

Beliefs and beyond : affect and the teaching and learning of mathematics

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Beliefs and beyond: affect and the teaching and learning of mathematics

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Abstract The importance of beliefs for the teaching and learning of mathematics is widely recognized among mathematics educators. In this special issue, we explicitly address what we call “beliefs and beyond” to indicate the larger field surrounding beliefs in mathematics education. This is done to broaden the discussion to related concepts (which may not originate in mathematics education) and to consider the interconnectedness of concepts. In particular, we present some new developments at the conceptual level, address different approaches to investigate beliefs, highlight the role of student beliefs in problem-solving activities, and discuss teacher beliefs and their significance for professional development. One specific intention is to consider expertise from colleagues in the fields of educational research and psychology, side by side with perspectives provided by researchers from mathematics education.

1 Introduction

Historically, mathematics-related research in the field of affect and beliefs goes back to the ‘failure of the problem-

solving based reforms’ of the mathematics curriculum in the USA in the late 1980s (Schoenfeld, 2007). Numerous studies have detected that the reasons for that ‘failure’ were to some extent the ‘inappropriate’ beliefs of teachers concerning mathematics in general, and the process of problem solving and characteristics of doing mathematics in particular, in addition to strong teacher convictions concerning students’ apparent lack of ability (cf. Schoenfeld, 1985, 1989, 1994; Frank, 1988, 1990; Garofalo, 1989; Mtetwa & Garofalo, 1989).

Since then, numerous studies have emphasized the essential role of beliefs in learning and teaching mathematics (cf. Thompson, 1992; Richardson, 1996; Philipp, 2007). Early on, researchers differentiated between beliefs of teachers and students and, at that time, focused on mathematical ‘world views’ to describe the different perspectives. Subsequently, beliefs were perceived as decisive for ‘misconceptions’ in mathematics education, and these, it was claimed at the time, had to be ‘changed’ or avoided (Thompson, 1992; Pajares, 1992). Although never conclusively proven, the hypothesis was that beliefs were the background and basis for actions, which indicated a connection between students’ poor performance and their beliefs. It was also assumed that there was a correlation between teachers’ and their students’ beliefs. However, the fundamental work by Schoenfeld (1998) and, particularly, his model of ‘Teaching-In-Context’ provided evidence for the claim that beliefs take center stage with respect to instantaneous teacher actions.

A subsequent step in binding together various approaches of beliefs and their influences (see also Ernest, 1989) was the issue of the ZDM in 1996, edited by Pehkonen and Toerner. The book edited by Leder, Pehkonen and Toerner 2002 reflected most of the views presented in the Special Issue, and beliefs were referred to as ‘Hidden variables’, with reference to a famous paper by Bauersfeld (1980).

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Considering the aforementioned papers by Thompson (1992) and Pajares (1992), which appeared almost at the same time, it is evident that the two researchers ‘originated’ in different fields of educational research: one in mathematics education; and the other in educational psychology. In his work, Pajares emphasizes the epistemological character of beliefs, whereas in the work of Thompson the word epistemology is not even mentioned. Moreover, it is apparent that similar constructs have been discussed, albeit with a different background, and different classifications have been produced (Sierpiska & Lerman, 1996). Approximately since 1990, a parallel intensive discussion about epistemology has taken place in the psychology community (e.g., Schommer, 1990; De Corte, Op’t Eynde & Verschaffel, 2002; Buehl & Alexander, 2005), which has hardly been acknowledged in mathematics education (cf. Muis, 2004). Over the years, new understandings have developed, in the sense that the discussions in both communities on epistemology in mathematics are now perceived essentially as beliefs discussions, or vice versa that beliefs about mathematics education are intrinsically connected to epistemological aspects. However, one seldom finds links to these studies conducted in the field of educational research and psychology.

Thus, one intention of this special issue is bridging the gap between mathematics education and the vast ‘continent’ of epistemology research conducted by educational psychologists. It is therefore no surprise that the Special Issue’s contributions developed not only from presentations of ICME 11’s Topic Study Group on affect (Mexico), mainly by mathematics educators, but also encompass perspectives discussed at a beliefs symposium at EARLI in Amsterdam, where selected researchers active in the field of educational psychology presented. Ultimately, we aim at developing a broader understanding of beliefs, the ways of ‘measuring’ them, and the ways beliefs may ‘play out’ in the mathematics classroom. The contributions thus address theoretical and methodological aspects and encompass issues of learning *and* teaching mathematics.

2 Developments at the conceptual level

Selected studies have investigated the role of teacher behavior in the mathematics classroom discussing teacher knowledge, teacher beliefs, and to a lesser extent teacher ‘instructional goals’ (e.g., Sowder, 2007; Philipp, 2007; Baumert et al., 2010; Roesken, 2011). Typically, these studies focus on one of the above-mentioned aspects without considering the interconnections between them. However, Schoenfeld (1998) provided a theory bringing together these different fields while modeling teacher actions as a function of teacher knowledge, goals, and beliefs. In the first article of

this Special Issue, Schoenfeld presents a modification of his initial theory and introduces a broader concept of resources to include teacher knowledge, in addition to orientations to encompass the fields of beliefs, values, biases, and dispositions. In particular, the article outlines and exemplifies how resources, goals, and orientations may shape teacher behavior. It indicates how the different aspects are interconnected, and why their evolution is necessarily slow. Finally, Schoenfeld suggests how these understandings may influence mathematics teacher professional development.

A large number of research studies address the theoretical clarification of the constructs of beliefs, emotions and attitudes, and their relationships. In this regard, McLeod (1994) proposes considering the affective domain as including emotions, attitudes, and beliefs. Thereby, the different constructs are named in increasing order of stability (over time), and with increasingly prominent cognitive elements (cf. Goldin, Rösken, & Törner, 2009). Expanding on this discussion, DeBellis and Goldin (2006) suggest including a fourth subdomain dealing with values, ethics, and morals, which is connected to the other three subdomains. In this respect, Di Martino and Zan investigate the deep interplay between beliefs and emotions, and they link both constructs to attitude. In their article in this issue, they elaborate on a three-dimensional model of attitude toward mathematics (Di Martino & Zan, 2010) that includes students’ emotional disposition, their vision of mathematics, and their perceived competence.

In the third paper, the role of emotion is explored in relation to students’ self-regulation as a key characteristic of learning in the field of mathematics education (De Corte et al. 2011). Although a strong focus on meta-cognitive aspects has accompanied the initial work in this field (Schoenfeld, 1992), many studies have included motivational and emotional aspects (e.g., McLeod 1994; Kloosterman, 1996). In their article in this issue, De Corte, Depaepe, Op’t Eynde, and Verschaffel present the ‘meta-emotional’ perspective as an essential component of a conceptual framework on self-regulation that fully acknowledges the role of emotions, and they exemplify their model with students working on school-related mathematical activities.

3 Approaches to capture beliefs

Studies to capture students’ beliefs are in most cases descriptive, for example reporting typical beliefs held by students (e.g., Ma & Kishor, 1997). Selected studies compare student beliefs in different countries (e.g., Pehkonen, 1994) or according to background variables such as gender (e.g., Frost, Hyde and Fennema, 1994), while a few contributions address explicitly belief systems (e.g., Op’t Eynde and De

Corte, 2003). Green (1971) introduced this term and although the importance of the systematic nature of beliefs is widely acknowledged (e.g., Schoenfeld, 1985), there is a lack of studies elaborating on belief systems. However, Op't Eynde, De Corte and Verschaffel (2002) provide a framework of students' mathematics-related beliefs and underline the interplay of different dimensions. In the fourth paper of this issue, Roesken, Hannula, and Pehkonen present a study investigating the complex structure of student views of themselves as learners of mathematics, taking into account the aforementioned framework. Different dimensions of such a view are presented for a representative sample of Finish students.

In the fifth article, Muis, Franco, and Gierus also present an approach that explores the systematic nature of beliefs. Students' epistemic beliefs are their focus, and these are theoretically considered, using a comparative approach, with respect to mathematics educationists' and educational psychologists' views on beliefs about mathematics. The authors examine whether students' epistemic beliefs differ as a function of variations in procedural versus conceptual knowledge in statistics. The study aims at exploring to what extent students' domain-specific epistemic beliefs are context specific.

While the first two papers of this section favor a quantitative approach to examine students' beliefs, Rolka and Halverscheid (in the sixth paper of the Special Issue) present an alternative approach to reveal students' beliefs using pictures, texts, and interviews as data sources for studying mathematical world views of fifth and sixth graders. Given that the results are not conclusive as they indicate 'mixed' world views of students, the authors have reflected on their methodological approach.

4 Student beliefs and mathematics learning

Much research has been conducted with respect to examining the role of affect in learning in general, and in social contexts such as the mathematics classroom in particular (Hannula, Evans, Philippou & Zan, 2004). It has been widely agreed that learning can be conceptualized as a social process, distributed over context, behavior, personal beliefs, and meanings (e.g., Lave & Wenger, 1991). However, critics still argue that not enough research results have reached the mathematics classroom and its practices (e.g., Lester, 1996).

From larger-scale studies such as PISA, it is evident that the development of a student identity as a learner of mathematics is likely to be different in different contexts, and may vary from country to country (e.g., Cogan & Schmidt, 1999). Although research on affect in mathematics education has benefited from including the influence of socio-cultural

aspects, it is surprising that there are so few international studies in the field. Thus, in the seventh paper, Pepin compares English and Norwegian pupils' attitude toward mathematics to develop a deeper understanding of the factors that may shape and influence 'pupil attitude toward mathematics'. One focus of this article claims, and supports, the conceptualization of affect as a socio-cultural construct embedded in and shaped by students' environment and context in which they learn mathematics.

The eighth paper of this Special Issue by Goldin, Epstein, Schorr, and Warner elaborates on beliefs that may influence students' mathematical learning and problem-solving activities, and how these are structured and intertwined with larger affective and cognitive structures. To capture this multifaceted view, the study theoretically explores a psychological concept, which the authors coin as 'engagement structure', and which according to their theory is intrinsically intertwined with beliefs.

The ninth paper also addresses student beliefs related to problem-solving behavior and activities. In particular, Csikos, Kelemen, and Verschaffel attend to subjective and implicit knowledge about what type of answers students consider appropriate in a given mathematical testing context, where students are presented with several types of supposedly non-solvable problems. The findings show an overall tendency to follow non-realistic approaches even when different response patterns are presented.

5 Role of teacher beliefs and their significance for professional development

In this section of the Special Issue, teacher beliefs are addressed, as well as how these may shape teacher attitude toward professional development in general, and more particularly in terms of their pedagogic practice in the mathematics classroom. Beliefs are often robust, or as Sowder (2007) puts it "many of teachers' core beliefs need to be challenged before change can occur" (p. 160). It has been acknowledged that any change or development in a teacher's beliefs is a long-term process, and that short-term professional development initiatives often remain at the surface and do not effect the desired change (cf. Pehkonen and Törner, 1999; Schommer-Aikins, 2004).

The article by Maass examines how teachers' beliefs may influence their intentions to implement change as suggested by a professional development initiative. Although based on a small sample of selected German mathematics teachers, the results provide interesting insights into teachers' perceptions of 'effective teaching' and the effect these may have on whether or not they intend to change their classroom practice as suggested by the professional development initiative.

Furinghetti and Morselli, in their paper, investigate the way secondary school teachers treat proof and develop an understanding of the factors that may influence such a treatment. In their theoretical framework, the authors combine selected results from the field of research on proof with those from research on teacher beliefs. In terms of results, Furinghetti and Morselli are able to detect the reasons behind teachers' choices in planning their work in the classroom and, perhaps more importantly, elements that help to unravel inconsistencies when using the construct of leading beliefs with respect to teacher work and mathematical proof.

In the last paper of this Special Issue, Roscoe and Sriraman propose a framework that views an individual's beliefs regarding mathematics on a continuum, which ranges from formal beliefs at one end to informal beliefs at the other. This study aims at developing an understanding of the relationship between participation in informal mathematics activities and the formal-to-informal beliefs of university teacher candidates in elementary education. In particular, student reflection upon personal experience derived from participation in the activities was analyzed for formal and informal belief statements.

In summary, the articles of this issue provide an encompassing view on research in the field 'beliefs and beyond', while elaborating in particular on the role of affect in relation to the teaching and learning of mathematics. Some new developments at the conceptual level are presented, methodological aspects in beliefs research are discussed, and students' and teachers' beliefs are analyzed with varying accentuations. The approaches and perspectives presented by researchers from mathematics education, educational researchers, and psychologists provide a multifaceted view on affect and beliefs, and bring together research findings developed in the different disciplines.

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