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Decomposing Conformance Checking on Petri Nets with Data

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Abstract. Process mining techniques relate observed behavior to modeled behavior, e.g., the automatic discovery of a Petri net based on an event log. Process mining is not limited to process discovery and also includes conformance checking. Conformance checking techniques are used for evaluating the quality of discovered process models and to diagnose deviations from some normative model (e.g., to check compliance). Existing conformance checking approaches typically focus on the control-flow, thus being unable to diagnose deviations concerning data. This paper proposes a technique to check the conformance of data-aware process models. We use so-called "data Petri nets" to model data variables, guards, and read/write actions. Additional perspectives such as resource allocation and time constraints can be encoded in terms of variables. Data-aware conformance checking problem may be very time consuming and sometimes even intractable when there are many transitions and data variables. Therefore, we propose a technique to decompose large data-aware conformance checking problems into smaller problems that can be solved more efficiently. We provide a general correctness result showing that decomposition does not influence the outcome of conformance checking. Moreover, two decomposition strategies are presented. The approach is supported through ProM plug-ins and experimental results show that significant performance improvements are indeed possible.

Keywords: Process Mining, Conformance Checking, Petri Net with Data, Process Model Decomposition

1 Introduction

The practical relevance of process mining is increasing as more and more event data becomes available. Process mining techniques aim to discover, monitor and improve real processes by extracting knowledge from event logs. The two most prominent process mining tasks are: (i) *process discovery:* learning a process model from example behavior recorded in an event log, and (ii) *conformance*

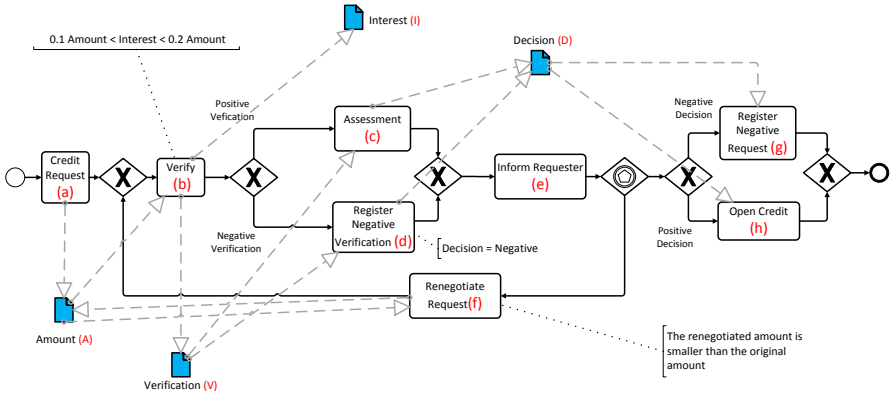


Fig. 1: Example of a (simplified) process to request loans. The dotted arcs going from a transition to a variable denote the writing operations; the reverse arcs denote the read operations, i.e. the transition requires accessing the current variables' value. In the paper, each transition is abbreviated with the lower-case letter in brackets and each variable with the upper-case letter in brackets.

checking: diagnosing and quantifying discrepancies between observed behavior and modeled behavior [1].

Most of the work done in conformance checking in the literature focuses on the control-flow of the underlying process. There are various approaches to computer the fraction of events or traces in the log that can be replayed by the model [2,3,4].

In a data-aware process model, each case, i.e. a process instance, is characterized by its case variables. Paths taken during the execution may be governed by guards and conditions defined over such variables. A process model specifies the set of variables and their possible values, guards, and write/read actions. Since existing conformance checking techniques typically completely abstract from data, resources, and time, many deviations remain undetected. Therefore, the event log may record executions of process instances that appear fully conforming, even when it is not the case. Only the analysis of the data perspective would be able to highlight the deviation.

Let us consider the process that is modeled as BPMN diagram in Figure 1. The process is concerned with customers requesting loans and is deliberately oversimplified to be able to explain the concepts more easily. The process starts with a credit request where the requestor provides some documents to demonstrate the capability of paying the loan back. These documents are verified and the interest amount is also computed. If the verification step is negative, a negative decision is made, the requestor is informed and, finally, the negative outcome of the request is stored in the system. If verification is positive, an assessment

is made to take a final decision. Independently of the assessment's decision, the requestor is informed. Moreover, even if the verification is negative, the requestor can renegotiate the loan (e.g. to have lower interests) by providing further documents or by asking for a smaller amount. In this case, the verification-assessment part is repeated. If both the decision and verification are positive and the requestor is not willing to renegotiate, the credit is opened.

Let us consider the following trace:[4]

$$\sigma_{ex} = \langle(\mathbf{a}, \emptyset, \{(A, 4000)\}), (\mathbf{b}, \{(A, 4000)\}, \{(I, 450), (V, \mathsf{false})\}), (\mathbf{c}, \{(V, \mathsf{false})\}, \{(D, \mathsf{true})\}), (\mathbf{e}, \emptyset, \emptyset), (\mathbf{f}, \{(A, 4000)\}, \{(A, 5000)\}), (\mathbf{b}, \{(A, 5000)\}, \{(I, 450), (V, \mathsf{false})\}), (\mathbf{d}, \{(V, \mathsf{false})\}, \{(D, \mathsf{false})\}), (\mathbf{e}, \emptyset, \emptyset), (\mathbf{h}, \{(D, \mathsf{true})\}, \emptyset)\rangle$$

Seen from a control-flow perspective only, the trace seems to be fully conforming. Nonetheless, a number of deviations can be easily noticed if the data perspective is considered. Firstly, if activity c is executed, previously activity b cannot result in a negative verification, i.e. V is set to false. Secondly, activity f cannot write value 5000 to variable A, as this new value is larger than the previous value, i.e. 4000. Furthermore, if the decision and verification are both negative, i.e. both V are D are set to false, then h cannot be executed at the end.

The identification of non-conforming traces clearly has value in itself. Nonetheless, organizations are often interested in explanations that can steer measures to improve the quality of the process. *Alignments* aim to support more refined conformance checking. An alignment aligns a case in the event log with an execution path of the process model as good as possible. If case deviates from the model, then it is not possible to perfectly align with the model and a best matching scenario is selected. Note that for the same deviation, multiple explanations can be given. For instance, the problem that h was executed when it was not supposed to happen can be explained in two ways: (1) h should not have occurred because V and D are both set to false ("control-flow is wrong") and (2) V and D should both have been set to true because h occurs ("data-flow is wrong"). In order to decide for the most reasonable explanation, costs are assigned to deviations and we aim to find the explanation with the lowest cost. For instance, if assigning a wrong value to V and D is less severe than executing h wrongly, the second explanation is preferred.

To the best of our knowledge, [5] is the only paper on data-aware conformance-checking analysis. Readers are referred to it for a state-of-the-art analysis. This work also shows that the problem of finding an alignment of an event log and a data-aware process model is NP-hard. In practice, it is exponential on the size of the model, i.e. the number of activities and data variables.

In this paper, we aim to speed up the computation of alignments by using a divide-and-conquer approach. The data-aware process model is split into smaller

[4] Notation $(\mathbf{act}, r, w)$ is used to denote the occurrence of activity *act* that writes and reads variables according to functions w and r, e.g., $(\mathbf{b}, \{(A, 4000)\}, \{(I, 450), (V, \mathsf{false})\})$ is an event corresponding to the occurrence of activity **b** while reading value 4000 for variable A and writing values 450 and false to variables I and V respectively. $(\mathbf{e}, \emptyset, \emptyset)$ corresponds to the occurrence of activity **e** without reading/writing any variables.

partly overlapping model fragments. For each model fragment a sublog is created by projecting the initial event log onto the activities used in the fragment. Given the exponential nature of conformance checking, this may significantly reduce the computation time. If the decomposition is done properly, then any trace that fits into the overall model also fits all of the smaller model fragments and vice versa.

Recently, several approaches have been proposed to decompose process mining problems, both for discovery and conformance checking. As described in [6,7] it is possible to decompose process mining problems in a variety of ways. Special cases of this more general theory are passages [8] and SESE-based decomposition [9,10]. However, these approaches are limited to control-flow. In this paper, we extend the general approach of [6] to Petri nets with data.

The decomposed data-aware conformance checking approach presented in this paper has been implemented using ProM framework and tested with several synthetic event logs. Experimental results show that data-aware decomposition may indeed be used to significantly reduce the time needed for conformance checking.

Preliminaries are presented in Section 2. Section 3 introduces our approach for data-aware decomposition. Section 4 describes different algorithms for instantiating the general results presented in Section 3. Section 5 reports on experimental results. Section 6 concludes the paper.

2 Preliminaries

In this section, we introduce preliminaries ranging from (data) Petri nets and event logs to data-aware alignments. Petri nets with data can be seen as a abstraction of high-level/colored Petri nets [11]. Petri nets with data provide precisely the information needed for conformance checking of data-aware models and logs.

2.1 System Nets

Petri nets and their semantics are defined as usual: A Petri net is a tuple (P, T, F) with P the set of places, T the set of transitions, $P \cap T = \emptyset$, and $F \subseteq (P \times T) \cup (T \times P)$ the flow relation. The marking of a Petri net is a multiset of tokens, i.e., $M \in \mathbb{B}(P)$. For some multiset $M \in \mathbb{B}(P)$, $M(p)$ denotes the number of times element p appears in M. The standard set operators can be extended to multisets, $M_1 \uplus M_2$ is the union of two multisets.

Definition 1 (Labeled Petri Net). *A labeled Petri net $PN = (P, T, F, l)$ is a Petri net (P, T, F) with labeling function $l \in T \not\rightarrow \mathcal{U}_A$ where $\mathcal{U}_A$ is some universe of activity labels.*

Definition 2 (System Net). *A system net $SN = (PN, M_{init}, M_{final})$ is a triplet where $PN = (P, T, F, l)$ is a labeled Petri net, $M_{init} \in \mathbb{B}(P)$ is the initial marking, and $M_{final} \in \mathbb{B}(P)$ is the final marking. $\mathcal{U}_{SN}$ is the* universe of system nets.

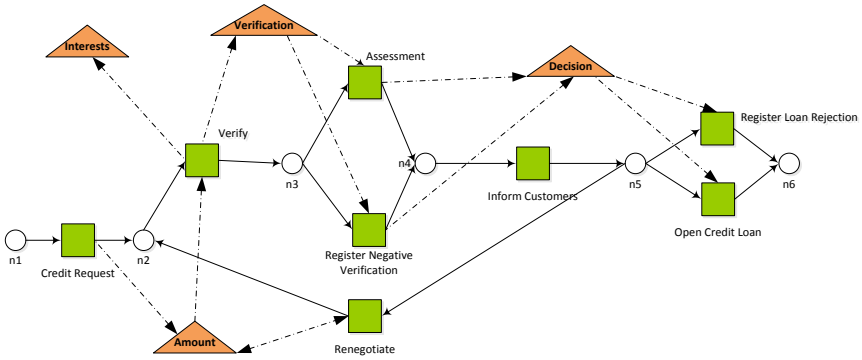


Fig. 2: Pictorial representation of a data Petri net that models the process earlier described in terms of BPMN diagram (cf. Figure 1). Places, transitions and variables are represented as circles, rectangles and triangles, respectively. The dotted arcs going from a transition to a variable denote the writing operations; the reverse arcs denote the read operations, i.e. the transition requires accessing the current variables' value.

Definition 3 (System Net Notations). *Let* $SN = (PN, M_{init}, M_{final}) \in \mathcal{U}_{SN}$ *be a system net with* $PN = (P, T, F, l)$.

- $T_v(SN) = dom(l)$ *is the set of* visible transitions *in* SN,
- $A_v(SN) = rng(l)$ *is the set of corresponding* observable activities *in* SN,
- $T_v^u(SN) = \{t \in T_v(SN) \mid \forall_{t' \in T_v(SN)}\ l(t) = l(t') \Rightarrow t = t'\}$ *is the set of* unique *visible transitions in* SN *(i.e., there are no other transitions having the same visible label), and*
- $A_v^u(SN) = \{l(t) \mid t \in T_v^u(SN)\}$ *is the set of corresponding* unique *observable activities in* SN.

To define the union of two nets, we need to merge the labeling functions. For any two partial functions $f_1 \in X_1 \not\to Y_1$ and $f_2 \in X_2 \not\to Y_2$: $f_3 = f_1 \oplus f_2$ is the union of the two functions. $f_3 \in (X_1 \cup X_2) \not\to (Y_1 \cup Y_2)$, $dom(f_3) = dom(f_1) \cup dom(f_2)$, $f_3(x) = f_2(x)$ if $x \in dom(f_2)$, and $f_3(x) = f_1(x)$ if $x \in dom(f_1) \setminus dom(f_2)$.

Definition 4 (Union of Nets). *Let* $SN^1 = (N^1, M^1_{init}, M^1_{final}) \in \mathcal{U}_{SN}$ *with* $N^1 = (P^1, T^1, F^1, l^1)$ *and* $SN^2 = (N^2, M^2_{init}, M^2_{final}) \in \mathcal{U}_{SN}$ *with* $N^2 = (P^2, T^2, F^2, l^2)$ *be two system nets.*

- $l^3 \in l^1 \oplus l^2$ *is the union of* l_1 *and* l_2,
- $N^1 \cup N^2 = (P^1 \cup P^2, T^1 \cup T^2, F^1 \cup F^2, l^3)$ *is the union of* N^1 *and* N^2, *and*
- $SN^1 \cup SN^2 = (N^1 \cup N^2, M^1_{init} \uplus M^2_{init}, M^1_{final} \uplus M^2_{final})$ *is the union of system nets* SN^1 *and* SN^2.

2.2 Data Petri Nets

A data Petri net is a Petri net having any number of variables.

Definition 5 (Variables and Values). $\mathcal{U}_{VN}$ *is the universe of variable names.* $\mathcal{U}_{VV}$ *is the universe of values.* $\mathcal{U}_{VM} = \mathcal{U}_{VN} \not\rightarrow \mathcal{U}_{VV}$ *is the universe of variable mappings.*

In such a net, transitions may read from and/or write to variables. Moreover, transitions are associated with guards over these variables. These define the conditions when these transitions can fire. Formally, a *Data Petri Net* (DPN) is defined as follows:

Definition 6 (Data Petri Net). *A Data Petri Net* $DPN = (SN, V, val, init, read, write, guard)$ *consists of*

- *a system net* $SN = (PN, M_{init}, M_{final})$ *based on labeled Petri net* $PN = (P, T, F, l)$,
- *a set* $V \subseteq \mathcal{U}_{VN}$ *of data variables,*
- *a function* $val \in V \rightarrow \mathcal{P}(\mathcal{U}_{VV})$ *that defines the values admissible for each variable, i.e.,* $val(v)$ *is the set of values that variable* v *can have,*[5]
- *a function* $init \in V \rightarrow \mathcal{U}_{VV}$ *that defines the initial value for each variable* v *such that* $init(v) \in val(v)$ *(initial values are admissible),*
- *a read function* $read \in T \rightarrow \mathcal{P}(V)$ *that labels each transition with the set of variables that it reads,*
- *a write function* $write \in T \rightarrow \mathcal{P}(V)$ *that labels each transition with the set of variables that it writes,*
- *a guard function* $guard \in T \rightarrow \mathcal{P}(\mathcal{U}_{VM} \times \mathcal{U}_{VM})$ *that associates a guard with each transition such that for any* $t \in T$ *and* $(r, w) \in guard(t)$*:*
  - $dom(r) = read(t)$*, i.e., transitions read the specified set of variables,*
  - $dom(w) = write(t)$*, i.e., transitions write the specified set of variables,*
  - *for any* $v \in read(t)$*:* $r(v) \in val(v)$*, i.e., all values read should be admissible, and*
  - *for any* $v \in write(t)$*:* $w(v) \in val(v)$*, i.e., all values written should be admissible.*

$\mathcal{U}_{DPN}$ *is the* universe of data Petri nets.

If $guard(t) = \emptyset$, then the guard of t is *false* and the transition can never fire. $RW(t) = \{(r, w) \in \mathcal{U}_{VM} \times \mathcal{U}_{VM} \mid dom(r) = read(t) \wedge dom(w) = write(t) \wedge \forall_{v \in read(t)}\ r(v) \in val(v) \wedge \forall_{v \in write(t)}\ w(v) \in val(v)\}$ denotes the weakest guard possible. If $guard(t) = RW(t)$, transition t can never block on data. This corresponds to the *true* guard. Note that if $read(t) = write(t) = \emptyset$, then $guard(t) = \{(\emptyset, \emptyset)\}$ corresponds to *true*.

The notion of bindings is essential for the remainder. A *binding* is a triplet (t, r, w) describing the execution of transition t while reading values r and writing values w. A binding (t, r, w) is valid if and only if $(r, w) \in guard(t)$.

[5] $\mathcal{P}(X)$ is the powerset of X, i.e., $Y \in \mathcal{P}(X)$ is and only if $Y \subseteq X$.

Table 1: Definitions of the guards of the transitions in Fig. 2. Variables and transition names are abbreviated as described in Figure 1.

Transition	Guard
Credit Request	$RW(\mathbf{a})$
Verify	$\{(r,w) \in RW(\mathbf{b}) \mid 0.1 \cdot r(A) < w(I) < 0.2 \cdot r(A)\}$
Assessment	$\{(r,w) \in RW(\mathbf{c}) \mid r(V) = \mathsf{true}\}$
Register Negative Verification	$\{(r,w) \in RW(\mathbf{d}) \mid r(V) = \mathsf{false} \wedge w(D) = \mathsf{false}\}$
Inform Requester	$RW(\mathbf{e})$
Renegotiate Request	$\{(r,w) \in RW(\mathbf{f}) \mid r(V) = \mathsf{false} \wedge w(A) < r(A)\}$
Register Negative Request	$\{(r,w) \in RW(\mathbf{g}) \mid r(D) = \mathsf{false}\}$
Open Credit	$\{(r,w) \in RW(\mathbf{h}) \mid r(D) = \mathsf{true}\}$

A marking (M, s) of a data Petri net DPN has two components: $M \in \mathbb{B}(P)$ is the *control-flow marking* and $s \in \mathcal{U}_{VM}$ with $dom(s) = V$ and $s(v) \in val(v)$ for all $v \in V$ is the *data marking*. The initial marking of a data Petri net DPN is $(M_{init}, init)$. Recall that *init* is a function that defines the initial value for each variable.

A binding $b = (t, r, w)$ is *enabled* in marking (M, s) of DPN, denoted as $(DPN, (M, s))[b\rangle$, if each of its input places $\bullet t$ contains at least one token (control-flow enabled), b is valid (i.e., the guard of t is satisfied: $(r, w) \in guard(t)$), and $s{\restriction}_{read(t)} = r$ (the actual values read match the binding).[6]

An enabled binding $b = (t, r, w)$ may *occur*, i.e., one token is removed from each of the input places $\bullet t$ and one token is produced for each of the output places $t\bullet$. Moreover, the variables are updated as specified by w. Formally: $M' = (M \setminus \bullet t) \uplus t\bullet$ is the control-flow marking resulting from firing enabled transition t in marking M (abstracting from data) and $s' = s \oplus w$ is the data marking where $s'(v) = w(v)$ for all $v \in write(t)$ and $s'(v) = s(v)$ for all $v \in V \setminus write(t)$. $(DPN, (M, s))[b\rangle(DPN, (M', s'))$ denotes that b is enabled in (M, s) and the occurrence of b results in marking (M', s').

Figure 2 shows a Data Petri Net DPN_{ex} that models the same process as represented in Figure 1 as BPMN diagram, and Table 1 illustrates the conditions of the guards of the transitions of this data Petri net. The labeling function l is such that the domain of l is the set of transitions of DPN_{ex} and, for each transition t of DPN_{ex}, $l(t) = t$. In other words, the set of activity labels coincides with the set of transitions.

Let $\sigma_b = \langle b_1, b_2, \ldots, b_n\rangle$ be a sequence of bindings. $(DPN, (M, s))[\sigma_b\rangle(DPN, (M', s'))$ denotes that there is a set of markings $(M_0, s_0), (M_1, s_1), \ldots, (M_n, s_n)$ such that $(M_0, s_0) = (M, s)$, $(M_n, s_n) = (M', s')$, and $(DPN, (M_i, s_i))[b_{i+1}\rangle (DPN, (M_{i+1}, s_{i+1}))$ for $0 \leq i < n$. A marking (M', s') is *reachable* from (M, s) if there exists a σ_b such that $(DPN, (M, s))[\sigma_b\rangle(DPN, (M', s'))$.

[6] $f{\restriction}_Q$ is the function projected on Q: $dom(f{\restriction}_Q) = dom(f) \cap Q$ and $f{\restriction}_Q(x) = f(x)$ for $x \in dom(f{\restriction}_Q)$. Projection can also be used for bags and sequences, e.g., $[x^3, y, z^2]{\restriction}_{\{x,y\}} = [x^3, y]$ and $\langle y, z, y\rangle{\restriction}_{\{x,y\}} = \langle y, y\rangle$.

$\phi_f(DPN) = \{\sigma_b \mid \exists_s\ (DPN, (M_{init}, init))[\sigma_b\rangle(DPN, (M_{final}, s))\}$ is the *set of complete binding sequences*, thus describing the behavior of DPN.

Definition 7 (Union of Data Petri Nets). *Let* $DPN^1 = (SN^1, V^1, val^1, init^1, read^1, write^1, guard^1)$ *and* $DPN^2 = (SN^2, V^2, val^2, init^2, read^2, write^2, guard^2)$ *with* $V^1 \cap V^2 = \emptyset$. $DPN^1 \cup DPN^2 = (SN^1 \cup SN^2, V^1 \cup V^2, val^1 \oplus val^2, init^1 \oplus init^2, read^3, write^3, guard^3)$ *is the union such that*

- $read^3(t) = read^1(t)$, $write^3(t) = write^1(t)$, *and* $guard^3(t) = guard^1(t)$ *if* $t \in T^1 \setminus T^2$,
- $read^3(t) = read^2(t)$, $write^3(t) = write^2(t)$, *and* $guard^3(t) = guard^2(t)$ *if* $t \in T^2 \setminus T^1$, *and*
- $read^3(t) = read^1(t) \cup read^2(t)$, $write^3(t) = write^1(t) \cup write^2(t)$, *and* $guard^3(t) = \{(r_1 \oplus r_2, w_1 \oplus w_2) \mid (r_1, w_1) \in guard^1(t) \wedge (r_2, w_2) \in guard^2(t)\}$ *if* $t \in T^1 \cap T^2$.

We use the notation $guard^3 = guard^1 \otimes guard^2$ *to describe this merge.*

2.3 Event Logs and Relating Models to Event Logs

Next we introduce *event logs* and relate them to the *observable* behavior of a data Petri net.

Definition 8 (Trace, Event Log with Data). *A trace* $\sigma \in (\mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*$ *is a sequence of activities with input and output data.* $L \in \mathbb{B}((\mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*)$ *is an event log with read and write information, i.e., a multiset of traces with data.*

Definition 9 (From Bindings to Traces). *Consider a data Petri net with transitions* T *and labeling function* $l \in T \not\to \mathcal{U}_A$. *A binding sequence* $\sigma_b \in (T \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*$ *can be converted into a trace* $\sigma_v \in (\mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*$ *by removing the bindings that correspond to unlabeled transitions and by mapping the labeled transitions onto their corresponding label.* $l(\sigma_b)$ *denotes the corresponding trace* σ_v.

Note that we overload the labeling function to binding sequences, $\sigma_v = l(\sigma_b)$. This can be used to define $\phi(DPN)$: the set of all visible traces.

Definition 10 (Observable Behavior of a Data Petri Net). *Let* DPN *be a data Petri net.* $(DPN, (M, s))[\sigma_v \rhd (DPN, (M', s'))$ *if and only if there is a sequence* σ_b *such that* $(DPN, (M, s))[\sigma_b\rangle(DPN, (M', s'))$ *and* $\sigma_v = l(\sigma_b)$. $\phi(DPN) = \{l(\sigma_b) \mid \sigma_b \in \phi_f(DPN)\}$ *is the set of* visible *traces starting in* $(M_{init}, init)$ *and ending in* (M_{final}, s) *for some data marking* s.

Definition 11 (Perfectly Fitting with Data). *A trace* $\sigma \in (\mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*$ *is perfectly fitting* $DPN \in \mathcal{U}_{DPN}$ *if* $\sigma \in \phi(DPN)$. *An event log* $L \in \mathbb{B}((\mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*)$ *is perfectly fitting* DPN *if all of its traces are perfectly fitting.*

Later, we will need to project binding sequences and traces onto subsets of transitions/activities and variables. Therefore, we introduce a generic projection operator $\Pi_{Y,V}(\sigma)$ that removes transitions/activities not in Y and variables not in V.

Definition 12 (Projection). *Let X be a set of transitions or activities (i.e., $X \subseteq T$ or $X \subseteq \mathcal{U}_A$. Let $Y \subseteq X$ be a subset and $V \subseteq \mathcal{U}_{VN}$ a subset of variable names. Let $\sigma \in (X \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*$ be a binding sequence or a trace with data. $\Pi_{Y,V}(\sigma) \in (Y \times (V \not\to \mathcal{U}_{VV}) \times (V \not\to \mathcal{U}_{VV}))^*$ is the projection of σ onto transitions/activities Y and variables V. Bindings/events unrelated to transitions/activities in Y are removed completely. Moreover, for the remaining bindings/events all read and write variables not in V are removed. $\Pi_{Y,V}(L) = [\Pi_{Y,V}(\sigma) \mid \sigma \in L]$ lifts the projection operator to the level of logs.*

2.4 Alignments

Conformance checking requires an *alignment* of event log L and process model DPN, that is the alignment of each single trace $\sigma \in L$ and process model DPN.

The events in the event log need to be related to transitions in the model, and vice versa. Such an alignment shows how the event log can be replayed on the process model. Building this alignment is far from trivial, since the log may deviate from the model at an arbitrary number of places.

We need to relate "moves" in the log to "moves" in the model in order to establish an alignment between a process model and an event log. However, it may be that some of the moves in the log cannot be mimicked by the model and vice versa. We explicitly denote such "no moves" by $\gg$.

An alignment is a sequence of moves:

Definition 13 (Legal alignment moves). *Let $DPN = (SN, V, val, init, read, write, guard)$ be a data Petri net, with $SN = (PN, M_{init}, M_{final})$ and $PN = (P, T, F, l)$. let $S_L = \mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM}$ be the universe of events. Let $S_{DPN} = T \times \mathcal{U}_{VM} \times \mathcal{U}_{VM}$ be the universe of bindings of DPN. Let be $S_{DPN}^{\gg} = S_{DPN} \cup \{\gg\}$ and $S_L^{\gg} = S_L \cup \{\gg\}$.*

A legal move in an alignment *is represented by a pair $(s_L, s_M) \in (S_L^{\gg} \times S_{DPN}^{\gg}) \setminus \{(\gg, \gg)\}$ such that*

- *(s_L, s_M) is a* move in log *if $s_L \in S_L$ and $s_M = \gg$,*
- *(s_L, s_M) is a* move in model *if $s_L = \gg$ and $s_M \in S_{DPN}$,*
- *(s_L, s_M) is a* move in both with correct read/write operations *if $s_M = (t, r, w) \in S_{DPN}$ and $s_L = (l(t), r, w) \in S_L$,*
- *(s_L, s_M) is a* move in both with incorrect read/write operations *if $s_M = (t, r, w) \in S_{DPN}$ and $s_L = (l(t), r', w') \in S_L$, and $r \neq r'$ or $w \neq w'$.*

All other moves are considered as illegal.

Definition 14 (Alignments). *Let $DPN = (SN, V, val, init, read, write, guard)$ be a data Petri net and $\sigma \in (S_L)^*$ be a event-log trace. Let $\mathcal{A}_{DPN}$ be the set of legal moves for DPN. A* complete alignment *of σ_L and DPN is a sequence $\gamma \in {\mathcal{A}_{DPN}}^*$ such that, ignoring all occurrences of $\gg$, the projection on the first element yields σ_L and the projection on the second yields a $\sigma_P \in \phi_f(DPN)$.*

Table 2: Examples of complete alignments of $\sigma_{example}$ and N. For readability, the read operations are omitted. Of course, read operations for any variable must match the most recent value for that variable.

(a)

Event-Log Trace	Process
(**a**, {(A,4000)})	(**a**, {(A,4000)})
(**b**, {(I,450),(V,false)})	(**b**, {(I,450),(V,true)})
(**c**, {(D,true)})	(**c**, {(D,true)})
(**e**, ∅)	(**e**, ∅)
(**f**, {(A,5000)})	(**f**, {(A,3000)})
(**b**, {(I,450),(V,false)})	(**b**, {(I,450),(V,false)})
(**d**, {(D,false)})	(**d**, {(D,false)})
(**e**, ∅)	(**e**, ∅)
(**h**, ∅)	≫
≫	(**g**, ∅)

(b)

Event-Log Trace	Process
(**a**, {(A,4000)})	(**a**, {(A,5100)})
(**b**, {(I,450),(V,false)})	(**b**, {(I,511),(V,true)})
(**c**, {(D,true)})	(**c**, {(D,true)})
(**e**, ∅)	(**e**, ∅)
(**f**, {(A,5000)})	(**f**, {(A,5000)})
(**b**, {(I,450),(V,false)})	(**b**, {(I,511),(V,false)})
(**d**, {(D,false)})	(**d**, {(D,false)})
(**e**, ∅)	(**e**, ∅)
(**h**, ∅)	≫
≫	(**g**, ∅)

Table 2 shows two complete alignments of the process model in Figure 2 and the log trace σ_{ex} from Section 1.

In order to define the severity of a deviation, we introduce a cost function on legal moves: $\kappa : \mathcal{A}_{DPN} \rightarrow \mathbb{R}_0^+$. This cost function can be used to favor one type of explanation for deviations over others. The cost of each legal move depends on the specific model and process domain and, hence, the cost function κ needs to be defined specifically for each setting. The cost function can be generalized to an alignment γ as the sum of the cost of each individual move: $\mathcal{K}(\gamma) = \sum_{(s_L, s_M) \in \gamma} \kappa(s_L, s_M)$.

However, we do not aim to find just any complete alignment. Our goal is to find a complete alignment of σ_L and DPN which minimizes the cost: an optimal alignment. Let $\Gamma_{\sigma_L, N}$ be the (infinite)set of all complete alignments of σ_L and $DPMN$. The alignment $\gamma \in \Gamma_{\sigma_L, DPN}$ is an *optimal alignment* if, for all $\gamma' \in \Gamma_{\sigma_L, N}$, $\mathcal{K}(\gamma) \leq \mathcal{K}(\gamma')$. Note that an optimal alignment does not need to be unique, i.e. multiple complete alignments with the same minimal cost may exist.

Let us consider again our example introduced above. Let us assume to have a cost function κ^s such that $\kappa^s(s_L, s_M) = 1$ if (s_L, s_M) is a move in log or visible move in process (i.e. $s_M = \gg$ or $s_L = \gg$ and s_M corresponds to a labeled transition, respectively) or a move in both with incorrect read/write operations and $\kappa^s(s_L, s_M) = 0$ in case of move in both without incorrect read/write operations or a move in model corresponding to an unlabeled transition. The alignment in Table 2a is associated with a cost 6 whereas that in Table 2b with a cost 8.[7] It follows that the former is a better alignment. As a matter of fact, it is also an optimal alignment, although it is not the only one. For instance, any variation of such an alignment where the move for f is of the form (now including

[7] They also include a cost of two accounted for incorrect read operations, not shown in the alignments, which are caused by incorrect write operations.

read operations) $((\mathbf{f}, \{(A, 4000)\}, \{(A, 5000)\})\ (\mathbf{f}, \{(A, 4000)\}, \{(A, x)\})\})$ with $2250 < x < 4000$ corresponds to an optimal alignment, as well.

3 Valid Decomposition of Multi-perspective Models

In [6] the author defines *valid decomposition* in terms of Petri nets: the overall system net SN is decomposed into a collection of subnets $\{SN^1, SN^2, \ldots, SN^n\}$ such that the union of these subnets yields the original system net. A decomposition is *valid* if the subnets "agree" on the original labeling function (i.e., the same transition always has the same label), each place resides in just one subnet, and also each invisible transition resides in just one subnet. Moreover, if there are multiple transitions with the same label, they should reside in the same subnet. Only unique visible transitions can be shared among different subnets.

Definition 15 (Valid Decomposition for Petri nets [6]). *Let $SN \in \mathcal{U}_{SN}$ be a system net with labeling function l. $D = \{SN^1, SN^2, \ldots, SN^n\} \subseteq \mathcal{U}_{SN}$ is a valid decomposition if and only if:*

- *$SN^i = (N^i, M^i_{init}, M^i_{final})$ is a system net with $N^i = (P^i, T^i, F^i, l^i)$ for all $1 \leq i \leq n$,*
- *$l^i = l|_{T^i}$ for all $1 \leq i \leq n$,*
- *$P^i \cap P^j = \emptyset$ for $1 \leq i < j \leq n$,*
- *$T^i \cap T^j \subseteq T^u_v(SN)$ for $1 \leq i < j \leq n$, and*
- *$SN = \bigcup_{1 \leq i \leq n} SN^i$.*

$\mathcal{D}(SN)$ is the set of all valid decompositions of SN.

From the definition the following properties follow:

1. each place appears in precisely one of the subnets, i.e., for any $p \in P$: $|\{1 \leq i \leq n \mid p \in P^i\}| = 1$,
2. each invisible transition appears in precisely one of the subnets, i.e., for any $t \in T \setminus T_v(SN)$: $|\{1 \leq i \leq n \mid t \in T^i\}| = 1$,
3. visible transitions that do not have a unique label (i.e., there are multiple transitions with the same label) appear in precisely one of the subnets, i.e., for any $t \in T_v(SN) \setminus T^u_v(SN)$: $|\{1 \leq i \leq n \mid t \in T^i\}| = 1$,
4. visible transitions having a unique label may appear in multiple subnets, i.e., for any $t \in T^u_v(SN)$: $|\{1 \leq i \leq n \mid t \in T^i\}| \geq 1$, and
5. each edge appears in precisely one of the subnets, i.e., for any $(x, y) \in F$: $|\{1 \leq i \leq n \mid (x, y) \in F^i\}| = 1$.

As shown in [6] these observations imply that conformance checking can be decomposed. Any trace that fits the overall process model can be decomposed into smaller traces that fit the individual model fragments. Moreover, if the smaller traces fit the individual model fragments, then they can be composed into an overall trace that fits into the overall process model. This result is the basis for decomposing process mining problems.

Theorem 1 (Conformance Checking Can be Decomposed [6]). *Let $L \in \mathbb{B}(A^*)$ be an event log with $A \subseteq \mathcal{U}_A$ and let $SN \in \mathcal{U}_{SN}$ be a system net. For any*

valid decomposition $D = \{SN^1, SN^2, \ldots, SN^n\} \in \mathcal{D}(SN)$*: L is perfectly fitting system net SN if and only if for all* $1 \leq i \leq n$*: the projection of L onto* $A_v(SN^i)$ *is perfectly fitting* SN^i*.*

In this paper, the definition of valid decomposition is extended to cover Petri nets with data.

Definition 16 (Valid Decomposition for Petri nets with Data). *Let* $DPN \in \mathcal{U}_{DPN}$ *be a data Petri net.* $D = \{DPN^1, DPN^2, \ldots, DPN^n\} \subseteq \mathcal{U}_{DPN}$ *is a valid decomposition if and only if:*

- *for all* $1 \leq i \leq n$*:* $DPN^i = (SN^i, V^i, val^i, init^i, read^i, write^i, guard^i)$ *is a data Petri net,* $SN^i = (PN^i, M^i_{init}, M^i_{final}) \in \mathcal{U}_{SN}$ *is a system net, and* $PN^i = (P^i, T^i, F^i, l^i)$ *is a labeled Petri net,*
- $D' = \{SN^1, SN^2, \ldots, SN^n\} \subseteq \mathcal{U}_{SN}$ *is a valid decomposition of* $\bigcup_{1 \leq i \leq n} SN^i$*,*
- $V^i \cap V^j = \emptyset$ *for* $1 \leq i < j \leq n$*,*
- $DPN = \bigcup_{1 \leq i \leq n} DPN^i$*.*

$\mathcal{D}(DPN)$ *is the set of all valid decompositions of* DPN*.*

Each variable appears in precisely one of the subnets. Therefore, there cannot be two fragments that read and or write the same data variables: $\bigcup_{t \in T^i} read^i(t) \cup write^i(t) \bigcap \bigcup_{t \in T^j} read^j(t) \cup write^j(t) = \emptyset$ for $1 \leq i < j \leq n$. Moreover, two guards in different fragments cannot refer to the same variable. If a transition t appears in multiple fragments, then it needs to have a visible unique label as shown in [6]. Such a uniquely labeled transition t shared among fragments, may use, read, or write different variables in different fragments. Since $DPN = \bigcup_{1 \leq i \leq n} DPN^i$, we know that the overall $guard = guard^1 \otimes guard^2 \otimes \ldots \otimes guard^n$. Without loss of generality we can assume that the first k fragments share t. Hence, $guard(t) = \{(r_1 \oplus r_2 \oplus \ldots \oplus r_k, w_1 \oplus w_2 \oplus \ldots \oplus w_k) \mid (r_1, w_1) \in guard^1(t) \wedge (r_2, w_2) \in guard^2(t) \wedge \ldots \wedge (r_k, w_k) \in guard^k(t)\}$. Hence, in a valid decomposition, the guard of a shared transition can only be split if the different parts do not depend on one another.

Based on these observations, we prove that we can decompose conformance checking also for Petri nets with data.

Theorem 2 (Conformance Checking With Data Can be Decomposed). *Let* $L \in \mathbb{B}((\mathcal{U}_A \times \mathcal{U}_{VM} \times \mathcal{U}_{VM})^*)$ *be an event log with information about reads and writes and let* $DPN \in \mathcal{U}_{DPN}$ *be a data Petri net. For any valid decomposition* $D = \{DPN^1, DPN^2, \ldots, DPN^n\} \subseteq \mathcal{U}_{DPN}$*: L is perfectly fitting data Petri net DPN if and only if for all* $1 \leq i \leq n$*:* $\Pi_{A_v(SN^i), V^i}(L)$ *is perfectly fitting* DPN^i*.*

Proof. Let $DPN = (SN, V, val, init, read, write, guard)$ be a data Petri net with $SN = (PN, M_{init}, M_{final})$ and $PN = (P, T, F, l)$. Let $D = \{DPN^1, DPN^2, \ldots DPN^n\}$ be a valid decomposition of DPN with $DPN^i = (SN^i, V^i, val^i, init^i, read^i, write^i, guard^i)$, $SN^i = (PN^i, M^i_{init}, M^i_{final}) \in \mathcal{U}_{SN}$, and $PN^i = (P^i, T^i, F^i, l^i)$.

($\Rightarrow$) Let $\sigma_v \in L$ be such that there exists a data marking s such that (DPN,

$(M_{init}, init))[\sigma_v\triangleright(DPN, (M_{final}, s))$. This implies that there exists a corresponding σ_b with $(DPN, (M_{init}, init))[\sigma_b\rangle(DPN, (M_{final}, s))$ and $l(\sigma_b) = \sigma_v$. For all $1 \leq i \leq n$, we need to prove that there is a σ_b^i with $(DPN^i, (M^i_{init}, init^i))$ $[\sigma_b^i\rangle(DPN^i, (M^i_{final}, s^i))$ for some s^i. This follows trivially because DPN^i can mimic any move of DPN with respect to transitions T^i: just take $\sigma_b^i = \Pi_{T^i,V^i}(\sigma_b)$. Note that guards can only become weaker by projection.
($\Leftarrow$) Let $\sigma_v \in L$. For all $1 \leq i \leq n$, let σ_b^i be such that $(DPN^i, (M^i_{init}, init^i))[\sigma_b^i\rangle$ $(DPN^i, (M^i_{final}, s^i))$ and $l^i(\sigma_b^i) = \Pi_{A_v(SN^i),V^i}(\sigma_v)$. The different σ_b^i sequences can be stitched together into an overall σ_b such that $(DPN, (M_{init}, init))[\sigma_b\rangle$ $(DPN, (M_{final}, s))$ with $s = s^1 \oplus s^2 \oplus \ldots \oplus s^n$. This is possible because transitions in one subnet can only influence other subnets through unique visible transitions and these can only move synchronously as defined by σ_v. Moreover, guards can only be split in independent parts (see Definition 16). Suppose that t appears in T_i and T_j, then $guard(t) = \{(r_i \oplus r_j, w_i \oplus w_j) \mid (r_i, w_i) \in guard^i(t) \wedge (r_j, w_j) \in guard^j(t)\}$. Hence, a read/write in subnet i cannot limit a read/write in subnet j. Therefore, we can construct σ_b and $l(\sigma_b) = \sigma_v$. □

4 Strategies for Realizing Valid Decomposition

In this section we present two different strategies to instantiate the valid decomposition definition over a Petri net with data presented in the previous section (cf. Def.16). The strategies proposed are: 1) *maximal* decomposition and 2) *SESE-based* decomposition The two strategies are based on partitioning the *arcs* of the Petri net with data, i.e., control-flow arcs of the Petri net F, and data arcs for variables result of the *read* and the *write* functions, $R = \{(v, t)|v \in read(t)\}$ and $W = \{(t, v)|v \in write(t)\}$. While the first strategy is devoted to finding subnets of minimal size thus alleviating the complexity of analyzing the derived subnets, the latter strategy is grounded in recent decomposition techniques focusing on the identification of meaningful subprocesses [12,9,10].

4.1 Maximal Valid Decomposition

In [6] a strategy to decompose the overall Petri net into the minimal subnets satisfying the valid decomposition definition without data was proposed. In this section, we present a similar strategy for decomposing Petri nets with data:

1. *Partitioning the arcs*: The construction of the maximal decomposition is based on partitioning the arcs. Each arc will end up in precisely one subnet. The partitioning is done such that all arcs connected to a variable are included in the same part. The same is done for the arcs connected to a place, invisible or duplicate transitions, in order to preserve Def. 16.
2. *Create and merge subnets if necessary*: Once the partitioning is done, a subnet is created for each part. Whenever a unique visible transition is shared, the guard of the original transition is split among these transitions. If this splitting contradicts some of the requirements of Def. 16, the transitions are merged (and subsequently the subnets).

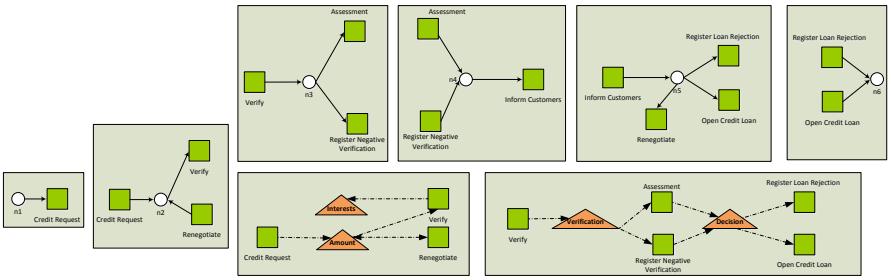


Fig. 3: Maximal decomposition for the running example.

Figure 3 shows the result of applying the maximal-decomposition technique for the Data Petri Net in Figure 2.

Decomposing an overall net into minimal subnets aims to reduce the computation time of checking its conformance, i.e., the simpler the subnets, the faster is to check its conformance. This makes this technique a clear option when dealing with large conformance instances. However, this maximal decomposition may not be optimal from a conformance diagnosis point of view, i.e, the components tend to be extremely simple and hence become meaningless (specially on the control-flow, where typically a subnet may include only a pair of arcs). In addition, a maximal decomposition may create a vast number of components, reducing the comprehension of the decomposition, and moreover may increase excessively the overhead for each component (e.g., creating the sublog for each subnet). In the remainder of this section we propose a different decomposition strategy in order to address these issues.

4.2 SESE-based Valid Decompositions

In the context of business processes, Single-Entry Single-Exit (SESE) components have been recently identified as meaningful fragments of a process model [12,9,10]. In the aforementioned work only the control flow dimension was considered. In the remaining of this section we propose to lift the SESE-based valid decomposition to also incorporate data.

We will now informally describe the necessary notions for understanding the proposed data-oriented SESE-based valid decomposition strategies described below. For the sake of clarity, we will focus on the control flow to illustrate the concepts, although the definitions will be extended at the end to also consider data.

Given Petri net $PN = (P, T, F, l)$, its *workflow graph* is the structural graph $WG = (V, E)$ with no distinctions between places and transitions, i.e., $V = P \cup T$ and $E = F$. For instance, Fig. 4(b) reports the workflow graph of the Petri net of Fig. 4(a) (corresponding with the control-flow part of the running example). Given a subset of edges $S \subseteq E$ of WG, the corresponding nodes $S\restriction_V$ can be partitioned into interior and boundary. Interior nodes have no connection with nodes outside $S\restriction_V$, while boundary nodes do. Furthermore, boundary nodes can be partitioned into entry (no incoming edge belongs to S), or exit (no outgoing

edge belongs to E'). $S \subseteq E$ is a *SESE* of WG iff the subnet derived from S has exactly two boundary nodes: one entry and one exit. Fig. 4(b) shows all non-trivial SESEs[8] of the Petri net of Fig. 4(a). For a formal definition we refer to [12].

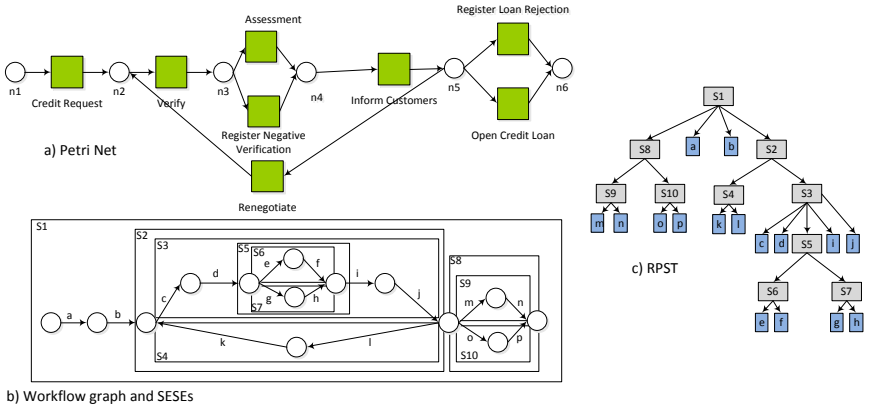


Fig. 4: A Petri net modeling the control-flow of the running example, its workflow graph and the RPST and SESE decomposition.

The decomposition based on SESEs is a well studied problem in the literature, and can be computed in linear time. In [13,14], efficient algorithms for constructing the *Refined Process Structure Tree (RPST)*, i.e., an hierarchical structure containing all the *canonical* SESEs of a model, were presented. Informally, an RPST is a tree where the nodes are canonical SESEs, such that the parent of a SESE S is the smallest SESE that contains S. Fig. 4(c) shows the RPST of the workflow graph depicted in Fig. 4(b). By selecting a particular set of SESEs in the RPST (e.g., k-decomposition [9]), it is possible to obtain a partitioning of the arcs. We refer the reader to the aforementioned work for a formal description of the SESE-based decomposition.

To extend the previous definitions to also account for data, one simply has to incorporate in the workflow graph the variables and read/write arcs, i.e., the *data workflow graph* of a data Petri net $(((P, T, F, l), M_{init}, M_{final}), V, val, init, read, write, guard)$ with data arcs $R = \{(v, t) | v \in read(t)\}$ and $W = \{(t, v) | v \in write(t)\}$ is $DWG = (V, E)$ with $V = P \cup T \cup V$ and $E = F \cup R \cup W$. The subsequent definitions after this extension (SESE, RPST) are analogous.

Similar to [9,10], we propose a SESE decomposition to analyze the conformance of Petri nets with data, but considering data workflow graph instead. Algorithm 1 describes the steps necessary to construct a SESE decomposition. The arcs are partitioned in SESEs by means of creating the RPST from the data workflow graph, and selecting a particular set of SESES over it. Once the partitioning is done, a subnet is created for each part. Subnets contradicting

[8] Note that by definition, a single edge is a SESE.

Algorithm 1 SESE-based Decomposition

1: Build data workflow graph DWG from F, R, W
2: Compute $RPST$ from DWG
3: Compute SESE decomposition D from the $RPST$
4: Compute and merge subnets if necessary to preserve valid decomposition.
5: **return** valid decomposition where perspectives are decomposed altogether

some of the requirements of Def. 16 (e.g, sharing places, invisible or duplicate transitions, variables, or transitions with non-splitting guards) are merged to preserve the valid decomposition definition.

Figure 5 shows the decomposition for the running example of Fig.2, where the RPST is partitioned using the 2-decomposition algorithm [9], i.e., SESEs of at most 2 arcs[9]. The merging at step 4 of Algorithm 1 combines multiple SESE fragments into larger ones, which are no more necessarily SESE, thus ensuring to always obtain valid decompositions.

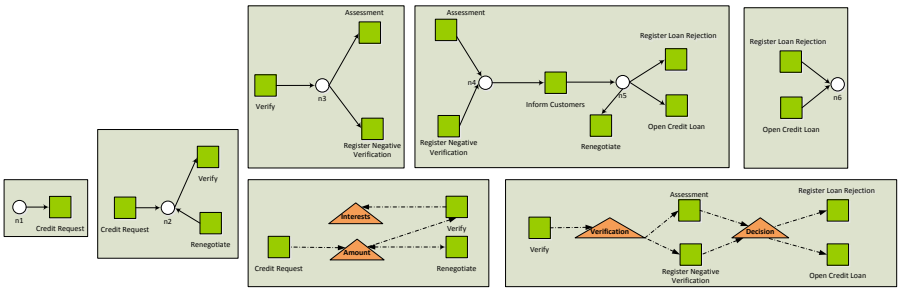


Fig. 5: SESE-based decomposition for the running example, with 2-decomposition.

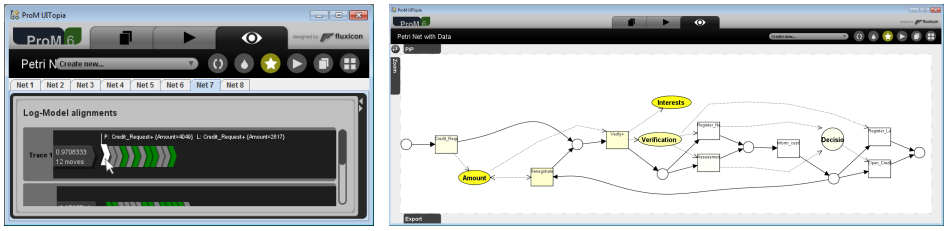
5 Implementation and Experimental Results

We have implemented the two instantiations of the framework of data-aware valid decomposition discussed in Section 4 as plug-ins for the open-source ProM framework.[10] ProM is a pluggable process mining framework. New process-mining algorithms can be easily developed as ProM plug-ins, thus avoiding re-implementing standard functionality.

Our implementation provides two outputs. The first type of output is shown in Figure 6a. The data Petri net is split in a number of fragments according to the decomposition's instantiation of choice and, for the SESE-based, the value chosen for parameter k. Each tab refers to a different fragment DPN_i; in each

[9] Although the SESEs have at most two arcs, this is not guaranteed for the final subnets, i.e., some subnets are merged to preserve the valid decomposition definition.

[10] `http://www.promtools.org`



(a) The visualization of the alignments. Each tab refers to a different fragment. Each sequence of triangle refers to the alignment of a trace with respect to that fragment, where events referring to transitions not present in the fragment are removed.

(b) The projection of the deviations on the process model. Due to implementation's issues, variables are not visualized as triangles but as ovals. We are currently working on fixing the visualization.

Fig. 6: Screenshots of the two types of output returned by the operationalization in the ProM framework.

tab, an optimal alignment is shown for each trace σ. In particular, we show the alignment of SN and the projection of σ onto the transitions of N. Optimal alignments are shown as a sequence of triangles, each representing an alignment's move. Each triangle is colored according to the move that it represents. The green and white colors are used to identify moves in both without or with incorrect write operations, respectively; yellow and purple are for moves in the log or in the process, respectively. Finally, the gray is used for moves for invisible transitions. When the user passes over a triangle with the mouse, the plug-in highlights the two transition firings s^L and s^P associated with the move (s^L, s^P). The value 0.97 associated with the trace is the fitness level, obtained by normalizing the alignment's cost between 0 and 1, where 1 indicates a zero cost (perfect fitness). We refer to [5] for details.

The second type of output is obtained by projecting every alignment for every fragment onto the data Petri net, as shown in Figure 6b. Transitions are colored according to the number of deviations: if no deviation occurs for a given transition, the respective box in the model is white-colored. The color shades towards red as a larger fraction of deviations occur for transition t, i.e. the number of moves in log, model or both with incorrect read/write operations for t divided by the total number of moves for t. Something similar is also done for variables: the more incorrect read/write operations occur for a variable, the more the variable is shown with a colour close to red. This visualization specializes what was proposed in [5] for the decomposition case. The main difference is that a transition t may appear in multiple fragments, say $DPN_1, \ldots, DPN_n$. Therefore,

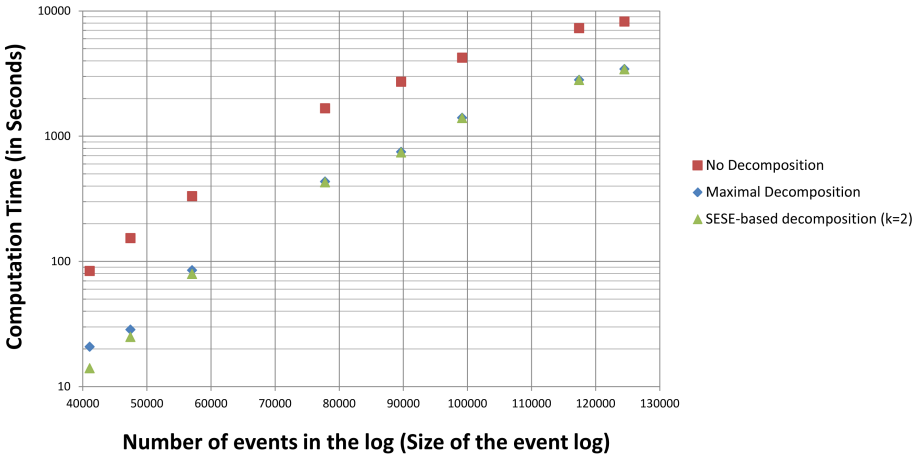


Fig. 7: Computation time for checking the conformance of the data Petri net in Figure 2 and event logs of different size. The Y axis is on a logarithmic scale.

an event for t in a trace σ is related to one move in n alignments, i.e. the alignments of σ with $DPN_1, \ldots, DPN_n$. To guarantee uniformity, the fraction of deviations is normalized by dividing it by n, i.e. the number of fragments in which t appears.

This output is extremely interesting from an end-user viewpoint as it allows for gaining a helicopter view on the main causes of deviations.

The plug-in has been evaluated using the model in Figure 2 and with a number of event logs that were artificially generated. In particular, we have generated different event logs with the same number of traces, 5000, but increasing number of events, meaning that, on average, traces were of different length. To simulate that, for each simulated process execution, an increasing number of renegotiations was enforced to happen. Traces were also generated so as to contain a number of deviations: the event logs were generated in a way that 25% of transitions fired violating the guards.

Figure 7 shows the results of checking for conformance of the different event logs and the process model, using both the maximal and the SESE-based decomposition (with $k = 2$). The decomposed nets are the same as in Figures 3 and 5. Regarding the cost function, we assign cost 1 to any deviation (this could be customized based on domain knowledge).

The results show that, for every combination of event log and process model, the decomposition significantly reduces the computation time and the improvement is exponential in the size of the event log. An interesting observation is that the SESE-based decomposition is faster for small log sizes. Initially, this may seem counterintuitive, since the SESE-based decomposition generates fragments of greater size. Nonetheless, it is easy to explain. As discussed, we employed

the conformance checker reported on in [5], which includes a overhead related to building one Integer Linear Problem for each trace. The SESE-based decomposition produces fewer data Petri nets to check the event-log conformance against. Since the number of traces is always 5000, a larger number of data Petri nets implies that the overhead is greater. If traces are not too long, the improvement in performance due to a maximal decomposition is lower than such an overhead.

For this particular example the computation time for the SESE-based and the maximal decomposition are comparable. This is due to the fact that the process model is too small to show dramatic differences between the two decompositions. The generation of synthetic event logs for large data Petri nets is far from being trivial, since there is no tool to automatically generate them, as it exists for normal Petri nets (e.g., [15]). However, in the future, we aim to perform extensive evaluations using much larger process models (e.g., with dozens or hundreds of transitions and process variables).

6 Conclusions and Future Work

Conformance checking is becoming more important for two reasons: (1) the volume of event data available for checking normative models is rapidly growing (the topic of "Big Data" is on the radar of all larger organizations) and (2) because of a variety of regulations there is a need to check compliance. Moreover, conformance checking is also used for the evaluation of process discovery algorithms. Also genetic process mining algorithms heavily rely on the efficiency of conformance checking techniques.

Thus far, lion's share of conformance checking techniques has focused on control-flow and relatively small event logs. As shown in this paper, abstracting from other perspectives may lead to misleading conformance results that are too optimistic. Moreover, as process models and event logs grow in size, divide-and-conquer approaches are needed to still be able to check conformance and diagnose problems. Perspectives such as work distribution, resource allocation, quality of service, temporal constraints, etc. can all be encoded as data constraints. Hence, there is an urgent need to support data-aware conformance checking in-the-large.

In this paper we demonstrated that data-aware decomposition can be used to speed up conformance checking significantly. As future work we would like to extend our experimental evaluation. For example, here we only used synthetic data. Moreover, we would like to explore alternative decomposition strategies using properties of the underlying data.

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