative error is approximately 0.5% (Law and Kelton, 2000). After some initial calibration, we were able to locate the optimal policy within four minutes on a personal computer (with 2.53Ghz CPU and 4GB RAM).

4.4 Numerical Results

Here we present and discuss the results of each experiment. Section 4.4.1 covers experiment 1, an analysis on the supplier's inclination to offer a higher service in return for reverse factoring. Section 4.4.2 covers experiment 2, an analysis on the maximum fill rate improvement that a retailer can ask in return for offering reverse factoring to a supplier. Finally, Section 4.4.3 discusses the results from our sensitivity analysis on the parameters.

4.4.1 Supplier's inclination to offer a higher service

All configurations of this experiment support the following assertion:

The change in the optimal base stock as a result of introducing reverse factoring is non-positive, $\Delta I^*_{C \to R} \leq 0$.

Figure 4.2 illustrates this for the base case (see Table 4.1) and the alternative cases with different lead times: $l = 2$ and 4 respectively. The figures illustrate the average financing costs per period (FC) and the average total costs per period (TC) as function of the base stock level for conventional financing with $\alpha = \beta = 5\%$ and for reverse factoring with $\gamma = 3\%$ and 1% respectively. The inventory cost as well as the service level of retailer A as function of the base stock remains equal across the difference financing scenarios. The alternative settings with positive mean demand for retailer B, i.e., $(\mu_A, \mu_B) = (5, 5)$, and higher demand variation for both retailers, i.e., $(c.v._A, c.v._B) = (0.5, 0.5)$ yield similar insights, hence are not presented.

In the base case (Figure 4.2b), the average financing costs decreases asymptotically to zero as the base stock increases. Indeed, recall that a firm that uses a periodic review base stock policy always orders what has been demanded the previous review period. Hence, when the payment term is equal to the lead time and deliveries are paid cash on delivery, the probability that replenishment cost exceed revenues decreases with the service level. When the fill rate is very close to 1, the probability of incurring a cash deficit reduces to almost zero. However, this is not the case when lead time is not equal to the payment term ($l \neq k$) anymore, as can be seen from Figure 4.2a and 4.2c respectively. The average financing cost decreases initially as function of the base stock, but it increases then asymptotically in the base stock. The latter is the result of diminishing marginal impact of service level on the volatility of the cash position of the firm; the expected back orders basically reduce to almost zero as the base stock increases. When $l \neq k$ the timings of the revenue receipts from demand realizations and the costs from replenishment orders are not synchronized, hence even when the service level is high there remains a positive probability of incurring cash deficit from demand uncertainty.

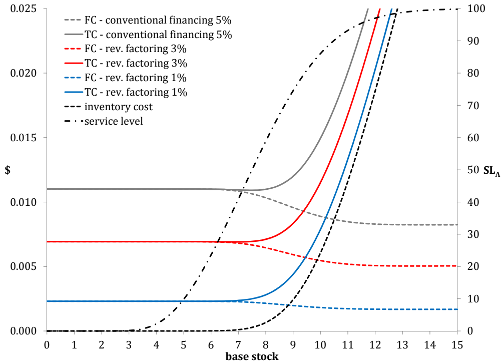
When considering the total average cost per period (TC), we observe that the offering of reverse factoring gives no cause for the supplier to raise its base stock level and thus offer a better service. Moreover, when the optimal fill rate is higher than the minimum fill rate, i.e., $SL_A(I^*) > MSL$ with conventional financing, the introduction of reverse factoring may even lower the fill rate level as the optimal base stock may decrease as a result of reverse factoring. This can be seen in Figure 4.2b. As the average financing cost decreases as a result of reverse factoring, the firm needs to invest less in inventory to hedge against the cost of cash volatility. In the extreme case, when cost of reverse factoring is zero and the firm can finance all of its cash deficits from receivables, the optimal base stock level is the base stock level that satisfies the minimum fill rate requirement. Indeed, the objective function consists of inventory costs only, and inventory cost ultimately increase in the base stock.

4.4.2 The maximum service level improvement that a retailer can ask

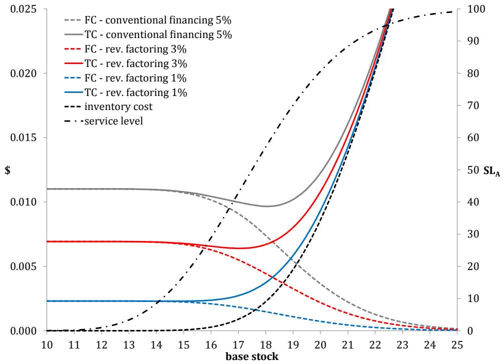
This experiment supports the following assertion:

The maximum fill rate improvement ($SL_A^* - MSL$) that a supplier can grant to retailer A in return for reverse factoring with rate γ:

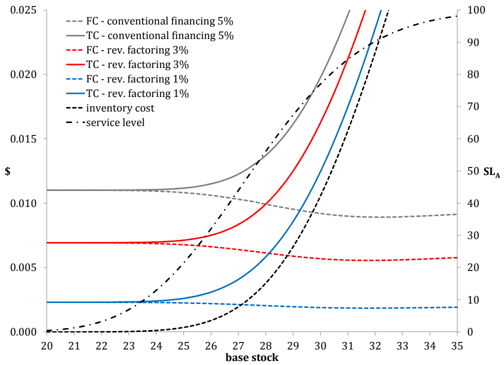
- *decreases in the coefficient of variation of demand of both retailers;*
- *increases in the difference between lead time and payment term, $|l - k|$;*
- *decreases in the relative size of the mean demand of retailer A, $\mu_A : \mu_B$;*
- *decreases in the minimum fill rate, MSL.*



(a) $l = 2$ $(l < k)$



(b) $l = 4$ $(l = k)$



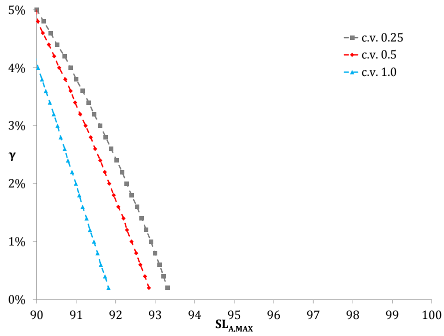
(c) $l = 6$ $(l > k)$

Figure 4.2 *Total average cost (TC), average financing cost (FC), average inventory cost (left axis) and average service level of A (right axis) as function of the base stock level for different levels of the replenishment lead time.*

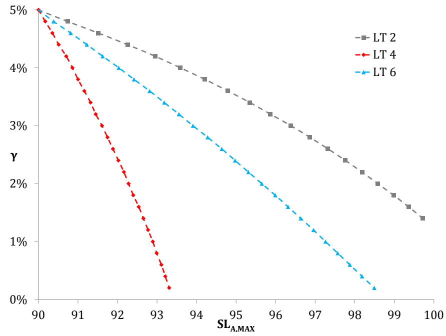
Figure 4.3 illustrates these findings. In Figure 4.3a, we observe that with high demand uncertainty and $MSL = 90\%$, the supplier may even not grant any service level improvement until some threshold reverse factoring rate is reached (e.g., see when $c.v. = 1$). From sample paths we observe that the supplier often needs to factor receivables with late maturities; sometimes it even runs out of receivables, such that the firm resorts to borrowing again. Despite having access to a cheaper rate, the firm appears not to be better off selling all of its receivables before resorting to borrowing when the credit differential, $(\beta - \gamma)$, is low. Indeed, if the firm sells all of its receivables due in the short term, it reaches a state in which no revenues are collected anymore, consequently, it will constantly sell 'expensively priced' receivables to meet deficits that are the result of having no maturing receivables anymore. The use of reverse factoring has thus a compounding effect on the firm's demand for cash; at some point it constantly needs to discount receivables and/or borrow money to compensate for not having incoming cash flows anymore.

That the maximum service level improvement is increasing in the difference between lead time and payment becomes intuitive when considering Figure 4.2 again. Indeed, when $l \neq k$ (i.e., revenues and replenishment cost from demand realizations are not synchronized), the financial saving from reverse factoring is still positive at higher base stock levels, but it diminishes in the base stock when $l = k$. As the marginal cost of higher service exceeds the marginal benefit from cheaper finance, the firm will not invest in higher service anymore. Note also from Figure 4.3b that the maximum service level improvement is higher for lead time $l = 2$ weeks than for lead time $l = 6$ weeks. Indeed, the safety stock requirements for meeting a certain service level in a base stock operation increases with the replenishment lead time (Zipkin, 2000), thus for $l = 2$ the marginal inventory cost of one unit service level improvement is lower than for $l = 6$. In other words, 'higher service' is cheaper for an inventory system with shorter lead time.

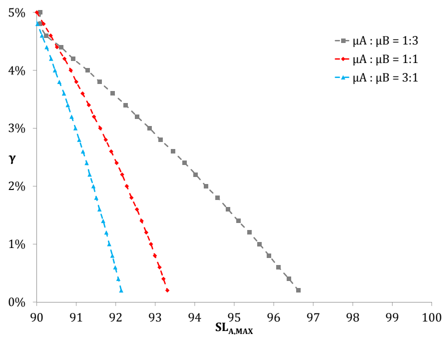
Our observation that the maximum service level improvement is decreasing in the relative mean demand size of retailer A might be surprising initially. Certainly, when we consider that more demands from retailer A gives more opportunities to sell receivables. Our finding becomes intuitive, however, when we consider that giving a higher service level to a smaller customer requires less additional inventory investment than to a large one. In Figure 4.3d it can be seen that the maximum fill rate improvement decreases in the minimum fill rate, MSL. Indeed, as the base stock gets high, the marginal total cost of giving one unit of extra service increases as a function of the initial service level. While inventory cost increases almost linearly in the base stock, the fill rate increases



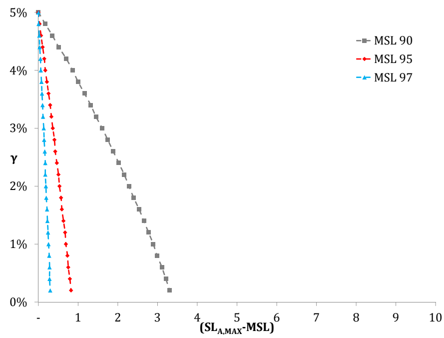
(a) $SL_A^*(\gamma)$ *as function of* γ *for c.v.* $=$ $0.25, 0.5$ *and* 1



(b) $SL_A^*(\gamma)$ *as function of* γ *for* $l = 2, 4$ *and* 6



(c) $SL_A^*(\gamma)$ *as function of* γ *for* $\mu_A : \mu_B = 1:3, 1:1,$ *and* $3:1$



(d) $SL_A^*(\gamma) - MSL$ *as function of* γ *for* $MSL = 0.90, 0.95$ *and* 0.97

Figure 4.3 *Trade off between reverse factoring rate* γ *and maximum fill rate improvement* $SL_A^*(\gamma)$ *for different settings of coefficient of demand variation (c.v.), lead time (l), relative demand size of retailer A* ($\mu_A : mu_B$)*, and minimum service level MSL. Fixed parameters* $p = 10$, $c = 9$, $k = 4$, *and* $\rho = \alpha = \beta = 5\%$

asymptotically to 1 (see Figure 4.2).

4.4.3 *Sensitivity Study*

We explore the sensitivity of our results to the firm's expected net profit margin, the operating leverage and to changes in the financing cost rates. These studies support the following assertion:

The maximum fill rate improvement ($SL_A^* - MSL$) *that a supplier can grant to retailer A in return for reverse factoring with rate* γ*:*

- *increases in the annual borrowing cost rate β;*
- *is non-decreasing in the opportunity cost rate for holding cash α;*
- *decreases in the opportunity cost rate for holding inventory ρ;*
- *decreases in the expected net profit margin ω;*
- *may decrease as well as increase in the operating leverage ω;*

The finding that the maximum fill rate improvement can also be constant in the cash holding cost rate can be explained by considering that the initial optimal cash target level can still be zero after decreasing the rate, i.e., $T_C^*(\alpha) = 0 \wedge T_C^*(\alpha_0) = 0$ with $\alpha > \alpha_0$. Indeed, no financial savings can then be realized on cash holding cost with reverse factoring.

The finding that the maximum fill rate improvement can increase as well as decrease in the operating leverage is initially surprising. Indeed, as the operating leverage increases, the firm's variable unit cost c, and thus also the inventory holding cost, decreases. Investing in extra service becomes thus cheaper. Our finding can be explained, however, by considering how the expected financing cost in conventional financing (C), $E[\alpha(m_t)^+ + \beta(-m_t)^+ \mid Z = Z^*]$ (i.e., the potential financial savings from reverse factoring introduction), behaves as function of the operating leverage ψ. Our simulation shows that the expected financing cost is convex in the operating leverage. This convexity appears to result from two competing effects of operating leverage on the expected cash of the firm, i.e., 'elevated exposure to deficit' and 'increased cash flow stability.' Indeed, while a higher portion of fixed cost elevates the exposure to incurring a cash deficit from uncertain revenues, it also mitigates the uncertainty in the outgoing expenditures. The latter subsequently mitigates the compounding effect of matching two (vs. one) highly uncertain streams of cash flows. Starting from operating $\psi = 0$ (and fixed net profit margin $\omega > 0$), the convexity ultimately causes the maximum fill rate improvement to initially decrease and subsequently increase in the operating leverage.

4.5 Conclusions and Managerial Insights

In this paper, we explore whether improved service and/or preferential treatment from a supplier is a natural outcome of granting cheaper finance to him by means of reverse factoring. We consider a periodic review infinite horizon base stock model serving two customers, one with reverse factoring and one without. We find that a supplier rather collects the financial savings made available from reverse factoring than give a higher service level to the retailer offering reverse factoring. Furthermore, we show that demand uncertainty, difference between the lead time and payment term, the relative size of the retailer's mean demand, and the initial minimum service level all play role in determining the maximum service level improvement that a retailer can ask for offering reverse factoring.

Our results suggest that if buyers aim to pursue a more reliable supply chain from implementing reverse factoring, they ought to consider contractually agreeing a higher service level rather than to leave it up to their supplier to decide whether he should offer a higher service. Furthermore, we show that significant opportunities are offered to firms to negotiate and improve their service levels rather than only payment terms. However, this means that operations and/or supply chain managers should be involved in the implementation of supply chain finance applications between firms.

Our research suggests several directions for future research. As noted in Chapter 3, the policies used in our models are not known to be optimal. This also includes the rationing policy that we have used. Indeed, rationing policies (like inventory management policies) are generally studied in a framework which only considers inventory (holding and backlog) costs or service levels targets, *not* cash management cost. While these allocation policies can thus perform well in minimizing inventory cost, they might function poorly from a cash management perspective. This is likely to be the case when the respective firm faces constraints on borrowing, or even cash retention (e.g., due to other operations within the firm). Research opportunities remain to develop and test rationing policies that account for a firm's cash management and see whether our insights still hold. As mentioned in the introduction of this chapter, a firm that has a borrowing constraint might favor a client that offers reverse factoring, as *more* receivables from the respective client gives more opportunity to avoid hitting the constraint. On the other hand, giving a client too much preference may result in an accumulation of backlog of another client, which eventually can become more costly to eliminate or even result in lost sales. There is thus still work left in making allocation policies 'smarter' concerning such trade-offs.

Chapter 5
Pooling Receivables and its Interaction with Pooling Investment

5.1 Introduction

Pooling of resources has been an important theme in operations management and research. For instance, inventory pooling, an arrangement in which entities share their inventories, is proven to be an effective strategy to reduce inventory levels (Eppen, 1979; Eppen and Schrage, 1981). Capacity pooling, the consolidation of capacity that resides in multiple facilities into a single one, is shown to improve service quality or to allow investment in less capacity for the same service quality (Yu et al., 2008). Manufacturing flexibility, i.e., allowing multiple product configurations to be manufactured in a single plant instead of having each plant dedicated to one configuration, improves a firm's market responsiveness (Jordan and Graves, 1995).

In pooling studies, the operational performance of the resources being pooled is usually the focal point of attention and many works show how resource efficiency and/or effectiveness behaves as a function of demand variability, the number of demands, correlation between demands, or other parameters. While some studies consider how the financial benefits of pooling can be shared between the participants (see, e.g., Karsten et al., 2015), pooling studies usually make no claims how the pooled and/or distributed system is financed, let alone whether the entities should pool their financing options. In a perfect capital market (i.e., a market with no information asymmetries, transaction costs, taxes, or other imperfections) this negligence should not raise any concern, as investment and financing decisions can then indeed be separated (Modigliani and Miller, 1958). In such a market the financing raised for an investment would exactly balance the value of the liability generated, so the decision to separate or pool investments would be irrelevant to how they are financed. Likewise, the decision to separate or pool financing options would be irrelevant to how much to invest in resources. Capital markets do display frictions, however, so firms may face different costs, conditions, and/or constraints for different sources of financing.

Reverse factoring, an arrangement in which a firm can cheaply sell future entitlements (accounts receivables) from its creditworthy buyer, forms the motivation of our study. Reverse factoring is an example of asset-based finance, a form of credit that is conditioned primarily on the risk of the specific assets being pledged or sold by the credit applicant rather than the overall creditworthiness of the applicant (Klapper, 2006). By focusing on a specific (set of) assets, financial intermediaries can resolve the opacity problems associated with credit types where the repayment ultimately depends on the financial strength of the applicant (Berger and Udell, 2006). Two particular features of asset-based finance allow credit applicants to obtain a better price for credit than in conventional forms of credit: (1) the possibility to efficiently transfer relevant information to assess the credit risk of the asset being pledged or sold (Tanrisever et al., 2012; Birge, 2014) and/or (2) the possibility of facilitating 'bankruptcy-remoteness' for a financial intermediary (Ayotte and Gaon, 2011). As far as the first feature is concerned, the intermediary's credit assessment activity in reverse factoring requires relatively low effort, since the creditworthy buyer gives an explicit guarantee of payment, and financial reports and credit rating for the buyer are available. Bankruptcy remoteness entails that collateralized or sold assets are isolated from a firm's estate if it were to file for bankruptcy. This protects existing creditors from dilution that could occur if the firm were to use existing assets to raise new funds in a potential reorganization. Indeed, contrary to the common assumption that financial intermediaries can just claim the collateralized assets from a bankrupt debtor, they can actually not do so unless these assets are declared bankruptcy-remote as part of a financial transaction.

In this paper, we explore how receivables pooling and investment pooling influence a firm's investment decision. Specifically, we consider how the 'indivisibility' of receivables in reverse factoring transactions restricts the full exploitation of cheaper financing by the owner of the receivable. Indeed, if a firm decides to sell its receivable in reverse factoring, it typically must sell the *whole* receivable (that is, the firm is not offered the option to sell fractions of its receivable). We illustrate how this friction provides a motive for the firm to pool its receivables with other firms. While indivisibility is not an issue for a firm for which the investment level and resulting capital need exceeds the value of a receivable, it can become costly when the value of the receivable exceeds the investment level. The firm is then forced to apply for more funding than it needs. If the receivable is too big, factoring the receivable may not be beneficial at all. A firm would benefit from a bulky factoring transaction as long as the multiplier of the amount of initial funding is no greater than the inverse of the multiplier of the initial financing rate. For instance, if the financing rate were halved from factoring, the firm still benefits from it even if the firm is forced to apply for twice the amount of funding it actually needs. While cash excesses from an indivisible transaction can itself generate a return (e.g., in an savings account or stocks and bonds), this may not compensate for the cost of financing.

There are many plausible arguments to explain why factoring companies do not offer perfect divisible transactions. Transaction costs, i.e., the costs associated with making an exchange, are pertinent. Klein (1973) shows that it is not optimal for financial intermediaries to offer perfect divisible financing due to transaction costs. Literature suggests that the actors of a financing transaction (i.e., creditors and/or debtors,) typically respond in similar fashion to transaction costs as the intermediary; they 'batch' their financial transactions. For instance, Lazimy and Levy (1983) demonstrate that transaction costs form one of the reasons why investors hold less diversified portfolios, i.e., they invest more in fewer types of assets. Bazdresch (2013) empirically demonstrates that the financial activity of firms is 'lumpy,' and even more lumpy than their investment activity. He attributes this to the transaction costs involved in making financial adjustments. In practice, smaller firms, i.e., firms that can sell only a small amount of high-quality receivables, are often excluded from participation by providers of reverse factoring, due to the relative cost of 'onboarding' them (Milne, 2009; Hurtrez and Salvadori, 2010). Financial indivisibility and/or batching of financial activities is thus an inevitable feature of a capital market with transaction costs.

Indivisibility causes inefficiencies for a firm in the sense that it cannot always exploit the full potential of opportunities for cheaper capital. In the context of reverse factoring, a firm may use the system less with indivisible receivables than with divisible receivables. We explore a measure that mitigates the adverse effect of the indivisibility of receivables in reverse factoring: pooling receivables among firms. This is not an unrealistic scenario. While perhaps receivables pooling can be more easily implemented among, e.g., divisions of a large corporation, technology is advancing such that the barriers and cost to receivables pooling between independent firms is becoming lower. For instance, web based market places, such as Receivables Exchange and MarketInvoice[1], are appearing in which companies can easily sell receivables to other companies and institutions. Furthermore, 'peer-to-peer finance', i.e., technologies which allow companies to lend money to each other via an online platform, are appearing and growing lately (Dunkley, 2014; Jenkins and Alloway, 2015). The idea behind those technologies is to bypass banks by directly linking companies who have enough money to invest to companies that need money.

By pooling receivables firms can: (i) improve transactional efficiency, by better utilizing the opportunity of cheaper capital, and (ii) improve liquidity, by allowing the value of non-cash assets to finance lucrative investment. While we expect firms to be able to pool high-quality receivables independently of any other arrangement, this choice may have additional value in a setting where firms already consider pooling in operational resources, such as labor, manufacturing or equipment, or R&D, as the examples mentioned at the outset. However, it is not clear how receivables pooling affects investment decision-making. Moreover, the interaction between the two pooling initiatives is unknown. Can firms expect a synergy benefit to arise from doing both types of pooling? Or, is the benefit of an investment pooling initiative dependent on implementing receivables pooling (and vice versa)? If so, under which conditions? These type questions are important to managers that are involved in evaluating one or both of the pooling concepts, e.g., as part of the consideration of engaging in a merger or alliance with another firm. Consequently, we explore the following research questions:

Research question 5.1. *If firms can invest to increase margin and factor receivables to reduce capital cost, what is the impact of pooling investment, pooling receivables, and pooling both on optimal investment?*

Research question 5.2. *If firms can pool investment in addition to pooling receivables, is the benefit from pooling both super-additive or sub-additive?*

We employ a model in which two identical make-to-order firms serve independent random orders from creditworthy customers. Before the orders are revealed, firms can invest in resources that allows them to reduce their unit production cost. The unit production cost diminishes with cumulative investment, and we introduce a parameter that specifies the amount of investment needed to reach an arbitrary level of unit production cost. This parameter proxies the technology maturity level of the firm's current assets relative to what is available

[1]See respectively www.recx.com and www.marketinvoice.com.

in the market. When the technology maturity level is low (high), the amount of investment needed to reach infinitesimal unit production cost is high (low). Investment must be initially financed from costly debt, but fulfilment of the order yields a reverse factoring option which allows to the firm repay (some of) the debt with cheaper money. Due to the indivisibility of the receivable, factoring only occurs to the extent that it reduces financing costs. The firms' goal is to maximize expected profit. Firms can operate in four modes: operate independently, pool receivables, pool investment, or pool both. By means of numerical experimentation, we explore the optimal investment level, expected profit, and expected return on investment (ROI) of the firms in each scenario.

Our work results in the following main conclusions. First, the technology maturity level threshold for which optimal investment becomes positive is larger when firms pool receivables, much larger when firms pool investment, and is the largest when firms pool both. Thus, while both receivables pooling and investment pooling extend the settings in which firms benefit from cost-reducing investment, their combination strengthens this ability further. Second, optimal investment may increase as well as decrease as a result of pooling receivables. As receivables allow cheaper financing, the observation that optimal investment can decrease as a result of pooling receivables might be initially surprising. The intuition behind this finding is that the marginal capital cost may exceed the marginal benefit from investment when firms pool receivables. Third, while the ROI is highest when firms pool both their investment and receivables, we find that the benefit from the two types of pooling can be either super-additive or sub-additive. When the technology maturity level of the firm's assets is low (high), and the firm can thus considerably (only marginally) decrease unit production cost through investment, the benefit from pooling both is super-additive (sub-additive). Overall, our results suggest that simultaneous evaluation of the pooling concepts can improve managerial decision making.

The remainder of this chapter is organized as follows. In Section 5.2, we review literature relevant to our problem. Subsequently, we provide the models of the pooling scenarios in Section 5.3. In Section 5.4, we discuss some analytical properties of our models and the results of our numerical experiments. Lastly, we provide our conclusions in Section 5.5.

5.2 *Literature*

Our work is related to three literature streams: asset-based finance, demand pooling, and the interface of operations and finance.

Asset-based finance, such factoring, reverse factoring, and asset-based lending, has become increasingly popular in industry (Ayotte and Gaon, 2011; Buzacott and Zhang, 2004). Despite the increased popularity, only a few studies have quantitatively analyzed asset-based finance techniques. Sopranzetti (1998) analyzes the impact of moral hazard on the conditions of factoring and supports this with empirical data. In another study, Sopranzetti (1999) shows how factoring can mitigate Myers' under-investment problem, an agency conflict that causes a firm to forego investment in positive net present value projects (Myers, 1977). Some studies have explored asset based finance in production or inventory settings. Buzacott and Zhang (2004) show in a multi-period setting how asset based financing can be incorporated into production decisions. Yang and Birge (2011) show that factoring can be perceived as a diversification mechanism for financing inventory. Ayotte and Gaon (2011) show how asset based finance allows firms to make more efficient investment decisions in bankruptcy. Alan and Gaur (2012) consider asset-based lending in a newsvendor setting and explore its implications on inventory investment, bankruptcy, and capital structure. Tanrisever et al. (2012) study the value of reverse factoring in a production setting. van der Vliet et al. (2015) study the trade-off between cheaper finance and trade credit extension when a firm is offered a reverse factoring in an inventory setting. However, most of the works that consider factoring, reverse factoring, or other asset-based finance techniques assume that financing transactions are perfectly divisible. We contribute to this literature by explicitly considering the impact of indivisibility.

The concept of pooling of common resources with uncertain demands, 'demand pooling,' originates from the work of Eppen (1979), that illustrates the benefit of pooling inventory locations. Since then, demand pooling has been studied for a wide variety of applications and model variations. Yang and Schrage (2009) mention at least four applications: product substitution, transhipment, postponement, and common components. Van Mieghem (1998) shows how demand pooling is applied in the context of flexible manufacturing capacity. The concept of demand pooling has also been recognized in queueing theory; queueing models are often used to illustrate the benefit of pooling manufacturing or service capacity (Smith and Whitt, 1981; Iyer and Jain, 2004). While a lot of knowledge has been documented on demand pooling, studies that consider interdependencies between different applications of pooling are rare. The work of Benjaafar et al. (2005) appears to be one of the few; they explore interdependency between capacity and inventory pooling. They find that while the relative benefit of capacity pooling increases with utilization, the relative benefit of inventory pooling decreases or remains invariant. We complement this research stream by considering the interaction between pooling financial assets, i.e., receivables, and investment in operational assets.

Literature on the interface of operations and finance studies the interaction between decisions from the respective disciplines and the benefit of seeking joint optima. The existence and relevance of these interactions are mostly motivated from capital market frictions, such as information asymmetries, agency conflicts, or transactions costs (Birge, 2014). Studies that illustrate this interaction appeared first in corporate finance journals. For instance, Hite (1977) shows how taxes induce an interaction between the optimal level of output, labor, capital, and leverage of the firm. Dotan and Ravid (1985) illustrate how taxes induce an interaction between the capacity investment and leverage. Some works even explore how to manage operations-finance interactions on the operational level. For instance, Baker and Damon (1977) provide a model for simultaneously planning the production, inventory, workforce, and working capital of a firm. More recently, operations researchers have begun to explore interactions in a framework that leverages existing knowledge from operations research (Birge et al., 2007; Birge, 2014). For instance, the interdependence between financing and ordering decisions has received much interest (Buzacott and Zhang, 2004; Xu and Birge, 2008; Dada and Hu, 2008; Kouvelis and Zhao, 2011). So far, most studies consider capital market frictions that cause investors or lenders to either introduce restrictions on the amount of financing accessible to a firm, i.e., credit limitations (see, e.g., Buzacott and Zhang, 2004; Kouvelis and Zhao, 2011), or require a higher rate than the rate that is offered in a frictionless capital market (see, e.g., Boyabatli and Toktay, 2011; Tanrisever et al., 2012). We extend this research by considering a friction that restricts ability to perfectly use a cheaper source of financing and a strategy to mitigate its adverse impact on investment: indivisibility and pooling of financing options respectively.

5.3 The Model

In this Section, we present the four models to evaluate the impact of receivables pooling on investment and the interaction of investment and receivables pooling. First, in Section 5.3.1, we discuss the preliminaries and assumptions concerning the pooling participants. Consequently, in Section 5.3.2, we provide the models for independent investment (I) and investment pooling (II) without receivables pooling. Finally, in Section 5.3.3, we provide the models for receivables pooling without investment pooling (III) and receivables pooling with investment pooling (IV) respectively.

5.3.1 Preliminaries and Assumptions

We consider a one-period model in which two identical firms with index $i \in \{1,2\}$ sell goods to creditworthy customers with i.i.d. random demands, $X_i \in \mathbb{R}^+$. Demand has density $f(x)$ and distribution $F(x)$ with mean μ and variance σ^2. Let $Y = X_1 + X_2$ be the joint demand level. The firms have ample capacity and negligible production and delivery lead time. Each unit can be initially produced for u_0 per unit and sold for p, with $u_0 \leq p$. Due to a credit period granted to the customer the firms must wait until the end of the period to collect their revenues. Likewise, payment obligations resulting from production activities are due at the end of the period.

By means of an additional investment $s \geq 0$ in assets, the firms can reduce their unit production cost to level $u(s) < u_0$ with $u(0) \equiv u_0$. However, the investment must be made before their demand arrives. The investment can be thought of an additional R&D facility, machinery, tool or software application; such assets usually require some time to be installed in the firm. In accordance with classic economic models on cost-reducing investment, we assume that unit production cost decreases with investment, but investment has diminishing returns, thus $u''(s) > 0$ (Flaherty, 1980; Brander and Spencer, 1983). Specifically, we assume that the relation between $u(s)$ and s is as follows:

$$u(s) = u_0 e^{-\frac{1}{\beta}s} \text{ with } \beta \in (0, \infty) \tag{5.1}$$

The 'cost reduction factor' β determines the amount of investment s required to reach an arbitrary level unit production cost ϵ, where $0 < \epsilon < u_0$. The cost reduction factor can be thought of as a proxy for the relative technology maturity of the firm's existing assets. As $\beta \uparrow$, the required investment to reach ϵ becomes larger; this represents the case in which a firm's *internal* technology is becoming relatively more advanced. To justify additional investment in cost-reducing assets the firm would need to expect larger demand. Conversely, as $\beta \downarrow$, the required investment to reach ϵ becomes smaller; this represents the case in which *external* technology is maturing such that units can be be produced at a lower cost and that using this technology requires less additional investment in assets. In this case, additional investment becomes increasingly justified.

The investment s creates a liability which is initially funded from a loan with rate r% per money unit per year. We assume that the loan does not significantly alter the firm's leverage position, thus r is invariant with s. While the loan is due at the end of the period, it has a flexible payment scheme, and interest is only incurred on the outstanding amount. Fulfilment of the order yields a receivable that can be factored at rate v% per money unit per year through a reverse factoring arrangement. The credit risk of the customer is lower than that of the firm, so $v < r$. The interest charge on the factoring transaction is subtracted from the cash disbursed to the firm. The remaining money, $xp(1-v)$, can be used instantly to reduce or even nullify the outstanding loan. The firm cannot, however, divide the receivable when exercising its factoring option, so it pays interest based on the full face value of the receivable, xp.

There is a demand level $x_k(s) = s/p(1-v)$ that allows the firm to nullify the loan from the cash payment from the factoring transaction. The demand can be arbitrarily large, however, and so can the receivable. At the demand level $x_l(s) = sr/pv$, factoring offers no benefit; the firm is thus better off borrowing from the more expensive capital source. Based on these two thresholds, we define the following parameter:

Definition 5.1. $q = \frac{r(1-v)}{v}$ *such that* $x_l = qx_k$ *and* $v = \frac{r}{q+r}$.

The 'interest rate ratio' q allows us to describe different cases concerning the relative position of x_k and x_l, and thus the demand region in which indivisibility is a problem. If $q = 1$ ($x_k = x_l$), the firm is indifferent between borrowing and factoring when $x \leq x_l$ and prefers borrowing when $x > x_l$. Indivisibility of receivables is thus not an issue. If $q < 1$ or $q > 1$, the firm prefers factoring when $x < x_l$, and prefers borrowing when $x > x_l$. If $q < 1$, the firm factors a too large receivable when $x \in (x_l, x_k]$, and if $q > 1$, the firm factors a too large receivable when $x \in (x_k, x_l]$. As in reverse factoring programs the factoring rate offered to SME's is often significantly lower than the rate they normally pay for financing (i.e., $r >> v$), we make the following assumption for the remainder of our study:

Assumption 5.1. $q > 1$ such that when

(1) $x \in [0, x_k)$ the firm factors but still has some portion of loan outstanding;

(2) $x \in [x_k, x_l)$ the firm factors and ends up with residual cash;

(3) $x \in [x_l, \infty)$ the firm does not factor, but leaves the full loan outstanding.

At the end of the period the firms salvage their investments together with their existing assets, and receive a cash flow $s(1-\gamma)$ where $0 < \gamma < 1$ is the salvage cost. We assume the return of re-investing the residual cash from a factoring transaction is zero. While introducing a positive return allows the firm to recoup the cost of being offered an indivisible transaction, it will not mitigate the burden completely, unless the return of re-investment is equal to the factoring rate v. A positive return could impact the factoring and thus the investment policy as it changes threshold values x_k and x_l, but it does not affect our qualitative insights on the impact of pooling concepts on investment and on the interaction between pooling concepts. We also assume that the value of the firm's existing assets is sufficiently large to capture losses that might occur from the additional investment made at the start of the period, which implies that receivables pooling entails no financial risk to the firms. While the model can be adapted to allow for a default risk, this forces much of our focus on managing the financial risk from pooling receivables, rather than on the interaction between investment and receivables pooling.

Both firms aim to maximize their expected profit. The expected profit function for each firm is as follows:

$$\pi(s) := (p - u(s))\mu_X - (E[c(s)] + s\gamma) \tag{5.2}$$

with

$$E[c(s)] = \int_0^{x_k(s)} (vpx + sr - rp(1-v)x)f(x)\mathrm{d}x + \int_{x_k(s)}^{x_l(s)} (vpx)f(x)\mathrm{d}x + \int_{x_l(s)}^{\infty} (sr)f(x)\mathrm{d}x. \tag{5.3}$$

In (5.2), the first term represents the expected profit margin and the second term the expected financing and salvage cost of the investment decision. The expected financing cost, $E[c(s)]$, is represented by (5.3); it consists of three integrals that reflect the financing policy and costs for the different demand regions.

The firm's objective is thus formulated as follows:

$$\max_s \pi(s). \tag{5.4}$$

Recall that we only consider positive investment levels, thus the firm's objective (5.4) is in fact constrained by $s \geq 0$. We say that an investment level s^* is optimal if $\pi(s^*) \geq \pi(s)$ for all $s \geq 0$. Furthermore, $\pi^* \equiv \max_s \pi(s)$.

Based on the foregoing considerations, we have four scenarios on how the two firms can operate. They can:

(I) invest and finance independently;

(II) pool investment;

(III) pool receivables;

(IV) or pool both investment and receivables.

In the remainder of this study, $\pi_j(s)$ denotes denotes the profit of each firm in operating scenario $j \in \{I, II, III, IV\}$, in which j is the scenario that corresponds to the same Roman numeral displayed above. Investment pooling allows each firm to benefit from a unit cost reduction that corresponds to the amount of resources that is invested jointly while only needing to finance its individual share. Receivables pooling allows the firms to benefit from a temporary exchange of cheap money between the firms. Specifically, receivables pooling improves:

(i) *transactional efficiency*, by reducing excessive factoring as a result of indivisibility;

(ii) *liquidity*, by allowing the value of non-cash assets to finance lucrative investment across the firms.

Note that in investment pooling (II) and in receivables pooling (III), the firms still respectively finance and invest in their operations independently. As the firms are identical and investments are made simultaneously, they pool on an equal sharing basis. In investment pooling, the total investment cost is shared equally. Likewise, in receivables pooling, the total financing cost is shared equally. Let S be the sum of investments made by the two firms. Due to the symmetry, we have $S = 2s$ in each operating scenario. Similarly, we say that a total investment level S^* is optimal in operating scenario j if $\pi_j(S^*) \geq \pi_j(S)$ for all $S \geq 0$.

5.3.2 *Independent Investment (I) and Investment Pooling (II) without receivables pooling*

If the firms invest and finance independently, the expected profit of each firm as function of the total investment level S is obtained by substituting $S/2$ for s in (5.2):

$$\pi_{\text{I}}(S) := (p - u(S/2))\mu_X - (E[c(S/2)] + (S/2)\gamma) \tag{5.5}$$

If the firms pool investment, they enjoy a cost reduction that corresponds to their joint investment level. They still incur financing costs and salvage costs independently. Consequently, only the first term (gross profit) in (5.5) changes, in order to give the expected profit of each firm as a function of total investment S:

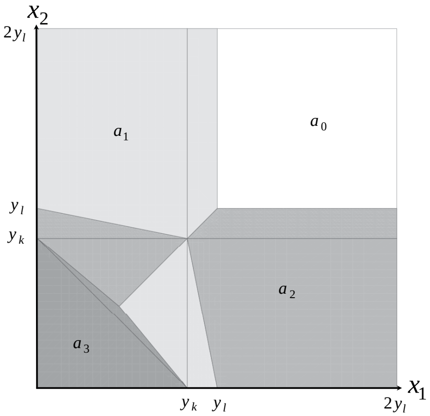
$$\pi_{\text{II}}(S) = (p - u(S))\mu_X - (E[c(S/2)] + (S/2)\gamma) \tag{5.6}$$

5.3.3 *Receivables Pooling without Investment Pooling (III) and Receivables Pooling with Investment Pooling (IV)*

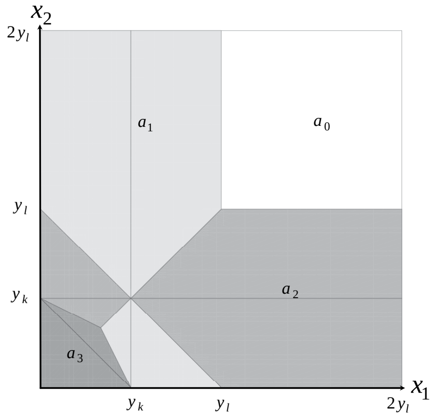
If firms pool receivables, they allow their receivables to be utilized across their investments. Rather than factoring to minimize the financing cost of their individual investments, firms centralize the factoring decision and aim to minimize the financing cost of their total investment.

Let $E[c_P(S)]$ be the combined expected financing cost for firms that pool receivables. To determine $E[c_P(S)]$ we need to know the factoring decision that leads to the smallest financing cost for every demand pair (x_1, x_2) given that firms invest S. Note that the expected financing cost is thus independent of whether firms pool investment or not. Let $\boldsymbol{a_j} := (a_{1j}, a_{2j})$ be factoring decision j, a vector of binary numbers which indicates whether the receivables from demand realizations x_1 and x_2 respectively are factored (1: yes and 0: no). We index the factoring decision by means of the set $A = \{\boldsymbol{a_0} : (0,0), \boldsymbol{a_1} : (1,0), \boldsymbol{a_2} : (0,1), \boldsymbol{a_3} : (1,1)\}$. Let $c_P(S, \boldsymbol{a_j}, x_1, x_2)$ be the financing cost for decision j when demand is $X_1 = x_1$ and $X_2 = x_2$. A decision $\boldsymbol{a}^*$ is optimal for demand pair (x_1, x_2) if $c_P(S, \boldsymbol{a^*}, x_1, x_2) \leq c_P(S, \boldsymbol{a_j}, x_1, x_2)$ for every $\boldsymbol{a_j} \in A$. By identifying $\boldsymbol{a}^*$ for each demand pair (x_1, x_2) and investment level S, we can determine $c_P(S, \boldsymbol{a}^*, x_1, x_2)$. We will refer to $c_P(S, \boldsymbol{a}^*, x_1, x_2)$ simply as $c_P(S)$ hereafter. Based on an extensive analysis on the optimal factoring decision $\boldsymbol{a}^*$ and the corresponding financing cost $c_P(S)$ for each possible demand pair (x_1, x_2), we were able to formulate $E[c_P(S)]$ as a sum of integrals. The analysis and formulation of $E[c_P(S)]$ is included in Appendix A. Next, we will describe the main insights from the analysis of the optimal factoring decision.

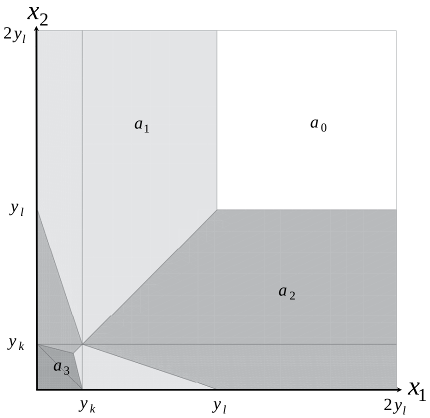
Figure 5.1 shows the optimal factoring decision $\boldsymbol{a}^*$ as a function of x_1 and x_2 for different values of the interest rate ratio q. Analogous to a setting in which only one firm is considered, there exists a demand level, $y_k(S) = S/p(1-v)$, for x_1, x_2, or y, at which the cash flow from exercising the factoring option is equal to the total investment level S. In addition, there exists a demand level, $y_l(S) = Sr/pv$, for x_1, x_2, or y, at which factoring the corresponding receivables offers no financial benefit anymore. We can observe from Figure 5.1 that while proportions change, the shape of the regions in which a particular factoring



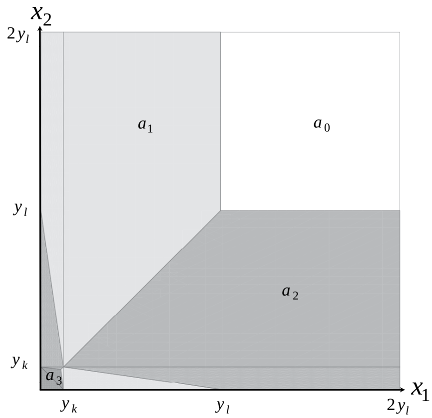
(a) $q = 1.2$



(b) $q = 2$



(c) $q = 4$



(d) $q = 8$

Figure 5.1 *Optimal factoring decision* $\boldsymbol{a}^*$ *as function of* x_1 *and* x_2 *for different settings of* q

decision is optimal remain qualitatively the same for different settings of q. If the demands of both firms are larger than y_l, the optimal decision is to factor none of the receivables. If $x_1 + x_2 \leq y_k$, the best decision is to factor both receivables. If both demands are smaller than y_k but the sum is larger than y_k, there exists a triangular region in which factoring both receivables is still the best decision. If one demand realization becomes too large it is optimal to factor just one receivable. If both demands are smaller than y_k it is optimal to select the largest receivable; if both demands are larger y_k than but smaller than y_l, it is optimal to select the smallest receivable. If one demand is smaller than y_k, while the other is larger than y_k but smaller than y_l, a trade-off arises between the additional borrowing cost when choosing for the smaller receivable and the excess factoring cost when choosing for the larger one. The optimal factoring decision then depends on q.

In summary, if firms pool receivables but invest independently, we obtain the expected profit for each firm as a function of total investment S by amending (5.5) as follows:

$$\pi_{\text{III}}(S) := (p - u(S/2))\mu_x - \left(\frac{E[c_P(S)]}{2} + (S/2)\gamma\right) \tag{5.7}$$

Note that besides utilizing function $c_P(.)$ instead of $c(.)$, the expected financing costs in are determined by the total investment level and subsequently divided between the firms, instead of resulting in the first place from the investment level of each firm.

If the firms pool both their investment and their receivables, the expected profit as function of total investment S is obtained by applying the same change to (5.6), giving:

$$\pi_{\text{IV}}(S) := (p - u(S))\mu_x - \left(\frac{E[c_P(S)]}{2} + (S/2)\gamma\right) \tag{5.8}$$

5.4 *Analysis and Numerical Results*

Here we present an analysis on the optimal investment and the results of the numerical studies for answering our research questions. In Section 5.4.1, we first some present properties of the firm's optimal investment (s^*) that help us understand the impact of pooling on optimal investment and allow us to efficiently search for the optimal investment level within a closed interval. Subsequently, in Section 5.4.2 we discuss the results of the numerical experiments.

5.4.1 *Properties of Investment*

From (5.2) we see that any closed form expression for the optimal investment level of a single firm will depend on the assumption made for the demand distribution. The same is of course also true for the four pooling scenarios, where the each firm's profit is described by the appropriate choice from (5.5) - (5.8). The expected financing cost, $E[c(s)]$ or $E[c_P(s)]$, depends both on the investment level s and the the demand density function, $f(x)$. However, by considering the single firm profit function (5.2) for $q = 1$ we are able to reveal some general properties of a firm's optimal investment level. These properties help us better understand the impact of investment and receivables pooling on investment and profit. Moreover, by considering the derivative of the profit function (5.2) in the limiting cases $s \to \infty$ and $\beta \to \infty$ respectively, we know in which interval the optimal investment level s^* is located. This allows us to search for s^* efficiently.

Property 1. *Let $q = 1$. Then optimal investment* $s^* = \begin{cases} -\beta \ln \left[\frac{\beta(r+\gamma)}{u_0 \mu_X} \right] & : \beta \leq \frac{u_0 \mu_X}{r+\gamma} \\ 0 & : \beta > \frac{u_0 \mu_X}{r+\gamma} \end{cases}$

Proof: If $q = 1$ the expected financing cost increases linearly in the investment level, i.e., $E[c(s)] = sr$. Indeed, note from (5.3) that the second integral collapses. The integrand of the first integral becomes sr. As $q = 1 \Rightarrow x_k(s) = x_l(s)$ the first and and third integral can be merged into one.

The expected profit function (5.2) therefore becomes:

$$\pi(s) = (p - u_0 e^{-\frac{1}{\beta}s})\mu_X - s(\gamma + r). \tag{5.9}$$

Taking the second derivative yields:

$$\frac{d^2 \pi(s)}{ds^2} = -\frac{u_0 e^{-\frac{s}{\beta}} \mu_X}{\beta^2}. \tag{5.10}$$

As the value of the second derivative is strictly negative, the firm's expected profit $\pi(s)$ is concave in s and thus we can derive an unique optimal investment s^* by finding the root of the first derivative:

$$\frac{d\pi(s)}{ds} = \frac{u_0 e^{-\frac{s}{\beta}} \mu_X}{\beta} - (r + \gamma). \tag{5.11}$$

Setting this equal to zero yields:

$$s^* = -\beta \ln \left[\frac{\beta(r+\gamma)}{u_0 \mu_X} \right]. \tag{5.12}$$

As $\frac{\beta(r+\gamma)}{u_0 \mu_X} \leq 1 \Leftrightarrow s^* \geq 0$, we have $\beta \leq \frac{u_0 \mu_X}{(r+\gamma)} \Leftrightarrow s^* \geq 0$. As we do not consider negative investment levels, we have $\beta > \frac{u_0 \mu_X}{(r+\gamma)} \Rightarrow s^* = 0$. □

Note that thus the optimal investment s^* increases as a function of initial unit production cost, u_0, and mean demand, μ_X, but decreases as a function of the borrowing rate, r, and the depreciation rate, γ. Intuitively, this makes sense. The greater initial cost and/or the expected demand, the greater the marginal benefit of investment. The greater the borrowing and/or depreciation rate, the greater the marginal cost of investment.

In contrast, the impact of β on optimal investment is not unidirectional. Taking the second derivative of (5.12) with respect to β yields:

$$\frac{d^2 s^*}{d\beta^2} = \begin{cases} -1/\beta & : \beta \leq \frac{u_0 \mu_X}{r+\gamma} \\ 0 & : \beta > \frac{u_0 \mu_X}{r+\gamma} \end{cases} \tag{5.13}$$

The optimal investment level is thus concave in β for $\beta \leq \frac{u_0 \mu_X}{r+\gamma}$. The intuition behind this is as follows. If β increases, the firm needs to invest more for the same level of unit production cost. The firm will invest more as long as the marginal benefit of regaining profit margin offsets the marginal cost of investment. The marginal profit increase from investment diminishes in β while the marginal (financing and salvage) cost is constant, hence, the marginal cost will exceed the marginal benefit some level of β. Beyond this level, the firm's optimal investment level decreases as β increases until the point is reached that additional investment will not generate additional profit anymore. For $q = 1$, this point is reached when $\beta = \frac{u_0 \mu_X}{r+\gamma}$.

While we do not have an explicit expression for the optimal investment for $q > 1$, we can derive the following general properties:

Property 2. *Let $q > 1$. There exists*

(a) an investment level $s_M \in [0, \infty)$ such that $\pi'(s) < 0$ when $s > s_M$.

(b) a cost reduction factor threshold $\beta_T \in (0, \infty)$ such that $s^* = 0$ when $\beta > \beta_T$;

PROOF: (a) Let $s_M \equiv \max\{s : \pi'(s) \geq 0\}$. Taking the derivative of the financing cost function (5.3) with respect to s yields:

$$\frac{dE[c(s)]}{ds} = r - r \int_{x_k(s)}^{x_l(s)} f(x)\mathrm{d}x = r(1 - F[x_l(s)] + F[x_k(s)]) \tag{5.14}$$

Let $\alpha(s) = (1 - F[x_l(s)] + F[x_k(s)])$. The first derivative of the expected profit function (5.2) with respect to s is then:

$$\frac{d\pi(s)}{ds} = \frac{u_0 \mu_x}{\beta} e^{-\frac{1}{\beta}s} - (\alpha(s) r + \gamma) \tag{5.15}$$

Note that $s \to \infty \Rightarrow \alpha(s) \to 1$, thus taking the limit $s \to \infty$ of (5.15) yields:

$$\lim_{s\to\infty} \left[\frac{d\pi(s)}{ds} \right] = -(r+\gamma). \tag{5.16}$$

We have two possible cases: (i) $\pi'(0) > 0$, or, (ii) $\pi'(0) \leq 0$. If $\pi'(0) > 0$, the optimal investment should be finite and positive, $\pi'(0) > 0 \Rightarrow 0 < s_M < \infty$. If $\pi'(0) \leq 0$, the optimal investment level can be *zero* or finite positive, $\pi'(0) < 0 \Rightarrow 0 \leq s_M < \infty$.

(b) Let $\beta_T \equiv \max\{\beta : s^* > 0\}$. Note from Property 1 that $q = 1 \Rightarrow \beta_T = \lim\limits_{\epsilon\to 0} \frac{u_0\mu_X}{r+\gamma} - \epsilon$ with $\epsilon > 0$, which implies $0 < \beta_T < \infty$. While we do not a explicit expression for s^* for the case $q > 1$, taking the limit $\beta \to \infty$ of (5.15) yields:

$$\lim_{\beta\to\infty} \left[\frac{d\pi(s)}{ds} \right] = \lim_{\beta\to\infty} \left[\frac{u_0\mu_x}{\beta} e^{-\frac{1}{\beta}s} - (\alpha(s)r+\gamma) \right] = -(\alpha(s)r+\gamma).$$

Note that by assumption 1 we have $0 \leq \alpha(s) < 1$, indeed, $q > 1 \Rightarrow F[x_l(s)] > F[x_k(s)]$ with $F[x_l(s)], F[x_k(s)] \in [0,1]$. Thus, $\lim\limits_{\beta\to\infty} \pi'(\beta) < 0$. We have $\lim\limits_{\beta\to 0} \pi'(\beta) > 0 \wedge \lim\limits_{\beta\to\infty} \pi'(\beta) < 0 \Rightarrow 0 < \beta_T < \infty$ □

Thus, regardless of the value of the cost reduction factor β, the expected marginal cost will exceed the expected marginal benefit of investment at some finite investment level and beyond. As investing more beyond this investment level only generates an additional expected loss, we can find the optimal investment s^* by searching within a closed interval $[0, s_M]$.

5.4.2 *Numerical Analysis*

Here, we present the results of the numerical studies for answering our research questions. Based on an extensive numerical exploration, we found functions (3)-(6) to be either unimodal or bimodal for a wide range of settings. Thus, we may need to evaluate multiple roots to find the optimal investment level S^*. As we have a bound, s_M, beyond which the marginal cost always exceeds the marginal benefit of investment, we can search for the roots within the interval $[0, s_M]$. An estimate of s_M, i.e., $\hat{s}_M$, can be obtained relatively quickly with Algorithm 3. In this algorithm, n is a tuning parameter that allows to adjust the precision of the search for s_M.

After locating $\hat{s}_M$, we use Newton's method for finding the roots within the interval $[0, s_M]$ in our numerical studies. To ensure that we indeed find the global

Algorithm 3: Algorithm for finding s_M

Choose an initial $\hat{s}_M$ such that $\pi(\hat{s}_M) << 0 \wedge \pi'(\hat{s}_M) \approx -(r+\gamma)$;
Choose $v = \hat{s}_M / n$ with $n \in \mathbb{N}^+$;
while $(\pi'(\hat{s}_M) < 0 \vee \hat{s}_M \neq 0)$ **do**
  $\hat{s}_M \leftarrow \hat{s}_M - v$;
  Evaluate $\pi'(\hat{s}_M)$;
return $\hat{s}_M + v$

maximum, we initiate multiple searches from four different starting points, i.e., from $\{\delta \hat{s}_M : \delta = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$, and subsequently evaluate the profit of the roots that are found. Finally, return the root that yields the greatest profit. This is the optimal investment level S^*.

In our initial exploration, we experimented with different density functions for demand and different settings for the variation coefficient σ_X/μ_X, selling price p, initial unit cost u_0, borrowing rate r, interest rate ratio q, and the cost reduction factor β. While the choice of the demand distribution and the settings for the different parameters indeed influence the optimal investment level and corresponding profit, we found that our research questions can be answered by considering only one setting for σ_X/μ_X, p, u_0, r and q, but a wide range of settings for β. The parameter settings that we use to answer our questions are displayed in Table 5.1. We vary β between $[10, 1000]$ in steps of 10 for each of the four scenarios. In total, we thus have the optimal investment level and corresponding profit for 4×100 settings.

Parameter	Setting
μ_X	25
σ_X	12.5
p	2
u_0	2
r	0.1
q	2
β	$[10, 1000]\backslash 10$

Table 5.1 *Input parameter settings.*

5.4.3 *The impact of pooling investment, pooling receivables, and pooling both on optimal investment*

To answer the first research question we explore the change in optimal investment level from pooling investment, pooling receivables, or pooling both. Figure 5.2(a) shows the optimal investment level S^* as function of the cost reduction factor β for the various pooling scenarios. Figure 5.2(b) shows the relative investment changes $\Delta S^*_m = \frac{S^*_m - S^*_{\text{I}}}{S^*_{\text{I}}}$ with $m \in \{\text{II}, \text{III}, \text{IV}\}$ for $\beta \in [100, 200]$. Note that in this interval, the optimal investment levels of all scenarios intersect with each other. Recall from Property 2 that β_T is the cost reduction factor threshold for which investment is still positive, i.e., $\beta_T \equiv \max\{\beta : s^* > 0\}$. Based on our experiments, we make the following observations on β_T:

The cost reduction factor threshold, β_T, is higher when firms pool receivables, much higher when firm pool their investments, the highest when firms pooling both. The optimal investment at the cost reduction factor threshold, $S^*(\beta_T)$, is lower when firms pool receivables, regardless whether firms pool their investments or not.

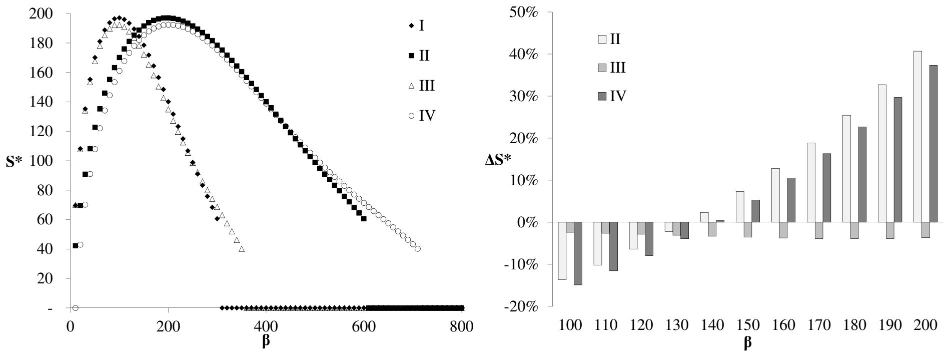
Indivisibility of receivables seems to cause an inaction region, which makes investment activity 'lumpy'. The optimal investment at β_T is considerably higher than zero for all four scenarios. Receivables pooling, however mitigates the adverse impact of indivisibility. Indeed, note from Figure 5.2(a) that investment becomes lucrative at a higher cost reduction factor threshold. Moreover, firms invest less at the threshold when they pool their receivables. Thus, receivables pooling reduces the inaction region that is induced by indivisibility; it allows firms to benefit from smaller adjustments in their investment level.

The optimal investment decreases (increases) from investment pooling when the cost reduction factor β is low (high). The optimal investment decreases (increases) from receivables pooling when β is low or moderate (high). The optimal investment decreases (increases) from pooling both when the cost reduction factor β is low (high).

Considering that receivables yield access to cheaper capital, the possibility that receivables pooling can lower optimal investment is somewhat surprising. It is understandable, however, if one considers that the marginal cost can exceed the marginal benefit of investment after firms pool their receivables. This implies that there is a delicate balance between the unit cost reduction and the financing cost of investment when firms pool receivables. As can be seen from Figure 5.2(b), receivables pooling can mitigate the amplification effect of investment pooling on investment, but it can also reinforce the mitigation effect of investment pooling on investment.

5.4.4 *The interaction between the benefit of investment pooling and receivables pooling*

To answer the second research question we first need to establish how we assess the benefit of pooling receivables and/or investment. The benefit of pooling can



(a) *Optimal investment S^* as function of β*

(b) *Change in optimal investment ΔS^* as function of β for II ,III, and IV*

Figure 5.2 *Optimal investment for: I-no pooling, II-investment pooling, III-receivables pooling, IV-pooling both*

be assessed purely based on the change in profit, $\Delta\pi_{b,j}(S^*)$, or based on the change in return on investment, $\Delta ROI_{b,j}(S^*)$:

$$\Delta\pi_j = \pi_j(S^*) - \pi_I(S^*), \tag{5.17}$$

$$\Delta ROI_{b,j} = ROI_j(S^*) - ROI_b(S^*) \tag{5.18}$$

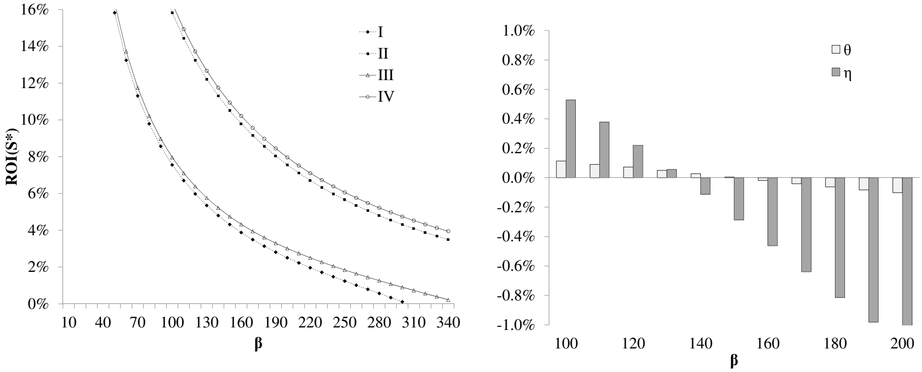
with $ROI(S^*) \equiv \frac{\pi(S^*)}{S^*}$, and $b, j \in \{\text{I, II, III, IV}\}$ and $b \neq j$. To evaluate the interaction between investment pooling and receivables pooling we can then use the following measures:

$$\eta = \frac{\Delta\pi_{\text{IV}} - (\Delta\pi_{\text{II}} + \Delta\pi_{\text{III}})}{(\Delta\pi_{\text{II}} + \Delta\pi_{\text{III}})}, \tag{5.19}$$

$$\theta = \Delta ROI_{\text{II, IV}} - \Delta ROI_{\text{I, III}}. \tag{5.20}$$

Here, the measure η indicates the relative change in profit from pooling *both* receivables and investment compared to sum of profits from pooling investment and receivables separately. If $\eta > 0$, the expected profit benefit is super-additive in pooling; if $\eta < 0$, it is sub-additive. The measure θ indicates the change in *ROI* from pooling receivables in pooled (vs. distributed) investment setting. If $\theta > 0$ ($\theta < 0$), it is more beneficial to pool receivables in a pooled (distributed) investment setting.

Figure 5.3(a) shows $ROI(S^*)$ as function of the cost reduction factor β for the



(a) *$ROI(S^*)$ as function of β* **(b)** *η and θ as function of β*

Figure 5.3 *$ROI(S^*)$, η, and θ for: I-no pooling, II-investment pooling, III-receivables pooling, IV-pooling both*

various pooling scenarios. Figure 5.3(b) shows η and θ as function of the cost reduction factor β for for $\beta \in [100, 200]$, i.e., the region in which the optimal investment levels intersect. We make the following observations:

The ROI is slightly higher when firms pool receivables, considerably higher when firms pool investment, and the highest when firms pool both receivables and investment.

Considering that *ROI* is a benchmark that consider both the efficiency and effectiveness of investment (Brealey et al., 2011), pooling both investment and receivables generates thus the best performance. This suggests that in resource pooling concepts, firms should consider pooling their financing options too.

The expected profit increase can be both sub- and super-additive in pooling. When the cost reduction factor β is low (high), the expected profit increase is super-additive (sub-additive). In addition, The ROI increase from pooling receivables can be both higher or lower in a pooled (vs. distributed) investment setting. When β is low (high), the ROI increase is higher in a pooled (distributed) investment setting.

The relative benefit of receivables pooling interacts thus with investment pooling. This can be explained by considering relative optimal investment level in a pooled investment setting for a particular cost reduction factor β. When the cost reduction factor is high, firms that pool investment invest more than firms that do not. Receivables pooling offers little benefit in terms of transactional efficiency; the likelihood that the size of the receivable exceeds the firms' financing need is relatively low. Firms are thus less 'burdened' by indivisibility. However, when the cost reduction factor is low, the opposite is the case. Firms that pool investment invest less than firms that do not pool investment. Receivables pooling then offers more transactional efficiency benefits in addition to the liquidity benefit.

5.5 Conclusions

In this paper, we examine the benefit of pooling receivables from creditworthy customers and its interaction with pooling of cost-reducing investment. We consider a setting in which a firms can sell high-quality receivables by means of reverse factoring. The sale allows the firm to obtain cheaper finance, but the firm must sell the whole receivable when it does so. As a consequence, the benefit from the factoring transaction is contingent on both on the firm's investment level and the demand realization. By means of a single period model in which two identical make-to-order firms can pool investment, pool receivables, or pool both, we evaluate the impact of pooling receivables on the optimal investment, expected profit, and expected return on investment. While the optimal investment level can increase from pooling receivables, we find that it decreases in most settings. Receivables pooling also extends the settings in which investment is lucrative. Furthermore, while receivables pooling provides a benefit regardless whether firms pool investment, the size of the benefit interacts with investment pooling.

Our study illustrates that the value of pooling operations and financing may interact when capital market fictions drive differences in the rates and the divisibility of different sources of financing. In our setting, receivables pooling aims to improve transactional efficiency and liquidity. Investment pooling aims to mitigate the demand uncertainty. In companies, the decision to pool financial assets or operational investments typically fall under the responsibility of financial and operations manager respectively. As the expected relative benefit can be both sub- and super-additive in pooling, closer cooperation between the finance and operations domain may improve decisions that depend on the correct valuation of pooling concepts.

There are a number of questions and issues left for further exploration. One is to explore how the benefit of receivables pooling behaves in a multi-period setting. Cozzolino (1971) shows how the re-investment of short-term cash excesses of a firm can be considered as a dynamic bin packing problem. Similarly, the redistribution of cash from indivisible receivables of a pool of firms can be studied as a packing problem. However, as the model would need to consider the cash dynamics of multiple firms, the analysis should become more complex.

Heuristic methods may then be employed to solve the problem concerning which receivables to factor in each period. Secondly, it would be interesting to explore whether restrictions on production capacity or borrowing amplify the interaction effects between the two pooling concepts. Thirdly, it would be interesting to explore how the size of the benefit behaves a function of the number firms, or the type of firms (e.g., in terms of size or demand uncertainty).

Appendix A
Analysis of the Optimal Factoring Decision

Recall that $c_P(S, \boldsymbol{a_j}, x_1, x_2)$ is the financing cost for decision j when demand is $X_1 = x_1$ and $X_2 = x_2$. If $a_{1j}x_1 + a_{2j}x_2 < y_k$ for decision $\boldsymbol{a_j}$ with $j \neq 0$, the total investment cannot be financed from factoring receivables and thus money needs to be borrowed by the firms. If $a_{1j}x_1 + a_{2j}x_2 \geq y_l$ for decision $\boldsymbol{a_j}$ with $j \neq 0$, decision $\boldsymbol{a_j}$ should not be chosen as $c_p(S, \boldsymbol{a_j}) \geq c_p(S, \boldsymbol{a_0})$. Find below the financing cost as function of the total investment S and factoring decision $\boldsymbol{a_j}$:

$$c_p(S, \boldsymbol{a_j}) = vp(a_{1j}x_1 + a_{2j}x_2) + \lambda[r(S - (a_{1j}x_1 + a_{2j}x_2)p(1-v))] \tag{A.1}$$
$$\lambda = \begin{cases} 1 & 0 \leq a_{1j}x_1 + a_{2j}x_2 < y_k \\ 0 & a_{1j}x_1 + a_{2j}x_2 \geq y_l \end{cases}$$

We are interested in $\boldsymbol{a}^*$ for every $x_1, x_2 \in [0, \infty\rangle$. Based on the joint demand level $Y = y$, we distinguish three main cases: A. $y \in [0, y_k\rangle$; B. $y \in [y_k, ky_l\rangle$; and C. $y \in [2y_l, \infty\rangle$. We define sub cases by fixing the range of x_1, and, consequently conditioning the value of x_2. Below we have listed the sub cases that cover all scenarios. To illustrate, Figure A.1 shows for $q = 2$ how all of the sub cases correspond to a region in 2 dimensional plane of x_1 and x_2.

A. $y \in [0, y_k\rangle$,

   A1. $0 \leq x_1 < y_k \wedge 0 \leq x_2 < y_k - x_1$

B. $y \in [y_k, 2y_l\rangle$,

   B1. $0 \leq x_1 < y_k \wedge y_k - x_1 \leq x_2 \leq y_k$

   B2. $0 \leq x_1 < y_k \wedge y_k < x_2 < 2y_l - x_1$

B3. $y_k \leq x_1 < y_l \wedge 0 \leq x_2 \leq y_k$

B4. $y_k \leq x_1 < y_l \wedge y_k < x_2 < 2y_l - x_1$

B5. $y_l \leq x_1 < 2y_l - y_k \wedge 0 \leq x_2 \leq y_k$

B6. $y_l \leq x_1 < 2y_l - y_k \wedge y_k < x_2 < 2y_l - x_1$

B7. $2y_l - y_k \leq x_1 < 2y_l \wedge 0 \leq x_2 < 2y_l - x_1$

C. $y \in [2y_l, \infty\rangle$,

C1. $0 \leq x_1 < y_k \wedge 2y_l - x_1 \leq x_2 \leq \infty$

C2. $y_k \leq x_1 < y_l \wedge 2y_l - x_1 \leq x_2 \leq \infty$

C3. $y_l \leq x_1 < 2y_l - y_k \wedge 2y_l - x_1 \leq x_2 \leq \infty$

C4. $2y_l - y_k \leq x_1 < 2y_l \wedge 2y_l - x_1 \leq x_2 \leq y_k$

C5. $2y_l - y_k \leq x_1 < 2y_l \wedge y_k < x_2 \leq \infty$

C6. $2y_l \leq x_1 \leq \infty \wedge 0 \leq x_2 \leq y_k$

C7. $2y_l \leq x_1 \leq \infty \wedge y_k < x_2 \leq \infty$

Case A1. $0 \leq x_1 < y_k \wedge 0 \leq x_2 < y_k - x_1$

We have $x_1 + x_2 < y_k \Rightarrow x_1 < y_k \wedge x_2 < y_k$, and thus $\lambda = 1$. By exploring the conditions in which a decision, say $\boldsymbol{a_k}$, outperforms another decision, $\boldsymbol{a_l}$ with $l \neq k$, for each decision $\boldsymbol{a_k} \in A$ we can distil the conditions in which decision

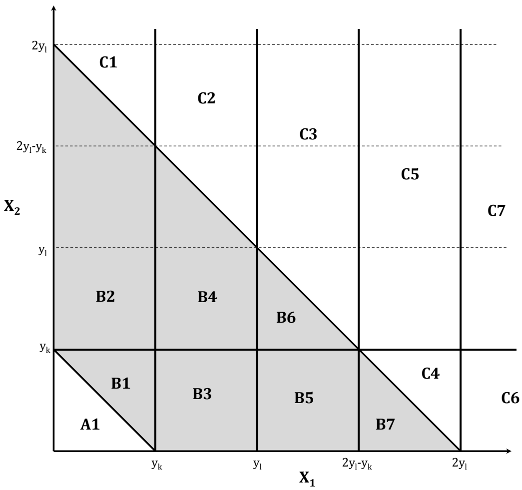


Figure A.1 *Regions corresponding to the sub cases for $q = 2$*

$\boldsymbol{a_k}$ is the optimal one. Table A.1 shows the conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$ for each decision $\boldsymbol{a_j} \in A$. We can observe that while factoring one receivable ($\boldsymbol{a_1}$ or $\boldsymbol{a_2}$) outperforms factoring none ($\boldsymbol{a_0}$) for $x_1 \geq 0$ $x_2 \geq 0$ respectively, however, factoring both receivables ($\boldsymbol{a_3}$) outperforms both factoring none as well as just one receivable for $x_1 \geq 0 \vee x_2 \geq 0$, hence we can conclude the following:

$$\begin{aligned} &0 \leq x_1 < y_k \wedge 0 \leq x_2 < y_k - x_1 \Rightarrow \\ &\boldsymbol{a}^* = \boldsymbol{a_3}, c_p(S, \boldsymbol{a}^*) = vp(x_1 + x_2) + r(S - (x_1 + x_2)p(1 - v)) \end{aligned} \tag{A.2}$$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \geq 0$	-	-	-
$\boldsymbol{a_2}$	$x_2 \geq 0$	$x_2 \geq x_1$	-	-
$\boldsymbol{a_3}$	$x_1 + x_2 \geq 0$	$x_2 \geq 0$	$x_1 \geq 0$	-

Table A.1 *A1. Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

Case B1: $0 \leq x_1 < y_k \wedge y_k - x_1 \leq x_2 \leq y_k$

As with the previous case, we explore a case in which both receivables are less than y_k, but the sum of the receivables is greater than y_k. Table A.2 shows the conditions in which the alternative decisions outperform each other.

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \geq 0$	-	-	-
$\boldsymbol{a_2}$	$x_2 \geq 0$	$x_2 \geq x_1$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq y_l - qx_1$	$x_2 \leq y_k - q^{-1}x_1$	-

Table A.2 *B1.Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

We observe that for $\boldsymbol{a_3}$ to outperform the other three decisions we have three conditions, $x_2 \leq y_l - x_1$, $x_2 \leq y_l - qx_1$ and $x_2 \leq y_k - q^{-1}x_1$ respectively. For $q > 1$ we know that $x_2 \leq y_l - x_1$ is already satisfied when $x_2 \leq y_l - qx_1$, hence we pay attention to the latter condition only.

We note that we have an intersection at $x_1 = \frac{y_l - y_k}{q - q^{-1}}$ with $\frac{y_l - y_k}{q - q^{-1}} \in \langle 0, y_k \rangle$[1] for which $y_k - q^{-1}x_1 = y_l - qx_1$. Let us call this intersection y_h. For $x_1 \leq y_h$ we have

[1] It can be shown that $\frac{y_l - y_k}{q - q^{-1}} < y_k$ results in the following condition $r(1 - v) < r(1 - v) + v$, which is true.

$y_k - q^{-1}x_1 \leq y_l - qx_1$ and $x_1 > y_h$ we have $y_k - q^{-1}x_1 > y_l - qx_1$. This means that for $0 \leq x_1 \leq y_h \wedge y_k - x_1 \leq x_2 \leq y_k - q^{-1}x_1$ and $y_h < x_1 \leq y_k \wedge y_k - x_1 \leq x_2 \leq y_l - qx_1$ we have $c_p(S, \boldsymbol{a_3}) \leq c_p(S, \boldsymbol{a_i})$ for $i \neq 3$, hence $\boldsymbol{a}^* = \boldsymbol{a_3}$. Thus:

$$0 \leq x_1 \leq y_h \wedge y_k - x_1 \leq x_2 \leq y_k - q^{-1}x_1 \Rightarrow \quad (A.3)$$
$$\boldsymbol{a}^* = \boldsymbol{a_3}, c_p(S, \boldsymbol{a}^*) = vp(x_1 + x_2)$$
$$y_h \leq x_1 < y_k \wedge y_k - x_1 \leq x_2 \leq y_l - qx_1 \Rightarrow \quad (A.4)$$
$$\boldsymbol{a}^* = \boldsymbol{a_3}, c_p(S, \boldsymbol{a}^*) = vp(x_1 + x_2)$$

For $0 \leq x_1 \leq y_h \wedge y_k - q^{-1}x_1 < x_2 \leq y_k$ we have $c_p(S, \boldsymbol{a_2}) \leq c_p(S, \boldsymbol{a_3})$. In addition, $x_1 \leq y_h \wedge x_2 > y_k - q^{-1}x_1 \Rightarrow x_2 \geq x_1$[2], which implies $c_p(S, \boldsymbol{a_2}) \leq c_p(S, \boldsymbol{a_1})$. Lastly, we have $c_p(S, \boldsymbol{a_2}) \leq c_p(S, \boldsymbol{a_0})$ for $x_2 \geq 0$. Thus, we can conclude:

$$0 \leq x_1 \leq y_h \wedge y_k - q^{-1}x_1 \leq x_2 < y_k \Rightarrow \quad (A.5)$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2 + r(S - x_2p(1 - v))$$

For $y_h < x_1 < y_k \wedge y_l - qx_1 \leq x_2 \leq y_k$ we have $c_p(S, \boldsymbol{a_1}) \leq c_p(S, \boldsymbol{a_3})$. However, depending on whether $x_2 \leq x_1$ or $x_2 > x_1$, we have $c_p(S, \boldsymbol{a_1}) \leq c_p(S, \boldsymbol{a_2})$ or $c_p(S, \boldsymbol{a_1}) > c_p(S, \boldsymbol{a_2})$ respectively. Lastly, we have $c_p(S, \boldsymbol{a_i}) \leq c_p(S, \boldsymbol{a_0})$ for $x_i \geq 0$ for $i = 1, 2$. Thus, we can conclude:

$$y_h < x_1 < y_k \wedge y_l - qx_1 < x_2 \leq x_1 \Rightarrow \quad (A.6)$$
$$\boldsymbol{a}^* = \boldsymbol{a_1}, c_p(S, \boldsymbol{a}^*) = vpx_1 + r(S - x_1p(1 - v))$$
$$y_h < x_1 < y_k \wedge x_1 < x_2 < y_k \Rightarrow \quad (A.7)$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2 + r(S - x_2p(1 - v))$$

Case B2. $0 \leq x_1 < y_k \wedge y_k < x_2 < 2y_l - x_1$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \geq 0$	-	-	-
$\boldsymbol{a_2}$	$x_2 \leq y_l$	$x_2 \leq y_l - (q-1)x_1$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq y_l - qx_1$	$x_1 \leq 0$	-

Table A.3 *B2. Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

[2]It can be shown that $y_k - q^{-1}x_1 > x_1 \Rightarrow x_1 < y_h$, which is true.

Table A.3 shows the conditions in which the alternative decisions outperform each other. As $x_1 \geq 0$ we have $c_p(S, \boldsymbol{a_1}) \leq c_p(S, \boldsymbol{a_0})$. However, depending on whether $x_2 \leq y_l - (1-q)x_1$ or $x_2 > y_l - (1-q)x_1$, we have $c_p(S, \boldsymbol{a_2}) \leq c_p(S, \boldsymbol{a_1})$ or $c_p(S, \boldsymbol{a_2}) > c_p(S, \boldsymbol{a_1})$ respectively. $x_2 < y_l - (q-1)x_1 \Rightarrow x_2 < y_l$, thus $x_2 < y_l - (q-1)x_1$ implies $c_p(S, \boldsymbol{a_2}) < c_p(S, \boldsymbol{a_0})$. As $x_1 \geq 0$ we have $c_p(S, \boldsymbol{a_2}) \leq c_p(S, \boldsymbol{a_3})$. Hence, $\boldsymbol{a}^* = \boldsymbol{a_2}$ for $y_k < x_2 \leq y_l - (q-1)x_1$. $x_2 > y_l - (q-1)x_1 \Rightarrow x_2 > y_l - qx_1$, thus $x_2 > y_l - (q-1)x_1$ implies $c_p(S, \boldsymbol{a_1}) < c_p(S, \boldsymbol{a_3})$. Thus, $\boldsymbol{a}^* = \boldsymbol{a_1}$ for $y_l - qx_1 < x_2 \leq 2y_l - x_1$. We can conclude:

$$0 \leq x_1 < y_k \wedge y_k < x_2 \leq y_l - (q-1)x_1 \Rightarrow \tag{A.8}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2$$
$$0 \leq x_1 < y_k \wedge y_l - (q-1)x_1 < x_2 < 2y_l - x_1 \Rightarrow \tag{A.9}$$
$$\boldsymbol{a}^* = \boldsymbol{a_1}, c_p(S, \boldsymbol{a}^*) = vpx_1 + r(S - x_1p(1-v))$$

Case B3. $y_k \leq x_1 < y_l \wedge 0 \leq x_2 \leq y_k$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \leq y_l$	-	-	-
$\boldsymbol{a_2}$	$x_2 \geq 0$	$x_2 \geq \frac{y_l - x_1}{q-1}$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_2 \leq \frac{y_l - x_1}{q}$	-

Table A.4 *B3. Conditions for which* $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ *with* $k > l$

Table A.4 shows the conditions in which the alternative decisions outperform each other for B3. We can conclude:

$$y_k \leq x_1 < y_l \wedge 0 \leq x_2 \leq \frac{y_l - x_1}{q-1} \Rightarrow \tag{A.10}$$
$$\boldsymbol{a}^* = \boldsymbol{a_1}, c_p(S, \boldsymbol{a}^*) = vpx_1$$
$$y_k \leq x_1 < y_l \wedge \frac{y_l - x_1}{q-1} < x_2 \leq y_k \Rightarrow \tag{A.11}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2 + r(S - x_2p(1-v))$$

Case B4. $y_k \leq x_1 < y_l \wedge y_k < x_2 < 2y_l - x_1$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \leq y_l$	-	-	-
$\boldsymbol{a_2}$	$x_2 \leq y_l$	$x_2 \leq x_1$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_1 \leq 0$	-

Table A.5 *B4. Conditions for which* $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ *with* $k > l$

Table A.5 shows the conditions in which the alternative decisions outperform each other for B4. We conclude:

$$y_k \leq x_1 < y_l \wedge y_k < x_2 \leq x_1 \Rightarrow \tag{A.12}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2$$
$$y_k \leq x_1 < y_l \wedge x_1 < x_2 < 2y_l - x_1 \Rightarrow \tag{A.13}$$
$$\boldsymbol{a}^* = \boldsymbol{a_1}, c_p(S, \boldsymbol{a}^*) = vpx_1$$

Case B5. $y_l \leq x_1 < 2y_l - y_k \wedge 0 \leq x_2 \leq y_k$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \leq y_l$	-	-	-
$\boldsymbol{a_2}$	$x_2 \geq 0$	$x_2 \geq \frac{y_l - x_1}{q-1}$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_2 \leq \frac{y_l - x_1}{q}$	-

Table A.6 *B5. Conditions for which* $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ *with* $k > l$

Table A.6 shows the conditions in which the alternative decisions outperform each other for B5. We conclude:

$$y_l \leq x_1 < 2y_l - y_k \wedge 0 \leq x_2 \leq y_k \Rightarrow \tag{A.14}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2 + r(S - x_2 p(1-v))$$

Case B6. $y_l \leq x_1 < 2y_l - y_k \wedge y_k < x_2 < 2y_l - x_1$

Table A.7 shows the conditions in which the alternative decisions outperform each other for B6. We can conclude:

$$y_l \leq x_1 < 2y_l - y_k \wedge y_k < x_2 \leq 2y_l - x_1 \Rightarrow \tag{A.15}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2$$

$a_k \setminus a_l$	a_0	a_1	a_2	a_3
a_0	-	-	-	-
a_1	$x_1 \leq y_l$	-	-	-
a_2	$x_2 \leq y_l$	$x_2 \leq x_1$	-	-
a_3	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_1 \leq 0$	-

Table A.7 *B6. Conditions for which $c_p(S, a_k) \leq c_p(S, a_l)$ with $k > l$*

Case B7. $2y_l - y_k \leq x_1 < 2y_l \wedge 0 \leq x_2 < 2y_l - x_1$

$a_k \setminus a_l$	a_0	a_1	a_2	a_3
a_0	-	-	-	-
a_1	$x_1 \leq y_l$	-	-	-
a_2	$x_2 \geq 0$	$x_2 \geq \frac{y_l - x_1}{q-1}$	-	-
a_3	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_2 \leq \frac{y_l - x_1}{q}$	-

Table A.8 *B7. Conditions for which $c_p(S, a_k) \leq c_p(S, a_l)$ with $k > l$*

We have $2y_l - y_k < x_1 < 2y_l \wedge 0 < x_2 \leq 2y_l - x_1 \Rightarrow x_2 < y_k$. Table A.8 shows the conditions in which the alternative decisions outperform each other for B7. We can conclude:

$$2y_l - y_k \leq x_1 < 2y_l \wedge 0 \leq x_2 < 2y_l - x_1 \Rightarrow \\ a^* = a_2, c_p(S, a^*) = vpx_2 + r(S - x_2 p(1-v)) \tag{A.16}$$

Case C1. $0 \leq x_1 < y_k \wedge 2y_l - x_1 \leq x_2 \leq \infty$

$a_k \setminus a_l$	a_0	a_1	a_2	a_3
a_0	-	-	-	-
a_1	$x_1 \geq 0$	-	-	-
a_2	$x_2 \leq y_l$	$x_2 \leq y_l - (q-1)x_1$	-	-
a_3	$x_2 \leq y_l - x_1$	$x_2 \leq y_l - qx_1$	$x_1 \leq 0$	-

Table A.9 *C1. Conditions for which $c_p(S, a_k) \leq c_p(S, a_l)$ with $k > l$*

Table A.9 shows the conditions in which the alternative decisions outperform each other for C1. We can conclude:

$$0 \leq x_1 < y_k \wedge 2y_l - x_1 \leq x_2 \leq \infty \Rightarrow \\ a^* = a_1, c_p(S, a^*) = vpx_1 + r(S - x_1 p(1-v)) \tag{A.17}$$

Case C2. $y_k \leq x_1 < y_l \wedge 2y_l - x_1 \leq x_2 \leq \infty$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \leq y_l$	-	-	-
$\boldsymbol{a_2}$	$x_2 \leq y_l$	$x_2 \leq x_1$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_1 \leq 0$	-

Table A.10 *C2. Conditions for which* $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ *with* $k > l$

Table A.10 shows the conditions in which the alternative decisions outperform each other for C2. We can conclude:

$$y_k \leq x_1 < y_l \wedge 2y_l - x_1 \leq x_2 \leq \infty \Rightarrow \tag{A.18}$$
$$\boldsymbol{a}^* = \boldsymbol{a_1}, c_p(S, \boldsymbol{a}^*) = vpx_1$$

Case C3. $y_l \leq x_1 < 2y_l - y_k \wedge 2y_l - x_1 \leq x_2 \leq \infty$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \leq y_l$	-	-	-
$\boldsymbol{a_2}$	$x_2 \leq y_l$	$x_2 \leq x_1$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_1 \leq 0$	-

Table A.11 *C3. Conditions for which* $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ *with* $k > l$

Table A.11 shows the conditions in which the alternative decisions outperform each other for C3. We can conclude:

$$y_l \leq x_1 < 2y_l - y_k \wedge 2y_l - x_1 \leq x_2 \leq y_l \Rightarrow \tag{A.19}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2$$
$$y_l \leq x_1 < 2y_l - y_k \wedge y_l < x_2 \leq \infty \Rightarrow \tag{A.20}$$
$$\boldsymbol{a}^* = \boldsymbol{a_0}, c_p(S, \boldsymbol{a}^*) = Sr$$

Case C4. $2y_l - y_k \leq x_1 < 2y_l \wedge 2y_l - x_1 \leq x_2 \leq y_k$

Table A.12 shows the conditions in which the alternative decisions outperform each other for C4. We can conclude:

$$2y_l - y_k \leq x_1 < 2y_l \wedge 2y_l - x_1 \leq x_2 \leq y_k \Rightarrow \tag{A.21}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2 + r(S - x_2p(1 - v))$$

$a_k \setminus a_l$	a_0	a_1	a_2	a_3
a_0	-	-	-	-
a_1	$x_1 \leq y_l$	-	-	-
a_2	$x_2 \geq 0$	$x_2 \geq \frac{y_l - x_1}{q-1}$	-	-
a_3	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_2 \leq \frac{y_l - x_1}{q}$	-

Table A.12 *C4. Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

Case C5. $2y_l - y_k \leq x_1 < 2y_l \wedge y_k < x_2 \leq \infty$

$a_k \setminus a_l$	a_0	a_1	a_2	a_3
a_0	-	-	-	-
a_1	$x_1 \leq y_l$	-	-	-
a_2	$x_2 \leq y_l$	$x_2 \leq x_1$	-	-
a_3	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_1 \leq 0$	-

Table A.13 *C5. Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

Table A.13 shows the conditions in which the alternative decisions outperform each other for C5. We can conclude:

$$2y_l - y_k \leq x_1 < 2y_l \wedge y_k < x_2 \leq y_l \Rightarrow \quad \text{(A.22)}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2$$
$$2y_l - y_k \leq x_1 < 2y_l \wedge y_l < x_2 \leq \infty \Rightarrow \quad \text{(A.23)}$$
$$\boldsymbol{a}^* = \boldsymbol{a_0}, c_p(S, \boldsymbol{a}^*) = Sr$$

Case C6. $2y_l \leq x_1 \leq \infty \wedge 0 \leq x_2 \leq y_k$

$a_k \setminus a_l$	a_0	a_1	a_2	a_3
a_0	-	-	-	-
a_1	$x_1 \leq y_l$	-	-	-
a_2	$x_2 \geq 0$	$x_2 \geq \frac{y_l - x_1}{q-1}$	-	-
a_3	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_2 \leq \frac{y_l - x_1}{q}$	-

Table A.14 *C6. Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

Table A.14 shows the conditions in which the alternative decisions outperform each other for C6. We can conclude:

$$2y_l \leq x_1 \leq \infty \wedge 0 \leq x_2 \leq y_k \Rightarrow \tag{A.24}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2 + r(S - x_2 p(1 - v))$$

Case C7. $2y_l \leq x_1 \leq \infty \wedge y_k \leq x_2 \leq \infty$

$\boldsymbol{a_k} \setminus \boldsymbol{a_l}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	-	-	-	-
$\boldsymbol{a_1}$	$x_1 \leq y_l$	-	-	-
$\boldsymbol{a_2}$	$x_2 \leq y_l$	$x_2 \leq x_1$	-	-
$\boldsymbol{a_3}$	$x_2 \leq y_l - x_1$	$x_2 \leq 0$	$x_1 \leq 0$	-

Table A.15 *C7. Conditions for which $c_p(S, \boldsymbol{a_k}) \leq c_p(S, \boldsymbol{a_l})$ with $k > l$*

Table A.15 shows the conditions in which the alternative decisions outperform each other for C7. We can conclude:

$$2y_l \leq x_1 \leq \infty \wedge y_k \leq x_2 \leq y_l \Rightarrow \tag{A.25}$$
$$\boldsymbol{a}^* = \boldsymbol{a_2}, c_p(S, \boldsymbol{a}^*) = vpx_2$$
$$2y_l \leq x_1 \leq \infty \wedge y_l < x_2 \leq \infty \Rightarrow \tag{A.26}$$
$$\boldsymbol{a}^* = \boldsymbol{a_0}, c_p(S, \boldsymbol{a}^*) = Sr$$

Now that we have identified the optimal factoring decision in all sub cases concerning the demands of the firms, we can formulate the expected financing cost $E[c_p(S)]$ as a sum of integrals:

$$E[c_p(S,\boldsymbol{a}^*)] = \int_0^{y_k}\int_0^{y_k-x_1} (vp(x_1+x_2)+rS-rp(1-v)(x_1+x_2))f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+ \tag{A.27}$$

$$\int_0^{y_h}\int_{y_k-x_1}^{y_k-q^{-1}x_1} (vp(x_1+x_2))f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_0^{y_h}\int_{y_k-q^{-1}x_1}^{y_k} (vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{y_h}^{y_k}\int_{y_k-x_1}^{y_l-qx_1} (vp(x_1+x_2))f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{y_h}^{y_k}\int_{y_l-qx_1}^{x_1} (vpx_1+rS-rp(1-v)x_1)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{y_h}^{y_k}\int_{x_1}^{y_k}(vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_0^{y_k}\int_{y_k}^{y_l-(q-1)x_1} (vpx_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_0^{y_k}\int_{y_l-(q-1)x_1}^{2y_l-x_1} (vpx_1+rS-rp(1-v)x_1)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{y_k}^{y_l}\int_0^{\frac{y_l-x1}{q-1}} (vpx_1)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{y_k}^{y_l}\int_{\frac{y_l-x1}{q-1}}^{y_k} (vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{y_k}^{y_l}\int_{y_k}^{x_1}(vpx_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{y_k}^{y_l}\int_{x_1}^{2y_l-x_1} (vpx_1)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{y_l}^{2y_l-y_k}\int_0^{y_k}(vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{y_l}^{2y_l-y_k}\int_{y_k}^{2y_l-x_1} (vpx_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{2y_l-y_k}^{2y_l}\int_0^{2y_l-x_1} (vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_0^{y_k}\int_{2y_l-x_1}^{\infty} (vpx_1+rS-rp(1-v)x_1)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{y_k}^{y_l}\int_{2y_l-x_1}^{\infty} (vpx_1)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{y_l}^{2y_l-y_k}\int_{2y_l-x_1}^{y_l} (vpx_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{y_l}^{2y_l-y_k}\int_{y_l}^{\infty}(rS)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{2y_l-y_k}^{2y_l}\int_{2y_l-x_1}^{y_k} (vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{2y_l-y_k}^{2y_l}\int_{y_k}^{y_l}(vpx_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{2y_l-y_k}^{2y_l}\int_{y_l}^{\infty}(rS)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{2y_l}^{\infty}\int_0^{y_k}(vpx_2+rS-rp(1-v)x_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1+$$

$$\int_{2y_l}^{\infty}\int_{y_k}^{y_l}(vpx_2)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1 + \int_{2y_l}^{\infty}\int_{y_l}^{\infty}(rS)f(x_2)f(x_1)\mathrm{d}x_2\mathrm{d}x_1.$$

Chapter 6
Conclusions, Implications, and Future Research

Supply chain finance (SCF) is an integral approach to financing within a supply chain. As intermediaries of capital and providers of technology expand their services to facilitate such an approach, SCF has gained considerable interest from industry. Reverse factoring is an example of such a service. In this arrangement, a financially strong buyer facilitates low cost capital to its supplier by confirming the status of its receivables to a financial intermediary. Despite the growing practical interest for SCF, scientific exploration is limited. Moreover, while emerging research shows that taking an integral approach to financing in conjunction with operations is practically relevant, research on supply chain finance often limits itself by merely considering the financial perspective. By developing concepts for SCF in general, and exploring trade-offs particularly related to reverse factoring, this thesis aims to contribute to filling the gap.

The main contributions of this thesis are as follows. First, we provide a framework that allows us to typify practises and to focus research efforts on key trade-offs of SCF. Secondly, we develop models that allow us to study trade-offs related to reverse factoring in an inventory management setting. We first study a trade-off that a supplier potentially faces on its payment terms. Subsequently, we study a buyer's trade-off that concerns the service level of its supplier. Thirdly, we demonstrate how receivables pooling, i.e., an arrangement in which suppliers share their receivables, can add value by mitigating a friction imposed in reverse factoring.

The goal of this chapter is to summarize our main findings and to provide a synthesis of our research. In Section 6.1, we reflect on each study and offer future directions by means of our framework. In Section 6.2, we reflect on our work, discuss some of its limitations, and conclude by providing future directions for research on SCF in general.

6.1 *Main Findings, Managerial implications, and Future Directions*

In this section, we reflect on what we have learned from our studies, note some managerial implications, and provide directions by means of our framework.

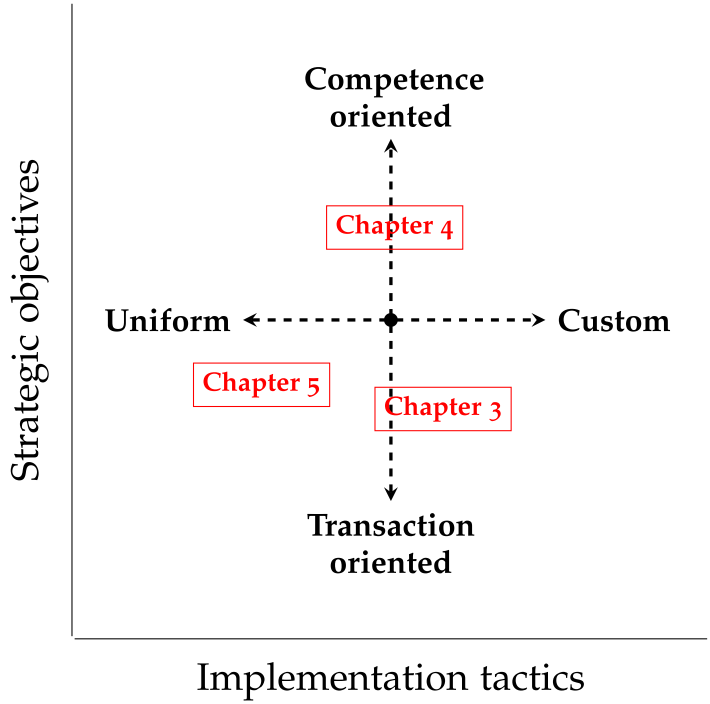


Figure 6.1 *A typology for SCF research*

In Chapter 2, we developed a conceptual framework that allows us to position SCF initiatives along two dimensions: strategic objectives and implementation tactics (see Figure 6.1). On the strategic dimension, we distinguish transaction-oriented from competence-oriented initiatives. While the improvement of supply chain competencies, such as operational efficiency or resilience, is the primary the focus in the latter, the transactional benefit that an arrangement brings is the focus in the first. On the tactical dimension, an initiative is classified as uniform or customized, based on the efforts to make an arrangement fit the nature of specific supply chain relationships.

While literature offers perspectives on how SCF can be customized based on the *financial* nature of the firms involved, works that show how firms' *operational* characteristics impact trade-offs related to SCF are limited. As shown in Chapter 4, while some firms do launch an SCF initiative to reduce supply chain risk, they in fact offer *passive* solutions when it comes to managing them. The lack of knowledge on how operational aspects may influence SCF trade-offs may have large impact on both users and providers of SCF. SCF users may assess the value of their decisions incorrectly which consequently results in making arrangements that do not yield the *most* value to the supply chain. SCF providers, on the other hand, may continue to develop SCF services that are *purely* financial arrangements, i.e., arrangements without any features that allow firms to actively improve supply chain competencies.

By exploring *tactical* trade-offs both in a transaction-oriented and in a competence-oriented SCF initiative (in Chapter 3 and 4 respectively) and investigating the value of a transaction-oriented pooling initiative (in Chapter 5), we offer insights on how operations matter to SCF decision-making (See Figure 6.1 for the placements of the chapters in the framework). These insights allow us to make better informed decisions in SCF, but also generate ideas on future directions.

In Chapter 3, we explore the maximum viable payment term extension that a supplier can grant for reverse factoring in an inventory management setting. We identify conditions where its maximum payment term depends on demand uncertainty, operating leverage, and profit margin. One of the key lessons from this chapter is that trade credit imposes an *additional* cost to a firm, i.e., a cost that is not assessed in conventional valuation methods. The cost arises from an additional exposure to cash *volatility* when allowing customers to pay later. Managers generally thus underestimate (overestimate) the cost of extending (shortening) their payment terms. We found the underestimation to be largest for firms that have high demand uncertainty, a large portion of variable cost, and a low profit margin.

While our research shows that operational and financial indicators are both relevant to the value of SCF, we have not yet explored the trade-off between applying uniform vs. custom SCF in a multi-party context (e.g., one buyer and multiple suppliers). Our case study in Chapter 2 for example shows that buyers may introduce reverse factoring to standardize payment terms to suppliers. The resulting uniformity may save transaction cost in, e.g., administration, processes, contracting, etc. On the other hand, the offering of an uniform arrangement may also result in price discrimination (see, e.g., Brennan et al., 1988), i.e., some will accept the arrangement, and some (can) not. Research that explores trade-offs on when to pursue customization in SCF, and to what extent, is still limited. Such research can help SCF providers to develop smarter solutions, but also support SCF users to develop better customization policies.

In Chapter 4, we explore whether a buyer that offers reverse factoring to a supplier can expect to be served better than other buyers. We find, in contrast to what practitioners believe, that a supplier is not naturally inclined to increase its service after reverse factoring is introduced. Indeed, a supplier rather collects the maximum possible financial savings from the scheme. On the other hand, a buyer may *contractually* require a significant service level improvement from offering reverse factoring, without hurting its supplier. Supply chain entities may thus forego value opportunities by not introducing the appropriate incentive mechanisms in competence-oriented SCF.

Now that we have explored *tactical* trade-offs in both transaction-oriented and competence-oriented SCF in Chapters 3 and 4 respectively, it would interesting to extend our research by investigating the *strategic* trade-off between transaction-oriented vs. competence-oriented SCF. For instance, under what conditions would it make more sense for a buyer to pursue a working capital vs. service level benefit in reverse factoring? To explore such a trade-off, however, we would need to model the operations between a buyer and its supplier from a richer perspective. For instance, to explore the monetary gain from a better supply service level we would need to know whether a buyer would use this as an opportunity to reduce its own inventory, or increase its own service to its customer.

In Chapter 5, we explore the value of a concept in which firms pool receivables that can be sold through reverse factoring. Indeed, the indivisibility of a receivable may give rise to still use a costlier, but flexible financing alternative. By pooling their receivables, firms can share and 're-distribute' their financing options offered in reverse factoring. In a stylized setting with two identical firms, we show how the value of receivables pooling interacts with investment pooling and identify conditions in which the benefit from the pooling initiatives is super-additive. Overall, our study suggests that firms would benefit from methods that would allow them identify the synergistic benefit of pooling financial as well operational resources. However, further study is needed on how to develop systems that can facilitate receivables pooling on shared resources or even in general between firms.

It would be interesting to extend our research by exploring a wider variety of pooling configurations on the financial and operational level. For instance, could firms generate value by diversifying their pooling arrangements? That is, pooling financially with a different set of firms than the firms they pool operational resources with. However, to answer such questions, we need to spend more thought on how such SCF arrangements could (should) be structured between firms. For instance, how should firms structure a receivables pooling arrangement that includes receivables with different default risk?

6.2 Synthesis, Limitations, and Future Research

In this final section, we reflect on our work and try to synthesize our findings. We also consider more broadly some of the limitations of our studies and discuss directions for future research on SCF.

At the start of this thesis, we explained how the consideration of financial decisions in general, thus also in conjunction with operational decisions, is theoretically justified based on imperfections in the capital market. Indeed, if the capital market would be perfect, firms would always get a premium that purely reflects the risk of the assets that they buy and modify (through operational decisions). However, capital markets in reality are imperfect. Capital providers may suffer from information asymmetry and adjust premiums accordingly. By informing capital providers, firms are able to mitigate this information asymmetry and thus facilitate cheaper capital within their supply chain. In operations management, however, models often do not consider the financing alternatives that are available to a firm, let alone the financing alternatives that arise from collaboration initiatives in the supply chain.

We developed models to explore trade-offs that concern the implementation of reverse factoring. In each of our studies, we discovered that it is challenging to derive structural properties, let alone derive optimal policies, when including financial dimensions in operational models. Specifically, the interactions between inventory and cash management precluded us from obtaining analytical results in Chapter 3 and 4. In Chapter 5, complexities also arose from considering alternative financing options as well as investment decisions of multiple firms. Nevertheless, in all studies, we were able obtain valuable insights by means of simulation-based optimization and numerical search methods. Given the presence of capital market frictions, we find that the significance of considering operations-finance interactions in business decisions is further amplified by demand uncertainty. For instance, in Chapter 3, we find that a base stock policy with trade credit ultimately propagates demand uncertainty to the firm's cash position. Furthermore, in Chapter 5, the possibility to collect an additional gain from implementing two interdependent risk pooling concepts is ultimately driven by demand uncertainty.

As is common in research, we made key assumptions and choices that may influence the generality, but not necessarily the practical relevance, of our results. First, in all of our models, we considered either the minimization of expected cost or the maximization of expected profit to be the firm's goal, rather than the total discounted cost or profit. In Chapters 3 and 4, we did this to avoid a direct dependence between trade-offs and the initial state (e.g., cash position) of the firm. In Chapter 5, the same choice was driven based on our focus on the sign (vs. the size) of the synergistic benefit. We do not expect that our main insights change when accounting for the time value of money by means of discounting. Secondly, the policies that we consider in our models in Chapter 3 and 4 are not proven to be optimal, and thus some caution must be exercised on the insights that we inferred. There may indeed exist an optimal policy for inventory and cash management that avoids the propagation of demand uncertainty on cash. Most likely, however, it is not a threshold policy type. Consequently, the optimal policy is potentially complex, and thus costly to implement[1].

As it is not unusual in scientific research, our studies altogether also raise issues on a more general level. Indeed, our research can be considered to be just an initial step in the exploration of trade-offs in SCF. One of the items that we consider to be important, but largely unexplored, is the potential *endogeneity* between the decisions and the state of the firm on the one hand, and the financing rates and the credit limits imposed by financial intermediaries on the other. Indeed, while we considered financing rates to be fixed in all of our studies, they may in fact be influenced by SCF. For instance, when using reverse factoring, a supplier's debt level may decrease, thus also its default risk. On the general level, interesting questions remain on how firms within a supply chain together (should) react to capital market imperfections. However, considering the default risk of firms in an operational context is challenging, and many researchers limit themselves to exploring trade-offs in a one-period setting (see, e.g., Xu and Birge, 2006; Kouvelis and Zhao, 2011; Alan and Gaur, 2012). Considering endogeneity in a multi-period setting could be a fruitful direction to better understand the value potential of SCF, but it raises many different questions on how to model firms correctly. For instance, while our models in Chapter 3, 4, and 5 could indeed include an (endogenously) set credit limit, the

[1] An alternative to this would be a payment (collection) policy that allows firms to smooth the cash flows from ordering decisions. Nevertheless, such a policy is likely not to be accepted freely by a supplier (customer).

firm's objective function should then include also some trade-off between its cost (or profit) and its default risk.

While it is difficult to predict what research on SCF will yield in the future, there is sufficient indication that it will flourish the coming years. A Google scholar search today (11 April 2015) shows that less than 200 scientific articles have the word "supply chain finance" in their title. Nearly, all of them appeared after 2007, which also illustrates the significance of the financial crisis for the interest level. Unfortunately, however, empirical research on what (and how) firms decide in SCF services based on secondary data is scarce. Such research could complement normative research on firms' trade-offs, but it could also shed new light on existing views in the literature. For instance, while economics research traditionally suggests that bank credit is redistributed from financially stronger firms to weaker firms via trade credit (see, e.g., Love et al., 2007), Klapper et al. (2012) recently found, based on the data from a SCF provider, that creditworthy buyers receive contracts with the longest payment terms from their least creditworthy suppliers. Research is still needed to reconcile reconcile such findings with theory; research that considers operational and/or behavioral aspects of firms.

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Summary

Concepts and Trade-offs in Supply Chain Finance

Supply chains are networks of interlinked companies. The various companies are interlinked such that each company can deliver a product or service that is valuable to at least one other company in the system. Ultimately, the companies furthest downstream in the supply chain deliver one or more final products for the consumer market. While the companies generally make investment and financing decisions independently from each other, they are interdependent from the perspective of effectiveness and efficiency. For instance, in order to be able respond rapidly to a client's demands, a company generally needs suppliers that can respond quickly to its own demands. Supply chain management is a field that provides concepts on how to best manage such interdependencies. The field became especially popular in the nineties among companies that faced increased complexity after having outsourced many of their non-core activities to suppliers overseas.

While the literature on supply chain management is rich in insights on how to best manage production, transportation, inventories, information, and information technology, within and between companies, it is still underdeveloped on financing. Albeit at a cost, it is generally assumed that the cash needed for a particular decision or policy is always available. The 2008 crisis shows, however, that cash can be expensive or constrained and that its cost or unavailability may severely impact the performance of the supply chain. For instance, many firms were forced to reduce capacity or inventory due to liquidity problems. As the crises hit many firms at once, supply risk management strategies, such as dual sourcing, provided an insufficient basis to sustain a reliable supply chain. Arrangements and activities that take a cooperative approach to financing in the supply chain, which are becoming known as Supply Chain Finance (SCF), may complement these strategies. For instance, to help suppliers to cope with their

liquidity problems, manufacturers such as Boeing, Rolls-Royce and Caterpillar introduced programs to offer them cheaper financing.

SCF has enjoyed considerable attention from industry in the last five years. The attention is primarily driven by the promotion of new services from financial intermediaries and technology providers. One of these services is reverse factoring. Reverse factoring is an arrangement where a creditworthy buyer facilitates early payment of its trade credit obligations to suppliers. It allows the supplier to sell receivables to a financial intermediary which already has received confirmations of the corresponding deliveries and resulting payment obligations from the buyer. The confirmations solves opacity problems that otherwise would exist between the supplier and the financial intermediary and allows thus the financial intermediary to offer the service cheaply. Like reverse factoring, several other services exist in which events or commitments in the supply chain serve the basis or trigger for financing.

In this thesis, we first develop a general framework to orient thought and research on SCF. Subsequently, we explore concepts and tradeoffs related to reverse factoring. Specifically, we explore contractual trade-offs on payment terms and on service levels and explore a concept in which suppliers pool receivables. Overall, our studies show that operations-finance interactions are relevant to trade-offs on the operational level, but also to valuing strategic initiatives, such as the initiative to pool financing options of business entities with shared resources. Furthermore, parameters such as demand uncertainty and the firm's cost structure, which are often overlooked in the context of supply chain finance, can influence the value of proposals significantly. While we focus on reverse factoring in this thesis, our insights are nevertheless relevant for the literature on operations-finance interactions in general. For instance, we show that trade credit decisions not only affect the expected amount of capital tied up in receivables (payables), but also the amount of capital needed to cope the propagation of demand uncertainty on the firm's cash flows.

The following paragraphs provide specific summaries of each of the four studies conducted for this thesis.

A Framework To Advance Research on Supply Chain Finance
We present a framework that allows to position SCF initiatives from firms and studies on SCF concepts. Emerging trends in the area SCF and a case study of two manufacturers served as basis for the framework. While literature emphasizes the financial motives of SCF, the trends illustrate SCF is becoming increasingly complex. The case study confirms this by showing the diverging perspectives of the firms on reverse factoring. The framework

allows SCF initiatives and concepts to be characterized based on two dimensions: strategic objectives and implementation tactics. On the strategic dimension, we distinguish transaction-oriented from competence-oriented SCF initiatives. While the improvement of supply chain competencies, such as agility, is the primary the focus in the latter, the transactional benefit that SCF brings is the focus in the first. On the tactical dimension, an initiative is classified as uniform or customized based on the customization efforts to make a SCF arrangement fit the nature of specific supply chain relationship. Having developed the framework, we show within it the positions of the three quantitative studies that we conduct in the remainder of this thesis .

The Price of Reverse Factoring: Financing Rates vs. Payment Delays
Reverse factoring allows a buyer to decouple the dates that its suppliers collect their entitlements from the dates that it meets the corresponding obligations. Therefore, buyers in reverse factoring often induce their suppliers to grant them more lenient payment terms. By means of a periodic review inventory model that includes alternative sources of financing, we explore the following question: what extensions of payment terms allow the supplier to benefit from reverse factoring? We obtain solutions by means of simulation optimization. We find that an extension of payment terms induces a non-linear financing cost for the supplier, beyond the opportunity cost of carrying additional receivables. Furthermore, we find that the size of the payment term extension that a supplier can accommodate depends on the demand uncertainty of the buyer and the cost structure of the supplier. Overall, our results show that the financial implications of an extension of payment terms needs careful assessment in stochastic settings.

Reverse Factoring and Service Levels: Let it happen or make it work?
While it increasingly suggested in the literature that improved service is a natural by-product of offering reverse factoring to a supplier, we aim to explore whether this is the case and how much service improvement can be contracted by the buyer. We consider a scenario where a supplier uses a base stock inventory system to serve demands from two retailers. One of the retailers (A) facilitates early payment to the supplier through reverse factoring; the other (B) pays the supplier with a fixed payment delay. We explore whether the supplier will naturally incline to provide a higher service level to retailer A than to retailer B. Consequently, we explore the maximum service level improvement that A can require, given the terms offered. We find that the optimal base stock decreases as a function of the reverse factoring rate, hence, a supplier does not 'naturally' offer a better service level for reverse factoring, but rather collects the financial savings. In addition, we find that the maximum service level improvement that can be contractually required by retailer A is conditioned on the relative size of

its mean demand, demand uncertainty, and the deviation between lead time and payment term. Our work yields managerial insights on when and how much retailers can operationally benefit from reverse factoring.

Pooling Receivables and its Interaction with Pooling Investment
While the other parts of this thesis derive primarily from the ability of reverse factoring to relax capital market frictions, our final study considers the fact that it in turn imposes a friction. Specifically, reverse factoring allows firms to finance investment more cheaply by selling high-quality receivables, but the transactions are indivisible: the full receivable must be sold. We consider how pooling receivables with other entities mitigates the adverse impact of this indivisibility. In a stochastic make-to-order setting in which firms can pool receivables and/or investment, we explore the following questions: (i) What is the impact of pooling investment, pooling receivables, or pooling both on the optimal investment level? (ii) is the benefit from the different pooling options super-additive? We find that, depending on changes in the marginal capital cost and the marginal investment benefit, the optimal investment level can increase as well as decrease from the two types pooling and thus also when they are implemented together. Furthermore, the benefit from the two pooling concepts can be sub- or super-additive. When the technology maturity level of the firm's assets is low (high), the benefit from pooling is super-additive (sub-additive). Our results thus indicate that simultaneous evaluation of pooling concepts is warranted.

About the author

Kasper van der Vliet was born in Amman (Jordan) on 15 December 1981. At a relatively young age, in 1986, he and his family moved to the Netherlands, where he grew up in Meerssen (Limburg). He received his master's in Industrial Engineering from the Eindhoven University of Technology in 2005. After finishing his studies, he first worked as a logistics consultant in The Hague. In 2007, he returned to Eindhoven where he joined the corporate supply management department of Philips. After having lead several cross-sector initiatives in the area of procurement, he became responsible for the launch of a supplier financing program for Europe and North America in 2009. Based on the impact and the cross-functional collaboration, his team was awarded the 1st prize in the Philips Supply Olympics by the CEO. The program also won several awards in international treasury conferences.

His experiences as project manager arouse general interest in supply chain finance, therefore, Kasper started a PhD under the supervision of Dr. Matthew Reindorp and Prof.dr.ir. Jan Fransoo in 2011. He worked on several research projects. One of his papers is published in the European Journal of Operational Research. His main interests includes how operational and financial management policies interact in supply chain settings, and how these interactions impact business performance and/or risk. His research was conducted in the context of 4C4More, a project supported by DINALOG (Dutch Institute for Advanced Logistics). In this project, he contributed to concepts and research on supply chain finance in horizontal collaboration. During his PhD, Kasper also served as a board member of the Student Supply Chain Forum, a branch of the European Supply Chain Forum.