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On the atomic line profiles in high pressure plasmas

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In a previous contribution to this journal [H. P. Stormberg, J. Appl. Phys. 51(4), 1963 (1980)], Stormberg presented an analytical expression for the convolution of Lorentz and Levy line profiles, which models atomic radiative transitions in high pressure plasmas. Unfortunately, the derivations are flawed with errors and the final expression, while correct, is accompanied by misguiding comments about the meaning of the symbols used therein, in particular the "complex error function." In this paper, we discuss the broadening mechanisms that give rise to Stormberg's model and present a correct derivation of his final result. We will also provide an alternative expression, based on the Faddeeva function, which has decisive computational advantages and emphasizes the real-valuedness of the result. The MATLAB/Octave scripts of our implementation have been made available on the publisher's website for future reference. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4829916]

I. INTRODUCTION

Modelling and numerical simulation have been widely used to aid the understanding and guide the development of High-Intensity Discharge (HID) lamps. An important aspect of such models is the transport of energy via radiation, which depends critically on broadening mechanisms. When multiple statistically independent broadening mechanisms are present, the resulting line profile is obtained as the convolution product of the individual line profiles (see Ref. 1, p. 56). A well-known example is the Voigt profile, which is obtained by convolving the Gaussian and Lorentzian profiles that result from Doppler and Resonance or Stark broadening, respectively. The Voigt profile[2–4] can be expressed as

$$P_V(z) = \sqrt{\ln 2}\frac{\mathcal{R}[w(z)]}{\sqrt{\pi}\gamma_G}, \tag{1}$$

where $z = \sqrt{\ln 2}\frac{\nu-\nu_0+i\gamma_L}{\gamma_G}$, γ_G and γ_L represent the Gaussian and Lorentzian half widths at half maxima (HWHM), ν is the frequency, ν_0 is the unperturbed frequency, and the Faddeeva function $w(z)$ is given by

$$w(z) = \exp(-z^2)\mathrm{Erfc}(-iz). \tag{2}$$

The function Erfc is the complementary error function, which is given by

$$\mathrm{Erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-t^2}dt = 1 - \mathrm{Erf}(z), \tag{3}$$

where the error function is given by

$$\mathrm{Erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt. \tag{4}$$

Stormberg[5] derived an analytical expression for the case that the centre of the spectral line can be described by a Lorentzian profile and the red wing by a van der Waals profile. This case is particularly relevant for atomic transitions in high-pressure plasmas. He showed that the resulting line profile is given by

$$P(\Delta\lambda) = \frac{1}{\pi\Delta\lambda_{1/2}(1+a^2)} - \frac{ic\pi}{2}\left[Z_1^{1.5}\exp(Z_1 b)\mathrm{Erfc}\left(\sqrt{Z_1 b}\right) - Z_2^{1.5}\exp(Z_2 b)\mathrm{Erfc}\left(\sqrt{Z_2 b}\right)\right], \tag{5}$$

where $\Delta\lambda_{1/2}$ represents the full width at half maximum (FWHM) of the Lorentzian profile and $\Delta\lambda_0$ the characteristic width of the van der Waals (also called Levy) profile. The auxiliary parameters a, b, c, and $Z_{1,2}$ are given by

$$a = \frac{\Delta\lambda}{\Delta\lambda_{1/2}}, \tag{6}$$

$$b = \frac{\pi\Delta\lambda_0}{4\Delta\lambda_{1/2}}, \tag{7}$$

$$c = \frac{\sqrt{\Delta\lambda_0}}{2\pi\left(\Delta\lambda_{1/2}\right)^{3/2}} = \frac{\sqrt{b}}{\pi^{3/2}\Delta\lambda_{1/2}}, \tag{8}$$

$$Z_{1,2} = \frac{-a\mp i}{1+a^2}. \tag{9}$$

Stormberg creates some confusion about the meaning of Erfc by mentioning a method for calculating the *complex error function* in the accompanying text, probably referring the evaluation of the error function for complex *arguments* instead.[6,7] But more commonly the term "complex error function" refers to the Faddeeva function.[2,4,7]

Most authors who cite Stormberg's article[5] merely indicate that they used his expression, without any further

remarks on the mistakes in Stormberg's paper or the meaning of Erfc. Only two articles known to the authors discuss Stormberg's expression any further. Weiß *et al.*[8] mention that they calculate a Faddeeva function, but do not even provide an expression in which that function occurs. Hartel *et al.*[9] present an interesting alternative form of Stormberg's expression that avoids the need to subtract two terms involving the complementary error function, the result is

$$P(\Delta\lambda) = \frac{1}{\pi\Delta\lambda_{1/2}(1+a^2)} + \pi c \exp\left(\frac{-ab}{1+a^2}\right) \times \mathcal{I}\left[Z^{3/2}\exp\left(\frac{-ib}{1+a^2}\right)\mathrm{Erfc}\left(\sqrt{Zb}\right)\right], \quad (10)$$

with $Z = Z_1$ and $\mathcal{I}$ denoting the imaginary part. Unfortunately, they also use the confusing term "complex error function" to refer to the complementary error function and do not provide a derivation of their result, or discuss its advantages.

In this paper, we will first provide a short introduction to the theory of line broadening that is relevant for the present discussion. We continue the text with a complete and corrected derivation of Stormberg's and Hartel's result, which shows that indeed Erfc represents the complementary error function in all expressions. We will then derive a novel expression for Stormberg's result that employs the Faddeeva function. This expression has decisive analytical and computational advantages: it highlights the real-valuedness of the result and avoids the subtraction of two possibly (nearly-)equal terms. The MATLAB/Octave source code that has been used in our tests has been made available on the publisher's website for future reference.

II. LINE BROADENING

The most important broadening mechanisms in high pressure plasmas are resonance, van der Waals, and Stark broadening. Resonance, Stark, and van der Waals broadening can be calculated in the impact approximation, which is valid when the collision time is much shorter than the time between collisions,[10,11] or $\lambda - \lambda_0 = \Delta\lambda < \Delta\lambda_L$, with

$$\Delta\lambda_L = \frac{\langle v\rangle\lambda^2}{2\pi c\rho_W}. \quad (11)$$

Here λ_0 is the wavelength of the unperturbed transition and $\langle v\rangle$ the average thermal velocity of the interacting particles. The potential of the perturbation is assumed to be of the form

$$V = h\frac{C_n}{r^n}, \quad (12)$$

where h is Planck's constant and C_n is the broadening constant with dimensions $\mathrm{m}^n\mathrm{s}^{-1}$. For such a potential, the Weißkopf radius is given by

$$\rho_W = \left(\frac{\alpha_n C_n}{\langle v\rangle}\right)^{\frac{1}{n-1}}, \quad (13)$$

with

$$\alpha_n = \sqrt{\pi}\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}, \quad (14)$$

and Γ is the gamma function. This results in a Lorentzian line profile,

$$P_{Lorentz}(\Delta\lambda) = \frac{\Delta\lambda_{1/2}}{\pi\left(\Delta\lambda_{1/2}^2 + \Delta\lambda^2\right)}, \quad (15)$$

with $\Delta\lambda_{1/2}$ the Lorentzian HWHM. The convolution of two Lorentzian profiles is again a Lorentzian profile with a HWHM that is equal to the sum of the HWHM of the individual profiles. In other words, the HWHM's are additive for all statistically independent mechanisms that result in a Lorentzian profile.

For resonance broadening in the impact approximation, the HWHM is given by

$$\Delta\lambda_{res} = \sum_i\sum_j C_{R,ji}n_i = \sum_i\sum_j \frac{\lambda^2}{2c_0}\pi C_{3,ji}n_i, \quad (16)$$

where $C_{R,ji}$ and $C_{3,ji}$ are resonance broadening constants for exchange of excitation energy (see Ref. 1, pp. 101–103) between states j and i, n_i is the density of the radiating particle in state i, and c_0 is the speed of light in vacuum. Laux *et al.*[12] note that the following perturbations in general are sufficient

$$\Delta\lambda_{res} = \frac{\lambda_{ul}^2\pi}{2c_0}(C_{3,lg}n_g + C_{3,ug}n_g + C_{3,ul}n_l), \quad (17)$$

where the indices l, u, and g refer to the lower state, upper state, and ground state of the radiating particle. The resonance broadening constant is given by

$$C_{3,ji} = k_{J_j,J_i}\frac{1}{64\pi^3}\left(\frac{g_j}{g_i}\right)A_{ji}\lambda_{ji}^3, \quad (18)$$

with k_{J_j,J_i} a constant which depends on the total angular momentum quantum numbers J_j and J_i. Corney (Ref. 13, p. 263) reports the value $k_{1,0} = 1.53$ for the case $J_j = 1$, $J_i = 0$. This value is in good agreement with experiments in helium where the transition probability is accurately known for the $^1\mathrm{P}_1$–$^1\mathrm{S}_0$ resonance line. An expression often used for the resonance broadening constant is the value from Griem,[1,8,14,15] which is given by

$$C_{3,ji} = \frac{1}{8\pi^4}\sqrt{\frac{g_j}{g_i}}A_{ji}\lambda_{ji}^3. \quad (19)$$

Although this expression is not as accurate as Corney's expression it has the advantage that it doesn't rely on the constant k_{J_j,J_i}. In case of resonance broadening by a $^1\mathrm{P}_1$–$^1\mathrm{S}_0$ line, the ratio of Griem's expression to Corney's expression

is only 0.961. It is also interesting to note that Lawler[16] states that the linear relation between density and resonance line width in Hg is valid at least up to 4×10^{25} m^{-3}. This density is outside the validity of the impact approximation. Lawler additionally claims that the Lorentzian line profile may also be valid to line offsets that are significantly larger than suggested by the validity of the impact theory.

In the impact approximation, the HWHM due to Stark broadening is calculated with various degrees of sophistication. Stormberg[17] uses a HWHM given by

$$\Delta\lambda_{Stark} = C_S n_e, \tag{20}$$

while Refs. 8, 14, and 15 use the model

$$\Delta\lambda_{Stark} = \frac{\lambda^2}{4\pi c_0} 11.37 C_4^{2/3} v_e^{1/3} n_e, \tag{21}$$

with C_S and C_4 the Stark broadening constants, v_e the thermal velocity of the electrons, and n_e the electron density. The second relation features a weak temperature dependency, which is not present in the first relation. Another expression for the HWHM of Stark broadening is found in Griem[10,18]

$$\Delta\lambda_{Stark} = [1 + 1.75\alpha(1 - 0.75r)]\omega, \tag{22}$$

with r the ratio of the mean distance between ions ρ_m and the Debye length ρ_D,

$$r = \frac{\rho_m}{\rho_D} = \frac{\left(\frac{4\pi n_i}{3}\right)^{-1/3}}{\left(\frac{\epsilon_0 k_B T}{e^2 n_e}\right)^{1/2}}, \tag{23}$$

where n_i is the ion density and α and ω are tabulated Stark broadening parameters by Griem.[18] Since all of these methods rely on input data, it is recommended to use the method that has the most accurate tabulations.

The contribution of van der Waals broadening in the impact approximation[8,14,15] is given by

$$\Delta\lambda_{vdW} = \frac{\lambda^2}{2\pi c_0} 4.04 \left(\frac{8 k_B T}{\pi}\right)^{3/10} \sum_i C_{6,i}^{2/5} \frac{n_i}{\mu_i^{3/10}}, \tag{24}$$

with C_6 the van der Waals broadening constant, n_i the density of the perturbing particle, and μ the reduced mass calculated for the radiating and the perturbing particle. The van der Waals broadening constant can be estimated based on a hydrogen like approximation as[14]

$$C_6 = \frac{1}{2h\epsilon_0} e^2 \alpha_{pert} |\langle r_u^2 \rangle - \langle r_l^2 \rangle|, \tag{25}$$

with α_{pert} the polarizability of the perturbing particle and $\langle r_i^2 \rangle$ the mean square radius of the atoms in state i, which is given by

$$\langle r_i^2 \rangle = a_0^2 \frac{(n_i^*)^2}{2} \langle 5(n_i^*)^2 + 1 - 3 l_i (l_i + 1) \rangle. \tag{26}$$

Here a_0 is the Bohr radius, l_i is the orbital quantum number of state i, and $\langle n_i^* \rangle^2$ is the effective quantum number

$$\langle n_i^* \rangle^2 = \frac{E_H}{E_{ion} - E_i}, \tag{27}$$

with E_H the ionization energy of hydrogen, E_{ion} the ionization energy of the radiating species, and E_i the energy of state i.

The van der Waals interaction can also be calculated in the quasistatic approximation. This approximation assumes that the radiating particles are perturbed by a slowly varying potential field which can be considered quasistatic. This approximation results in a Levy profile,[10,14]

$$P(\Delta\lambda) = \begin{cases} \frac{\sqrt{\Delta\lambda_0}}{2(\Delta\lambda)^{3/2}} \exp\left(-\frac{\pi\Delta\lambda_0}{4\Delta\lambda}\right) & \Delta\lambda > 0 \\ 0 & \Delta\lambda \le 0, \end{cases} \tag{28}$$

with $\Delta\lambda_0$ given by

$$\Delta\lambda_0 = \sum_{pert} C_{W,pert} n_{pert}^2, \tag{29}$$

$$= \sum_{pert} \frac{\lambda^2}{2\pi c_0} C_{6,pert} \left(\frac{4}{3}\pi n_{pert}\right)^2, \tag{30}$$

with C_W and C_6 van der Waals broadening constants and n_{pert} the perturber density.

III. STORMBERG'S EXPRESSION

A comparison with experiments shows that the Lorentz profile describes the centre of the line profile accurately while the Levy profile is an accurate estimate of the red wing. Stormberg determines the total line profile by taking the convolution of the Lorentz and the Levy profile. In this section, we present a corrected derivation of Stormberg's expression, derive Hartel's result, and provide a numerically superior expression, based on the Faddeeva function. The starting point is the convolution integral

$$P(\Delta\lambda) = \int_{-\infty}^{\infty} P_{Levy}(\Delta\lambda) P_{Lorentz}(\epsilon - \Delta\lambda) d\epsilon. \tag{31}$$

Substitution of Eqs. (15) and (28) yields

$$P(\Delta\lambda) = \frac{\sqrt{\Delta\lambda_0}}{2\pi(\Delta\lambda_{1/2})^{3/2}} \sqrt{\Delta\lambda_{1/2}} \times \int_0^{\infty} \frac{1}{(\epsilon)^{1.5}} \exp\left(-\frac{\pi\Delta\lambda_0}{4\Delta\lambda_{1/2}} \frac{\Delta\lambda_{1/2}}{\epsilon}\right) \frac{1}{1 + \left(\frac{\Delta\lambda - \epsilon}{\Delta\lambda_{1/2}}\right)^2} d\epsilon, \tag{32}$$

and by changing to the integration variable

$$y = \frac{\Delta\lambda_{1/2}}{\epsilon}, \tag{33}$$

and using the definitions (6) and (7) one obtains

$$P(\Delta\lambda) = c\int_0^\infty \sqrt{y}\frac{y}{y^2+(ya-1)^2}\exp(-by)dy. \tag{34}$$

Using definition (9), one gets

$$\frac{y}{y^2+(ya-1)^2} = \frac{-i}{2(1+a^2)}\left(\frac{a+i}{y+Z_1}+\frac{-a+i}{y+Z_2}\right), \tag{35}$$

which allows us to write Eq. (34) as

$$\begin{aligned} P(\Delta\lambda) = {} & \frac{ic}{2(1+a^2)} \\ & \times\int_0^\infty\left[\frac{(-a-i)\sqrt{y}}{y+Z_1}\exp(-by)\right. \\ & \left.-\frac{(-a+i)\sqrt{y}}{y+Z_2}\exp(-by)\right]dy. \end{aligned} \tag{36}$$

This equation has the form of a (unilateral) Laplace transform,

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\exp(-st)dt, \tag{37}$$

and in a book of Laplace tables like[19] one finds that

$$\int_0^\infty \frac{\sqrt{y}}{y+Z}\exp(-by)dy = \sqrt{\frac{\pi}{b}} - \pi\sqrt{Z}\exp(Zb)\mathrm{Erfc}\left(\sqrt{Zb}\right), \tag{38}$$

where Erfc is the *complementary* error function. This result is valid under the conditions that

$$\mathcal{R}(b) = \mathcal{R}\left(\frac{\pi\Delta\lambda_0}{4\Delta\lambda_{1/2}}\right) > 0, \tag{39}$$

$$|\arg(Z)| = \left|\arg\left(\frac{-a\mp i}{1+a^2}\right)\right| < \pi, \tag{40}$$

which are always met because of the definitions of b and $Z_{1,2}$. Note that $\mathcal{R}$ is used to refer to the real part of an expression. By substituting Eq. (38) in Eq. (36) Stormberg's original expression (5) can be obtained, with the observation that Erfc is the *complementary* error function.

In the remaining part of this section, an alternative expression will be derived that is based on the Faddeeva function. We start by rearranging the Laplace transform as

$$\begin{aligned} & \sqrt{\frac{\pi}{b}} - \pi\sqrt{Z}\exp(Zb)\mathrm{Erfc}\left(\sqrt{Zb}\right) \\ & \quad = \sqrt{\frac{\pi}{b}}\left(1-\sqrt{\pi}\sqrt{Zb}\exp(Zb)\mathrm{Erfc}\left(\sqrt{Zb}\right)\right). \end{aligned} \tag{41}$$

Using

$$\sqrt{Zb} = -i^2\sqrt{Zb} = -id, \tag{42}$$

with $d = i\sqrt{Zb}$, the Laplace transform can be expressed in terms of d as

$$\begin{aligned} & \sqrt{\frac{\pi}{b}}\left(1+i\sqrt{\pi}d\exp(-d^2)\mathrm{Erfc}(-id)\right) \\ & \quad = \sqrt{\frac{\pi}{b}}\left(1+i\sqrt{\pi}dw(d)\right), \end{aligned} \tag{43}$$

where we have introduced the Faddeeva function Eq. (2). The final line profile can then be rewritten as

$$\begin{aligned} \frac{P(\Delta\lambda)}{P_{Lor}(\Delta\lambda)} = {} & \frac{-i(+a+i)}{2}\left(1+i\sqrt{\pi}d_1w(d_1)\right) \\ & +\frac{-i(-a+i)}{2}\left(1+i\sqrt{\pi}d_2w(d_2)\right) \\ = {} & 1 \\ & -\frac{i(1+a^2)d_1^2}{2b}i\sqrt{\pi}d_1w(d_1) \\ & +\frac{i(1+a^2)d_2^2}{2b}i\sqrt{\pi}d_2w(d_2) \\ = {} & 1+\frac{(1+a^2)\sqrt{\pi}}{2b}\left\{d_1^3w(d_1)-d_2^3w(d_2)\right\}. \end{aligned} \tag{44}$$

The term $d_1^3w(d_1)$ can be related to $d_2^3w(d_2)$ by noting that

$$\overline{d_1} = \overline{i\sqrt{Z_1b}} = -i\sqrt{\overline{Z_1}b} = -i\sqrt{Z_2b} = -d_2, \tag{45}$$

as a result of which

$$\overline{d_1^3w(d_1)} = \overline{d_1^3}\,\overline{w(d_1)} = \overline{d_1}^3w\left(\overline{-d_1}\right) = -d_2^3w(d_2). \tag{46}$$

In the last step, we have used $w(\bar{z}) = \overline{w(-z)}$ (see Ref. 20, Eq. 7.1.12). Then the final expression can be simplified to

$$\frac{P(\Delta\lambda)}{P_{Lor}(\Delta\lambda)} = \left[1+\frac{(1+a^2)\sqrt{\pi}}{b}\mathcal{R}\left\{d_1^3w(d_1)\right\}\right]. \tag{47}$$

When the Faddeeva function is expressed in terms of the complementary error function this yields Hartel's expression.

Equation (47) has decisive analytical and computational advantages, compared with Stormberg's and Hartel's expressions. It is more compact, emphasizes that the line profile is real-valued, and avoids the subtraction of possibly (nearly) equal terms. Moreover, there is no need to evaluate the product of a complex exponent and an error function, since specialized algorithms exist for the numerically stable evaluation of the Faddeeva function itself.[21]

Before discussing the numerical stability further, it is instructive to explore the limiting cases in which one of the broadening mechanisms is absent. The Lorentzian profile can be retrieved trivially by using

$$\lim_{\Delta\lambda_0\downarrow 0} b = \frac{\pi\Delta\lambda_0}{4\Delta\lambda_{1/2}} = 0, \tag{48}$$

which implies that

$$\lim_{\Delta\lambda_0\downarrow 0}\frac{d_{1,2}^3}{b} = \lim_{\Delta\lambda_0\downarrow 0}\frac{[i(-a\mp i)b]^3}{(1+a^2)^{3/2}b} = 0. \tag{49}$$

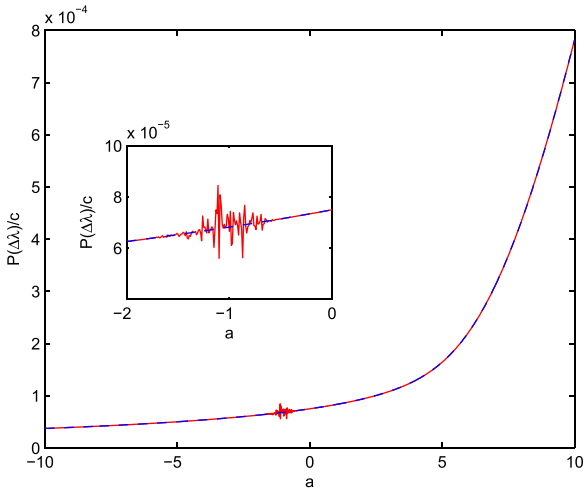

FIG. 1. For $b = 50$, the line profile can become numerically unstable when Stormberg's original expression (5) is used (solid line). The new expression (47) is stable for all b values (dashed line).

Recovering the Levy profile is more tricky, since $\lim_{\Delta\lambda_{1/2}\downarrow 0} a = \infty$, $\lim_{\Delta\lambda_{1/2}\downarrow 0} b = \infty$ and $\lim_{\Delta\lambda_{1/2}\downarrow 0} c = \infty$. But since

$$\lim_{\Delta\lambda_{1/2}\downarrow 0} \frac{a}{b} = \frac{4\Delta\lambda}{\pi\Delta\lambda_0}, \tag{50}$$

is constant, for $\Delta\lambda_{1/2} \downarrow 0$ we can easily derive that

$$P(\Delta\lambda) = \frac{4}{\pi^2\Delta\lambda_0}\left(\frac{b}{a}\right)^{1.5} \times \left[\frac{\sqrt{\pi}}{2}\left\{w\left(\sqrt{\frac{b}{a}}\right) + w\left(-\sqrt{\frac{b}{a}}\right)\right\}\right]. \tag{51}$$

Using $w(z) + w(-z) = 2\exp(-z^2)$ (see Ref. 20, Eq. 7.1.11) and the definitions of a and b, we arrive at the expression for the Levy profile.

IV. NUMERICAL STABILITY

The line profile calculated with Eq. (5) shows oscillations for large values of b. First, an example is given of a physical situation where these high values of b can occur. After that an implementation of Eq. (47) in MATLAB and its numerical stability are discussed.

The line profile can be characterized with the variables $\Delta\lambda_{1/2}$ and $\Delta\lambda_0$. A more insightful description is given by the parameter b which contains the ratio of the impact and quasistatic linewidths. By only taking into account resonance broadening in the impact limit and van der Waals broadening in the quasistatic limit, this ratio can be expressed in terms of the broadening constants as

$$b = \frac{\pi}{4}\frac{16C_6 n_p^2}{9C_3 n_r}, \tag{52}$$

with n_r the density of radiating species and n_p the density of perturbing species. In a situation where the radiating particles are not the same as the perturbing particles a high value of b can occur. For example in a high pressure mercury discharge containing sodium iodide, the ratio can become large at lower temperatures were the sodium is bound in molecules. In methods like raytracing,[22] a correct calculation of the line profile at lower temperatures is also required to account for the absorption accurately.

The calculation of the line profile[21] according to Eq. (5) in MATLAB requires an implementation of the error function. Since this is not a built-in function, we used the

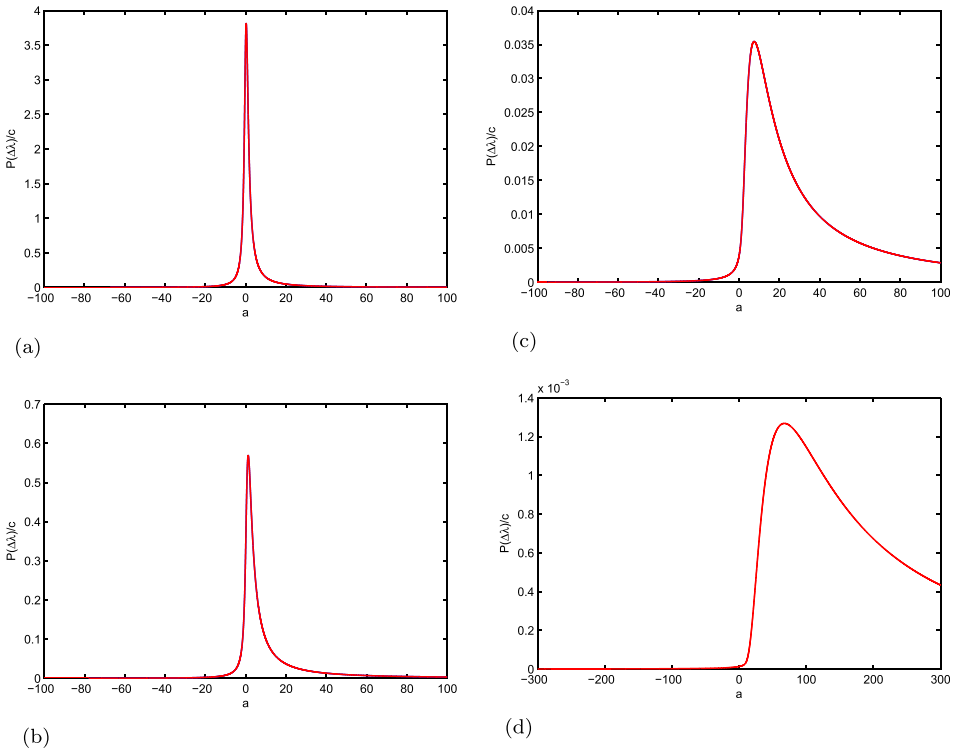

FIG. 2. The function $P(\Delta\lambda)/c$ for various values of b.

implementation from Leutenegger[21] for our tests. For $b > 50$, these revealed numerical artifacts in the line shape that can be attributed to inaccuracies in Leutenegger's implementation. Our new expression (47) that was tested with the implemented Faddeeva function of Ikuma[21] did not have such problems.

These results are shown in Figure 1. Since the line profiles are proportional to c, we have plotted $P(\Delta\lambda)/c$ in these graphs. The figure clearly shows the oscillations for $b = 50$ when Stormberg's original expression is used with Leutenegger's algorithm for calculating the error function; the noise rapidly increases in magnitude and width when b is increased further.

For a few values of b, the resulting line profiles are shown in Figure 2. These figures show that the oscillations which start to occur around $b = 50$ arise when the line profile is dominated by the Levy contribution. Wharmby[23] already mentioned that impact and quasistatic theory have their limitations and that for high densities the Levy profile can cause unrealistically large red wings. He was still able to obtain accurate results by convolving again with a Gaussian profile to artificially limit these wings. A correct calculation of the line profile requires to take into account accurate potential curves of the interacting species. Such curves are not always available and in some cases a more simple calculation of the line profile is preferred. For these cases, Stormberg's line profile can be used as a first order approximation.

V. CONCLUSION

We have evaluated the dominant broadening mechanisms of atomic lines in high pressure plasma. We have shown that when Stormberg's original expression is used in conjunction with a popular implementation of the error function in MATLAB/Octave, intolerable numerical errors are observed when the broadening is dominated by the Levy contribution. We have presented a novel expression that is based on the Faddeeva function, which is more elegant, computationally efficient and accurate under all circumstances. The MATLAB/Octave scripts of our implementation have been made available on the publisher's website for future reference.[21]

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