

## **Nested and dynamic documentation systems in/for teaching and teacher learning : re-conceptualising mathematics curriculum resources and their use.**

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# Afterword: Using and Designing Resources for Practice

Deborah Loewenberg Ball

## Closing Reaction

The chapter authors in this volume examine and theorize about the nature, role, and use of resources in instruction. The range of “resources” they investigate is vast—from commercial to teacher-made curriculum, to videos, to technology-based tools and environments, to artifacts—and generated by a range of creators, from teachers to professional designers and researchers.

Certainly textbooks remain a mainstay of mathematics instruction. Research on modal mathematics classrooms suggests that much teaching is “text-driven.” Reformers often turn their attention to the design of text materials as a means of leveraging teaching and thus improving learning. Others praise teachers who do not “follow” curriculum materials, but who invent their own curriculum, lessons, and examples. At the same time, the range of resource material used for instruction is expanding to many other forms. That said, understanding resources-in-use, or “lived” resources, matters across these forms. Both the concepts of *resources* and *use* are fundamental to the inquiry.

The improvement of learning depends on many factors, but clearly, the resources used by learners and their teachers form a vital medium of instruction. At their best, resources are both attentive to learners’ ideas and responsible to mathematical learning goals. Not all curriculum resources are created by outsiders: In highly responsive and interactive teaching, teacher-invented materials constitute the pertinent resources. The authors of this volume consider what counts as a “curriculum resource” for mathematics instruction and examine how design and use interact in real-time teaching and learning.

As the chapter authors make visible, “using a textbook”—or any curriculum resource—is a process that is both interpretive and dynamic. Teachers read and make sense of curriculum developers’ ideas, adapting them to their own ideas and contexts. Learners, too, interpret and use textbooks, not necessarily as writers intended

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D.L. Ball (✉)

School of Education, University of Michigan, Ann Arbor, MI 48109-1259, USA  
e-mail: dball@umich.edu

or envisioned. The difference between the text as written and the text as enacted is significant. The chapters in this book support an important direction in the field, toward curriculum resources that are designed for reasoned and responsive use, not for control or for loose and unspecified improvisation. This is the heart of the notion of curriculum as “lived resources.”

Three questions stand out to which the authors of this book make vital contributions:

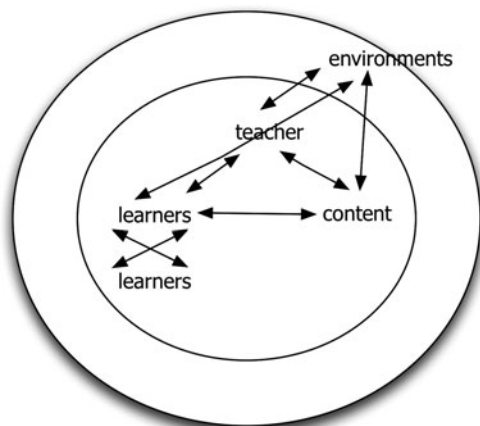
1. What counts as a “curriculum resource” for teaching mathematics?
2. How are resources used in instruction?
3. How can resources be designed for use, and for learning, in and from practice?

First, what counts as a “curriculum resource”? Nominal categories such as textbooks, programs, problem sets, learning goals, artifacts, and tasks insufficiently specify the notion of a “resource.” Helping learners develop understanding and skill with mathematics depends on the development of pathways, spaces, and tools in and with which to work on key ideas and processes. This translation of disciplinary content into forms that are accessible and manipulable by those learning the subject is an old and core problem of instruction. On the one hand, such translations between learners’ thinking and the mature ideas of the field involve managing between what John Dewey (1902/1956) referred to as the “psychological and logical aspects” of the subject. Dewey wrote with sympathy about the challenges inherent in managing these two different aspects of the subject. Young children thinking about integers do not have the real line as a mental object, yet developing their understanding requires sensitivity both to their current ways of thinking (numbers refer to “real” counts of objects or measures of actual lengths) and to an eye on their mathematical horizons, as well as a focus on the integrity of the subject matter itself (Ball, 1993). Curriculum resources, of many different forms, comprise a wide range of translational materials and objects, but all designed to support this fundamental challenge of building bridges between learners and the discipline.

Designing such resources is no small task and requires substantial knowledge and skill, and deep understanding of the nature of instruction, or teaching and learning. Instruction consists of interactions among teachers and learners, around content, in environments (Fig. 1, from Cohen, Raudenbush, & Ball, 2003).

Teachers interpret learners, learners interpret their teachers, and both bring past experience and understandings of the material. As teachers and learners work with curricular resources, whether texts, or problems, or software environments, these mutual interpretations, in context, shape what these resources become in practice. Two teachers, teaching with the same text, will make different decisions, some deliberate and some as a product of their beliefs and assumptions, about everything from what to emphasize and what to omit, to how to modify and where to stick close to the material, to how to “speak” the tools and how to engage learners with them. These interpretations profoundly affect the resource as “lived” out in class; further, learners also interpret and shape resources-in-use. Two different groups of learners will work differently with the same lesson as offered by the same teacher, as a function

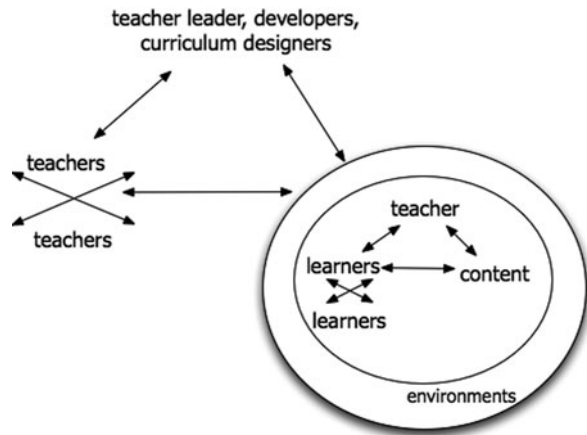
**Fig. 1** The instructional triangle: instruction as interaction in environments



of individual and collective and mutual interpretation. As such, as the authors of this volume make clear, curriculum tools are converted in real-time use from potential to actual resources. Consequently, design produces material with potential for use; it cannot determine actual use.

What then does this conception of curriculum resources, and this understanding of their use in practice, suggest about their design? The chapters in this book reveal many subtleties inherent in the crucial work of design. First, for curriculum resources to support and guide deliberate use as envisioned for learning, designers must have a sensitivity for practice and its demands. For example, in real-time teaching, teachers cannot read detailed instructions as they listen to and interpret learners and manage the trajectory of content through the discourse and activity of the class. How then can their decision-making be supported? Further, how can additional examples, questions, and guidance be designed in ways that are usable in practice? Another concern regards learners: they say and do many things that are predictable and patterned; they also produce unexpected and novel ideas and conceptions. Designers can seek to provide forecasts and guidance for the predictable and open teachers' readiness for the unanticipated. Thoughtfulness here can increase the support provided through design. And other issue pertains to the content: the mathematics itself is often also complex, and support for teachers' learning often weak. Designers can develop usable opportunities for teachers' own learning, but how can this be done well? Building curriculum resources with an eye toward their potential to support teachers' development requires a multifocal approach, with an eye on the mathematics, on learners, on teachers, and on their learning and interactions. Such demands point to the importance of educationally oriented design, based on backward mapping from an understanding of practice and of resources-in-use, or "lived resources," in order to support resource use for improved learning. Figure 2 proposes an expansion of the instructional dynamic represented in Fig. 1, which affords a view of the dynamics of supporting instruction and of teachers'

**Fig. 2** The instructional triangle of teachers' learning in and from practice



professional learning of and from practice. This learning occurs through interaction with resources and with other professionals, as well as in and from practice itself.

At the heart of improving learning is to understand instruction as the complex weave of interactions and interpretations. It is on this foundation that this volume provides rich analyses and examples for the development of curriculum resources designed to be used, or “lived,” and from which both teachers and their pupils can learn. Bringing together such sophisticated design with a detailed understanding of practice can contribute to both better research on curriculum and its use, as well as better resources for use, and better outcomes for learners. This volume is itself a wonderful resource for this important agenda.

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# Conclusions

Ghislaine Gueudet, Birgit Pepin, and Luc Trouche

## Dynamic and Nested Documentation Systems in/for Teaching and Teacher Learning: Re-conceptualising Mathematics Curriculum Resources and Their Use

“An organised being is then not a mere machine, for that has merely *moving* power, but it possesses in itself *formative* power of a self-propagating kind which it communicates to its materials though they have it not of themselves; it organises them, in fact, and this cannot be explained by mere mechanical faculty of motion.” Immanuel Kant (in “Critique of judgment”)

Reading through the 17 chapters and reactions in the book, we are impressed by the rich and varied perspectives provided by the authors and reactants. This underlines the fruitfulness of the position proposed at the beginning of this book: viewing teachers as designers and creative users of their own resources, considering the implications of teacher ‘interactions’ with resources for teacher professional development and hence the deepening of our understanding of ‘teacher documentation’.

The authors have considered a great variety of resources, encompassing and re-conceptualising artefacts and tools: from clay tablets, to textbooks and websites, including student work, and language; to name but a few. They have explored these resources in a creative and encompassing way, and their findings evidence the richness that lies in seeing resources as ‘lived resources’, when teachers work with them in their resource systems, and how these processes become part of teacher professional development.

In this respect the use of digital resources raises particular questions. For example, some software is difficult to integrate into a teacher’s resource system, whilst other online resources are widely used and contribute to create new networks and

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G. Gueudet (✉)

CREAD, Université de Bretagne Occidentale, IUFM Bretagne site de Rennes, 35043 Rennes Cedex, France

e-mail: ghislaine.gueudet@bretagne.iufm.fr

communities. Studying interactions between teachers and (digital) resources, and various types of collectives, helps to deepen our understandings of such phenomena.

This book has also provided evidence for the contextual nature of the processes involved in teacher–resource interactions. Interestingly, whilst being influenced by the context, there is evidence that these interactions are not restricted to particular countries considered to be ‘developed’. Furthermore, in any context, teacher documentation work, teacher agency and their interaction with teacher knowledge and professional development are evident.

The aim of this conclusion chapter is to draw together the book’s chapters and synthesise the main results, and hence develop a deeper understanding of ‘teacher documentation’ as a construct and with respect to teacher learning. We propose four key issues that permeate the four sections of the book:

1. The intentions and affordances of a resource in terms of its use;
2. The adaptation, appropriation and work with resources;
3. From resources to orchestration and collaborative use of resources;
4. Interrelations between documentation process and teacher knowledge.

In the following, we will attend to the four themes in turn, giving and relating to examples from the book’s chapters, before providing the conclusive remarks.

## **Theme 1: The Characteristics, Intentions and Affordances of a Resource in Terms of Its ‘Use’**

We regard the affordances of a resource as the attributes and characteristics of the resource which provide potential for its use with peers/colleagues and students/pupils in the course of teachers’ work. This means that by virtue of their support for particular actions in a setting, the affordances may foster particular actions, and inhibit other actions which are less desirable. However, affordances of resources must also be considered in relation to the intentions of the participants in the activity they support. Thus, affordances, in this view, are ‘potentials’ or pre-conditions for activity. A particular resource provides an affordance for some activity; this does not imply that the activity will occur, although it may contribute to the likelihood of that activity. Additional conditions include ‘characteristics’ of an ‘agent’, that is beliefs and principles of practice of the teacher with respect to the resource (and its affordances). An example is Ruthven’s (Chapter 5) description of the ‘resource system’ and its affordances, shaping the integration of technology, or indeed, in Pepin’s chapter (Chapter 7), the (mathematical task analysis) tool’s affordances in terms of reflection and feedback. Schmidt (Chapter 8) investigates the affordances of school mathematics textbooks in terms of opportunities to learn demanding and engaging mathematics, and hence students having different experiences in school mathematics courses. The most striking example is probably provided by Proust (Chapter 9), who examines ancient Mesopotamian resources.

Here the resources, the master and the tablet (including the text), may have different affordances and intentions (as defined in [Chapter 4](#)), depending on who has written the tablet and for what purpose (e.g. mathematics teaching, providing cultural background). Interestingly, Remillard ([Chapter 6](#)) uses the notion of ‘positioning’ to analyse affordances of curriculum materials. She contends that curriculum materials have particular ‘modes of address’, ways of ‘talking’ to teachers, and that these prescribe particular roles for teachers. This links to Sensevy’s contention ([Chapter 3](#)) that documents have particular ‘pedagogic intentions’. The question remains where the agency of the teacher lies.

## **Theme 2: The Adaptation, Appropriation and Work with the Resource – Its ‘Use’**

This theme is at the heart of the documentation process and runs through most of the book’s chapters. It relates to the *instrumental approach* introduced by Verillon and Rabardel (1995) where the subject (in our case, the teacher) plays a crucial role in creating, modifying and using tools as instruments. Verillon and Rabardel claim that *instruments* are created when they are used and integrated into the subject’s activities – this process, the *instrumental genesis*, is linked to the tool’s characteristics and affordances (or constraints) and to the subject/teacher’s knowledge and principles of practice. According to this approach, there is an inter-relationship between the tool and the subject/teacher: the subject/teacher uses the tool and in the process evolves and develops, and in turn the *instrument* evolves. Two processes are crucial here: *instrumentation*, that is the implicit modes of actions and knowledge, and *instrumentalisation*, that is how the subject/teacher shapes the tool. In [Chapter 2](#), Gueudet and Trouche develop these ideas in their *documentational genesis* approach where teachers interact with resources, select and work with/on them. The work in [Chapter 4](#) (by Mariotti & Maracci) is sensitive to the semiotic aspects and potential of an artefact, and the authors explore how such an artefact (e.g. ICT tool) can be a resource for the teacher. In [Chapter 13](#) Trigueros and Lozano describe a case of *documentational genesis* when working with teachers in ‘Enciclomedia’: teachers analysed and transformed texts in particular ways due to the resources affordances. In [Chapter 7](#) by Pepin teachers’ work with the tool changed the tool, to become a ‘catalytic tool’, and in the process it changes its character, from tool as artefact to ‘epistemic object’ at the interface between task design and enactment. Kieran et al. ([Chapter 10](#)) theorise how teachers adapt ‘researcher-designed’ resources considering teachers’ own beliefs, knowledge and principles of practice.

## **Theme 3: From Resources to Orchestration and Collaborative Use of Resources**

Documentation can be considered ([Chapters 2 and 16](#)) as a continuous process, the work in class being only one of its components. However, in most contexts, most of the interactions between teachers and students appear to happen in class. This in turn



confers a particular status to the classroom and leads us to focus on the *orchestration* of resources as a central part of the documentation process. Originally introduced by Trouche (2004), *orchestration* can here be at different levels: at the level of documents, or sets of documents, or at the level of the participant (e.g. teacher) working with, and relating to, the documents/sets of documents. In [Chapter 14](#) Drijvers reports on teachers privileging orchestrations where students work individually or in pairs, and he contends that teacher beliefs and agency play an important role in the development and enactment of the processes involved in transforming resources into orchestrations. The collaborative use of resources relates to collaborative work of teachers in terms of resources and in the larger frame of scaling-up of the process of documentation and use of resources. In terms of teacher learning, collaborative use of resources is illustrated when groups of teachers work together on documents (likely to be important for their teaching) to analyse, search for understanding and meaning, and to create a common resource of their learning. Sensevy ([Chapter 3](#)) develops an understanding of collective thought (influenced by the *institutional thought style*) by identifying ‘patterns of didactic intentions’ which in fact are said to lie in the documents (used by teachers) and the positioning of teachers towards these documents. Linking this to ICT communication, collaborative learning networks can develop, via electronic dialogue, and where participants share a common purpose of/for documentation. In [Chapter 16](#) (Gueudet & Trouche) the common ‘purpose’ is Sésamath, both an individual and a collective resource. The processes involved in collective documentation are exemplified by Gueudet and Trouche, when ‘sharing’ turns into ‘cooperation & sharing’, into ‘collaboration & cooperation & sharing’ before another cycle develops. The scaling-up collaborative process is evident in [Chapter 17](#) (Visnovska, Cobb, & Dean), where the authors drew on a five-year-long interventionist professional development study where teachers collectively (e.g. in a professional development group) designed resources for teaching of a statistics unit and at the same time made meaning of the objectives prescribed by the State. Interestingly, Winsløw ([Chapter 15](#)) compared two very different genres of teacher collaborative work (using the frame of paradidactic infrastructure): the Japanese lesson study and the Danish teacher collaboration in ‘multidisciplinary modules’. He concludes that collaborative work forms, also for documentation work, are influenced by the cultural and educational traditions of the country concerned and that particular practices would be ‘unthinkable’ in certain environments, whereas in others they are common practice – hence the importance and influence of the context in which the documentation process is taking place.

#### **Theme 4: Interrelations Between Documentation Process and Teacher Knowledge**

Teachers working with resources, we have presumed, is an interactive and dialectic process: teachers shape the resources, and the documentation processes involved influence teachers in turn. Teachers, it is argued, develop deeper understandings with

respect to particular resources, and they may adopt new roles in their interactions with the resources initiating or constructing new processes in terms of learning situations, or indeed they may communicate and interact in particular collaborative ways with their colleagues – all acts of teacher learning that are connected to the documentation process. In [Chapter 1](#), Adler argues for ‘professional knowledge in use’, and in her study illuminates ‘knowledge resources in use’ in two different pedagogic practices. Pepin ([Chapter 7](#)) claims that the task analysis ‘tool’ provided feedback to teachers, at four different levels, and in turn helped them to develop deeper understandings. Interestingly, Forest and Mercier ([Chapter 11](#)) provide evidence for using video as a tool for professional development, in particular considering the teacher’s attitudes and gestures as resources and connecting them with the use of language in the mathematics classroom.

Whilst these four themes capture most of the authors’ work, there is a permeating strand that runs through all of the chapters: the pupils’ influence and involvement in the documentation process. As an example, Rezat’s work ([Chapter 12](#)) considers the orchestration of resources in and outside the classroom when exploring pupil/student use of the textbook as resource, which in turn is said to have an influence on teacher use. Interestingly, Sensevy ([Chapter 3](#)), as well as Forest and Mercier ([Chapter 11](#)), conceptualise the teaching processes as the joint didactic action of the teacher and the students. Schmidt ([Chapter 8](#)) develops a way to quantify student curricular experiences in different courses, their exposure to particular curriculum materials, which in turn is likely to have an influence on their opportunities to work with and learn from mathematics resources.

## Concluding Remarks and Looking Ahead

Considering the issues raised by authors and reactants, one wonders what makes a ‘documentation system’, and how does such a system evolve? It seems that the key factors that can be argued to explain the ‘workability’ of a documentation system are the nature of the system, its constituents and the feedback ‘loops’ that characterise and shape such a system. In each study an important step to develop a documentation system appeared to have been when reflective capacity was built, such as between teachers and resources, and/or amongst peers, and/or between teachers and academics. With this reflective capacity, the participants of the system had information about the nature of the resources and their potential dynamics (also with participants). However, it is not evident that this reflective capacity develops as a matter of course. As Visnovska et al. ([Chapter 17](#)) point out, teachers need support to design and implement ‘coherent instructional sequences’. Moreover, the participants of such a ‘workable’ documentation system need a shared purpose (see Kieran et al.’s [Chapter 10](#)), and it appears to develop more ‘easily’ in collectives (see [Chapters 3](#) and [16](#)). It can be argued that the documentation system needs a ‘minding of the system’ (Vickers, 1995) in order to be workable.

Systems such as documentation systems have to be acknowledged to be inherently turbulent, and also inherently unique in the way that they adapt to external intervention (such as inclusion of web books, web-based learning groups, etc.). The different sub-systems can be regarded as inter-dependent, or inter-related – the concept often used is that of *nested systems*, with each system nested within a larger one.

This book constitutes one step in an ongoing work, and the key issues and results outlined above need to be further investigated. Considering the new perspectives crucial questions emerge and these need further investigation:

- (1) Many resources are available for mathematics teachers, but which resources do they crucially need for their work? Are there resources that could be regarded ‘universal’, as ‘resources of the (mathematics teaching) profession’? What are the national and cultural differences among resources, what are the individual differences? In which ways could such resources be designed, and differences catered for? How could they be made available to all teachers (e.g. ‘broadcasted’)?
- (2) Considering Shulman’s (1986) major categories of teacher knowledge in connection with Ball, Thames, & Phelps’s (2008) categories, one wonders where the ‘documentation process knowledge’ is situated. In particular its dynamic and creative nature, in addition to its ‘position’ at the interface between design and enactment, does not make it ‘fit in easily’. We contend that an additional teacher knowledge category (perhaps ‘hors categorie’) may be necessary, which we call *documentation knowledge* and which would include knowledge about resources/materials in use, individually or collectively, and their interaction with the teaching/learning process of both teachers and learners (including the teacher as learner).
- (3) All the book’s chapters focus on the teaching of mathematics. In mathematics the documentation work of teacher educators, or of mathematicians (Chapter 9), may be similar, or different, to teachers’ documentation work. Turning to other subject areas, similar (or different) phenomena may be evident for teacher documentation work in other domains. Investigating these is likely to deepen our understandings of the documentation process.

In conclusion, closing the cycle and linking to the book’s title, we have developed deeper understandings about mathematics curriculum materials as ‘lived’ resources – which points to their use in the past. We now suggest viewing them as ‘living resources’ emphasising their present and continuous use in teachers’ work. Teacher documentation, we have learnt, is a creative and dynamic process where participants work in a collaborative system and with the aim of teacher learning – this provides challenges, and at the same time a positive outlook both for teachers and reformers.

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