

Adjust or invest : what is the best option to green a supply chain?

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Adjust or Invest: What is the Best Option to Green a Supply Chain?

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Abstract: Greening a supply chain can be achieved by considering several options. However, companies lack of clear guidelines to assess and compare these options. In this paper, we propose to use multiobjective optimization to assess operational adjustment and technology investment options in terms of cost and carbon emissions. Our study is based on a multiobjective formulation of the economic order quantity model called the sustainable order quantity model. The results show that both options may be effective to lower the impacts of logistics operations. We also provide analytical conditions under which an option outperforms the other one for two classical decision rules, i.e. the carbon cap and the carbon tax cases. The results allow deriving some interesting and potentially impacting practical insights.

Keywords: Green supply chain, inventory control, multiobjective optimization, sustainable order quantity.

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1 Introduction

Environmental awareness has considerably increased since the Brundtland's report publication (World Commission on Environment and Development, 1987). Nowadays, customers, investors, employees and other stakeholders consider that greening the supply chain is a key issue for companies. In response, two thirds of the European companies have for instance intensified their green actions over the past three years (Bearing Point, 2010). One of the main challenges when greening a supply chain consists in reducing carbon emissions. The logistics industry is indeed responsible for around 5.5% of global greenhouse gas emissions worldwide. These emissions are mainly generated by transportation. Nevertheless, warehousing contributes to 13% of the sector's carbon footprint mainly due to indirect emissions from electricity consumption (World Economic Forum, 2009). This paper thus focuses on inventory models as they aim at finding a good balance between transportation and warehousing impacts.

When intending to reduce the carbon footprint of a supply chain, companies first focus on reorganization projects that quickly lead to win-win situations, i.e. projects that contribute to reduce both cost and carbon emissions in the short term. For instance, a transportation optimization project that decreases the travelled distance will quickly reduce cost and carbon emissions. Nowadays, companies have however begun to exhaust these low-hanging fruits leading to short term win-win situations and start thinking that "sustainability can only be attained by optimizing seemingly conflicting targets" (DHL, 2010). The Bearing Point 2010 survey highlights that "more than one third of the 582 interviewed companies declare being ready to start up environmental actions in spite of their low present profitability, provided they create value in the medium term" (Bearing Point, 2010). This paper thus focuses on situations where carbon footprint reduction can only be achieved by increasing the operating cost.

Going beyond the win-win situations, which may be reached by obvious actions, does not seem so trivial. The decision makers are generally willing to invest in the latest carbon-reducing technology. For instance, a third-party logistics company can invest in greener trucks. In the short term, this investment will increase the cost of operations while reducing the carbon footprint of the supply chain. However, it may be profitable for the company in the

long term. Several types of technology investment may be applied to transportation and warehousing in order to reduce the carbon footprint of the supply chain. Another option is proposed by Benjaafar et al. (2010) who “study the extent to which carbon reduction requirements can be addressed by operational adjustments, as an alternative (or a supplement) to costly investments in carbon-reducing technologies”. Chen et al. (2011) have indeed demonstrated that significant reductions in carbon emissions can be obtained without significantly increasing cost by making only adjustments in the order quantities for the economic order quantity (EOQ) model.

This paper thus intends to assess operational adjustment and technology investment options in terms of cost and carbon emissions. To do so, we model both options in a multiobjective formulation of the EOQ model called the sustainable order quantity (SOQ) model (Bouchery et al., 2011). The results show that both options should be taken into consideration when intending to green the supply chain. It gives additional flexibility to supply chain managers who are likely to be focused on investing in carbon reducing technology. We also provide analytical conditions under which an option outperforms the other one for two classical decision rules.

The paper is organized as follows. Section 2 is devoted to the presentation of the model and to multiobjective optimization results. Operational adjustment and technology investment options are first expressed in the SOQ framework. Then we show that both options can be of interest to reduce the carbon footprint of the supply chain. Section 3 is devoted to the study of two common decisions rules. The first one consists of choosing an upper limit on carbon emissions and the second one is based on carbon pricing. For both of them, we provide analytical conditions under which an option outperforms the other one. Finally, the advantages and drawbacks of several regulatory policies are discussed in Section 4.

2 Model formulation

2.1 Modeling carbon emissions in the EOQ framework

The EOQ model is a rather simple inventory model that balances ordering and warehousing costs. After a short presentation of the traditional model, we show how carbon emissions can be included in this framework.

In the EOQ model, the final demand is assumed to be constant and approximated as continuous, the ordering leadtime is fixed and no shortage is allowed. The average total cost per time unit has the following expression:

$$Z_c(Q) = P_c D + \frac{Q}{2} h_c + \frac{D}{Q} O_c, \quad (1)$$

with:

Q = batch quantity (decision variable),

D = demand per time unit,

h_c = constant inventory holding cost per product unit and time unit,

P_c = Purchasing cost per product unit,

O_c = fixed ordering or setup cost.

The cost function Z_c is strictly convex for $Q \in \mathfrak{R}_+^*$ and the optimal batch quantity can then be expressed as follows:

$$Q_c^* = \sqrt{\frac{2O_c D}{h_c}}. \quad (2)$$

As the fixed purchasing cost P_c does not affect the order quantity, it will be omitted in what follows.

Including carbon footprint concerns into inventory models is a new challenge that triggers more and more research. Several papers model carbon emissions in the EOQ framework. Bonney and Jaber (2011) briefly study the impact of including vehicle emissions cost into the EOQ model. Emissions associated with the storage of products are not taken into account. The order quantity is thus larger than the classical EOQ. Hua et al. (2011) study how the carbon emissions trading mechanism can influence optimal order quantity in the EOQ framework. Carbon emissions issued from both transportation and warehousing are taken into account. The author present analytical and numerical results and provide some managerial insights. Arslan and Turkay (2010) include carbon emissions and working hours into the EOQ model. Five regulatory policies are studied, i.e. the carbon tax policy, the carbon cap one, the cap-and-trade system, the possibility to invest in carbon offsets and the case where the carbon footprint is treated as an additional source of economic cost. Finally, Chen et al. (2011) investigate how operational adjustment can be used to reduce carbon emissions under a constraint on carbon emissions in the EOQ model.

As for the papers cited above (except for Bonney and Jaber (2011) where emissions associated with the storage of the products are not considered), we adopt here the following expression of the average carbon footprint per time unit:

$$Z_E(Q) = P_E D + \frac{Q}{2} h_E + \frac{D}{Q} O_E, \quad (3)$$

with:

Q = batch quantity (decision variable),

D = demand per time unit,

h_E = constant inventory holding emissions per product unit and time unit,

P_E = Carbon emissions per purchased product unit,

O_E = fixed ordering or setup emissions.

The fixed amount of carbon emissions per order O_E represents the emissions related to order processing and transportation. An amount of carbon emissions h_E is also associated with the storage of each unit per time unit. This amount can become important in case of refrigeration. Finally, a fixed amount of emissions per purchased unit P_E is also used. It represents the emissions associated with the manufacturing of the product. These emissions parameters correspond to both direct emissions from fuel consumption and indirect emissions from electricity consumption. The emissions function Z_E is strictly convex for $Q \in \mathfrak{R}_+^*$ and the optimal batch quantity can then be expressed as follows:

$$Q_E^* = \sqrt{\frac{2O_E D}{h_E}}. \quad (4)$$

As P_E does not affect the order quantity, it will be omitted in what follows.

2.2 Operational adjustment

In all the papers presented in Section 2.1, the technical implications of including carbon footprint into the EOQ model consist in penalizing the cost objective function and / or adding a new constraint to the model. However, we believe that a more general approach may be to consider that minimizing carbon emissions is, in itself, an objective for the company like the economic cost of operations. In this case, two seemingly conflicting objectives have to be minimized.

Bouchery et al. (2011) thus study a multiobjective version of the EOQ called the SOQ model. In a multiobjective optimization problem, a set of alternatives (operational decisions) A is evaluated on a family of n objectives $Z_1; Z_2; \dots; Z_n$ with $Z_i : A \rightarrow \mathfrak{R}$ for all $i \in [1, n]$. In most of the cases, it is impossible to find an alternative that is optimal for all objectives. Multiobjective optimization thus aims at identifying all the interesting solutions, i.e. the solutions that represent potentially attracting trade-offs. In this paper, the objectives must be minimized. An alternative $a \in A$ is thus said to be dominated if there exists $b \in A$ such that $Z_i(b) \leq Z_i(a)$ for all $i \in [1, n]$ with at least one strict inequality. Multiobjective optimization consists in identifying all the non-dominated alternatives called efficient solutions or Pareto optimal solutions. In this paper, $n = 2$ objectives (the cost and the carbon footprint) are taken into consideration.

In the EOQ model, the set of possible values for Q is $A = \mathfrak{R}_+^*$. Let $Z : A \rightarrow \mathfrak{R}^2$, $Z(a) = \{Z_C(a); Z_E(a)\}$, for all $a \in A$, with Z_C defined by Formula 1 representing the total cost of operations and Z_E defined by Formula 3 representing the total carbon emissions. $Z(A) = \{(Z_C(Q); Z_E(Q)) | Q \in A\}$ is the image of A in the criterion space (evaluation space). The set of efficient solutions also called the efficient frontier is a subset of A noted E . Its image in the criterion space is $Z(E)$. The following Theorem identifies analytically this efficient frontier in function of Q_C^* and Q_E^* , the optimal quantities defined by Formulas 2 and 4 respectively.

Theorem 1. Let E be the efficient frontier of the problem, then:

$$E = [\min(Q_C^*, Q_E^*); \max(Q_C^*, Q_E^*)].$$

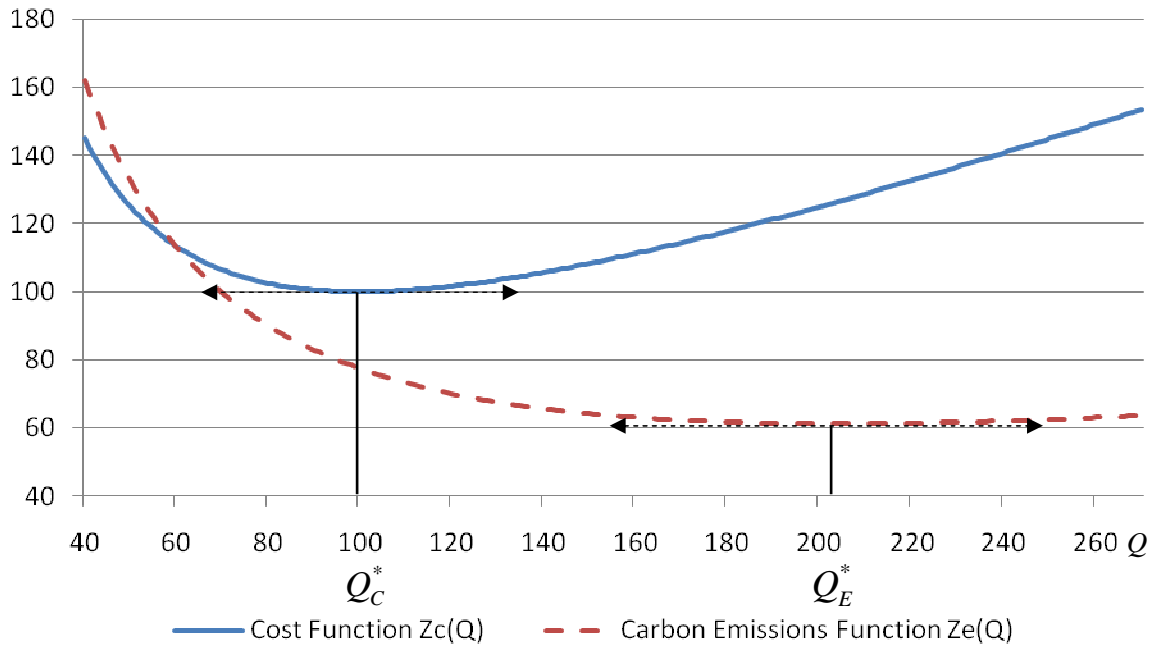
This result is proven in Appendix A. It shows that it is possible to reduce the carbon emissions of a supply chain by modifying the batch size (from the economic order quantity) if $Q_E^* \neq Q_C^*$. This condition is equivalent to:

$$\frac{O_E}{h_E} \neq \frac{O_C}{h_C}. \quad (5)$$

In what follows, this batch size modification is called an operational adjustment.

Let us consider the following illustrative example. Let $D = 25$ product units per time unit, $O_C = 200$, $h_C = 1$, $O_E = 250$ and $h_E = 0.3$. It can be noticed that the parameters' units are not given. Indeed, they are not useful as only the ratios O/h matter. Applying Formula 2 and 4 implies that $Z_C(Q)$ is minimum for $Q_C^* \approx 100$ and $Z_E(Q)$ for $Q_E^* \approx 204$. Figure 1 illustrates the results.

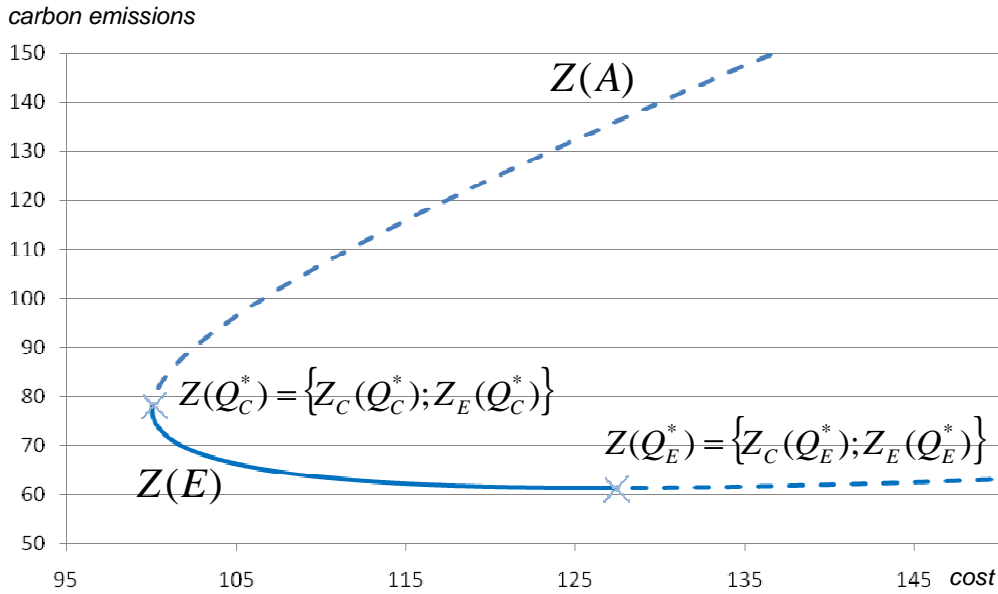
Figure 1
Cost and carbon emissions in function of the batch size



By applying Theorem 1, we obtain that $E = [\min(Q_C^*, Q_E^*); \max(Q_C^*, Q_E^*)] = [100; 204]$. Figure 2 displays the results in the criterion space.

Assume that the current situation is cost optimized. Figure 2 shows that a significant carbon emissions reduction can be achieved by increasing the batch size starting from Q_C^* . Moreover, the required financial effort remains reasonable for a significant carbon emissions reduction. For instance, the carbon emissions can be reduced by almost 15% for a 5% cost increase in the presented example. This feature can be explained with Figure 1 as the flat region of the cost function coincides with a steeper region of the emissions function. Chen et al. (2011) provide conditions under which the relative reduction in emissions is greater than the relative increase in cost for the EOQ model. On the opposite, the financial effort will increase as Q is getting closer to Q_E^* , the batch size that minimizes the amount of carbon emissions.

Figure 2
The images of the feasible solutions and the efficient solutions in the criterion space



2.3 Technology investment

In the previous section, we define the operational adjustment option and we illustrate how it can be used to reduce the carbon footprint of a supply chain. However, companies can also invest in carbon-reducing technologies to curb emissions. In this section, we show how to model a green technology investment option in the SOQ framework.

A technology investment is either a tactical or a strategic decision. However, it has an impact on operational decisions. In the SOQ framework, carbon emissions result from both ordering and warehousing. An investment in a carbon-reducing technology can then modify the ordering and / or the holding parameters of the model. For instance, investing in the latest available lighting technology can reduce the electricity consumption of a warehouse as up to 80% of electricity consumption in a logistics facility typically comes from lighting (DHL, 2010). This investment thus decreases the carbon footprint of holding products. However, it increases the inventory holding cost as this investment is included in the cost of operating the warehouse. Other examples of green technology investments for warehouses are improvements on heating and cooling systems and investments in better insulation. Some green technology investments can also decrease the ordering carbon emissions parameter. For instance, investing in hybrid or electric vehicles will decrease the emissions related to transportation. Investment can also be done to improve the aerodynamics of the transportation

vehicle. This investment can be done directly by the company but it can also be made by a supplier. A third party logistics provider may for instance be asked to use greener trucks. The logistics provider may thus charge the customers with a fixed cost per delivery to support this investment.

In summary, a technology investment enables reducing a carbon emissions parameter (either O_E or h_E) by requiring an increase in a cost parameter (either O_C or h_C). In what follows, we focus on ordering parameters as transportation is recognized as a major source of carbon emissions in supply chains.

An investment in a carbon-reducing technology T may thus be modeled as follows:

- The new fixed ordering carbon emissions parameter is O_E^T with $O_E^T < O_E$,
- the new fixed ordering cost parameter is O_C^T with $O_C^T > O_C$.

The new average cost function is:

$$Z_C^T(Q) = \frac{Q}{2} h_C + \frac{D}{Q} O_C^T, \quad (6)$$

and the new average carbon emissions function has the following expression:

$$Z_E^T(Q) = \frac{Q}{2} h_E + \frac{D}{Q} O_E^T. \quad (7)$$

By directly applying the results of Sections 2.1 and 2.2, we obtain that:

$$Q_C^{T*} = \sqrt{\frac{2O_C^T D}{h_C}} > Q_C^*, \quad (8)$$

$$Q_E^{T*} = \sqrt{\frac{2O_E^T D}{h_E}} < Q_E^*, \quad (9)$$

$$\text{and } E^T = [\min(Q_C^{T*}, Q_E^{T*}); \max(Q_C^{T*}, Q_E^{T*})], \quad (10)$$

with E^T being the efficient frontier of the SOQ problem in the technology investment case.

As $O_E^T < O_E$, the following expression holds:

$$Z_E^T(Q_E^{T*}) = \sqrt{2O_E^T D h_E} < Z_E(Q_E^*) = \sqrt{2O_E D h_E}. \quad (11)$$

Finally, as $O_c^T > O_c$, we obtain that:

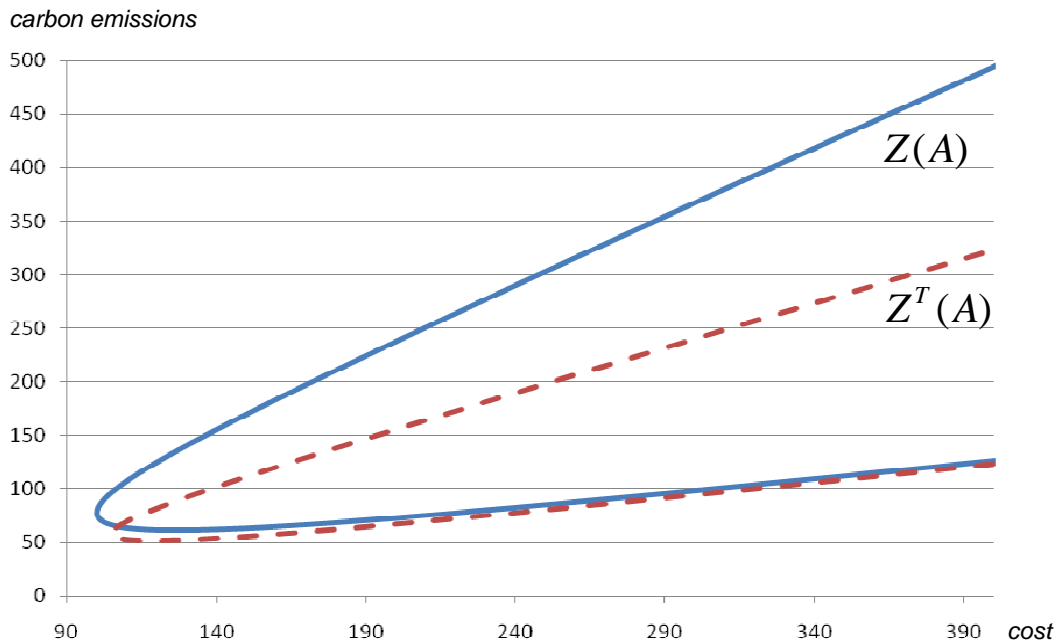
$$Z_c^T(Q_c^{T*}) = \sqrt{2O_c^T Dh_c} > Z_c(Q_c^*) = \sqrt{2O_c Dh_c} . \quad (12)$$

In what follows, we intend to compare technology investment and operational adjustment options in terms of cost and carbon emissions. It is important to note that in the technology investment case, an operational adjustment may also be needed as a supplement. The case where an operational adjustment is made solely is thus referred to as the operational adjustment case.

2.4 Operational adjustment option versus technology investment option

Let us assume that a company is considering both operational adjustment and technology investment options to green its supply chain. To illustrate the situation, the example of Section 2.2 is adapted by assuming that the company has also the possibility to invest in a technology with the following parameters: $O_c^T = 220 (> O_c = 200)$ and $O_E^T = 180 (< O_E = 250)$. Figure 3 represents the image of the feasible solutions in the criterion space for both the operational adjustment option and the technology investment one.

Figure 3
operational adjustment case and technology investment case in the criterion space



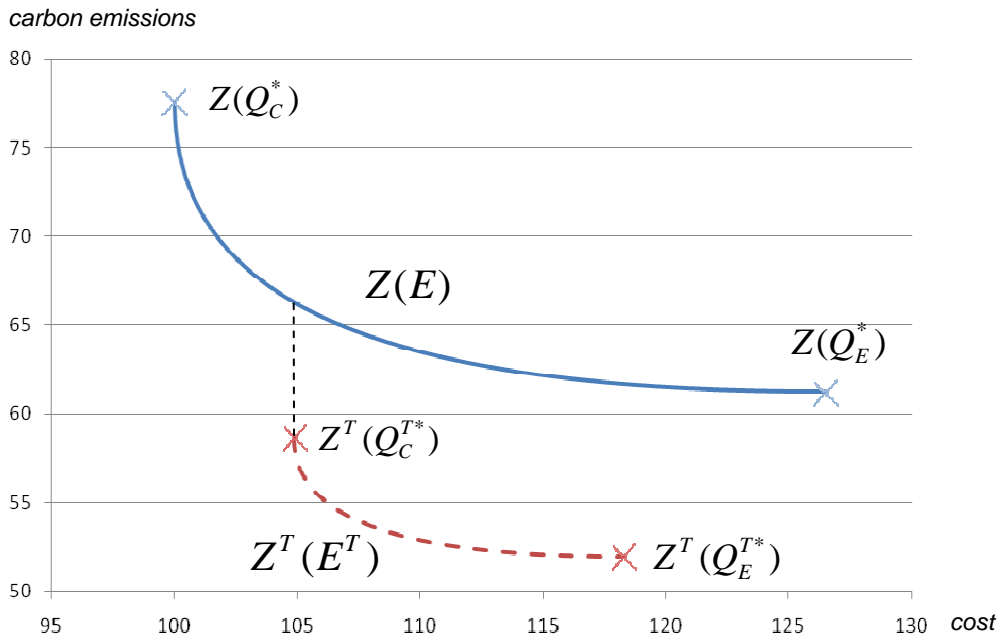
Note that $Z^T(A) = \{(Z_C^T(Q); Z_E^T(Q)) \mid Q \in A\}$ corresponds to the image of the feasible solutions for the technology investment option. It can be noticed in Figure 3 that there is a single intersection point between $Z(A)$ and $Z^T(A)$. More generally, the following result holds:

Theorem 2. Let $Z(A)$ and $Z^T(A)$ be the images of the feasible solutions for the operational adjustment option and the technology investment option:

- If $\frac{O_E^T}{O_C^T} \leq \frac{h_E}{h_C} \leq \frac{O_E}{O_C}$ then $Z^T(A) \cap Z(A)$ is empty,
- else $Z^T(A) \cap Z(A)$ is a singleton.

This result is proven in Appendix A. Figure 4 illustrates the trade-offs that a company can face when deciding on technology investment and on the batch size. In general, the image of the global problem efficient frontier is included into $Z(E) \cup Z^T(E^T)$. However, we cannot assert that all elements of $Z(E)$ and $Z^T(E^T)$ are efficient.

Figure 4
Images of the efficient frontiers in the criterion space



In this example, the elements of $Z(E)$ at the right of the vertical dash line starting from $Z^T(Q_C^{T*})$ are indeed dominated by this latter solution. By using Formula 11 and 12, we can

assert that in the general case, the image of the global problem efficient frontier contains at least one element of $Z(E)$ and at least one element of $Z^T(E^T)$. Multiobjective optimization provides a strong support for decision makers as it enables determining and visualizing the interesting trade-offs (efficient solutions). However, the best option to green a supply chain will depend on the chosen trade-off. Two common decisions rules are thus studied in the following section. The first one consists of choosing an upper limit on carbon emissions and the second one is based on carbon pricing.

3 The best option to green a supply chain

3.1 The carbon cap case

This paper aims at evaluating operational adjustment and technology investment options with respect to both cost and carbon emissions. Results of Section 2 show that both options are effective to reduce the carbon emissions of operations. However, the best option to green a supply chain cannot be determined without deciding on a trade-off between cost and carbon emissions.

In this section, we consider that the decisions rule consists of choosing an upper limit on carbon emissions. This decision can either come from a voluntary effort of the company or it can be imposed by government regulations. This upper limit is noted CAP and is expressed in the same unit as h_E , O_E and O_E^T . We further assume that $CAP \geq Z_E^T(Q_E^{T*})$, otherwise, no feasible solution exists for the given technology investment option. In this context, operational adjustment will perform better if the carbon cap is high enough and technology investment is the best option for a low value of CAP . This result is stated in Theorem 3.

Theorem 3. Assume that the company faces an upper limit on carbon emissions noted CAP , then there exists a threshold L_E on carbon emissions such that:

- If $CAP > L_E$, operational adjustment performs better than technology investment,
- if $CAP < L_E$, technology investment is the best option.

Theorem 3 is proven in Appendix A. In what follows, L_E is analytically determined so that the results of Theorem 3 are fully useful in practice. Two cases must be considered depending on the efficiency of $Z(Q_E^*)$ for the global problem.

Case 1:

If $Z(Q_E^*)$ is an efficient solution for the global problem, then $L_E = Z_E(Q_E^*) = \sqrt{2O_E D h_E}$.

As $Z(Q_E^*)$ is included into $Z(E)$, it can only be dominated by an element of $Z^T(E^T)$. Moreover, due to the properties of Z and Z^T demonstrated in Appendix A, $Z(Q_E^*)$ is dominated if and only if there exists $Q_D \in \mathfrak{R}_+^*$ such that

$$\begin{cases} Z_C^T(Q_D) = Z_C(Q_E^*) \\ Z_E^T(Q_D) < Z_E(Q_E^*) \end{cases} \quad (13)$$

The condition “ $Z(Q_E^*)$ is an efficient solution for the global problem” can thus be expressed as follows:

$$Z_C^T(Q) = Z_C(Q_E^*) \Rightarrow Z_E^T(Q) > Z_E(Q_E^*) \text{ for all } Q \in \mathfrak{R}_+^*. \quad (14)$$

In Expression 14, the equation $Z_C^T(Q) = Z_C(Q_E^*)$ is equivalent to:

$$\frac{h_C}{2} Q^2 - \sqrt{\frac{D}{2O_E h_E}} (O_C h_E + O_E h_C) Q + O_C^T D = 0. \quad (15)$$

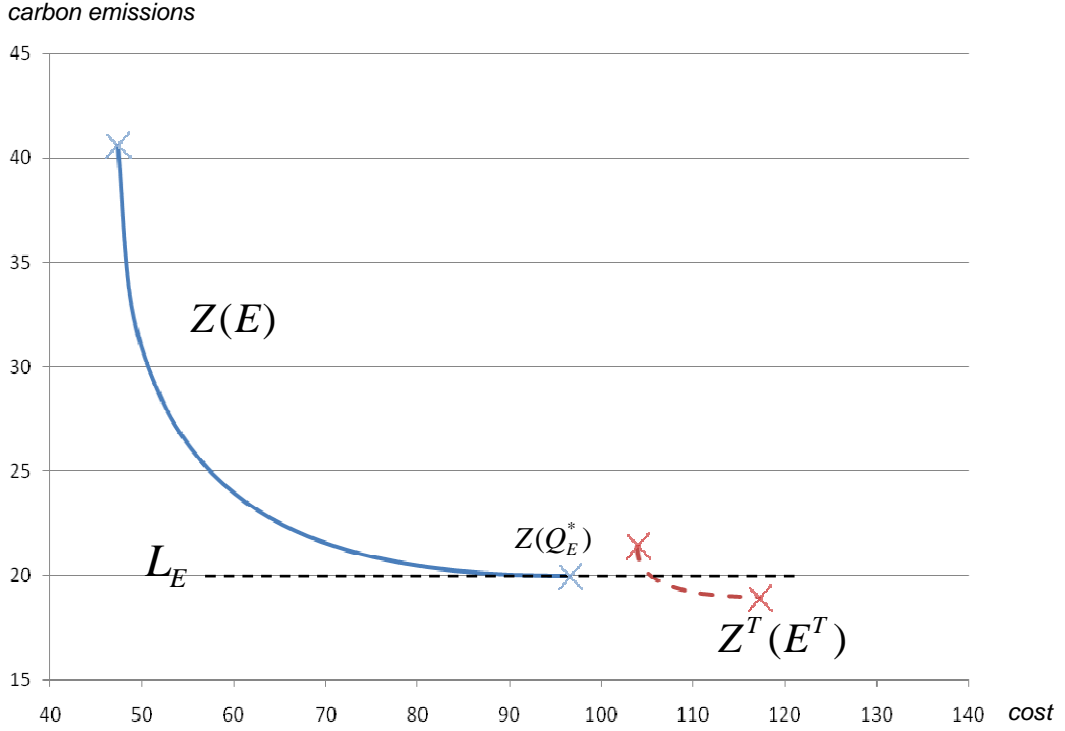
If Equation 15 does not have any feasible solution then Expression 14 is verified. Else, assume that Q_1 and Q_2 are the roots of Equation 15 (not necessarily distinct). By calculating $Z_E^T(Q_1)$, $Z_E^T(Q_2)$ and $Z_E(Q_E^*)$, Condition 14 can be easily verified.

Case 1 is illustrated with the following example. Let $D = 10$ product units per time unit, $O_C = 125$, $O_C^T = 600$, $h_C = 0.9$, $O_E = 200$, $O_E^T = 180$ and $h_E = 0.1$. Figure 5 illustrates the situation. For this example, the discriminant of Equation 15 is negative. $Z(Q_E^*)$ is thus efficient for the global problem, then $L_E = Z_E(Q_E^*) = 20$.

This means that according to the relative position of the emissions limit *CAP* regarding L_E , one of these two options will be preferred. It can also be noticed that in the case where

$CAP = L_E$, operational adjustment is the best option if $Z(Q_E^*)$ is an efficient solution for the global problem.

Figure 5
Illustration of Case 1



Case 2:

If $Z(Q_E^*)$ is not an efficient solution for the global problem, two subcases should be considered.

Case 2.1:

If $Z(E) \cap Z^T(E^T)$ is non empty, then the single intersection point is noted $\{C_\cap; E_\cap\}$ and $L_E = E_\cap$.

By applying theorem 2, we know that if $\frac{O_E^T}{O_C^T} \leq \frac{h_E}{h_C} \leq \frac{O_E}{O_C}$, then $Z(E) \cap Z^T(E^T)$ is empty.

Else, there exists a single solution $(Q; Q^T)$ such that:

$$\begin{cases} Z_C(Q) = Z_C^T(Q^T) \\ Z_E(Q) = Z_E^T(Q^T) \end{cases} \quad (16)$$

If $Q \in E$ and $Q^T \in E^T$ then, $L_E = Z_E(Q) = Z_E^T(Q^T)$. Else $Z(E) \cap Z^T(E^T)$ is empty.

Case 2.1 is illustrated in the following example. Let $D = 15$ product units per time unit, $O_C = 400$, $O_C^T = 450$, $h_C = 0.8$, $O_E = 50$, $O_E^T = 35$ and $h_E = 0.25$. By using Theorem 1 and Formula 10, we get that $E \approx [77;122]$ and $E^T \approx [65;130]$. Figure 6 illustrates the images of the efficient frontiers in the criterion space.

Figure 6
Illustration of Case 2.1

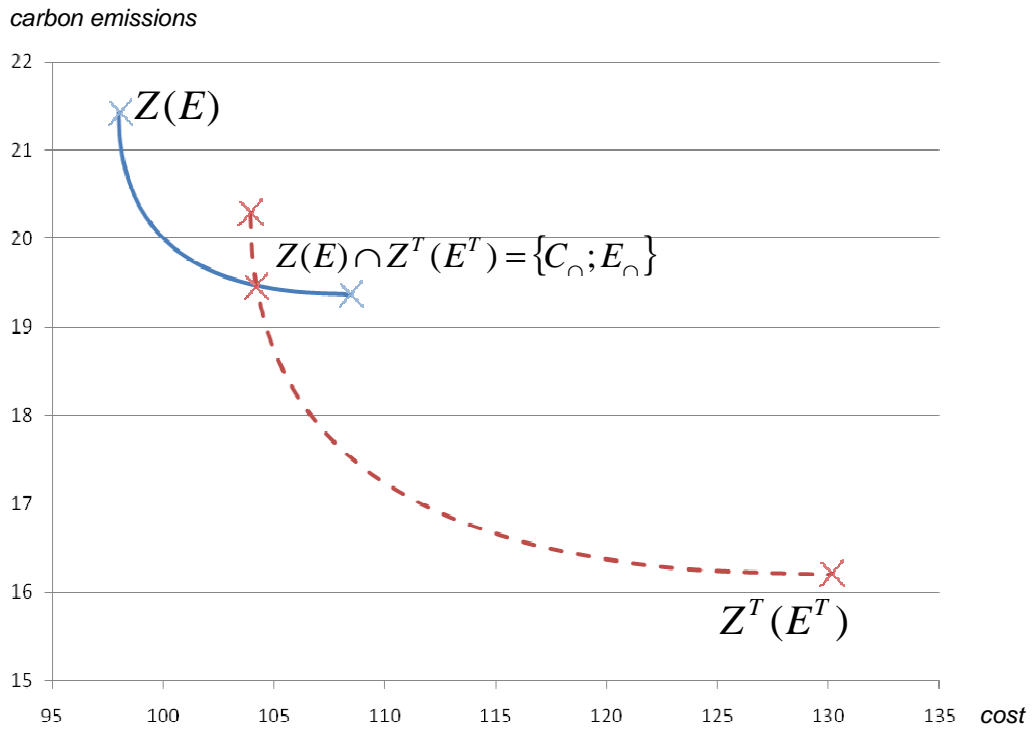


Figure 6 shows that there exists an intersection between $Z(E)$ and $Z^T(E^T)$. By solving System 16, we obtain that $Q \approx 86 \in E$, $Q^T \approx 121 \in E^T$ and $\{C_\cap; E_\cap\} \approx \{104; 19.5\}$. It can then be concluded that $L_E = E_\cap \approx 19.5$.

It can also be noticed that in the case where $CAP = L_E$, both operational adjustment and technology investment options allow to achieve the emissions limit CAP . Nevertheless, operational adjustment may be preferred in this case as this operational decision can be quickly reassessed relatively to technology investment option.

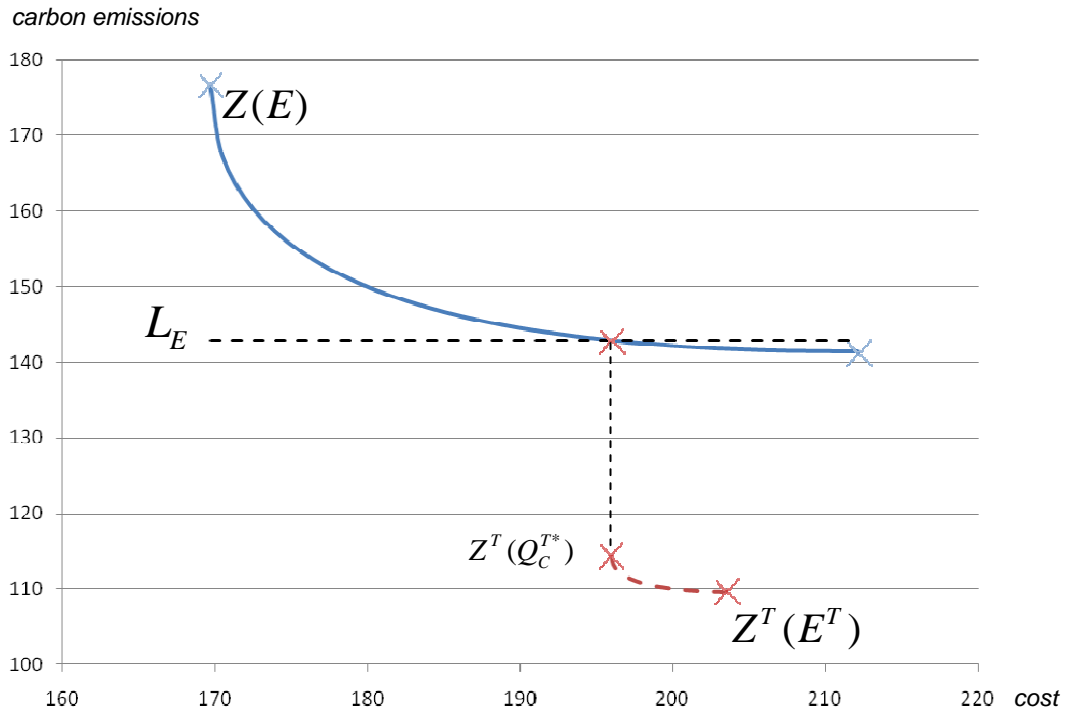
Case 2.2:

If $Z(Q_E^*)$ is not an efficient solution for the global problem and if $Z(E) \cap Z^T(E^T)$ is empty, then there exists Q_{L_E} such that $Z_C(Q_{L_E}) = Z_C^T(Q_C^{T*}) = \sqrt{O_C^T D h_C}$ and $L_E = Z_E(Q_{L_E})$.

$$\text{Moreover, } Q_{L_E} = \arg \min \left(Z_E \left(\sqrt{\frac{2D}{h_C}} (\sqrt{O_C^T} - \sqrt{O_C^T - O_C}) \right); Z_E \left(\sqrt{\frac{2D}{h_C}} (\sqrt{O_C^T} + \sqrt{O_C^T - O_C}) \right) \right).$$

Case 2.2 is illustrated with $D = 40$ product units per time unit, $O_C = 300$, $O_C^T = 400$, $h_C = 1.2$, $O_E = 500$, $O_E^T = 300$ and $h_E = 0.5$. The images of the efficient frontiers are sketched in Figure 7.

Figure 7
Illustration of Case 2.2



$Z(E) \cap Z^T(E^T)$ is clearly empty (System 16 could also be solved). To verify that $Z(Q_E^*)$ is not efficient, we first compute the discriminant of Equation 15 that is equal to 6600. Equation 15 then possesses two distinct roots $Q_1 \approx 109$ and $Q_2 \approx 244$. $Z_E(Q_E^*) = 141$, $Z_E^T(Q_1) \approx 137$ and $Z_E^T(Q_2) \approx 110$ so Condition 14 is not verified. We can then conclude that $Z(Q_E^*)$ is not an efficient solution for the global problem (as it can be graphically noticed).

$$Q_{L_E} = \arg \min \left(Z_E \left(\sqrt{\frac{2D}{h_C}} \left(\sqrt{O_C^T} - \sqrt{O_C^T - O_C} \right) \right); Z_E \left(\sqrt{\frac{2D}{h_C}} \left(\sqrt{O_C^T} + \sqrt{O_C^T - O_C} \right) \right) \right) \approx 245 \quad \text{then}$$

$L_E = Z_E(Q_{L_E}) = 143$. It can be noticed that if $CAP = L_E$ in Case 2.2, the technology investment option outperforms the operational adjustment one.

We have thus shown how to determine L_E in all the situations. In the next section, we will focus on another decision rule based on a carbon tax.

3.2 The carbon tax case

In this section, we prove that the best option among operational adjustment and technology investment is obtained by verifying a simple condition on the company parameters through a carbon tax policy. So let us consider that a cost is associated to carbon emissions. This cost can be imposed to the company in the case of a carbon tax. However, it can also come from an internal evaluation from the company, by considering the cost of the energy used or the cost issued from an environmental accounting analysis. This cost per amount of carbon emissions is noted $\alpha \in [0; \infty)$. The decision problem can then be formulated as determining:

$$\min_{Q \in \mathfrak{X}_+^*} (Z_C(Q) + \alpha Z_E(Q); Z_C^T(Q) + \alpha Z_E^T(Q)). \quad (17)$$

In this context, there exists a value $L_C \in (0; \infty)$ such that if $\alpha < L_C$, the operational adjustment option performs better than the technology investment one. On the opposite, the technology investment option is the best option if $\alpha > L_C$. Moreover, $L_C = \frac{O_C^T - O_C}{O_E - O_E^T}$. This result is stated

in Theorem 4.

Theorem 4. Assume that a carbon cost noted $\alpha \in [0; \infty)$ is given, then:

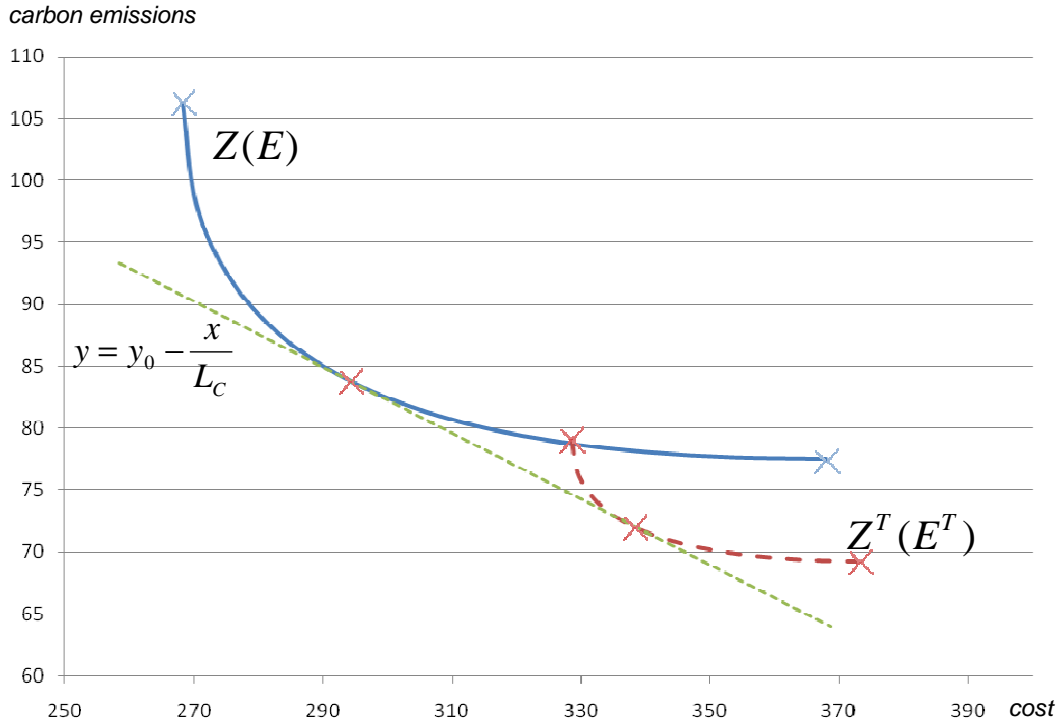
- If $\alpha < L_C = \frac{O_C^T - O_C}{O_E - O_E^T}$, then the operational adjustment option outperforms

the technology investment one,

-if $\alpha > L_C = \frac{O_C^T - O_C}{O_E - O_E^T}$, then technology investment is the best option.

Theorem 4 is proven in Appendix A. This result is illustrated with the following example. Assume that $D = 150$ product units per time unit, $O_C = 300$, $O_C^T = 450$, $h_C = 0.8$, $O_E = 200$, $O_E^T = 160$ and $h_E = 0.1$. For this example, $L_C = \frac{O_C^T - O_C}{O_E - O_E^T} = 3.75$. The situation is illustrated in Figure 8.

Figure 8
The carbon tax case



In the criterion space, for $\alpha \in (0; \infty)$, the problem stated in Formula 17 is equivalent to find the tangent points between $Z(E) \cup Z^T(E^T)$ and a straight line of slope $-\frac{1}{\alpha}$. It is thus equivalent to minimize $y_0 \in \mathfrak{R}$ such that $\left\{x \in \mathfrak{R}; y = y_0 - \frac{x}{\alpha}\right\} \cap \{Z(E) \cup Z^T(E^T)\}$ is not empty. If $\alpha < L_C$, $-\frac{1}{\alpha} < -\frac{1}{L_C}$ then the problem stated in Formula 17 is solved with an operational adjustment. On the other hand, if $\alpha > L_C$, $-\frac{1}{\alpha} > -\frac{1}{L_C}$ then the problem stated in Formula 17 is solved with a technology investment.

The case where $\alpha = L_C = \frac{O_C^T - O_C}{O_E - O_E^T}$ is particularly interesting. Operational adjustment and technology investment indeed give the same overall result (cost of operations + tax) and the optimal order quantity is also the same:

$$Q^* = \sqrt{\frac{2(O_C^T O_E - O_C O_E^T)D}{h_C(O_E - O_E^T) + h_E(O_C^T - O_C)}}. \quad (18)$$

In the previous example, we obtain that $Q^* \approx 518$, $Z_C(Q^*) + L_C \cdot Z_E(Q^*) \approx 294 + 3.75 \cdot 84 \approx 608$ and $Z_C^T(Q^*) + L_C \cdot Z_E^T(Q^*) \approx 337 + 3.75 \cdot 72 \approx 608$. As the overall cost is the same for both options, the company can arbitrarily adopt one of these two solutions. However, each option corresponds to different impacts in terms of cost and carbon footprint. In the previous example, the cost of operations is equal to $Z_C(Q^*) \approx 294$ for the operational adjustment option and $Z_C^T(Q^*) \approx 337$ for the technology investment one. The carbon emissions also differ with $Z_E(Q^*) \approx 84$ for the operational adjustment option and $Z_E^T(Q^*) \approx 72$ for the technology investment one.

Two different decision rules were studied in this section. For both the carbon cap and the carbon tax cases, we have proven that there exists a limit value that allows deciding between the operational adjustment option and the technology investment one. In the next section, the advantages and drawbacks of several regulatory policies are illustrated using the results of Section 3.

4 Discussion and conclusion

This paper uses a multiobjective formulation of the EOQ model called the SOQ model to evaluate how operational adjustment and technology investment can be used to green the supply chain. In Section 2, we prove that the efficient frontier of the global problem contains some technology investment solutions as well as some operational adjustment ones. Both options are thus effective to reduce carbon emissions. Two common decisions rules are then studied in Section 3. In the carbon cap case, we prove that the best option among operational adjustment and technology investment is obtained by verifying a simple condition on the

company parameters. The same kind of result is also demonstrated in the carbon tax case. These results show that both technology investment and operational adjustment should be taken into consideration when intending to green a supply chain. It gives additional flexibility to supply chain managers who are likely to be focused on investing in carbon reducing technology. Note that the results of this paper can also be directly extended to the case where several technologies are available.

Let us now focus on situations where carbon emissions have to be reduced in response to regulatory policies. In this case, two types of questions must be answered. First, policy makers should determine and implement the most effective regulatory policy. Then companies have to react by identifying the best option to comply with the regulation. The results presented in this paper clearly answer to the second question. However, they can also be used to discuss the first question. Our results indeed show that controlling emissions via a carbon price has some technical drawbacks. Carbon emissions are controlled by a carbon price for the carbon tax policy as well as for the cap and trade system. Hua et al. (2011) have indeed proven that emissions levels depend only on the carbon price in the EOQ model with a fixed carbon price under the cap and trade system. In this case, the minimum amount of emissions cannot be achieved as it would imply an infinite carbon price. Moreover, the financial effort will considerably increase as getting closer to the minimum amount of emissions as both operational cost and emissions cost will significantly increase. The case

where the carbon cost is $\alpha = \frac{O_C^T - O_C}{O_E - O_E^T}$ studied in Section 3.2 reveals another drawback of the

carbon tax policy and the cap and trade system. For this given carbon price, operational adjustment and technology investment give the same overall result with different cost and carbon emissions levels. At a macroeconomic level, this operational flexibility implies that the total amount of carbon emissions is hardly controllable by setting a carbon price.

Whatever the chosen value of α , some companies may face the case $\alpha = \frac{O_C^T - O_C}{O_E - O_E^T}$. These

companies may thus be able to choose among several carbon emissions levels. However, governments are interested in designing regulatory policies that enable to predict and manage the global amount of carbon emissions as many countries have ratified the Kyoto protocol mainly based on a negotiated carbon cap for each country (UNFCCC, 1997). A regulatory policy based on a carbon price gives unexpected flexibility to companies but, on the other hand, it limits the possibilities. Some interesting operational solutions are indeed ruled out

whatever the chosen carbon price is. In Figure 8, each efficient solution with an emissions level between (72;84) is unreachable for any given value of $\alpha \in [0; \infty)$. This can be seen as a limitation induced by setting a carbon price. As a result, using an upper limit on carbon emissions seems to be more effective to green supply chains as the previous drawbacks are avoided. Moreover, using a carbon cap is in accordance with the concept of sustainability that states that some ecological services are critical to life support and cannot be substituted (Neumayer, 2004). However, this kind of regulatory policy may be harder to implement as a cap has to be set up for each company.

Several research directions can then be considered. First, other inventory models could be revisited by including carbon emissions and other sustainability objectives. For instance, Benjaafar et al. (2010) as well as Absi et al. (2011) incorporate carbon emissions constraints on single and multi-stage lot-sizing models with a cost minimization objective. Both papers highlight the difficulty that appears when focusing on more sophisticated inventory models. More research should be carried out on this direction.

This paper is also based on several assumptions that could also be relaxed in future research. First, our study exclusively focuses on cost and carbon emissions as greenhouse gas reduction is nowadays a key issue. Some other sustainable objectives may certainly be incorporated to the model to adopt a holistic view of sustainable development. We recall that the SOQ model can be used with any number of objectives. Moreover, we have modeled technology investment by modifying ordering parameters as transportation is recognized as a major source of carbon emissions in supply chains. Studying the effects of other types of investment can be of interest. Note that modeling a technology investment by modifying holding parameters would give similar results by reasoning with ordering frequencies instead of batch size. In this case, the average total cost per time unit has the following expression:

$$Z_c(N) = P_c D + \frac{D}{2N} h_c + N O_c, \quad (19)$$

with:

N = ordering frequency (decision variable).

Finally, we have modeled carbon emissions by Formula 3 in accordance with the existing literature. A more accurate evaluation of the carbon footprint including vehicle capacity for instance could be of practical interest.

Appendix A

Proof of Theorem 1 (Adapted from Bouchery et al. (2011)):

If $Q_C^* = Q_E^*$, $E = Q_C^*$ as Q_C^* is the optimal batch size for both cost and carbon emissions.

Assume that $Q_C^* < Q_E^*$:

- $Z_C(Q)$ is strictly increasing on $[Q_C^*, Q_E^*]$,
 - $Z_E(Q)$ is strictly decreasing on $[Q_C^*, Q_E^*]$,
 - $Z_C(Q)$ and $Z_E(Q)$ are strictly increasing on $[Q_E^*, \infty)$ and strictly decreasing on $(0, Q_C^*]$
- then the solution is dominated if $Q \notin [Q_C^*, Q_E^*]$.

By using the same argumentation for $Q_E^* < Q_C^*$, it follows that:

$$E = [\min(Q_C^*, Q_E^*); \max(Q_C^*, Q_E^*)].$$

□

Proof of Theorem 2:

Convexity:

Let $Z_{\text{int}} = \{(x_C, x_E) \in \mathfrak{R}_+^2 \mid x_C \geq Z_C(Q_C^*), \exists (x_C, x_E^+), (x_C, x_E^-) \in Z(A) \times Z(A) \mid x_E^- \leq x_E \leq x_E^+\}$

and $Z_{\text{int}}^T = \{(x_C, x_E) \in \mathfrak{R}_+^2 \mid x_C \geq Z_C^T(Q_C^{T*}), \exists (x_C, x_E^+), (x_C, x_E^-) \in Z^T(A) \times Z^T(A) \mid x_E^- \leq x_E \leq x_E^+\}$

(the symbol \exists corresponds to “there exists”).

Z_{int} is a convex set as Z_C and Z_E are convex functions.

Z_{int}^T is a convex set as Z_C^T and Z_E^T are convex functions.

Limits:

$\frac{Z_E(Q)}{Z_C(Q)}$ tends to $\frac{h_E}{h_C}$ as Q tends to infinity. $\frac{Z_E^T(Q)}{Z_C^T(Q)}$ tends to $\frac{h_E}{h_C}$ as Q tends to infinity, so

$Z(A)$ and $Z^T(A)$ have a common asymptote with equation $y = \frac{h_E}{h_C}x$ as Q tends to infinity.

$\frac{Z_E(Q)}{Z_C(Q)}$ tends to $\frac{O_E}{O_C}$ as Q tends to zero. $\frac{Z_E^T(Q)}{Z_C^T(Q)}$ tends to $\frac{O_E^T}{O_C^T}$ as Q tends to zero. Moreover,

$\frac{O_E^T}{O_C^T} < \frac{O_E}{O_C}$ thus the asymptote of $Z(A)$ as Q tends to zero is steeper than the asymptote of

$Z^T(A)$ as Q tends to zero.

Intersection point:

By using the previous results, it follows that:

- If $\frac{O_E^T}{O_C^T} \leq \frac{h_E}{h_C} \leq \frac{O_E}{O_C}$, $Z(A) \cap Z^T(A)$ is empty.

- If $\frac{h_E}{h_C} < \frac{O_E^T}{O_C^T} < \frac{O_E}{O_C}$, $Z(A) \cap \{Z^T(Q) | Q \in [Q_E^{T*}, \infty)\}$ and $Z^T(A) \cap \{Z(Q) | Q \in (0, Q_C^*]\}$ are empty.

Moreover $\{Z(Q) | Q \in [Q_C^*, \infty)\} \cap \{Z^T(Q) | Q \in (0, Q_E^{T*}]\}$ is a singleton.

- If $\frac{O_E^T}{O_C^T} < \frac{O_E}{O_C} < \frac{h_E}{h_C}$, $Z(A) \cap \{Z^T(Q) | Q \in (0, Q_E^{T*}]\}$ and $Z^T(A) \cap \{Z(Q) | Q \in [Q_C^*, \infty)\}$ are empty.

Moreover $\{Z(Q) | Q \in (0, Q_C^*]\} \cap \{Z^T(Q) | Q \in [Q_E^{T*}, \infty)\}$ is a singleton. □

Proof of Theorem 3:

By using the results of Theorem 2, we obtain that:

- If there exist a value L_E^+ such that the operational adjustment option is the best one for $CAP = L_E^+$, then the operational adjustment option is the best one for all values of $CAP \geq L_E^+$,

- If there exist a value L_E^- such that the technology investment option is the best one for $CAP = L_E^-$, then the technology investment option is the best one for all values of $CAP \leq L_E^-$.

If $CAP = Z_E(Q_C^*)$, the operational adjustment option is the best option then we can choose

$$L_E^+ = Z_E(Q_C^*).$$

If $CAP = Z_E^T(Q_E^{T*})$, the technology investment option is the best option then we can choose

$$L_E^- = Z_E^T(Q_E^{T*}).$$

It can then be concluded that there exists L_E with $L_E^- \leq L_E \leq L_E^+$ that allow deciding among the two options. □

Proof of Theorem 4:

By using the same argumentation as in Theorem 3, we obtain that:

- If there exists $L_C^- \in \mathfrak{R}_+^*$ such that $\min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + L_C^- \cdot Z_E(Q)) \leq \min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + L_C^- \cdot Z_E^T(Q))$, then for

all $\alpha \in \mathfrak{R}_+$ such that $\alpha < L_C^-$, $\min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + \alpha \cdot Z_E(Q)) < \min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + \alpha \cdot Z_E^T(Q))$.

- If there exists L_C^+ such that $\min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + L_C^+ \cdot Z_E^T(Q)) \leq \min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + L_C^+ \cdot Z_E(Q))$, then for all

$\alpha > L_C^+$, $\min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + \alpha \cdot Z_E^T(Q)) < \min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + \alpha \cdot Z_E(Q))$.

Let $L_C = \frac{O_C^T - O_C}{O_E - O_E^T}$, $\min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + L_C \cdot Z_E(Q)) = \min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + L_C \cdot Z_E^T(Q))$, then:

- For all $\alpha \in \mathfrak{R}_+$ such that $\alpha < L_C$, $\min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + \alpha \cdot Z_E(Q)) < \min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + \alpha \cdot Z_E^T(Q))$.

- For all $\alpha > L_C$, $\min_{Q \in \mathfrak{R}_+^*} (Z_C(Q) + \alpha \cdot Z_E(Q)) < \min_{Q \in \mathfrak{R}_+^*} (Z_C^T(Q) + \alpha \cdot Z_E^T(Q))$. □

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