Transverse and longitudinal noise

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Transverse and longitudinal noise

L. K. J. Vandamme and L. P. J. Kamp

Eindhoven University of Technology, Department of Electrical Engineering, Eindhoven, Netherlands

Analytic functions are derived for the spectral noise voltage density between two circular sensor electrodes on a two-dimensional isotropic conductor, placed either transversely or longitudinally to a homogeneous electric field. The sensor electrodes are far removed from both the bias electrodes and the boundaries. The relations have been checked experimentally for $1/f$ conductivity fluctuations. Both the transverse and longitudinal noise are proportional to $I^{-2}\int_s(\mathbf{J}\cdot\tilde{\mathbf{J}})^2\,ds$ where $\mathbf{J}\cdot\tilde{\mathbf{J}}$ represents the dot (scalar) product of the homogeneous current density $\mathbf{J}$ and the adjoint current density $\tilde{\mathbf{J}}$, and the current through the bias electrodes is I. It is found that the transverse noise is somewhat smaller than the longitudinal noise.

I. INTRODUCTION

This paper is concerned with the effects of electrode position and diameter on the noise voltages measured between the electrodes. The systems have the sensor electrodes far removed from bias electrodes and boundaries, in a uniform isotropic material in a homogeneous electric field. The electrodes are either placed precisely in line with the direction of bias current (longitudinally) or on a line perpendicular to the direction of the bias current (transversely). The noise between transversely or longitudinally placed sensor electrodes is often called transverse and longitudinal noise, respectively. The electrode configurations are given in the inset of Fig. 1. Transverse and longitudinal noise consists of a thermal noise term and a conduction noise term. The thermal noise is proportional to the real part of the impedance between the sensor electrodes. For sensors far removed from bias electrodes and from the boundaries, the impedance between the sensors is a function only of the sheet resistivity, the sensor diameter, and the distance between the sensors. So, the transverse thermal noise and the longitudinal thermal noise are the same when the sensor diameter $2r$ and the distance $2b$ are kept constant. The resistance R between such an electrode pair is given by Vandamme and Groot[1] and the thermal noise between the sensors is then $4kTR$.

When a constant current is passed through the sample, the conductivity fluctuations cause voltage perturbations which can be observed either in the direction of the current flow or at right angles to it. The conduction noise term is proportional to the square of the bias current I. Thus, above a certain current level the noise at the sensors is dominated by the conduction noise term.

Here we consider only transverse noise and longitudinal noise due to conductivity fluctuations. The advantages of measuring transverse noise rather than longitudinal noise at very low frequencies were already mentioned by Hawkins and Bloodworth.[2] General relations for the noise voltage between arbitrarily shaped and placed sensor electrodes on a conductor when a constant current or voltage is applied to another pair of arbitrarily shaped and placed driver electrodes is given by Vandamme and van Bokhoven.[3] A computer approach solving the transverse noise and longitudinal noise in samples submitted to homogeneous fields is also treated.[3]

We report on analytic functions and measurements for
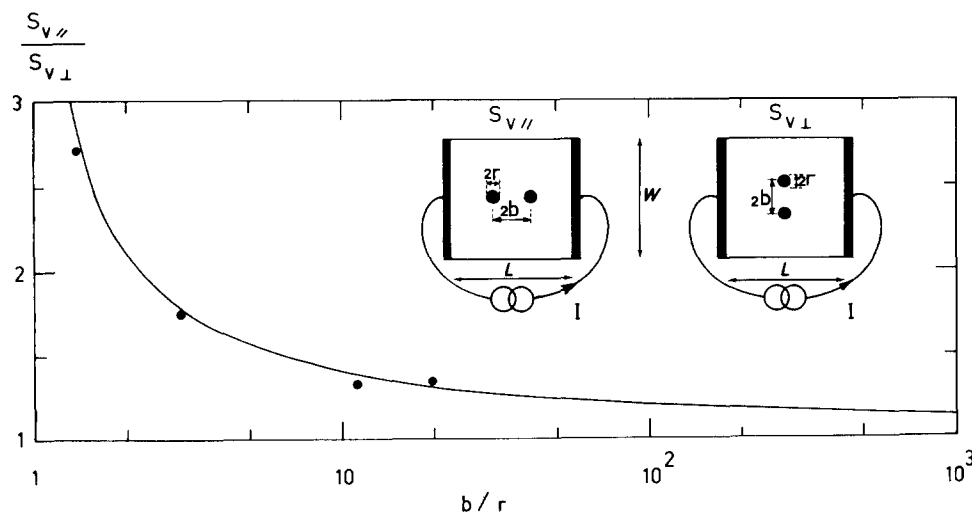
FIG. 1. The inset shows the electrode placement for the longitudinal noise $S_{v\parallel}$ and the transverse noise $S_{v\perp}$ measurement. The sample is biased with a constant current I at the strip-shaped driver electrodes. The line represents the ratio of the longitudinal noise to the transverse noise as a function of the ratio of the distance between the sensors and the sensor diameter. The dots represent the experimental results.

the transverse noise and longitudinal noise between two circular sensor electrodes in two-dimensional conductor submitted to a uniform field when the noise is dominated by $1/f$ noise. The modification required in the equations for a generation-recombination spectrum is presented.

II. CALCULATION OF TRANSVERSE NOISE AND LONGITUDINAL NOISE POWER DENSITIES $S_{v\perp}$ AND $S_{v\parallel}$

The inset of Fig. 1 shows circular sensors having diameters $2r$, with $2b$ the distance between the centers. The length and the width are denoted by L and W, respectively. The conductor is assumed to be homogeneous and isotropic in its macroscopic as well as its statistical properties. We assume the conductance fluctuations between very small subareas are uncorrelated.

The electrodes are assumed to be ideal, which means that the resistivity is negligible. We take the sensors far from the boundaries and the driver electrodes, and the diameter $2r$ is less than one-tenth of the sample dimensions L and W. The ratio b/r is chosen larger than 1.4, to avoid deviations of the homogeneous field around the sensors. The noise due to conduction fluctuations is calculated, using the general relations[3] based on the sensitivity theorem. For a two-dimensional conductor the spectral power density of the voltage between the sensors becomes for $1/f$ noise

$$S_v = \frac{\alpha \rho^2}{nfI^2} \iint_{\substack{\text{total surface} \\ \text{except electrodes}}} |\mathbf{J} \cdot \tilde{\mathbf{J}}|^2 \, ds, \qquad (1)$$

where α is an empirical constant,[4] ρ is the two-dimensional resistivity with the dimension Ω, n is the free charge-carrier concentration per unit area (cm^{-2}), I is the current through the driver electrodes, f is the frequency, and the integrand is the square of the scalar product of the homogeneous current density $\mathbf{J}$ and the adjoint current density $\tilde{\mathbf{J}}$. The adjoint density $\tilde{\mathbf{J}}$ is the current density that exists in an experiment of thought after switching the current source from the driver over to the sensor electrodes. The integral in Eq. (1) must be taken over the whole conductor, except the noiseless electrodes. A proof for Eq. (1) was given by van Bokhoven.[5]

Now, the scalar product of $\mathbf{J}$ and $\tilde{\mathbf{J}}$ for the transversely and longitudinally placed electrodes is calculated in order to calculate $S_{v\perp}$ and $S_{v\parallel}$. Owing to the homogeneity of the field, the integrand in Eq. (1) reduces to $|\mathbf{J}_x \cdot \tilde{\mathbf{J}}_x|^2$ or $(I/W)^2 \tilde{\mathbf{J}}_x^2$, because J_Y equals zero by an appropriate choice of the coordinate system. For a pair of point electrodes at a distance $2a$ apart and placed longitudinally to $\mathbf{J}$ the x component of the adjoint current density $\tilde{\mathbf{J}}$ becomes

$$\tilde{\mathbf{J}}_{x\parallel}(x,y) = \frac{Ia(y^2 + a^2 - x^2)}{\pi r_1^2 r_2^2}, \qquad (2)$$

where r_1 and r_2 are the distances from a point (x,y) in the two-dimensional conductor to the point sources at $(-a,0)$ and $(a,0)$. Equation (2) follows from simple superposition of current densities due to two-point current sources of opposite sign placed at a distance $2a$ apart. If a pair of point electrodes

at a distance $2a$ is placed transversely to J, then $\tilde{\mathbf{J}}_x$ becomes

$$\tilde{\mathbf{J}}_{x\perp}(x,y) = \frac{I \, 2axy}{\pi r_1^2 r_2^2}. \qquad (3)$$

In order to introduce circular sensors with radius r at a distance between centers of $2b$ and to simplify the integral borders around the sensor electrodes, Eqs. (2) and (3) are converted into bipolar coordinates u and v. The same reasoning was followed in Ref. 1. Using the conversion formulas between the Cartesian and the bipolar coordinate systems[1] the relations for $\tilde{J}_{x\parallel}$ and $\tilde{J}_{x\perp}$ in bipolar coordinates becomes

$$\tilde{J}_{x\parallel}(u,v) = (I/2\pi a)(1 - \cosh v \cos u), \qquad (4)$$

$$\tilde{J}_{x\perp}(u,v) = (I/2\pi a) \sinh v \sin u. \qquad (5)$$

An elementary area ds in bipolar coordinates becomes

$$ds = a^2 \, du \, dv / (\cosh v - \cos u), \qquad (6)$$

where $v = \text{const}$ describes equipotential circles and $u = \text{const}$ describes the field lines. The relation between r and v, a on the one hand, and the relation between b and v, a on the other, are given in Ref. 1. Using Eqs. (1), (4), (5), and (6) and considering the symmetry over the four quadrants, we obtain

$$S_{v\parallel} = \frac{\alpha \rho^2 I^2}{nfW^2\pi^2} \int_0^{v_1} \int_0^\pi \left(\frac{1 - \cosh v \cos u}{\cosh v - \cos u} \right)^2 du \, dv, \qquad (7)$$

$$S_{v\perp} = \frac{\alpha \rho^2 I^2}{nfW^2\pi^2} \int_0^{v_1} \int_0^\pi \left(\frac{\sinh v \sin u}{\cosh v - \cos u} \right)^2 du \, dv. \qquad (8)$$

The evaluation of the definite integrals over u in Eqs. (7) and (8) is achieved by using the residue theorem together with a suitable function $f(z)$ and a suitable closed path C. Let $z = \exp(iu)$, then $\sin u = (z - z^{-1})/2i$, $\cos u = \frac{1}{2}(z + z^{-1})$, and $du = dz/iz$. Here C is the unit circle with its center at the origin, and the functions $f(z)$ are single valued and analytic inside and on the unit circle C, except at the singularities $z = 0$ and $z = -\cosh v + [(\cosh^2 v - 1)]^{1/2}$ inside C. Using the residue theorem which states that $\oint_C f(z)dz$ equals $2\pi i \Sigma$ residues inside C leads to

$$S_{v\parallel} = \frac{\alpha \rho^2 I^2}{nfW^2\pi^2} \int_0^{v_1} \pi (\cosh v)(\cosh v - \sinh v) \, dv, \qquad (9)$$

$$S_{v\perp} = \frac{\alpha \rho^2 I^2}{nfW^2\pi^2} \int_0^{v_1} \pi (\sinh v)(\cosh v - \sinh v) \, dv. \qquad (10)$$

Evaluating these integrals to v leads to

$$S_{v\parallel} = \frac{\alpha \rho^2 I^2}{nfW^2} \left(\frac{1}{2\pi} (-\sinh^2 v_1 + \tfrac{1}{2} \sinh 2v_1 + v_1) \right), \qquad (11)$$

$$S_{v\perp} = \frac{\alpha \rho^2 I^2}{nfW^2} \left(\frac{1}{2\pi} (\sinh^2 v_1 - \tfrac{1}{2} \sinh 2v_1 + v_1) \right). \qquad (12)$$

For generation-recombination noise the factor α/nf in Eqs. (11) and (12) must be replaced by $4\tau^2/\{[1 + (2\pi f\tau)^2]\tau_1 n\}$, where τ_1 is the free-carrier lifetime and τ the g-r relaxation time.[6] The factors in large parentheses in Eqs. (11) and (12) give the ratios between the noise at the sensor electrodes and the noise at the current-carrying driver electrodes if $W = L$. In general the ratio between the noise at the sensors and the voltage noise at the driver electrodes S_{vD} is given by

$$\frac{S_{v\parallel}}{S_{vD}} = \frac{W}{L}\left(\frac{1}{2\pi}\left(-\sinh^2 v_1 + \tfrac{1}{2}\sinh 2 v_1 + v_1\right)\right) \quad (13)$$

and

$$\frac{S_v}{S_{vD}} = \frac{W}{L}\left(\frac{1}{2\pi}\left(\sinh^2 v_1 - \tfrac{1}{2}\sinh 2 v_1 + v_1\right)\right). \quad (14)$$

The function between b/r and v_1 is given by[1]

$$v_1 = \ln\{b/r + [(b/r)^2 - 1]^{1/2}\}. \quad (15)$$

Using Eqs. (11) and (12), the ratio $S_{v\parallel}/S_{v\perp}$ becomes

$$\frac{S_{v\parallel}}{S_{v\perp}} = \frac{2v_1 + [1 - \exp(-2 v_1)]}{2 v_1 - [1 - \exp(-2v_1)]}. \quad (16)$$

This ratio is plotted versus that of the distance between the centers of the sensor electrodes $2b$ and the sensor diameter $2r$, as a line in Fig. 1. The dots represent the experimental results on resistors made of carbon sheet which is something like Teledeltos. The distance between the sensors was constant. The diameter of the sensors was increased step by step.

III. CONCLUSIONS

The longitudinal noise $S_{v\parallel}$ is at least as large as the transverse noise $S_{v\perp}$ in homogeneous and isotropic samples submitted to uniform fields. The ratio $S_{v\parallel}/S_{v\perp}$ is independent of the type of conduction fluctuations ($1/f$ or generation-recombination noise).

The value of the integrand in Eq. (1) is higher in the neighborhood of the spot sensors due to a higher value of $\tilde{\mathbf{J}}$. So the conductivity fluctuations in the neighborhood of the spot sensors contribute more than conductivity fluctuations far away from the sensors. The small contribution from areas far away from the sensors is even smaller in the calculation of $S_{v\perp}$ due to the fact that in the area between the sensor, $\tilde{\mathbf{J}}$ is about perpendicular to $\mathbf{J}$. This leads to a very small dot product $\mathbf{J}\cdot\tilde{\mathbf{J}}$ which explains that $S_{v\perp}$ is always smaller than $S_{v\parallel}$.

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