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by

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# Threshold effects in zero range processes

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## Abstract

We study a zero range process characterized by the presence of a threshold switching the particle dynamics from the independent particle model to the simple exclusion process. The setting is relevant to pedestrian dynamics in obscured corridors. We investigate the hydrodynamic limit of the model considering both symmetric and asymmetric jump probabilities, and highlight the effect of the threshold parameter on the resulting behavior of the diffusion coefficient and of the outgoing current.

## 1 Introduction

We consider a stochastic interacting particle model known as zero range process, originally proposed by Spitzer [15], and discuss the effect induced, on the dynamics, by a threshold parameter. Within our framework, the threshold allows one to switch from an independent motion of the particles to a motion that can be mapped to a simple exclusion process. When taking the hydrodynamic limit of the model, the resulting macroscopic dynamics exhibits a non-trivial dependence on the threshold and, to our knowledge, it is yet unexplored.

Our motivation for this study stems from our investigation of the motion of pedestrian flows in dark or in obscured corridors, where the internal dynamics of pedestrians (particles) can change depending on the willingness to cooperate (here: to adhere to large groups) or to be selfish (here: to perform independent random walks); see [5, 6, 13] for more details in this direction. To be able to understand the behavioural change leading individuals from cooperation to selfishness and eventually backwards, we consider, here, the presence of the threshold in a zero range process. The driving question is which values of the threshold yield higher evacuation fluxes (currents), or, in other words, allow for lower (average) residence times.

It is worth noting that this particular traffic scenario is intimately related to the dynamics of molecular motors seen from the perspective of processivity (cf., e.g., [10]). In this context,

one distinguishes between processive and non-processive motors. The processive ones perform best when working in small groups (porters), while the non-processive motors work best in large groups (rowers). Their joint collective dynamics has been investigated in [2]. If the motors suddenly change their own processivity from porters to rowers (for instance, due to particular environmental conditions, or due to a command control from a hierarchical structure), then our approach based on zero range processes with threshold approximates conceptually well the changing-in-processivity dynamics.

Threshold effects are not new in microscopic dynamics: as already said above, they are introduced to model dynamics undergoing sudden changes when some dynamical observable exceeds an a priori prescribed value. A natural application of this point of view is present in infections propagation models, where an individual get infected if the number of infected neighbours is large enough. A very well studied application is the Bootstrap Percolation problem [4] in which, for instance, on a square lattice, a site becomes infected as soon as the number of its neighbouring infected sites is larger than a fixed threshold value. In this problem the most interesting and surprising situation is the one in which the threshold is precisely half of the total neighbouring sites. In such a case new scaling laws have been discovered in the infinite volume limit [1, 3].

This work focuses on the hydrodynamic limit of zero range processes characterized by a threshold, subjected to periodic boundary conditions and equipped with either symmetric or asymmetric jump probabilities. We explicitly highlight the effect of the threshold on the macroscopic transport equations and discuss, in particular, the dependence of the diffusion coefficient and of the outgoing current on the threshold. Our analysis allows one to recover some known analytical results available for the independent particle model and for the simple exclusion process, and sets also the stage for a deeper understanding of the hydrodynamic limit of zero range processes characterized by an arbitrary value of the threshold.

## 2 The model

We consider a positive integer  $L$  and define a *zero range* [9, 12] stochastic process (ZRP) on the finite torus (periodic boundary conditions)  $\Lambda_L := \{1, \dots, L\} \subset \mathbb{Z}$ .

We fix  $N \in \mathbb{Z}_+$  and consider the finite *state* or *configuration space*  $\Omega_{N,L} := \{0, \dots, N\}^\Lambda$ . Given  $\omega = (\omega_1, \dots, \omega_L) \in \Omega_{N,L}$  the integer  $\omega_x$  is called *number of particle* at site  $x \in \Lambda$  in the *state* or *configuration*  $\omega$ .

We pick  $T \in \{1, \dots, N\}$ , the *threshold*, and define the *intensity function*

$$I_T(k) = \begin{cases} 0 & \text{if } k = 0 \\ 1 & \text{if } 1 \leq k \leq T \\ k - T + 1 & \text{if } k > T \end{cases} \quad (1)$$

for each  $k \in \mathbb{Z}_+$ .

The *zero range process* we consider in this paper is the Markov process  $\omega_t \in \Omega_{N,L}$ , with  $t \geq 0$ , such that each site  $x \in \Lambda$  is updated with intensity  $I_T(\omega_x(t))$  and, once such a site  $x$  is chosen, a particle jump either to the neighbouring left  $x - 1$  or neighbouring right  $x + 1$  site with equal probability  $1/2$  (recall periodic boundary conditions are imposed). For more details we refer the reader to [11, 12].

Given the threshold  $T$ , the intensity function is constantly equal to one up to  $T$  and then it increases linearly with the number of particles occupying the site. In other words, all sites with number of particles smaller or equal to  $T$  are treated equally by the dynamics, whereas the updating of those sites with more than  $T$  particles is favoured. We note that in the limiting case  $T = 1$  the intensity function becomes  $I_1(k) = k$ , for  $k > 0$ , and thus the well known *independent particle* model is recovered. A different limiting situation is the one in which the intensity function is constantly equal to 1 for any  $k \geq 1$  and equal to zero for  $k = 0$ . In this case a zero range process whose configurations can be mapped to the simple exclusion model states, see [9], is found. We shall refer to the latter case as to the *simple exclusion*-like model. Such a model is found, in our set-up, when  $T \rightarrow \infty$  (in the sequel, with a slight abuse of notation, we simply write  $T = \infty$ ). We stress that one of the interesting issues of our model is the fact that it is able to tune between two very different dynamics: the independent particle and simple exclusion-like behavior.

We let the *Gibbs measure* with *fugacity*  $z \in \mathbb{R}$  of the ZRP introduced above be the product measure on  $\mathbb{N}^\Lambda$

$$\prod_{x=1}^L \nu_{z,T}(\eta_x) \quad \text{for any } \eta = (\eta_1, \dots, \eta_L) \in \mathbb{N}^\Lambda \quad (2)$$

with

$$\nu_{z,T}(0) = C_{z,T} \quad \text{and} \quad \nu_{z,T}(k) = C_{z,T} \frac{z^k}{I_T(1) \cdots I_T(k)} \quad \text{for } k \geq 1, \quad (3)$$

where  $C_{z,T}$  is a normalizing factor depending, in general, on  $L$ ,  $z$ , and  $T$ .

It can be proven, we refer again the reader to [12] for more details, that the invariant measure of the ZRP process can be obtained by conditioning the above Gibbs measure to the event  $\eta_1 + \cdots + \eta_L = N$ , namely, to the subspace of  $\mathbb{N}^\Lambda$  made of those configurations with  $N$  particles. It is then found that the invariant measure of the ZRP process with periodic boundary conditions on  $\Lambda$  and  $N$  particles is given by

$$\mu_{N,L,T}(\omega) = \frac{1}{Z_{N,L,T}} \prod_{\substack{x=1, \dots, L \\ \omega_x \neq 0}} \frac{1}{I_T(1) \cdots I_T(\omega_x)} \quad (4)$$

for any  $\omega \in \Omega_{N,L}$ , where the partition function  $Z_{N,L,T}$  is the normalization constant. For this computation we refer the reader to [9].

We conclude this Section by explicitly evaluating the expression of the normalization factor  $C_{z,T}$  in (3) for different values of the threshold  $T$ . For the independent particle model, one has  $T = 1$  and  $I_T(k) = k$ . The normalization factor in (3) can be then evaluated explicitly:

$$1 = \sum_{k=0}^{\infty} \nu_{z,1}(k) = C_{z,1} \sum_{k=0}^{\infty} \frac{z^k}{k!} \implies C_{z,1} = e^{-z} . \quad (5)$$

For the simple exclusion-like process, one takes  $I_\infty(k) = 1$ . Proceeding as previously indicated, we find that for  $z < 1$  it holds

$$1 = \sum_{k=0}^{\infty} \nu_{z,\infty}(k) = C_{z,\infty} \sum_{k=0}^{\infty} z^k \implies C_{z,\infty} = 1 - z . \quad (6)$$

When considering an arbitrary value for the threshold  $1 \leq T < \infty$ , the intensity function obeys the definition given in (1), and we find

$$1 = \sum_{k=0}^{\infty} \nu_{z,T}(k) = C_{z,T} \left[ \sum_{k=0}^T z^k + \sum_{k=T+1}^{\infty} \frac{z^k}{(k-T+1)!} \right] = C_{z,T} \left[ \sum_{k=0}^T z^k + z^{T-1} \sum_{k=2}^{\infty} \frac{z^k}{k!} \right]$$

This yields

$$C_{z,T} = \left[ \frac{1 - z^{T-1}}{1 - z} + z^{T-1} e^z \right]^{-1} \quad (7)$$

It is straightforward to verify that, for  $T = 1$ , the equation (5) is recovered. Moreover, taking the limit  $T \rightarrow \infty$  in (7), with  $z < 1$ , one finds (6).

Finally, it is of interest to compute the mean value (against the Gibbs measure) of the intensity function  $I_T$ . Indeed, as it will be clear in the following, such a quantity has a relevant physical meaning. By using (1) and (3) we get

$$\nu_{z,T}[I_T(\omega_1)] = \sum_{k=0}^{\infty} \nu_{z,T}(k) I_T(k) = C_{z,T} \left[ \sum_{k=1}^T z^k + \sum_{k=T+1}^{\infty} \frac{z^k}{(k-T+1)!} (k-T+1) \right] = z \quad (8)$$

where we have used that  $I_T(0) = 0$  and we have omitted the final algebraic computation. It is relevant to stress that the expression of such a mean, as a function of the activity, does not depend on the threshold.

### 3 Hydrodynamic limit for the ZRP with threshold

The evolution of the distribution of the particles on the space  $\Lambda$  under the zero range process with threshold  $T$  introduced above can be described in the diffusive hydrodynamic limit via the time evolution of the *density function*  $\rho(x, t)$  with the space variable  $x$  varying in the interval  $[0, 1]$  and  $t \geq 0$ .

We again refer to [12] for a detailed and precise derivation of the hydrodynamic limit. Here, we just recall the main results: we first define the function

$$\bar{\rho}_T(z) = \sum_{k=0}^{\infty} k \nu_{z,T}(k) \quad (9)$$

Note that  $\bar{\rho}_T$  is defined for any positive  $z$  for finite  $T$ , whereas, in the simple exclusion-like model, we have the limitation  $z < 1$  on the fugacity. It is possible to prove a nice expression for  $\bar{\rho}_T$ , indeed, recalling (3), equation (9) can be rewritten as

$$\bar{\rho}_T(z) = C_{z,T} \sum_{k=1}^{\infty} k \frac{z^k}{I_T(1) \cdots I_T(k)} = z C_{z,T} \frac{d}{dz} \sum_{k=1}^{\infty} \frac{z^k}{I_T(1) \cdots I_T(k)} = z C_{z,T} \frac{d}{dz} \frac{1}{C_{z,T}} \sum_{k=1}^{\infty} \nu_{z,T}(k)$$

which implies

$$\bar{\rho}_T(z) = z C_{z,T} \frac{d}{dz} \frac{1}{C_{z,T}} = -\frac{z}{C_{z,T}} \frac{d}{dz} C_{z,T} \quad (10)$$

The function  $\bar{\rho}_T(z)$  will be computed explicitly below for any value of the threshold  $T$ , see (15), (18), (21), and plotted in Fig. 1.

It is not difficult to prove that, for any  $L$  and  $T$ , the function  $\bar{\rho}_T(z)$  is an increasing function of the fugacity. Indeed, given  $L$  and  $T$ , with some straightforward algebra, one can prove that

$$\frac{\partial}{\partial z} \bar{\rho}_T(z) = \frac{\partial}{\partial z} C_{z,T} \sum_{k=1}^{\infty} \frac{kz^k}{I_T(1) \cdots I_T(k)} = \frac{1}{z} [\nu_{z,T}(\eta_1^2) - (\nu_{z,T}(\eta_1))^2] > 0 . \quad (11)$$

A similar result holds in the simple exclusion-like model. It is then possible to invert the function  $\bar{\rho}_T$ ; such an inverse function will be denoted by  $\bar{z}_T$ .

In [12] it is proven that, for symmetric jump probabilities, i.e.  $p = \frac{1}{2}$ , the continuous space density  $\rho(x, t)$  is the solution of the partial differential equation

$$\frac{\partial}{\partial t} \rho = - \frac{\partial}{\partial x} J_T(\rho) \quad (12)$$

where the *macroscopic flux*  $J_T(\rho)$  is defined as

$$J_T(\rho) = - \frac{1}{2} D_T(\rho) \frac{\partial}{\partial x} \rho \quad (13)$$

with the *diffusion coefficient*  $D_T$  given by

$$D_T(\rho) = \frac{\partial}{\partial \rho} \nu_{\bar{z}_T(\rho), T} [I_T(\omega_1)] . \quad (14)$$

Note that the diffusion coefficient is computed in terms of the mean of the intensity function evaluated against the single site Gibbs measure with fugacity corresponding to the local value of the density.

The subscript  $T$  in  $D_T$  reminds that the expression of the diffusion coefficient will crucially depend of the value of the threshold. The main point in this paper is precisely to discuss the effect of the threshold on the diffusion coefficient.

We shall first consider the limiting cases ( $T = 1$  and  $T = \infty$ ) and finally we shall compute the diffusion coefficient for any value of the threshold.

*Independent particle model.* In the case  $T = 1$ , and thus  $I_1(k) = k$  for any  $k > 0$ , recalling (5) and (10), one has

$$\bar{\rho}_1(z) = z \quad (15)$$

Hence, recalling (8) and the definition of  $\bar{z}_T$  given below (11), one finds

$$\nu_{\bar{z}_1(\rho), 1} [I_1(\omega_1)] = \nu_{\rho, 1} [I_1(\omega_1)] = \rho \quad (16)$$

Thus, by using (14), the diffusion coefficient reads

$$D_1(\rho) = \frac{\partial}{\partial \rho} \nu_{\bar{z}_1(\rho), 1} [I_1(\omega_1)] = 1 \quad (17)$$

*Simple exclusion-like model.* In the case  $I_\infty(k) = 1$  for any  $k \geq 1$  and  $I_\infty(0) = 0$ , recalling (6) and (10), one has

$$\bar{\rho}_\infty(z) = \frac{z}{1 - z} \quad (18)$$



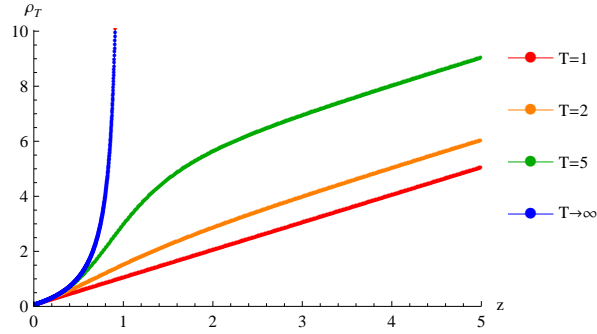


Figure 1: Graph of the function  $\bar{\rho}_T(z)$  for different values of the threshold, i.e.,  $T = 1, 2, 5, \infty$ .

Since  $\bar{z}_T$  is defined as the inverse of the function  $\bar{\rho}_T$ , we have that  $\bar{z}_\infty(\rho) = \rho/(1 + \rho)$ . Thus, by (8), we find

$$\nu_{\bar{z}_\infty(\rho), \infty} [I_\infty(\omega_1)] = \nu_{\rho/(1+\rho), \infty} [I_\infty(\omega_1)] = \frac{\rho}{1 + \rho} . \quad (19)$$

Hence, recalling (14), the diffusion coefficient reads

$$D_\infty(\rho) = \frac{1}{(1 + \rho)^2} \quad (20)$$

which is a well known result [8].

*Model with an arbitrary threshold.* When considering an arbitrary value of the threshold  $T \geq 1$ , we use (7) and (10) to compute

$$\bar{\rho}_T(z) = z \left[ \frac{1 - z^{T-1}}{1 - z} + z^{T-1} e^z \right]^{-1} \left[ \frac{1 + (T-2)z^{T-1} - (T-1)z^{T-2}}{(1-z)^2} + e^z z^{T-2} (z + T - 1) \right] \quad (21)$$

It is straightforward to verify that for  $T = 1$  the equation (15) is recovered.

Moreover, taking the limit  $T \rightarrow \infty$  in (21), with  $z < 1$ , one also recovers (18).

The expression of the diffusion coefficient can be then obtained using the general recipe in equation (14) and recalling (8). Indeed,

$$D_T(\rho) = \frac{\partial}{\partial \rho} \nu_{\bar{z}_T(\rho), T} [I_T(\omega_1)] = \frac{\partial}{\partial \rho} \bar{z}_T(\rho) = \left( \frac{\partial}{\partial z} \bar{\rho}_T(z) \right)^{-1} \Big|_{z=\bar{z}_T(\rho)} \quad (22)$$

We remark that the explicit expression of the quantity  $\bar{D}_T(z) = \partial \bar{\rho}_T(z) / \partial z$  appearing in (22) is lengthy and, therefore, it is omitted here.

We now discuss the above results in detail. The graph of the function  $\bar{\rho}_T(z)$  for different values of the threshold, i.e.,  $T = 1, 2, 5, \infty$ , is plotted in Fig. 1. First, we remark that, as proven in (11), the function  $\bar{\rho}_T(z)$  is a monotonic increasing function of the fugacity. It is worth noting that the parameter  $T$  tunes between the independent particle and the simple exclusion-like limiting models. In other words, our model interpolates between the two different regimes corresponding to  $T = 1$  and  $T = \infty$ , thus allowing to recover the expressions given, respectively, in (15) and (18). We also observe that, for any finite threshold, if  $z$  is large the effect of the

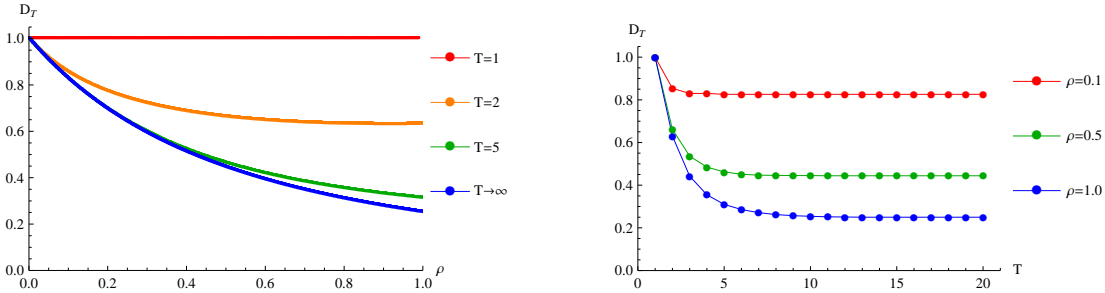


Figure 2: *Left panel:* Behavior of the diffusion coefficient  $D_T(\rho)$  vs.  $\rho$  for different values of the threshold, i.e.,  $T = 1, 2, 5, \infty$ . *Right panel:* Behavior of the diffusion coefficient  $D_T(\rho)$  vs.  $T$  for different values of the local density, i.e.,  $\rho = 0.1, 0.5, 1.0$ .

threshold becomes negligible, because, as shown in Fig. 1, the function  $\bar{\rho}_T(z)$  starts increasing linearly (similarly to the independent particle case) as  $\bar{\rho}_T(z) \approx z + (T - 1)$ .

Figure 2 shows the behavior of the diffusion coefficient as a function of the local density (left panel) and of the threshold (right panel). We notice that, in the independent particle case (i.e.,  $T = 1$ ), the diffusion coefficient is constant with respect to the local density. This is a very intuitive result, indeed the rate at which a site in the microscopic zero range process is updated is essentially proportional to the number of particles at that site.

On the other hand, as soon as the threshold  $T$  is larger than one, the diffusion coefficient is a monotonic decreasing function of the local density. This is due to the fact that, up to the threshold  $T$  on the number of particle occupying a site, the rate at which sites are updated does not depend on the number of particles on the site. This means that the dynamics favors the formation of piles on sites. This effect is larger and larger when  $T$  is increased. The limiting behavior is attained in the simple exclusion-like case (i.e.,  $T = \infty$ ), when all the sites, whatever is the number of particle on each of them, are updated with the same probability.

The picture in the right panel of Fig. 2 has a similar interpretation. Indeed, there it is shown that, fixed the local density, the corresponding diffusion coefficient is a decreasing function of the threshold. In other words, increasing the threshold has the effect of slowing down the dynamics.

## 4 Hydrodynamic limit for the ZRP with threshold under drift

In the section above we have discussed the effect of the threshold on the diffusion equation describing the macroscopic behavior of the system in the hydrodynamic limit. In this section we investigate how the dynamics depends on the threshold under the effect of an external field inducing a nonvanishing outgoing current.

We pick  $0 < p < 1$ , consider the model defined in Section 2 and assume that, once a site  $x$  of the torus  $\Lambda$  is selected, a particle leaves such a site and jumps to its right, i.e., to the site  $x + 1$ , with probability  $p$  and to its left, i.e., to the site  $x - 1$ , with probability  $1 - p$ . Recall that we assume periodic boundary condition.

The evolution of the distribution of the particles on the space  $\Lambda$  in a zero range process with threshold  $T$  under drift can be described, in the hydrodynamic limit, via the time evolution of the *density function*  $\rho(x, t)$  with the space variable  $x$  varying in the interval  $[0, 1]$  and  $t \geq 0$ .

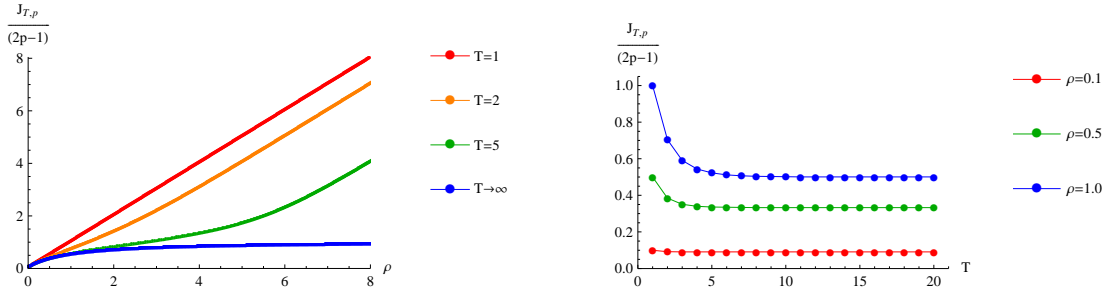


Figure 3: *Left panel:* Behavior of the macroscopic flux  $J/(2p-1)$  defined in (23) vs.  $\rho$  for different values of the threshold, i.e.,  $T = 1, 2, 5, \infty$ . *Right panel:* Behavior of the macroscopic flux  $J/(2p-1)$  defined in (23) vs.  $T$  for different values of the local density, i.e.,  $\rho = 0.1, 0.5, 1.0$ .

It can be then proven that the equation governing the evolution of the macroscopic local density  $\varrho$  is (12) with the *macroscopic flux*  $J_{T,p}(\varrho)$  defined as

$$J_{T,p}(\varrho) = (2p-1)\nu_{\bar{z}_T(\rho),T}[I(\omega_1)] \quad (23)$$

where, we recall, the intensity function is defined in (1) and the Gibbs measure is defined in (3), see [7, equation (1.3)].

We can now use our results of Section 3 to derive the explicit expression for the macroscopic flux under an external drift. First of all, we note that, for any value of the threshold, by (8), it follows that the macroscopic flux is given by

$$J_T(\rho) = (2p-1)\bar{z}_T(\rho) \quad . \quad (24)$$

It is not possible to write such an expression explicitly, since it is not possible to solve explicitly (21) with respect to  $z$ . But, in the independent particle and simple exclusion-like cases, from (15) and (18), we find

$$J_1(\rho) = \rho \quad \text{and} \quad J_\infty(\rho) = \frac{\rho}{1+\rho} \quad , \quad (25)$$

respectively.

Figure 3 shows the behavior of the macroscopic flux under drift as a function of the local density (left panel) and of the threshold (right panel). We notice that, in the independent particle case (i.e.,  $T = 1$ ), the macroscopic flux increases linearly with the local density and this is a very intuitive result.

On the other hand, as soon as the threshold  $T$  is larger than one, the macroscopic flux becomes a sub-linear increasing function of the local density, with a milder and milder behavior for large  $T$ . For  $T \rightarrow \infty$  the curve tends to the simple exclusion-like behavior.

Again, this effect can be ascribed to the fact that, up to the threshold  $T$  on the number of particle occupying a site, the rate at which sites are updated does not depend on the number of particles on the site. This means that the dynamics favors the formation of piles on sites resulting in a general slowing down of the dynamics.

Moreover, fixed the threshold, if the local density is large enough, the effect of the threshold becomes negligible, since the flux increases linearly as in the independent particle case. This follows immediately from (24) and from the fact that, as remarked in Section 3,  $\bar{\rho}_T(z) \approx z + (T-1)$  for  $z$  large, so that  $J_{T,p}(\rho) \approx (2p-1)[\rho - (T-1)]$  for  $\rho$  large.

## 5 Conclusions

We considered one-dimensional zero range processes under periodic boundary conditions with symmetric and asymmetric transition probabilities.

The novelty of our approach stems from the introduction of a parameter  $T$ , called threshold. The threshold can be tuned so as to control the magnitude of the intensity function, thus allowing one to span a broad variety of zero range dynamics, ranging from the independent particle models (for  $T = 1$ ) to the simple exclusion processes (attained in the limit  $T \rightarrow \infty$ ).

We then investigated the hydrodynamic limit of the considered zero range processes for an arbitrary value of the threshold, and discussed the effect of the latter parameter on some macroscopic quantities, e.g. the diffusion coefficient, the particle density and the outgoing current. We recover known results in the limiting scenarios, and also provide explicit formulae for the arbitrary threshold case. Our investigation thus provides a noteworthy bridge between the features of the underlying microscopic dynamics and some macroscopic quantities relevant in the hydrodynamic limit of the model, which are also experimentally accessible. Further investigation is called, next, to extend our results to the even more challenging scenario characterized by non-periodic boundary conditions.

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