Maintenance optimization for multi-component systems under condition monitoring

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Maintenance Optimization for Multi-component Systems under Condition Monitoring

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Contents

Chapter 1

Introduction

"An ounce of prevention is worth a pound of cure."

Benjamin Franklin

1.1 Definition of Failures and Maintenance Actions under Various Maintenance Policies

When a system or component is unable to deliver its satisfactory function, we define this undesirable situation as a failure [26]. In practice, two types of failures can be recognized: hard failure and soft failure [36, 50]. When a component stops functioning or the system breaks down, such a failure is a hard failure. For example, power outage due to a short circuit can lead to the complete down of a production system. When a system or component can continue its operation with a lower performance compared with its normal standard, such a failure is a soft failure. For example, a worn bearing connecting to the main shaft in a wind turbine will reduce the efficiency of energy generation, while the wind turbine is still operating. This lower performance results in a loss in terms of revenue generation, which we may also see as quality loss costs.

To restore a failed system or component to its satisfactory performance level, a relatively expensive corrective maintenance (CM) action is taken, which also results in unplanned downtime. Instead of waiting for failures to happen and taking CM actions, a more proactive approach is to take preventive maintenance (PM) actions before failures happen, which is often cheaper than taking CM actions. However, the drawback of a PM action is that components are replaced earlier than their actual failure time. In this case, only part of their life durations is utilized and remaining useful lifetimes of components are wasted. To avoid high wastes of remaining useful lifetimes, the actual degradations of components should be monitored. According to the actual conditions of the degradation, also known as condition-based maintenance policies, one can take PM actions at much later moments than in the situation of age-based maintenance policies, so that less remaining useful lifetimes are wasted. The details of age-based and condition-based maintenance policies are described in Subsection 1.3.

1.2 Maintenance Costs and Service Contracts in the High-Tech Industry

In this thesis, we focus on the high-tech industry. When the maintenance/downtime costs of equipments with long life cycles are very high, there is a huge economic incentive to improve/optimize the maintenance policy [38]. According to the research in 2006 by Cohen et al. [12], "American business and consumers spend approximately 1 trillion Dollars on the assets they already own". Hence, original equipment manufacturers (OEMs) in the high-tech industry consider maintenance and services in the after-sales market as very important issues.

Service contracts are often used in the high-tech industry. This means that operators do not only buy equipments from OEMs, but also packages/contracts covering the maintenance/services for the long operational period of the equipments. In this case, the after-sales services can be guaranteed or even regulated in the service contracts, which can result in high system availability/uptime for the operator. Under such contracts, OEMs have a stronger motivation to improve the quality of their products and to optimize the maintenance policy (e.g., by implementing more advanced preventive maintenance actions and techniques). These service contracts can lead to win-win results for both OEMs and operators. Hence, it is beneficial to improve maintenance policies for both OEMs and operators.

High-tech capital goods (e.g., aircraft engines, wind turbines, semiconductor production systems, MRI scanners) nowadays have high production efficiencies and long life times. The common trends of these complex high-tech systems are: (i) the structure of the system and the dependency between components are very complicated, so that it becomes harder or even impossible for operators to do quick-and-easy maintenance by themselves; (ii) it is very expensive when a system is down; (iii) while buying new systems, operators consider total cost of ownership. During the long exploitation phases of capital goods, the maintenance costs including downtime and setup costs are often very high (e.g., twice as high as the purchase price in the case study reported in Öner et al. [38]). Two types of costs can be very high in the high-tech industry:

- *Downtime costs*: Because of the high production efficiency of high-tech systems, interruptions are very costly. Every hour of downtime can easily cause a profit reduction of many thousands of Euros.
- *Setup costs*: The fixed cost before taking maintenance actions. For example, it may be expensive and time-consuming to send maintenance personnel and equipments to a remote site.

As shown in Table 1.1, these costs happen to different systems in various high-tech industries.

Compared with notoriously expensive downtime, much less attention is drawn to the high setup costs for maintenance of these complex high-tech systems. Due to a large amount of components in the systems, it is often economically beneficial to perform maintenance actions of multiple components simultaneously. Hence, we focus on the setup of maintenance actions in this research, which are very expensive in the high-tech industry. When maintenance actions are taken, a maintenance crew and equipments have to be sent to the field and the operation of the system is interrupted. Consequently, a high fixed setup cost is charged for the visited system. This setup cost refers to a fixed cost that is incurred for a maintenance visit regardless

Table 1.1 Downtime costs and setup costs in the high-tech industries

Complex systems	Downtime costs	Setup costs
Wind turbines in the energy industry	Loss of energy generation	Sending maintenance crews and equipments to the field by jack-up vessels or helicopters
Airplanes and trains in the transportation industry	Loss of goodwill, compensations to passengers, delays on other tracks	Sending airplanes or trains to the maintenance depot for repair.
MRI scanners in the health care industry	Loss of patients and even human lives	Sending maintenance crews and equipments
Micro chips production systems	Loss of production time	Maintenance crews need time to enter the clean room.

what maintenance actions are performed. For a production line, it includes the cost of sending a maintenance team to the site, stopping the production, resetting the production environment, etc. For a transportation system (e.g., aircrafts, trains or trucks), it includes the cost of bringing the system to a maintenance depot/hangar. These complex high-tech systems consist of large numbers of components. Hence, it is often economically beneficial to perform maintenance actions of multiple components simultaneously. If we decide to take a maintenance visit for a single component, we need to pay such a fixed setup cost per visit. However, if we decide to take a maintenance visit to conduct the maintenance activities for several components at one joint maintenance interval, we only have the setup cost once.

Various maintenance policies are proposed to schedule the joint maintenance actions in multi-component systems, in order to minimize cost. These policies are also different from one to another, due to different maintenance techniques described in Subsection 1.3.

1.3 Maintenance Policies and Underlying Techniques

Regarding the policies used in maintenance, we distinguish three categories in our research, i.e., condition-based, age-based (or time-based) and failure-based maintenance policies. According to Jardine et al. [25], the earliest maintenance technique is run-to-failure or failure-based, where a corrective maintenance action is taken only at each time that a failure happens. Under such a failure-based maintenance (FBM) policy, the maintenance actions are unplanned and the system downtime is unscheduled, which may be very expensive. To avoid expensive failures, preventive maintenance actions are planned either at a certain moment in time or based on the age of components/ systems. In this case, one can use the life time distribution to optimize the maintenance policy, which is also known as age-based (or time-based) maintenance optimization. Compared with a FBM policy, there are more planned maintenance actions and scheduled system downtimes under an age-based maintenance (ABM) policy. However, the physical degradation (e.g., temperature of engine, wearing of a brake) of components/ systems is not considered to determine the actual health status. The rapid development of advanced sensor and ICT technology makes the remote acquisition of condition monitoring data less costly. Based on the condition data, a large amount of unnecessary maintenance tasks can be avoided, by taking maintenance actions only when the physical degradations of the critical conditions are evident. This is known as condition-based maintenance (CBM), which helps to reduce maintenance related costs further [25, 42]. Hence, considerable attention from researchers has been drawn to study CBM policies.

In a CBM framework, there are several key steps, i.e., data acquisition, data processing, diagnostics and prognostics. Using the information from prognostics, maintenance decisions can be optimized to minimize the maintenance costs or maximize the availability/reliability of systems [25]. The main difference between the conventional maintenance models and CBM models is the utilization of condition measurements [42]. In this case, a failure is identified when the degradation level of a component reaches its failure threshold level. Consequently, an expensive CM action is triggered. To have cheaper PM actions before its degradation exceeds its failure threshold level, a control limit (lower than the failure threshold level) on the degradation of each component is introduced. On one hand, if the control limit is high, the risk that failures occur will be higher, which is expensive. On the other hand, if the control limit is low, the component will be maintained much earlier than necessary, which is also expensive. Due to this trade-off, it is important to determine the optimal control limit to minimize the maintenance cost at the component level.

Regarding degradation modeling, Si *et al.* [49] distinguished two types of probability models of RUL estimation: directly observed CBM models (e.g., regression-based models [33], Wiener processes [21], Gamma processes [59], Markovian-based models [27]) and indirectly observed CBM models (e.g., stochastic filtering-based models [63], covariation-based hazard models [60], hidden Markov models [32]). In this research, we consider the situation that the degradation data can be observed continuously by micro-sensors, which is a typical feature of systems in the industry of advance capital goods. When a specific physics of failures is given by the engineering department of the companies, a random coefficient model [33] can be used, due to its flexibility of describing various degradation paths derived from laws of mechanical engineering and material science. Alternatively, the Gamma process [59] is a very popular tool to model the degradation in literature, due to its mathematical properties (e.g., memoryless, additive, etc). We use both the random coefficient model and the Gamma process to model the degradation path in our research (for more details, see Appendix A).

Our research objective is to develop maintenance optimization models for multi-component systems based on remotely monitored condition data, which helps to minimize the average cost rate of the entire systems in a long run.

1.4 Literature Review

In the existing literature on age-based and condition-based maintenance, much more attention has been paid to single-component problems (blocks A and E in Table 1.2) than to multi-component problems (blocks B, C and D in Table 1.2) [11, 17, 25, 37, 42, 51, 61]. As mentioned in our research objective, we focus on maintenance policies for multi-component systems. According to Wang's review paper [61], multi-component maintenance optimization models can be classified as group maintenance policies and opportunistic maintenance policies (see Table 1.2). Group maintenance models are often used to cluster components in a system into different sub groups/coalitions, where joint maintenance actions are taken. In practice, we also observe that the failures of a few critical components have large impacts in terms of costs. In this case, the opportunistic maintenance policies are more useful for synchronizing the maintenance actions of those components, together with the maintenance actions of the entire systems.

Table 1.2 Categories of literature

	Single-component	Multi-component	
		Opportunistic maintenance	Group maintenance
Condition-based	A	B (Chapter 3)	C (Chapter 2)
Age-based	D	E (Chapter 4)	F
Mixed	-	-	G (Chapter 5)

For condition-based maintenance for single components (block A in Table 1.2), various methods are used to model degradation paths. According to Si *et al.* [49], diverse degradation models in the literature can be classified as indirectly observed CBM models and directly observed CBM models. Regarding the indirectly observed CBM models, the condition of components/systems are not physical parameters (e.g., temperature, pressure, etc) that can be observed directly. For example, the proportional hazard model (PHM) is a popular model that is often used to relate the system's condition to the hazard function of a system, so that the maintenance policies can be optimized with respect to the optimal risk value of the hazard function [24, 60]. In contrast, the condition in the literature of directly observed CBM models are physical parameters. Within the directly observed CBM models, the degradation can be modeled by diverse methods. For monotonic stochastic deteriorations, the Gamma process is a popular degradation model in the literature [59]. Based on the Gamma process, the CBM models were developed to have a single-level control limit [18, 40, 41] or a multi-level control limit [22] under the scenarios of periodic inspection [40], aperiodic inspection [18, 22] or continuous monitoring [31, 41]. If the physical degradation is based on the physics of failures, the random coefficient model (RCM) [33] is a more convenient model to describe degradation formula from mechanical engineering or law of physics. Based on the RCM model, Wang [62] proposed a CBM model to determine the optimal control limit and the inspection interval in terms of cost, downtime and reliability. Gebraeel *et al.* [21] extended the general degradation model to estimate the RUL distribution from sensor signals, using a Wiener process and Bayesian updating. Using this technique, a single-unit replacement problem is formulated as a Markov decision process to develop a structured replacement policy [20]. For the degradation process modeled by discrete states, Markovian-based models are often applied. The optimal replacement policies were derived from observable Markov processes [27, 34] or the evolution of the hidden states [7, 32]. More CBM literature can be found in review papers within the area of prognostics [25, 42, 61].

Although many models have been proposed for single-component systems, they cannot be applied directly for multi-component systems, because one has to deal with the economic, structural and/or stochastic dependencies among the components [11, 17, 37, 51]. According to Wang's review paper [61], the CBM literature for multi-component systems is classified into block B and C in Table 1.2. Regarding the group maintenance models in block C, Bouvard *et al.* [6] converted a condition-based maintenance problem into a similar age-based maintenance clustering problem [65], which yielded an optimal schedule with a dynamic maintenance interval. By including a random failure threshold and imperfect maintenance as an extension of Bouvard *et al.*, van Horenbeek and Pintelon [58] proposed a dynamic scheduling model, based on simulation. They also compared their maintenance policy with five different policies, which shows a significant cost savings. Wijnmalen and Hontelez [64] used a heuristic algorithm to compute control limits for components in systems under different discounted scenarios, which is formulated within a Markov decision framework. Castanier *et al.* [10] introduced a model to coordinate inspection/replacement of a two-component system via a Markov renewal process and minimize the long-run maintenance cost. However, it becomes intractable for extending to systems consisting of a large amount of components. Similarly, Barata *et al.* [2] simulated the degradation process by Monte Carlo simulation and optimized the control limit of each component in a two-component series system. To solve large-scale problems for systems with many components, Marseguerra *et al.* [35] used Monte Carlo simulation and Genetic Algorithms. They used a Markov model to define the state transition probability of the condition degradation and formulate a multi-objective optimization problem with control limits on the states of components. Alternatively, Tian *et al.* proposed two maintenance policies for multi-component systems using a Proportional Hazard Model [53] and an Artificial Neural Network [52], where the control limits of components are not based on the degradations of their physical conditions. By assuming identical components in the system, they [53] studied two-component and three component systems respectively. The other model [52] optimized the control limits of failure probabilities based on simulation, where a case study of wind turbines is performed. Regarding the opportunistic maintenance model in block B in Table 1.2, it is surprising to find that very little attention has been paid on CBM models in the context of opportunistic maintenance, which also coincides with the findings of Koochaki *et al.* [28]. Koochaki *et al.* studied the cost effectiveness of condition-based and age-based opportunistic maintenance policy, by considering a three-component series system via simulation.

Compared with the existing CBM models for multi-component systems, there is more abundant literature about age/time-based maintenance models for multi-component systems. Similarly, we use the same classification of Wang's review paper [61] (see blocks E and F in Table 1.2).

Regarding the opportunistic maintenance models in block E, many models considered either scheduled or unscheduled opportunities. For the models with unscheduled opportunities only, Radner and Jorgenson [45] introduced an (n, N) policy with a proof of optimality. They distinguished two types of components, 0 and 1, where n is the age threshold for opportunistic replacements of component 0 when component 1 fails and N is the preventive replacement threshold of component 0 when component 1 is good. Zheng [67] introduced a $(T-w, T)$ policy. If the ages of components exceed T, preventive maintenance actions will be taken, which are also considered to be opportunities to preventively replace other components with their ages between $T-w$ and T. This policy is similar to the (n, N) policy, but based on renewal theory. Pham and Wang [43] proposed a (τ, T) policy. According to this policy, no preventive maintenance is taken and only minimum repairs are performed on failures in the period $(0, \tau]$. In the period $(\tau, T]$, if k components fail, those k components are replaced and all other components are preventively replaced; if there are less than k failed components in the system, all components are preventively replaced at time point T. Moreover, Dagpunar[13] proposed a policy with a control limit on age, based on assuming opportunity process is Poisson. Similarly, exponentially distributed times between opportunities were assumed by Dekker and Dijkstra [15]. As a different policy, they proposed a so-called "one-opportunity-look-ahead" policy;

namely, one can make a decision on taking either the current opportunity or the next opportunity after X time units, based upon a marginal cost function and the distribution of time between maintenance opportunities. For the model with scheduled opportunities only, Dekker and Smeitink extended this "one-opportunity-look-ahead" policy in the context of block replacement. Regarding the literature including both scheduled and unscheduled opportunities, two models [29, 50] are able to solve the optimization problem of small-scale systems by simulation. Taghipour and Banjevic [50] proposed a model that considers both scheduled inspection and non-scheduled failures of systems as opportunities to perform inspections on soft-failure components. For hard failure components, preventive maintenance actions are taken at scheduled inspections depending on their condition. Its objective function in a finite-horizon setting is evaluated by a simulation algorithm. Similarly, Laggoune *et al.* [29] developed a different dynamic clustering model based on simulation. In this model, preventive maintenance is scheduled at each fixed time point $k\tau, k \in \mathbb{N}$, and each component j can be preventively replaced at a multiple of τ, $k_j\tau, k_j \in \mathbb{N}$. If unscheduled system downs occur, a decision on taking the opportunity or not will be made, according to marginal costs.

Regarding the group maintenance models in block F, we can categorize them in two types: exact methods and heuristics. The exact methods are often aimed for the analytical results of optimality and insights, where the numbers of components in systems are limited. For systems with a large number of components, the exact models become intractable. In contrast, various heuristics are proposed to solve the optimization problem for systems with a large number of components. Regarding the exact methods, the models of Haurie [23] and Ozekici [39] are able to find the exact optimal solution for small scale systems (e.g., two-component system). For the k-out-of-n systems, Popova and Wilson [44] provided a (k, T) policy. This policy suggests the replacement of all components either at the time of the kth failure or time T, whichever occurs first. Close-form results are shown in the case of a three-component system. A generalized group maintenance policy $(T, T + w, k)$ was introduced [48], which includes k failures as a decision variable also. In the period $(0, T]$, this policy distinguishes two types of failures: i) "minor" failures that will be fixed by minimum repair and ii) "catastrophic" failures that will be fixed by replacements. In the period $(T, T + w]$, if k "catastrophic" failures happen, all components are jointly replaced; otherwise, this joint maintenance will be delayed till $T + w$. To reduce the complexity of the large-scale optimization problem, Dekker and Wildeman [17, 65] developed a maintenance clustering method to coordinate maintenance tasks at the system level, considering the penalty cost of deviating with the maintenance schedule from the optimal maintenance interval of individual components. By assuming the expected deterioration cost function based on a Weibull process, they proved the structure of their clustering policy is optimal. As alternatives to reduce the computational complexity, various heuristics were proposed [1, 55, 56]. Van der Duyn Schouten and Vanneste developed a model for systems consisting of M identical components [55]. The accuracy of approximate results was validated via comparison with simulation. In the work of van Dijkhuizen and van Harten [56], a greedy-heuristic with a branch and bound procedure was proposed. Based on 100 randomly generated test problem with 10 set-up and 30 maintenance jobs, the heuristic was able to find the optimal solutions in 47 out 100 tests. The average computation time was in terns of seconds. Moreover, as the extension of block replacement policy, Berg and Epstein [4] introduced a (b, t) model, where t is the fixed maintenance interval and b is a control limit in terms of age. At each point $nt, n \in \mathbb{N}$, preventive maintenance is performed on the components whose ages are larger than b. A heuristic was also developed to extend the (b, t) model for multi-component systems [1].

To sum up, there is much less literature at the multi-component level on condition-based maintenance (block B and C) than on age/time-based maintenance models (block E and F). To the best of our knowledge, there is no literature considering a system that consists of a large number of components with a mixture of CBM, ABM and FBM policies (block G).

1.5 Contribution and Structure of the Thesis

The overall position of this entire research is pinpointed in Table 1.2. To optimize maintenance policies of different complex systems with different features, we developed four models [68, 69, 70, 71] in four chapters respectively. In this section, the difference and connection with our models are described. Each model has its unique scientific contribution, which is briefly explained. The more detailed comparison with its relevant literature for each model is elaborated in the introduction of each chapter (see Sections 2.1, 3.1, 4.1 and 5.1).

According to the classification of Wang's review paper [61], our research outcomes are two group maintenance models for multi-component systems (see Chapter 2 and 5) and two opportunistic maintenance models (see Chapter 3 and 4). To develop the model in Chapter 5, we develop these two opportunistic maintenance models for a single component in a multi-component system as building blocks (see Chapter 3 and 4). In Chapter 2, we consider soft failures and quality loss cost, and only allow maintenance actions at predetermined scheduled downs with a static time interval. In contrast, we consider hard failures in Chapter 5. In this case, system downs occur not only at predetermined scheduled downs, but also at the unscheduled moments that the hard failures of other components in the system occur. These scheduled and unscheduled downs are considered as opportunities for preventive maintenance, which generates dynamic intervals for the joint maintenance actions. Moreover, the system in Chapter 2 consists of condition-based components only, while the system in Chapter 5 has a mixture of CBM, ABM and FBM components. As the building blocks of the model in Chapter 5, Chapter 3 describes a CBM model and Chapter 4 describes an ABM model. Figure 1.1 provides an overview on the structure of the thesis.

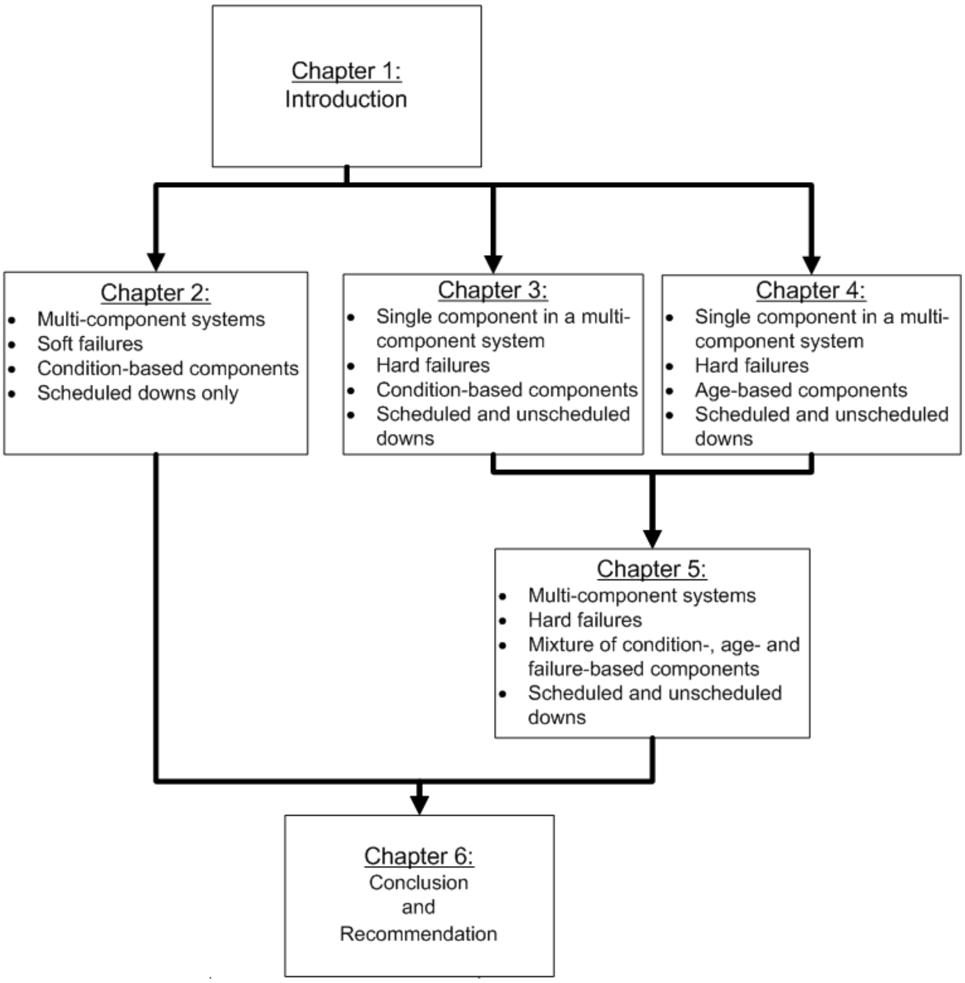


Figure 1.1 Structure of the thesis

Regarding the contribution of Chapter 2, we propose a new CBM model for multi-component systems consisting of a large number of components. To reduce the high setup cost of maintenance, a joint maintenance interval is introduced by setting up periodic scheduled downs to take maintenance actions of multiple components simultaneously. We assume all failures of components are soft failures with quality/performance loss costs per time unit, before maintenance actions are taken at the scheduled downs. In this case, we can decompose the main problem at the system level to subproblems at the component level, which allows exact evaluation of the objective function (i.e., the average cost rate). With the maintenance interval at the system level and the control limit on the degradation of each component as decision variables, we develop a model to minimize the long-run average maintenance cost rate of the systems. Moreover, a numerical study of a production system consisting of a large number of non-identical components is presented, including a sensitivity analysis. Finally, our policy is compared to a failure-based policy and an age-based policy, in order to evaluate the cost-saving potential.

Regarding the contribution of Chapter 3, to the best of our knowledge, our policy is the first opportunistic CBM policy that considers both the scheduled and unscheduled downs of a complex system as free opportunities. In practice, unscheduled downs can also happen if hard failures occur for other components, instead of soft failures as assumed in Chapter 2. Both scheduled and unscheduled downs can be considered as free opportunities for monitored components to perform preventive maintenance, so that no additional setup cost and downtime cost are charged on the monitored component. This model determines the optimal control limit of a critical component monitored continuously, in order to decide the timing of taking opportunistic maintenance, where the long-run average cost rate is minimized. In this chapter, the end points of maintenance actions are not always at scheduled system downs, because the scheduled downs are not rescheduled after each maintenance action of the CBM component. Hence, our cost rate evaluation based on renewal theory is not exact, but approximate. The accuracy of this approximation is verified via the comparison of approximate results and simulated results. Moreover, a case study on lithography machines in the semiconductor industry is provided. Finally, numerical experiments are performed to investigate the accuracy of our model and the cost-saving potential of our optimal policy under various parameter settings.

As explained in the existing literature of opportunistic ABM models in block E in Table 1.2, most of them considered either scheduled or unscheduled opportunities. Only two simulation-based models consider both types of opportunities. The simulation-based models have computational limitations for systems consisting of a large number of components. However, the model proposed in Chapter 4, with both the scheduled and unscheduled opportunities, can be used as a building block to solve optimization problem for systems consisting of a large number of components. In practice, it is not always feasible to monitor components remotely, due to physical constraints from the design of the system. For example, a sensor can hardly be installed in an enclosed space (e.g., inside a gearbox). In this case, ABM may be implemented, instead of CBM as in Chapter 3. Hence, we develop this ABM model in Chapter 4 to optimize the age limit on the age of the age-based component, instead of the control limit on the degradation of the condition-based component. The optimal age limit helps to decide the timing of taking opportunities, in order to minimize the average long-run cost rate. Moreover, a numerical study is conducted to show the usage of the model. Similar to the CBM model in Chapter 3, the cost evaluation is also an approximation and its accuracy is verified via the comparison with simulated results. Under various numerical experiments, we investigate the accuracy of our model and the cost-saving potential of our optimal policy.

In the high-tech industry, we observe that complex engineering systems are often with a mixture of components under different maintenance policies (e.g., ABM, CBM and FBM). However, to the best of our knowledge, this maintenance optimization problem with the coordination of maintenance actions under the different policies (block G in Table 1.2) has not been studied in the literature. The model proposed in Chapter 5 is able to solve maintenance optimization problems for a system with such a mixture of components under CBM, ABM and FBM policies, by using the CBM and ABM model in Chapter 3 and 4 as building blocks. This mixture also can better represent multi-component systems in real life, which usually consist of components under different maintenance disciplines. To be able to solve large-scale problems in real life, where systems consist of large numbers of components, we develop a maintenance optimization model with a heuristic procedure to optimize 1) the control limits of condition-based components, 2) the age limits of age-based components, and 3) the maintenance interval for scheduled downs of the entire system. Similar to Chapter 2, we decompose the main problem at the system level to subproblems at the component level. Via an iterative procedure, in a relatively short time, we are able to find a heuristic solution with a close-to-minimal average cost rate for the entire system, under the assumed policy structure. Moreover, we also simulate the true average cost rate by using the same heuristic solution, to compare with the cost rates obtained via our approximation.

Regarding the scope of our research, first of all, we only consider perfect quality of maintenance actions. Each maintenance action will restore components to its original performance level. Hence, we do not consider the aging effect of components and imperfect maintenance actions. Secondly, we assume small time intervals spent on maintenance actions, because most of the maintenance actions in the practice of high-tech systems are replacements, instead of repair on site. In other words, when a maintenance action is triggered on a failed component in a system, a new component is brought to the field and exchanged with the failed component. The actual repair of the failed component will be carried out in the repair shop. This procedure results in a short downtime for the system operation. Thirdly, the coordination of maintenance actions for multi-component systems is difficult, due to the economic, structural or stochastic dependencies among the components [11, 17, 37, 51]. Amongst the literature of multi-component problem, we focus on the economic dependency, and not on the structural and stochastic dependencies. Moreover, in practice, the maintenance actions are planned with a smaller interval (in terms of weeks), compared with the long life cycles (from 10 to 40 years) of complex systems. Hence, an infinite time horizon is considered in this research. Finally, the application of micro-sensors and other techniques enable the remote collection of condition data in a real time, which motivate the assumption that the degradation of components can be monitored continuously.

Chapter 2

A Condition-Based Maintenance Policy for Multi-Component Systems with a High Maintenance Setup Cost

"In my opinion, all things in nature occur mathematically."

René Descartes

2.1 Introduction

Compared with single-component systems, the maintenance optimization of multi-component systems in a CBM framework is much more complicated because of economic, structural or stochastic dependencies among the components [11, 17, 37]. In this chapter, we focus on economic dependency and propose a new CBM policy for multi-component systems with stochastic and continuous deteriorations. To reduce the setup cost of maintenance for multi-component systems, we propose a joint maintenance interval to synchronize the maintenance activities for all degrading components in a system. Maintenance strategies with static joint maintenance intervals are often applied in the industries of advance capital goods (e.g., aviation, oil-gas refinery, renewable energy and chemical process) due to the convenience of static intervals for the operations planning and coordination of maintenance resources (e.g., service engineers, maintenance equipments, spare parts) [17].

Table 2.1 Summary of literature about condition-based maintenance models for multi-component systems (Y=yes, N=no, L=large, S=small, Con=continuous, Dis=discrete, Sim=simulation, Ana=analysis, Heu=heuristic)

	[64]	[10]	[2]	[35]	[6]	[58]	[52]	[53]	[28]	This chapter
Assumptions:										
Monotonic degradation	Y	Y	Y	Y	Y	Y	N	N	Y	Y
Repair as good as new	Y	Y	N	N	Y	N	Y	Y	Y	Y
Negligible repair/replacement time	Y	Y	N	Y	Y	N	N	Y	Y	Y
Infinite time horizon	N	Y	N	N	N	N	N	N	N	Y
Directly observable degradation	Y	Y	Y	Y	N	Y	N	N	Y	Y
Independence	Y	N	Y	N	Y	N	Y	Y	N	Y
Features:										
Time	Dis	Dis	Dis	Con	Con	Dis	Con	Con	Con	Con
Scale of problem	L	S	S	L	L	L	L	L	S	L
Solution method	Heu	Ana	Sim	Sim + GA	Sim + Ana	Sim	Sim	PHM	Sim	Ana

To position our research among existing works, we summarize the literature of condition-based maintenance model for multi-component systems in Table 2.1. Via the comparison of existing research, our contribution is that we develop a new mathematical model to optimize the condition-based maintenance policy for systems with a large number of identical/non-identical components (rather than small-scale systems in [2, 10, 28, 58]). Instead of using simulation as in many papers in the literature [2, 6, 28, 35, 58], our model is based on analysis and our cost rate evaluation is exact. Instead of a finite-horizon setting as in [2, 6, 28, 35, 64], we consider an infinite horizon. Unlike [2, 10, 58, 64], our degradation path is continuous. Our maintenance decision is optimized based on the directly observable measurements of physical degradations, instead of using indirect measurements (e.g., risk threshold, hazard rate, etc) as decision variables [6, 52, 53, 58]. Therefore, we can conclude our model has a unique contribution among the existing literature.

To avoid high setup costs, our model coordinates the maintenance tasks at the system level by introducing a static joint maintenance interval. The components are jointly maintained at the next upcoming maintenance time point if their physical conditions exceed the specified control limits, which can be easily implemented in the industries of advance capital goods. Under this structure, we develop a nested enumeration approach to minimize the long-run average cost rate by specifying the control limits of degrading components and the fixed joint maintenance interval. This model is capable of dealing with systems consisting of a large number of identical/non-identical components, because the setup cost of maintenance visits and the variable cost of maintenance visits can be evaluated in separate terms in the objective function: (i) the setup cost is related to the joint maintenance interval, which can be optimized at the outer loop of the optimization algorithm (ii) the variable cost, which is dependent on the types of maintenance activities (preventive or corrective) and the amount of components involved, can be evaluated separately for each component using renewal theory. Due to this decomposition, for a given maintenance interval, we can first optimize the control limits of components and then specify the optimal joint maintenance interval at the system level. For different degradation processes, the structure of the model and the algorithm of optimization will not be changed, although the probability expressions will be different for different degradation models. Notice that our model is not only adaptable for components with different degradation processes (e.g., random coefficient models, Wiener processes and Gamma processes), but also applicable to systems composed of components with different types of maintenance policies (e.g., age-based maintenance or periodic inspections).

The outline of this chapter is as follows. The description of the system and the assumptions are given in Section 2.2. The details of the mathematical model are explained in Section 2.3. In Section 2.4, a numerical study of a semiconductor production system is performed. Moreover, in Section 2.5, a sensitivity analysis is performed. In Section 2.6, our optimal policy is compared with the optimal solutions of a failure-based maintenance policy and an age-based maintenance policy, in order to evaluate the cost-saving potential. Finally, the conclusions are stated in Section 2.7.

2.2 System Description

Consider a system consisting of m subsystems. The set $J = \{1, 2..., m\}$ denotes the set of subsystems. Subsystem $j \in J$ consists of l_j components. For the system, all components of all systems are numbered from 1 to k and $I = \{1, 2, ..., k\}$ denotes the set of components, where $k = \sum_{j \in J} l_j$.

When maintenance actions are taken, a maintenance crew and equipment have to be sent to the field and the operation of the system is interrupted. Consequently, a high fixed setup cost S is charged on the system for maintenance actions on its components. The setup cost S refers to a fixed cost that is incurred for a maintenance visit regardless of what maintenance actions are performed. For a production line, it includes the cost of sending a maintenance team to the site, stopping the production, resetting the production environment, etc. Hence, it is often economically beneficial to perform maintenance actions of multiple components simultaneously. If we decide to take a maintenance visit for a single component, we need to pay such a fixed cost S. However, if we decide to take a maintenance visit to conduct the maintenance activities for n components at one joint maintenance interval, we only need to pay one fixed cost S. In this case, we save $n-1$ setup costs for the system, compared with taking maintenance visits separately for each component at different time moments. This is the economic dependency that we are dealing with.

Due to the convenience of implementation, maintenance policies with a *fixed interval* are commonly adopted in practice, which is also referred as block replacement policy in literature. For example, in the industry of semiconductor, a periodic maintenance visit will be scheduled at fixed time points. We consider such a policy with a *fixed maintenance interval* τ (a decision variable). Namely, it is possible to set up maintenance actions only at time points $n\tau$, $n \in \mathbb{N}$. In practice, the maintenance interval (in terms of weeks) is small compared with the long life cycles (from 10 to 40 years) of complex systems. Hence, an infinite time horizon is assumed in this chapter.

At the component level, we can continuously monitor the degradation of a certain physical parameter; e.g., the wearing of a braking system, the cracks of a stringer. Such physical conditions degrade over time monotonically and restored by maintenance actions only. For each component $i \in I$, $X_i(t)$ is the degradation path over time $t \in [0, \infty)$ (see Figure 2.1). In this chapter, we assume a *soft failure*, which means that a component continues functioning with a lower performance when its degradation exceeds its soft failure threshold H_i (i.e., $X_i(t) > H_i$). Such soft failures usually happen to components with mechanical/thermal-stress degradation [8]. For example, i) the cutting tools are not able to deliver satisfactory performance after a certain percentage of the metal material is worn, which can result in a lower throughput of production line; ii) an overpowered laser beam generated by a degraded laser unit may lead to imprecise cutting and high scrap rate in production. Both of them can

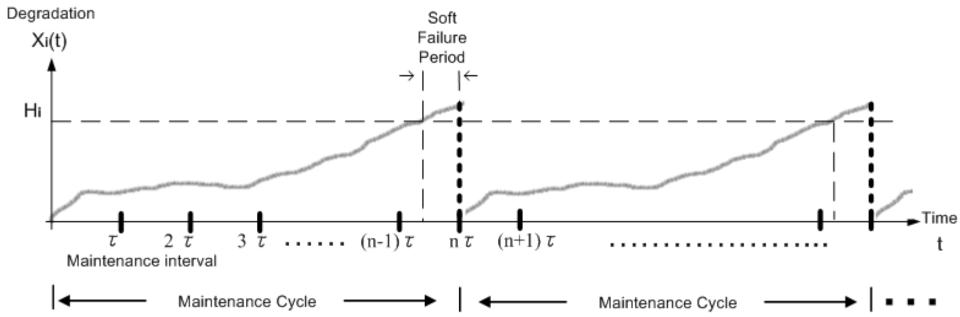


Figure 2.1 Condition-based maintenance of single components with corrective maintenance only

be considered as soft failures. When $X_i(t)$ exceeds H_i, a soft failure is observed between two maintenance points $(n-1)\tau$ and $n\tau$ and a corrective maintenance (CM) action (with a cost c_i^{CM}) on the failed component is taken at the maintenance point $n\tau$. The period from the time point when the soft failure occurs till the maintenance point $n\tau$ is the *soft failure period* (see Figure 2.1). Such a period can cause quality loss in production or lower performance in operation with a cost rate c_i^P. For instance, in a semiconductor production system, if the laser power output exceeds a certain limit, the silicon wafers will not be cut precisely, which will cause a higher scrap rate. Hence, this quality loss/low performance cost is equal to the length of soft failure period multiplies c_i^P.

In order to avoid a high corrective maintenance cost c_i^{CM} and quality loss costs when $X_i(t)$ exceeds H_i, it is economically beneficial to take maintenance actions pro-actively, which is known as preventive maintenance (PM), with a lower cost c_i^{PM} ($c_i^{PM} < c_i^{CM}$). Thus, for each component, we introduce a control limit C_i to trigger PM actions at the next closest maintenance point, before its degradation exceeds H_i ($C_i < H_i$), as shown in Figure 2.2. When the stochastic degradation increases fast and exceeds both C_i and H_i at the next closest maintenance point $n\tau$, a CM action will be taken (see Figure 2.2 (B)). Nevertheless, if the stochastic degradation increases slowly and the degradation level is between C_i and H_i at the next closest maintenance point $n\tau$; a PM action with a lower cost will be taken (see Figure 2.2 (A)). Notice that both τ and C_i, $i \in I$, are the decision variables of the optimization model. After a maintenance action is taken, the condition of the component is restored to the initial degradation level (also known as "Repair-As-New") and the component continues its operation till the next maintenance action is taken. This renewal cycle will repeat itself throughout the infinite time horizon. The period between two consecutive maintenance actions for a component is defined as a *maintenance cycle* (see Figure 2.1), which is also called as a renewal cycle. The beginning of each cycle is a so-called renewal point. According to renewal theory, the average cost rate over an infinite time horizon is equal to the average cost rate over one maintenance (renewal) cycle, $Z_i(\tau, C_i)$. The expected maintenance cost per cycle and the expected maintenance cycle length are derived in Section 2.3.2.

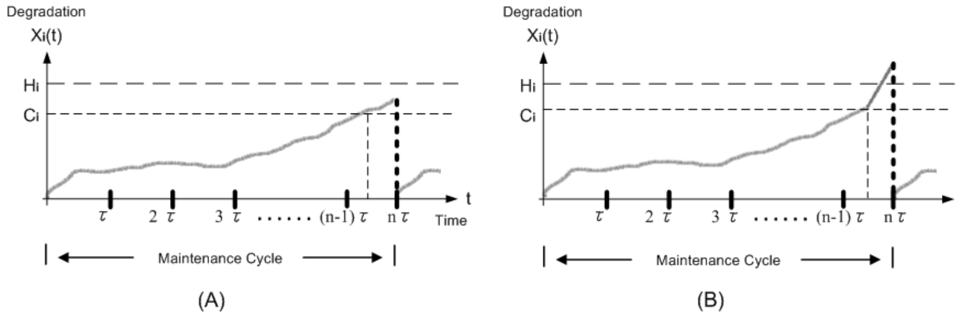


Figure 2.2 Condition-based maintenance of single components with preventive and corrective maintenance: (A) a PM action is taken at the next maintenance point if $H_i \geq X_i(t) > C_i$; (B) a CM action is taken at the next maintenance point if $X_i(t) > H_i$

In this chapter, we assume components are independent with each other. Several real life applications satisfy this assumption. For example, in a lithography machine consisting of many components and modules [69], the degradations of the modules (e.g., laser units, micro mirrors, etc.) are independent. They are independent because there is no joint environmental factor, since the operation of the machine requires a clean-room or vacuum environment. The degradations of the lighting systems in a building are independent, because the degradation of a light bulb will not affect the degradation of another light bulb. In a multi-stage production system, the mechanical components degrade over time (e.g., the cutting tools, the transmitting chains, the rotating/moving components). Most of the components are independent, because the degradation of the mechanical components in one stage will not affect the degradation of the mechanical components in another stage.

To solve the maintenance problem for systems with a large number of components, we propose a nested enumeration approach, because the setup cost of maintenance visits and the variable cost of maintenance visits can be evaluated separately. We first decompose the optimization of the system into optimization problems at individual component level to find the optimal control limit of each component C_i^* for a given τ by minimizing the average cost rate of each component, $Z_i(\tau, C_i)$. Afterwards, we can find the optimal τ by minimizing the average cost rate of the system $Z_{syst}(\tau)$. We assume that the system is composed of a large number of components, so that the probability of no component failure within one maintenance interval is negligible. Hence, a setup of maintenance actions is always needed at each static maintenance point and the average setup cost rate can be modeled as $\frac{S}{\tau}$. Furthermore, since the degradation processes of components are assumed to be independent, the variable cost rate of maintenance visits equals the summation of the variable cost rates of all the individual components, which can be evaluated by using renewal theory. Consequently, the average cost rate on the system level for a given τ is

$$Z_{syst}(\tau) = \frac{S}{\tau} + \sum_{i \in I} Z_i^*(\tau) \tag{2.1}$$

where $Z_i^*(\tau) = Z_i(\tau, C_i^*)$, which is the minimum average cost rate excluding setup costs for each component with an optimal control limit C_i^* for a given τ.

2.2.1 Notation

i : index of components in the system
n : index of maintenance intervals over the planning horizon
$X_i(t)$: degradation of component i on a physical condition
τ : maintenance interval at the system level (decision variable)
C_i : control limit on the degradation level of component i (decision variable)
H_i : (predetermined) soft failure threshold on the degradation level of component i
Z_i : average cost rate of component i (without setup costs)
Z_{syst} : average cost rate of the system
c_i^{PM} : cost per PM action taken on component i
c_i^{CM} : cost per CM action taken on component i
c_i^{P} : soft failure cost rate on component i
S : cost per setup action taken at the system level

2.2.2 Assumptions

1) Maintenance actions are set up at fixed maintenance points $n\tau, n \in \mathbb{N}$.
2) The time horizon is infinite
3) Maintenance actions restore the conditions of components back to their initial degradation levels. (also known as "repair-as-new").
4) The components in the system are independent of each other.
5) The condition degrades over time monotonically.
6) The system continues its operation with a lower performance when the degradation of components exceeds the failure thresholds (also known as "soft failure").

2.3 Model Formulation and Analysis

Before optimizing the maintenance policy at the system level (see Equation (2.1)), the degradation process of a single component within a single maintenance cycle is introduced in Subsection 2.3.1. Afterwards, the optimization model is formulated both at the component and system level in Subsection 2.3.2.

2.3.1 Degradation model

As mentioned in the literature in Section 2.1, there are several approaches to modeling the stochastic degradation paths of components (e.g., Random Coefficient Model, Gamma process, Brownian Motion or Markov Process). In this chapter, we use the Random Coefficient Model [33], because it is relatively flexible and convenient for describing the degradation derived from physics of failures, such as law of physics and material science. According to the Random Coefficient Model, the degradation level of component i at time $\hat{t} \in [0, \infty)$ in a single maintenance cycle, $X_i(\hat{t}; \Phi_i, \Theta_i)$, is a random variable, given a set of constant parameters $\Phi_i = \{\phi_{i,1}, ..., \phi_{i,Q}\}, Q \in \mathbb{N}$, and a set of random parameters $\Theta_i = \{\theta_{i,1}, ..., \theta_{i,V}\}, V \in \mathbb{N}$, following certain probability distributions. The probability that the degradation at time $\hat{t}$ exceeds a threshold χ is equal to the probability that the passage time T_χ over the threshold χ is less than time $\hat{t}$

$$Pr\{T_\chi < \hat{t}\} = Pr\{X(\hat{t}; \Phi_i, \Theta_i) > \chi\}, \qquad \forall i \in I. \tag{2.2}$$

EXAMPLE 1: In order to clarify the model, a simple example is given. Consider a component i in the system with a degradation path $X_i(t; \Phi_i, \Theta_i) = \phi_{i,1} + \theta_{i,1} t^{\phi_{i,2}}$ where $\Phi_i = \{\phi_{i,1}, \phi_{i,2}\}$ and $\Theta_i = \{\theta_{i,1}\}$. Equation (2.2) can be written in terms of $F_{\theta_{i,1}}$ (the cumulative density function of random variable $\theta_{i,1}$, $\theta_{i,1} \geq 0$):

$$\begin{aligned} Pr\{T_\chi < \hat{t}\} &= Pr\{\phi_{i,1} + \theta_{i,1}\hat{t}^{\phi_{i,2}} > \chi\} \\ &= Pr\{\theta_{i,1} > \frac{\chi - \phi_{i,1}}{\hat{t}^{\phi_{i,2}}}\} \\ &= 1 - F_{\theta_{i,1}}\Big(\frac{\chi - \phi_{i,1}}{\hat{t}^{\phi_{i,2}}}\Big) \end{aligned} \tag{2.3}$$

$\diamondsuit$

For component $i \in I$, the cumulative density functions of passage time T_{C_i} and T_{H_i} (when the degradation level exceeds C_i and H_i) can be derived based on Equation (2.2) given the degradation path function $X_i(\hat{t}; \Phi_i, \Theta_i)$ and the probability distributions of Θ_i. Recalling the proposed policy explained in Section 2.2 (see Figure 2.2), maintenance actions are taken at fixed time points. Hence, the probability that the control limit C_i is reached between time point $(n-1)\tau$ and $n\tau$ can be expressed as

$$Pr\{X_i\big((n-1)\tau;\Phi_i,\Theta_i\big) \leq C_i < X_i(n\tau;\Phi_i,\Theta_i)\} = Pr\{(n-1)\tau \leq T_{C_i} < n\tau\}, \quad \forall n \in \mathbb{N}. \tag{2.4}$$

The probability that soft failure threshold H_i is reached before time point $n\tau$ can be expressed as

$$Pr\{X_i(n\tau;\Phi_i,\Theta_i) > H_i\} = Pr\{T_{H_i} < n\tau\}, \qquad \forall n \in \mathbb{N}, \quad i \in I \tag{2.5}$$

where $C_i < H_i$ and $T_{C_i} \leq T_{H_i}$, since the degradation path is assumed to be monotonic. After C_i is reached between $(n-1)\tau$ and $n\tau$, there are two possibilities for the maintenance action at $n\tau$ as mentioned in Section 2: preventive maintenance (PM) if $C_i \leq X_i(n\tau) < H_i$ and corrective maintenance (CM) if $X_i(n\tau) \geq H_i$. Thus, the probability that PM occurs at time $n\tau$ after the degradation level of component i has reached its control limit C_i between time $(n-1)\tau$ and $n\tau$, can be derived based on Equations (2.2), (2.4) and (2.5) as

$$Pr\{PM \ at \ n\tau\} = Pr\{T_{H_i} > n\tau, \ (n-1)\tau \leq T_{C_i} < n\tau\}. \tag{2.6}$$

Similarly, for CM,

$$Pr\{CM \ at \ n\tau\} = Pr\{T_{H_i} \leq n\tau, \ (n-1)\tau \leq T_{C_i} < n\tau\}. \tag{2.7}$$

EXAMPLE 1 (continued): According to Equations (2.3) and (2.4), the probability of reaching the control limit C_i between $(n-1)\tau$ and $n\tau$ can be obtained as

$$Pr\{(n-1)\tau \leq T_{C_i} < n\tau\} = F_{\theta_{i,1}}\Big(\frac{C_i-\phi_{i,1}}{\big((n-1)\tau\big)^{\phi_{i,2}}}\Big) - F_{\theta_{i,1}}\Big(\frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big), \quad \forall n \in \mathbb{N}. \tag{2.8}$$

For component $i \in I$, the probability that either PM or CM occurs at time point $n\tau$ after the degradation reaches C_i between $(n-1)\tau$ and $n\tau$ can be derived from Equations (2.6) and (2.7):

$$\begin{aligned} Pr\{PM \ at \ n\tau\} &= Pr\Big\{\theta_{i,1} < \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}, \ \frac{C_i-\phi_{i,1}}{\big((n-1)\tau\big)^{\phi_{i,2}}} \geq \theta_{i,1} > \frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big\} \qquad (2.9)\\ &= \begin{cases} F_{\theta_{i,1}}\Big(\frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}\Big) - F_{\theta_{i,1}}\Big(\frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big), & if \ \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} > \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}} \\ F_{\theta_{i,1}}\Big(\frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big) - F_{\theta_{i,1}}\Big(\frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big), & if \ \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} \leq \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}. \end{cases} \end{aligned}$$

Similarly, for CM,

$$\begin{aligned} Pr\{CM \ at \ n\tau\} &= Pr\Big\{\theta_{i,1} \geq \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}, \ \frac{C_i-\phi_{i,1}}{\big((n-1)\tau\big)^{\phi_{i,2}}} \geq \theta_{i,1} > \frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big\} \qquad (2.10)\\ &= \begin{cases} 0, & if \ \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} > \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}} \\ F_{\theta_{i,1}}\Big(\frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}\Big) - F_{\theta_{i,1}}\Big(\frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\Big), & if \ \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} \leq \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}. \end{cases} \end{aligned}$$

Regardless of the distribution of $\theta_{i,1}$, the sum of the probabilities of PM and CM in the interval of $n\tau$ is equal to the probability of reaching C_i between $(n-1)\tau$ and $n\tau$, as derived in Equation (2.8) . $\diamondsuit$

2.3.2 Evaluation and optimization

As mentioned in Section 2, we propose a nested approach to find the optimal maintenance policy (i.e., the control limits C_i of each degrading component and the joint maintenance interval τ) by minimizing the long-run average cost rate of the system.

Evaluation and optimization for each component:
We first evaluate the long-run average variable cost rate for component $i \in I$ incurred by preventive maintenance, corrective maintenance and soft failure. The variable cost of one maintenance visit is dependent on the type of maintenance activities (preventive or corrective) on various components. Suppose that we add all the variable costs for all the maintenance visits over an infinite time horizon together. The summation of the variable costs over an infinite time horizon is dependent on the different frequencies of replacements of the components and the different proportions of the types of maintenance activities (preventive or corrective) over a long term for different components. Notice that we assume the degradation processes of the components are independent. Therefore, the frequencies of replacements for different components are independent under a given τ and the given control limits. These frequencies can be evaluated by the lengths of renewal cycles, using renewal theory separately. The proportions of the types of maintenance activities (preventive or corrective) can be calculated by deriving the probability of soft failures for a renewal cycle. Therefore, the variable cost can be evaluated separately for different components, given the values of τ and control limits. According to renewal theory, the long-run average cost $Z_i(\tau, C_i)$ is equal to the expected maintenance cost per cycle $\mathbb{E}\Big[K_i(\tau, C_i)\Big]$ divided by the expected cycle length $\mathbb{E}\Big[L_i(\tau, C_i)\Big]$. The expected maintenance cost per cycle $\mathbb{E}\Big[K_i(\tau, C_i)\Big]$ is given as

$$\mathbb{E}\Big[K_i(\tau, C_i)\Big] = \sum_{n\in\mathbb{N}} \Bigg[Pr\{PM\ at\ n\tau\}c_i^{PM} + Pr\{CM\ at\ n\tau\}c_i^{CM}\Bigg] + \mathbb{E}\Big[D_i(\tau, C_i)\Big]c_i^{P}, \tag{2.11}$$

where $Pr\{PM\ at\ n\tau\}$ and $Pr\{CM\ at\ n\tau\}$ can be obtained from Equations (2.6) and (2.7). The costs of preventive maintenance and corrective maintenance on component

i are denoted by c_i^{PM} and c_i^{CM} respectively. The soft failure cost in Equation (2.11) is evaluated by the product of the expected soft failure period $\mathbb{E}\Big[D_i(\tau, C_i)\Big]$ and the penalty cost rate c_i^P, as described in Section 2. The expected soft failure period $\mathbb{E}\Big[D_i(\tau, C_i)\Big]$ can be derived as

$$\mathbb{E}\Big[D_i(\tau, C_i)\Big] = \sum_{n\in\mathbb{N}} \int_{(n-1)\tau}^{n\tau} \left(\int_x^{n\tau} (n\tau - y) f_{T_{H_i}|T_{C_i}}(y|x)dy \right) f_{T_{C_i}}(x)dx, \quad \forall\, i \in I, \tag{2.12}$$

where $f_{T_{C_i}}(x)$ is the probability density function of passage time T_{C_i} and $f_{T_{H_i}|T_{C_i}}(y|x)$ is the conditional probability density function of passage time T_{H_i}, given that $T_{C_i} = x$.

Moreover, the expected cycle length $\mathbb{E}\Big[L_i(\tau, C_i)\Big]$ is given as

$$\mathbb{E}\Big[L_i(\tau, C_i)\Big] = \sum_{n\in\mathbb{N}} n\tau Pr\{(n-1)\tau \leq T_{C_i} < n\tau\}, \qquad \forall i \in I. \tag{2.13}$$

EXAMPLE 1 (continued): Assuming that the degradation rate $\theta_{i,1}$ follows a Weibull distribution with α_i and β_i, the distribution of passage time T_{C_i} can be derived as

$$f_{T_{C_i}}(x) = \phi_{i,2}\beta_i \Big(\frac{C_i - \phi_{i,1}}{\alpha_i}\Big)^{\beta_i} x^{-(\phi_{i,2}\beta_i+1)} exp\Big[-\big(\frac{C - \phi_{i,1}}{\alpha_i x^{\phi_{i,2}}}\big)^{\beta_i}\Big], \qquad \forall i \in I. \tag{2.14}$$

According to Equation (2.12), the expected soft failure period can be derived as

$$\mathbb{E}\Big[D_i(\tau, C_i)\Big] = \sum_{n\in\mathbb{N}} \left[\int_{(n-1)\tau}^{n\tau} \Big[n\tau - \Big(\frac{H_i - \phi_{i,1}}{C_i - \phi_{i,1}}\Big)^{(1/\phi_{i,2})} x\Big]^+ f_{T_{C_i}}(x)dx \right], \quad \forall i \in I. \tag{2.15}$$

$\diamondsuit$

Hence, the optimization for C_i is formulated as

$$\begin{aligned} \min_{C_i} \qquad & Z_i(\tau, C_i) = \frac{\mathbb{E}\Big[K_i(\tau, C_i)\Big]}{\mathbb{E}\Big[L_i(\tau, C_i)\Big]} \\ \text{s.t.} \qquad & 0 < C_i < H_i \qquad \forall i \in I \end{aligned}$$

Notice that the maintenance interval τ is treated as a given parameter, instead of a decision variable in this subproblem; so that the optimal control limit $C_i^*(\tau)$ can be obtained for each component for a given τ.

Evaluation and optimization of the system:
For each τ value, component i has its corresponding control limit $C_i^*(\tau)$ and optimal long-run average cost rate excluding setup cost $Z_i^*(\tau)$. Hence, the long-run average cost rate of the system $Z_{syst}(\tau)$ can be minimized by enumerating τ. $Z_{syst}(\tau)$ includes not only the sum of the minimum average cost rates of all components $\sum_{i\in I} Z_i^*(\tau)$, but also the average setup cost rate $\frac{S}{\tau}$. Hence, the optimization model is

$$\begin{aligned} \min_{\tau} \qquad & Z_{syst}(\tau) = \frac{S}{\tau} + \sum_{i\in I} Z_i^*(\tau) \\ \text{s.t.} \qquad & 0 < \tau < M_\tau \end{aligned}$$

where M_τ is the upper bound of the maintenance interval τ. In practice, there can be a limit on τ suggested by manufacturers or industry regulations. The detailed explanation of the algorithm is elaborated in Subsection 2.A.3.

2.4 Numerical Study

To demonstrate the use of our model, we provide a general numerical study of a complex engineering system. One can consider a production system, with 60 individual components ($i \in I = \{1, ..., 60\}$). For each component, micro-sensors can be installed to continuously monitor the degradation. The degradation $X_i(t; \phi_{i,1}, \phi_{i,2}, \theta_{i,1})$ can be described by the Random Coefficient Model [33]:

$$X_i(t; \phi_{i,1}, \phi_{i,2}, \theta_{i,1}) = \phi_{i,1} + \theta_{i,1} * t^{\phi_{i,2}}, \qquad \forall i \in I$$

where t is the operation time and $\theta_{i,1}$ is the positive random parameter. The constant parameter $\phi_{i,2}$ is an acceleration factor and the constant parameter $\phi_{i,1}$ is the initial degradation level. Notice that the production system will generate products with low quality, when the degradation $X_i(t)$ reaches a threshold H at a passage time T_H. Hence, this threshold H is considered as the soft failure threshold. The degradations of components are stochastically independent. We assume that the distribution of $\theta_{i,1}$ follows a Weibull distribution with two parameters: α_i and β_i, which can be obtained by condition data fitting [33]. Therefore, we can use the mathematical expressions of Example 1 in Subsection 2.3.1 (Equations (2.8), (2.9), (2.10) and (2.15)) to formulate the degradation path of the component.

The parameter setting is shown in Table 2.2. Notice that these 60 components are from three different component types, so that their parameters in Table 2.2 are not identical. Moreover, on the system level, a very expensive setup cost S is charged, which includes the traveling cost of maintenance crews and resources, the cost of production disturbance and downtime, the resetting cost of manufacturing environment, etc. To solve this maintenance optimization problem, we use the approach proposed in Subsection 2.3.2.

Table 2.2 The parameter setting

Parameter	Explanation	Type x $i \in \{1, ..., 20\}$	Type y $i \in \{21, ..., 40\}$	Type z $i \in \{41, ..., 60\}$
c_i^{PM}	PM cost [thousand Euro]	$c_i^{PM} = 7$	$c_i^{PM} = 15$	$c_i^{PM} = 10$
c_i^{CM}	CM cost [thousand Euro]	$c_i^{CM} = 30$	$c_i^{CM} = 70$	$c_i^{CM} = 50$
c_i^P	Soft failure cost rate [thousand Euro]	$c_i^P = 7.2$	$c_i^P = 7.2$	$c_i^P = 7.2$
S	Setup cost, S=50 [thousand Euro]	-	-	-
α_i	Scale parameter of Weibull distribution	$\alpha_i = 2.12$	$\alpha_i = 2.52$	$\alpha_i = 1.02$
β_i	Shape parameter of Weibull distribution	$\beta_i = 7.9$	$\beta_i = 7.5$	$\beta_i = 6.9$
H_i	Soft failure threshold	$H_i = 10$	$H_i = 20$	$H_i = 15$
$\phi_{i,1}$	Initial degradation level	$\phi_{i,1} = 1$	$\phi_{i,1} = 2$	$\phi_{i,1} = 3$
$\phi_{i,2}$	Constant parameter for different rotational mechanisms	$\phi_{i,2} = 0.33$	$\phi_{i,2} = 0.41$	$\phi_{i,2} = 0.51$
G_i	Expected passage time (the first moment of Equation (2.14)) of H_i [days]	$G_i = 116.12$	$G_i = 141.11$	$G_i = 143.43$

By the nested enumeration algorithm (see Subsection 2.A.3), the optimal maintenance policy is found and shown in Table 2.3. The optimal policy is to set the maintenance interval at 36.1 days and the control limits on the physical condition of the three types of components are 8.11 (out of 10), 17.12 (out of 20) and 12.72 (out of 15) respectively. The resulting average maintenance cost rate of this production system is 7424 Euros per day. The computation performance is given in Subsection 2.A.4, which shows the computational benefit of our algorithm compared with the algorithms that don't use decomposition.

Table 2.3 The optimal maintenance policy of the numerical example in Table 2.2 (index: x for $i \in \{1, ..., 20\}$; y for $i \in \{21, ..., 40\}$; z for $i \in \{41, ..., 60\}$)

Optimal Policy	Values	Explanation
$Z_{syst}(\tau^*)$	7424	the minimum average cost rate of the system [Euro / day]
τ^*	36.0	the optimal maintenance interval of the system [day]
$\{C_x^*(\tau^*), C_y^*(\tau^*), C_z^*(\tau^*)\}$	$\{8.11, 17.12, 12.72\}$	the optimal control limits of each component
$\{Z_x(\tau^*), Z_y(\tau^*), Z_z(\tau^*)\}$	$\{94.3, 126.2, 81.2\}$	the minimum average variable cost rate of each component [Euro / day].

In Figure 2.3, we depict the average cost rate of the system, $Z_{syst}(\tau)$, as a function of the maintenance interval τ, which includes the sum of two elements: the setup cost rate S/τ and the variable maintenance cost rate of all components $\sum_{i\in I} Z_i^*(\tau)$. When τ increases, S/τ decreases due to the less frequent setups of maintenance actions on one hand; on the other hand, $\sum_{i\in I} Z_i^*(\tau)$ increases due to the higher probability that CM occurs in a maintenance interval and higher expected soft failure costs.

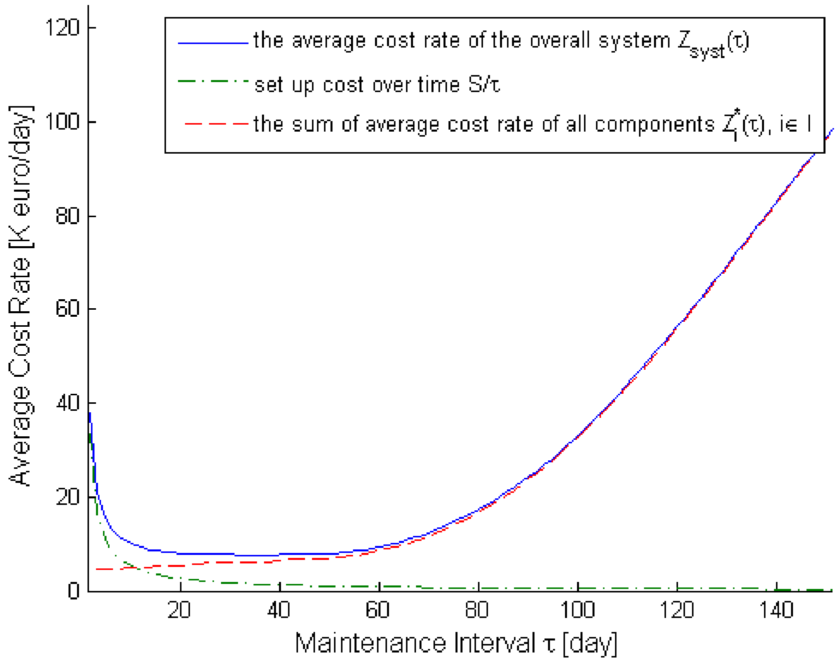


Figure 2.3 Average cost rate [thousand Euro per day] at the system level over τ [days]

To obtain further insight, the optimal solution of a single component is analyzed. Taking component 1 as an example, we investigate the changes of the average variable maintenance cost rate $Z_1(\tau, C_1)$ under given τ values over the control limit C_1 as shown in Figure 2.4. For $\tau = 15, 20$ and 25, the optimal control limit $C_1^*(\tau)$ is $9.28, 8.92$, and 8.83 respectively and the minimum average cost rate $Z_1(\tau, C_1^*)$ is $75.0, 82.2$ and 91.9 Euros per day respectively. We can observe a higher $Z_1(\tau, C_1^*)$ and a lower C_1^* at larger τ values. This is because the probability that CM occurs in a maintenance interval increases and the expected soft failure cost becomes higher. Consequently, the average variable cost rate of maintenance for each component increases, even though lower control limits are set on the degradation levels. (The plot of Z_1 under a larger τ value is included in Subsection 2.A.2)

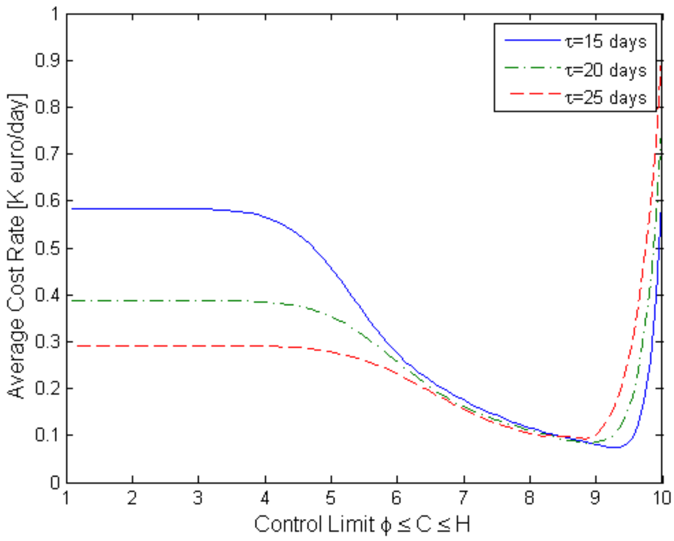


Figure 2.4 Average cost rate [thousand Euro per day] on component 1 over C_1 for various τ value

2.5 Sensitivity Analysis

A sensitivity analysis is performed based on varying the four parameters c_i^{PM}, c_i^P, S and β_i in Table 2.2 by $\pm 50\%$ and the rest of the parameter setting remains unchanged. We choose each parameter equal to 50%, 100% and 150% of its original value, and a full factorial test bed is set up by considering all combinations. To simplify the notation, we define the factors of the test bed as $a = (c_1^{PM}, c_2^{PM}, ..., c_{60}^{PM})$, $b = (c_1^P, c_2^P, ..., c_{60}^P)$, $c = S$ and $d = (\beta_1, \beta_2, ..., \beta_{60})$. Also we define a test bed of instances Λ with elements $(a_j, b_l, c_k, d_m), \forall j, l, k, m \in \{1, 2, 3\}$, where $a_1 = 50\% \times a, a_2 = 100\% \times a$, and $a_3 = 150\% \times a$; and similarly for b_l, c_k and d_m. This test bed consists of 81 instances. The output of each instance consists of the minimum average cost rate of the system, the optimal joint maintenance interval and control limits of components, which is denoted by $(\widehat{Z}_{syst}(\widehat{\tau}^*), \widehat{\tau}^*, \widehat{C}_x^*, \widehat{C}_y^*, \widehat{C}_z^*)$. The components from supplier x, y and z are grouped as: x for $i \in \{1, ..., 20\}$; y for $i \in \{21, ..., 40\}$; z for $i \in \{41, ..., 60\}$. In Table 2.4, the relative ratios between the optimal policies in the test bed and the optimal policy under the original parameter setting (see Table 2.3) are given.

The results in Table 2.4 match our intuition. They show that the joint optimal

Table 2.4 Results of the test bed (the percentages is the relative ratio dividing the new optimal solutions by the original optimal solutions in Table 2.3.

Λ	$(\frac{\widehat{Z}_{syst}(\widehat{\tau}^*)}{Z_{syst}(\tau^*)}, \frac{\widehat{\tau}^*}{\tau^*}, \frac{\widehat{C}_x^*}{C_x^*}, \frac{\widehat{C}_y^*}{C_y^*}, \frac{\widehat{C}_z^*}{C_z^*})$	Λ	$(\frac{\widehat{Z}_{syst}(\widehat{\tau}^*)}{Z_{syst}(\tau^*)}, \frac{\widehat{\tau}^*}{\tau^*}, \frac{\widehat{C}_x^*}{C_x^*}, \frac{\widehat{C}_y^*}{C_y^*}, \frac{\widehat{C}_z^*}{C_z^*})$
(a_1, b_1, c_1, d_1)	(48%, 36%, 113%, 117%, 119%)	(a_2, b_2, c_2, d_3)	(97%, 113%, 96%, 96%, 95%)
(a_1, b_1, c_1, d_2)	(49%, 95%, 101%, 101%, 101%)	(a_2, b_2, c_3, d_1)	(113%, 48%, 111%, 114%, 115%)
(a_1, b_1, c_1, d_3)	(49%, 110%, 97%, 96%, 96%)	(a_2, b_2, c_3, d_2)	(106%, 101%, 99%, 99%, 99%)
(a_1, b_1, c_2, d_1)	(63%, 50%, 110%, 113%, 114%)	(a_2, b_2, c_3, d_3)	(103%, 114%, 96%, 95%, 95%)
(a_1, b_1, c_2, d_2)	(57%, 97%, 101%, 101%, 101%)	(a_2, b_3, c_1, d_1)	(80%, 38%, 113%, 117%, 118%)
(a_1, b_1, c_2, d_3)	(54%, 112%, 97%, 96%, 96%)	(a_2, b_3, c_1, d_2)	(93%, 95%, 101%, 101%, 101%)
(a_1, b_1, c_3, d_1)	(75%, 58%, 109%, 111%, 112%)	(a_2, b_3, c_1, d_3)	(91%, 111%, 97%, 96%, 96%)
(a_1, b_1, c_3, d_2)	(64%, 101%, 99%, 99%, 99%)	(a_2, b_3, c_2, d_1)	(97%, 37%, 113%, 117%, 118%)
(a_1, b_1, c_3, d_3)	(60%, 114%, 96%, 95%, 95%)	(a_2, b_3, c_2, d_2)	(96%, 68%, 106%, 108%, 109%)
(a_1, b_2, c_1, d_1)	(48%, 37%, 113%, 117%, 119%)	(a_2, b_3, c_2, d_3)	(97%, 112%, 97%, 96%, 96%)
(a_1, b_2, c_1, d_2)	(48%, 67%, 107%, 109%, 110%)	(a_2, b_3, c_3, d_1)	(113%, 48%, 111%, 114%, 115%)
(a_1, b_2, c_1, d_3)	(49%, 110%, 97%, 97%, 96%)	(a_2, b_3, c_3, d_2)	(107%, 99%, 100%, 100%, 100%)
(a_1, b_2, c_2, d_1)	(63%, 48%, 111%, 114%, 116%)	(a_2, b_3, c_3, d_3)	(103%, 113%, 96%, 96%, 95%)
(a_1, b_2, c_2, d_2)	(57%, 95%, 101%, 101%, 101%)	(a_3, b_1, c_1, d_1)	(102%, 18%, 118%, 122%, 124%)
(a_1, b_2, c_2, d_3)	(55%, 111%, 97%, 96%, 96%)	(a_3, b_1, c_1, d_2)	(133%, 103%, 99%, 99%, 99%)
(a_1, b_2, c_3, d_1)	(78%, 60%, 108%, 110%, 111%)	(a_3, b_1, c_1, d_3)	(128%, 74%, 105%, 106%, 107%)
(a_1, b_2, c_3, d_2)	(64%, 97%, 101%, 101%, 101%)	(a_3, b_1, c_2, d_1)	(127%, 33%, 114%, 117%, 119%)
(a_1, b_2, c_3, d_3)	(61%, 113%, 97%, 96%, 95%)	(a_3, b_1, c_2, d_2)	(140%, 106%, 98%, 98%, 98%)
(a_1, b_3, c_1, d_1)	(48%, 38%, 113%, 117%, 118%)	(a_3, b_1, c_2, d_3)	(136%, 77%, 105%, 106%, 107%)
(a_1, b_3, c_1, d_2)	(48%, 63%, 107%, 109%, 111%)	(a_3, b_1, c_3, d_1)	(145%, 36%, 113%, 117%, 118%)
(a_1, b_3, c_1, d_3)	(49%, 109%, 97%, 97%, 97%)	(a_3, b_1, c_3, d_2)	(146%, 106%, 98%, 98%, 98%)
(a_1, b_3, c_2, d_1)	(64%, 48%, 111%, 114%, 115%)	(a_3, b_1, c_3, d_3)	(145%, 94%, 101%, 101%, 101%)
(a_1, b_3, c_2, d_2)	(58%, 93%, 101%, 101%, 102%)	(a_3, b_2, c_1, d_1)	(99%, 20%, 117%, 121%, 124%)
(a_1, b_3, c_2, d_3)	(55%, 110%, 97%, 96%, 96%)	(a_3, b_2, c_1, d_2)	(135%, 100%, 99%, 99%, 99%)
(a_1, b_3, c_3, d_1)	(77%, 50%, 110%, 113%, 115%)	(a_3, b_2, c_1, d_3)	(134%, 114%, 96%, 95%, 95%)
(a_1, b_3, c_3, d_2)	(65%, 95%, 101%, 101%, 101%)	(a_3, b_2, c_2, d_1)	(127%, 33%, 114%, 118%, 120%)
(a_1, b_3, c_3, d_3)	(61%, 112%, 97%, 96%, 96%)	(a_3, b_2, c_2, d_2)	(141%, 101%, 99%, 99%, 99%)
(a_2, b_1, c_1, d_1)	(78%, 26%, 115%, 120%, 122%)	(a_3, b_2, c_2, d_3)	(139%, 115%, 96%, 95%, 95%)
(a_2, b_1, c_1, d_2)	(92%, 101%, 99%, 99%, 99%)	(a_3, b_2, c_3, d_1)	(145%, 37%, 113%, 117%, 119%)
(a_2, b_1, c_1, d_3)	(91%, 112%, 97%, 96%, 95%)	(a_3, b_2, c_3, d_2)	(147%, 103%, 99%, 99%, 99%)
(a_2, b_1, c_2, d_1)	(97%, 38%, 113%, 117%, 118%)	(a_3, b_2, c_3, d_3)	(145%, 115%, 96%, 95%, 95%)
(a_2, b_1, c_2, d_2)	(99%, 101%, 99%, 99%, 99%)	(a_3, b_3, c_1, d_1)	(99%, 22%, 117%, 121%, 124%)
(a_2, b_1, c_2, d_3)	(97%, 115%, 96%, 95%, 95%)	(a_3, b_3, c_1, d_2)	(119%, 37%, 113%, 117%, 119%)
(a_2, b_1, c_3, d_1)	(112%, 48%, 111%, 114%, 116%)	(a_3, b_3, c_1, d_3)	(134%, 112%, 97%, 96%, 95%)
(a_2, b_1, c_3, d_2)	(105%, 103%, 99%, 99%, 99%)	(a_3, b_3, c_2, d_1)	(127%, 33%, 114%, 118%, 120%)
(a_2, b_1, c_3, d_3)	(102%, 116%, 96%, 95%, 94%)	(a_3, b_3, c_2, d_2)	(142%, 100%, 99%, 99%, 99%)
(a_2, b_2, c_1, d_1)	(78%, 30%, 115%, 119%, 120%)	(a_3, b_3, c_2, d_3)	(140%, 114%, 96%, 96%, 95%)
(a_2, b_2, c_1, d_2)	(85%, 38%, 113%, 117%, 118%)	(a_3, b_3, c_3, d_1)	(145%, 38%, 113%, 117%, 119%)
(a_2, b_2, c_1, d_3)	(91%, 112%, 97%, 96%, 96%)	(a_3, b_3, c_3, d_2)	(148%, 99%, 100%, 100%, 100%)
(a_2, b_2, c_2, d_1)	(97%, 36%, 113%, 118%, 119%)	(a_3, b_3, c_3, d_3)	(145%, 115%, 96%, 95%, 95%)
(a_2, b_2, c_2, d_2)	(100%, 100%, 100%, 100%, 100%)		

maintenance interval $\widehat{\tau}^*$ increases when c_i^P and c_i^{PM} decrease or when β_i and S increase. These findings are sensible because: 1) it is economically beneficial to have a longer maintenance interval or less frequent maintenance setups when soft failure costs c_i^P are less expensive, or when the setup cost S is more expensive; 2) if preventive maintenance costs c_i^{PM} are more expensive, optimal control limits $\widehat{C}_i^*$ become larger to have less PM actions and reduce average cost rate at the component level. In this case, the probability of corrective maintenance and expected soft failure cost also increases, so that a shorter $\widehat{\tau}^*$ can help reduce average cost rate at the system level; 3) a larger β_i leads to a lower variance in the distribution of degradation rate, and it is economically beneficial to have a higher $\widehat{\tau}^*$ at the system level and a lower $\widehat{C}_i^*$ at the component level in this case. Moreover, we also observe that the optimal control limits $\widehat{C}_i^*$ decrease when $\widehat{\tau}^*$ increases. When maintenance intervals are larger at the system level, more corrective maintenance and soft failures will occur at individual component level. To reduce these high costs, it is sensible to keep control limits lower.

Table 2.5 Summary of sensitivity analysis

	$\widehat{Z}_{syst}(\widehat{\tau}^*)/Z_{syst}(\tau^*)$			$\widehat{\tau}^*/\tau^*$		
Λ	Mean	Min	Max	Mean	Min	Max
Λ_{a_1}	58%	48%	78%	83%	36%	114%
Λ_{a_2}	98%	78%	113%	81%	26%	116%
Λ_{a_3}	134%	99%	148%	76%	18%	115%
Λ_{b_1}	97%	48%	146%	81%	18%	116%
Λ_{b_2}	97%	48%	147%	81%	20%	115%
Λ_{b_3}	99%	48%	148%	78%	22%	114%
Λ_{c_1}	85%	48%	135%	72%	18%	114%
Λ_{c_2}	98%	55%	142%	82%	33%	115%
Λ_{c_3}	107%	61%	148%	87%	36%	116%
Λ_{d_1}	95%	48%	145%	39%	18%	60%
Λ_{d_2}	98%	48%	148%	92%	37%	106%
Λ_{d_3}	97%	49%	145%	110%	75%	116%

In Table 2.5, we categorize the instances of Table 2.4 containing a specific level of a factor into a subset. For example, a subset of instances containing a_1 is defined as $\Lambda_{a_1} = \{(a_1, b_j, c_l, d_k)|j, l, k \in \{1, 2, 3\}\}$. Table 2.5 shows the mean, minimum and maximum levels of $\widehat{Z}_{syst}(\widehat{\tau}^*)/Z_{syst}(\tau^*)$ and $\widehat{\tau}^*/\tau^*$ for these 12 subsets. In general, we observe that the mean value of $\widehat{Z}_{syst}(\widehat{\tau}^*)/Z_{syst}(\tau^*)$ increases when cost parameters, i.e., c_i^{PM}, c_i^P and S, are higher. Among them, the variation of c_i^{PM} leads to the largest variation on the mean value of $\widehat{Z}_{syst}(\widehat{\tau}^*)/Z_{syst}(\tau^*)$. Also notice that c_i^{PM} has a relatively low difference between the minimum and maximum of $\widehat{Z}_{syst}(\widehat{\tau}^*)/Z_{syst}(\tau^*)$, compared with c_i^P, S and β_i. Regarding the mean of $\widehat{\tau}^*/\tau^*$, the variation of β_i leads to the largest variation, on one hand. On the other hand, the difference between the minimum and maximum in the case of β_i is much lower than c_i^{PM}, c_i^P and S.

2.6 Performance Evaluation

To evaluate the cost reduction potential of our model, we compare our optimal solution in Table 2.3 with the optimal solutions of two other maintenance policies: i) *failure-based maintenance policy*: a condition-based maintenance policy without control limits C_i for PM actions, i.e., there are only CM actions for components; and ii) *age-based maintenance policy*: similar to our condition-based maintenance policy, the decision variables are PM age limits A_i on the ages, instead of the physical condition, at the component level and the optimal maintenance interval $\widetilde{\tau}$ of the system; which is a modification of Berg and Epstein's policy [4]. The detailed description and model formulation of these two policies are given in Subsection 2.A.1. Regarding the design of our experiments, we only know the failure time distribution in the case of the age-based maintenance policy, instead of the degradation level over time in the case of a condition-based maintenance policy. In the experiments, we are trying to evaluate the value of advanced information for the optimization of maintenance policies. Therefore, the changing factor for the two cases in our experiment design is the fact that the age-based maintenance policy does not have the advanced information, whereas the condition-based maintenance policy has such information. In order to have a fair comparison, we need to keep the other factors fixed according to the one-factor-at-a-time method [14]. Hence, the failure time distribution for the age-based maintenance policy is the same failure time distribution generated by the degradation processes in condition-based maintenance policy.

The motivations of such comparisons are: 1) to show the economic benefits of implementing condition-based maintenance and remote monitoring to decision makers in industry, via the comparisons with current policies, i.e., *failure-based maintenance policy* and *age-based maintenance policy*; 2) to fill the literature gap on the comparison of condition-based maintenance and age-based maintenance. This comparison is scientifically interesting in the context of systems with a large amount of components.

Based on the same parameter setting in Table 2.2, the optimal solutions of these two policies are shown in Table 2.6. We denote these two policies as Policy (i) and (ii).

Comparing those two policies in Table 2.6 with our policy in Table 2.3, our policy shows a considerable cost-saving potential. Notice that we use the percentage of extra cost incurred by using Policy (i) or (ii), comparing with the minimum cost rate obtained via our proposed policy

$$\triangle = \frac{\widetilde{Z}_{syst}(\widetilde{\tau}^*) - Z_{syst}(\tau^*)}{Z_{syst}(\tau^*)},$$

as the performance indicator. Policy (i) suggests a joint maintenance interval $\widetilde{\tau}^*$ of 5.98 days and the average cost rate $\widetilde{Z}_{syst}(\widetilde{\tau}^*)$ is 36817 Euros per day. The maintenance

Table 2.6 The optimal solutions of Policy (i): failure-based maintenance policy and Policy (ii): age-based maintenance policy (index: x = Main Bearing, $i \in \{1, ..., 20\}$; y = Gearbox, $i \in \{21, ..., 40\}$; z = Generator, $i \in \{41, ..., 60\}$)

Policy (i)	Values	Explanation
$\widetilde{Z}_{syst}(\widetilde{\tau}^*)$	36817	the minimum average cost rate of the system [Euro / day]
$\widetilde{\tau}^*$	5.98	the optimal maintenance interval of the system [day]
$\{\widetilde{Z}_x(\widetilde{\tau}^*), \widetilde{Z}_y(\widetilde{\tau}^*), \widetilde{Z}_z(\widetilde{\tau}^*)\}$	$\{432.1, 553.8, 438.3\}$	the minimum average maintenance cost rate of each component [Euro / day].
Policy (ii)	Values	Explanation
$\widetilde{Z}_{syst}(\widetilde{\tau}^*)$	12431	the minimum average cost rate of the system [Euro / day]
$\widetilde{\tau}^*$	25.50	the optimal maintenance interval of the system [day]
$\{A^*_x(\widetilde{\tau}^*), A^*_y(\widetilde{\tau}^*), A^*_z(\widetilde{\tau}^*)\}$	$\{51.0, 76.5, 76.5\}$	the optimal PM threshold on the age of each component [day]
$\{\widetilde{Z}_x(\widetilde{\tau}^*), \widetilde{Z}_y(\widetilde{\tau}^*), \widetilde{Z}_z(\widetilde{\tau}^*)\}$	$\{172.4, 217.3, 133.8\}$	the minimum average maintenance cost rate of each component [Euro / day].

interval of Policy (i) is much smaller than our policy, because a shorter maintenance interval helps to decrease the expected soft failure costs when no PM actions are taken. However, the setup cost rate becomes higher when τ is smaller, which further increases the cost rate at the system level. Policy (ii) suggests a joint maintenance interval $\widetilde{\tau}^*$ of 25.50 days and PM thresholds on age $A^*_i(\widetilde{\tau}^*)$ of $\{51.0, 76.5, 76.5\}$ days. The average cost rate $\widetilde{Z}_{syst}(\widetilde{\tau}^*)$ is 12431 Euros per day. A shorter maintenance interval increases the setup cost rate, which leads to a higher cost rate at the system level. Policy (ii) performs worse than our condition-based maintenance policy, because the maintenance optimization is solely based on the failure time distribution, instead of the continuously monitored condition. In this numerical example, our policy with $Z_{syst}(\tau^*) = 7424$ outperforms not only Policy (i) with $\widetilde{Z}_{syst}(\widetilde{\tau}^*) = 36817$, but also Policy (ii) with $\widetilde{Z}_{syst}(\widetilde{\tau}^*) = 12431$.

To show the cost saving potential under different parameter settings, we used the same test bed design as in Section 2.5. For each instance, the minimum average cost rate $\widetilde{Z}^*_{syst}$ of Policy (i) and (ii) is compared with the minimum average cost rate of our proposed model Z^*_{syst} under the same parameter setting. The percentage of extra cost $\triangle$ and the optimal joint maintenance interval of two policies $\widetilde{\tau}^*$ are presented in Table 2.7.

As shown in Table 2.7, the first insight is that all percentages of extra costs are positive. The average of the percentages is 448% compared with Policy (i) and 43% compared with Policy (ii). Hence, we conclude that the cost saving potential of our proposed policy is considerable under various parameter settings. In the test bed, we have 3 levels for each factor. In the summarized results in Table 2.8, for

Table 2.7 Results of the test bed of cost saving potential.

Λ	Policy (i) $\{\triangle, \widetilde{\tau}^*\}$	Policy (ii) $\{\triangle, \widetilde{\tau}^*\}$	Λ	Policy (i) $\{\triangle, \widetilde{\tau}^*\}$	Policy (ii) $\{\triangle, \widetilde{\tau}^*\}$
(a_1, b_1, c_1, d_1)	$\{528\%, 6.72\}$	$\{116\%, 47.6\}$	(a_2, b_2, c_2, d_3)	$\{442\%, 5.48\}$	$\{16\%, 49.5\}$
(a_1, b_1, c_1, d_2)	$\{667\%, 5.80\}$	$\{32\%, 27.7\}$	(a_2, b_2, c_3, d_1)	$\{294\%, 8.02\}$	$\{57\%, 35.7\}$
(a_1, b_1, c_1, d_3)	$\{738\%, 5.65\}$	$\{17\%, 48.3\}$	(a_2, b_2, c_3, d_2)	$\{417\%, 6.95\}$	$\{24\%, 45.6\}$
(a_1, b_1, c_2, d_1)	$\{441\%, 9.44\}$	$\{76\%, 48.3\}$	(a_2, b_2, c_3, d_3)	$\{461\%, 6.73\}$	$\{15\%, 49.6\}$
(a_1, b_1, c_2, d_2)	$\{652\%, 8.32\}$	$\{24\%, 44.2\}$	(a_2, b_3, c_1, d_1)	$\{374\%, 3.78\}$	$\{104\%, 34.3\}$
(a_1, b_1, c_2, d_3)	$\{731\%, 8.10\}$	$\{16\%, 48.5\}$	(a_2, b_3, c_1, d_2)	$\{407\%, 3.20\}$	$\{27\%, 44.8\}$
(a_1, b_1, c_3, d_1)	$\{395\%, 11.6\}$	$\{57\%, 48.9\}$	(a_2, b_3, c_1, d_3)	$\{445\%, 3.09\}$	$\{16\%, 49.1\}$
(a_1, b_1, c_3, d_2)	$\{626\%, 10.2\}$	$\{22\%, 44.4\}$	(a_2, b_3, c_2, d_1)	$\{362\%, 5.31\}$	$\{78\%, 34.5\}$
(a_1, b_1, c_3, d_3)	$\{708\%, 9.95\}$	$\{16\%, 48.7\}$	(a_2, b_3, c_2, d_2)	$\{475\%, 4.54\}$	$\{31\%, 45.0\}$
(a_1, b_2, c_1, d_1)	$\{616\%, 4.66\}$	$\{127\%, 44.1\}$	(a_2, b_3, c_2, d_3)	$\{497\%, 4.41\}$	$\{16\%, 49.2\}$
(a_1, b_2, c_1, d_2)	$\{809\%, 3.97\}$	$\{35\%, 43.5\}$	(a_2, b_3, c_3, d_1)	$\{342\%, 6.50\}$	$\{60\%, 34.7\}$
(a_1, b_2, c_1, d_3)	$\{840\%, 3.84\}$	$\{17\%, 48.1\}$	(a_2, b_3, c_3, d_2)	$\{474\%, 5.60\}$	$\{24\%, 45.1\}$
(a_1, b_2, c_2, d_1)	$\{531\%, 6.56\}$	$\{84\%, 44.6\}$	(a_2, b_3, c_3, d_3)	$\{524\%, 5.41\}$	$\{15\%, 49.3\}$
(a_1, b_2, c_2, d_2)	$\{766\%, 5.65\}$	$\{24\%, 43.8\}$	(a_3, b_1, c_1, d_1)	$\{205\%, 6.72\}$	$\{117\%, 39.3\}$
(a_1, b_2, c_2, d_3)	$\{859\%, 5.48\}$	$\{16\%, 48.3\}$	(a_3, b_1, c_1, d_2)	$\{190\%, 5.80\}$	$\{25\%, 47.5\}$
(a_1, b_2, c_3, d_1)	$\{470\%, 8.02\}$	$\{59\%, 45.1\}$	(a_3, b_1, c_1, d_3)	$\{220\%, 5.65\}$	$\{19\%, 50.9\}$
(a_1, b_2, c_3, d_2)	$\{751\%, 6.95\}$	$\{22\%, 44.0\}$	(a_3, b_1, c_2, d_1)	$\{169\%, 9.44\}$	$\{76\%, 39.6\}$
(a_1, b_2, c_3, d_3)	$\{851\%, 6.73\}$	$\{16\%, 48.4\}$	(a_3, b_1, c_2, d_2)	$\{208\%, 8.32\}$	$\{24\%, 47.7\}$
(a_1, b_3, c_1, d_1)	$\{684\%, 3.78\}$	$\{134\%, 42.2\}$	(a_3, b_1, c_2, d_3)	$\{234\%, 8.10\}$	$\{16\%, 51.0\}$
(a_1, b_3, c_1, d_2)	$\{885\%, 3.20\}$	$\{35\%, 43.2\}$	(a_3, b_1, c_3, d_1)	$\{155\%, 11.6\}$	$\{60\%, 39.8\}$
(a_1, b_3, c_1, d_3)	$\{916\%, 3.09\}$	$\{20\%, 30.6\}$	(a_3, b_1, c_3, d_2)	$\{217\%, 10.2\}$	$\{23\%, 47.8\}$
(a_1, b_3, c_2, d_1)	$\{601\%, 5.31\}$	$\{89\%, 42.6\}$	(a_3, b_1, c_3, d_3)	$\{236\%, 9.95\}$	$\{13\%, 51.1\}$
(a_1, b_3, c_2, d_2)	$\{851\%, 4.54\}$	$\{24\%, 43.4\}$	(a_3, b_2, c_1, d_1)	$\{252\%, 4.66\}$	$\{128\%, 37.2\}$
(a_1, b_3, c_2, d_3)	$\{956\%, 4.41\}$	$\{16\%, 48.1\}$	(a_3, b_2, c_1, d_2)	$\{224\%, 3.97\}$	$\{26\%, 46.6\}$
(a_1, b_3, c_3, d_1)	$\{551\%, 6.50\}$	$\{67\%, 43.0\}$	(a_3, b_2, c_1, d_3)	$\{245\%, 3.84\}$	$\{15\%, 50.4\}$
(a_1, b_3, c_3, d_2)	$\{843\%, 5.60\}$	$\{22\%, 43.6\}$	(a_3, b_2, c_2, d_1)	$\{216\%, 6.56\}$	$\{83\%, 37.4\}$
(a_1, b_3, c_3, d_3)	$\{957\%, 5.41\}$	$\{16\%, 48.2\}$	(a_3, b_2, c_2, d_2)	$\{255\%, 5.65\}$	$\{25\%, 46.7\}$
(a_2, b_1, c_1, d_1)	$\{288\%, 6.72\}$	$\{98\%, 36.7\}$	(a_3, b_2, c_2, d_3)	$\{278\%, 5.48\}$	$\{15\%, 50.5\}$
(a_2, b_1, c_1, d_2)	$\{320\%, 5.80\}$	$\{27\%, 45.9\}$	(a_3, b_2, c_3, d_1)	$\{206\%, 8.02\}$	$\{66\%, 37.6\}$
(a_2, b_1, c_1, d_3)	$\{349\%, 5.65\}$	$\{16\%, 49.7\}$	(a_3, b_2, c_3, d_2)	$\{272\%, 6.95\}$	$\{24\%, 46.9\}$
(a_2, b_1, c_2, d_1)	$\{253\%, 9.44\}$	$\{69\%, 37.0\}$	(a_3, b_2, c_3, d_3)	$\{298\%, 6.73\}$	$\{14\%, 50.5\}$
(a_2, b_1, c_2, d_2)	$\{336\%, 8.32\}$	$\{25\%, 46.1\}$	(a_3, b_3, c_1, d_1)	$\{285\%, 3.78\}$	$\{133\%, 36.0\}$
(a_2, b_1, c_2, d_3)	$\{369\%, 8.10\}$	$\{15\%, 49.8\}$	(a_3, b_3, c_1, d_2)	$\{299\%, 3.20\}$	$\{44\%, 46.0\}$
(a_2, b_1, c_3, d_1)	$\{230\%, 11.6\}$	$\{53\%, 37.3\}$	(a_3, b_3, c_1, d_3)	$\{273\%, 3.09\}$	$\{15\%, 50.0\}$
(a_2, b_1, c_3, d_2)	$\{341\%, 10.2\}$	$\{24\%, 46.3\}$	(a_3, b_3, c_2, d_1)	$\{252\%, 5.31\}$	$\{88\%, 36.2\}$
(a_2, b_1, c_3, d_3)	$\{377\%, 9.95\}$	$\{15\%, 50.0\}$	(a_3, b_3, c_2, d_2)	$\{289\%, 4.54\}$	$\{25\%, 46.1\}$
(a_2, b_2, c_1, d_1)	$\{346\%, 4.66\}$	$\{106\%, 35.2\}$	(a_3, b_3, c_2, d_3)	$\{317\%, 4.41\}$	$\{15\%, 50.1\}$
(a_2, b_2, c_1, d_2)	$\{414\%, 3.97\}$	$\{39\%, 45.3\}$	(a_3, b_3, c_3, d_1)	$\{244\%, 6.50\}$	$\{70\%, 36.3\}$
(a_2, b_2, c_1, d_3)	$\{404\%, 3.84\}$	$\{16\%, 49.4\}$	(a_3, b_3, c_3, d_2)	$\{313\%, 5.60\}$	$\{24\%, 46.2\}$
(a_2, b_2, c_2, d_1)	$\{314\%, 6.56\}$	$\{74\%, 29.8\}$	(a_3, b_3, c_3, d_3)	$\{343\%, 5.41\}$	$\{15\%, 50.2\}$
(a_2, b_2, c_2, d_2)	$\{396\%, 5.98\}$	$\{67\%, 25.5\}$			

Table 2.8 Summary of cost saving

	Policy (i)			Policy (ii)		
	$\triangle_{mean}$	$\triangle_{min}$	$\triangle_{max}$	$\triangle_{mean}$	$\triangle_{min}$	$\triangle_{max}$
Λ_{a_1}	712%	395%	957%	45%	16%	134%
Λ_{a_2}	382%	230%	525%	41%	15%	106%
Λ_{a_3}	249%	155%	344%	45%	13%	133%
Λ_{b_1}	378%	155%	738%	41%	13%	117%
Λ_{b_2}	455%	206%	859%	44%	15%	127%
Λ_{b_3}	510%	244%	957%	46%	15%	134%
Λ_{c_1}	453%	190%	916%	56%	15%	134%
Λ_{c_2}	449%	169%	956%	41%	15%	89%
Λ_{c_3}	441%	155%	957%	34%	13%	70%
Λ_{d_1}	356%	155%	684%	87%	53%	134%
Λ_{d_2}	473%	190%	885%	28%	22%	44%
Λ_{d_3}	514%	220%	957%	16%	13%	20%

each level of a certain factor, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing a_1 is defined as $\Lambda_{a_1} = \{(a_1, b_j, c_l, d_k) | \forall j, l, k \in \{1,2,3\}\}, \Lambda_{a_1} \subset \Lambda$. Table 2.8 shows the means, minimums and maximums of extra cost percentages ($\triangle_{mean}$, $\triangle_{min}$ and $\triangle_{max}$ respectively) of these 12 subsets. Generally speaking, Policy (ii) with preventive maintenance outperforms Policy (i) without preventive maintenance, which is intuitively sensible. Also notice that if β_i is larger or the variance of the life time distribution is lower, the mean of $\triangle$ in comparison with Policy (ii) is significantly lower, which makes our policy much less attractive.

Also shown in Table 2.7, for both policies, the optimal maintenance interval $\widetilde{\tau}^*$ decreases and cost saving potential $\triangle$ increases when c_i^P increases. This implies that it is economically beneficial to have shorter maintenance intervals and more frequent maintenance setups to avoid increasing soft failure costs. On the contrary, it is more sensible to have longer maintenance intervals and less frequent maintenance setups when the setup cost S is more expensive. A larger β_i implies a lower variance in the distribution of degradation rate. In Policy (i), a larger β_i leads to lower $\widetilde{\tau}^*$. This is because the expected maintenance cycle length decreases in a higher β_i and the decreasing rate becomes faster in a higher $\widetilde{\tau}$ (see Subsection 2.A.1). Hence, the average cost rate at the component level grows increasingly fast over $\widetilde{\tau}$. To have a lower average cost rate at the system level, a lower $\widetilde{\tau}^*$ is more economically beneficial. Also notice that there is no control limit or PM actions in Policy (i). On one hand, the optimal cost rate $\widetilde{Z}^*_{syst}$ and $\widetilde{\tau}^*$ remain unchanged in Policy (i). On the other hand, as explained in Section 2.5, the optimal cost rate of our proposed policy Z^*_{syst} increases with a higher c_i^{PM}. Hence, $\triangle$ decreases when c_i^{PM} increases.

2.7 Conclusions

In this chapter, we proposed a new condition-based maintenance model for multi-component systems with continuous stochastic deteriorations. In order to reduce the high setup cost of maintenance for multi-component systems, we used a joint maintenance interval τ to coordinate the maintenance tasks. In addition, we introduced the control limits C_i on the degradation levels of components to trigger the preventive maintenance actions. The optimal maintenance control limits of components and the optimal joint maintenance interval were determined by minimizing the long-run average cost rate related to maintenance and failures. A nested enumeration approach was proposed to solve this large-scale optimization problem. We first decomposed the optimization of the system into the optimization at the individual component level to obtain the optimal C_i for a given τ. Afterwards, we enumerated τ to find the minimum average maintenance cost rates of the system. The numerical example for a production system demonstrated that our model and the nested enumeration approach can be applied on complex systems with a large number of non-identical components. Comparing with a failure-based maintenance policy and age-based maintenance policy, our maintenance policy has a considerable cost-saving potential. Moreover, a sensitivity analysis of full factorial design was conducted to investigate the influence of different parameter settings on the optimal solutions.

Our model can be utilized to solve the maintenance scheduling problems of various engineering systems with a large number of non-identical components (e.g., production lines), because 1) it is convenient in practice to implement such a static maintenance interval for planning; 2) different physics of failures and degradations models can be adopted by the formulation of our optimization model; 3) our model can be integrated with different maintenance policies (e.g., age-based maintenance, periodic inspection) due to the static maintenance interval.

The limitation of our model includes 1) the degradation processes of components are assumed to be independent; 2) the effect of hard failures has not been taken into account. For future research, the maintenance interval can be dynamic, rather than static, in order to further reduce the long-run average cost rate. Another possible extension of the model is to consider the system structures or the dependency of components in the systems. Moreover, the effect of hard failures on the maintenance policies of complex systems can also be investigated, since many components in a system are subject to multiple failure processes (e.g., random shocks, wear-out, and crack growth).

2.A Appendices

2.A.1 Description of two comparison policies

1) Failure-based maintenance policy:

When the degradation of one component $X_i(t)$ in the system reaches H_i, a CM action is taken. For each component $i \in I$, the failure-based maintenance policy implies that there is no PM action taken, so that no control limit C_i is set on the degradation before H_i is reached (see Figure 2.1). Or equivalently, $C_i = H_i$. The optimization algorithm of our model in Subsection 2.3.2 remains unchanged in essence. Equation (2.6), (2.7), (2.11), (2.12) and (2.13) are derived as follows,

$$\begin{aligned} &Pr\{PM \ at \ n\tilde{\tau}\} = 0 \\ &Pr\{CM \ at \ n\tilde{\tau}\} = Pr\{(n-1)\tilde{\tau} \leq T_{H_i} < n\tilde{\tau}\} \end{aligned}$$

$$\begin{aligned} \mathbb{E}\Big[K_i(\tilde{\tau})\Big] &= \sum_{n \in \mathbb{N}} \Bigg[Pr\{PM \ at \ n\tilde{\tau}\}c_i^{PM} + Pr\{CM \ at \ n\tilde{\tau}\}c_i^{CM} + \mathbb{E}\Big[D_i(\tilde{\tau})\Big]c_i^P\Bigg] \\ &= c_i^{CM} + \left(\sum_{n \in \mathbb{N}} \mathbb{E}\Big[D_i(\tilde{\tau})\Big]\right)c_i^P \end{aligned}$$

$$\mathbb{E}\Big[D_i(\tilde{\tau})\Big] = \int_{(n-1)\tilde{\tau}}^{n\tilde{\tau}} (n\tilde{\tau} - x) \ f_{T_{H_i}}(x)dx$$

$$\mathbb{E}\Big[L_i(\tilde{\tau})\Big] = \sum_{n \in \mathbb{N}} n\tilde{\tau} Pr\{(n-1)\tilde{\tau} \leq T_{H_i} < n\tilde{\tau}\}$$

2) Age-based maintenance policy:

Unlike the failure-based maintenance policy, PM actions are taken at joint maintenance time point $n\tilde{\tau}, n \in \mathbb{N}$ according to a threshold A_i on the age of component i. It is almost the same to our proposed policy in Section 2.2, except the ages of components are observed, instead of the condition degradation. Notice the assumptions in Subsection 2.2.2 are also valid. Since PM actions are taken at a joint maintenance time point to save setup costs, the decision variable A_i should be a multiple of $\tilde{\tau}$, i.e., $A_i = k_i\tilde{\tau}$. Hence, if there is no failure before A_i, a PM action will be performed at A_i which is also a joint maintenance point. Otherwise, if there is a failure, a CM action will be performed at the next closest joint maintenance point, similarly to the maintenance policy proposed in this chapter. The optimization algorithm of this age-based maintenance policy is also similar to the one proposed in Subsection 2.3.2, except Equation (2.11), (2.12) and (2.13) are derived as follows,

$$\mathbb{E}\Big[K_i(\tilde{\tau}, A_i)\Big] = \int_{k_i\tilde{\tau}}^{\infty} f_{T_{H_i}}(x)dx\ c_i^{PM} + \int_0^{k_i\tilde{\tau}} f_{T_{H_i}}(x)dx\ c_i^{CM} + \mathbb{E}\Big[D_i(\tilde{\tau}, A_i)\Big]c_i^{P},$$

$$\mathbb{E}\Big[L_i(\tilde{\tau}, A_i)\Big] = \int_{k_i\tilde{\tau}}^{\infty} k_i\tilde{\tau} f_{T_{H_i}}(x)dx + \sum_{n=1}^{k_i}\int_{(n-1)\tilde{\tau}}^{n\tilde{\tau}} n\tilde{\tau} f_{T_{H_i}}(x)dx,$$

$$\mathbb{E}\Big[D_i(\tilde{\tau}, A_i)\Big] = \sum_{n=1}^{k_i}\int_{(n-1)\tilde{\tau}}^{n\tilde{\tau}} (n\tilde{\tau} - x) f_{T_{H_i}}(x)dx,$$

and $f_{T_{H_i}}(x)$ is the probability density function of the failure time ($C_i = H_i$ in Equation (2.14)). Notice that the distribution of the failure time is the same as the distribution of the passage time of H_i, because a soft failure occurs when the degradation process crosses the threshold H_i.

2.A.2 The average cost rate of a single component over two decision variables C_i and τ

To show how the objective function varies with two decision variables C_i and τ, we plot the average cost rate of component 1, a function of C_1 and τ in Figure 2.5, as an example.

2.A.3 Optimization algorithm

The procedure of the nested enumeration algorithm can be summarized in Algorithm 1. Notice different grid sizes can be used for optimizing C_i and τ , which will also affect the computational duration. In this chapter, we use the grid size $H_i/100$ and $M_\tau/300$ for C_i and τ respectively. The upper bound M_τ is a very large value (at least larger than $max_{i\in I}\{G_i\}$.). In this chapter, we choose $M_\tau = 300$ days.

2.A.4 Computation performance

Instead of optimizing $C_i(\tau)$ and τ simultaneously, we used a nested approach. Namely, we i) optimize $C_i(\tau)$ for each component under a given τ and then ii) optimize τ for the system. The motivation of such a decomposition is to reduce the computation time of large-scale problems. When the amount of components in a system is large,

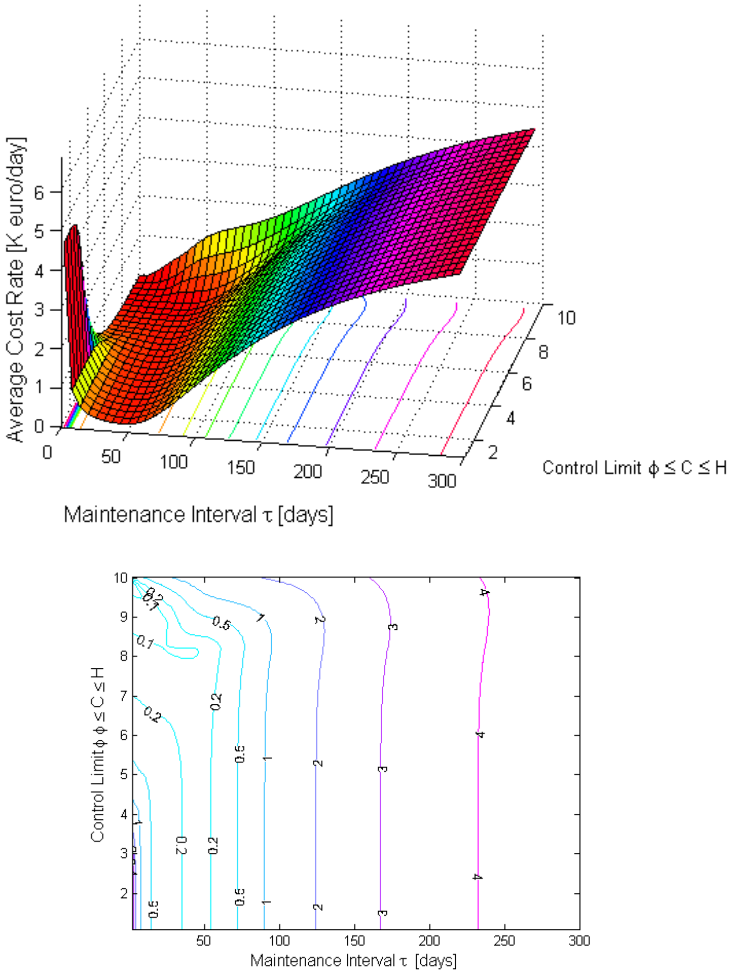


Figure 2.5 Average cost rate on component 1 over τ and C_1 (Top: 3D plot; Down: contour plot)

the solution space of decision variables increases dramatically.

For example, a system consisting of two components ($i \in \{1, 2\}$) is considered in our optimization model. For each component, we optimize the $C_i(\tau) \in (0, H)$. Suppose we discretise the degradation range $(0, H)$ into 10 grids with a grid size $H/10$. The size of the solution space (C_1, C_2) at a given τ value is 10^2. In the case of this two-component system, it is plausible to optimize τ and C_i simultaneously. However, if a

Algorithm 1 Nested optimization algorithm.

Initialize
for all $\tau \in (0, M_\tau]$ **do**
  for all $i \in I$ **do**
    for all $C_i \in [\phi_{i,1}, H_i]$ **do**
      $Z_i(\tau, C_i) = \frac{\mathbb{E}\Big[K_i(\tau, C_i)\Big]}{\mathbb{E}\Big[L_i(\tau, C_i)\Big]}$
    end for
    $C_i^*(\tau) = argmin\{Z_i(\tau, C_i)\}, \quad i \in I$
  end for
  $Z_{syst}(\tau) = \frac{S}{\tau} + \sum_{i \in I} Z_i^*(\tau)$
end for
Find $\tau^* = argmin\{Z_{syst}(\tau)\}$
Results: optimal maintenance policy $\{\tau^*, C_i^*(\tau^*)\}, \forall i \in I$

system consists of 50 components, then the size of its solution space will be 10^{50} under each given τ, which is nearly impossible to solve within a short period. Therefore, it is not efficient to optimize τ and C_i simultaneously. To solve such a large-scale problem within a reasonable computation time, we propose a nested approach to decompose the problem at system level into component level (see Section 2.3.2). This approach will reduce the solution space to 10×50 under a given τ. Regarding the numerical example in Section 2.4, the code is built in MATLAB with the runtime of 4.6×10^3 seconds (by a computer with a 2.5 GHz processor and 4 G RAM) .

Chapter 3

An Opportunistic Maintenance Policy for Components Under Condition Monitoring in Complex Systems

"There is only one kind of shock worse than the totally unexpected: the expected for which one has refused to prepare."

Mary Renault

3.1 Introduction

In the industry of advance capital goods (e.g., aviation, oil-gas refinery, energy plant, automotive), it is usually not feasible to implement CBM for all components in a complex engineering system. Instead, there are only a few very critical components in the system that are under condition monitoring continuously. However, these critical components have large impacts on the system in terms of costs. In this case, the opportunistic maintenance policies are more useful for synchronizing the maintenance actions of those components, together with the maintenance actions of the entire systems. The rest of the components in the system may be subject to different maintenance policies (e.g., failure-based maintenance, periodic preventive maintenance, etc). Hence, it is a challenging problem to coordinate different maintenance actions for a complex engineering system.

Before solving a multi-component problem, one needs to solve a single-component problem first. The literature of condition-based maintenance models at single-component level has been explained in Section 1.4 (see block A in Table 1.2). Wang proposed a CBM model based on the general random coefficient model [33] to determine the optimal control limit and the monitored interval in terms of cost, downtime and reliability. This work is closely related with our model in this chapter. For example, in both models, degradation paths are modeled by random coefficient model with 1) a pre-set failure level and 2) a preventive maintenance level as a decision variable. The clear difference is that our model included the opportunities from other components in the system, but Wang's work did not. After reviewing the CBM literature for multi-component systems (block B and C in Table 1.2 of Section 1.4), it is surprising to find that very little attention has been paid on CBM models in the context of opportunistic maintenance, which also coincides with the findings of Koochaki *et al.* [28]. They studied the cost effectiveness of condition-based and age-based maintenance in the context of opportunistic maintenance, by considering a three-component series system. Unlike their research considering only the unscheduled opportunities, our model included both scheduled and unscheduled opportunities. There are also some studies including both scheduled and unscheduled opportunities (see [29, 50]). However, they are not condition-based models, but age-based models.

Regarding the contribution of this chapter, we propose a new opportunistic maintenance policy for a monitored component to minimize the downtime cost and setup cost of maintenance. The uniqueness of our opportunistic maintenance policy is the coordination of maintenance actions for a single CBM component, by considering opportunities from both i) scheduled system downs at predetermined time points and ii) unscheduled system downs at random time points (i.e., a large portion of the components in the system are subject to failure-based maintenance policies and periodic preventive maintenance policies). This coordination has rarely been discussed in the literature of CBM policies. However, for a complex engineering system in practice, different maintenance policies are employed for different components due to the diverse characteristics of components. For example, some electronic parts (e.g., circuit board, current adapter) can be under the failure-based maintenance policy, since their failure times follow exponential distributions. On the other hand, some parts in the system can be under the periodic preventive maintenance policy due to the fact that the conditions of the components are too difficult to be measured. Under such circumstances, if we can combine the CBM activities of this monitored component with other components that are under failure-based maintenance policy (i.e., unscheduled opportunities) and periodic preventive maintenance policy (i.e., scheduled opportunities), the downtime cost and setup cost of maintenance for this monitored component will be reduced/eliminated. Thus, we introduce a control limit for the monitored component, so that when the degradation level of this component exceeds the control limit we will take the appeared opportunities from other maintenance policies and jointly maintain this monitored component with other components. Based on renewal theory, the long-run average cost rate of maintenance for this component is evaluated and minimized by optimizing the control limit of opportunistic maintenance. Notice that the cost rate evaluation is approximate, because renewal theory implies that the time points of periodic preventive maintenance are rescheduled after each maintenance action taken. However, in practice, the periodic preventive maintenance actions are planned in advance. Hence, we also verify the accuracy of the approximate evaluation by comparing with simulated evaluation under various parameter settings. Moreover, we investigate the cost saving potential of using opportunities at USDs or/and SDs under various parameter settings.

The outline of this chapter is as follows. The description of the system and the assumptions are explained in Section 3.2. The details of the mathematical model are given in Section 3.3. In Section 3.4, a numerical case of lithography machines in semiconductor industry is studied. Moreover, in Section 3.5, numerical experiments are performed to investigate the accuracy of our model and the cost-saving potential under various parameter settings. Finally, the conclusions are given in Section 3.6.

3.2 System Description

Consider a complex engineering system consisting of multiple components. One critical component is monitored continuously and maintained according to a condition-based maintenance policy. We call such a component a “CBM component”. The degradation state of the CBM component $X(t)$ can be monitored continuously over time t, $t \in [0, \infty)$. When the degradation state $X(t)$ exceeds a predetermined warning limit H, the system operates under an unsatisfied condition. Hence, a maintenance action will be triggered immediately to restore the degradation level of the CBM component to its initial level. Such a system down due to the maintenance of the CBM component is called "CBMD" (see Figure 3.1). In this model, the warning limit H is a given parameter from the experts, who have the knowledge on the physics of failures.

Apart from this CBM component, all other components in the system are subject to either a corrective maintenance or a periodic maintenance policy:

- *Failure-based maintenance policy*: For the components that are under a failure-based maintenance policy, the maintenance or replacement will be conducted immediately after the failure of the component. This will lead to *unscheduled downs* (USDs) of the system (see Figure 3.1). We assume that the inter arrival time of the failures follow an exponential distribution with its rate λ, so that the corrective maintenance actions causes USDs that can be modeled as an homogeneous Poisson process. According to the Palm-Khintchine theorem [46], even if the failure times of some components do not follow exponential distributions, the combination of a large amount of non-Poisson renewal processes will still have Poisson properties. Hence, this assumption about corrective maintenance is realistic if a sufficiently large amount of components in the system is under a failure-based maintenance policy.

- *Periodic maintenance policy*: In the industries of advance capital goods (e.g., aviation, oil-gas refinery, energy, automotive), periodic maintenance actions (e.g., inspection, cleaning, lubrication) for the system are taken every fixed interval τ [54]. This is a common practice in industry, due to the convenience of planning and coordination of maintenance resources (e.g., service engineers, maintenance equipments, spare parts). τ is a given parameter in our model, which can be determined by industrial regulations. For example, the automotive industry often recommends annual inspections on cars ($\tau = 1$ year), which leads to *scheduled downs* (SDs) of the system (see Figure 3.1).

When a system down occurs (e.g., USD, SD, or CBMD), the system operation will be interrupted and it will cause a high downtime cost for the system. Also, a setup cost of maintenance will be incurred, such as sending maintenance crews to the field. To save the downtime costs and setup costs for the multi-component system, it can be beneficial to combine preventive maintenance actions of multiple components opportunistically, which is also known as opportunistic maintenance. In this model, we use the system downs caused by corrective maintenance (at USD) and periodic maintenance (at SD) as opportunities to do preventive maintenance actions for this CBM component before $X(t)$ reaches the warning limit H (see Figure 3.2). Consequently, the setup cost and downtime cost of this CBM component will be

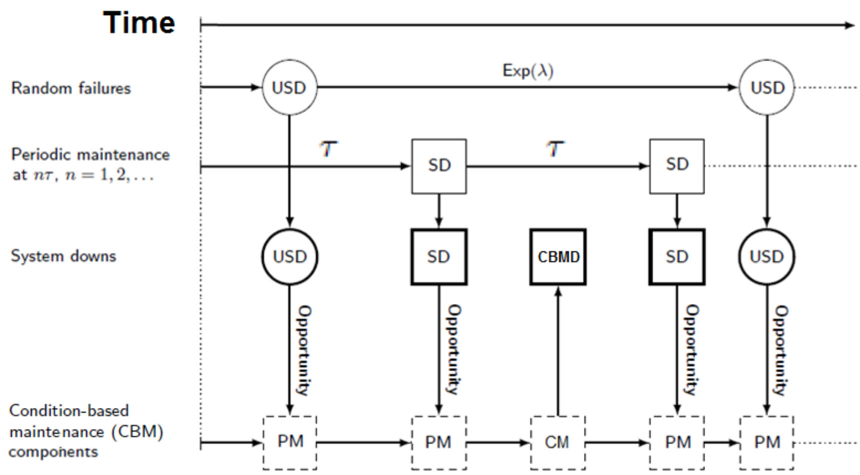


Figure 3.1 The maintenance policy of one CBM component, given the reliability information of the system

reduced by taking advantage of the opportunities. As a drawback of this opportunistic preventive maintenance, the useful lifetime of this CBM component will be shortened. In this chapter, we distinguish three types of maintenance actions on this CBM component:

1. Corrective Maintenance at a CBMD (CM): when the system stops due to a CBMD, namely, at the time point $t = \inf\{t : X(t) > H\}$ (see Figure 3.2), a corrective maintenance (CM) action is taken with a cost c^{CM}, which includes maintenance setup cost and downtime cost.

2. Preventive Maintenance at an USD (PM-USD): when the system stops at time t due to an USD, it provides an opportunity for the CBM component to be maintained together with components under the failure-based maintenance policy at this USD. If the degradation $X(t)$ exceeds a control limit C ($X(t) \geq C$, see Figure 3.2), a preventive maintenance (PM) action will be taken with a cost c^{PM-USD}. Notice that $c^{PM-USD} < c^{CM}$, because the maintenance setup cost and downtime cost of the CBM component can be eliminated or reduced, if we take the opportunity at USD to jointly maintain this CBM part. This opportunity will not be taken by the CBM component if $X(t) < C$.

3. Preventive Maintenance at a SD (PM-SD): when the system stops due to a SD at time $n\tau$, $n \in \mathbb{N}$; it provides an opportunity for the CBM component to be maintained together with components under the periodic maintenance policy at

this SD. If the degradation $X(t)$ exceeds a control limit C ($X(t) \geq C$, see Figure 3.2), a preventive maintenance (PM) action will be taken at this SD with a cost c^{PM-SD}. Notice that $c^{PM-SD} < c^{CM}$, because the maintenance setup cost and downtime cost of the CBM component will be eliminated or reduced, if we take the opportunity at SD to jointly maintain this CBM part. This opportunity will not be taken by the CBM component if $X(t) < C$.

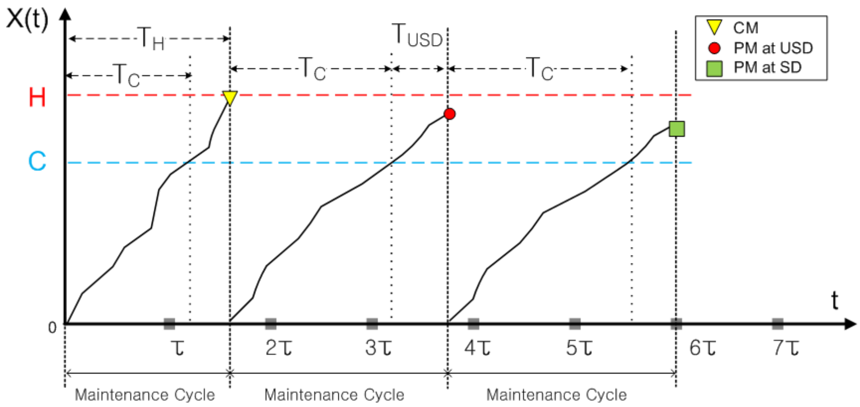


Figure 3.2 The degradation of the CBM component with three maintenance actions in practice

The periodic maintenance at time points $n\tau$, $n \in \mathbb{N}$ with maintenance interval τ (in terms of days or weeks) is small compared with the long life cycles (from 10 to 20 years) of complex engineering systems. Hence, an infinite time horizon is assumed. Moreover, we assume that the CBM component is restored as good as new by any maintenance action (CM, PM-USD or PM-SD), as shown in Figure 3.2. The intervals between two consecutive maintenance actions is defined as *maintenance cycles*. Hence, the maintenance cycle length of the CBM component depends on the ending point of the previous maintenance cycle and the maintenance action in current maintenance cycle (see Figure 3.2):

1. if a corrective maintenance action is taken on the CBM component, the maintenance cycle length is equal to the passage time that $X(t)$ exceeds H (i.e., T_H);

2. if a preventive maintenance action is taken at an USD, the maintenance cycle ends at the time point that the first USD of other components occurs after the degradation exceeds C (i.e., $T_C + T_{USD}$, where T_{USD} is exponentially distributed with a rate λ) due to the memoryless property of the Poisson process.

3. if a preventive maintenance action is taken at a SD, the maintenance cycle ends at the time point that the first SD occurs after the degradation exceeds C.

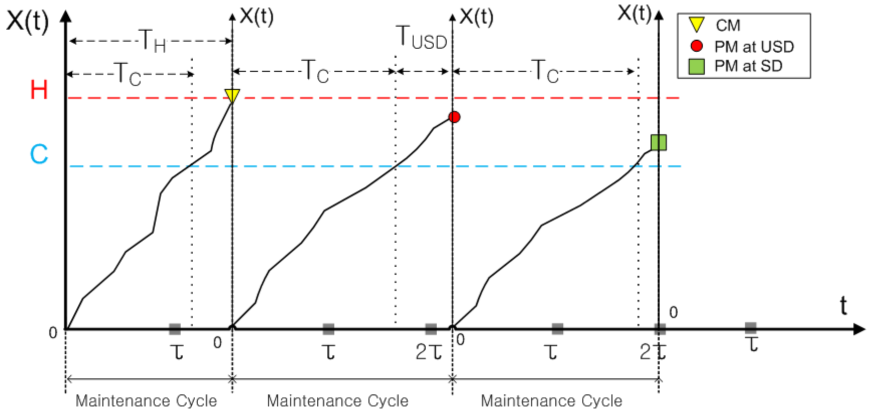


Figure 3.3 The degradation of the CBM component with three maintenance actions in renewal theory

Notice that if we assume that periodic maintenance is rescheduled at the end of each maintenance cycle of the CBM component (see Figure 3.3), the renewal theory can be applied to evaluate the long-run average cost rate of the CBM component. Consequently, the end points of maintenance cycles are the renewal points. However, the schedule of periodic maintenance for other components is usually planned in advance, which can not be changed due to the maintenance of the CBM component (see Figure 3.2). In other word, the renewal theory is exact only in the case that the previous maintenance cycle ends with a PM-SD. Hence, the renewal theory is not an exact method to evaluate the long-run average cost rate of the CBM component, but an approximation. In this chapter, we first assume the schedule of periodic maintenance restarts at every maintenance point of the CBM component (see Figure 3.3), so that renewal theory can be used to evaluate the long-run average cost rate approximately. This implies that we assume all maintenance cycles start at time points $n\tau, n \in \mathbb{N}$. Based on this approximate evaluation, an optimization model of the opportunistic maintenance policy is proposed to minimize the long-run average cost rate by specifying the control limit C. The simulated evaluation of the long-run average cost rate (see Figure 3.2) is performed in the case study in Section 3.4.

3.2.1 Notation

$X(t)$: degradation of the CBM component over time t
τ : interval of scheduled downs
λ : arrival rate of unscheduled downs (a Poisson process)
C : control limit on the degradation level (decision variable)
H : CM threshold on the degradation level

$Z(C)$: average cost rate of the CBM component
c^{PM-USD} : PM cost of the CBM component at unscheduled system downs
c^{PM-SD} : PM cost of the CBM component at scheduled system downs
c^{CM} : CM cost of the CBM component

3.2.2 Assumptions

1) The degradation of the CBM component is independent of scheduled and unscheduled downs caused by other components in the system.
2) The time horizon is infinite.
3) Maintenance actions restore the conditions of components back to their initial degradation levels. (also known as "repair-as-new").

3.3 Approximate Evaluation

The probabilities of the three maintenance actions on the CBM component in a maintenance cycle mentioned in Section 3.2 are derived in Subsection 3.3.1. Using the analytical results obtained in Subsection 3.3.1, we evaluate the long-run average cost rate of the CBM component in Subsection 3.3.2, by deriving the expected cost in a maintenance cycle and the expected cycle length. The optimization model is formulated at the end of this section.

3.3.1 Degradation model

Let $X(\hat{t})$ denotes the degradation of the CBM component at time $\hat{t} \in [0, \infty)$ in one maintenance cycle. Notice that the degradation process can be described by many different kinds of stochastic processes, e.g., random coefficient model, Gamma process, Brownian Motion or Markov Process. If the degradation process is monotonic, the probability that the degradation at time $\hat{t}$ exceeds a threshold χ is equal to the probability that the passage time T_χ of the threshold χ is less than time $\hat{t}$:

$$Pr\{T_\chi \leq \hat{t}\} = Pr\{X(\hat{t}) \geq \chi\}, \tag{3.1}$$

which is also equal to $F_{T_\chi}(\hat{t})$, the cumulative density function (c.d.f.) of the passage time T_χ. Hence, the c.d.f. and p.d.f. (probability density function) of the passage T_C and T_H can be derived based on the degradation process $X(\hat{t})$, given C and H respectively. Since we assume the degradation $X(\hat{t})$ is monotonic, $X(\hat{t})$ will first cross the control limit C before reaching H (i.e., $T_C < T_H$). The CBM component is eligible for preventive maintenance, only if $C < X(\hat{t}) < H$. In other words, if there are opportunities between T_C and T_H for the CBM component to do joint maintenance with other components, we will take the first opportunity to maintain the CBM component preventively, together with other components. If no opportunity appeared between T_C and T_H, we have to maintain the CBM component by CM, once $X(\hat{t})$ crosses the warning limit H (i.e., at the time point T_H).

We consider T_C occurs in a certain interval between the two periodic maintenance actions $(n-1)\tau \leq T_C < n\tau, n \in \mathbb{N}$, namely, when $X(\hat{t})$ reaches C at the time point $u \in \big[(n-1)\tau, n\tau\big)$. The p.d.f. of T_C is $f_{T_C}(u)\ du$. Notice the passage time T_H depends on the T_C. Given that $T_C = u$, the conditional p.d.f of T_H is $f_{T_H|T_C}(v|u)$, where $v \in [u, \infty)$. The probabilities of the maintenance actions are analyzed under the two scenarios:

Scenario 1: $(n-1)\tau \leq T_C < n\tau$ ***and*** $T_H < n\tau$
Given $(n-1)\tau \leq T_C < n\tau$, if $X(\hat{t})$ passes H at the time point v before $n\tau$, i.e., $T_H = v$ and $v \in [u, n\tau)$, there will be no opportunity due to periodic maintenance. Hence, it is only possible to take the first opportunity due to corrective maintenance. This will happen if $T_C + T_{USD} \leq T_H$, with a probability $Pr\{T_{USD} \leq v - u\} = 1 - e^{-\lambda(v-u)}$. Notice that this probability is the conditional probability given that $T_C = u$ and $T_H = v$. Hence, PM-USD happens in this scenario with a probability:

$$\int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} (1 - e^{-\lambda(v-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

On the other hand, if no opportunity is taken (i.e., $T_C + T_{USD} \geq T_H$), a CM will be taken once $X(\hat{t})$ reaches H, with a probability:

$$\int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

Scenario 2: $(n-1)\tau \leq T_C < n\tau$ ***and*** $T_H \geq n\tau$
Given $(n-1)\tau \leq T_C < n\tau$, if $X(\hat{t})$ passes H at the time point v after $n\tau$, i.e., $T_H = v$ and $v \in [n\tau, \infty)$, there will never be a CM. Instead, the first opportunity caused by either periodic maintenance or corrective maintenance of other components will be taken immediately after $X(\hat{t})$ exceeds C. Hence, if $T_C + T_{USD} \leq n\tau$, a PM-USD will be taken on the CBM component. Notice that this probability depends on $T_C = u$ and $n\tau$ with a conditional probability $Pr\{T_{USD} \leq n\tau - u\} = 1 - e^{-\lambda(n\tau-u)}$. Hence, PM-USD happens in this scenario with a probability:

$$\int_{u=(n-1)\tau}^{u=n\tau} (1 - e^{-\lambda(n\tau-u)}) \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

On the other hand, if $T_C + T_{USD} \geq n\tau$), a PM-SD will be taken at $n\tau$. This happens with a probability

$$\int_{u=(n-1)\tau}^{u=n\tau} e^{-\lambda(n\tau-u)} \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

To summarize, the choice maintenance actions (CM, PM-SD and PM-USD) depend on which one among T_H, $n\tau$ and $T_C + T_{USD}$ happens first (see in Section 3.2). The probabilities of those three types of maintenance actions within a periodic maintenance interval $(n-1)\tau$ and $n\tau$ are

$$\begin{aligned}
Pr\Big\{PM-USD\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= Pr\big\{(n-1)\tau \le T_C < T_C + T_{USD} < \min\,(T_H,\ n\tau)\big\} \\
Pr\Big\{PM-SD\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= Pr\big\{(n-1)\tau \le T_C < n\tau < \min\,(T_H,\ T_C + T_{USD})\big\} \\
Pr\Big\{CM\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= Pr\big\{(n-1)\tau \le T_C < T_H < \min\,(n\tau,\ T_C + T_{USD})\big\}
\end{aligned}$$

Here we define $P_1 = \sum_{n=1}^{\infty} Pr\Big\{PM-USD\ in\ \big[(n-1)\tau, n\tau\big)\Big\}$, $P_2 = \sum_{n=1}^{\infty} Pr\Big\{PM-SD\ in\ \big[(n-1)\tau, n\tau\big)\Big\}$ and $P_3 = \sum_{n=1}^{\infty} Pr\Big\{CM\ in\ \big[(n-1)\tau, n\tau\big)\Big\}$.

$$\begin{aligned}
P_1 &= \sum_{n=1}^{\infty} \Big\{ \int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} (1 - e^{-\lambda(v-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\
&+ \int_{u=(n-1)\tau}^{u=n\tau} (1 - e^{-\lambda(n\tau-u)}) \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \Big\} \\
P_2 &= \sum_{n=1}^{\infty} \int_{u=(n-1)\tau}^{u=n\tau} e^{-\lambda(n\tau-u)} \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\
P_3 &= \sum_{n=1}^{\infty} \int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du
\end{aligned} \tag{3.2}$$

The sum of those three probabilities (P_1, P_2 and P_3) is also equal to one. Notice the aggregation of $(n-1)\tau \le T_C < n\tau$ ($n \in \mathbb{N}$, $\tau \in \Re$) implies $T_C \in [0, \infty)$ and $\sum_{n=1}^{\infty} \int_{u=(n-1)\tau}^{u=n\tau} f_{T_C}(u)\ du = 1$

3.3.2 Evaluation and optimization

As explained in Section 3.2, the evaluation of the average cost rate via renewal theory is an approximation, which will be compared with simulation results. The minimization of the cost rate can be based on the approximate evaluation $Z(C)$ and simulation $\hat{Z}(\hat{C})$, which leads to their optimal control limits C^* and $\hat{C}^*$ respectively.

According to Equation (3.2), the expected cycle cost $K(C)$ can be derived:

$$
\begin{aligned}
K(C) &= P_1\ c^{PM-USD} + P_2\ c^{PM-SD} + P_3\ c^{CM} \\
&= \sum_{n=1}^{\infty} \int_{u=(n-1)\tau}^{u=n\tau} \Big\{ \int_{v=u}^{v=n\tau} \left[c^{PM-USD}(1-e^{-\lambda(v-u)}) + (c^{CM})e^{-\lambda(v-u)} \right] \cdot \\
&\quad f_{T_H|T_C}(v|u)\, dv + \left[c^{PM-USD}\left(1-e^{-\lambda(n\tau-u)}\right) + (c^{PM-SD})e^{-\lambda(n\tau-u)} \right] \cdot \\
&\quad \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\, dv \Big\} f_{T_C}(u)\, du
\end{aligned}
$$

and similarly the expected cycle length $L(C)$ is (also see Subsection 3.A.1)

$$
\begin{aligned}
L(C) &= \sum_{n=1}^{\infty} \int_{u=(n-1)\tau}^{u=n\tau} \left\{ u + \int_{v=u}^{v=n\tau} \left(\frac{1}{\lambda} \left(1 - e^{-\lambda(v-u)} \right) \right) f_{T_H|T_C}(v|u)\, dv \right. \\
&\quad \left. + \frac{1}{\lambda} \int_{v=n\tau}^{v=\infty} \left(1 - e^{-\lambda(n\tau-u)} \right) f_{T_H|T_C}(v|u)\, dv \right\} f_{T_C}(u)\, du
\end{aligned}
\tag{3.3}
$$

According to the renewal theory, the expected cost rate of the CBM component $Z(C)$ is equal to $K(C)/L(C)$. Hence, the optimization model is formulated as

$$
\begin{aligned}
\min_{C} \quad & Z(C) = \frac{K(C)}{L(C)} \\
\text{s.t.} \quad & 0 < C < H
\end{aligned}
$$

The objective function is non-linear and different when degradation paths are modeled by different degradation models. Hence, several non-linear optimization methods may be used [5], depending on different degradation models.

3.4 Case Study

As a demonstration of our model, we provide a case of lithography machines in semiconductor industry. The machines are complex engineering systems processing the pure-silicon-made wafers to semiconductor integrated circuits, also known as micro-chips. The laser unit in the machine is considered as one of the most important components, whose degradation is continuously monitored. The measurement of its physical condition is the output power in Watts. When the degradation of output power exceeds a certain limit, bad chips are produced and a maintenance action is needed. Considering the laser unit as the CBM component, the degradation of output power over time is obtained from the historical data of n laser units. For each laser unit $j = \{1, 2, ..., n\}$, the degradation level $x_{k,j}$ is measured at minute k, $k = \{1, 2, ..., m\}$, where $m \in \mathbb{N}$. The time of the last degradation measurement m is

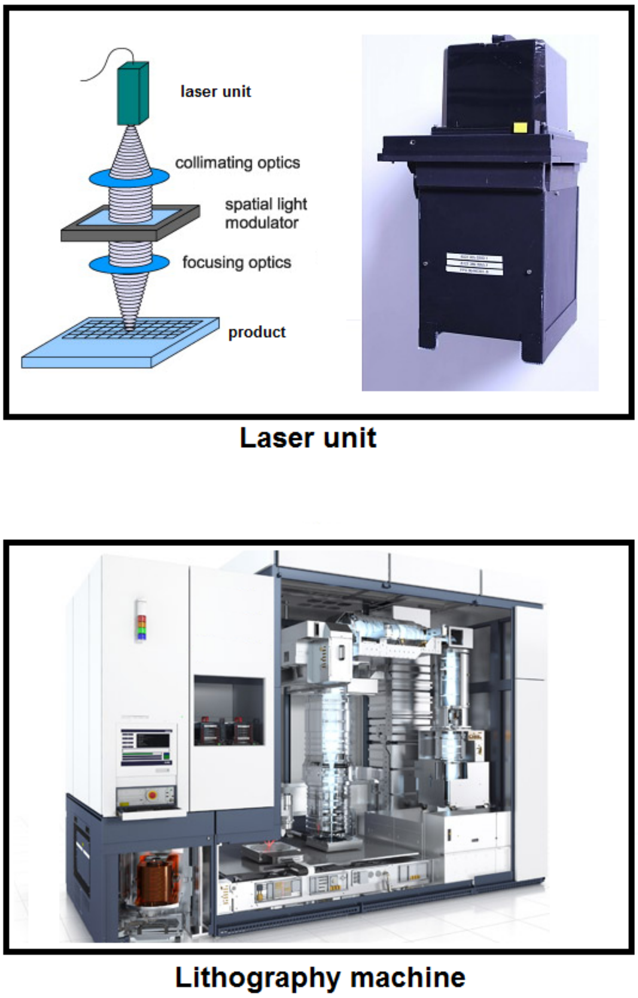


Figure 3.4 A laser unit in a lithography machine[3]

the same for all laser units.

As mentioned in the literature review in Section 3.1, there are several approaches to model the stochastic degradation paths of a component (e.g., random coefficient model, Gamma process, Wiener process or Markov Process). To validate our model for various degradation paths, we model $X(\hat{t})$ by two approaches: i) *Random coefficient model* [33], because it is relatively flexible and convenient for describing the degradation paths derived from physics of failures, such as laws of physics and material science; ii) *Gamma process* [59], due to its popularity in the literature.

Fitting Option 1 - *Random coefficient model:* $X(\hat{t};\Phi,\Theta)$ *is a random variable given a set of constant parameters* $\Phi = \{\phi_1, ..., \phi_Q\}, Q \in \mathbb{N}$*; and a set of random parameters,* $\Theta = \{\theta_1, ..., \theta_V\}, V \in \mathbb{N}$*, following certain probability distributions. In order to clarify the model, we start with a simple degradation path* $X(\hat{t};\Phi,\Theta) = \phi_1 + \theta_1 \hat{t}^{\phi_2}$*, where* $\Phi = \{\phi_1, \phi_2\}$ *and* $\Theta = \{\theta_1\}$*. Equation (3.1) can be written in terms of* F_{θ_1} *(the cumulative density function of random variable* θ_1*,* $\theta_1 \geq 0$*) as:*

$$\begin{aligned} Pr\{T_\chi \leq \hat{t}\} &= Pr\{\phi_1 + \theta_1 \hat{t}^{\phi_2} \geq \chi\} \\ &= Pr\{\theta_1 \geq \frac{\chi - \phi_1}{\hat{t}^{\phi_2}}\} \\ &= 1 - F_{\theta_1}\Big(\frac{\chi - \phi_1}{\hat{t}^{\phi_2}}\Big) \end{aligned} \tag{3.4}$$

For example, if the degradation rate θ_1 *follows a Weibull distribution with a scale parameter* α *and a shape parameter* β*, then the probability density function of the passage time* T_χ *is*

$$f_{T_\chi}(\hat{t}) = \frac{\phi_2 \beta \alpha}{\chi - \phi_1} \left(\frac{\chi - \phi_1}{\alpha \hat{t}^{\phi_2}}\right)^{\beta+1} exp\{-\left(\frac{\chi - \phi_1}{\alpha \hat{t}^{\phi_2}}\right)^{\beta}\}, \quad t > 0. \tag{3.5}$$

Notice that $\phi_1 = 0$ *and* $\phi_2 = 1$ *in the case of this laser unit and the degradation path reduces to* $X(\hat{t}) = \theta_1 \hat{t}$*. Hence, only the parameters* α *and* β *need to be estimated.* $\diamondsuit$

Fitting Option 2 - *Gamma process: if* $X(\hat{t})$ *is a Gamma distribution with its initial degradation level* x_0 *at* $\hat{t} = 0$*. The random increments throughout the process are independently and identically distributed (i.i.d.) according to a Gamma process with a scale parameter* η *and a shape parameter* γ*. The cumulative density function of the passage time* T_χ *is*

$$F_{T_\chi}(\hat{t}) = \frac{\Gamma(\gamma\hat{t}, \eta(\chi - x_0))}{\Gamma(\gamma\hat{t})}, \tag{3.6}$$

where $\Gamma(\gamma\hat{t}) = \int_0^\infty y^{\gamma\hat{t}-1} e^{-y} dy$ *and* $\Gamma(\gamma\hat{t}, \eta(\chi - x_0)) = \int_{\eta(\chi-x_0)}^\infty y^{\gamma\hat{t}-1} e^{-y} dy$ *[47].* $\diamondsuit$

Besides the degradation parameters (i.e., α, β, γ and η) estimated from the data, the rest of the input parameters in Table 3.1 are given by the company of lithography machines [54]. The parameter estimation of the degradation path follows the standard methods in the literature (see Section A.1 in Appendix A).

Given the input parameters [1] in Table 3.1, the optimal maintenance policy of the laser unit can be obtained for both the random coefficient model (see Fitting Option 1) and the Gamma process (see Fitting Option 2).

[1] Due to the regulation of confidentiality from the company, the parameters in this table are dummy values.

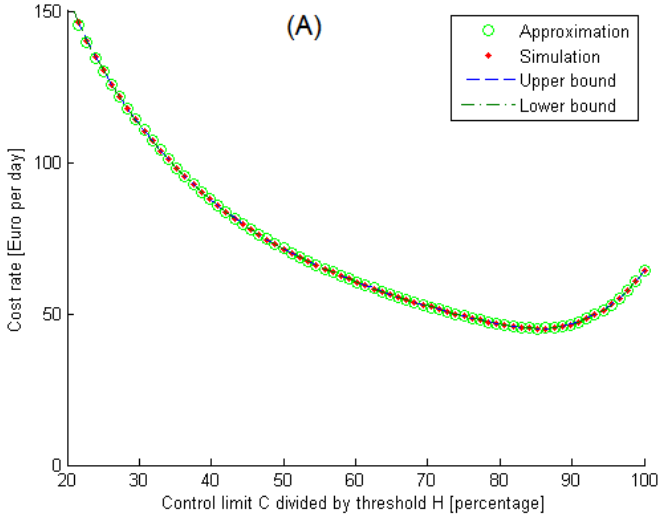
Table 3.1 The parameter setting

Parameter	Explanation
$c^{PM-SD} = 26.5$	Preventive maintenance due to scheduled downs [thousand Euro]
$c^{PM-USD} = 28.8$	Preventive maintenance due to unscheduled downs [thousand Euro]
$c^{CM} = 44.5$	Corrective maintenance [thousand Euro]
$\tau = 91$	The interval of scheduled downs [day]
$\alpha = 0.159$	Scale parameter of the Weibull distribution
$\beta = 3.73$	Shape parameter of the Weibull distribution
$\{\phi_1, \phi_2\} = \{0, 1\}$	Constant parameters
$\lambda = 8.86 * 10^{-3}$	Poisson arrival rate of unscheduled downs [per day]
$H = 88$	Failure threshold [Watt]
$\gamma = 0.221$	Shape parameter of the Gamma distribution
$\eta = 1.85$	Scale parameter of the Gamma distribution

The optimal control limit C^* in terms of a percentage of H can be found by minimizing the average cost rate $Z(C^*)$ via the approximate evaluation (see Subsection 3.3.2). As a comparison, we simulate the average cost rate $\hat{Z}$ (see Subsection 3.A.2) given C^* as the control limit. Figure 3.5 illustrates the changes of the average cost rate over the control limit C (relative to H). The results obtained from both the approximate evaluation and simulation, are shown in Figure 3.5, where the random coefficient model is used in Figure 3.5 (A) and the Gamma process is used in Figure 3.5 (B).
The numerical results are also given in Table 3.2. Under the use of the random coefficient model, the optimal maintenance policy obtained via the approximate evaluation has a control limit that is 85.71% of the threshold ($C^*/H = 85.71\%$) and a minimum cost rate of 45.09 euro per day (see Figure 3.5-A). In the case of the Gamma process, the optimal maintenance policy obtained via the approximate evaluation has a control limit that is 87.18% of H with a minimum cost rate around 40.99 euro per day (see Figure 3.5-B). The confidence interval is also very small in Figure 3.5. (More details in Subsection 3.A.2).

To further investigate the differences between our approximation model and the simulation model, Table 3.2 shows: i) the optimal policy obtained via the approximate evaluation, including the optimal control limit C^*, its minimum cost rate $Z(C^*)$, its probabilities of three maintenance actions $\left\{P_1, P_2, P_3\right\}$ and its expected cycle length $L(C^*)$; ii) the simulation results under the optimal control limit C^* obtained via approximate evaluation, where $\hat{Z}(C^*)$ denotes the average cost rate, $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ denotes the probabilities of three maintenance actions and $\hat{L}(C^*)$ denotes the mean cycle length; iii) the optimal control limit $\hat{C}^*$ obtained via simulation-based optimization with its minimum cost rate $\hat{Z}(\hat{C}^*)$, its probabilities of three maintenance actions $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ and its mean cycle length $\hat{L}(\hat{C}^*)$.

Based on the results in Table 3.2, we observe that the absolute value $|\big(\hat{Z}(C^*) -$



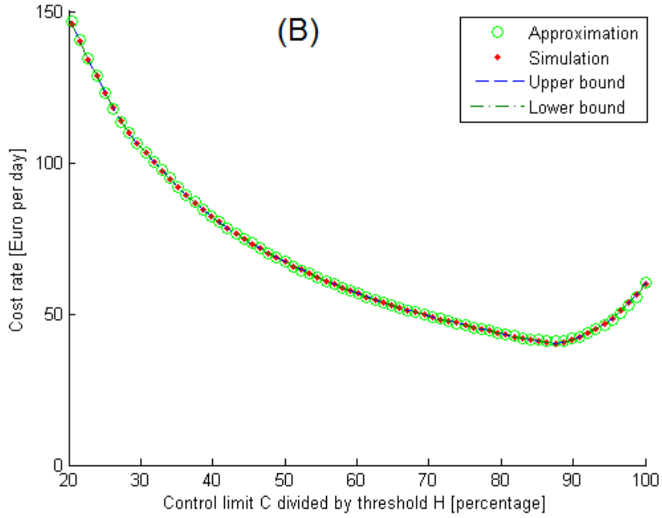


Figure 3.5 Average cost rate [euro per day] over various $\frac{C}{H}$. (A) by the random coefficient model and (B) by the Gamma process

$Z(C^*)\big)/\hat{Z}(C^*)|$, denoted by Gap 1, is only 0.16% in the RCM case and 0.05% in the GP case, which shows that our approximate evaluation model is very close to the simulation model, under the same C^* value. The absolute value $|\big(\hat{Z}(\hat{C}^*) - \hat{Z}(C^*)\big)/\hat{Z}(\hat{C}^*)|$, denoted by Gap 2, is only 0.04% in the RCM case and 1.06% in the GP case. This implies that the deviation of C^* from $\hat{C}^*$ does not lead to a large deviation on the simulated cost rate, which is due to the fact that the cost rate $\hat{Z}(C)$ is

Table 3.2 The optimal maintenance policies under the parameter setting in Table 3.1 ($\{P_1, P_2, P_3\}$ and $\{\hat{P}_1, \hat{P}_2, \hat{P}_3\}$ are the probabilities of taking PM-USD, PM-SD and CM actions obtained via the approximate evaluation and simulation respectively)

in the case of the random coefficient model (RCM)	
Approximation Result	Simulation Result 1
$Z(C^*) = 45.09$ [euro per day] $C^*/H = 85.71\%$ $\left\{P_1, P_2, P_3\right\} = \{0.3075, 0.6350, 0.0576\}$ $L(C) = 627.4$ [day]	$\hat{Z}(C^*) = 45.16 \pm 0.024$ [euro per day] $C^*/H = 85.71\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3062, 0.6333, 0.0605\}$ $\hat{L}(C) = 627.6$ [day]
Simulation Result 2	
$\hat{Z}(\hat{C}^*) = 45.14 \pm 0.021$ [euro per day] $\hat{C}^*/H = 85.23\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3086, 0.6412, 0.0502\}$ $\hat{L}(\hat{C}^*) = 623.8$ [day]	$\|Gap1\|$: $\left\|\frac{\left(\hat{Z}(C^*)-Z(C^*)\right)}{\hat{Z}(C^*)}\right\| = 0.16\%$ $\|Gap2\|$: $\left\|\frac{\left(\hat{Z}(\hat{C}^*)-\hat{Z}(C^*)\right)}{\hat{Z}(\hat{C}^*)}\right\| = 0.04\%$

in the case of the Gamma process (GP)	
Approximation Result	Simulation Result 1
$Z(C^*) = 40.99$ [euro per day] $C^*/H = 87.18\%$ $\left\{P_1, P_2, P_3\right\} = \{0.3102, 0.6563, 0.0335\}$ $L(C) = 679.76$ [day]	$\hat{Z}(C^*) = 41.01 \pm 0.051$ [euro per day] $C^*/H = 87.18\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3096, 0.6512, 0.0392\}$ $\hat{L}(C) = 681.98$ [day]
Simulation Result 2	
$\hat{Z}(\hat{C}^*) = 40.57 \pm 0.038$ [euro per day] $\hat{C}^*/H = 85.75\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3122, 0.6635, 0.0243\}$ $\hat{L}(\hat{C}^*) = 682.79$ [day]	$\|Gap1\|$: $\left\|\frac{\left(\hat{Z}(C^*)-Z(C^*)\right)}{\hat{Z}(C^*)}\right\| = 0.05\%$ $\|Gap2\|$: $\left\|\frac{\left(\hat{Z}(\hat{C}^*)-\hat{Z}(C^*)\right)}{\hat{Z}(\hat{C}^*)}\right\| = 1.06\%$

flat in the neighborhood of its minimum. Hence, in practice, the optimal maintenance policy of our approximate evaluation will result in an average cost rate that is very close to the true minimum cost rate. Also notice that the values of $\left\{P_1, P_2, P_3\right\}$ and $L(C)$ via approximation are very close to the simulated values $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ and $\hat{L}(C)$. Therefore, we can conclude that the gaps are small and our approximate evaluation is accurate in this case study.

3.5 Numerical Experiments

To validate our model under various parameter settings, we conduct the following numerical experiments based on full factorial test beds. In Section 3.5.1 and 3.5.2, we investigate the accuracy of our approximate evaluation. In Section 3.5.3, we evaluate the cost reduction potential of our proposed policy under various parameter settings.

3.5.1 Accuracy of the approximate evaluation

The accuracy of our approximate evaluation is assessed based on the gap between the simulation result $\hat{Z}(C)$ and the approximation result $Z(C)$. We vary four factors in our test bed: the variable C and three parameters τ, λ and σ [2]. Three different levels of the control limit C as the percentage of the threshold H, $\{30\%, 50\%, 70\%\}$, are chosen, and each of the other three parameters has a basic value multiplied by a set of factors, $\{50\%, 100\%, 150\%\}$. The basic values are found back in Table 3.3. Hence, a full factorial test bed is set up and a space of instances is defined, $(C_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda, \forall j, l, k, m = \{1, 2, 3\}$, which leads to $|\Lambda| = 81$ instances in the test bed.

Table 3.3 Parameter setting of the test bed

Parameter	Explanation
$\tau = 0.2 * \{50\%, 100\%, 150\%\}$	The interval of scheduled downs
$\lambda = 2 * \{50\%, 100\%, 150\%\}$	Poisson arrival rate of unscheduled downs
$\sigma = 1/2 * \{50\%, 100\%, 150\%\}$	Standard deviation of component life time
$C = \{30\%, 50\%, 70\%\}$	Control limit on the degradation level
$E[T_H] = 1$	Expected component life time
$H = 100\%$	Failure threshold

We set the expected life time $E[T_H]$ of the CBM component to be equal to 1, which normalizes the time unit. By fitting the two moments of the component life time, the shape and scale parameters of the Weibull distribution in the RCM case (see Fitting Option 1 in Subsection 3.3.1) and of the Gamma distribution in the GP case (see Fitting Option 2 in Subsection 3.3.1) can be estimated. To show the variance of the degradation path, we choose the standard deviation σ as a varying parameter. Moreover, τ and λ are varied, because they determine the frequency of the opportunities from PM-USD and PM-SD events (see Section 3.2). The design of the full factorial test bed is shown in Table 3.3.

Notice that no cost parameter is chosen as factors in this test bed, since the objective function is fully determined by the probabilities of the three maintenance actions and the expected cycle length. This also helps to reduce the size of the test bed. To compare the approximation results and simulation results, we compare the probabilities of PM-USD, PM-SD and CM and the expected cycle length obtained by the approximate evaluation $\big(P_1, P_2, P_3, L(C)\big)$ and the simulation $\big(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\big)$

[2] $\sigma^2 = E[T_H^2] - E[T_H]^2$, where $E[T_H]$ and $E[T_H^2]$ are the 1^{st} and 2^{nd} moment of the component life time. σ is the standard deviation of the component life time distribution T_H (see Equation (3.5) and (3.6)). The larger σ is, the larger the variance of the degradation path is.

respectively; which is similar to Table 3.2 in Section 3.4. To see how much the approximation results deviate from the simulation results, we define a deviation vector $\left[\delta_1, \delta_2, \delta_3, \delta_4\right] = \left[\hat{P}_1 - P_1, \hat{P}_2 - P_2, \hat{P}_3 - P_3, (\hat{L}(C) - L(C))/\hat{L}(C)\right]$. The deviation vectors of 81 instances are shown in Table 3.8, 3.9, 3.10 and 3.11 of Subsection 3.A.3. There are three levels for each factor in $(C_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda, \forall j, l, k, m = \{1, 2, 3\}$. We categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing C_1 is defined as $\Lambda_{C_1} = \big\{(C_1, \sigma_l, \lambda_k, \tau_m) | l, k, m \in \{1, 2, 3\}\big\}$. For each of these subsets, the average of the absolute deviation vectors (denoted by AAD) and the maximum of the absolute deviation vectors (denoted by MAD) are summarized in Table 3.4.

Table 3.4 The evaluation of the average absolute difference (AAD) and maximum absolute difference (MAD) between the simulation results and the approximation results; in the case of the random coefficient model (RCM) and the Gamma process (GP)

	RCM case			
	$\lvert\delta_1\rvert$ $\{AAD, MAD\}$	$\lvert\delta_2\rvert$ $\{AAD, MAD\}$	$\lvert\delta_3\rvert$ $\{AAD, MAD\}$	$\lvert\delta_4\rvert$ $\{AAD, MAD\}$
Λ_{C_1}	$\{0.0047, 0.0156\}$	$\{0.0048, 0.0160\}$	$\{0.0003, 0.0004\}$	$\{0.52\%, 1.48\%\}$
Λ_{C_2}	$\{0.0030, 0.0143\}$	$\{0.0031, 0.0149\}$	$\{0.0005, 0.0007\}$	$\{0.21\%, 1.01\%\}$
Λ_{C_3}	$\{0.0020, 0.0168\}$	$\{0.0039, 0.0295\}$	$\{0.0023, 0.0127\}$	$\{0.12\%, 0.62\%\}$
Λ_{σ_1}	$\{0.0043, 0.0168\}$	$\{0.0057, 0.0295\}$	$\{0.0017, 0.0127\}$	$\{0.33\%, 1.40\%\}$
Λ_{σ_2}	$\{0.0028, 0.0148\}$	$\{0.0034, 0.0151\}$	$\{0.0009, 0.0053\}$	$\{0.26\%, 1.30\%\}$
Λ_{σ_3}	$\{0.0026, 0.0150\}$	$\{0.0028, 0.0154\}$	$\{0.0005, 0.0014\}$	$\{0.26\%, 1.48\%\}$
Λ_{λ_1}	$\{0.0012, 0.0048\}$	$\{0.0019, 0.0143\}$	$\{0.0010, 0.0095\}$	$\{0.17\%, 0.58\%\}$
Λ_{λ_2}	$\{0.0031, 0.0109\}$	$\{0.0038, 0.0209\}$	$\{0.0011, 0.0122\}$	$\{0.29\%, 1.06\%\}$
Λ_{λ_3}	$\{0.0054, 0.0168\}$	$\{0.0061, 0.0295\}$	$\{0.0011, 0.0127\}$	$\{0.39\%, 1.48\%\}$
Λ_{τ_1}	$\{0.0005, 0.0014\}$	$\{0.0007, 0.0019\}$	$\{0.0005, 0.0007\}$	$\{0.07\%, 0.19\%\}$
Λ_{τ_2}	$\{0.0031, 0.0140\}$	$\{0.0031, 0.0138\}$	$\{0.0004, 0.0007\}$	$\{0.25\%, 0.95\%\}$
Λ_{τ_3}	$\{0.0060, 0.0168\}$	$\{0.0081, 0.0295\}$	$\{0.0022, 0.0127\}$	$\{0.53\%, 1.48\%\}$
Λ	$\{0.0032, 0.0168\}$	$\{0.0039, 0.0295\}$	$\{0.0010, 0.0127\}$	$\{0.28\%, 1.48\%\}$
	GP case			
	$\lvert\delta_1\rvert$ $\{AAD, MAD\}$	$\lvert\delta_2\rvert$ $\{AAD, MAD\}$	$\lvert\delta_3\rvert$ $\{AAD, MAD\}$	$\lvert\delta_4\rvert$ $\{AAD, MAD\}$
Λ_{C_1}	$\{0.0017, 0.0055\}$	$\{0.0016, 0.0040\}$	$\{0.0008, 0.0040\}$	$\{0.16\%, 0.50\%\}$
Λ_{C_2}	$\{0.0017, 0.0078\}$	$\{0.0069, 0.0314\}$	$\{0.0066, 0.0294\}$	$\{0.20\%, 0.44\%\}$
Λ_{C_3}	$\{0.0052, 0.0186\}$	$\{0.0298, 0.0600\}$	$\{0.0339, 0.0635\}$	$\{0.37\%, 1.12\%\}$
Λ_{σ_1}	$\{0.0021, 0.0078\}$	$\{0.0080, 0.0504\}$	$\{0.0073, 0.0511\}$	$\{0.19\%, 0.43\%\}$
Λ_{σ_2}	$\{0.0034, 0.0186\}$	$\{0.0125, 0.0516\}$	$\{0.0141, 0.0598\}$	$\{0.26\%, 0.88\%\}$
Λ_{σ_3}	$\{0.0032, 0.0154\}$	$\{0.0177, 0.0600\}$	$\{0.0198, 0.0635\}$	$\{0.29\%, 1.12\%\}$
Λ_{λ_1}	$\{0.0019, 0.0077\}$	$\{0.0148, 0.0600\}$	$\{0.0150, 0.0635\}$	$\{0.24\%, 0.70\%\}$
Λ_{λ_2}	$\{0.0030, 0.0186\}$	$\{0.0119, 0.0509\}$	$\{0.0137, 0.0598\}$	$\{0.26\%, 1.12\%\}$
Λ_{λ_3}	$\{0.0038, 0.0184\}$	$\{0.0115, 0.0479\}$	$\{0.0125, 0.0575\}$	$\{0.24\%, 0.88\%\}$
Λ_{τ_1}	$\{0.0013, 0.0030\}$	$\{0.0041, 0.0207\}$	$\{0.0033, 0.0191\}$	$\{0.18\%, 0.50\%\}$
Λ_{τ_2}	$\{0.0026, 0.0097\}$	$\{0.0144, 0.0600\}$	$\{0.0148, 0.0635\}$	$\{0.22\%, 0.55\%\}$
Λ_{τ_3}	$\{0.0048, 0.0186\}$	$\{0.0197, 0.0535\}$	$\{0.0231, 0.0607\}$	$\{0.34\%, 1.12\%\}$
Λ	$\{0.0029, 0.0186\}$	$\{0.0127, 0.0600\}$	$\{0.0137, 0.0635\}$	$\{0.25\%, 1.12\%\}$

The first insight from Table 3.4 is that the AAD and MAD of δ_1, δ_2, δ_3 and δ_4 are small, which implies that our approximate evaluation is accurate under all parameter settings (including the control limit C). We also observe that the AAD and MAD in the RCM case are at the magnitude of 10^{-3} and 10^{-2}, respectively. Compared with the RCM case, our model in the GP case is less accurate, as its AAD and MAD are at the magnitude of 10^{-2} and 10^{-2} respectively. Notice that the AAD and MAD for both RCM and GP become larger when τ is larger. This is due to the assumption that each maintenance cycle starts at a scheduled down in the approximate evaluation. The larger τ is, the larger the effect of this assumption is.

3.5.2 Heuristic optimization based on the approximate evaluation

The results in Table 3.2 of Section 3.4 show that the optimal policies obtained via our approximate evaluation and simulation are close to each other, which verifies the accuracy of the approximate evaluation in the case of lithography machines. In this subsection, we intend to verify these results further under various parameter settings in this test bed. Similar to Table 3.2, we evaluate two gaps: i) Gap 1, $\big(\hat{Z}(C^*) - Z(C^*)\big)/\hat{Z}(C^*)$, shows how much the true cost rate deviates from the cost rate of the approximate evaluation, by using the optimal control limit of our approximate evaluation; and ii) Gap 2, $\big(\hat{Z}(\hat{C}^*) - \hat{Z}(C^*)\big)/\hat{Z}(\hat{C}^*)$, shows how much the optimal maintenance policy of our approximate evaluation deviates from the true optimal policy; where C^* and $\hat{C}^*$ are the optimal control limits of the approximate evaluation and simulation respectively.

Since we will compare the optimal policies via approximate evaluation with simulation, the control limit C is no longer a factor in the test bed and Table 3.3. Hence, the space of instances is $(\sigma_l, \lambda_k, \tau_m) \in \Omega, \forall l, k, m = \{1, 2, 3\}$, which leads to $|\Omega| = 27$ instances in the test bed. The cost factors are found back in Table 3.1. The deviation vectors of 27 instances are shown in Table 3.12 of Subsection 3.A.3. For each factor, we categorize the instances containing a specific level into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \big\{(\sigma_1, \lambda_k, \tau_m) | \forall k, m \in \{1, 2, 3\}\big\}$. For each of these subsets, the average of absolute deviation is denoted by AAD and the maximum of the absolute deviation vectors is denoted by MAD. These results are summarized in Table 3.5 for RCM and GP.

The first insight from Table 3.5 is that the AAD and MAD are reasonably small, which verifies the accuracy of our approximate evaluation in the neighborhood of the optimal solution. Regarding Gap 1, the AAD for RCM is smaller than the AAD for GP, which means the accuracy of the approximate evaluation is higher in the RCM case than GP. Regarding Gap 2, the AAD for RCM is smaller than the AAD for GP, which is caused by the lower deviations of $\hat{C}^*$ from C^* for RCM than for GP. Compared Gap 1 with Gap 2 for both cases of RCM and GP, we observe that the AAD

Table 3.5 The average absolute difference (AAD) and the maximum absolute difference (MAD) between the simulation results and the approximation results for Gap 1, Gap 2 and $(\hat{C}^*-C^*)/H$; in the case of the random coefficient model (RCM) and the Gamma process (GP)

	RCM case		
	$(\hat{C}^* - C^*)/H$	$Gap1$	$Gap2$
	$\{AAD, MAD\}$	$\{AAD, MAD\}$	$\{AAD, MAD\}$
Ω_{σ_1}	$\{0.97\%, 6.57\%\}$	$\{0.20\%, 0.30\%\}$	$\{0.12\%, 0.41\%\}$
Ω_{σ_2}	$\{0.55\%, 1.36\%\}$	$\{0.20\%, 0.64\%\}$	$\{0.10\%, 0.19\%\}$
Ω_{σ_3}	$\{0.90\%, 3.72\%\}$	$\{0.17\%, 0.35\%\}$	$\{0.14\%, 0.33\%\}$
Ω_{λ_1}	$\{0.83\%, 3.72\%\}$	$\{0.14\%, 0.30\%\}$	$\{0.12\%, 0.33\%\}$
Ω_{λ_2}	$\{1.13\%, 6.57\%\}$	$\{0.21\%, 0.63\%\}$	$\{0.15\%, 0.41\%\}$
Ω_{λ_3}	$\{0.45\%, 1.36\%\}$	$\{0.22\%, 0.64\%\}$	$\{0.10\%, 0.17\%\}$
Ω_{τ_1}	$\{0.31\%, 1.36\%\}$	$\{0.11\%, 0.35\%\}$	$\{0.10\%, 0.21\%\}$
Ω_{τ_2}	$\{0.73\%, 3.72\%\}$	$\{0.15\%, 0.26\%\}$	$\{0.15\%, 0.33\%\}$
Ω_{τ_3}	$\{1.37\%, 6.57\%\}$	$\{0.32\%, 0.64\%\}$	$\{0.11\%, 0.41\%\}$
Ω	$\{0.81\%, 6.57\%\}$	$\{0.19\%, 0.64\%\}$	$\{0.12\%, 0.41\%\}$
	GP case		
	$(\hat{C}^* - C^*)/H$	$Gap1$	$Gap2$
	$\{AAD, MAD\}$	$\{AAD, MAD\}$	$\{AAD, MAD\}$
Ω_{σ_1}	$\{1.76\%, 3.82\%\}$	$\{3.61\%, 4.93\%\}$	$\{0.68\%, 1.69\%\}$
Ω_{σ_2}	$\{2.67\%, 5.45\%\}$	$\{3.98\%, 6.76\%\}$	$\{1.03\%, 1.74\%\}$
Ω_{σ_3}	$\{3.71\%, 7.33\%\}$	$\{3.18\%, 7.13\%\}$	$\{1.92\%, 3.93\%\}$
Ω_{λ_1}	$\{2.72\%, 6.65\%\}$	$\{3.98\%, 7.13\%\}$	$\{1.44\%, 3.93\%\}$
Ω_{λ_2}	$\{2.53\%, 5.14\%\}$	$\{3.56\%, 6.90\%\}$	$\{1.11\%, 2.79\%\}$
Ω_{λ_3}	$\{2.89\%, 7.33\%\}$	$\{3.23\%, 6.52\%\}$	$\{1.08\%, 2.58\%\}$
Ω_{τ_1}	$\{1.21\%, 1.79\%\}$	$\{1.11\%, 2.36\%\}$	$\{0.68\%, 1.69\%\}$
Ω_{τ_2}	$\{2.17\%, 4.30\%\}$	$\{3.59\%, 4.70\%\}$	$\{1.33\%, 2.73\%\}$
Ω_{τ_3}	$\{4.77\%, 7.33\%\}$	$\{6.06\%, 7.13\%\}$	$\{1.62\%, 3.93\%\}$
Ω	$\{2.72\%, 7.33\%\}$	$\{3.59\%, 7.13\%\}$	$\{1.21\%, 3.93\%\}$

of Gap 2 is smaller than Gap 1. This implies that the deviation of $\hat{C}^*$ from C^* does not lead to much deviation on the simulated cost rate under various parameter settings; even though the inaccuracy of the approximate evaluation (or Gap 1) is sightly higher. This is due to the fact that the neighborhood of the minimum $\hat{Z}$ is flat. Hence, in practice, the optimal maintenance policy of our approximate evaluation is only an suboptimal solution, where its cost rate is very close to the true optimal cost rate. Also notice that the AAD and MAD in both cases of RCM and GP become larger when τ is larger. This is sensible, because we assume that scheduled downs restart at the end of each maintenance cycle in our approximate evaluation, as explained in Section 3.2. The larger τ is, the lower the probability that a maintenance cycle ends at a multiple of τ. Hence, we can conclude that our approximate evaluation is more accurate at a smaller τ.

3.5.3 Cost reduction potential

To show the cost benefits of including the opportunities at USDs and SDs for a continuously monitored component, three policies are considered: 1) an *only-SD-opportunistic policy*, which means that only SDs are considered as opportunities and no opportunistic preventive maintenance actions are taken at USDs; 2) an *only-USD-opportunistic policy*, which means that only USDs are considered as opportunities and no SDs are planned; and 3) a *failure-based policy*, which means that neither USDs or SDs are considered as opportunities for preventive maintenance. Notice that Policy 1 can be analyzed as a special case of our policy, where $\lambda = 0$; Policy 2 can be analyzed as a special case of our policy, where $\tau = \infty$; and Policy 3 can be analyzed as a special case of our policy, where $C = H$.

To show the cost benefits under various parameter settings, we use the same test bed and parameter settings as in Section 3.5.2. The minimum average cost rate of our policy that includes opportunities at both USDs and SDs is denoted by Z. The minimum average cost rates of Policy 1 and 2 are denoted by $\tilde{Z}_1$ and $\tilde{Z}_2$ respectively. Notice that no opportunity is considered in Policy 3; so that its cost rate $\tilde{Z}_3$ remains unchanged under different parameter settings, which is 44.5 thousand Euro. We use $\tilde{Z}_3$ as the basis of the comparison.

In total, we have three comparisons: A) the cost saving percentage of including opportunities at both USDs and SDs, denoted by $\triangle_A = (\tilde{Z}_3 - Z)/\tilde{Z}_3$; B) the cost saving percentage of using only opportunities at SDs (i.e., Policy 1), denoted by $\triangle_B = (\tilde{Z}_3 - \tilde{Z}_1)/\tilde{Z}_3$; C) the cost saving percentage of using only opportunities at USDs (i.e., Policy 2), denoted by $\triangle_C = (\tilde{Z}_3 - \tilde{Z}_2)/\tilde{Z}_3$. Similar to Subsection 3.5.2, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \big\{(\sigma_1, \lambda_k, \tau_m) | \forall k, m \in \{1, 2, 3\}\big\}$. The means, minimums and maximums of the cost saving percentages of these 9 subsets for both cases of RCM and GP are summarized in Table 3.6. The result of each instance is shown in Table 3.13 in Subsection 3.A.3

The first observation from Table 3.6 is that our policy (using opportunities at both SDs and USDs) has a higher cost-saving potential than Policy 1 (using opportunities at SDs only), Policy 2 (using opportunities at USDs only) and Policy 3 (using no opportunities). The mean values of $\triangle_A$ are bigger than $\triangle_B$ and $\triangle_C$, because more opportunities for preventive maintenance (cheaper than corrective maintenance) are included in our policy than in Policy 1 and 2. The mean values of $\triangle_B$ are bigger than $\triangle_C$, because the cost of preventive maintenance at a SD is cheaper than at an USD. Regarding the variation of the mean values under various parameter settings, we observe that i) $\triangle_A$ is inversely proportional to σ, λ and τ. ii) $\triangle_B$ is inversely proportional to σ and τ. It remains unchanged to λ, because no USD opportunity is considered in Policy 1. iii) $\triangle_C$ is proportional to λ and inversely proportional to

Table 3.6 Summary of the cost saving percentages by using opportunities at USDs and SDs; including the mean, minimum and maximum values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ respectively

	Cost saving percentage in the case of RCM								
	$\triangle_A$			$\triangle_B$			$\triangle_C$		
	mean	min	max	mean	min	max	mean	min	max
Ω_{σ_1}	29.7%	23.0%	35.3%	23.4%	11.6%	31.6%	8.6%	5.1%	11.7%
Ω_{σ_2}	28.7%	22.7%	34.7%	22.5%	14.6%	29.7%	8.4%	5.0%	11.5%
Ω_{σ_3}	28.5%	23.0%	34.3%	21.9%	16.2%	29.5%	8.2%	4.8%	11.3%
Ω_{λ_1}	29.5%	23.7%	35.3%	22.6%	11.6%	31.6%	5.0%	4.8%	5.1%
Ω_{λ_2}	28.8%	22.7%	35.0%	22.6%	11.6%	31.6%	8.7%	8.5%	8.9%
Ω_{λ_3}	28.5%	22.7%	34.7%	22.6%	11.6%	31.6%	11.5%	11.3%	11.7%
Ω_{τ_1}	34.5%	33.7%	35.3%	30.3%	29.5%	31.6%	8.4%	4.8%	11.7%
Ω_{τ_2}	29.0%	27.8%	30.9%	23.4%	20.2%	27.0%	8.4%	4.8%	11.7%
Ω_{τ_3}	23.4%	22.7%	25.0%	14.1%	11.6%	16.2%	8.4%	4.8%	11.7%
Ω	28.9%	22.7%	35.3%	22.6%	11.6%	31.6%	8.4%	4.8%	11.7%
	Cost saving percentage in the case of GP								
	$\triangle_A$			$\triangle_B$			$\triangle_C$		
	mean	min	max	mean	min	max	mean	min	max
Ω_{σ_1}	27.4%	23.6%	31.4%	21.0%	16.5%	25.9%	11.5%	8.8%	14.0%
Ω_{σ_2}	26.3%	23.0%	30.1%	20.8%	15.4%	26.4%	10.5%	7.5%	13.1%
Ω_{σ_3}	26.1%	22.9%	29.7%	20.7%	12.2%	27.9%	9.5%	6.4%	12.4%
Ω_{λ_1}	26.7%	22.9%	31.4%	20.8%	12.2%	27.9%	7.6%	6.4%	8.8%
Ω_{λ_2}	26.6%	22.9%	31.2%	20.8%	12.2%	27.9%	10.8%	9.8%	11.8%
Ω_{λ_3}	26.5%	23.0%	31.0%	20.8%	12.2%	27.9%	13.2%	12.4%	14.0%
Ω_{τ_1}	30.2%	29.5%	31.4%	26.7%	25.9%	27.9%	10.5%	6.4%	14.0%
Ω_{τ_2}	26.3%	25.6%	27.4%	21.1%	20.6%	21.9%	10.5%	6.4%	14.0%
Ω_{τ_3}	23.2%	22.9%	24.0%	14.7%	12.2%	16.5%	10.5%	6.4%	14.0%
Ω	26.6%	22.9%	31.4%	20.8%	12.2%	27.9%	10.5%	6.4%	14.0%

σ. It remains unchanged to τ, because no SD opportunity is considered in Policy 2. A higher σ means a higher variance in the lifetime distribution of the component, which leads to a higher probability of having corrective maintenance (more expensive than preventive maintenance). Hence, the mean values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ decrease when σ increases. Moreover, the cost of taking opportunities at SDs is cheaper than at USDs. On one hand, a higher λ leads to more opportunities at expensive USDs; on the other hand, a higher τ leads to less opportunities at cheaper SDs. Hence, $\triangle_A$ decreases when λ or τ increases. For the same reason, a higher τ leads to a lower $\triangle_B$. However, $\triangle_C$ increases at a higher λ, because only opportunities at USDs are considered in $\triangle_C$. In this case, a higher λ leads to more opportunities to take, so that higher cost saving percentages can be observed. Notice that the aforementioned observations appear in both cases of RCM and GP.

3.6 Conclusions

In this chapter, we proposed a new opportunistic maintenance policy for a monitored component in a complex system, under given scheduled and unscheduled downs of the system. This opportunistic maintenance policy can be utilized in the context of a mixture of different maintenance policies, such as failure-based maintenance policies or/and periodic preventive maintenance policies. As the decision variable of the model, a control limit is introduced to decide the timing of taking opportunities to maintain together with other components in the system, which saves the downtime cost and setup cost of the monitored component. The optimal control limit is determined such that it minimizes long-run average cost rate of the monitored component under an infinite time horizon setting.

To validate our model, we compared our approximate evaluation results with the simulation results. In a case study of lithography machines in the semiconductor industry, our approximate evaluation is very accurate and the cost-saving potential of our model is considerable. To verify this finding further, numerical experiments are executed based on a full factorial test bed. Under various parameter settings, our model shows a good accuracy and a considerable cost-saving potential.

Our model can be applied to different types of monitored critical components in different complex engineering systems, because i) different physics of failures and various degradations models (as Subsection 3.3.1) can be plugged directly into the optimization model (as Subsection 3.3.2) and ii) our model can be used as a building block for multi-component systems with a mixture of different maintenance policies (not only condition-based, but also age-based maintenance or/and periodic inspection).

3.A Appendices

3.A.1 Derivation of the expected cycle length

$L(C)$ is formulated as

$$\sum_{n=1}^{\infty}\int_{u=(n-1)\tau}^{u=n\tau}\left\{\int_{v=u}^{v=n\tau}\left(\int_{s=0}^{s=v-u}(u+s)\lambda e^{-\lambda s}\,ds+v\int_{s=v-u}^{s=\infty}\lambda e^{-\lambda s}\,ds\right)f_{T_H|T_C}(v|u)\,dv\right.$$
$$\left.+\int_{v=n\tau}^{v=\infty}\left(\int_{s=0}^{s=n\tau-u}(u+s)\lambda e^{-\lambda s}\,ds+n\tau\int_{s=n\tau-u}^{s=\infty}\lambda e^{-\lambda s}\,ds\right)f_{T_H|T_C}(v|u)\,dv\right\}$$
$$\times f_{T_C}(u)\,du \tag{3.7}$$

Using integration by part, we can obtain:

$$\int_{s=0}^{s=v-u}(u+s)\lambda e^{-\lambda s}\,ds+v\int_{s=v-u}^{s=\infty}\lambda e^{-\lambda s}\,ds=u+\frac{1}{\lambda}\left(1-e^{-\lambda(v-u)}\right)$$

and

$$\int_{s=0}^{s=n\tau-u}(u+s)\lambda e^{-\lambda s}\,ds+n\tau\int_{s=n\tau-u}^{s=\infty}\lambda e^{-\lambda s}\,ds=u+\frac{1}{\lambda}\left(1-e^{-\lambda(n\tau-u)}\right)$$

Hence, $L(C)$ can be rewritten as

$$\sum_{n=1}^{\infty}\int_{u=(n-1)\tau}^{u=n\tau}\left\{u+\int_{v=u}^{v=n\tau}\frac{1}{\lambda}\left(1-e^{-\lambda(v-u)}\right)f_{T_H|T_C}(v|u)\,dv\right.$$
$$\left.+\frac{1}{\lambda}\int_{v=n\tau}^{v=\infty}\left(1-e^{-\lambda(n\tau-u)}\right)f_{T_H|T_C}(v|u)\,dv\right\}f_{T_C}(u)\,du$$

3.A.2 Simulation procedures

As explained in Section 3.1, the periodic maintenance planned on the schedule will be shifted after each maintenance cycle in our approximate evaluation model, which is not the case in practice. To evaluate the accuracy of the approximate evaluation, we run a simulation to compare with the approximation results. We simulate i) the random failures by a Poisson process with a rate λ and ii) the degradation models under certain distributions.

$$I_k^{PM-USD} = \begin{cases} 1 & \text{if a PM-USD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_k^{PM-SD} = \begin{cases} 1 & \text{if a PM-SD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_k^{CM} = \begin{cases} 1 & \text{if a CM action is taken} \\ 0 & \text{otherwise} \end{cases}$$

There are m seeds in the simulation. In each seed $i \in \{1, 2, ..., m\}$, we simulate k_i renewal/maintenance cycles, where $k \in \{0, 1, 2, ..., k_i\}$ is the index of the cycles. Each seed i consists of: 1) a Poisson process with random arrival time points $A = \{a_1, a_2, ..., a_x\} \in \Re_+^x, x \in \mathbb{N}$, where $\Re_+ = [0, \infty)$; 2) a set of random passage times $T_{C_{k,i}}$ and $T_{H_{k,i}} \in \Re+, \forall k \in \mathbb{N}$, according to the degradation process; and 3) a constant set $B = \{\tau, 2\tau, ..., n\tau\}$, $n \in \mathbb{N}$ on a time horizon T_{max} that is sufficiently large to simulate the infinite time horizon (e.g., 10^6 times larger than $L(C)$).ε is a very small number ($\varepsilon = e^{-4}$)

By running the algorithm in Table 3.7 iteratively with m seeds, the final result of the simulation $\hat{Z} = \frac{\sum_{i=1}^m \hat{Z}_i}{m}$ with a $100(1-\alpha)\%$ confidence interval is expressed as follows [30]:

$$\hat{Z} \pm t(1-\alpha/2, m-1)\sqrt{\frac{S^2}{m}}$$

where $S = \sum_{i=1}^m \frac{(\hat{Z}_i - \hat{Z})^2}{m-1}$ and $t(1-\alpha/2, m-1)$ is the upper $1-\alpha/2$ critical point for the t-distribution with $(m-1)$ degrees of freedom (in our case, $m = 100$ and $\alpha = 5\%$). The expected cost rate for each simulation run is:

$$\hat{Z}_i = \frac{\sum_{k=1}^{k_i} \left(I_{k,i}^{PM-USD} c^{PM-USD} + I_{k,i}^{PM-SD} c^{PM-SD} + I_{k,i}^{CM} c^{CM} \right)}{R_{k_i,i}}, \qquad \forall i \in \{1, 2, ..., m\}$$

```
For i = 1 : m,
  Initialize k = 0: Ẑ_0 = 0 and R_{0,i} = 0
    While R_{k,i} < T_max and |(Z_{k,i} − Z_{k−1}) / Z_{k,i}| ≥ ε
    k_i = k_i + 1
      If ∃ a non-empty subset {A_{k,i}} ⊆ A : {A_{k,i}} ⊆ [R_{k−1,i} + T_{C_{k,i}}, R_{k−1,i} + T_{H_{k,i}}),
        If ∃ a non-empty subset {B_{k,i}} ⊆ B : {B_{k,i}} ⊆ [R_{k−1,i} + T_{C_{k,i}}, R_{k−1,i} + T_{H_{k,i}}),
          If min{A_{k,i}} > min{B_{k,i}},
            Calculate Ẑ_{k,i}; given (I_{k,i}^{PM−USD}, I_{k,i}^{PM−SD}, I_{k,i}^{CM}) = (0,1,0) and R_{k,i} = min{B_{k,i}}
          Else Calculate Ẑ_{k,i}; given (I_{k,i}^{PM−USD}, I_{k,i}^{PM−SD}, I_{k,i}^{CM}) = (1,0,0) and R_{k,i} = min{A_{k,i}}
        Else Calculate Ẑ_{k,i}; given (I_{k,i}^{PM−USD}, I_{k,i}^{PM−SD}, I_{k,i}^{CM}) = (1,0,0) and R_{k,i} = min{A_{k,i}}
      Else
        If ∃ a a non-empty subset {B_{k,i}} ⊆ B : {B_{k,i}} ⊆ [R_{k−1,i} + T_{C_{k,i}}, R_{k−1,i} + T_{H_{k,i}})
          Calculate Ẑ_{k,i}; given (I_{k,i}^{PM−USD}, I_{k,i}^{PM−SD}, I_{k,i}^{CM}) = (0,1,0) and R_{k,i} = min{B_{k,i}}
        Else Calculate Ẑ_{k,i}; given (I_{k,i}^{PM−USD}, I_{k,i}^{PM−SD}, I_{k,i}^{CM}) = (0,0,1) and R_{k,i} = R_{k−1,i} + T_{H_{k,i}}
      End if
    End while
  Obtain Ẑ_i = Ẑ_{k,i} and R_{k_i,i} = R_{k,i}, where k = k_i
End
```

Table 3.7 Simulation procedure

Moreover, the probabilities of PM-USD, PM-SD and CM $[\hat{P}_1, \hat{P}_2, \hat{P}_3]$; and the expected cycle length $[\hat{L}(C)]$ are:

$$\hat{P}_1 = \frac{\sum_{i=1}^{m}\left[\sum_{k=1}^{k_i}\left(\frac{I_{k,i}^{PM-USD}}{I_{k,i}^{PM-USD}+I_{k,i}^{PM-SD}+I_{k,i}^{CM}}\right)\right]}{m}$$

$$\hat{P}_2 = \frac{\sum_{i=1}^{m}\left[\sum_{k=1}^{k_i}\left(\frac{I_{k,i}^{PM-SD}}{I_{k,i}^{PM-USD}+I_{k,i}^{PM-SD}+I_{k,i}^{CM}}\right)\right]}{m}$$

$$\hat{P}_3 = \frac{\sum_{i=1}^{m}\left[\sum_{k=1}^{k_i}\left(\frac{I_{k,i}^{CM}}{I_{k,i}^{PM-USD}+I_{k,i}^{PM-SD}+I_{k,i}^{CM}}\right)\right]}{m}$$

and

$$\hat{L} = \frac{\sum_{i=1}^{m} \left(\frac{R_{k_i,i}}{k_i} \right)}{m}$$

3.A.3 Detail results of Test bed 1 and 2

Detail results of Table 3.4, 3.5 and 3.6 are given in the following tables

Table 3.8 A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximate evaluation; in the case of random coefficient model (RCM)

Λ	RCM case	
	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.044, 0.956, 0.000, 0.344\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.104, 0.896, 0.000, 0.403\}$	$\{0.000, -0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.109, 0.890, 0.000, 0.409\}$	$\{0.003, -0.003, 0.000, 0.6\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.085, 0.914, 0.000, 0.343\}$	$\{0.000, 0.000, 0.000, 0.2\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.193, 0.807, 0.000, 0.397\}$	$\{-0.002, 0.002, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.201, 0.799, 0.000, 0.401\}$	$\{0.008, -0.008, 0.000, 1.0\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.125, 0.875, 0.000, 0.342\}$	$\{0.001, -0.001, 0.000, 0.2\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.273, 0.727, 0.000, 0.391\}$	$\{-0.003, 0.003, 0.000, -0.3\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.278, 0.722, 0.000, 0.393\}$	$\{0.016, -0.016, 0.000, 1.4\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.000, 0.348\}$	$\{0.000, 0.000, 0.000, 0.0\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.109, 0.891, 0.000, 0.409\}$	$\{-0.001, 0.001, 0.000, -0.3\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.108, 0.891, 0.000, 0.408\}$	$\{0.002, -0.003, 0.000, 0.5\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.092, 0.907, 0.000, 0.347\}$	$\{-0.001, 0.001, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.201, 0.799, 0.000, 0.400\}$	$\{-0.005, 0.005, 0.000, -0.6\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.202, 0.798, 0.000, 0.401\}$	$\{0.009, -0.009, 0.000, 1.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.000, 0.346\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.279, 0.721, 0.000, 0.393\}$	$\{-0.011, 0.011, 0.000, -0.9\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.280, 0.720, 0.000, 0.393\}$	$\{0.015, -0.015, 0.000, 1.3\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.050, 0.950, 0.000, 0.350\}$	$\{0.000, 0.000, 0.000, 0.0\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.109, 0.891, 0.000, 0.410\}$	$\{-0.002, 0.002, 0.000, -0.4\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.108, 0.891, 0.000, 0.409\}$	$\{0.002, -0.002, 0.000, 0.5\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.096, 0.903, 0.000, 0.348\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.201, 0.799, 0.000, 0.401\}$	$\{-0.008, 0.007, 0.000, -0.7\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.202, 0.797, 0.000, 0.401\}$	$\{0.008, -0.008, 0.000, 0.9\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.140, 0.860, 0.000, 0.346\}$	$\{0.000, 0.000, 0.000, -0.1\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.279, 0.721, 0.000, 0.394\}$	$\{-0.014, 0.014, 0.000, -1.0\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.282, 0.717, 0.000, 0.395\}$	$\{0.015, -0.015, 0.000, 1.5\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.049, 0.950, 0.000, 0.549\}$	$\{0.000, -0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.107, 0.892, 0.000, 0.606\}$	$\{0.000, 0.000, 0.000, -0.2\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.128, 0.871, 0.001, 0.627\}$	$\{0.004, -0.005, 0.001, 0.5\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.096, 0.904, 0.000, 0.548\}$	$\{0.000, -0.001, 0.000, 0.1\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.196, 0.803, 0.000, 0.598\}$	$\{-0.005, 0.004, 0.000, -0.3\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.240, 0.759, 0.001, 0.620\}$	$\{0.011, -0.011, 0.001, 0.8\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.138, 0.862, 0.000, 0.546\}$	$\{-0.001, 0.000, 0.000, 0.0\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.272, 0.727, 0.000, 0.591\}$	$\{-0.010, 0.010, 0.000, -0.5\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.334, 0.666, 0.001, 0.613\}$	$\{0.014, -0.015, 0.001, 1.0\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.050, 0.950, 0.000, 0.548\}$	$\{0.001, -0.002, 0.000, 0.0\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.097, 0.902, 0.000, 0.599\}$	$\{-0.002, 0.001, 0.000, -0.1\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.139, 0.861, 0.001, 0.638\}$	$\{0.001, -0.002, 0.001, 0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.095, 0.905, 0.000, 0.547\}$	$\{0.001, -0.002, 0.000, 0.0\%\}$

Table 3.9 (Continued) a full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximate evaluation; in the case of random coefficient model (RCM)

Λ	RCM case	
	Simulation	Deviation
	$\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	$\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.184, 0.816, 0.000, 0.592\}$	$\{-0.002, 0.001, 0.000, -0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.255, 0.745, 0.001, 0.627\}$	$\{0.003, -0.003, 0.001, 0.2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.000, 0.546\}$	$\{0.001, -0.001, 0.000, 0.1\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.255, 0.745, 0.000, 0.586\}$	$\{-0.006, 0.006, 0.000, -0.1\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.351, 0.648, 0.001, 0.618\}$	$\{0.003, -0.004, 0.001, 0.2\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.049, 0.951, 0.000, 0.549\}$	$\{0.001, -0.001, 0.000, 0.2\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.094, 0.906, 0.000, 0.595\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.145, 0.855, 0.001, 0.645\}$	$\{-0.001, 0.000, 0.001, 0.0\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.000, 0.547\}$	$\{0.001, -0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.000, 0.588\}$	$\{-0.001, 0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.263, 0.736, 0.001, 0.632\}$	$\{-0.002, 0.002, 0.001, -0.2\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.000, 0.546\}$	$\{0.001, -0.001, 0.000, 0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.245, 0.754, 0.000, 0.583\}$	$\{-0.003, 0.003, 0.000, 0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.360, 0.640, 0.001, 0.620\}$	$\{-0.005, 0.005, 0.001, -0.3\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.048, 0.952, 0.001, 0.748\}$	$\{0.000, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.098, 0.902, 0.001, 0.797\}$	$\{0.000, -0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.145, 0.762, 0.093, 0.847\}$	$\{-0.005, 0.014, -0.01, -0.3\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.093, 0.907, 0.001, 0.747\}$	$\{-0.001, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.181, 0.819, 0.001, 0.791\}$	$\{-0.001, 0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.263, 0.670, 0.067, 0.832\}$	$\{-0.009, 0.021, -0.012, -0.5\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.137, 0.863, 0.001, 0.745\}$	$\{0.001, -0.002, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.254, 0.746, 0.001, 0.785\}$	$\{-0.003, 0.002, 0.001, -0.1\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.354, 0.599, 0.048, 0.818\}$	$\{-0.017, 0.029, -0.013, -0.6\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.048, 0.952, 0.001, 0.749\}$	$\{-0.001, 0.000, 0.001, 0.1\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.092, 0.908, 0.001, 0.792\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.132, 0.790, 0.079, 0.833\}$	$\{-0.002, 0.006, -0.004, -0.1\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.093, 0.906, 0.001, 0.748\}$	$\{0.000, 0.000, 0.001, 0.2\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.172, 0.827, 0.001, 0.786\}$	$\{0.000, -0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.240, 0.702, 0.058, 0.822\}$	$\{-0.003, 0.008, -0.005, 0.1\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.001, 0.746\}$	$\{0.000, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.244, 0.755, 0.001, 0.782\}$	$\{0.002, -0.002, 0.001, 0.2\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.332, 0.623, 0.045, 0.811\}$	$\{-0.001, 0.005, -0.004, -0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.001, 0.749\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.092, 0.906, 0.002, 0.792\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.128, 0.802, 0.070, 0.829\}$	$\{0.000, 0.002, -0.001, 0.0\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.001, 0.747\}$	$\{0.000, -0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.172, 0.826, 0.002, 0.787\}$	$\{0.000, -0.001, 0.001, 0.2\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.237, 0.708, 0.055, 0.818\}$	$\{0.002, -0.002, 0.000, 0.1\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.001, 0.745\}$	$\{-0.001, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.243, 0.755, 0.001, 0.782\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.325, 0.632, 0.043, 0.809\}$	$\{0.002, -0.003, 0.001, 0.2\%\}$

Table 3.10 A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximate evaluation; in the case of Gamma process (GP)

Λ	GP case	
	Simulation	Deviation
	$\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	$\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.047, 0.953, 0.000, 0.360\}$	$\{-0.001, 0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.095, 0.905, 0.000, 0.406\}$	$\{0.001, -0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.132, 0.868, 0.000, 0.446\}$	$\{-0.001, 0.001, 0.000, 0.3\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.092, 0.908, 0.000, 0.358\}$	$\{-0.002, 0.001, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.000, 0.400\}$	$\{-0.001, 0.001, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.241, 0.759, 0.000, 0.432\}$	$\{-0.001, 0.001, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.138, 0.861, 0.000, 0.357\}$	$\{0.002, -0.002, 0.000, 0.1\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.246, 0.753, 0.000, 0.393\}$	$\{-0.003, 0.003, 0.000, -0.4\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.330, 0.669, 0.000, 0.421\}$	$\{-0.001, 0.001, 0.000, -0.2\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.051, 0.949, 0.000, 0.371\}$	$\{0.002, -0.002, 0.000, -0.1\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.092, 0.907, 0.000, 0.416\}$	$\{-0.001, 0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.131, 0.868, 0.001, 0.457\}$	$\{-0.002, 0.003, -0.001, 0.1\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.092, 0.908, 0.000, 0.370\}$	$\{-0.002, 0.001, 0.000, 0.2\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.000, 0.412\}$	$\{0.000, 0.000, 0.000, 0.4\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.244, 0.755, 0.001, 0.446\}$	$\{0.001, 0.000, -0.001, 0.3\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.137, 0.863, 0.000, 0.368\}$	$\{0.000, -0.001, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.244, 0.756, 0.000, 0.406\}$	$\{-0.004, 0.004, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.338, 0.661, 0.001, 0.435\}$	$\{0.003, -0.003, 0.000, 0.2\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.049, 0.951, 0.000, 0.385\}$	$\{0.001, -0.001, 0.000, 0.5\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.094, 0.905, 0.001, 0.430\}$	$\{0.000, 0.001, -0.001, 0.4\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.136, 0.861, 0.003, 0.469\}$	$\{0.001, 0.003, -0.004, -0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.000, 0.382\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.177, 0.822, 0.001, 0.423\}$	$\{0.001, 0.000, -0.001, 0.0\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.251, 0.747, 0.003, 0.457\}$	$\{0.004, 0.000, -0.004, -0.2\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.134, 0.865, 0.000, 0.380\}$	$\{-0.002, 0.002, 0.000, 0.0\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.252, 0.747, 0.001, 0.417\}$	$\{0.003, -0.003, -0.001, -0.1\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.345, 0.653, 0.002, 0.448\}$	$\{0.005, -0.002, -0.003, 0.1\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.045, 0.954, 0.000, 0.555\}$	$\{-0.003, 0.003, 0.000, -0.2\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.095, 0.905, 0.000, 0.599\}$	$\{0.001, -0.001, 0.000, -0.4\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.131, 0.868, 0.001, 0.643\}$	$\{-0.003, 0.007, -0.003, 0.0\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.093, 0.906, 0.000, 0.556\}$	$\{0.000, 0.000, 0.000, 0.2\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.178, 0.821, 0.001, 0.595\}$	$\{0.002, -0.003, 0.000, -0.2\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.249, 0.751, 0.001, 0.632\}$	$\{0.003, 0.000, -0.003, 0.2\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.001, 0.554\}$	$\{-0.001, 0.000, 0.001, 0.2\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.240, 0.759, 0.001, 0.593\}$	$\{-0.008, 0.008, 0.000, 0.3\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.339, 0.661, 0.001, 0.622\}$	$\{0.001, 0.001, -0.002, 0.3\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.049, 0.950, 0.001, 0.565\}$	$\{0.001, -0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.094, 0.906, 0.001, 0.607\}$	$\{0.000, 0.004, -0.004, -0.4\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.134, 0.865, 0.001, 0.653\}$	$\{-0.002, 0.019, -0.017, 0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.093, 0.907, 0.001, 0.562\}$	$\{-0.001, 0.001, 0.000, -0.2\%\}$

Table 3.11 (Continued) A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximate evaluation; in the case of Gamma process (GP)

Λ	GP case	
	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.001, 0.605\}$	$\{0.000, 0.003, -0.003, 0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.246, 0.753, 0.001, 0.641\}$	$\{-0.001, 0.015, -0.014, 0.2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.001, 0.561\}$	$\{0.000, 0.000, 0.000, -0.1\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.250, 0.749, 0.000, 0.600\}$	$\{0.003, 0.000, -0.003, 0.2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.343, 0.656, 0.001, 0.630\}$	$\{0.004, 0.008, -0.012, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.048, 0.952, 0.000, 0.572\}$	$\{-0.001, 0.003, -0.002, -0.3\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.095, 0.904, 0.001, 0.618\}$	$\{0.002, 0.010, -0.012, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.132, 0.863, 0.005, 0.660\}$	$\{-0.002, 0.031, -0.029, 0.1\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.001, 0.571\}$	$\{0.000, 0.002, -0.002, -0.2\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.176, 0.823, 0.001, 0.614\}$	$\{0.001, 0.009, -0.010, 0.3\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.244, 0.752, 0.004, 0.650\}$	$\{0.000, 0.025, -0.025, 0.4\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.000, 0.570\}$	$\{-0.001, 0.003, -0.002, -0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.246, 0.753, 0.001, 0.608\}$	$\{-0.001, 0.010, -0.009, 0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.339, 0.658, 0.003, 0.635\}$	$\{0.003, 0.018, -0.022, -0.2\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.001, 0.754\}$	$\{0.000, 0.001, 0.000, 0.1\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.091, 0.908, 0.000, 0.797\}$	$\{-0.002, 0.022, -0.020, -0.1\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.133, 0.826, 0.041, 0.840\}$	$\{0.001, 0.050, -0.051, 0.4\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.091, 0.908, 0.001, 0.754\}$	$\{-0.003, 0.003, 0.000, 0.3\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.176, 0.823, 0.001, 0.791\}$	$\{0.002, 0.015, -0.017, -0.1\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.244, 0.721, 0.035, 0.829\}$	$\{0.002, 0.039, -0.041, 0.4\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.133, 0.866, 0.001, 0.748\}$	$\{-0.003, 0.003, 0.000, -0.3\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.247, 0.752, 0.001, 0.787\}$	$\{0.000, 0.015, -0.015, 0.1\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.340, 0.637, 0.023, 0.816\}$	$\{0.007, 0.033, -0.040, 0.1\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.001, 0.761\}$	$\{0.000, 0.008, -0.009, 0.4\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.095, 0.904, 0.001, 0.805\}$	$\{0.003, 0.043, -0.047, 0.5\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.136, 0.802, 0.063, 0.843\}$	$\{0.008, 0.052, -0.059, 0.7\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.091, 0.908, 0.001, 0.757\}$	$\{-0.002, 0.010, -0.008, 0.1\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.182, 0.817, 0.002, 0.799\}$	$\{0.010, 0.032, -0.042, 0.4\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.253, 0.703, 0.044, 0.832\}$	$\{0.019, 0.041, -0.060, 0.6\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.135, 0.865, 0.001, 0.754\}$	$\{-0.001, 0.008, -0.008, -0.1\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.240, 0.759, 0.001, 0.793\}$	$\{-0.003, 0.041, -0.037, 0.2\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.342, 0.621, 0.037, 0.825\}$	$\{0.018, 0.034, -0.052, 0.9\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.050, 0.949, 0.001, 0.764\}$	$\{0.002, 0.017, -0.019, 0.2\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.094, 0.901, 0.006, 0.806\}$	$\{0.004, 0.060, -0.064, 0.2\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.133, 0.786, 0.081, 0.846\}$	$\{0.007, 0.053, -0.061, 0.7\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.090, 0.909, 0.001, 0.763\}$	$\{-0.002, 0.021, -0.018, 0.3\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.177, 0.818, 0.005, 0.798\}$	$\{0.007, 0.051, -0.058, -0.2\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.243, 0.693, 0.064, 0.839\}$	$\{0.012, 0.046, -0.059, 1.1\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.136, 0.864, 0.001, 0.762\}$	$\{0.001, 0.016, -0.018, 0.3\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.245, 0.751, 0.004, 0.799\}$	$\{0.005, 0.048, -0.053, 0.5\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.333, 0.616, 0.050, 0.828\}$	$\{0.015, 0.042, -0.057, 0.8\%\}$

Table 3.12 Detail results of Table 3.5: the optimal maintenance policy of approximate evaluation and simulation in the case of random coefficient model (RCM) and Gamma process (GP).

Instances	by RCM $\{\frac{\hat{C}^*-C^*}{H}, Gap1, Gap2\}$	by GP $\{\frac{\hat{C}^*-C^*}{H}, Gap1, Gap2\}$
$(\sigma_1, \lambda_1, \tau_1)$	$\{-0.03\%, 0.06\%, 0.01\%\}$	$\{1.79\%, 2.36\%, 1.69\%\}$
$(\sigma_1, \lambda_1, \tau_2)$	$\{0.28\%, 0.25\%, 0.05\%\}$	$\{-0.26\%, 4.70\%, 0.07\%\}$
$(\sigma_1, \lambda_1, \tau_3)$	$\{0.35\%, 0.30\%, 0.01\%\}$	$\{-3.82\%, 4.80\%, 0.72\%\}$
$(\sigma_1, \lambda_2, \tau_1)$	$\{-0.03\%, 0.11\%, 0.02\%\}$	$\{1.54\%, 1.69\%, 1.20\%\}$
$(\sigma_1, \lambda_2, \tau_2)$	$\{0.28\%, 0.24\%, 0.24\%\}$	$\{-1.12\%, 4.25\%, 0.04\%\}$
$(\sigma_1, \lambda_2, \tau_3)$	$\{6.57\%, 0.09\%, 0.41\%\}$	$\{-1.83\%, 4.86\%, 0.54\%\}$
$(\sigma_1, \lambda_3, \tau_1)$	$\{-0.03\%, 0.24\%, 0.07\%\}$	$\{1.29\%, 1.43\%, 1.08\%\}$
$(\sigma_1, \lambda_3, \tau_2)$	$\{0.28\%, 0.26\%, 0.10\%\}$	$\{-0.64\%, 3.44\%, 0.60\%\}$
$(\sigma_1, \lambda_3, \tau_3)$	$\{-0.88\%, 0.27\%, 0.17\%\}$	$\{-3.57\%, 4.93\%, 0.19\%\}$
$(\sigma_2, \lambda_1, \tau_1)$	$\{0.58\%, 0.06\%, 0.19\%\}$	$\{1.32\%, 1.75\%, 0.39\%\}$
$(\sigma_2, \lambda_1, \tau_2)$	$\{-0.50\%, 0.01\%, 0.10\%\}$	$\{-1.47\%, 4.61\%, 1.35\%\}$
$(\sigma_2, \lambda_1, \tau_3)$	$\{0.35\%, 0.11\%, 0.04\%\}$	$\{-4.59\%, 6.76\%, 1.74\%\}$
$(\sigma_2, \lambda_2, \tau_1)$	$\{-0.20\%, 0.03\%, 0.09\%\}$	$\{1.02\%, 1.05\%, 0.36\%\}$
$(\sigma_2, \lambda_2, \tau_2)$	$\{-0.50\%, 0.08\%, 0.15\%\}$	$\{-2.39\%, 4.27\%, 0.96\%\}$
$(\sigma_2, \lambda_2, \tau_3)$	$\{-0.96\%, 0.63\%, 0.06\%\}$	$\{-5.14\%, 6.34\%, 1.44\%\}$
$(\sigma_2, \lambda_3, \tau_1)$	$\{1.36\%, 0.12\%, 0.11\%\}$	$\{0.74\%, 0.88\%, 0.40\%\}$
$(\sigma_2, \lambda_3, \tau_2)$	$\{0.28\%, 0.12\%, 0.16\%\}$	$\{-1.94\%, 3.65\%, 1.17\%\}$
$(\sigma_2, \lambda_3, \tau_3)$	$\{-0.18\%, 0.64\%, 0.05\%\}$	$\{-5.45\%, 6.52\%, 1.50\%\}$
$(\sigma_3, \lambda_1, \tau_1)$	$\{-0.20\%, 0.01\%, 0.10\%\}$	$\{0.29\%, -0.55\%, 0.38\%\}$
$(\sigma_3, \lambda_1, \tau_2)$	$\{3.72\%, 0.24\%, 0.33\%\}$	$\{-4.30\%, 3.18\%, 2.73\%\}$
$(\sigma_3, \lambda_1, \tau_3)$	$\{1.48\%, 0.25\%, 0.20\%\}$	$\{-6.65\%, 7.13\%, 3.93\%\}$
$(\sigma_3, \lambda_2, \tau_1)$	$\{-0.20\%, 0.35\%, 0.21\%\}$	$\{-1.32\%, -0.26\%, 0.20\%\}$
$(\sigma_3, \lambda_2, \tau_2)$	$\{-0.50\%, 0.14\%, 0.16\%\}$	$\{-3.92\%, 2.45\%, 2.45\%\}$
$(\sigma_3, \lambda_2, \tau_3)$	$\{-0.96\%, 0.25\%, 0.05\%\}$	$\{-4.52\%, 6.90\%, 2.79\%\}$
$(\sigma_3, \lambda_3, \tau_1)$	$\{-0.20\%, 0.02\%, 0.10\%\}$	$\{-1.61\%, 0.06\%, 0.43\%\}$
$(\sigma_3, \lambda_3, \tau_2)$	$\{0.28\%, 0.02\%, 0.09\%\}$	$\{-3.47\%, 1.80\%, 2.58\%\}$
$(\sigma_3, \lambda_3, \tau_3)$	$\{0.60\%, 0.29\%, 0.04\%\}$	$\{-7.33\%, 6.33\%, 1.76\%\}$

Table 3.13 The cost saving potential of including opportunities at USDs and SDs: the mean, minimum and maximum values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ respectively; in the case of the random coefficient model (RCM) and the Gamma process (GP).

Ω	$\triangle_A$		$\triangle_B$		$\triangle_C$	
	by RCM	by GP	by RCM	by GP	by RCM	by GP
$(\sigma_1, \lambda_1, \tau_1)$	35.3%	31.4%	31.6%	25.9%	5.1%	8.8%
$(\sigma_1, \lambda_1, \tau_2)$	30.9%	27.4%	27.0%	20.6%	5.1%	8.8%
$(\sigma_1, \lambda_1, \tau_3)$	25.0%	24.0%	11.6%	16.5%	5.1%	8.8%
$(\sigma_1, \lambda_2, \tau_1)$	35.0%	31.2%	31.6%	25.9%	8.9%	11.8%
$(\sigma_1, \lambda_2, \tau_2)$	30.2%	27.1%	27.0%	20.6%	8.9%	11.8%
$(\sigma_1, \lambda_2, \tau_3)$	23.0%	23.7%	11.6%	16.5%	8.9%	11.8%
$(\sigma_1, \lambda_3, \tau_1)$	34.7%	31.0%	31.6%	25.9%	11.7%	14.0%
$(\sigma_1, \lambda_3, \tau_2)$	29.6%	26.9%	27.0%	20.6%	11.7%	14.0%
$(\sigma_1, \lambda_3, \tau_3)$	23.1%	23.6%	11.6%	16.5%	11.7%	14.0%
$(\sigma_2, \lambda_1, \tau_1)$	34.7%	30.1%	29.7%	26.4%	5.0%	7.5%
$(\sigma_2, \lambda_1, \tau_2)$	29.0%	26.2%	23.0%	20.7%	5.0%	7.5%
$(\sigma_2, \lambda_1, \tau_3)$	24.0%	23.1%	14.6%	15.4%	5.0%	7.5%
$(\sigma_2, \lambda_2, \tau_1)$	34.4%	29.9%	29.7%	26.4%	8.7%	10.7%
$(\sigma_2, \lambda_2, \tau_2)$	28.6%	26.0%	23.0%	20.7%	8.7%	10.7%
$(\sigma_2, \lambda_2, \tau_3)$	22.7%	23.0%	14.6%	15.4%	8.7%	10.7%
$(\sigma_2, \lambda_3, \tau_1)$	34.1%	29.8%	29.7%	26.4%	11.5%	13.1%
$(\sigma_2, \lambda_3, \tau_2)$	28.2%	25.9%	23.0%	20.7%	11.5%	13.1%
$(\sigma_2, \lambda_3, \tau_3)$	22.7%	23.0%	14.6%	15.4%	11.5%	13.1%
$(\sigma_3, \lambda_1, \tau_1)$	34.3%	29.7%	29.5%	27.9%	4.8%	6.4%
$(\sigma_3, \lambda_1, \tau_2)$	28.4%	25.8%	20.2%	21.9%	4.8%	6.4%
$(\sigma_3, \lambda_1, \tau_3)$	23.7%	22.9%	16.2%	12.2%	4.8%	6.4%
$(\sigma_3, \lambda_2, \tau_1)$	34.0%	29.6%	29.5%	27.9%	8.5%	9.8%
$(\sigma_3, \lambda_2, \tau_2)$	28.1%	25.7%	20.2%	21.9%	8.5%	9.8%
$(\sigma_3, \lambda_2, \tau_3)$	23.1%	22.9%	16.2%	12.2%	8.5%	9.8%
$(\sigma_3, \lambda_3, \tau_1)$	33.7%	29.5%	29.5%	27.9%	11.3%	12.4%
$(\sigma_3, \lambda_3, \tau_2)$	27.8%	25.6%	20.2%	21.9%	11.3%	12.4%
$(\sigma_3, \lambda_3, \tau_3)$	23.0%	23.0%	16.2%	12.2%	11.3%	12.4%

Chapter 4

An Age-Based Maintenance Policy Using the Opportunities of Scheduled and Unscheduled System Downs

"All knowledge degenerates into probability."

David Hume

4.1 Introduction

Due to physical constraints from the design of the system, many critical components can not be monitored remotely. For example, a sensor can hardly be installed in an enclosed space (e.g., inside a gearbox). In this case, we develop a new age-based maintenance (ABM) model, which also includes both scheduled and unscheduled downs as free opportunities for a single ABM component to perform opportunistic maintenance.

There are many age/time-based models proposed for multi-component systems considering the economic dependence, which has been explained in Section 1.4 (see block E and F in Table 1.2). Most of them considered only one types of opportunities from either scheduled system downs (see [16]) or unscheduled system downs [13, 15, 39, 67]. There are also some studies including both scheduled and unscheduled opportunities [29, 50]. Laggoune *et al.* [29] developed a different dynamic clustering model based on simulation. In this model, preventive maintenance is scheduled at each fixed time point $k\tau, k \in \mathbb{N}$, and each component j can only be preventively replaced at a multiple of τ, $k_j\tau, k_j \in \mathbb{N}$. If unscheduled system downs occur, a decision on taking the opportunity or not will be made, according to marginal costs. Taghipour and Banjevic [50] proposed a model that considers both scheduled inspection and non-scheduled failures of systems as opportunities to perform inspections on soft-failure components. For hard failure components, preventive maintenance actions are taken at scheduled inspections depending on their condition. Its objective function in a finite-horizon setting is evaluated by a simulation algorithm.

Different from the aforementioned literature including one type of opportunities, we consider two types of opportunities to jointly maintain single components under an age-based maintenance policy and other components in a system, i.e., the opportunities from the scheduled system downs and unscheduled system downs due to random failures of other components. It is important to consider both the scheduled and unscheduled system downs as opportunities for joint maintenance when the fixed setup costs of maintenance are high. The high setup costs of maintenance will be reduced further by considering multiple types of opportunities together in the joint maintenance model, compared with using only one type of opportunities. For example, lithography machines in chip factories normally operate continuously over time (24 hours/day,7 days/week), except during the scheduled system downs per month and unscheduled system downs due to random failures. In order to conduct maintenance tasks for the machine, the machine needs to be shut down and the production of chips will be interrupted, which will cause a significant economic loss per hour downtime. Some lithography machines require a special manufacturing environment (e.g., vacuum environment), which makes the setup of maintenance even more costly. Therefore, it can be beneficial to take the opportunities of both the

scheduled and unscheduled system downs to jointly perform preventive maintenance tasks of components under age-based maintenance policies. Compared with the literature that includes both scheduled and unscheduled opportunities (see [29, 50]), simulation algorithms were used to evaluate the objective function. In contrast, our objective function is evaluated by stochastic renewal theory. Notice that renewal theory implies that the time points of periodic preventive maintenance are rescheduled after each maintenance action taken. However, in practice, periodic preventive maintenance actions are planned in advance. Hence, the evaluation is approximate and we investigate the accuracy of the approximate evaluation by comparing with simulated evaluation under various parameter settings. Finally, we investigate the cost saving potential of using opportunities at USDs or/and SDs under various parameter settings.
The outline of this chapter is as follows. The description of the system and the assumptions are given in Section 4.2. The details of the mathematical model are explained in Section 4.3. In Section 4.4, a numerical example is provided to show the utility of the model. Moreover, in Section 4.5, numerical experiments are performed to investigate the accuracy of our approximate evaluation and the cost-saving potential under various parameter settings. Finally, the conclusions are given in Section 4.6.

4.2 System description

We propose an age-based maintenance policy for a single component in a multi-component system, given the information of scheduled downs (SDs) and unscheduled downs (USDs) caused by other components in the system. This component is an ABM component.

Two preventive maintenance actions at USDs and SDs are considered:

- *Preventive Maintenance at an USD (PM-USD)*: when the system stops due to an USD, it provides an opportunity for an ABM component to be maintained together with other components in the system. If this opportunity is taken, then a PM-USD action will be taken on this ABM component. This will incur a cost c^{PM-USD} that includes the repair cost of the component on one hand. On the other hand, the downtime cost and setup cost caused by this ABM component can be saved by conducting its maintenance action simultaneously with other components that cause this USD.

- *Preventive Maintenance at a SD (PM-SD)*: when the system stops at time t due to a SD, it provides an opportunity for an ABM component to be maintained together with other components in the system. If this opportunity is taken, then a PM-SD action will be taken on this ABM component. This will incur a cost c^{PM-SD} that includes the repair cost of the component on one hand. On the other hand, the downtime cost and setup cost caused by this ABM component can be saved by conducting its maintenance action simultaneously with other components that cause this USD.

Suppose the life time of this ABM component T follows a certain distribution with p.d.f. $f(T)$. If failures occur before taking USD or SD opportunities for preventive maintenance, corrective maintenance (CM) actions are taken with the costs c^{CM}, which consists of not only the repair costs c^R of the component, but also the unscheduled setup costs including downtime costs, denoted by S^{USD}; namely, c^{CM}=S^{USD}+c^R. Notice that c^{PM-USD} and c^{PM-SD} are smaller than c^{CM}. In other words, if an ABM component is jointly maintained at an opportunity, instead of taking a CM action separately, the maintenance setup cost and downtime cost of this component can be saved at the system level, on one hand. On the other hand, the maintenance cost rate increases while taking opportunities too frequently due to the wasted usage lifetime of this component. Hence, we have to make decisions on the timing of taking opportunities for joint maintenance actions, in order to minimize the cost rate of the system. An age limit A on the age of the component t is introduced as a decision variable, which gives us a decision rule as follows:

- If an opportunity at SD or USD appears at time $t < A$, do nothing at this opportunity.
- If an opportunity at SD or USD appears at time $t \geq A$, take this opportunity to do preventive maintenance.

We define the interval between two consecutive maintenance actions as a *maintenance cycle*. Similar to the condition-based policy in Chapter 3, if scheduled downs are rescheduled at the end of each maintenance cycle of the ABM component (see Figure 4.1-(A)), renewal theory can be applied to evaluate the long-run average cost rate of the component. However, in practice, the scheduled downs are planned in advance (see Figure 4.1-(B)), which can not restart according to the maintenance actions of the component. Hence, we develop an approximate evaluation procedure, which is still based on renewal theory.

To improve the cost rate evaluation further, we propose an approximate evaluation method considering the distribution of the ending points of maintenance cycles with respect to SDs for an infinite horizon. We denote the deviation of a renewal point from SD as ξ, where $0 \leq \xi < \tau$ (see Figure 4.2)). The conditional average cycle cost and cycle length can be evaluated in an exact way, given that the maintenance cycle starts at a time point that is ξ time units away from the previous SD. Then we approximate

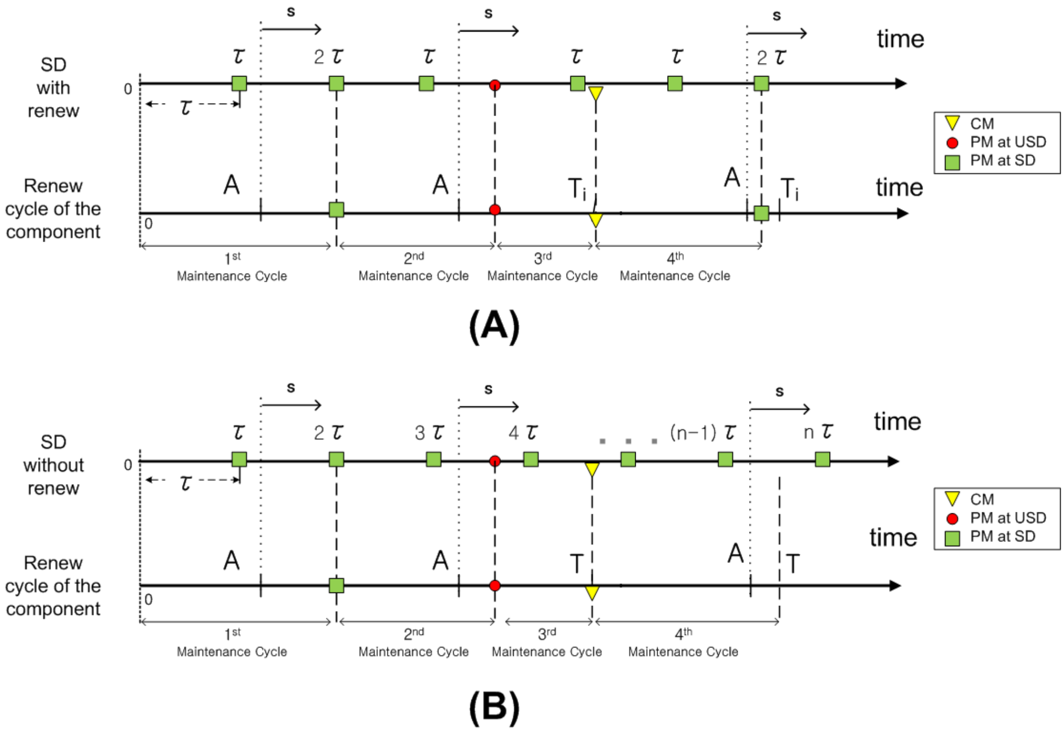


Figure 4.1 The three maintenance actions of an age-based component in the context of renewal theory (A) and practical situation (B)

the distribution of the deviation ξ over $[0, \tau)$ by analyzing the behavior of the renewal process. Finally we evaluate the average maintenance cycle cost and cycle length based on the approximate distribution of the deviateion ξ, and the conditional average cycle cost and cycle length. To verify our approximate model, we use simulation to check the accuracy of our approximate evaluation.

4.2.1 Notation

τ : interval of scheduled downs
λ : arrival rate of unscheduled downs (a Poisson process)
A : age limit on the age of component (decision variable)
$Z(A)$: average cost rate of the ABM component
c^{PM-USD} : PM cost of the ABM component at an USD
c^{PM-SD} : PM cost of the ABM component at a SD

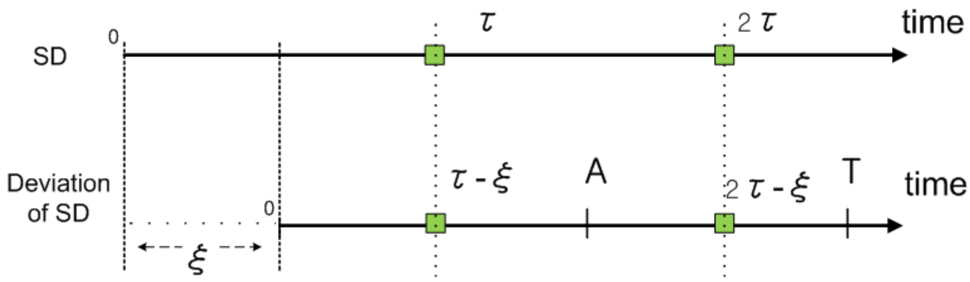


Figure 4.2 The deviation of the renewal point ξ

c^{CM} : CM cost of the ABM component

4.2.2 Assumptions

1) The life time of the ABM component is independent of SDs and USDs caused by other components in the system.
2) The time horizon is infinite
3) Maintenance actions restore components as new.

4.3 Approximate Evaluation Procedure

As shown in Figure 2, suppose a maintenance cycle starts at ξ time units away from the previous SD. The SDs in this maintenance cycle will occur at time points $n\tau - \xi$ till the end of this maintenance cycle, where $n \in \mathbb{N}$. The age limit A of opportunistic maintenance will be in a certain interval between two SDs, i.e., $(n-1)\tau - \xi < A \leq n\tau - \xi$ (notice that given ξ and τ, n is dependent on the decision variable A and $n = \lceil \frac{A+\xi}{\tau} \rceil$). Let u denote the random lifetime of the ABM component. We have 3 scenarios as follows:

Scenario 1: $u \leq A$
Possibility 1.1: if a failure occurs at time $u \leq A$, then no opportunities is taken for preventive maintenance, and a corrective maintenance CM action is taken. The corresponding probability is

$$P_{[1.1]} = Pr\Big\{u < A\Big\} = \int_{u=0}^{u=A} f(u)\ du, \tag{4.1}$$

and the contribution to the expected cycle length $\dot{L}(A|\xi)$ is

$$\dot{L}\big(A|\xi,\ in\ 1.1\big)P_{[1.1]} = \int_{u=0}^{u=A} uf(u)du. \tag{4.2}$$

Scenario 2: $u > n\tau - \xi$ ***and*** $u > A$

Possibility 2.1: If i) no failure of the component occurs before $n\tau-\xi$ and ii) at the same time no USD occurs before $n\tau - \xi$, then a PM-SD action will be taken at time point $n\tau-\xi$. Since the arrivals of the USDs are assumed to follow a Poisson process with an arrival rate λ, the time period from A to the first arrival of USDs after A follows an exponential distribution according to the memoryless property. Thus the probability that there's no USDs between A and $n\tau-\xi$ is equal to $\int_{n\tau-\xi-A}^{\infty} g(s)\ ds = e^{-\lambda(n\tau-\xi-A)}$, where $g(s)$ is the p.d.f. of the exponential distribution with a parameter λ. Hence, a PM-SD happens in this scenario. The corresponding probability is

$$P_{[2.1]} = \int_{u=n\tau-\xi}^{u=\infty} \int_{s=n\tau-\xi-A}^{s=\infty} g(s)\ ds\ f(u)\ du, \tag{4.3}$$

and the contribution to the expected cycle length $\dot{L}(A|\xi)$ is

$$\dot{L}\big(A|\xi,\ in\ 2.1\big)P_{[2.1]} = (n\tau - \xi) \int_{u=n\tau-\xi}^{u=\infty} \int_{s=n\tau-\xi-A}^{s=\infty} g(s)dsf(u)du. \tag{4.4}$$

Possibility 2.2: However, if there is an USD before $n\tau - \xi$, with a probability $\int_0^{n\tau-\xi-A} g(s)\ ds = 1 - e^{-\lambda(n\tau-\xi-A)}$, a PM-USD happens in this scenario. The corresponding probability is

$$P_{[2.2]} = \int_{u=n\tau-\xi}^{u=\infty} \int_{s=0}^{s=n\tau-\xi-A} g(s)\ ds\ f(u)\ du, \tag{4.5}$$

and the contribution to the expected cycle length $\dot{L}(A|\xi)$ is

$$\dot{L}\big(A|\xi,\ in\ 2.2\big)P_{[2.2]} = \int_{u=n\tau-\xi}^{u=\infty} \int_{s=0}^{s=n\tau-\xi-A} (A+s)g(s)dsf(u)du. \tag{4.6}$$

Scenario 3: $u \leq n\tau - \xi$ ***and*** $u > A$

Possibility 3.1: If there is a failure before $n\tau-\xi$, no PM-SD action is possible. The first arrival of the USDs after A may occur before u with a probability $\int_0^{u-A} g(s)\ ds = 1 - e^{-\lambda(u-A)}$. Hence, a PM-USD happens in this scenario. The corresponding probability is

$$P_{[3.1]} = \int_{u=A}^{u=n\tau-\xi} \int_{s=0}^{s=u-A} g(s)\ ds\ f(u)\ du, \tag{4.7}$$

and the contribution to the expected cycle length $\dot{L}(A|\xi)$ is

$$\dot{L}\big(A|\xi,\ in\ 3.1\big)P_{[3.1]} = \int_{u=A}^{u=n\tau-\xi} \int_{s=0}^{s=u-A} (A+s)g(s)dsf(u)du. \tag{4.8}$$

Possibility 3.2: Or if there is no USD before u, with a probability $\int_{u-A}^{\infty} g(s)\ ds = e^{-\lambda(u-A)}$, a CM happens in this scenario. The corresponding probability is

$$P_{[3.2]} = \int_{u=A}^{u=n\tau-\xi} \int_{s=u-A}^{s=\infty} g(s)\ ds\ f(u)\ du, \tag{4.9}$$

and the expected cycle length is

$$\dot{L}\big(A|\xi,\ in\ 3.2\big)P_{[3.2]} = \int_{u=A}^{u=n\tau-\xi} u \int_{s=u-A}^{s=\infty} g(s)dsf(u)du. \tag{4.10}$$

To summarize, we have 5 possibilities in total. Under our opportunistic maintenance policy for the component, the occurrence of the three maintenance actions at the end of every maintenance cycle depends on which event happens first, u, $n\tau - \xi$, or the first USD after A. The probability of PM-USD, $\dot{P}_1$, is the sum of $P_{[2.2]}$ and $P_{[3.1]}$; the probability of PM-SD, $\dot{P}_2$, is equal to $P_{[2.1]}$; the probability of CM, $\dot{P}_3$, is the sum of $P_{[1.1]}$ and $P_{[3.2]}$. Notice that the sum of $\dot{P}_1$, $\dot{P}_2$ and $\dot{P}_3$ is equal to one.

$$\begin{aligned}
\dot{P}_1(\xi) &= \int_{u=n\tau-\xi}^{u=\infty} \int_{s=0}^{s=n\tau-\xi-A} g(s)\ ds\ f(u)\ du + \int_{u=A}^{u=n\tau-\xi} \int_{s=0}^{s=u-A} g(s)\ ds\ f(u)\ du,\\
\dot{P}_2(\xi) &= \int_{u=n\tau-\xi}^{u=\infty} \int_{s=n\tau-\xi-A}^{s=\infty} g(s)\ ds\ f(u)\ du.\\
\dot{P}_3(\xi) &= \int_{u=A}^{u=n\tau-\xi} \int_{s=u-A}^{s=\infty} g(s)\ ds\ f(u)\ du + \int_{u=0}^{u=A} f(u)\ du
\end{aligned} \tag{4.11}$$

and the expected cycle length is the sum of the contribution in all possibilities,

$$\begin{aligned}
\dot{L}(A|\xi) &= \dot{L}\big(A|\xi,\ in\ 1.1\big)P_{[1.1]} + \dot{L}\big(A|\xi,\ in\ 2.1\big)P_{[2.1]} + \dot{L}\big(A|\xi,\ in\ 2.2\big)P_{[2.2]}\\
&+ \dot{L}\big(A|\xi,\ in\ 3.1\big)P_{[3.1]} + \dot{L}\big(A|\xi,\ in\ 3.2\big)P_{[3.2]}\\
&= \int_{u=A}^{u=n\tau-\xi} \left(\int_{s=0}^{s=u-A} (A+s)g(s)ds + u\int_{s=u-A}^{s=\infty} g(s)ds\right) f(u)du\\
&+ \int_{u=n\tau-\xi}^{u=\infty} \left((n\tau-\xi)\int_{s=n\tau-\xi-A}^{s=\infty} g(s)ds + \int_{s=0}^{s=n\tau-\xi-A} (A+s)g(s)ds\right) f(u)du\\
&+ \int_{u=0}^{u=A} uf(u)du.
\end{aligned} \tag{4.12}$$

As mentioned previously in Section 4.2, to evaluate the average cost rate over an infinite horizon, we need to propose an approximate evaluation method to characterize the difference between the ending points of maintenance cycles and SDs, i.e., ξ. The appearance of ξ also depends on the three maintenance actions at the end of every maintenance cycle. If a PM-SD action is taken, the ξ for the next maintenance cycle will be equal to 0. If a PM-USD action or a CM action is taken, ξ for the next maintenance cycle will be any possible value in $[0, \tau)$. Since the arrivals of USDs follow a Poisson process, ξ for the next maintenance cycle is assumed to take any value in $[0, \tau)$ with equal chances, given that a PM-USD action is taken at the end of this maintenance cycle. Furthermore, if we assume that the intervals of SDs are relatively small compared with the average value of the failure time u, the ξ will also be approximately evenly-distributed over $[0, \tau)$, given that an CM action is taken at the end of this maintenance cycle. Hence, the following distribution function can be used to describe the random variable ξ,

$$H(\xi) = \begin{cases} 0 & \text{if } \xi < 0 \\ q + \frac{(1-q)\xi}{\tau} & \text{if } 0 \leq \xi < \tau \\ 1 & \text{if } \xi \geq \tau \end{cases} \tag{4.13}$$

where $H(\xi)$ is uniformly distributed on $[0, \tau)$ with a positive probability mass q at $\xi = 0$. The probability q is the probability that an arbitrary cycle ends with a PM-SD. Notice that q can not be calculated directly from $\dot{P}_2(\xi)$ in Equation (4.11), because $\dot{P}_2(\xi)$ is the conditional probability of PM-SD given that the renewal point starts at ξ time units away from the previous SD.

Suppose we could approximate the value of q, we can also obtain the unconditional probabilities of PM-USD, PM-SD and CM by multiplying $f(\xi)$ with the conditional probabilities of PM-USD, PM-SD and CM, denoted by P_1, P_2 and P_3 respectively:

$$\begin{aligned} P_1 &= \dot{P}_1(0)q + \int_0^\tau \dot{P}_1(\xi)\frac{(1-q)}{\tau}\, d\xi, \\ P_2 &= \dot{P}_2(0)q + \int_0^\tau \dot{P}_2(\xi)\frac{(1-q)}{\tau}\, d\xi, \\ P_3 &= \dot{P}_3(0)q + \int_0^\tau \dot{P}_3(\xi)\frac{(1-q)}{\tau}\, d\xi \end{aligned} \tag{4.14}$$

Similarly, the expected cycle length denoted by $L(A|\xi)$ are derived as

$$L(A) = \dot{L}(A|\xi = 0)q + \int_0^\tau \dot{L}(A|\xi)\frac{(1-q)}{\tau}\, d\xi. \tag{4.15}$$

The determination of q is done by iteratively calculating P_2, starting with $q = \dot{P}_2$ at $\xi = 0$. The details of the algorithm are elaborated in Subsection 4.A.1.

According to Equation (4.14), the expected cycle cost $K(A)$ can be derived as:

$$K(A) = P_1\ C_{opm,usd} + P_2\ C_{opm,sd} + P_3\ C_{CM}. \tag{4.16}$$

According to the renewal theory, the expected total maintenance cost rate of the component $Z(A)$ is equal to $K(A)/L(A)$. Hence, the optimization model is formulated as

$$\begin{aligned} \min_{A} \qquad & Z(A) = \frac{K(A)}{L(A)} \\ \text{s.t.} \qquad & 0 < A < \infty \end{aligned}$$

The objective function is non-linear. Hence, several non-linear optimization methods may be used (e.g., local search, interior point method, first-order-condition method)[5], depending on different degradation models. As explained in Section 4.2, the evaluation of the average cost rate via renewal theory is an approximation, which will be compared with simulation results. The minimization of the cost rate can be based on the approximate evaluation $Z(A)$ and simulation $\hat{Z}(\hat{A})$, which leads to their optimal age limits A^* and $\hat{A}^*$ respectively.

4.4 Numerical Example

To demonstrate the usage of the model, we show a simple numerical example. The life time distribution of the component is subject to a Weibull distribution with a scale parameter α and a shape parameter β. In this case, the probabilities of PM-USD, PM-SD and CM are

$$\begin{aligned} \dot{P}_1(\xi) &= \int_{u=A}^{u=n\tau-\xi} \left(1 - e^{-\lambda(u-A)}\right) \left(\frac{\beta u^{(\beta-1)}}{\alpha^\beta} e^{-(\frac{u}{\alpha})^\beta}\right) du \\ &+ \left(1 - e^{-\lambda(n\tau-\xi-A)}\right) \left(1 - F(n\tau - \xi)\right) \\ \dot{P}_2(\xi) &= \left(e^{-\lambda(n\tau-\xi-A)}\right) \left(1 - F(n\tau - \xi)\right) \\ \dot{P}_3(\xi) &= \int_{u=A}^{u=n\tau-\xi} \left(e^{-\lambda(u-A)}\right) \left(\frac{\beta u^{(\beta-1)}}{\alpha^\beta} e^{-(\frac{u}{\alpha})^\beta}\right) du \ + \ F(A) \end{aligned} \tag{4.17}$$

where $F(u) = 1 - e^{-(\frac{u}{\alpha})^\beta}$ is the c.d.f. of the Weibull distribution.

The input parameters given in Table 4.1 are not from a real case. We set the shape ($\beta = 2.101$) and scale parameter ($\alpha = 1.129$) of the Weibull distribution, so that the expected life time $E[T]$ is equal to 1 year, which normalizes the time unit.

The optimal age limit A^* can be found by minimizing the average cost rate $Z(A^*)$ via the approximate evaluation (see Section 4.3). As a comparison, we simulate the

Table 4.1 The parameter setting

Parameter	Explanation
$c^{PM-SD} = 1$	Preventive maintenance due to scheduled downs [thousand Euro]
$c^{PM-USD} = 2$	Preventive maintenance due to unscheduled downs [thousand Euro]
$c^{CM} = 10$	Corrective maintenance [thousand Euro]
$\tau = 0.2$	The interval of scheduled downs [year]
$\alpha = 1.129$	Scale parameter of Weibull distribution
$\beta = 2.101$	Shape parameter of Weibull distribution
$\lambda = 2$	Poisson arrival rate of unscheduled downs [per year]

average cost rate $\hat{Z}$ (see Subsection 4.A.2) under a given A. Figure 4.3 illustrates the changes of the average cost rate over the age limit A via the approximate evaluation and the simulation with a 95% confidence interval.

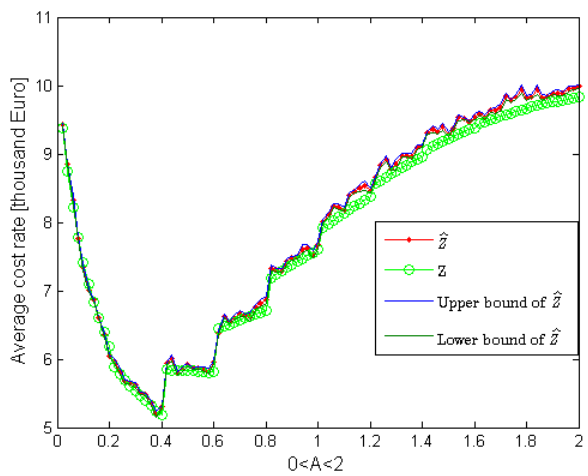


Figure 4.3 Average cost rate [thousand euro per year] over A [year]. The approximate result Z is compared with the simulated result $\hat{Z}$ in a confidence interval with a lower and upper bound

The first observation is that the curves of the average cost rate obtained via the approximate evaluation and simulation are very close, which means our approximate evaluation is relatively accurate. The optimal maintenance policy via the approximate evaluation has a age limit that is $A^* = 0.400$ year and a minimum cost rate $Z(A^*) = 5.189$ euro per day (see Figure 4.3), which is slightly different from the simulation results $\hat{A}^* = 0.380$ and $\hat{Z}(\hat{A}^*) = 5.185 \pm 0.006$ in Table 4.2. Moreover, the confidence interval is very small in Figure 4.3 (More details in Subsection 4.A.2).

Table 4.2 shows i) the optimal policy via the approximate evaluation, including the optimal age limit A^*, its minimum cost rate $Z(A^*)$, its probabilities of three maintenance actions $\left\{P_1, P_2, P_3\right\}$ and its expected cycle length $L(A^*)$; ii) the simulation results under the optimal age limit A^* obtained via the approximate evaluation, where $\hat{Z}(A^*)$ denotes the average cost rate, $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ denotes the probabilities of three maintenance actions and $\hat{L}(A^*)$ denotes the mean cycle length; iii) the optimal age limit $\hat{A}^*$ obtained via simulation-based optimization with its minimum cost rate $\hat{Z}(\hat{A}^*)$, its probabilities of three maintenance actions $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ and its mean cycle length $\hat{L}(\hat{A}^*)$.

Table 4.2 The optimal maintenance policies under the parameter setting in Table 4.1 ($\{P_1, P_2, P_3\}$ and $\{\hat{P}_1, \hat{P}_2, \hat{P}_3\}$ are the probabilities of taking PM-USD, PM-SD and CM actions by approximate evaluation and simulation respectively)

Approximation Result	Simulation Result 1
$Z(A^*) = 5.189$ [K euro per year] $A^* = 0.400$ [year] $\left\{P_1, P_2, P_3\right\} = \{0.0269, 0.8570, 0.1161\}$ $L(A^*) = 0.3993$ [day]	$\hat{Z}(A^*) = 5.289 \pm 0.008$ [K euro per year] $A^* = 0.400$ [year] $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.0601, 0.8132, 0.1267\}$ $\hat{L}(A^*) = 0.4161$ [day]
Simulation Result 2	
$\hat{Z}(\hat{A}^*) = 5.185 \pm 0.006$ [K euro per year] $\hat{A}^* = 0.380$ [year] $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.0485, 0.8420, 0.1095\}$ $\hat{L}(\hat{A}^*) = 0.3923$ [day]	$\lvert Gap1\rvert$: $\lvert\frac{\left(\hat{Z}(C^*)-Z(C^*)\right)}{\hat{Z}(C^*)}\rvert = 1.88\%$ $\lvert Gap2\rvert$: $\lvert\frac{\left(\hat{Z}(\hat{C}^*)-\hat{Z}(C^*)\right)}{\hat{Z}(\hat{C}^*)}\rvert = 1.96\%$

Based on the results in Table 4.2, we observe that the absolute value $|\big(\hat{Z}(A^*) - Z(A^*)\big)/\hat{Z}(A^*)|$, denoted as Gap 1, is only 1.88%, which shows that our approximate evaluation is very close to the simulation results, under the same A^* value. The absolute value $|\big(\hat{Z}(\hat{A}^*) - \hat{Z}(A^*)\big)/\hat{Z}(\hat{A}^*)|$, denoted as Gap 2, is only 1.96%. This implies that the deviation of A^* from $\hat{A}^*$ does not lead to a large deviation on the simulated cost rate, which is due to the fact that the cost rate $\hat{Z}(A)$ is flat in the neighborhood of its minimum. Hence, in practice, the optimal maintenance policy of our approximate evaluation will result in an optimal solution having an average cost rate that is very close to the true minimum cost rate. Also notice that the values of $\left\{P_1, P_2, P_3\right\}$ and L via the approximate evaluation are very close to the simulated values of $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ and $\hat{L}$. Therefore, we can conclude that the gaps are small and our approximate evaluation is relatively accurate in this numerical example.

It is very interesting to observe that the average cost rates of approximate evaluation and simulation are not smooth curves. To have insights further on the spikes of the curves, we plot the probabilities P_1, P_2 and P_3 in Figure 4.4 and the expected cycle

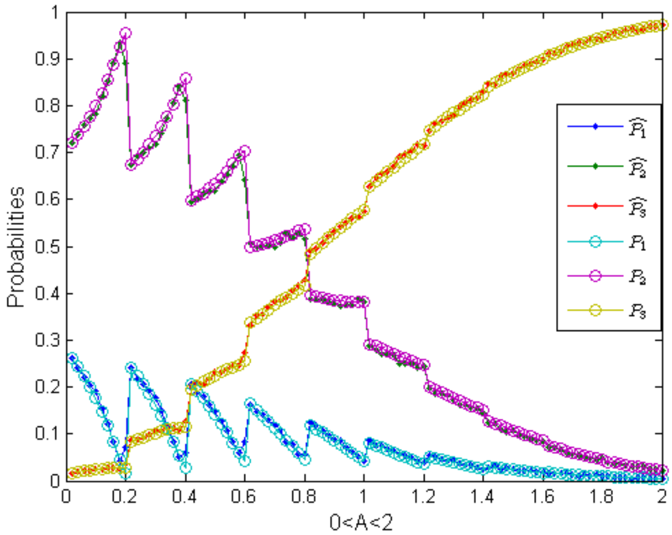


Figure 4.4 The probabilities of three maintenance actions (i.e., PM-USD, PM-SD and CM) over A [year]. The approximate result $\left\{P_1, P_2, P_3\right\}$ is compared with the simulated result $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$

length L in Figure 4.5.

The first observation is that the differences between the value of P_1, P_2, P_3 and L obtained via the approximate evaluation and simulation are very small, which verifies the accuracy of our approximate evaluation further. The spikes in Figure 4.4 and the jump in Figure 4.5 appear at scheduled downs $n\tau$, where $\tau = 0.2$ and $n \in \mathbb{N}$. The reason is that our model has a strict age limit for taking opportunities. Consider two cases: 1) when A is just before $n\tau$, or $A + \epsilon = n\tau$ where ϵ is infinitely small and positive, the next opportunity after A is almost certain to be PM-SD, because $n\tau$ is just behind A. 2) However, if A is just after $n\tau$ or $A = n\tau + \epsilon$, then the next PM-SD opportunity is at $(n + 1)\tau$, instead of $n\tau$. Therefore, P_2 is much larger in Case 1 than in Case 2 when A is smaller than the expected life time ($\mathbb{E}[T] = 1$). At the same time, P_1 and P_3 are much smaller in Case 1 compared with Case 2 when A is small. For the larger values of A, e.g., $A = 1.4, 1.6, 1.8$ or 2.0; much less USD and SD opportunities are taken and the probability of CM becomes larger. This means that the renewal cycles end with failures more often, and the ξ has less chance to take the value zero. Therefore, the spikes in Figure 4.4 becomes less sharp when A increases. For the same reason, the magnitude of the jumps of expected cycle length in Figure 4.5 also becomes less at a larger A.

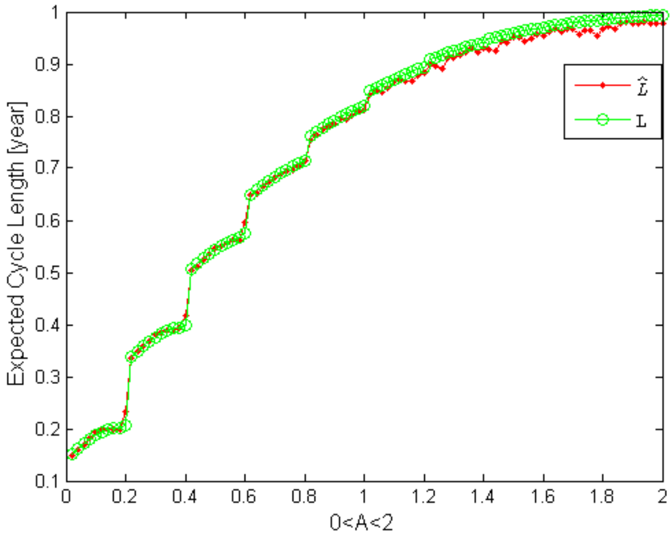


Figure 4.5 Expected cycle length [year] over A [year]. The approximate result L is compared with the simulated result $\hat{L}$

4.5 Numerical Experiments

To validate our model under various parameter settings, we conduct the following numerical experiments based on full factorial test beds. In Section 4.5.1 and 4.5.2, we investigate the accuracy of our approximate evaluation. In Section 4.5.3, we evaluate the cost reduction potential of our proposed policy that includes both opportunities at USDs and SDs.

4.5.1 Accuracy of the approximate evaluation

The accuracy of our approximate evaluation is assessed based on the gap between the simulation result $\hat{Z}(A)$ and the approximation result $Z(A)$. We vary four factors in our test bed: the variable A and three parameters τ, λ and σ [1]. Three different levels of the age limit A, $\{0.5, 1.0, 1.5\}$, are chosen. For each of the other three parameters, three different levels are obtained by multiplying a base value with a set of coefficients,

[1] $\sigma^2 = E[T^2] - E[T]^2$, where $E[T]$ and $E[T^2]$ are the 1^{st} and 2^{nd} moment of the component life time. σ is the standard deviation of the component life time distribution T

$\{50\%, 100\%, 150\%\}$ (see Table 4.3). Hence, a full factorial test bed is set up and a space of instances is defined, $(A_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda, \forall j, l, k, m = \{1, 2, 3\}$, which leads to $|\Lambda| = 81$ instances in the test bed.

Table 4.3 Parameter setting of the test bed

Parameter	Explanation
$\tau = 0.2 * \{50\%, 100\%, 150\%\}$	The interval of scheduled downs [year]
$\lambda = 2 * \{50\%, 100\%, 150\%\}$	Poisson arrival rate of unscheduled downs [per year]
$\sigma = 1/2 * \{50\%, 100\%, 150\%\}$	Standard deviation of component life time
$A = \{0.5, 1.0, 1.5\}$	Age limit values
$E[T] = 1$	Expected component life time [year]

We set the expected life time $E[T]$ of the component to be equal to 1, which normalizes the time unit. By fitting the two moments of the component life time, the shape and scale parameters of the Weibull distribution can be estimated. To show the variance of the life time distribution, we choose the standard deviation σ as a varying parameter. Moreover, τ and λ are varied, because they determine the frequency of the opportunities from PM-USD and PM-SD events (see Section 4.2). The design of the full factorial test bed is shown in Table 4.3.

Notice that no cost parameters are chosen as factors in this test bed, since the objective function is fully determined by the probabilities of the three maintenance actions and the expected cycle length. This also helps to reduce the size of the test bed. To compare the approximation results and simulation results, we compare the probabilities of PM-USD, PM-SD and CM and the expected cycle length obtained by the approximate evaluation $\big(P_1, P_2, P_3, L(A)\big)$ and the simulation $\big(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(A)\big)$; which is similar to Table 4.2 in Section 4.4. To see how much the approximation results deviate from the simulation results, we define a deviation vector $\big[\delta_1, \delta_2, \delta_3, \delta_4\big] = \big[\hat{P}_1 - P_1, \hat{P}_2 - P_2, \hat{P}_3 - P_3, (\hat{L}(A) - L(A))/\hat{L}(A)\big]$. The deviation vectors of 81 instances are shown in Table 4.9 and 4.10 of Subsection 4.A.4. There are three levels for each factor in $(A_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda, \forall j, l, k, m = \{1, 2, 3\}$. We categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing A_1 is defined as $\Lambda_{A_1} = \big\{(A_1, \sigma_l, \lambda_k, \tau_m) | l, k, m \in \{1, 2, 3\}\big\}$. For each of these subsets, the average of the absolute deviation vectors (denoted by AAD) and the maximum of the absolute deviation vectors (denoted by MAD) are summarized in Table 4.4.

The first insight from Table 4.4 is that the AADs and MADs of δ_1, δ_2, δ_3 and δ_4 are small, which implies that our approximate evaluation is accurate under all parameter settings (including the age limit A). We also observe that the AADs and MADs of δ_1, δ_2 and δ_3 are at the magnitude of 10^{-3} and 10^{-2} respectively. The AADs of δ_4 are less than 1% and the MAD of δ_4 are less than 3%. Notice that the AADs and MADs is larger, when A is the multiple of τ. Unlike our approximate evaluation, SDs in the

Table 4.4 The average absolute difference (AAD) and the maximum absolute difference (MAD) between the simulation results and the approximation results

	$\lvert\delta_1\rvert$ $\{AAD, MAD\}$	$\lvert\delta_2\rvert$ $\{AAD, MAD\}$	$\lvert\delta_3\rvert$ $\{AAD, MAD\}$	$\lvert\delta_4\rvert$ $\{AAD, MAD\}$
Λ_{A_1}	$\{0.0065, 0.0261\}$	$\{0.0087, 0.032\}$	$\{0.0033, 0.0085\}$	$\{0.33\%, 0.82\%\}$
Λ_{A_2}	$\{0.0021, 0.0076\}$	$\{0.0027, 0.0087\}$	$\{0.0031, 0.0114\}$	$\{1.05\%, 2.54\%\}$
Λ_{A_3}	$\{0.001, 0.0036\}$	$\{0.0032, 0.0075\}$	$\{0.0033, 0.0067\}$	$\{1.1\%, 2.85\%\}$
Λ_{σ_1}	$\{0.0036, 0.0261\}$	$\{0.005, 0.032\}$	$\{0.003, 0.0114\}$	$\{0.74\%, 1.54\%\}$
Λ_{σ_2}	$\{0.0032, 0.021\}$	$\{0.005, 0.0171\}$	$\{0.0031, 0.0069\}$	$\{0.83\%, 2.4\%\}$
Λ_{σ_3}	$\{0.0028, 0.0157\}$	$\{0.0046, 0.0184\}$	$\{0.0036, 0.0089\}$	$\{0.92\%, 2.85\%\}$
Λ_{λ_1}	$\{0.0015, 0.0063\}$	$\{0.0031, 0.0087\}$	$\{0.0027, 0.0085\}$	$\{0.7\%, 2.4\%\}$
Λ_{λ_2}	$\{0.0029, 0.011\}$	$\{0.0044, 0.0159\}$	$\{0.0032, 0.0089\}$	$\{0.87\%, 2.85\%\}$
Λ_{λ_3}	$\{0.0053, 0.0261\}$	$\{0.0071, 0.032\}$	$\{0.0038, 0.0114\}$	$\{0.9\%, 2.54\%\}$
Λ_{τ_1}	$\{0.0008, 0.0023\}$	$\{0.0033, 0.0087\}$	$\{0.0033, 0.0089\}$	$\{0.71\%, 2.54\%\}$
Λ_{τ_2}	$\{0.0047, 0.0261\}$	$\{0.0065, 0.032\}$	$\{0.003, 0.006\}$	$\{1.02\%, 2.85\%\}$
Λ_{τ_3}	$\{0.0041, 0.021\}$	$\{0.0047, 0.0193\}$	$\{0.0034, 0.0114\}$	$\{0.75\%, 2.4\%\}$
Λ	$\{0.0032, 0.0261\}$	$\{0.0049, 0.032\}$	$\{0.0032, 0.0114\}$	$\{0.83\%, 2.85\%\}$

simulation are not rescheduled, as explained in Section 4.2. In the simulation, if the previous maintenance cycle does not end at the multiple of $n\tau$, the starting point of the next maintenance cycle shift. Hence, even A in the next cycle is equal to $n\tau$, A may still be far away from the actual SD in the simulation. This difference between the approximation and simulation at $A = n\tau$ results in the larger AADs and MADs.

4.5.2 Heuristic optimization based on the approximate evaluation

The results in Table 4.2 of Section 4.4 show that the optimal policies via the approximate evaluation and simulation are close to each other, which verifies the accuracy of our approximate evaluation. In this subsection, we intend to verify these results further under various parameter settings. Similar to Table 4.2, we evaluate two gaps: i) Gap 1, $\big(\hat{Z}(A^*) - Z(A^*)\big)/\hat{Z}(A^*)$, shows how much the true cost rate deviates from the cost rate of approximate evaluation, while using the optimal age limit of our approximate evaluation model; and ii) Gap 2, $\big(\hat{Z}(\hat{A}^*) - \hat{Z}(A^*)\big)/\hat{Z}(\hat{A}^*)$, shows how much the optimal maintenance policy of our approximate evaluation deviates from the true optimal policy.

Since we will compare the optimal policies via the approximation with the simulation results, the age limit A is no longer a factor in the test bed. Hence, the space of instances is $(\sigma_l, \lambda_k, \tau_m) \in \Omega, \forall l, k, m = \{1, 2, 3\}$; which leads to $|\Omega| = 27$ instances in the test bed. The cost factors are found back in Table 4.1. The deviation vectors of 27 instances are shown in Table 4.11 of Subsection 4.A.4. For each factor, we categorize the instances containing a specific level into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \big\{(\sigma_1, \lambda_k, \tau_m) | \forall k, m \in \{1, 2, 3\}\big\}$.

Table 4.5 The average absolute difference (AAD) and the maximum absolute difference (MAD) between the simulation results and the approximation results for Gap 1, Gap 2 and $(\hat{A}^* - A^*)/\hat{A}^*$

	$(\hat{A}^* - A^*)/\hat{A}^*$ $\{AAD, MAD\}$	$Gap1$ $\{AAD, MAD\}$	$Gap2$ $\{AAD, MAD\}$
Ω_{σ_1}	$\{4.07\%, 6.67\%\}$	$\{1.83\%, 3.21\%\}$	$\{1.63\%, 3.04\%\}$
Ω_{σ_2}	$\{4.35\%, 6.67\%\}$	$\{2.31\%, 3.13\%\}$	$\{2.28\%, 3.01\%\}$
Ω_{σ_3}	$\{10.6\%, 25.0\%\}$	$\{3.19\%, 4.79\%\}$	$\{2.67\%, 5.21\%\}$
Ω_{λ_1}	$\{6.57\%, 25.0\%\}$	$\{1.99\%, 4.38\%\}$	$\{2.19\%, 5.21\%\}$
Ω_{λ_2}	$\{6.20\%, 25.0\%\}$	$\{2.50\%, 3.84\%\}$	$\{2.15\%, 3.10\%\}$
Ω_{λ_3}	$\{6.20\%, 25.0\%\}$	$\{2.84\%, 4.79\%\}$	$\{2.23\%, 4.82\%\}$
Ω_{τ_1}	$\{10.6\%, 25.0\%\}$	$\{2.16\%, 3.21\%\}$	$\{1.59\%, 2.63\%\}$
Ω_{τ_2}	$\{3.61\%, 5.00\%\}$	$\{2.80\%, 4.79\%\}$	$\{2.71\%, 5.21\%\}$
Ω_{τ_3}	$\{4.81\%, 6.67\%\}$	$\{2.37\%, 3.82\%\}$	$\{2.26\%, 3.10\%\}$
Ω	$\{6.33\%, 25.0\%\}$	$\{2.44\%, 4.79\%\}$	$\{2.19\%, 5.21\%\}$

For each of these subsets, the average of absolute deviation is denoted by AAD and the maximum of the absolute deviation vectors is denoted by MAD. These results are summarized in Table 4.5.

In Table 4.5, we observe that the AADs and MADs of Gap 1 and Gap 2 are small, even the AADs and MADs of $(\hat{A}^* - A^*)/A^*$ are relatively larger. This implies two points: 1) the neighborhood of the minimum $\hat{Z}$ is flat and our approximate evaluation is robust; 2) our approximate evaluation is accurate in the neighborhood of the optimal solution. Comparing Gap 1 with Gap 2, we observe that the AADs of Gap 2 are smaller than Gap 1. This implies that the deviation of A^* from $\hat{A}^*$ does not lead to much deviation on the simulated cost rate under various parameter settings; even though the inaccuracy of the approximate evaluation (or Gap 1) is slightly higher. Hence, in practice, the optimal maintenance policy of our approximate evaluation will lead to a suboptimal cost rate that is very close to the true optimal cost rate.

4.5.3 Cost reduction potential

To show the cost benefits of including the opportunities at USDs and SDs for the ABM component, three policies are considered: 1) an *only-SD-opportunistic policy*, which means that only SDs are considered as opportunities and no opportunistic preventive maintenance actions are taken at USDs; 2) an *only-USD-opportunistic policy*, which means that only USDs are considered as opportunities and no SDs are planned; and 3) a *failure-based policy*, which means that neither USDs or SDs are considered as opportunities for preventive maintenance. Notice that Policy 1 can be analyzed as a special case of our policy, where $\lambda = 0$; Policy 2 can be analyzed as a special case of our policy, where $\tau = \infty$; and Policy 3 can be analyzed as a special case of our policy, where $A = \infty$.

To show the cost benefits under various parameter settings, we use the same test bed and parameter settings as in Section 4.5.2. The minimum average cost rate of our policy that includes opportunities at both USDs and SDs is denoted by Z. The minimum average cost rates of Policy 1 and 2 are denoted by $\tilde{Z}_1$ and $\tilde{Z}_2$ respectively. Notice that no opportunity is considered in Policy 3; so that its cost rate $\tilde{Z}_3$ remains unchanged under different parameter settings, which is 10 thousand Euro. We use $\tilde{Z}_3$ as the basis of the comparison.

Table 4.6 Summary of the cost saving percentages by using opportunities at USDs and SDs; including the mean, minimum and maximum values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ respectively.

	$\triangle_A$			$\triangle_B$			$\triangle_C$		
	mean	min	max	mean	min	max	mean	min	max
Ω_{σ_1}	63.7%	60.5%	65.4%	58.8%	55.9%	60.3%	28.7%	19.6%	36.1%
Ω_{σ_2}	47.8%	45.9%	48.9%	41.0%	39.7%	41.7%	19.7%	14.1%	24.3%
Ω_{σ_3}	32.7%	30.7%	33.9%	24.2%	23.0%	25.0%	12.2%	9.1%	14.6%
Ω_{λ_1}	48.5%	32.2%	65.4%	41.3%	23.0%	60.3%	14.3%	9.1%	19.6%
Ω_{λ_2}	48.1%	31.4%	65.3%	41.3%	23.0%	60.3%	21.4%	12.9%	30.4%
Ω_{λ_3}	47.7%	30.7%	65.2%	41.3%	23.0%	60.3%	25.0%	14.6%	36.1%
Ω_{τ_1}	49.2%	33.4%	65.4%	42.3%	25.0%	60.3%	20.2%	9.1%	36.1%
Ω_{τ_2}	48.7%	32.6%	65.2%	42.1%	24.7%	60.2%	20.2%	9.1%	36.1%
Ω_{τ_3}	46.3%	30.7%	61.4%	39.5%	23.0%	55.9%	20.2%	9.1%	36.1%
Ω	48.1%	30.7%	65.4%	41.3%	23.0%	60.3%	20.2%	9.1%	36.1%

In total, we have three comparisons: A) the cost saving percentage of including opportunities at both USDs and SDs, denoted by $\triangle_A = (\tilde{Z}_3 - Z)/\tilde{Z}_3$; B) the cost saving percentage of using only opportunities at SDs (i.e., Policy 1), denoted by $\triangle_B = (\tilde{Z}_3 - \tilde{Z}_1)/\tilde{Z}_3$; C) the cost saving percentage of using only opportunities at USDs (i.e., Policy 2), denoted by $\triangle_C = (\tilde{Z}_3 - \tilde{Z}_2)/\tilde{Z}_3$. Similar to Subsection 4.5.2, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \{(\sigma_1, \lambda_k, \tau_m)|\forall k, m \in \{1,2,3\}\}$. The means, minimums and maximums of the cost saving percentages of these 9 subsets are summarized in Table 4.6. The result of each instance is shown in Table 4.12 in Subsection 4.A.4

The first observation from Table 4.6 is that our policy (using opportunities at both SDs and USDs) has a higher cost-saving potential than Policy 1 (using opportunities at SDs only), Policy 2 (using opportunities at USDs only) and Policy 3 (using no opportunities). The mean values of $\triangle_A$ are bigger than $\triangle_B$ and $\triangle_C$, because more opportunities for preventive maintenance (cheaper than corrective maintenance) are included in our policy than in Policy 1 and 2. The mean values of $\triangle_B$ are bigger than $\triangle_C$, because the cost of preventive maintenance at a SD is cheaper than at an USD. Regarding the variation of the mean values under various parameter settings, we observe that i) $\triangle_A$ is inversely proportional to σ, λ and τ. ii) $\triangle_B$ is inversely proportional to σ and τ. It remains unchanged to λ, because no USD opportunity is considered in Policy 1. iii) $\triangle_C$ is proportional to λ and inversely proportional to σ. It remains unchanged to τ, because no SD opportunity is considered in Policy 2. A higher σ means a higher variance in the lifetime distribution of the component, which leads to a higher probability of having corrective maintenance (more expensive than preventive maintenance). Hence, the mean values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ decrease when σ increases. Moreover, the cost of taking opportunities at SDs is cheaper than at USDs. On one hand, a higher λ leads to more opportunities at expensive USDs; on the other hand, a higher τ leads to less opportunities at cheaper SDs. Hence, $\triangle_A$ decreases when λ or τ increases. For the same reason, a higher τ leads to a lower $\triangle_B$. However, $\triangle_C$ increases at a higher λ, because only opportunities at USDs are considered in $\triangle_C$. In this case, a higher λ leads to more opportunities to take, so that higher cost saving percentages can be observed.

4.6 Conclusions

In this chapter, we proposed a new opportunistic maintenance policy for a single ABM component in a complex system, under given scheduled and unscheduled downs of the system. This opportunistic maintenance policy can be utilized in the context of a mixture of different maintenance policies, such as failure-based maintenance policies or/and periodic preventive maintenance policies. As the decision variable of the model, an age limit is introduced to decide the timing of taking opportunities to maintain the ABM component together with other components in the system, which saves the downtime cost and setup cost of the ABM component. The optimal age limit is determined with respect to minimum long-run average cost rate of the component over an infinite time horizon.

To validate our model, we compared our approximate evaluation results with the simulation results under various parameter settings. In numerical experiments based on a full factorial test bed, our model shows a good accuracy and a considerable cost-saving potential. It is also interesting to observe the spikes and jumps of the average cost rate when the age limit is a multiple of the schedule down interval; which is unexpected, but sensible. Finally, our model can be applied to different complex engineering systems, because it can be used as a building block for multi-component systems with a mixture of different maintenance policies.

4.A Appendices

4.A.1 Iteration algorithm

We use an iteration algorithm to find the value of q in Equation (4.13). We start with an initial value $q_0 = \dot{P}_2(\xi = 0)$ and iterate k times until $q_k - q_{k-1} < \epsilon$, where ϵ is a very small positive value (e.g., $\epsilon = 10^{-8}$). The procedure of the iteration algorithm is summarized in Table 4.7.

`Initialize` $q_0 = \dot{P}_2(\xi = 0)$
$k = 1$, $q_1 = q_0 \dot{P}_2(\xi = 0) + \int_0^\tau \dot{P}_2(\xi) f(\xi)\ d\xi$, where $f(\xi) = (1 - q_0)/\tau$
`While` $q_k - q_{k-1} < \epsilon$
$k = k + 1$
$q_k = q_{k-1} \dot{P}_2(\xi = 0) + \int_0^\tau \dot{P}_2(\xi) f(\xi)\ d\xi$, where $f(\xi) = (1 - q_{k-1})/\tau$
`End while`
Obtain $q = q_k$
`End`

Table 4.7 Iteration Algorithm

4.A.2 Simulation procedures

As explained in Section 4.2, the schedule downs will be shifted after each maintenance cycle in our approximate evaluation model, which is not the case in practice. To evaluate the accuracy of the approximate evaluation, we run a simulation to compare with the approximate evaluation results.

$$I_k^{opm,usd} = \begin{cases} 1 & \text{if a PM-USD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_k^{opm,sd} = \begin{cases} 1 & \text{if a PM-SD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_k^{CM} = \begin{cases} 1 & \text{if a CM action is taken} \\ 0 & \text{otherwise} \end{cases}$$

There are m seeds in the simulation. In each seed $i \in \{1, 2, ..., m\}$, we simulate k_i renewal/maintenance cycles, where $k \in \{0, 1, 2, ..., k_i\}$ is the index of the cycles. Hence, each seed consists of 1) a Poisson process with a rate λ and random arrival time points $D = \{d_1, d_2, ..., d_x\} \in \Re_+^x, x \in \mathbb{N}$, where $\Re_+ = [0, \infty)$; 2) a set of random failure times $T_{k,i} \in \Re+, \forall k \in \mathbb{N}$; and 3) a constant set $B = \{\tau, 2\tau, ..., n\tau\}$, $n \in \mathbb{N}$ on a time horizon T_{max} that is sufficiently large to simulate the infinite time horizon (e.g., 10^6 times larger than $L(A)$).

By running the algorithm in Table 4.8 iteratively with m seeds, the final result of the simulation $\hat{Z} = \frac{\sum_{i=1}^{m} \hat{Z}_i}{m}$ with a $100(1-\alpha)\%$ confidence interval is expressed as follows [30]:

$$\hat{Z} \pm t(1 - \alpha/2, m - 1)\sqrt{\frac{S^2}{m}}$$

where $S = \sum_{i=1}^{m} \frac{(\hat{Z}_i - \hat{Z})^2}{m-1}$ and $t(1 - \alpha/2, m - 1)$ is the upper $1 - \alpha/2$ critical point for the t-distribution with $(m-1)$ degrees of freedom (in our case, $m = 100$ and $\alpha = 5\%$). The expected cost rate for each simulation run is:

$$\hat{Z}_i = \frac{\sum_{k=1}^{k_i} \left(I_{k,i}^{opm,usd} C_{opm,usd} + I_{k,i}^{opm,sd} C_{opm,sd} + I_{k,i}^{CM} C_{CM} \right)}{R_{k_i,i}}, \qquad \forall i \in \{1, 2, ..., m\}$$

Moreover, the probabilities of PM-USD, PM-SD and CM $\left[\hat{P}_1, \hat{P}_2, \hat{P}_3\right]$; and the expected cycle length $\left[\hat{L}(A)\right]$ are:

```
For i = 1 : m,
  Initialize k = 0: Ẑ_0 = 0 and R_{0,i} = 0
  While R_{k,i} < T_max
    k_i = k_i + 1
    If R_{k-1,i} + A ≥ R_{k-1,i} + T_{k,i}
    Calculate Ẑ_{k,i}; given (I^{opm,usd}_{k,i}, I^{opm,sd}_{k,i}, I^{CM}_{k,i}) = (0,0,1) and R_{k,i} = R_{k-1,i} + T_{k,i}
    Else
      If ∃ a non-empty subset {D_{k,i}} ⊆ D : {D_{k,i}} ⊆ [R_{k-1,i} + A, R_{k-1,i} + T_{k,i}),
        If ∃ a non-empty subset {B_{k,i}} ⊆ B : {B_{k,i}} ⊆ [R_{k-1,i} + A, R_{k-1,i} + T_{k,i}),
          If min{D_{k,i}} > min{B_{k,i}},
            Calculate Ẑ_{k,i}; given (I^{opm,usd}_{k,i}, I^{opm,sd}_{k,i}, I^{CM}_{k,i}) = (0,1,0) and R_{k,i} = min{B_{k,i}}
          Else Calculate Ẑ_{k,i}; given (I^{opm,usd}_{k,i}, I^{opm,sd}_{k,i}, I^{CM}_{k,i}) = (1,0,0) and R_{k,i} = min{D_{k,i}}
        Else Calculate Ẑ_{k,i}; given (I^{opm,usd}_{k,i}, I^{opm,sd}_{k,i}, I^{CM}_{k,i}) = (1,0,0) and R_{k,i} = min{D_{k,i}}
      Else
        If ∃ a a non-empty subset {B_{k,i}} ⊆ B : {B_{k,i}} ⊆ [R_{k-1,i} + A, R_{k-1,i} + T_{k,i})
          Calculate Ẑ_{k,i}; given (I^{opm,usd}_{k,i}, I^{opm,sd}_{k,i}, I^{CM}_{k,i}) = (0,1,0) and R_{k,i} = min{B_{k,i}}
        Else Calculate Ẑ_{k,i}; given (I^{opm,usd}_{k,i}, I^{opm,sd}_{k,i}, I^{CM}_{k,i}) = (0,0,1) and R_{k,i} = R_{k-1,i} + T_{k,i}
      End if
    End if
  End while
  Obtain Ẑ_i=Ẑ_{k,i} and R_{k_i,i} = R_{k,i}, where k = k_i
End
```

Table 4.8 Simulation algorithm

$$\hat{P}_1 = \frac{\sum_{i=1}^{m}\left[\sum_{k=1}^{k_i}\left(\frac{I_{k,i}^{opm,usd}}{I_{k,i}^{opm,usd}+I_{k,i}^{opm,sd}+I_{k,i}^{CM}}\right)\right]}{m}$$

$$\hat{P}_2 = \frac{\sum_{i=1}^{m}\left[\sum_{k=1}^{k_i}\left(\frac{I_{k,i}^{opm,sd}}{I_{k,i}^{opm,usd}+I_{k,i}^{opm,sd}+I_{k,i}^{CM}}\right)\right]}{m}$$

$$\hat{P}_3 = \frac{\sum_{i=1}^{m}\left[\sum_{k=1}^{k_i}\left(\frac{I_{k,i}^{CM}}{I_{k,i}^{opm,usd}+I_{k,i}^{opm,sd}+I_{k,i}^{CM}}\right)\right]}{m}$$

and

$$\hat{L} = \frac{\sum_{i=1}^{m}\left(\frac{R_{k_i,i}}{k_i}\right)}{m}$$

4.A.3 Results without the iteration procedure

As explained in Section 4.3, we use an iterative procedure to calculate P_1, P_2, P_3 and L (see Equations (4.14) and (4.15)) based on a probability density function $H(\xi)$ (see Equation (4.13)). To show the effectiveness of the iterative procedure, we take the same numerical case in Section 4.4 as an example and evaluate the average cost rate Z without the iterative procedure, where $q = 1$ in $H(\xi)$. This will lead to $P_1 = \dot{P}_1$, $P_2 = \dot{P}_2$, $P_3 = \dot{P}_3$ and $L = \dot{L}$ (see Equations (4.14) and (4.15)), where $\dot{P}_1$, $\dot{P}_2$, $\dot{P}_3$ and $\dot{L}$ are derived as Equations (4.11) and (4.12). The results without the iterative procedure are given in Figure 4.6 and Figure 4.7.

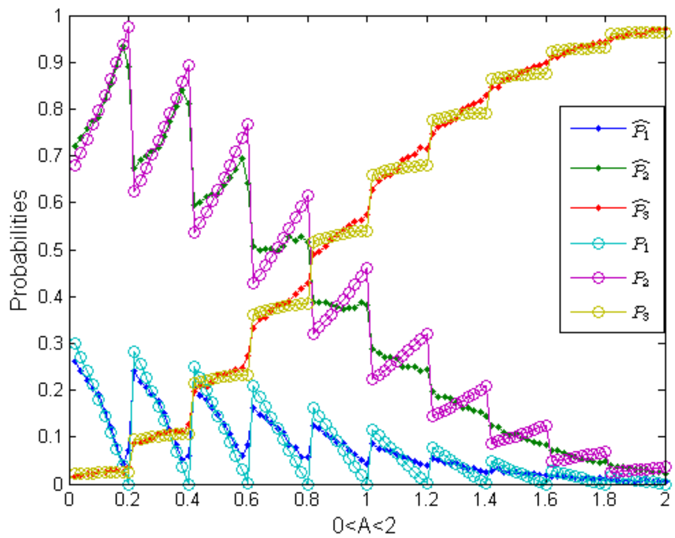


Figure 4.6 The probabilities of three maintenance actions (i.e., PM-USD, PM-SD and CM) over A [year]. The approximate result (without the iterative procedure) $\left\{P_1, P_2, P_3\right\}$ is compared with the simulated result $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$

Similar to Figure 4.4 and Figure 4.5 in Section 4.4, we plot the probabilities P_1, P_2 and P_3 in Figure 4.6 and the expected cycle length L in Figure 4.7. By comparing i) Figure 4.4 against Figure 4.6 and ii) Figure 4.5 against Figure 4.7, it is obvious that our approximate evaluation with the iterative procedure is more accurate than the case without the iterative procedure.

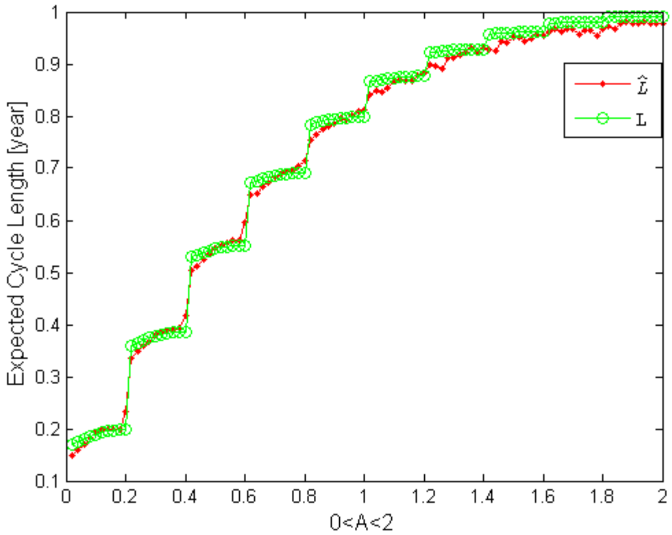


Figure 4.7 Expected cycle length [year] over A [year]. The approximate result (without the iterative procedure) L is compared with the simulated result $\hat{L}$

4.A.4 Detail results of Test bed 1 and 2

Detail results of Tables 4.4, 4.5 and 4.6 are given in the following tables

Table 4.9 A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(A)\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximation

	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(A)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(A_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.003, 0.915, 0.082, 0.493\}$	$\{-0.001, 0.0000.000 - 0.2\%\}$
$(A_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.090, 0.775, 0.135, 0.577\}$	$\{0.006, -0.009, 0.003, 0.6\%\}$
$(A_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.093, 0.766, 0.142, 0.583\}$	$\{-0.001, 0.0000.001, -0.1\%\}$
$(A_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.006, 0.914, 0.080, 0.493\}$	$\{-0.001, 0.003, -0.002, -0.2\%\}$
$(A_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.169, 0.697, 0.133, 0.574\}$	$\{0.011, -0.015, 0.004, 0.8\%\}$
$(A_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.184, 0.677, 0.139, 0.582\}$	$\{0.004, -0.003, -0.001, 0.4\%\}$
$(A_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.009, 0.912, 0.079, 0.493\}$	$\{-0.001, 0.004, -0.003, -0.2\%\}$
$(A_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.250, 0.617, 0.133, 0.569\}$	$\{0.026, -0.032, 0.006, 0.8\%\}$
$(A_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.275, 0.585, 0.140, 0.579\}$	$\{0.016, -0.019, 0.003, 0.5\%\}$
$(A_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.007, 0.824, 0.169, 0.478\}$	$\{0.0000.002, -0.002, -0.1\%\}$
$(A_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.081, 0.693, 0.226, 0.548\}$	$\{0.006, -0.004, -0.003, 0.2\%\}$
$(A_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.085, 0.678, 0.238, 0.558\}$	$\{-0.001, 0.004, -0.003, 0.0\%\}$
$(A_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.014, 0.815, 0.171, 0.477\}$	$\{0.0000.0000.000 - 0.400\%\}$
$(A_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.151, 0.618, 0.231, 0.546\}$	$\{0.010, -0.016, 0.006, 0.5\%\}$
$(A_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.172, 0.589, 0.239, 0.557\}$	$\{0.007, -0.008, 0.001, 0.4\%\}$
$(A_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.018, 0.815, 0.166, 0.479\}$	$\{-0.002, 0.006, -0.004, -0.1\%\}$
$(A_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.214, 0.562, 0.223, 0.541\}$	$\{0.015, -0.016, 0.002, 0.4\%\}$
$(A_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.257, 0.513, 0.230, 0.553\}$	$\{0.021, -0.017, -0.004, 0.3\%\}$
$(A_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.009, 0.748, 0.243, 0.462\}$	$\{0.000 - 0.008, 0.008, -0.4\%\}$
$(A_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.072, 0.634, 0.294, 0.526\}$	$\{0.004, -0.007, 0.003, 0.3\%\}$
$(A_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.076, 0.611, 0.313, 0.533\}$	$\{-0.005, -0.004, 0.009, -0.6\%\}$
$(A_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.015, 0.752, 0.233, 0.462\}$	$\{-0.002, 0.004, -0.002, -0.3\%\}$
$(A_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.139, 0.569, 0.292, 0.523\}$	$\{0.009, -0.014, 0.005, 0.5\%\}$
$(A_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.154, 0.541, 0.305, 0.531\}$	$\{-0.001, -0.004, 0.004, -0.3\%\}$
$(A_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.024, 0.746, 0.230, 0.463\}$	$\{-0.002, 0.007, -0.005, -0.1\%\}$
$(A_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.199, 0.514, 0.287, 0.517\}$	$\{0.016, -0.018, 0.003, 0\%\}$
$(A_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.228, 0.473, 0.299, 0.529\}$	$\{0.008, -0.010, 0.002, -0.1\%\}$
$(A_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.012, 0.451, 0.537, 0.861\}$	$\{0.000 - 0.003, 0.003, -0.9\%\}$
$(A_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.023, 0.416, 0.562, 0.871\}$	$\{0.0000.000 - 0.001, -1.0\%\}$
$(A_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.061, 0.279, 0.660, 0.908\}$	$\{0.003, -0.005, 0.002, -0.8\%\}$
$(A_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.022, 0.447, 0.531, 0.864\}$	$\{-0.001, 0.004, -0.003, -0.5\%\}$
$(A_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.041, 0.395, 0.564, 0.871\}$	$\{-0.003, 0.0000.003, -0.9\%\}$
$(A_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.111, 0.249, 0.640, 0.902\}$	$\{0.005, -0.002, -0.003, -0.9\%\}$
$(A_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.035, 0.435, 0.529, 0.857\}$	$\{0.001, 0.003, -0.004, -1.4\%\}$
$(A_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.062, 0.377, 0.561, 0.873\}$	$\{-0.002, 0.0000.002, -0.7\%\}$
$(A_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.138, 0.219, 0.643, 0.894\}$	$\{-0.008, -0.004, 0.011, -1.3\%\}$
$(A_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.011, 0.431, 0.558, 0.804\}$	$\{-0.001, 0.002, -0.001, -0.7\%\}$
$(A_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.023, 0.397, 0.580, 0.813\}$	$\{0.001, -0.002, 0.002, -1.0\%\}$
$(A_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.061, 0.291, 0.648, 0.849\}$	$\{0.003, -0.008, 0.005, -0.9\%\}$
$(A_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.022, 0.420, 0.557, 0.806\}$	$\{-0.001, 0.002, -0.001, -0.6\%\}$
$(A_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.043, 0.377, 0.580, 0.812\}$	$\{0.000 - 0.003, 0.003, -1.0\%\}$

Table 4.10 (continued) A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(A)\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximation

	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(A)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(A_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.110, 0.264, 0.626, 0.849\}$	$\{0.004, 0.002, -0.007, -0.3\%\}$
$(A_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.035, 0.408, 0.557, 0.800\}$	$\{0.002, 0.000, -0.002, -1.2\%\}$
$(A_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.062, 0.366, 0.572, 0.811\}$	$\{-0.002, 0.006, -0.004, -1.1\%\}$
$(A_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.146, 0.225, 0.629, 0.830\}$	$\{0.001, -0.006, 0.004, -2\%\}$
$(A_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.011, 0.408, 0.581, 0.762\}$	$\{0.000, -0.002, 0.002, -0.8\%\}$
$(A_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.023, 0.382, 0.595, 0.771\}$	$\{0.001, -0.001, 0.000, -1.0\%\}$
$(A_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.058, 0.296, 0.646, 0.809\}$	$\{0.001, -0.001, 0.000, -0.5\%\}$
$(A_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.022, 0.390, 0.588, 0.758\}$	$\{0.000, -0.009, 0.009, -1.2\%\}$
$(A_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.039, 0.365, 0.596, 0.762\}$	$\{-0.004, 0.002, 0.002, -2.0\%\}$
$(A_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.108, 0.256, 0.635, 0.800\}$	$\{0.006, -0.003, -0.003, -0.9\%\}$
$(A_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.034, 0.389, 0.577, 0.749\}$	$\{0.001, 0.001, -0.001, -2.5\%\}$
$(A_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.059, 0.343, 0.598, 0.764\}$	$\{-0.004, -0.001, 0.005, -1.7\%\}$
$(A_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.144, 0.226, 0.630, 0.799\}$	$\{0.003, -0.003, -0.001, -0.5\%\}$
$(A_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.004, 0.068, 0.929, 0.982\}$	$\{0.000, 0.003, -0.004, -0.8\%\}$
$(A_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.005, 0.049, 0.946, 0.983\}$	$\{-0.001, 0.001, -0.001, -0.9\%\}$
$(A_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.007, 0.043, 0.950, 0.991\}$	$\{0.000, 0.003, -0.004, -0.2\%\}$
$(A_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.005, 0.057, 0.938, 0.979\}$	$\{-0.001, -0.004, 0.005, -1.0\%\}$
$(A_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.011, 0.042, 0.947, 0.980\}$	$\{0.001, -0.003, 0.002, -1.2\%\}$
$(A_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.014, 0.035, 0.951, 0.982\}$	$\{0.001, -0.001, 0.000, -1.0\%\}$
$(A_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.008, 0.053, 0.938, 0.974\}$	$\{-0.001, -0.006, 0.007, -1.5\%\}$
$(A_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.015, 0.038, 0.947, 0.981\}$	$\{0.000, -0.004, 0.004, -1.1\%\}$
$(A_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.020, 0.032, 0.949, 0.990\}$	$\{0.002, -0.002, 0.000, -0.2\%\}$
$(A_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.005, 0.138, 0.857, 0.948\}$	$\{-0.001, -0.004, 0.005, -0.5\%\}$
$(A_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.013, 0.114, 0.873, 0.945\}$	$\{0.000, -0.003, 0.003, -1.6\%\}$
$(A_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.015, 0.111, 0.873, 0.940\}$	$\{0.000, 0.004, -0.004, -2.4\%\}$
$(A_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.013, 0.136, 0.851, 0.954\}$	$\{0.001, 0.001, -0.002, 0.1\%\}$
$(A_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.023, 0.102, 0.874, 0.941\}$	$\{0.000, -0.005, 0.006, -2.0\%\}$
$(A_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.027, 0.094, 0.879, 0.948\}$	$\{-0.001, -0.003, 0.004, -1.4\%\}$
$(A_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.018, 0.133, 0.849, 0.940\}$	$\{0.001, 0.002, -0.003, -1.3\%\}$
$(A_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.036, 0.092, 0.872, 0.954\}$	$\{0.003, -0.008, 0.005, -0.5\%\}$
$(A_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.042, 0.085, 0.873, 0.947\}$	$\{0.003, -0.003, 0.000, -1.4\%\}$
$(A_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.007, 0.175, 0.818, 0.905\}$	$\{0.000, -0.001, 0.001, -1.2\%\}$
$(A_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.016, 0.150, 0.834, 0.917\}$	$\{0.001, 0.000, -0.001, -0.9\%\}$
$(A_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.021, 0.146, 0.833, 0.924\}$	$\{0.003, 0.004, -0.007, -0.4\%\}$
$(A_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.013, 0.167, 0.820, 0.910\}$	$\{0.000, -0.002, 0.003, -0.7\%\}$
$(A_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.032, 0.137, 0.831, 0.899\}$	$\{0.002, 0.000, -0.002, -2.8\%\}$
$(A_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.035, 0.124, 0.842, 0.915\}$	$\{0.000, -0.004, 0.004, -1.3\%\}$
$(A_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.020, 0.170, 0.810, 0.915\}$	$\{0.000, 0.007, -0.007, -0.1\%\}$
$(A_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.043, 0.121, 0.836, 0.902\}$	$\{0.001, -0.006, 0.005, -2.4\%\}$
$(A_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.052, 0.117, 0.831, 0.916\}$	$\{0.004, 0.001, -0.005, -1.0\%\}$

Table 4.11 Detail results of Table 4.5: the optimal maintenance policy of approximation and simulation.

Ω	$\{(\hat{A}^* - A^*)/A^*, Gap1, Gap2\}$
$(\sigma_1, \lambda_1, \tau_1)$	$\{5.00\%, 2.07\%, 2.63\%\}$
$(\sigma_1, \lambda_1, \tau_2)$	$\{2.50\%, 0.51\%, 0.33\%\}$
$(\sigma_1, \lambda_1, \tau_3)$	$\{6.67\%, 1.40\%, 1.34\%\}$
$(\sigma_1, \lambda_2, \tau_1)$	$\{2.50\%, 0.10\%, 0.30\%\}$
$(\sigma_1, \lambda_2, \tau_2)$	$\{2.50\%, 2.75\%, 3.04\%\}$
$(\sigma_1, \lambda_2, \tau_3)$	$\{3.33\%, 2.41\%, 2.27\%\}$
$(\sigma_1, \lambda_3, \tau_1)$	$\{2.50\%, 3.21\%, 2.17\%\}$
$(\sigma_1, \lambda_3, \tau_2)$	$\{5.00\%, 2.29\%, 1.46\%\}$
$(\sigma_1, \lambda_3, \tau_3)$	$\{6.67\%, 1.76\%, 1.12\%\}$
$(\sigma_2, \lambda_1, \tau_1)$	$\{2.50\%, 1.96\%, 2.54\%\}$
$(\sigma_2, \lambda_1, \tau_2)$	$\{2.50\%, 1.61\%, 1.67\%\}$
$(\sigma_2, \lambda_1, \tau_3)$	$\{6.67\%, 1.83\%, 3.01\%\}$
$(\sigma_2, \lambda_2, \tau_1)$	$\{5.00\%, 2.81\%, 2.14\%\}$
$(\sigma_2, \lambda_2, \tau_2)$	$\{5.00\%, 1.89\%, 1.96\%\}$
$(\sigma_2, \lambda_2, \tau_3)$	$\{6.67\%, 2.74\%, 2.83\%\}$
$(\sigma_2, \lambda_3, \tau_1)$	$\{2.50\%, 2.24\%, 0.93\%\}$
$(\sigma_2, \lambda_3, \tau_2)$	$\{5.00\%, 3.13\%, 2.94\%\}$
$(\sigma_2, \lambda_3, \tau_3)$	$\{3.33\%, 2.54\%, 2.47\%\}$
$(\sigma_3, \lambda_1, \tau_1)$	$\{-25.0\%, 2.74\%, 1.27\%\}$
$(\sigma_3, \lambda_1, \tau_2)$	$\{5.00\%, 4.38\%, 5.21\%\}$
$(\sigma_3, \lambda_1, \tau_3)$	$\{3.33\%, 1.42\%, 1.69\%\}$
$(\sigma_3, \lambda_2, \tau_1)$	$\{-25.00\%, 2.12\%, 0.76\%\}$
$(\sigma_3, \lambda_2, \tau_2)$	$\{2.50\%, 3.84\%, 3.00\%\}$
$(\sigma_3, \lambda_2, \tau_3)$	$\{3.33\%, 3.82\%, 3.10\%\}$
$(\sigma_3, \lambda_3, \tau_1)$	$\{-25.0\%, 2.21\%, 1.61\%\}$
$(\sigma_3, \lambda_3, \tau_2)$	$\{2.50\%, 4.79\%, 4.82\%\}$
$(\sigma_3, \lambda_3, \tau_3)$	$\{3.33\%, 3.40\%, 2.56\%\}$

Table 4.12 The cost saving potential of including opportunities at USDs and SDs: $\triangle_A$, $\triangle_B$ and $\triangle_C$.

Ω	$\triangle_A$	$\triangle_B$	$\triangle_C$
$(\sigma_1, \lambda_1, \tau_1)$	65.4%	60.3%	19.6%
$(\sigma_1, \lambda_1, \tau_2)$	65.2%	60.2%	19.6%
$(\sigma_1, \lambda_1, \tau_3)$	61.4%	55.9%	19.6%
$(\sigma_1, \lambda_2, \tau_1)$	65.3%	60.3%	30.4%
$(\sigma_1, \lambda_2, \tau_2)$	64.9%	60.2%	30.4%
$(\sigma_1, \lambda_2, \tau_3)$	61.0%	55.9%	30.4%
$(\sigma_1, \lambda_3, \tau_1)$	65.2%	60.3%	36.1%
$(\sigma_1, \lambda_3, \tau_2)$	64.8%	60.2%	36.1%
$(\sigma_1, \lambda_3, \tau_3)$	60.5%	55.9%	36.1%
$(\sigma_2, \lambda_1, \tau_1)$	48.9%	41.7%	14.1%
$(\sigma_2, \lambda_1, \tau_2)$	48.5%	41.4%	14.1%
$(\sigma_2, \lambda_1, \tau_3)$	47.1%	39.7%	14.1%
$(\sigma_2, \lambda_2, \tau_1)$	48.7%	41.7%	20.8%
$(\sigma_2, \lambda_2, \tau_2)$	48.1%	41.3%	20.8%
$(\sigma_2, \lambda_2, \tau_3)$	46.4%	39.7%	20.8%
$(\sigma_2, \lambda_3, \tau_1)$	48.5%	41.7%	24.3%
$(\sigma_2, \lambda_3, \tau_2)$	47.7%	41.4%	24.3%
$(\sigma_2, \lambda_3, \tau_3)$	45.9%	39.7%	24.3%
$(\sigma_3, \lambda_1, \tau_1)$	33.9%	25.0%	9.1%
$(\sigma_3, \lambda_1, \tau_2)$	33.6%	24.7%	9.1%
$(\sigma_3, \lambda_1, \tau_3)$	32.2%	23.0%	9.1%
$(\sigma_3, \lambda_2, \tau_1)$	33.7%	25.0%	12.9%
$(\sigma_3, \lambda_2, \tau_2)$	33.1%	24.7%	12.9%
$(\sigma_3, \lambda_2, \tau_3)$	31.4%	23.0%	12.9%
$(\sigma_3, \lambda_3, \tau_1)$	33.4%	25.0%	14.6%
$(\sigma_3, \lambda_3, \tau_2)$	32.6%	24.7%	14.6%
$(\sigma_3, \lambda_3, \tau_3)$	30.7%	23.0%	14.6%

Chapter 5

An Opportunistic Maintenance Model for Multi-Component Systems under a Mixture of Different Maintenance Policies

"Simplicity is the ultimate sophistication."

Leonardo da Vinci

5.1 Introduction

Due to the diverse characteristics of components, complex engineering systems in the high-tech industry generally have a mixture of different components (e.g., age-based, condition-based, and failure-based components). For example, for some electronic parts (e.g., circuit board, current adapter), we may want to use a failure-based maintenance policy (FBM), since their failures occur according to a constant failure rate. On the other hand, for some parts in the system, it is desired to use an age-based maintenance (ABM) policy due to the fact that they have an increasing failure rate and the conditions of the components are too difficult to be measured. Under such circumstances, if we can combine the CBM activities of the monitored component with other components for which an age-based or failure-based maintenance policy is followed, the downtime cost and setup cost of maintenance for the entire system will be further reduced/eliminated. In this way, the system downs and maintenance setups for other components can serve as opportunities of a certain component to perform the preventive maintenance action without setup/downtime cost.

In this chapter, we consider multi-component systems with a mixture of components that are under CBM, ABM and FBM policies. Even though various maintenance optimization models of CBM and ABM at single-component level are available [11, 17, 25, 37, 42, 51], they cannot be applied directly to multi-component systems, because one has to deal with the economic, structural or stochastic dependencies among the components [11, 51, 61]. In this chapter, we consider the economic dependence only. There are many ABM and CBM models for multi-component systems that has been elaborated in Section 1.4 (see block B, C, E and F in Table 1.2). However, none of them has consider a multi-component system consisting of 1) such a mixture of CBM, ABM and FBM policies and 2) a large number of components. As the only relevant work in the literature, Koochaki *et al.* [28] evaluated the cost effectiveness of ABM and CBM policies for a three-component system in the context of opportunistic maintenance, which is based on simulation due to the complexity of the analysis. Different from the work of Koochaki *et al*, we propose a maintenance policy for multi-component systems consisting of a large number of components under a mixture of CBM, ABM and FBM policies. As independent building blocks, the CBM and ABM are opportunistic maintenance policies (see Chapter 3 and 4). To the best of our knowledge, the coordination of maintenance actions under such a mixture (block G in Table 1.2 in Section 1.4) has not been studied in the literature.

In this new policy, we introduce control limits for CBM components and age limits for ABM components to determine when to take these opportunities. When the degradation level of a CBM component exceeds its control limit, we will take the scheduled/unscheduled opportunities from other components and jointly maintain this CBM component with other components. Similarly, when the age of an ABM component exceeds its age limit, we will take the appeared opportunities to do joint maintenance on this ABM component together with other components.

To be able to solve large-scale problems in real life, we develop a maintenance optimization model with a heuristic procedure to minimize the long-run average cost rate of the system by optimizing 1) the control limits of CBM components, 2) the age limits of ABM components, and 3) the maintenance interval for scheduled downs of the entire system. In this case, we can decompose the main problem at the system level to subproblems at the component level. Regarding the reasons of using a heuristic, instead of an exact solution method; i) it is hard to obtain a fast and exact evaluation procedure for the objective function per component, because renewal theory can not be directly applied due to the fixed intervals for scheduled downs (also see Chapter 3 and 4). ii) When the number of components is large, the exact methods become intractable. iii) The objective function is non-linear and non-convex. Therefore, we can use a heuristic via an iterative procedure to find the heuristic solution under the structure of our proposed policy within a reasonable computation time.

The outline of this chapter is as follows. The problem description and formulation are given in Section 5.2. In Section 5.3, the solution approach is given based on new CBM and ABM models. For the details of the mathematical analysis, we will refer to Chapter 3 and 4. Moreover, we show a numerical example in Section 5.4, including three computation experiments. Finally, we give conclusions in Section 5.5.

5.2 Problem Description and Formulation

Consider a system consisting of multiple components that are subject to failures. The set I denotes the set of all components, and the components are numbered as $\{1, \ldots, |I|\}$, see Figure 5.1. Within the system, components follow different maintenance policies. We denote components that are under condition-based maintenance policy by subset I_{CBM}. Components that are under age-based maintenance policy are denoted by I_{ABM}. Components that are under failure-based maintenance policy are denoted by I_{FBM}. It holds that $I_{CBM} \cap I_{ABM} \cap I_{FBM} = \emptyset$ and $I_{CBM} \cup I_{ABM} \cup I_{FBM} = I$. We are in particular interested in the cases that have many components in a system (i.e., a high $|I|$), for which we have large-scale optimization problems at the system level.

For such multi-component systems, we often see two types of system downs, especially in the high-tech industry:

- *Scheduled downs*: For many systems, periodic maintenance actions for components/systems (e.g., inspection, cleaning, and lubrication) are executed at

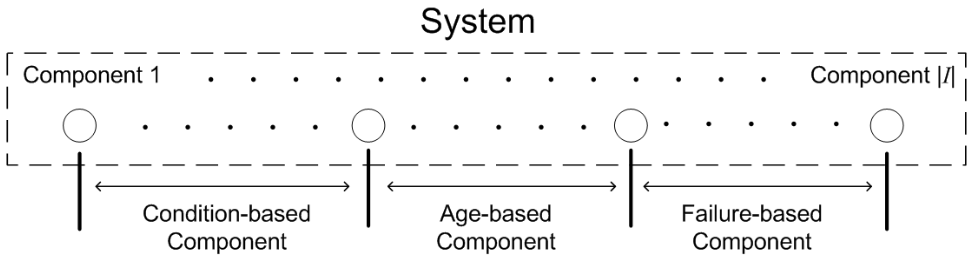


Figure 5.1 Structure of a typical multi-component system

predetermined time points with fixed time intervals in between. This leads to *scheduled downs* (SDs) of the system. This is a common practice, because it facilitates the planning and coordination of maintenance resources (e.g., service engineers, maintenance tools, spare parts). For example, the production lines of food processing industry often schedule preventive maintenance activities periodically [57].

- *Unscheduled downs*: Beside the scheduled downs, we will also have unexpected system downs when hard failures of components occur. The *unscheduled downs* (USDs) may come from the three different types of components in Figure 5.1:

  - For FBM components, maintenance actions will be conducted immediately after the failures occur. Thus the unexpected failures of components in I_{FBM} lead to a proportion of the *unscheduled downs*. For example, the electronics of a system are often under failure-based maintenance policies. When a short-circuit failure occurs, the system goes down due to power loss.

  - For ABM components, maintenance actions will be conducted preventively when the age of a component passes a certain limit and at the same time an opportunity appears for joint maintenance. But due to the randomness of failure times and the arrivals of opportunities, unexpected failures may still happen, which lead to *unscheduled downs*. For example, the loading structures, such as trusses, are often under age-based policies. When a failure due to material fatigue occurs, a truss fails before its preventive maintenance action. The system then goes down due to a structure deformation.
  - For CBM components, their conditions are monitored continuously by micro sensors. Opportunistic preventive maintenance will be conducted if

the degradation level of a component crosses a certain limit. However, due to the randomness of the opportunities, it is possible that the degradation of a component's condition exceeds the predetermined failure threshold[1]. Then the performance of the system becomes unacceptable (e.g., yield rate decreases or product defect rate increases), which causes a stop of the operation.

Every scheduled/unscheduled down will cause a period of downtime and a setup cost for maintenance actions. For the overall maintenance policy, in order to reduce the total system downs and the setup costs of maintenance, a smart clustering of maintenance actions (also known as joint/group maintenance) is required. The objective is to minimize the long-run average maintenance costs of the system. To achieve this, we can combine the preventive maintenance activities of age-based and condition-based components with the scheduled downs and unscheduled downs.

We assume that we have scheduled downs at fixed time points $\{\tau, 2\tau, 3\tau, \ldots, \infty\}$ over an infinite time horizon. The scheduled downs can be used by age-based and condition-based components as opportunities for preventive maintenance actions. The fixed interval τ for scheduled downs is a decision variable. Apart from combining the preventive maintenance actions with scheduled downs, the unscheduled downs can also be used by ABM and FBM components as opportunities for preventive maintenance actions. We could further reduce the total system downs and maintenance setups by this type of opportunistic clustering with unscheduled downs. The unscheduled downs may come from the FBM, ABM and CBM components. We approximate the arrivals of unscheduled downs using a Poisson process, which is reasonable when the number of components in the system is large.

Each scheduled down has a fixed setup cost S^{SD}, which includes the cost of the downtime and the setup of maintenance actions (e.g., sending a maintenance engineer to the system site). Similarly, each unscheduled down has a fixed cost S^{USD}. These latter costs are assumed to be higher than the scheduled downtime cost S^{SD}, because: (i) generally the hourly cost of unscheduled downtime is more expensive than the hourly cost of scheduled downtime, since the operation or production gets disturbed by the unscheduled downs; (ii) often, an unscheduled down lasts longer because a service engineer (and other resources) cannot be planned beforehand and it takes a certain amount of waiting time to have them available; (iii) extra costs may be incurred to get the maintenance equipments available within a short time period.

Next, we describe the details of the opportunistic maintenance policies for individual components.

For each component $i \in I_{FBM}$, the lifetime of component i is a random variable with its probability density function $f_i(x)$ and expected life time μ_i. For this component,

[1]This threshold is often determined by the experts in the design and engineering department

we never execute a preventive maintenance action. We wait until the component fails and conduct a corrective maintenance action. This leads to a cost S^{USD} for the unscheduled down and a cost c_i^{Rep} for the corrective replacement.

For each component $i \in I_{ABM}$, the lifetime of component i is also a random variable with probability density function $f_i(x)$. We introduce an age limit A_i and execute a preventive maintenance action if the lifetime is larger or equal to A_i when the opportunities of joint maintenance arrive (i.e., when the scheduled or unscheduled downs of other components appear). Since these preventive maintenance actions are performed within the time periods of the scheduled downs or unscheduled downs from other components, the cost of downtime can be saved for component $i \in I_{ABM}$ in this case. The setup cost of maintenance for component i is also reduced by this joint maintenance, due to the fact that we only need one setup for this joint maintenance activity. Thus, the corresponding preventive maintenance costs are equal to c_i^{PM-SD} and c_i^{PM-USD}, for scheduled downs and unscheduled downs respectively; generally, $c_i^{PM-SD} \leq c_i^{PM-USD}$. When component i fails i) before the age A_i has been reached or ii) before an opportunity appears to do a preventive replacement while the age A_i has been reached; a corrective maintenance action is taken on component i, which leads to a corrective replacement cost c_i^{Rep} and a fixed cost S^{USD} for this unscheduled down caused by component i (namely, $c_i^{CM} = S^{USD} + c_i^{Rep}$). Generally speaking, c_i^{CM} is larger than c_i^{PM-SD} and c_i^{PM-USD}, since preventive maintenance is normally cheaper than corrective maintenance. Notice that $A_i = \infty$ implies that we will never have a preventive maintenance; in that special case, the ABM policy reduces to a FBM policy.

For each component $i \in I_{CBM}$, the degradation level is measured continuously and the degradation level at time t is denoted by $X_i(t)$, $t \geq 0$. We assume that a higher level of $X_i(t)$ corresponds to a higher level of degradation. A failure threshold H_i is given by the experts in the design and engineering department. If the degradation level $X_i(t)$ reaches level H_i, one is not allowed to continue the operation and has to do an immediate repair. Hence, when $X_i(t)$ reaches level H_i, we see this as a failure and the repair is seen as a corrective maintenance action. The repair itself may be done by simply replacing the failed component by a ready-for-use spare part. This corrective maintenance action of component i leads to a cost c_i^{CM}, which is composed of a corrective replacement cost c_i^{Rep} and a fixed cost S^{USD} related to the downtime and setup of maintenance for this unscheduled down. In order to avoid these relatively high costs, we introduce a control limit C_i ($\leq H_i$). We replace component i preventively if $X_i(t) \geq C_i$ when an opportunity of scheduled/unscheduled downs appears at time t. The cost of a preventive replacement at a scheduled and unscheduled down is equal to c_i^{PM-SD} and c_i^{PM-USD}, respectively. Both cost factors are smaller than c_i^{CM}, and generally $c_i^{PM-SD} \leq c_i^{PM-USD}$. Notice that $C_i = H_i$ implies that we will never have a preventive replacement; in that special case, the CBM policy reduces to a FBM policy.

Given the above structure for the overall maintenance policy, we can distinguish two types of decision variables: (i) the fixed interval τ for scheduled downs (at the system level); (ii) the control limits C_i, $i \in I_{CBM}$, and the age limits A_j, $j \in I_{ABM}$ (at the component level). We can determine them in a nested way. For a given τ, we can first optimize the control limits C_i and the age limits A_j, $\forall i \in I_{CBM} and j \in I_{ABM}$. Next one can optimize τ. This leads to subproblems at the component level and a main problem at the system level that have to be solved.

Let us first formulate the subproblem for a given τ. Let $\mathbf{C}$ and $\mathbf{A}$ be vectors consisting of all $C_i (i \in I_{CBM})$ and all $A_j (j \in I_{ABM})$, respectively. The long-run average cost rate under a given choice for $\mathbf{C}$ and $\mathbf{A}$ is denoted by $Z_{syst}(\tau, \mathbf{C}, \mathbf{A})$. They consist of the average cost rate of scheduled downs S^{SD}/τ and all other cost rates that can be directly coupled to individual components. Let $Z_i(\tau, \mathbf{C}, \mathbf{A})$ be the cost rates that are coupled to component i, $i \in I$. Then it holds that

$$Z_{syst}(\tau, \mathbf{C}, \mathbf{A}) = \frac{S^{SD}}{\tau} + \sum_{i \in I} Z_i(\tau, \mathbf{C}, \mathbf{A}).$$

We assume the degradation processes or lifetimes of components are independent of other components, and components are as good as new after repair or replacement. Thus, the average cost rate $Z_i(\tau, \mathbf{C}, \mathbf{A})$ can be determined for each component $i \in I$. The average cost rate $Z_i(\tau, \mathbf{C}, \mathbf{A})$ consists of the costs due to corrective maintenance actions for all $i \in I_{FBM}$, with a cost of $S^{USD} + c_i^{Rep}$ per corrective maintenance action. Further, for all $i \in I_{CBM} \cup I_{ABM}$, they consist of the costs due to preventive maintenance actions at scheduled downs (with a cost c_i^{PM-SD} per action), the costs due to preventive maintenance actions at unscheduled downs (with a cost c_i^{PM-USD} per action), and the costs due to corrective maintenance actions (with a cost $S^{USD} + c_i^{Rep}$ per corrective maintenance action). Notice that the average cost rate $Z_i(\tau, \mathbf{C}, \mathbf{A}) = (S^{USD} + c_i^{Rep})/\mu_i$ for a failure based component $i \in I_{FBM}$ are independent of $\mathbf{C}$ and $\mathbf{A}$; hence, we may also write $Z_i(\tau, \mathbf{C}, \mathbf{A}) = Z_i(\tau)$ for all $i \in I_{FBM}$. For each condition-based component $i \in I_{CBM}$, we can find the optimal control limits C_i^* that minimizes the cost rate $Z_i^*(\tau, \mathbf{C}, \mathbf{A}) = Z_i\big(C_i^*(\tau)\big)$, given a τ. For each age-based component $i \in I_{ABM}$, we can find the optimal age limits A_i^* that minimizes the cost rate $Z_i^*(\tau, \mathbf{C}, \mathbf{A}) = Z_i\big(A_i^*(\tau)\big)$, given a τ. Let $\mathbf{C}^*(\tau)$ and $\mathbf{A}^*(\tau)$ be the optimal vectors consisting of all $C_i^*(i \in I_{CBM})$ and all $A_j^*(j \in I_{ABM})$ which minimize $Z_{syst}(\tau, \mathbf{C}, \mathbf{A})$ under a given τ. Then the minimum average cost rates for a given τ are equal to

$$Z_{syst}(\tau) = Z_{syst}(\tau, \mathbf{C}^*(\tau), \mathbf{A}^*(\tau)).$$

The subproblem for a given τ is to determine the optimal control limits $\mathbf{C}^*(\tau)$ and the age limits $\mathbf{A}^*(\tau)$ and the corresponding average cost rates $Z_{syst}(\tau)$.

If we are able to solve the subproblem for a given τ, then the next step is to solve the main problem at the system level:

$$(P): \qquad \min_{\tau} \qquad Z_{syst}(\tau)$$

$$\text{s.t.} \qquad \tau^{LB} < \tau < \tau^{UB}.$$

We assume both a lower bound τ^{LB} (≥ 0) and an upper bound τ^{UB} (maybe equal to ∞) for τ. The lower bound τ^{LB} is specified due to the fact that the user of the system does not allow too frequent scheduled downs or capacity limitations of service engineers may prohibit a too small value for τ. The upper bound τ^{UB} may come from regulations that prescribe the minimal frequencies of regular inspections (e.g., the annual inspection for personal cars implies at least one scheduled system down per year). $\tau^* \in [\tau^{LB}, \tau^{UB}]$ is the optimal maintenance interval that gives the minimum average cost rates of the system denoted by $Z^*_{syst} = Z_{syst}(\tau^*)$.

5.2.1 Notation

τ : interval of scheduled downs
A_i : age limit on the age of component $i \in I_{ABM}$ (decision variable)
C_i : control limit on the degradation level of component $i \in I_{CBM}$ (decision variable)
H_i : CM threshold on the degradation of component $i \in I_{CBM}$
μ_i : expected life time of component $i \in I_{FBM}$ Z_i : average cost rate of component $i \in I$
Z_{syst} : average cost rate of the system
c_i^{PM-USD} : PM cost of component $i \in \{I_{CBM} \cup I_{ABM}\}$ at unscheduled system downs
c_i^{PM-SD} : PM cost of component $i \in \{I_{CBM} \cup I_{ABM}\}$ at scheduled system downs
c_i^{Rep} corrective replacement cost of component $i \in I$
S^{USD} : setup cost and downtime cost at unscheduled downs
S^{SD} : setup cost and downtime cost at scheduled downs
c_i^{CM} : CM cost ($S^{USD} + c_i^{Rep}$) of component $i \in I$

5.2.2 Assumptions

1) The life time of each ABM component is independent of scheduled and unscheduled downs caused by other components in the system.
2) The degradation of each CBM component is independent of scheduled and unscheduled downs caused by other components in the system.
3) The time horizon is infinite
4) Maintenance actions restore components as new.
5) The system is composed of a large number of components.

5.3 Solution Approach

In this section, we first describe the solution procedure of the subproblem for a given τ. Next, we describe how the main problem is solved.

Consider the subproblem for a given τ. It is hard to obtain an exact and fast evaluation procedure for the cost function $Z_{syst}(\tau, \mathbf{C}, \mathbf{A})$, since the renewal theory can not be directly applied due to the fixed intervals for scheduled downs. And, even if we would have such an evaluation procedure, it is still relatively difficult to have an exact and efficient optimization algorithm to obtain the optimal control limits $\mathbf{C}$ and age limits $\mathbf{A}$, since the cost rate functions $Z_i(\tau, \mathbf{C}, \mathbf{A})$ are nonlinear and not convex. We therefore develop a heuristic procedure, based on approximate evaluations of the $Z_i(\tau, \mathbf{C}, \mathbf{A})$. In this heuristic procedure, we specify the control limits and age limits one by one in an iterative way. In each iteration, we take the view of an individual component $i \in I_{CBM} \cup I_{ABM}$ and consider scheduled downs with fixed intervals of length τ and unscheduled downs at certain random time points as opportunities for preventive maintenance. Notice that these unscheduled downs occur because of failures of all other components $j \in \{I \setminus i\}$. For each other component j, they occur according to a certain renewal process. The total process of unscheduled downs of all other components consists of the merge of multiple renewal processes, and therefore this total process will be close to a Poisson process when we have sufficiently many components. In any case, we approximate the total process as a Poisson process.

The first step of our heuristic procedure is the initialization step. In this step, we first consider each FBM component $i \in I_{FB}$ with its lifetime distribution denoted by $f_i(x)$. The component will generate unscheduled downs with rate $\lambda_i = 1/\mu_i$. Next, for each CBM component $i \in I_{CBM}$, we set the control limit C_i equal to H_i, in which case this CBM component also behaves as a FBM component. And we determine the rate λ_i with which unscheduled downs will be generated. Similarly, for each age-based component $i \in I_{ABM}$, we set the control limit A_i equal to ∞, in which case this ABM component also behaves as a FBM component. And we determine the rate λ_i with which unscheduled downs are generated.

After the initialization, we optimize each of the control limits C_i, $i \in I_{CBM}$, and the age limits A_i, $i \in I_{ABM}$, in an iterative way. Per iteration, we consider each of the components in $I_{CBM} \cup I_{ABM}$:

- Consider a component $i \in I_{CBM}$. This component sees scheduled downs with intervals of length τ and unscheduled downs with rate $\sum_{j \in \{I \setminus i\}} \lambda_j$ as opportunities for preventive maintenance actions. We want to choose C_i such that $Z_i(\tau, \mathbf{C}, \mathbf{A})$ is minimized. This problem for component i has been studied in Chapter 3. In that chapter, an accurate and efficient approximation procedure has been derived for the evaluation of $Z_i(\tau, \mathbf{C}, \mathbf{A})$, and the control limit C_i is optimized on the basis of this approximate function. We determine A_i and $Z_i(\tau, \mathbf{C}, \mathbf{A})$, and we also update the rate λ_i with which unscheduled downs are generated by component i.

- Consider a component $i \in I_{ABM}$. This component sees scheduled downs with intervals of length τ and unscheduled downs with rate $\sum_{j \in \{I \setminus i\}} \lambda_j$ as opportunities for preventive maintenance actions. We want to choose A_i such that $Z_i(\tau, \mathbf{C}, \mathbf{A})$ is minimized. This problem for component i has been studied in Chapter 4. In that chapter, an accurate and efficient approximation procedure has been derived for the evaluation of $Z_i(\tau, \mathbf{C}, \mathbf{A})$, and the age limit A_i is optimized on the basis of this approximate function. We determine A_i and $Z_i(\tau, \mathbf{C}, \mathbf{A})$, and we also update the rate λ_i with which unscheduled downs are generated by component i.

We continue with these iterations until λ_i converges for all components with the total cost $Z_{syst}(\tau, \mathbf{C}, \mathbf{A})$. Notice that the decision variable of component i (i.e., C_i or A_i) also converges when λ_i converges. Next, we also determine $Z_{syst}(\tau, \mathbf{C}, \mathbf{A})$.

We have no guarantee for the convergence of the above procedure, but it is likely that the convergence is obtained in general. The procedure leads to a heuristic solution $(\tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ and an approximated $Z_{syst}(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ for the function $Z_{syst}(\tau)$. The heuristic algorithm is summarized in Subsection 5.A.1.

The main problem (P) is solved on the basis of the approximation $Z_{syst}(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$. Here, we simply apply enumeration to minimize $Z_{syst}(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ over τ.

5.4 Numerical Study

To demonstrate the usage of the model, we start with a simple example: a system consisting of 5 condition-based components ($I_{CBM} = \{1, 2, 3, 4, 5\}$) and 5 age-based components ($I_{ABM} = \{6, 7, 8, 9, 10\}$) and 40 failure-based components ($I_{FBM} = \{11, 12, ...49, 50\}$). To show our model is able to deal with systems consisting of both identical and non-identical components, we consider the case where i) component $i \in I_{ABM}$ and $i \in I_{CBM}$ are non-identical. ii) components $i \in I_{FBM}$ consisting of 4 identical subgroups $I_{FBM_1} = \{11, .., 20\}$, $I_{FBM_2} = \{21, .., 30\}$, $I_{FBM_3} = \{31, .., 40\}$ and $I_{FBM_4} = \{41, .., 50\}$. Each of these identical subgroups consists of 10 non-identical components with their expected life time $\{\mu_{11}, ..., \mu_{20}\}$=$\{\mu_{21}, ..., \mu_{30}\}$=$\{\mu_{31}, ..., \mu_{40}\}$=$\{\mu_{41}, ..., \mu_{50}\}$ = {1.33, 1.47, 1.60, 1.73, 1.87, 2.00, 2.13, 2.27, 2.40, 2.53 } in years and their corrective maintenance cost $\{c_{11}^{CM}, ..., c_{20}^{CM}\}$ =$\{c_{21}^{CM}, ..., c_{30}^{CM}\}$ = $\{c_{31}^{CM}, ..., c_{40}^{CM}\}$ = $\{c_{41}^{CM}, ..., c_{50}^{CM}\}$ = { 6.00, 6.10, 6.20, 6.30, 6.40, 6.50, 6.60, 6.70, 6.80, 6.90} in thousand Euro. Notice that we can aggregate the unscheduled down rate λ_i generated by each FBM component

$i \in I_{FBM}$. This is because each FBM component is modeled by a renewal process and we can merge these 40 renewal processes of 40 FBM components into one aggregated FBM component with an exponentially distributed lifetime and its failure rate $\sum_{i \in I_{FBM}} \lambda_i = 21.56$ (also see Palm-Khintchine theorem [46]). Similarly, the average cost rates of these 40 FBM components can be also aggregated, $\sum_{i \in I_{FBM}} Z_i = 137.82$ thousand Euro per year.
For component $i \in I_{CBM}$, we use the random coefficient model mentioned in Section 3.4 of Chapter 3 to model its degradation path. The slope of the degradation path is a Weibull distribution with its scale and shape parameter α_i^{CBM} and β_i^{CBM}, and the constant parameters $\{\phi_{1_i}, \phi_{2_i}\} = \{0, 1\}, \forall i \in I_{CBM}$. For component $i \in I_{ABM}$, we assume the lifetime distribution is a Weibull distribution with its scale and shape parameter α_i^{ABM} and β_i^{ABM}, which is also described in Section 4.4 of Chapter 4. Notice that the CM cost $c_i^{CM} = c_i^{Rep} + S^{USD}$ must be higher than the PM cost (c_i^{PM-USD} or c_i^{PM-SD}), since PM uses unscheduled or scheduled downs as opportunities to save downtime and setup costs. For $i \in I_{FBM}$, the expected lifetime μ_i is needed for cost rate evaluation. At the system level, the fixed cost of scheduled down S^{SD} is smaller than the fixed cost of unscheduled down S^{USD}. The input parameters at the component level and the system level are given in Table 5.1. To solve this maintenance optimization problem, we use the approach proposed in Subsection 5.3.

5.4.1 Optimal maintenance policy

By solving the main problem (see Subsection 5.A.1), the heuristic solution of our maintenance policy is found and shown in Table 5.2. This solution suggests to set up scheduled maintenance actions every 0.5 year. To take scheduled or unscheduled downs as opportunities for joint maintenance, the control limits on the physical condition of the CBM components, $\{\tilde{C}_i^*(\tau^*), \forall i \in I_{CBM}\}$, are $\{81.1\%, 78.5\%, 73.2\%, 73.4\%, 75.3\%\}$; and the age limits of ABM components, $\{\tilde{A}_i^*(\tau^*), \forall i \in I_{ABM}\}$, are $\{0.5, 0.5, 0.5, 0.5, 0.5\}$ year, which is a multiple of τ^* in this case. The minimum maintenance cost rate of the system $Z_{syst}(\tau^*)$ is 187.88 thousand Euro per year. Notice that the control limits are presented as percentages of the failure thresholds H_i (see Section 3.4 and Chapter 3). More details of the heuristic solutions at the component level can be found in Table 5.7 in Subsection 5.A.3.

Moreover, we evaluate the gap between the average cost rate of the heuristics $Z_{syst}(\tau)$ and the average cost rate of the simulated results $\hat{Z}_{syst}(\tau)$. In the simulation, we use the control limits $\{\tilde{C}_i^*(\tau^*), \forall i \in I_{CBM}\}$ and age limits $\{\tilde{A}_i^*(\tau^*), \forall i \in I_{ABM}\}$ obtained via the heuristic procedure under various τ values to evaluate the average cost rate $\hat{Z}_{syst}(\tau)$. Figure 5.2 shows $Z_{syst}(\tau)$ and $\hat{Z}_{syst}(\tau)$ over various τ values, where we can observe a small gap between them. Especially, the gap at the optimal solution

Table 5.1 The parameter setting

Parameters	Explanation	Value for $i = \{1, .., 5\}$ I_{ABM}	Value for $i = \{6, .., 10\}$ I_{CBM}
c_i^{PM-USD}	PM cost at unscheduled down [thousand Euro]	$2.00, 2.10, 2.20, 2.30, 2.40$	$2.50, 2.60, 2.70, 2.80, 2.90$
c_i^{PM-SD}	PM cost at scheduled down [thousand Euro]	$1.00, 1.05, 1.10, 1.15, 1.20$	$1.25, 1.30, 1.35, 1.40, 1.45$
c_i^{Rep}	Replacement cost (in a CM) [thousand Euro]	$5.00, 5.50, 6.00, 6.50, 7.00$	$7.50, 8.00, 8.50, 9.00, 9.50$
α_i^{ABM}	Scale parameter of Weibull distribution for component i's lifetime ($i \in I_{ABM}$)	$1.13, 1.24, 1.35, 1.46, 1.58$	-
β_i^{ABM}	Shape parameter of Weibull distribution for component i's lifetime ($i \in I_{ABM}$)	$2.10, 2.31, 2.52, 2.73, 2.94$	-
α_i^{CBM}	Scale parameter of Weibull distribution for component i's degradation rate ($i \in I_{CBM}$)	-	$1.13, 1.24, 1.35, 1.46, 1.58$
β_i^{CBM}	Shape parameter of Weibull distribution for component i's degradation rate ($i \in I_{CBM}$)	-	$6.01, 6.61, 7.21, 7.81, 8.41$
H_i	Replacement threshold (for condition-based components)	-	$H_i = 100\%$
S^{USD}	Unscheduled setup cost (in a CM) [thousand Euro]	5	5
$\sum_{i \in I_{FBM}} \lambda_i$	the unscheduled down rate generated by 40 FBM components [per year]	21.56	21.56
S^{SD}	Scheduled setup cost [thousand Euro]	3	3

Table 5.2 The heuristic solution of our maintenance policy, where the average cost rate of the system is minimized. The solution contains: 1) the minimum average cost rate [thousand Euro / year] of the system Z_{syst} and the component Z_i, 2) optimal maintenance interval of the system [year] τ^* and 3) optimal control limits $\tilde{\mathbf{C}}$ for CBM components and age limits $\tilde{\mathbf{A}}$ for ABM components

Policy	Values
$Z_{syst}(\tau^*)$	187.88
$\{Z_i(\tau^*, \tilde{\mathbf{C}}(\tau^*), \tilde{\mathbf{A}}(\tau^*)), \forall i \in I_{CBM}\}$	$\{3.13, 3.75, 4.25, 4.61, 4.97\}$
$\{Z_i(\tau^*, \tilde{\mathbf{C}}(\tau^*), \tilde{\mathbf{A}}(\tau^*)), \forall i \in I_{ABM}\}$	$\{6.29, 5.24, 4.45, 3.85, 3.50\}$
$\sum_{i \in I_{FBM}} Z_i(\tau^*, \tilde{\mathbf{C}}(\tau^*), \tilde{\mathbf{A}}(\tau^*))$	137.82
τ^*	0.5
$\tilde{\mathbf{C}}(\tau^*) = \{\tilde{C}_i^*(\tau^*), \forall i \in I_{CBM}\}$	$\{81.1\%, 78.5\%, 73.2\%, 73.4\%, 75.3\%\}$
$\tilde{\mathbf{A}}(\tau^*) = \{A_i^*(\tau^*), \forall i \in I_{ABM}\}$	$\{0.5, 0.5, 0.5, 0.5, 0.5\}$

$\tau^* = 0.5$ (see Table 5.2) is only 0.14% $((\hat{Z}_{syst}(\tau^*) - Z_{syst}(\tau^*))/\hat{Z}_{syst}(\tau^*)$=(188.15 − 187.88)/188.15 = 0.14%). Also, we observe small confidence intervals of the simulated results.

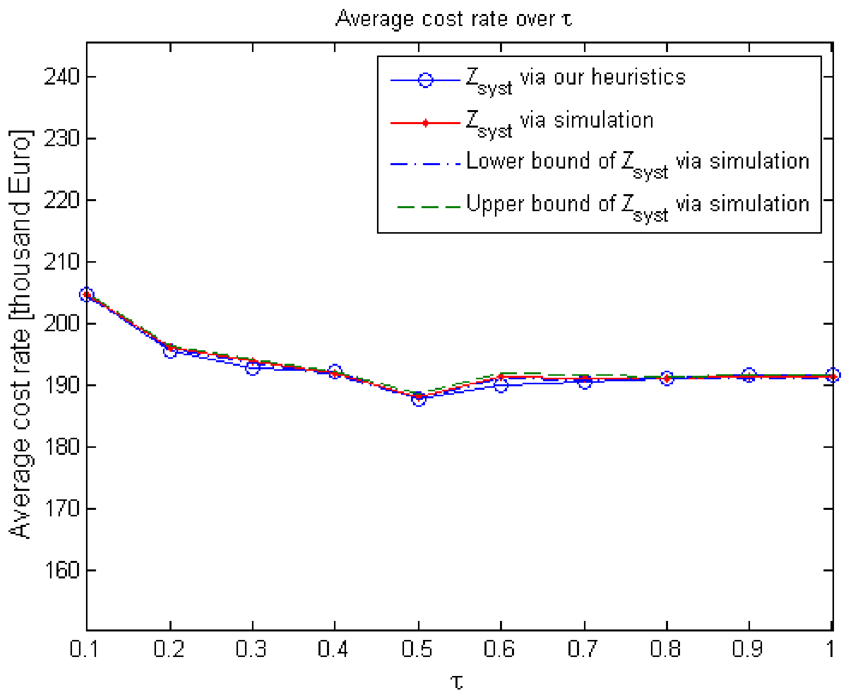


Figure 5.2 Average cost rate [thousand Euro per year] at the system level over τ [year]. The cost rate obtained via heuristics ($Z_{syst}(\tau)$) is compared with the simulated result ($\hat{Z}_{syst}(\tau)$) in a confidence interval with a lower and upper bound

The cost rate shown in Figure 5.2 is intuitively sensible. On one hand, when τ is small, the frequency of scheduled downs at the system level is high. This leads to a high cost rate for scheduled downs in $Z_{syst}(\tau)$. On the other hand, when τ is large, the opportunities of scheduled down become less, and the probabilities of PM-USD and CM actions with higher cost per action ($c_i^{PM-SD} \leq c_i^{PM-USD} < c_i^{Rep} + S^{USD}$) become larger, which makes the expected maintenance cost per cycle more expensive. This also results in a higher $Z_{syst}(\tau)$. The control limits and age limits at the component level also vary with different τ values, which is shown in Table 5.6 and elaborated in Subsection 5.A.2.

5.4.2 Convergence in the heuristic algorithm

Table 5.3 The convergence of $\lambda_{i,k}$, $\{C^*_{i,k}, \forall i \in I_{CBM}\}$ and $\{A^*_{i,k}, \forall i \in I_{ABM}\}$ at kth iteration, given $\tau^* = 0.5$

	Iteration $k =$	1	2	3	4
$i \in I_{CBM}$	$\lambda_{i,k}$				
1		1.0000	0.0135	0.0196	0.0195
2		1.1146	0.0166	0.0240	0.0240
3		1.2290	0.0199	0.0288	0.0287
4		1.3431	0.0237	0.0343	0.0342
5		1.4571	0.0281	0.0406	0.0406
$i \in I_{ABM}$	$\lambda_{i,k}$				
6		1.0000	0.3739	0.3828	0.3827
7		0.9088	0.3192	0.3267	0.3267
8		0.8317	0.2248	0.2311	0.2311
9		0.7658	0.1978	0.2032	0.2032
10		0.7091	0.1756	0.1432	0.1431
$i \in I_{CBM}$	$C^*_{i,k}$				
1		100.0%	84.8%	81.1%	81.1%
2		100.0%	83.2%	78.5%	78.5%
3		100.0%	80.7%	73.2%	73.2%
4		100.0%	77.9%	73.4%	73.4%
5		100.0%	78.7%	75.3%	75.3%
$i \in I_{ABM}$	$A^*_{i,k}$				
6		∞	0.500	0.500	0.500
7		∞	0.500	0.500	0.500
8		∞	0.500	0.500	0.500
9		∞	0.500	0.500	0.500
10		∞	0.500	0.500	0.500

According to Section 5.3, the control limits $\left\{\tilde{C}^*_i(\tau), \forall i \in I_{CBM}\right\}$ and age limits $\left\{\tilde{A}^*_i(\tau), \forall i \in I_{ABM}\right\}$ obtained via the heuristic procedure under a given τ can be obtained through our iteration algorithm. The convergence of the unscheduled down rate generated by component i, λ_i, can be observed. To show the details of the convergence, we take the solution in Table 5.2 as an example, given $\tau^* = 0.5$. Notice that the rate of unscheduled downs λ_i of component i at the kth iteration is denoted by $\lambda_{i,k}$. Given $\tau = 0.5$, we observe in Table 5.3 that $\lambda_{i,k}$ converges quickly after 4 iterations for all components in $\{I_{CBM}\}$ and $\{I_{ABM}\}$. Moreover, we also observe that the control limits $\{C^*_{i,k}, \forall i \in I_{CBM}\}$ and the age limits $\{A^*_{i,k}, \forall i \in I_{ABM}\}$ at kth iteration converge when $\lambda_{i,k}$ converges. This matches our expectation explained in Section 5.3. Also notice that $C^*_{i,4} = \tilde{C}^*_i(\tau^*), \forall i \in I_{CBM}$ and $A^*_{i,4} = \tilde{A}^*_i(\tau^*), \forall i \in I_{CBM}$; which are the same results in Table 5.2.

5.4.3 Computational experiment 1

As mentioned previously, a smaller τ leads to more opportunities from scheduled downs to take preventive maintenance, which is cheaper at the component level because $c_i^{PM-SD} < c_i^{PM-USD} < c_i^{CM}$. However, on the other hand, a smaller τ leads to a higher cost rate of scheduled down S^{SD}/τ at the system level. To investigate the changes of the optimal policy under this tradeoff, we set the cost of scheduled down S^{SD} at different levels (i.e., $\{0.1, 3, 6\}$), and plot the average cost rates of the system over various τ values in Figure 5.3. The results match our expectations. When S^{SD} is relatively small ($S^{SD} = 0.1$), S^{SD}/τ has less impact on the total cost rate, so that a smaller τ^* is preferable. In contrast, when S^{SD} is relatively large ($S^{SD} = 6$), S^{SD}/τ has larger impacts on the total cost rate, so that a larger τ^* is preferable.

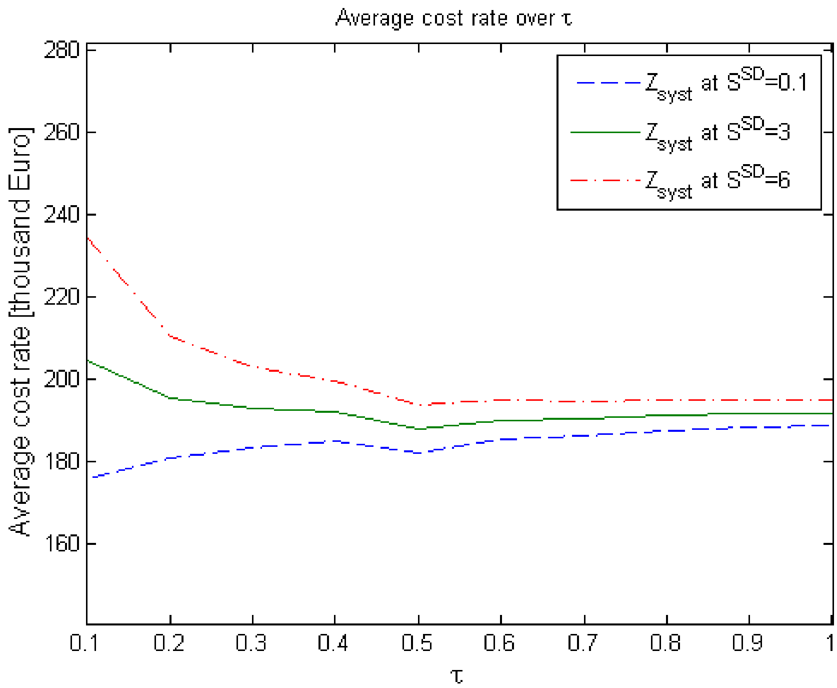


Figure 5.3 Average cost rate [thousand Euro per year] at the system level over τ [year]

5.4.4 Computational experiment 2

Instead of optimizing the decision variables at the component level (i.e., $C_i(\tau)$ and $A_i(\tau)$) simultaneously with the decision variable τ at the system level, which leads to a dramatic increment of the solution space of decision variables while the amount of components in a system is large; our approach in Section 5.3 decomposes the system optimization problem into the subproblems of individual components. Namely, we i) optimize $C_i(\tau)$ and $A_i(\tau)$ for each component under a given τ and then ii) optimize τ for the system. The motivation of such a decomposition approach is to reduce the computation time of large-scale problems.

To see the performance of our solution approach in terms of computation time, we run our model for several systems consisting of different amounts of FBM, ABM and CBM components. This experiment is performed under the same computing environment as before (by a computer with a 2.5 GHz processor and 4 G RAM). In Table 5.4, the computation time grows with an increasing rate with respect to the total number of components in a system. The computation times are in the magnitude of hours, which shows that our model can be applied to optimize the maintenance policy of a real life system that has a large number of components.

Table 5.4 The computation time of different systems

System	$\lvert I_{FBM}\rvert$	$\lvert I_{ABM}\rvert$	$\lvert I_{CBM}\rvert$	Computation Time [hours]
1)	40	5	5	1.908
2)	80	10	10	4.638
3)	120	15	15	10.056
4)	160	20	20	22.011

5.4.5 Computational experiment 3

As a trend observed in practice, CBM policies are implemented for more and more components in complex systems, which gradually replaces ABM policies. To know if it is economically attractive to do so, we show the cost difference between using our CBM policy and our ABM policy. In this case, we consider two scenarios based on the original system consisting of 5 ABM, 5 CBM and 40 FBM components (see Subsection 5.4.2), with the input parameter settings in Table 5.1.

- Scenario 1: By using the CBM policy for the 5 ABM components of the original system; the system in this scenario consists of no ABM, 10 CBM and 40 FBM components. Suppose the underlying degradation processes of these 5 ABM components are the same random coefficient model of the CBM components in the original system (see Fitting Option 1 in Section 3.4). According to the parameter fitting procedure explained in [66], we can convert the ABM components with their life time distributions into 5 CBM components with the underlying degradation processes, so that the CBM policy can be used for the ABM components.

- Scenario 2: By using the ABM policy for the 5 CBM components of the original system; the system in this scenario consists of 10 ABM, no CBM and 40 FBM components. We use the first passage time of H for CBM components described in Equation (3.5) as their life time distributions in the ABM policy.

Notice that the optimization problem at the system level remains unchanged and the approach proposed in this chapter can be used directly. Similar to Table 5.2, the heuristic solutions for both scenarios are shown in Table 5.5.

Table 5.5 The heuristic solution of our maintenance policy, where the average cost rate of the system is minimized. The solution contains: 1) the minimum average cost rate [thousand Euro / year] of the system Z_{syst} and the component Z_i, 2) optimal maintenance interval of the system [year] τ^* and 3) optimal control limits $\tilde{\mathbf{C}}$ for CBM components and age limits $\tilde{\mathbf{A}}$ for ABM components

Policy	Scenario 1	Scenario 2
$Z_{syst}(\tau^*)$	169.24	193.50
For $i \in I_{CBM}$, $\{Z_i(\tau^*, \tilde{\mathbf{C}}(\tau^*), \tilde{\mathbf{A}}(\tau^*))\}$	$\{1.25, 1.27, 1.29, 1.27, 1.29$ $3.12, 3.51, 3.89, 4.39, 5.15\}$	-
For $i \in I_{ABM}$, $\{Z_i(\tau^*, \tilde{\mathbf{C}}(\tau^*), \tilde{\mathbf{A}}(\tau^*))\}$	-	$\{6.26, 5.27, 4.44, 3.91, 3.53,$ $3.56, 3.94, 4.66, 5.96, 8.15\}$
For $\sum_{i \in I_{FBM}} Z_i$	137.82	137.82
τ^*	0.6	0.5
$\tilde{\mathbf{C}}(\tau^*)$	$\{82.7\%, 83.9\%, 85.1\%, 86.2\%, 87.0\%,$ $77.7\%, 76.8\%, 78.0\%, 79.3\%, 79.4\%\ \}$	-
$\tilde{\mathbf{A}}(\tau^*)$	-	$\{0.5, 0.5, 0.5, 0.5, 0.5,$ $0.5, 0.5, 0.5, 0.5, 0.5\}$

By comparing these two scenarios in Table 5.5, we observe that the minimum average cost rate of the system Z_{syst} is 169.24 thousand Euro per year in Scenario 1 and

193.50 thousand Euro per year in Scenario 2. For the original system consisting of 5 ABM, 5 CBM and 40 FBM components, $Z_{syst}(\tau^*) = 187.88$ is between Z_{syst} in Scenario 1 and Scenario 2. Notice that the cost rate of the 40 FBM components remains unchanged in both scenarios, which is the same as in Table 5.2. Hence, the cost difference is caused by 1) the interval of setup τ^* and 2) the cost rate of ABM and CBM components Z_i, $i = \{1, 2, ..., 10\}$. As shown in Table 5.5, τ^* is 0.6 year in Scenario 1 and 0.5 year in Scenario 2, which implies that the frequency of setup and the setup cost rate are lower in Scenario 1 than Scenario 2. The sum of $\sum_{i \in I_{CBM} \cup I_{ABM}} Z_i$ is 26.43 and 49.69 thousand Euro per year in Scenario 1 and 2 respectively, which results in $(49.69 - 26.43)/49.69 = 46.8\%$ cost reduction. The main cause of this cost different is that i) the expected maintenance cycle lengths of components are much shorter and ii) more CM actions occurs at the end of renewal cycles in Scenario 2 than in Scenario 1. Hence, economically speaking, it is more efficient to use the CBM policy than the ABM policy. More details of the heuristic solutions at the component level in both scenarios can be found in Tables 5.8 and 5.9 in Subsection 5.A.3.

5.5 Conclusions

For a multi-component system with a mix of CBM, ABM, and FBM components, we have proposed a class of opportunistic maintenance policies in Chapter 3 and 4 using both scheduled and unscheduled downs. To solve the large-scale optimization problem, we integrated these policies and developed an efficient heuristic procedure to determine the interval length for the scheduled downs, the control limits of the CBM components, and the age limits of ABM components. Via the numerical example of a system consisting of 40 FBM, 5 CBM and 5 ABM components, we show that this heuristic can be applied to optimize the maintenance policy of a real life system that has a large number of components (e.g., semiconductor production systems, food production lines, trucks), within an acceptable computation time. Finally, we find small gaps by comparing the simulated average cost rates of using the same heuristic solution with the cost rates obtained via our heuristic.

5.A Appendices:

5.A.1 Heuristic algorithm

Our heuristic algorithm is explained below. Notice that we use k as the index of iteration number. For example, $C_{i,k}$ is the control limit of component i at kth iteration.

Step 1 Evaluate the cost rate for main problem (P) $Z_{syst}(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ under a given τ:

Step 1.1 Evaluate the cost rate of each subproblem (SP) $Z_i(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ under a given τ

Step 1.1.1 Initiation $k = 1$: Set $C_{i,k} = H_{i,k}$, $\forall i \in I_{CBM}$ and $A_i = \infty$, $\forall i \in I_{ABM}$ and calculate the $\lambda_{i,k} = 1/L_{i,k}$ of all component $i \in I$ and the initial rate of the entire system at $k = 1$ is $\sum_{i \in I} \lambda_{i,k}$, where $L_{i,k}$ the expected cycle length of component i at kth iteration. For $i \in I_{CBM}$, $L_{i,k}(\tau, C_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k})$ is derived in Equation (3.3) of Section 3.3.1. For $i \in I_{ABM}$, $L_{i,k}(\tau, A_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k})$ is derived in Equation (4.15) of Section 4.3.

Step 1.1.2 Iteration $k = k + 1$:

- For $i \in I_{CBM}$: By using the rate $\sum_{j \in \{I \backslash i\}} \lambda_{j,k}$, we find optimal control limit $C^*_{i,k}$, $\forall i \in I_{CBM}$ and update

$$\lambda_{i,k} = P^3_{i,k}(\tau, C^*_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k})/L_{i,k}(\tau, C^*_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k}),$$

  where $P^3_{i,k}(\tau, C^*_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k})$ is the probability of CM (see Equation (3.2) in Section 3.3.1).

- For $i \in I_{ABM}$: By using the rate $\sum_{j \in \{I \backslash i\}} \lambda_{j,k}$, we find optimal age limit $A^*_{i,k}$, $\forall i \in I_{ABM}$and update

$$\lambda_{i,k} = P^3_{i,k}(\tau, A^*_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k})/L_{i,k}(\tau, A^*_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k}),$$

  where $P^3_{i,k}(\tau, A^*_{i,k}, \sum_{j \in \{I \backslash i\}} \lambda_{j,k})$ is the probability of CM (see Equation (4.14) in Section 4.3).

Step 1.1.3 Stop iteration:

Keep iterating **Step 1.1.2** with the updated $\lambda_{i,k}$ and stop at iteration $k = n$, where $|\lambda_{i,n} - \lambda_{i,n-1}| < \varepsilon$ for all components. In this case,

$\tilde{\mathbf{C}}(\tau) = \{\tilde{C}^*_{i,n},\ \forall i \in I_{CBM}\}$ and $\tilde{\mathbf{A}}(\tau) = \{\tilde{A}^*_{i,n},\ \forall i \in I_{ABM}\}$ also converge, where $Z_i(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ is minimized for this given τ.

Step 1.2 Obtain the minimum cost rate of each subproblem (SP) $Z_i(\tau, \tilde{\mathbf{C}}(\tau), \tilde{\mathbf{A}}(\tau))$ under a given τ

Step 2 Optimize τ with respect to $Z_{syst}(\tau, \mathbf{C}, \mathbf{A})$. Given the optimal interval τ^*, $\left\{\tilde{C}^*_i(\tau^*), \forall i \in I_{CBM}\right\}$ and $\left\{\tilde{A}^*_i(\tau^*), \forall i \in I_{ABM}\right\}$ are obtained in **Step 1.2**

5.A.2 Control limits and age limits at various τ values

The control limits $\tilde{C}_i^*(\tau)$ and age limits $\tilde{A}_i^*(\tau)$ obtained via our heuristic procedure at the component level for various τ values are shown in Table 5.6. Generally speaking, we observe that the control limit $\tilde{C}_i^*(\tau)$ is higher at smaller τ (e.g., $\{\tilde{C}_i^*(0.025); \forall i \in I_{CBM}\} = \{96.2\%, 96.1\%, 95.8\%, 95.6\%, 95.2\%\}$) and is lower at larger τ (e.g., $\{\tilde{C}_i^*(1.00); \forall i \in I_{CBM}\} = \{81.2\%, 79.4\%, 77.7\%, 76.2\%, 74.9\%\}$). This general trend is sensible because a smaller τ leads to more opportunities of scheduled downs at the system level, which allows a higher $\tilde{C}_i^*(\tau)$. However, this trend does not hold over all τ values, because the approximated evaluation takes only the renewal cycles that start with scheduled downs into account, which leads to small approximation errors in evaluating the objective functions.

Unlike $\tilde{C}_i^*(\tau)$, when τ is relatively small (e.g., $\tau \leq 0.65$ in Table 5.6), $\tilde{A}_i^*(\tau)$ is a multiple of τ (i.e., $\tilde{A}_i^*(\tau) = k\tau$, $k \in \mathbb{N}$) to have a higher probability of taking PM-SD actions, which are cheaper than other maintenance actions. This is also explained in Section 4.4 of Chapter 4. When τ is relatively large (e.g., $\tau > 0.65$ in Table 5.6), $\tilde{A}_i^*(\tau)$ is not always a multiple of τ. The reason is that a large τ can lead to a low frequency of opportunities from scheduled downs. To have more opportunities for PM actions (i.e., PM-SD and PM-USD) at the end of a renewal cycle, opportunities from unscheduled downs should be taken more often by setting up an age limit $\tilde{A}_i^*(\tau)$ smaller than τ.

τ	$\{\tilde{A}_1^*(\tau), \tilde{A}_2^*(\tau), \tilde{A}_3^*(\tau), \tilde{A}_4^*(\tau), \tilde{A}_5^*(\tau), \tilde{C}_1^*(\tau), \tilde{C}_2^*(\tau), \tilde{C}_3^*(\tau), \tilde{C}_4^*(\tau), \tilde{C}_5^*(\tau)\}$
0.025	{0.375, 0.425, 0.500, 0.500, 0.625, 96.2%, 96.1%, 95.8%, 95.6%, 95.2%}
0.050	{0.400, 0.500, 0.500, 0.500, 0.500, 93.0%, 92.3%, 92.3%, 91.6%, 90.9%}
0.075	{0.375, 0.375, 0.375, 0.525, 0.525, 90.0%, 88.8%, 88.8%, 87.5%, 87.4%}
0.100	{0.400, 0.500, 0.500, 0.500, 0.500, 87.5%, 86.6%, 85.7%, 84.7%, 83.3%}
0.125	{0.375, 0.375, 0.500, 0.500, 0.625, 85.7%, 84.2%, 83.3%, 81.1%, 79.9%}
0.150	{0.450, 0.450, 0.450, 0.750, 0.750, 84.2%, 83.3%, 82.0%, 81.6%, 76.8%}
0.175	{0.525, 0.525, 0.525, 0.525, 0.525, 83.4%, 82.8%, 79.9%, 78.3%, 79.0%}
0.200	{0.400, 0.400, 0.400, 0.400, 0.800, 82.8%, 80.5%, 80.3%, 80.8%, 80.4%}
0.225	{0.450, 0.450, 0.450, 0.675, 0.675, 82.1%, 81.8%, 81.6%, 78.7%, 72.2%}
0.250	{0.250, 0.500, 0.500, 0.500, 0.500, 82.8%, 81.9%, 77.4%, 74.6%, 75.8%}
0.275	{0.550, 0.550, 0.550, 0.550, 0.550, 82.7%, 79.2%, 76.9%, 77.8%, 79.2%}
0.300	{0.300, 0.300, 0.600, 0.600, 0.600, 80.8%, 78.5%, 79.0%, 80.0%, 79.9%}
0.325	{0.325, 0.650, 0.650, 0.650, 0.650, 80.4%, 80.2%, 80.8%, 80.5%, 78.5%}
0.350	{0.350, 0.700, 0.700, 0.700, 0.700, 81.3%, 81.5%, 81.1%, 79.4%, 76.8%}
0.375	{0.375, 0.375, 0.375, 0.375, 0.750, 82.0%, 81.6%, 80.1%, 77.7%, 75.3%}
0.400	{0.400, 0.400, 0.400, 0.400, 0.800, 82.4%, 81.2%, 79.1%, 76.8%, 65.7%}
0.425	{0.425, 0.425, 0.425, 0.425, 0.425, 82.3%, 80.5%, 78.3%, 74.6%, 68.0%}
0.450	{0.450, 0.450, 0.450, 0.450, 0.900, 82.0%, 80.0%, 77.5%, 70.0%, 70.6%}
0.475	{0.475, 0.475, 0.475, 0.475, 0.475, 81.5%, 79.3%, 74.7%, 71.4%, 73.0%}
0.500	{0.500, 0.500, 0.500, 0.500, 0.500, 81.1%, 78.5%, 73.2%, 73.4%, 75.3%}
0.525	{0.525, 0.525, 0.525, 0.525, 0.525, 80.6%, 76.9%, 74.2%, 75.4%, 77.3%}
0.550	{0.550, 0.550, 0.550, 0.550, 0.550, 80.0%, 76.1%, 75.7%, 77.2%, 78.9%}
0.575	{0.575, 0.575, 0.575, 0.575, 0.575, 79.2%, 76.6%, 77.2%, 78.7%, 79.8%}
0.600	{0.600, 0.600, 0.600, 0.600, 0.600, 78.7%, 77.6%, 78.6%, 79.8%, 79.9%}
0.625	{0.625, 0.625, 0.625, 0.625, 0.625, 78.8%, 78.7%, 79.8%, 80.4%, 79.4%}
0.650	{0.650, 0.650, 0.650, 0.650, 0.650, 79.3%, 79.7%, 80.6%, 80.4%, 78.5%}
0.675	{0.525, 0.675, 0.675, 0.675, 0.675, 80.0%, 80.6%, 80.9%, 80.0%, 77.6%}
0.700	{0.500, 0.700, 0.700, 0.700, 0.700, 80.7%, 81.2%, 81.0%, 79.3%, 76.9%}
0.725	{0.525, 0.550, 0.725, 0.725, 0.725, 81.3%, 81.5%, 80.7%, 78.6%, 76.3%}
0.750	{0.500, 0.575, 0.750, 0.750, 0.750, 81.8%, 81.7%, 80.3%, 78.1%, 75.9%}
0.775	{0.525, 0.550, 0.625, 0.775, 0.775, 82.2%, 81.6%, 79.8%, 77.6%, 75.6%}
0.800	{0.525, 0.575, 0.625, 0.800, 0.800, 82.4%, 81.4%, 79.4%, 77.2%, 75.4%}
0.825	{0.500, 0.550, 0.625, 0.825, 0.825, 82.5%, 81.1%, 79.0%, 77.0%, 75.2%}
0.850	{0.525, 0.575, 0.600, 0.700, 0.850, 82.4%, 80.7%, 78.6%, 76.7%, 75.1%}
0.875	{0.500, 0.575, 0.625, 0.675, 0.875, 82.3%, 80.4%, 78.4%, 76.6%, 75.1%}
0.900	{0.500, 0.550, 0.600, 0.700, 0.900, 82.1%, 80.1%, 78.2%, 76.5%, 75.0%}
0.925	{0.525, 0.575, 0.625, 0.700, 0.750, 81.8%, 79.9%, 78.0%, 76.4%, 75.0%}
0.950	{0.525, 0.575, 0.625, 0.675, 0.725, 81.6%, 79.7%, 77.9%, 76.3%, 74.9%}
0.975	{0.500, 0.550, 0.600, 0.700, 0.750, 81.4%, 79.5%, 77.8%, 76.3%, 74.9%}
1.000	{0.525, 0.575, 0.625, 0.675, 0.725, 81.2%, 79.4%, 77.7%, 76.2%, 74.9%}

Table 5.6 The control limits and age limits obtained via our heuristic procedure, $\{\tilde{C}_i^*(\tau), \forall i \in I_{CBM}\}$ and $\{\tilde{A}_i^*(\tau), \forall i \in I_{ABM}\}$ under various τ values

5.A.3 Detail of the solution obtained via our heuristic procedure

The solution obtained via our heuristic procedure under a given τ^* is shown in Table 5.2 in Subsection 5.4.2. In this subsection, we show the details of the solution at the component level in Table 5.7. This system consists of 5 CBM, 5 ABM and 40 FBM components. For each component, Table 5.7 include i) the probabilities of having PM-USD, PM-SD and CM denoted by P_1, P_2 and P_3; and ii) the expected cycle length denoted by L. For each CBM component, P_1, P_2, P_3 and L are functions of the control limit C (see Equations (3.2) and (3.3) in Chapter 3). For each ABM component, P_1, P_2, P_3 and L are functions of the age limit A (see Equations (4.14) and (4.15) in Chapter 4).

	P_1	P_2	P_3	L
$i \in I_{CBM}$				
1	0.938	0.045	0.017	0.852
2	0.957	0.027	0.016	0.747
3	0.884	0.109	0.006	0.635
4	0.806	0.188	0.006	0.583
5	0.762	0.230	0.008	0.551
$i \in I_{ABM}$				
6	0.457	0.361	0.182	0.492
7	0.348	0.527	0.125	0.497
8	0.256	0.661	0.083	0.500
9	0.167	0.780	0.054	0.500
10	0.125	0.840	0.035	0.501

Table 5.7 The detail of the solution obtained via our heuristic procedure in Table 5.2, given $\{\tilde{C}_i^*(\tau^*), \forall i \in I_{CBM}\}$ and $\{\tilde{A}_i^*(\tau^*), \forall i \in I_{ABM}\}$. This system consists of 5 CBM, 5 ABM and 40 FBM components

	P_1	P_2	P_3	L
$i \in I_{CBM}$				
1	0.922	0.069	0.009	0.820
2	0.853	0.140	0.007	0.729
3	0.807	0.184	0.009	0.672
4	0.791	0.196	0.013	0.627
5	0.810	0.168	0.021	0.583
6	0.909	0.077	0.014	1.624
7	0.916	0.072	0.013	1.678
8	0.917	0.071	0.012	1.751
9	0.914	0.075	0.011	1.834
10	0.913	0.078	0.009	1.922

Table 5.8 The detail of the solution obtained via our heuristic procedure in Table 5.5, given $\{\tilde{C}_i^*(\tau^*), \forall i \in I_{CBM}\}$ and $\{\tilde{A}_i^*(\tau^*), \forall i \in I_{ABM}\}$. This system consists of 10 CBM, 0 ABM and 40 FBM components

	P_1	P_2	P_3	L
$i \in I_{ABM}$				
1	0.093	0.873	0.034	0.502
2	0.076	0.879	0.045	0.500
3	0.099	0.836	0.064	0.501
4	0.159	0.738	0.101	0.502
5	0.223	0.610	0.167	0.502
6	0.447	0.372	0.181	0.491
7	0.360	0.516	0.125	0.497
8	0.249	0.669	0.083	0.499
9	0.193	0.753	0.054	0.501
10	0.137	0.829	0.035	0.501

Table 5.9 The detail of the solution obtained via our heuristic procedure in Table 5.5, given $\{\tilde{C}_i^*(\tau^*), \forall i \in I_{CBM}\}$ and $\{\tilde{A}_i^*(\tau^*), \forall i \in I_{ABM}\}$. This system consists of 0 CBM, 10 ABM and 40 FBM components

Similarly, we show the details of the heuristic solution of Table 5.5 in Subsection 5.4.5. In Scenario 1, the system consists of 10 CBM, 0 ABM and 40 FBM components. In Scenario 2, the system consists of 0 CBM and 10 ABM and 40 FBM components. For each component, P_1, P_2, P_3 and L are shown in Table 5.8 and 5.9.

5.A.4 Simulation procedures

To compare with the heuristic results of our model, we run a simulation to calculate the true cost rate of the system $\hat{Z}_{syst}(\tau)$ for given τ values. There are m different seeds in the simulation. In each seed $j \in \{1, 2, ..., m\}$, we simulate $k_i \in \mathbb{N}$ maintenance cycles of component $i \in I_{CBM} \cup I_{ABM}$. Regarding the opportunities for preventive maintenance, each seed j consists of: 1) a Poisson process with random arrival time points $\Omega_j^{usd} = \{a_1, a_2, ..., a_x\} \in \Re_+^x, x \in \mathbb{N}$, where $\Re_+ = [0, \infty)$; and 2) a constant set $\Omega_j^{sd} = \{\tau, 2\tau, ..., n\tau\}$, $n \in \mathbb{N}$; 3) a set of random failure times $T_{k_i,i,j} \in \Re+$ for $i \in I_{ABM}$ and a set of random passage times $T_{C_{k_i,i,j}}$ and $T_{H_{k_i,i,j}} \in \Re+$ for $i \in I_{CBM}$, according to the degradation process.

The k^{th} maintenance action of component i is taken on at a maintenance point $R_{k_i,i,j}$. Notice that the maintenance point $R_{k_i,i,j}$ is dependent on the previous maintenance action at $R_{k_i-1,i,j}$. To simplify the notation in the simulation algorithm, we define a range $[LB_{k_i,i,j}, UB_{k_i,i,j})$, where 1) $LB_{k_i,i,j} = R_{k_i-1,i,j} + A_i$ and $UB_{k_i,i,j} = R_{k_i-1,i,j} + T_{k_i,i,j}$ for $i \in I_{ABM}$; and 2) $LB_{k_i,i,j} = R_{k_i-1,i,j} + T_{C_{k_i,i,j}}$ and $UB_{k_i,i,j} = R_{k_i-1,i,j} + T_{H_{k_i,i,j}}$, for $i \in I_{CBM}$. Moreover, for each component $i \in I_{CBM} \cup I_{ABM}$, we define three sets: $\mathbb{A}_{k_i,i,j} = \{\Omega_j^{usd} \cap [LB_{k_i,i,j}, UB_{k_i,i,j})\}$, $\mathbb{B}_{k_i,i,j} = \{\Omega_j^{sd} \cap [LB_{k_i,i,j}, UB_{k_i,i,j})\}$ and $\mathbb{C}_{k_i,i,j} = \{UB_{k,l,j} \cap [LB_{k_i,i,j}, UB_{k_i,i,j}); l \in I \setminus i\}$. The binary parameters $I_{k_i,i,j}^{pm,usd}$, $I_{k_i,i,j}^{pm,sd}$ and $I_{k_i,i,j}^{cm}$ are defined as follows:

$$I_{k_i,i,j}^{pm,usd} = \begin{cases} 1 & \text{if a PM-USD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{k_i,i,j}^{pm,sd} = \begin{cases} 1 & \text{if a PM-SD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{k_i,i,j}^{cm} = \begin{cases} 1 & \text{if a CM action is taken} \\ 0 & \text{otherwise} \end{cases}$$

We use the algorithm in Table 5.11 as a building block to construct the simulation algorithm of a multi-component system. The interaction between components is the most difficult part in the simulation algorithm. For example, Component 1 with a longer lifetime has its 1^{st} maintenance at the time point $R_{1,1,j}$ according to the algorithm in Table 5.11. However, the Component 2 with shorter lifetime may have its 10^{th} maintenance at $R_{10,2,j}$, which can be just before $R_{1,1,j}$. In this case, $R_{10,2,j}$ can also be an opportunity to do preventive maintenance for Component 1. Hence, the maintenance points $R_{k_i,i,j}$ obtained via Table 5.11 may not be the true maintenance point $\hat{R}_{k_i,i,j}$ in a multi-component setting. To deal with this problem, we construct the simulation algorithm of a multi-component system in Table 5.10. The true cost rate of component i at k_ith maintenance point denoted by $\hat{Z}_{k_i,i,j}$ is:

$$\hat{Z}_{k_i,i,j} = \frac{\sum_{l=1}^{k_i}\left(I_{l,i,j}^{pm,usd}c_i^{PM-USD} + I_{l,i,j}^{pm,sd}c_i^{PM-SD} + I_{l,i,j}^{cm}c_i^{CM}\right)}{\hat{R}_{k_i,i,j}}.$$

By running the algorithm in Table 5.10 iteratively with m seeds, the final result of the simulation $\hat{Z}_{syst} = \frac{\sum_{i=1}^{m}\hat{Z}_j}{m}$ with a $100(1-\alpha)\%$ confidence interval is expressed as follows [30]:

$$\hat{Z} \pm t(1-\alpha/2, m-1)\sqrt{\frac{S^2}{m}}$$

where $S = \sum_{j=1}^{m}\frac{(\hat{Z}_j-\hat{Z})^2}{m-1}$ and $t(1-\alpha/2, m-1)$ is the upper $1-\alpha/2$ critical point for the t-distribution with $(m-1)$ degrees of freedom (in our case, $m = 100$ and $\alpha = 5\%$).

```
For j = 1 : m,
  Initiation: k_i = 0, Ẑ_{0,i,j} = 0 and R̂_{0,i,j} = 0, ∀i, j.
    While (Z_{k_i,i,j} − Z_{k_i−1,i,j}) / Z_{k_i,i,j} ≥ ε

      calculate {𝔸_{k_i,i,j}, 𝔹_{k_i,i,j}, ℂ_{k_i,i,j}}

        For i = 1 : |I_CBM ∪ I_ABM|
            Case 1: for component i ∈ I_CBM, use Block in Table 5.11
            Case 2: for component i ∈ I_ABM,
                If A_i < T_{k_i,i,j}
                Use Block in Table 5.11
                Else
                (I^{pm,usd}_{k_i,i,j}, I^{pm,sd}_{k_i,i,j}, I^{cm}_{k_i,i,j}) = (0, 0, 1) and R_{k_i,i,j} = R_{k_i−1,i,j} + T_{k_i,i,j}
                End if
        End For

        Obtain {R_{k_i,i,j} | ∀i ∈ I_CBM ∪ I_ABM}
        For i = 1 : |I_CBM ∪ I_ABM|
            If R_{k_i,i,j} = min{R_{k_i,i,j}, ∀i ∈ I_CBM ∪ I_ABM}
                R_{k_i,i,j} = R̂_{k_i,i,j} and k_i = k_i + 1
            Else
                R_{k_i,i,j} = R_{k_i,i,j} and k_i = k_i
            End if
        Calculate Ẑ_{k_i,i,j}
        End For

    End while

  Obtain K_i = k_i, ∀i ∈ I_CBM ∪ I_ABM and calculate Ẑ_j = Σ_{i=1}^{|I_CBM ∪ I_ABM|} Ẑ_{k_i,i,j}
End For
```

Table 5.10 Simulation algorithm for multi-component systems

Block
`If` $\min\{\mathbb{A}_{k_i,i,j}, \mathbb{B}_{k_i,i,j}, \mathbb{C}_{k_i,i,j}\} \neq \emptyset$,
$R_{k_i,i,j} = \min\{\mathbb{A}_{k_i,i,j}, \mathbb{B}_{k_i,i,j}, \mathbb{C}_{k_i,i,j}\}$ with 3 cases:
Case 1: if $R_{k_i,i,j} \in \mathbb{A}_{k_i,i,j}$, then $(I^{pm,usd}_{k_i,i,j}, I^{pm,sd}_{k_i,i,j}, I^{cm}_{k_i,i,j}) = (1,0,0)$
Case 2: if $R_{k_i,i,j} \in \mathbb{B}_{k_i,i,j}$, then $(I^{pm,usd}_{k_i,i,j}, I^{pm,sd}_{k_i,i,j}, I^{cm}_{k_i,i,j}) = (0,1,0)$
Case 3: if $R_{k_i,i,j} \in \mathbb{C}_{k_i,i,j}$, then $(I^{pm,usd}_{k_i,i,j}, I^{pm,sd}_{k_i,i,j}, I^{cm}_{k_i,i,j}) = (1,0,0)$
`Else`
$R_{k_i,i,j} = UB_{k_i,i,j}$, then $(I^{pm,usd}_{k_i,i,j}, I^{pm,sd}_{k_i,i,j}, I^{cm}_{k_i,i,j}) = (0,0,1)$
`End If`

Table 5.11 Simulation algorithm for single components

Chapter 6

Conclusion and Recommendation

"Essentially, all models are wrong, but some are useful. However, the approximate nature of the model must always be borne in mind."

George Box

Our research objective was to develop maintenance optimization models for multi-component systems based on remotely-monitored condition data, in order to minimize the long-run average cost rate of entire systems. In total, we developed four models: two group maintenance models (see Chapter 2 and 5) and two opportunistic maintenance models (see Chapter 3 and 4). These models help to optimize maintenance policies of complex systems from different industries with different features. Beside the utilities in practice, each model has its unique scientific contribution (see Section 2.1, 3.1, 4.1 and 5.1).

For multi-component systems in which soft failures of components occur and the degradation of each component is monitored continuously, we proposed a new CBM policy to reduce the high setup cost of maintenance actions in Chapter 2. A joint maintenance interval was introduced to take maintenance actions of multiple components simultaneously at periodic scheduled downs. In this case, we decomposed the main problem at the system level into subproblems at the component level, which allows an exact analysis of the cost rate evaluation. By optimizing the maintenance interval at the system level and the control limit on the degradation of each component, our model minimizes the long-run average cost rate. A numerical study of a production system consisting of 60 components is presented, including a sensitivity analysis. By comparing our policy against a failure-based maintenance policy and an age-based maintenance policy, we showed a considerable cost-saving potential of implementing our policy.

For multi-component systems in which hard failures of components occur, the systems stop at scheduled and unscheduled downs, which can be considered as free opportunities for a monitored component to perform preventive maintenance. In this case, no additional setup cost and downtime cost are charged on the monitored component. In Chapter 3, we proposed a new CBM policy for a critical component being monitored continuously, given the scheduled and unscheduled downs of a complex system as free opportunities. This model determines the optimal control limit of the monitored component, in order to decide the timing of taking opportunistic maintenance and minimize the long-run average cost rate of the component. Notice that the cost rate evaluation is not exact, but approximate. Via the comparison between the approximate results and simulated results in various numerical experiments, the high accuracy of our approximate evaluation was shown under different parameter settings. Moreover, a case study on lithography machines within the semiconductor industry was performed to demonstrate the utilization of our model in real life. Finally, by comparing with three different maintenance policies, our policy showed a considerable cost-saving potential under various parameter settings.

For some systems, their critical components can not be monitored remotely (e.g., because of physical constraints from the design of the system). For this case, we developed a new ABM model in Chapter 4. Similar to the CBM model in Chapter 3, both scheduled and unscheduled downs are considered as free opportunities to perform opportunistic maintenance. Unlike the CBM model in Chapter 3, this ABM model optimizes the age limit on the age of the component, instead of the control limit on the degradation of the condition-based component. The optimal age limit helps to decide the timing of taking opportunities, in order to minimize the cost rate. The cost rate evaluation is also an approximation and its high accuracy was shown via the comparison with simulation results under various parameter settings. Moreover, a numerical study was conducted under different parameter settings, to show the usage of the model. By comparing our policy with three different maintenance policies, our policy showed a considerable cost-saving potential under various parameter settings.

The high accuracy of the approximate evaluation in the CBM and ABM models (see Chapter 3 and 4) enabled the use of the CBM and ABM models as building blocks for an integrated maintenance policy for multi-component systems in Chapter 5. This model helps to coordinate different maintenance actions for a system with a mixture of components under CBM, ABM and FBM. Such a mixture can better represent multi-component systems in real life, which usually consist of components under different maintenance policies. We developed a new maintenance optimization model with a heuristic procedure to find a heuristic solution with a close-to-minimal average cost rate for the entire system under the assumed policy structure. This solution includes 1) the control limits of CBM components, 2) the age limits of ABM components, and 3) the maintenance interval for scheduled downs of the entire system. Moreover, we provided a numerical example of a system with 40 FBM components, 5 CBM components and 5 ABM components. We also simulated the average cost rate for the heuristic solution. The difference between the cost rates obtained via our heuristic and simulation was small. Finally, we used two scenarios to show the cost difference between implementing the CBM policy and the ABM policy.

The current work describes a maintenance model for multi-component systems. Via additional research, the underlying building blocks can be improved and other extension can be made. We suggest the following topics for further research.

1. The decision of preventive maintenance actions on a CBM component in Chapter 2 and 3 is based on a simple control limit. There are two ways to improve the decision rule of the preventive maintenance, mentioned as follows:

    - We use one control limit in this research, because the advantage of taking opportunities (regardless at SDs or USDs) is to save additional setup and downtime costs. These costs are very high compared with the actual repair costs of the components in the high-tech capital goods industry, so that the cost difference between preventive maintenance actions at SD and USD opportunities is small. However, in other industries, this cost difference between preventive maintenance actions at SDs and USDs can be much larger. In this case, one may also use two separate control limits for opportunities at SDs and USDs respectively.

- It is possible to consider the control limit as a time-dependent variable. For example, a CBM component degrades over time t, where $(n-1)\tau \leq t < n\tau$ and $n \in \mathbb{N}$. Within two consecutive SDs, i.e., $(n-1)\tau$ and $n\tau$, when t gets closer to $n\tau$ (i.e., the opportunity at the next SD), the attractiveness of taking opportunities at an USD is lower. In this case, a higher control limit will be more economically beneficial. As an extreme example, when t is just before $n\tau$, the control limit should be set as high as possible.

The advantage is that a higher cost reduction can be achieved. The disadvantage is that the mathematical analysis will be more complex at the component level, which will further increase the complexity of the optimization problem at the system level (e.g., the number of components in a multi-component system is limited).

The same recommendation holds for the ABM component in Chapter 4 with a single age limit.

2. The decisions regarding setups at the system level can be improved by adding extra flexibility $\Delta\tau$ to the fixed maintenance interval τ. For example, when there is an USD occurring just before the next SD with a very short time difference $\Delta\tau$, it is sensible to take this opportunity at the USD to do preventive maintenance actions for other components. In this case, the next SD that is very close to the USD can be skipped, so that the setup and downtime cost for that SD can be saved. Such a $\Delta\tau$ can also be a decision variable. Obviously, this will lead to additional complexity for the analysis and for finding a close-to-optimal policy.

3. We consider the economic dependency only, and the structures of systems are not considered. However, in some cases, the degradation of a component may depend on the degradation of other components in the system. By investigating such structure and stochastic dependencies, the maintenance policy of a system can be further optimized.

4. The degradation paths of components in our research are derived via statistical estimation from the history data. However, the actual degradation may deviate from the historical behaviors of degradation. In this case, it is helpful to include a Bayesian updating mechanism in the estimation of degradation paths, which enables more dynamic maintenance decision making.

5. The maintenance actions are assumed to be perfect, i.e., each maintenance action will restore a component to its original performance level. In practice, the more often a component/system has been repaired, the shorter its life time

is and the older the component is. In other words, an old component may have a faster degradation rate or a lower performance level than a brand new component, which is also known as aging effects. Hence, it is sensible to perform preventive maintenance actions more frequently on old components/systems. For expensive components, one may decide if it is economically beneficial to repair the old components or buy new ones. This tradeoff naturally triggers a "repair-or-scrap" decision.

Appendix A

Appendix

"Assumptions eat mathematical models for breakfast."

Qiushi Zhu

A.1 Parameter estimation of the degradation model

The purpose of this section is to demonstrate how to convert condition data collected by micro-sensors into a mathematical degradation path. The first step is to monitor a correct physical parameter, which reflects the true "health" or status of a component. In practice, the experts of diagnostics and trouble shooting from the engineering department are able to provide the information about which physical parameter has the most significant causal-effect relationship to failures. Then we start to collect the degradation data of this physical parameter in real time and use them to estimate the parameters of the degradation path statistically.

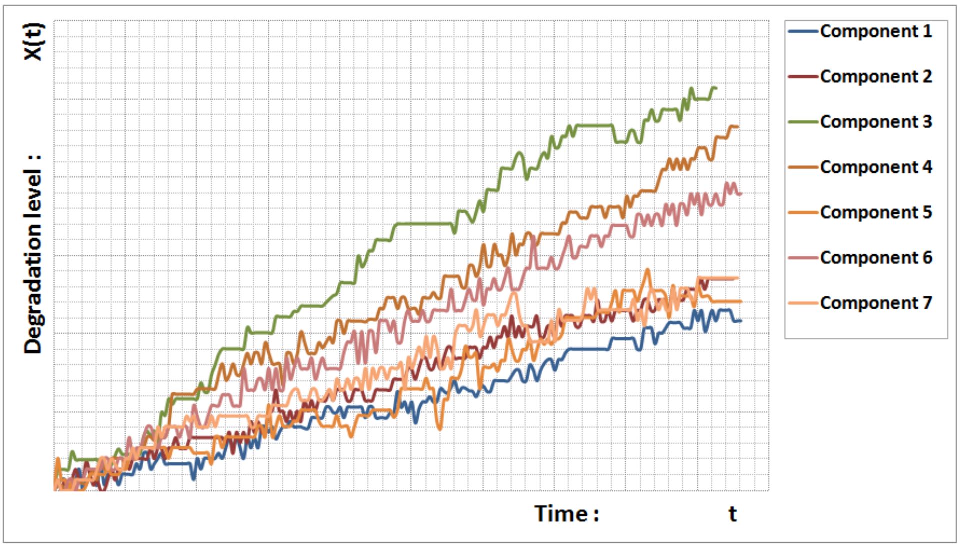


Figure A.1 Degradation level of a physical parameter over time, measured on several components

The degradation data is collected in the following format

$$\begin{array}{ccccc} t_1 & x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ t_2 & x_{2,1} & x_{2,2} & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ t_m & x_{m,1} & \cdots & \cdots & x_{m,n} \end{array}$$

where t_i is the i-th time point and x_{ij} is the degradation level at time point t_i for component j ($i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$). Each component j has a sample path that consists of the degradation levels $x_{i,j}$ at time points $i = 1, 2, \ldots m$.

As shown in Figure A.1, the degradation paths vary from one component to another. In this research, we model the degradation level $X(t)$ over time t by two approaches: i) *Random coefficient model*[33], because it is relatively flexible and convenient for describing the degradation paths derived from physics of failures, such as laws of physics and material science; ii) *Gamma process* [59], which is popular in the literature, due to the nice mathematical properties (e.g., the step increments are memoryless, the aggregation of multiple Gamma distribution is also a Gamma distribution, etc).

As an example, we use the raw degradation data from 73 identical laser components (i.e., $n = 73$). The degradation are recorded over 51 time points, namely, $m = 51$. Also notice the initial degradation levels of laser components are 0, namely, $X(0) = 0$.

A.1.1 Random coefficient model

The random coefficient model [33] is a convenient model for describing the degradation paths derived from the physics of failures, such as laws of physics and material science. As shown in Figure A.1, the degradations of identical units are different. This component-to-component variation leads to different trends on degradation paths, as seen in Figure A.2. To depict the degradation path of one type of components/units, we can statistically estimate the expected trend of the degradation level over time.

For all components in the group J, we can use the least-square method [36, pp. 337] to fit the degradation data and obtain 73 degradation paths for these 73 laser components. To show a simple example, we use Fitting Option 1 in Section 3.4 (more general forms of the random coefficient model are explained in Section A.2). The slope of the degradation path Θ is estimated statistically, according to [36]. The estimated slopes $\hat{\Theta}_j$ of component j are shown in Table A.1 with high R^2 values. The conditions of the components deteriorate over time, so that their degradations increase over time. In this case, we can assume that Θ is a positive random variable following a Weibull distribution, with its scale parameter α and shape parameter β.

Via the maximum likelihood estimation [19, pp. 409-412], we find the estimated parameters $\hat{\alpha} = 0.194$ and $\hat{\beta} = 2.350$ for the Weibull distribution of the estimated slopes $\hat{\Theta}_j$.

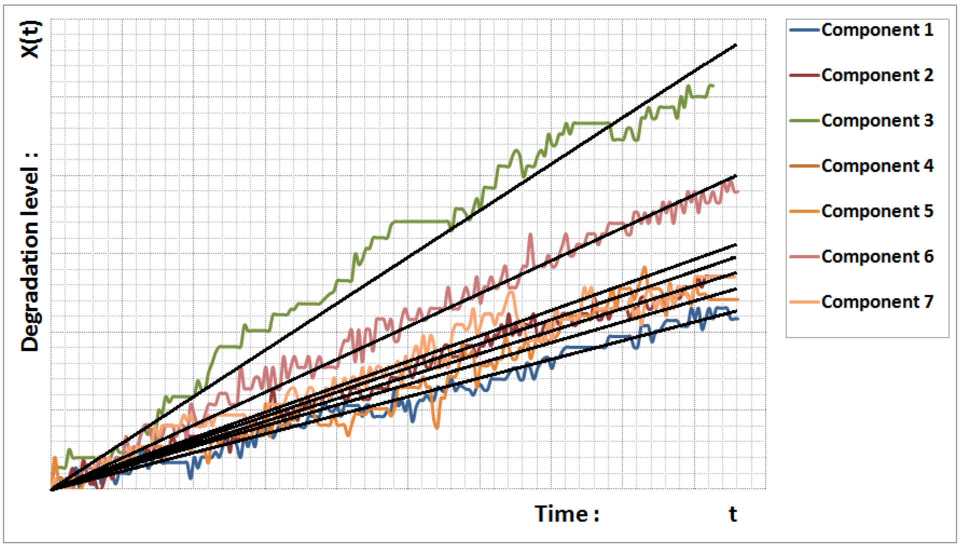


Figure A.2 The different trends of degradation level over time, measured on several identical components / units.

A.1.2 Gamma process

The Gamma stochastic process is a popular model to describe monotonic degradation paths in the literature [59]. Suppose the degradation level $X(t_i)$ is monitored over time t. In a Gamma process shown in Figure A.3, the degradation increments $X(t_i) - X(t_{i-1})$, between two time points t_i and t_{i-1}, are independent and identically distributed according to a Gamma distribution with its shape parameter $\gamma(t_i - t_{i-1})$ and scale parameter η. An extensive explanation of the Gamma process with its properties (e.g., memoryless, monotonic, additive, etc) is available in [47]. In the raw data of 73 components, the degradation level is measured once per time point; namely, $\Delta t = t_i - t_{i-1}$ for all $i = \{1, ..., m\}$. Unlike the component-to-component variation mentioned in the random coefficient model, it is not necessary to distinguish which degradation increment is from which component in a Gamma process. Hence, the index of a component, j, is not used in this subsection. Instead, the degradation increments of all components denoted by a set $G = \big\{x_{i,j} - x_{i-1,j}; \forall i = \{1, ..., m\}, j = \{1, ..., n\}\big\}$ are converted into a set $\big\{\Delta x_k, \forall k = \{1, ..., |G|\}\big\}$. Notice that Δx_k are i.i.d according to a $\Gamma(\gamma \Delta t, \eta)$ distribution.

Via the maximum likelihood estimation for a Gamma process [47, pp. 9-12], the estimated parameters $\hat{\gamma} = 0.404$ and $\hat{\eta} = 2.287$ can be found by solving the following equations [47]:

Table A.1 Estimated slope of the degradation paths of 73 laser components

Path i	Degradation rate $\hat{\Theta}_j$	R_i^2	Path i	Degradation rate $\hat{\Theta}_j$	R_i^2
1	0.1929	0.8741	37	0.2514	0.9511
2	0.1954	0.9589	38	0.1044	0.8283
3	0.1330	0.8730	39	0.1878	0.9304
4	0.2027	0.9837	40	0.2979	0.9885
5	0.2440	0.9639	41	0.1109	0.5916
6	0.2341	0.7504	42	0.0823	0.7512
7	0.1271	0.9236	43	0.3132	0.9824
8	0.1003	0.8452	44	0.2264	0.9843
9	0.2112	0.6806	45	0.0838	0.9418
10	0.1086	0.9343	46	0.1256	0.5189
11	0.0613	0.4871	47	0.1388	0.7561
12	0.1588	0.9824	48	0.2388	0.9569
13	0.1172	0.7286	49	0.0699	0.7708
14	0.0974	0.8662	50	0.1145	0.9214
15	0.1509	0.9411	51	0.2243	0.9666
16	0.1741	0.8479	52	0.0977	0.9324
17	0.2295	0.7963	53	0.0975	0.7067
18	0.2132	0.9626	54	0.2197	0.8415
19	0.1133	0.8742	55	0.1452	0.9333
20	0.1144	0.8610	56	0.1784	0.9208
21	0.1134	0.5778	57	0.2119	0.8885
22	0.1492	0.8691	58	0.2427	0.9867
23	0.1940	0.9727	59	0.2502	0.9748
24	0.1376	0.9697	60	0.0775	0.4710
25	0.1385	0.9503	61	0.3845	0.8288
26	0.1187	0.9530	62	0.3423	0.8586
27	0.1088	0.9869	63	0.1494	0.9653
28	0.0820	0.6466	64	0.1431	0.9601
29	0.1445	0.9646	65	0.0817	0.9414
30	0.0919	0.7603	66	0.1378	0.5280
31	0.1131	0.9578	67	0.2834	0.9541
32	0.0840	0.9800	68	0.1793	0.9300
33	0.2413	0.8723	69	0.1410	0.9102
34	0.3991	0.9094	70	0.3246	0.8483
35	0.1371	0.8080	71	0.3508	0.9923
36	0.1909	0.8208	72	0.1727	0.9601
			73	0.1133	0.8534

$$|G| \ln\left(\hat{\gamma}\frac{|G|\Delta t}{\sum_{k=1}^{|G|}\Delta x_k}\right) + \Delta t \sum_{k=1}^{|G|} [\ln(\Delta x_k) - \Psi(\hat{\gamma}\Delta t)] = 0$$

and

$$\hat{\eta} = \hat{\gamma}\frac{(|G|)\Delta t}{\sum_{k=1}^{|G|}\Delta x_k}$$

where $\Psi(\hat{\gamma}\Delta t) = \Gamma'(\hat{\gamma}\Delta t)/\Gamma(\hat{\gamma}\Delta t)$.

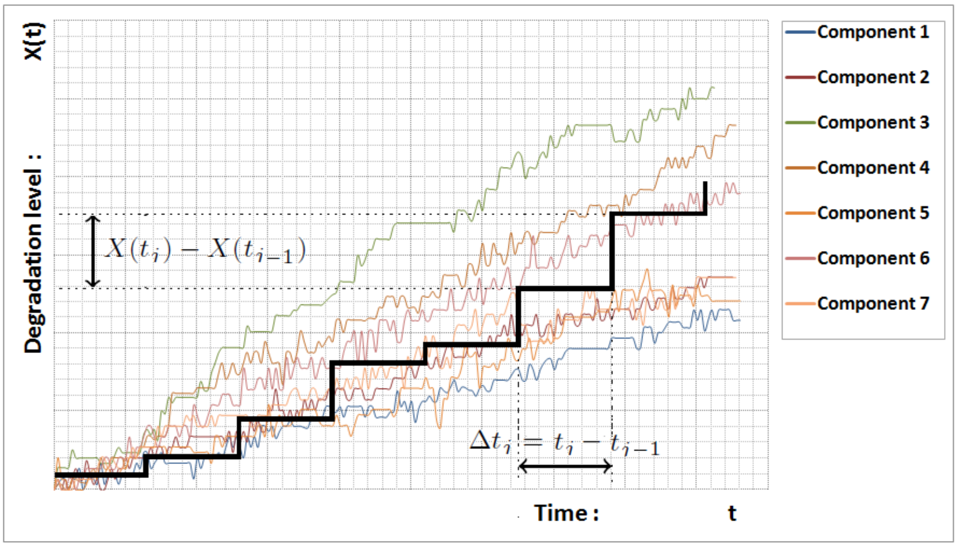


Figure A.3 Degradation level of several identical components/units over time, modeled by a Gamma process.

A.2 Multiple variables in the random coefficient model

In this section, we will give a more general form of the random coefficient model. The degradation path $X(t;\Phi,\Theta)$ at time $t \in [0,\infty)$ is a random variable given a set of constant parameters $\Phi = \{\phi_1, ..., \phi_m\}, m \in \mathbb{N}$; and a set of random parameters, $\Theta = \{\theta_1, ..., \theta_n\}, n \in \mathbb{N}$, following certain probability distributions. To simplify the notation, we consider $X(t;\Phi,\Theta)$ as a function of n random variables $\Theta = \{\theta_1, ..., \theta_n\}, n \in \mathbb{N}$. The probability that the degradation path $X(\theta_1, ..., \theta_n)$ exceeds a threshold χ is

$$\begin{aligned} Pr\{X(\theta_1,...,\theta_n) > \chi\} &= 1 - F_X(\chi), \\ &= Pr\{(\theta_1,...,\theta_n) \in D_X\} \\ &= \underbrace{\int ... \int_{(\theta_1,...,\theta_n)\in D_X}}_{n} f_\Theta(\theta_1,...,\theta_n)\, d\theta_1 \, ... \, d\theta_n \end{aligned} \tag{A.1}$$

where D_X is the domain such that $X(\theta_1, ..., \theta_n) \geq \chi$ and F_X is the cumulative distribution function of X. The joint probability density distribution function of the random variables $\{\theta_1, ..., \theta_n\}$ is denoted by $f_\Theta(\theta_1, ..., \theta_n)$. If the random variables $\{\theta_1, ..., \theta_n\}$ are independent, then

$$f_\Theta(\theta_1,...,\theta_n) = f_{\theta_1}(\theta_1) f_{\theta_2}(\theta_2) \, ... \, f_{\theta_n}(\theta_n)$$

where $f_{\theta_i}(\theta_i)$ is the probability density function of the random variable θ_i, $\forall i \in \{1, 2..., n\}$. Notice that the random variables $\{\theta_1, ..., \theta_n\}$ are not necessarily independent. In this case, more details can be found in [9].

EXAMPLE 1: *As a simple example of two independent random variables θ_1 and θ_2, suppose a degradation path $X = \theta_1 t + \theta_2$; where i) $\theta_1 \in [0, \infty)$ follows a Weibull distribution with a shape parameter β_1 and a scale parameters α_1, ii) $\theta_2 \in [0, \infty)$ follows a Weibull distribution with a shape parameter β_2 and a scale parameters α_2, iii) $t \in [0, \infty)$. The probability of $X(\theta_1, \theta_2) < \chi$ can be derived according to Equation A.1:*

$$\begin{aligned} Pr\{X(\theta_1, \theta_2) \leq \chi\} &= F_X(\chi) \\ &= \int_{u=0}^{u=\frac{\chi}{t}} f_{\theta_1}(u) \left(\int_{v=0}^{v=\chi-ut} f_{\theta_2}(v)\ dv \right) du \\ &= \int_{u=0}^{u=\frac{\chi}{t}} \left(\frac{\beta_1}{\alpha_1} (\frac{u}{\alpha_1})^{\beta_1 - 1} e^{(-\frac{u}{\alpha_1})^{\beta_1}} \right) \left(1 - e^{(-\frac{\chi-ut}{\alpha_2})^{\beta_2}} \right) du \end{aligned} \tag{A.2}$$

where F_X is the cumulative distribution function of the random variable $X = \theta_1 t + \theta_2$. $\diamondsuit$

Bibliography

[1] T.W. Archibald and R. Dekker. Modified block-replacement for multiple-component systems. *IEEE Transactions on Reliability*, 45(1):75–83, Mar 1996.

[2] J. Barata, C.G. Soares, M. Marseguerra, and E. Zio. Simulation modelling of repairable multi-component deteriorating systems for 'on condition' maintenance optimisation. *Reliability Engineering & System Safety*, 76(3):255–264, Jun 2002.

[3] C.W.J. Berendsen, M. Skeren, D. Najdek, and F. Cerny. Superhydrophobic surface structures in thermoplastic polymers by interference lithography and thermal imprinting. *Applied Surface Science*, 255(23):9305 – 9310, Jul 2009.

[4] M. Berg and B. Epstein. Modified block replacement policy. *Naval Research Logistics*, 23(1):15–24, 1976.

[5] D.P. Bertsekas. *Nonlinear Programming.* Athena Scientific, 1995.

[6] K. Bouvard, S. Artus, C. Berenguer, and V. Cocquempot. Condition-based dynamic maintenance operations planning & grouping: Application to commercial heavy vehicles. *Reliability Engineering & System Safety*, 96(6, SI):601–610, Jun 2011.

[7] C. Bunks, D. McCarthy, and T. Al-Ani. Condition-based maintenance of machines using Hidden Markov Models. *Mechanical Systems & Signal Processing*, 14(4):597–612, Jul 2000.

[8] W.D. Callister and D.G. Rethwisch. *Materials Science and Engineering: An Introduction, Sixth Edition.* John Wiley & Sons, 2003.

[9] G. Casella and R.L. Berger. *Statistical Inference.* Thomson Learning, 2002.

[10] B. Castanier, A. Grall, and C. Berenguer. A condition-based maintenance policy with non-periodic inspections for a two-unit series system. *Reliability Engineering & System Safety*, 87(1):109–120, Jan 2005.

[11] D.I. Cho and M. Parlar. A survey of maintenance models for multiunit systems. *European Journal of Operational Research*, 51(1):1–23, Mar 1991.

[12] M. A. Cohen, N. Agrawal, and V. Agrawal. Winning in the Aftermarket. *Harvard Business Review*, 84(5):129–138, May 2006.

[13] J.S. Dagpunar. A maintenance model with opportunities and interrupt replacement options. *Journal of the Operational Research Society*, 47(11):1406–1409, Nov 1996.

[14] C. Daniel. One-at-a-time plans. *Journal of the American Statistical Association*, 68(342):353–360, 1973.

[15] R. Dekker and M.C. Dijkstra. Opportunity-based age replacement - exponentially distributed times between opportunities. *Naval Research Logistics*, 39(2):175–190, Mar 1992.

[16] R. Dekker and E. Smeitink. Opportunity-based block replacement. *European Journal of Operational Research*, 53(1):46–63, Jul 1991.

[17] R Dekker, R.E. Wildeman, and F.A. van der Duyn Schouten. A review of multi-component maintenance models with economic dependence. *Mathematical Methods of Operations Research*, 45(3):411–435, Jan 1997.

[18] L. Dieulle, C. Berenguer, A. Grall, and M. Roussignol. Sequential condition-based maintenance scheduling for a deteriorating system. *European Journal of Operational Research*, 150(2):451–461, Oct 2003.

[19] C.E. Ebeling. *An Introduction to Reliability and Maintainability Engineering.* Waveland Press Inc., 2009.

[20] A.H. Elwany, N.Z. Gebraeel, and L.M. Maillart. Structured replacement policies for components with complex degradation processes and dedicated sensors. *Operations Research*, 59(3):684–695, May 2011.

[21] N.Z. Gebraeel, M.A. Lawley, R. Li, and J.K. Ryan. Residual-life distributions from component degradation signals: A Bayesian approach. *IIE Transactions*, 37(6):543–557, Jun 2005.

[22] A. Grall, C. Berenguer, and L. Dieulle. A condition-based maintenance policy for stochastically deteriorating systems. *Reliability Engineering and System Safety*, 76(2):167 – 180, May 2002.

[23] A. Haurie and P. L'ecuyer. A stochastic-control approach to group preventive replacement in a multicomponent system. *IEEE Transactions on Automatic Control*, 27(2):387–393, Apr 1982.

[24] A.K.S. Jardine, P. M. Anderson, and D. S. Mann. Application of the weibull proportional hazards model to aircraft and marine engine failure data. *Quality and Reliability Engineering International*, 3(2), Apr 1987.

[25] A.K.S. Jardine, D. Lin, and D. Banjevic. A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical Systems and*

Signal Processing, 20(7):1483–1510, Oct 2006.

[26] A.K.S. Jardine and A.H.C. Tang. *Maintenance, Replacement, and Reliability: Theory and Applications.* Taylor & Francis Group, 2006.

[27] J.P. Kharoufeh, C.J. Solo, and M. Ulukus. Semi-markov models for degradation-based reliability. *IIE Transactions*, 42(8):599–612, May 2010.

[28] J. Koochaki, J. Bokhorst, H. Wortmann, and W. Klingenberg. Condition based maintenance in the context of opportunistic maintenance. *International Journal of Production Research*, 50(23):6918–6929, 2012.

[29] R. Laggoune, A. Chateauneuf, and D. Aissani. Opportunistic policy for optimal preventive maintenance of a multi-component system in continuous operating units. *Computers & Chemical Engineering*, 33(9):1499–1510, Sep 2009.

[30] A.M. Law and W.D. Kelton. *Simulation modeling and analysis, 3rd edition.* McGraw-Hill, 2000.

[31] H.T. Liao, A. Elsayed, and L.Y. Chan. Maintenance of continuously monitored degrading systems. *European Journal of Operational Research*, 175(2):821 – 835, Dec 2006.

[32] D.M. Lin and V. Makis. Recursive filters for a partially observable system subject to random failure. *Advances in Applied Probability*, 35(1):207–227, Mar 2003.

[33] C.J. Lu and W.Q. Meeker. Using degradation measures to estimate a time-to-failure distribution. *Technometrics*, 35(2):161–174, May 1993.

[34] V. Makis and X. Jiang. Optimal replacement under partial observations. *Mathematics of Operations Research*, 28(2):382–394, May 2003.

[35] M. Marseguerra, E. Zio, and L. Podofillini. Condition-based maintenance optimization by means of genetic algorithms and Monte Carlo simulation. *Reliability Engineering & System Safety*, 77(2):151–165, Jul 2002.

[36] W.Q. Meeker and L.A. Escobar. *Statistical methods for reliability data.* John Wiley & Sons, 1998.

[37] R.P. Nicolai and R. Dekker. A review of multi-component maintenance models. In *Risk, Reliability And Societal Safety, Vols 1-3*, Proceedings and Monographs in Engineering, Water and Earth Sciences, pages 289–296, 2007.

[38] K.B. Oner, R. Franssen, G.P. Kiesmuller, and G.J. van Houtum. Life cycle costs measurement of complex systems manufactured by an engineer-to-order company. *The 17th International Conference on Flexible Automation and Intelligent Manufacturing,Philadelphia*, pages 589–596, Feb 2007.

[39] S. Ozekici. Optimal periodic replacement of multicomponent reliability systems. *Operations Research*, 36(4):542–552, Jul 1988.

[40] K.S. Park. Optimal continuous-wear limit replacement under periodic inspections. *IEEE Transactions on Reliability*, 37(1):97 –102, Apr 1988.

[41] K.S. Park. Optimal wear-limit replacement with wear-dependent failures. *IEEE Transactions on Reliability*, 37(3):293–294, Aug 1988.

[42] Y. Peng, M. Dong, and M.J. Zuo. Current status of machine prognostics in condition-based maintenance: A review. *International Journal of Advanced Manufacturing Technology*, vol.50(1-4):pp.297–313, Sep 2010.

[43] H. Pham and H.Z. Wang. Optimal (τ, T) opportunistic maintenance of a k-out-of-n : G system with imperfect PM and partial failure. *Naval Research Logistics*, 47(3):223–239, Apr 2000.

[44] E. Popova and J.G. Wilson. Group replacement policies for parallel systems whose components have phase distributed failure times. *Annals of Operations Research*, 91:163–189, 1999.

[45] R. Radner and D.W. Jorgenson. Opportunistic replacement of a single part in the presence of several monitored parts. *Management Science*, 10(1):70–84, Oct 1963.

[46] S.M. Ross. *Stochastic Processes, Second Edition*. John Wiley & Sons, 1996.

[47] M. Roussignol. Gamma stochastic process and application to maintenance, 2009.

[48] S.H. Sheu and J.P. Jhang. A generalized group maintenance policy. *European Journal of Operational Research*, 96(2):232–247, Jan 1997.

[49] X.S. Si, W. Wang, C.H. Hu, and D.H. Zhou. Remaining useful life estimation - A review on the statistical data driven approaches. *European Journal of Operational Research*, 213(1):1–14, Aug 2011.

[50] S. Taghipour and D. Banjevic. Optimal inspection of a complex system subject to periodic and opportunistic inspections and preventive replacements. *European Journal of Operational Research*, 220(3):649–660, Aug 2012.

[51] L.C. Thomas. A survey of maintenance and replacement models for maintainability and reliability of multi-item systems. *Reliability Engineering*, 16(4):297–309, May 1986.

[52] Z.G. Tian, T.D. Jin, B.R Wu, and F.F. Ding. Condition based maintenance optimization for wind power generation systems under continuous monitoring. *Renewable Energy*, 36(5):1502–1509, May 2011.

[53] Z.G. Tian and H.T. Liao. Condition based maintenance optimization for multi-component systems using proportional hazards model. *Reliability Engineering & System Safety*, 96(5):581–589, May 2011.

[54] B. Timmermans. Development and application of decision model for synchronizing condition-based maintenance. *Master of Science thesis,*

Department of Industrial Engineering, Eindhoven University of Technology, 2012.

[55] F.A. van der Duyn Schouten and S.G. Vanneste. Two simple control policies for a multicomponent maintenance system. *Operations Research*, 41(6):1125–1136, Nov 1993.

[56] G. van Dijkhuizen and A. van Harten. Optimal clustering of frequency-constrained maintenance jobs with shared set-ups. *European Journal of Operational Research*, 99(3):552–564, Jun 1997.

[57] N. van Dorst. Component Condition Monitoring and Remaining Useful Life prediction. *Master of Science thesis, Department of Industrial Engineering, Eindhoven University of Technology*, 2014.

[58] A. van Horenbeek and L. Pintelon. A dynamic predictive maintenance policy for complex multi-component systems. *Reliability Engineering & System Safety*, 120:39–50, 2013.

[59] J.M. van Noortwijk. A survey of the application of gamma processes in maintenance. *Reliability Engineering & System Safety*, 94(1, SI):2–21, Jan 2009.

[60] P.J. Vlok, J.L. Coetzee, D. Banjevic, A.K.S. Jardine, and V. Makis. Optimal component replacement decisions using vibration monitoring and the proportional-hazards model. *Journal of The Operational Research Society*, 53(2):193–202, Feb 2002.

[61] H.Z. Wang. A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139(3):469–489, Jun 2002.

[62] W. Wang. A model to determine the optimal critical level and the monitoring intervals in condition-based maintenance. *International Journal of Production Research*, 38(6):1425–1436, Apr 2000.

[63] W. Wang and A.H. Christer. Towards a general condition based maintenance model for a stochastic dynamic system. *The Journal of the Operational Research Society*, 51(2):145–155, Feb 2000.

[64] D.J.D Wijnmalen and J.A.M. Hontelez. Coordinated condition-based repair strategies for components of a multi-component maintenance system with discounts. *European Journal of Operational Research*, 98(1):52–63, Apr 1997.

[65] R.E. Wildeman, R. Dekker, and A.C.J.M. Smit. A dynamic policy for grouping maintenance activities. *European Journal of Operational Research*, 99(3):530–551, Jun 1997.

[66] H.F. Yu and S.T. Tseng. Designing a degradation experiment with a reciprocal weibull degradation rate. *Journal of the American Statistical Association*, 1(1):47–63, 2004.

[67] X.T. Zheng. All opportunity-triggered replacement policy for multiple-unit

systems. *IEEE Transactions on Reliability*, 44(4):648–652, Dec 1995.

[68] Q. Zhu, H. Peng, and G.J. van Houtum. A condition-based maintenance policy for multi-component systems with a high maintenance setup cost. *Working paper, BETA research school*, Nr. 400, Eindhoven University of Technology 2013.

[69] Q. Zhu, H. Peng, and G.J. van Houtum. An opportunistic maintenance policy for components under condition monitoring in complex systems. *Working paper, Eindhoven University of Technology*, , 2013.

[70] Q. Zhu, H. Peng, and G.J. van Houtum. An age-based maintenance policy under scheduled and unscheduled opportunities for preventive maintenance actions. *Working paper, Eindhoven University of Technology*, 2014.

[71] Q. Zhu, H. Peng, and G.J. van Houtum. An opportunistic maintenance model for multi-component systems under a mixture of different maintenance policies. *Working paper, Eindhoven University of Technology*, 2014.

Summary

High-tech capital goods (e.g., aircraft engines, wind turbines, semiconductor production systems, MRI scanners) nowadays have high production efficiencies and long life times. The common trends of these complex high-tech systems are: (i) the structure of the system and the dependency between components are very complicated, so that it becomes harder or even impossible for operators to do quick-and-easy maintenance by themselves; (ii) it is very expensive when a system is down; (iii) while buying new systems, operators consider total cost of ownership. During the long exploitation phases of capital goods, the maintenance costs including downtime and setup costs are often very high. Hence, for systems consisting of large numbers of components, it is often economically beneficial to perform the maintenance actions of multiple components simultaneously, in order to save setup costs and downtime.

As traditional approaches, one can synchronize the maintenance actions at certain moments in time or based on the age/usage of components/systems, known as age/usage-based maintenance. In this case, the maintenance actions are not based on the actual physical conditions of systems/components and thus one may lose significant parts of the useful lifetime. Due to the rapid development of advanced sensor and ICT technology, nowadays, it is less costly to acquire the physical conditions of systems/components remotely. Based on these condition data, a large amount of unnecessary maintenance tasks can be avoided, by taking maintenance actions only when the physical degradations of the critical components are close to the failure levels. This is known as condition-based maintenance (CBM), by which maintenance costs can be reduced significantly in comparison to age/usage-based maintenance.

In the existing literature, much more attention has been paid to single-component systems than to multi-component systems. Within the literature of multi-component systems, much less research efforts at the multi-component level are spent on condition-based maintenance than on age-based maintenance models.

Therefore, our research objective is to develop condition-based maintenance optimization models for multi-component systems, which helps to minimize the average

cost rate of the entire systems in a long run.

To optimize maintenance policies of different complex systems with different features, we developed four models in four chapters, respectively. Each of them has not only unique scientific contributions, but also value for practical applications.

- Regarding the contribution of Chapter 2, we proposed a new CBM model for multi-component systems consisting of a large number of components. To reduce the high setup cost of maintenance, a joint maintenance interval was introduced by setting up periodic scheduled downs to take maintenance actions of multiple components simultaneously. We assumed all failures of components were *soft failures* with quality/performance loss costs per time unit, before maintenance actions taken at the scheduled downs. In this case, we decomposed the main problem at the system level to subproblems at the component level, which allows exact evaluation of the objective function (i.e., the average cost rate). With the maintenance interval at the system level and the control limit on the degradation of each component as decision variables, we developed a model to minimize the long-run average maintenance cost rate of the systems. Moreover, a numerical study of a production system consisting of 60 components was presented, including a sensitivity analysis. By comparing our policy against a failure-based maintenance policy and an age-based maintenance policy, we showed a considerable cost-saving potential of implementing our policy.

- Regarding the contribution of Chapter 3, to the best of our knowledge, our policy is the first opportunistic CBM policy that considers both the scheduled and unscheduled downs of a complex system as free opportunities. In practice, unscheduled downs can happen if *hard failures* occur for other components. Both scheduled and unscheduled downs are considered as free opportunities for monitored components to perform preventive maintenance, so that no additional setup cost and downtime cost are charged on the monitored component. This model determines the optimal control limit of a critical component monitored continuously, in order to minimize the long-run average cost rate. In this chapter, our cost rate evaluation based on renewal theory is not exact, but approximate. The accuracy of this approximation was verified via the comparison of approximate results and simulated results. Moreover, a case study on lithography machines in the semiconductor industry was provided. Finally, by comparing with three different maintenance policies, our policy showed a considerable cost-saving potential under various parameter settings.

- In practice, it is not always feasible to monitor components remotely, due to physical constraints from the design of the system. In this case, ABM may be implemented, instead of CBM as in Chapter 3. Hence, we developed this

ABM model in Chapter 4 to optimize the age limit on the age of the component, instead of the control limit on the degradation of the component. Unlike most of existing works considering either scheduled or unscheduled opportunities, this model proposed in Chapter 4 includes both the scheduled and unscheduled opportunities. The optimal age limit helps to decide the timing of taking opportunities, in order to minimize the average long-run cost rate. Moreover, a numerical study was conducted to show the usage of the model. Similar to the CBM model in Chapter 3, the cost evaluation is also an approximation and its accuracy was verified via the comparison with simulated results. By comparing our policy with three different maintenance policies, we showed that the new policy has a considerable cost-saving potential.

- In the high-tech industry, we observe that complex engineering systems are often with a mixture of components under different maintenance policies (e.g., ABM, CBM and FBM). However, to the best of our knowledge, this maintenance optimization problem with the coordination of maintenance actions under the different policies has not been studied in the literature. The model proposed in Chapter 5 is able to solve maintenance optimization problems for a system with such a mixture of components under CBM, ABM and FBM policies, by using the CBM and ABM model in Chapter 3 and 4 as building blocks. To be able to solve large-scale problems in real life, where systems consist of large numbers of components, we developed a maintenance optimization model with a heuristic procedure to optimize 1) the control limits of condition-based components, 2) the age limits of age-based components, and 3) the maintenance interval for scheduled downs of the entire system. Via an iterative procedure, in a relatively short time, we are able to find a heuristic solution with a close-to-minimal average cost rate for the entire system (under the assumed policy structure). Moreover, we provided a numerical example of a system with 40 FBM components, 5 CBM components and 5 ABM components, where we also simulated the average cost rate for the heuristic solutions. The difference between the cost rates obtained via our heuristic and simulation was small. Finally, we used two scenarios to show the cost difference between implementing the CBM policy and the ABM policy.

About the author

Qiushi Zhu was born in Wuhan, China, on August 29, 1988. In 2005, he came to the Netherlands for higher education. In 2008, he obtained his Bachelor (BSc) diploma from the Aerospace Engineering Faculty of Delft University of Technology. His BSc graduation project was about the design of multi-functional unmanned aerial vehicles. He was also awarded by the faculty with a twelve-thousand-euro scholarship to stay in the faculty and to proceed his Master study. In 2010, he obtained his Master (MSc) diploma with Cum Laude from the Aerospace Engineering Faculty of Delft University of Technology. His MSc graduation project was sponsored by Fokker Services B.V with a topic on "Reliability monitoring and predicting system with cost-effectiveness analysis".

From October 2010 to October 2014, Qiushi was a PhD student in the Department of Industrial Engineering and Innovation Sciences, at Eindhoven University of Technology, under the supervision of prof.dr.ir. Geert-Jan van Houtum and dr. Hao Peng. During his PhD, he also visited Tsinghua University in Beijing, China, and worked with dr. Kaibo Wang from March 2014 to April 2014.

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Since December 2014, Qiushi has started at his new job in the department of integration management at Vanderlande Industries.