An exact approach for the pollution-routing problem

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An Exact Approach for the Pollution-Routing Problem

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The Pollution-Routing Problem (PRP) is a recently introduced problem in the field of green road freight transportation. It concerns routing a number of vehicles serving a set of geographically dispersed customers within their time windows, jointly with determining their speed on each route so as to minimize a function comprising fuel and driver costs. Due to its complexity, all known solution methods are based on (meta-)heuristics. This paper presents an exact solution approach based on a branch-and-price algorithm. The master problem is a set-partitioning problem, and the pricing problem is a speed- and departure time-dependent elementary shortest path problem with resource constraints. The master problem is solved by means of column generation, and a tailored labeling algorithm is used to solve the pricing problem. New dominance criteria are developed to discard more labels during the column generation process by exploiting the structure of the ready time and the fuel consumption functions. Results of extensive computational experimentations confirm the efficiency of the algorithm. We are able to solve small- and moderate-size instances to optimality within reasonable execution time. More specifically, our algorithm solves 46 out of 56 instances with 25 customers, 18 instances with 50 customers and 4 instances with 100 customers on the adjusted Solomon data sets.

Key words: Green road freight transportation; Fuel consumption; Vehicle speed and departure time optimization; Vehicle routing problem; Column generation; Branch and price

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1. Introduction

Green road freight transportation concerns dispatching goods in a sustainable way. With a world-wide increasing concern for the environment, logistics service providers and freight carriers pay more and more attention to the carbon dioxide (CO_2) emissions of their operations. The carbon dioxide equivalent (CO_2e) measures how much global warming a given type and amount of greenhouse gases (GHGs) may cause, using the functionally equivalent amount or concentration of CO_2 as the reference. The CO_2e emissions are generally proportional to the amount of fuel consumed by a vehicle, which depends on a variety of vehicle, environment and traffic-related parameters, such as vehicle speed, load and acceleration (Demir et al. 2011, 2014a).

The Pollution-Routing Problem (PRP) is an extension of the Vehicle Routing Problem with Time Windows (VRPTW). The VRPTW is concerned with the routing of a fleet of homogeneous vehicles, each with a finite capacity, to serve a set of customers. Each customer has a certain demand and must be served within a predefined time window, and the capacity must be respected on each route. The traditional objective in the VRPTW is to minimize the total distance traveled by all vehicles. The PRP aims at minimizing an objective function comprising the fuel and duration costs of all routes. The fuel consumption along a route and its duration depend on the traveling speed. The route duration is the difference between its start and end time at the depot. Therefore, in the PRP, next to the sequence in which customers are visited, vehicles' speed are considered as decision variables. In this paper, departure time at the depot is also considered as a decision variable.

The existing literature on green road freight transportation (see, e.g., Demir et al. 2014a), mainly focuses on heuristic solutions. In this paper, we solve the PRP to optimality using a branch-and-price algorithm. In each branch-and-bound node, the master problem is solved by column generation. While the master problem of the column generation approach remains unchanged, compared to that of the VRPTW, the pricing problem is a speed- and departure time-dependent elementary shortest path problem with resource constraints (SDESPPRC).

Note that our pricing problem is more complicated, compared to that of the VRPTW, because we use minimization of fuel consumption and duration costs as our objective, together with the speed-dependent travel times. If the traditional VRPTW objective of minimizing the total distance was kept together with fuel cost minimization, then the pricing problem can be modeled with non-decreasing resource functions. This means that a standard labeling algorithm for the pricing problem of the VRPTW is applicable by simply changing the travel time calculation for a given speed. Minimization of total fuel consumption and duration of all routes is also not too difficult in case of speed independent travel times. Irnich (2008) shows how to deal with the case of duration minimization.

To solve the SDESPPRC, we develop a tailored labeling algorithm. In each label, we store functions that compute when the vehicle is ready to leave the last node, and the fuel consumption along a partial path, given a departure time at the depot and a traveling speed. The labeling algorithm generates columns that have negative reduced cost together with their optimal departure time at the depot and optimal traveling speed. To improve the performance of the labeling algorithm, new dominance criteria are introduced to discard labels not leading to routes in the optimal solution.

To accelerate the branch-and-price algorithm, two heuristics are designed to find columns with negative reduced cost. Furthermore, we relax the pricing problem by allowing non-elementary paths. Although the non-elementary SDSPPRC results in worse lower bounds, it is easier to solve and integrality of the master problem is still guaranteed by the branch-and-bound.

The scientific contribution of this study is three-fold: *i*) We present an exact method for the PRP. A branch-and-price algorithm is proposed to determine the set of routes that minimizes the sum of fuel consumption and duration cost of all routes. *ii*) The pricing problem that arises is a SDESPPRC, which is more complicated than the ESPPRC, and is solved by a tailored labeling algorithm. *iii*) We introduce new dominance criteria by exploiting the structure of the ready time and the fuel consumption functions.

The remainder of this paper is organized as follows. Section 2 briefly reviews the literature on green logistics and exact approach methodologies. Section 3 presents the PRP along with its all dimensions. In Section 4 and 5, we present a set partitioning problem and pricing problem, respectively. Section 6 provides extensive computational results. Conclusions are stated in Section 7.

2. Literature Review

In this section, we look at the existing studies on green road freight transportation as well as column generation algorithms, particularly for the VRPTWs.

2.1. Green Road Freight Transportation

The PRP is introduced by Bektaş and Laporte (2011). Their model approximates the total amount of energy consumed on a arc, which directly translates into the fuel consumption and further into GHG emissions. Moreover, it minimizes the fuel consumption, emissions and driver costs on a arc. In Demir et al. (2012), a meta-heuristics that iterates between the solution to the VRPTW and a speed optimization problem is introduced. The VRPTW is solved by an Adaptive Large Neighborhood Search (ALNS). The speed optimization problem is solved by means of a procedure that runs in polynomial time. In a related study, Demir et al. (2014b) investigate the trade-offs between fuel consumption and driving time. They show that trucking companies need not compromise greatly in terms of driving time in order to achieve a significant reduction in fuel consumption and CO_2 emissions. Franceschetti et al. (2013) extend the PRP to a time-dependent setting by capturing traffic congestion that limits vehicles in their flexibility of choosing the traveling speed. The authors consider a two-period planning horizon; one congested and the other not. The authors identify conditions under which it is optimal to wait at certain locations in order to avoid congestion and to reduce emissions cost. Kramer et al. (2014) propose a method that combines a local search-based meta-heuristic with an integer programming approach over a set covering formulation and a recursive speed optimization algorithm.

Closely related to the PRP, Figliozzi (2010) introduce the emissions vehicle routing problem (EVRP) which concerns the minimization of emissions and fuel consumption. Their model is based on the work by (Hickman et al. 1999). In the proposed algorithm, a partial EVRP is first solved to minimize the number of vehicles, and then emissions are optimized subject to a fleet size constraint. Furthermore, departure times at customers are also optimized.

2.2. Column Generation Algorithms

Several column generation algorithms have been successfully implemented to solve combinatorial optimization problems. For a detailed picture of column generation algorithms, the reader is referred to Lübbecke and Desrosiers (2005). In the context of the VRPTW, column generation was first introduced by Desrochers et al. (1992). Later, Kohl et al. (1999) introduced subtour elimination constraints and 2-path cuts in a column generation framework, and Cook and Rich (1999) applied the more general $k-$path cuts. In the nineties, the pricing problem was the shortest path problem with resource constraints (SPPRC) and two cycle elimination, which was extended by Irnich and Villeneuve (2006) to a SPPRC with k-cycle elimination leading to tighter bounds. Feillet et al. (2004) and Chabrier (2006) proposed algorithms for the ESPPRC, which further improved the lower bounds.

In order to speed up the ESPPRC algorithm, Righini and Salani (2006, 2008) proposed various techniques, including bi-directional search and decremental state space relaxation. Furthermore, cutting planes are used to tighten the lower bounds when solving the master problem by column generation. Jepsen et al. (2008) were the first to introduce valid inequalities based on the master problem variables in the context of VRPTW. To further accelerate the pricing problem, Desaulniers et al. (2008) proposed a tabu search heuristic for the ESPPRC. Furthermore, elementarity was relaxed for a subset of nodes, and both 2-path and subset-row inequalities were used. Baldacci et al. (2011) introduced a new route relaxation, called ng-route, used to solve the pricing problem. Their framework proved to be very effective in solving difficult instances of the VRPTW with wide time windows. It is also worth mentioning the column generation algorithm of Bettinelli et al. (2011) that considered the dispatch time from the depot as a decision variable but assumed constant travel times. Dabia et al. (2013) applied column generation to the time-dependent VRPTW where the duration of all routes is minimized. Their pricing problem is a ESPPRC with time-dependent travel times. To handel travel time functions, they encountered issues comparable to ours.

3. Problem Description

Consider a graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N} = \{0, ..., n, n+1\}$ is the set of nodes, $\mathcal{N}_0 = \mathcal{N} \setminus \{0, n+1\}$ is the set of customers, where 0 and $n+1$ are the start and the end depots of each route, respectively. Let d_{ij} be the distance between nodes i and j. Let $\tau_{ij}(v) = \frac{d_{ij}}{v}$ be the travel time function between nodes i and j, where v is the traveling speed along arc (i,j). Moreover, v is assumed to be in some interval $[v_{min}, v_{max}]$. An unlimited fleet of homogeneous vehicles $\mathcal{K}$ is available at the depot, each vehicle has capacity Q. Let q_i be the demand, s_i be the service time and $[a_i, b_i]$ be the time window of node i. The set of feasible arcs can be defined as $\mathcal{A} = \{(i,j) \in \mathcal{N} \times \mathcal{N} : i \neq j \text{ and } a_i + s_i + \tau_{ij}(v_{max}) \leq b_j \;\wedge\; q_i + q_j \leq Q\}$. Furthermore, we assume that $s_0 = s_{n+1} = q_0 = q_{n+1} = a_0 = 0$.

3.1. Fuel Consumption Function

The fuel consumption is based on the *comprehensive emissions model* (CMEM) described by Barth et al. (2005), Scora and Barth (2006), and Barth and Boriboonsomsin (2008). According to the CMEM, the fuel rate (in liter/second) is given by

$$f_r = \frac{\xi}{\kappa\psi}\left\{kNV + \frac{1}{\eta}\left(\frac{((w+q)(\chi + g\sin\theta + gC_r\cos\theta) + 0.5C_d\rho A v^2)v}{1000\eta_{tf}} + P_{acc}\right)\right\}. \tag{1}$$

All parameters along with typical values are provided in Table 1. Moreover, the column "Classification" shows the type of each parameter.

Table 1 Parameters

Notation	Classification	Description	Typical values
ξ	E, V, S	Fuel-to-air mass ratio	1
κ	E, S	Heating value of a typical diesel fuel (kJ/g)	44
ψ	S	Conversion factor (g/s to L/s)	737
k	V	Engine friction factor (kJ/rev/liter)	0.23
N	V	Engine speed (rev/s)	37
V	V	Engine displacement (liters)	5
η	E, V	Efficiency parameter for diesel engines	0.9
w	V	Curb-weight (kg)	6350
q	S, V	Payload (kg)	0-3650
χ	S	Instantaneous acceleration (m/s^2)	0
θ	E, S	Road gradient (%)	0
C_d	V	Coefficient of aerodynamic drag	0.7
ρ	E	Air density (kg/m^3)	1.2041
A	V	Frontal surface area (m^2)	3.912
g	E	Gravitational constant (m/s^2)	9.81
C_r	V	Coefficient of rolling resistance	0.01
n_{tf}	S, V	Vehicle drive train efficiency	0.4
P_{acc}	S, V	Engine power demand for accessories	0
c_f	S	Fuel cost per liter (€)	1.4
c_d	S	Driver wage per (€/s)	0.0033
v_{min}	S	Lower speed limit (m/s)	16.7 (=$(c_2/2c_3)^{1/3}$ or ≈60 km/h)
v_{max}	S	Upper speed limit (m/s)	25 (or 90 km/h)

E: Environment related, S: Scenario related, V: Vehicle related

For a path $p = (i_0, ..., i_k)$ traversed with speed v, the fuel consumption cost (in euros) on this path can be expressed as

$$F_p(v) = \left(c_0 + \frac{c_2}{v} + c_3 v^2\right) A_p^0 + c_1 A_p^1, \tag{2}$$

where $c_0 = \frac{c_f w\xi(\chi + g\sin\theta + gC_r\cos\theta)}{1000\kappa\psi n_{tf}\eta}$, $c_1 = \frac{c_0}{w}$, $c_2 = \frac{c_f\xi kNV}{\kappa\psi}$ and $c_3 = c_f\frac{\xi 0.5C_d\rho A}{1000\kappa\psi n_{tf}\eta}$. The parameters c_0, c_1, c_2 and c_3 depend on inertia force, rolling resistance, wind resistance and vehicle characteristics. $A_p^0 = \sum_{l=1}^{k} d_{i_{l-1}i_l}$ is the total distance traveled along path p (in meters) , and $A_p^1 = \sum_{l=1}^{k} q_{i_l}\sum_{m=1}^{l} d_{i_{m-1}i_m}$ is the loaded distance traveled (in kilogram meters).

The fuel cost function $F_p(v)$ is convex in v. The optimal speed v_F^* is calculated by solving the equation $\frac{dF_p(v)}{dv} = 0$ resulting in $v_F^* = \left(\frac{c_2}{2c_3}\right)^{\frac{1}{3}}$. Note that it is never optimal for a vehicle to travel with a speed lower than v_F^* (Demir et al. 2012). For path p, the fuel consumption function can be completely described by A_p^0 and A_p^1 and the minimum speed $v_{min}^p \geq v_F^*$ by which path p can be traversed and still be feasible (i.e., time windows are respected).

3.2. Ready Time Function

The travel time function $\tau_{ij}(v) = \frac{d_{ij}}{v}$ for some arc $(i,j) \in \mathcal{A}$ is a non-increasing convex function of the speed v. Given a partial path $p = (i_0, \ldots, i_k)$ with $i_0 = 0$, we define $\delta_{i_k}^p(t,v)$ as a function that gives the ready time (when service is completed) at node i_k for a departure time t at the depot and a traveling speed v. Obviously, if a route is infeasible for some departure time t at the depot and traveling speed v (i.e., time windows are violated), it will also be infeasible for any dispatch time $t' \geq t$ at the depot and speed $v' \leq v$. We can recursively express the ready time at each node of the path as follows:

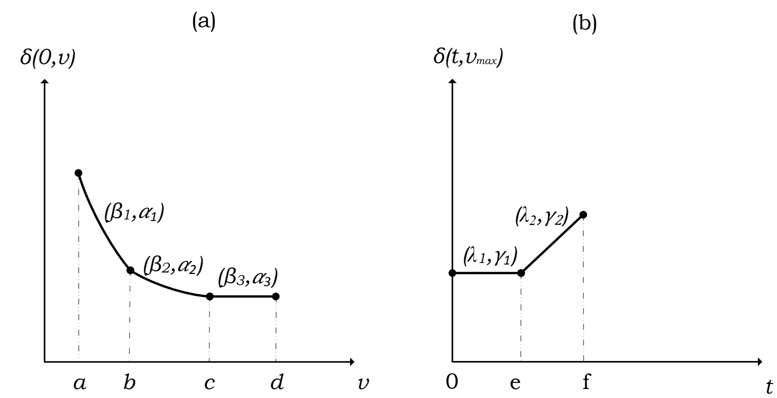
$$\delta_{i_l}^p(t,v) = \begin{cases} t & \text{if } l = 0 \\ \max\left\{a_{i_l} + s_{i_l}, \delta_{i_{l-1}}^p(t,v) + \tau_{i_{l-1},i_l}(v) + s_{i_l}\right\} & \text{if } l \in \{1,\ldots,k\}. \end{cases} \tag{3}$$

The duration of path p is $\delta_{i_k}^p(t,v) - t$. The optimal departure time at the depot t^* and traveling speed v^* that result in the cheapest path satisfy:

$$(t^*, v^*) = \underset{(t,v)\in \mathrm{dom}_\delta}{\arg\min} \left\{c_d(\delta_{i_k}^p(t,v) - t) + F_p(v)\right\} \tag{4}$$

where dom_δ is the domain of the function $\delta_{i_k}^p(t,v)$. The calculation of the ready time function $\delta_{i_k}^p(t,v)$ using the recursive Equation (3) is not straightforward because it is a complicated two variables function and because of time windows. If we fix the departure time at the depot t, the ready time function will be a non-increasing piecewise convex function as shown in Figure 1.a for the case of $t = 0$. It can be completely represented on a set of *segments* $\mathcal{S}$. Each segment $s \in \mathcal{S}$ is defined by a speed interval $\mathcal{V}_s$ and two coefficients β_s and α_s. For every speed $v \in \mathcal{V}_s$, the ready time is calculated by the equation $\frac{\beta_s}{v} + \alpha_s$. In Figure 1.a, the ready time function for $t = 0$ is represented by three segments. For example, the first segment has an equation $\frac{\beta_1}{v} + \alpha_1$ for all $v \in \mathcal{V}_1 = [a,b]$. If we fix the speed v, the ready time function is a non-decreasing piecewise convex linear function as shown in Figure 1.b for the case of $v = v_{max}$. Similarly, the ready time function can be completely described by a set of linear segments L^s, where each segment is defined on a time interval $\mathcal{T}_s$ with a linear equation $\lambda_s t + \gamma_s$ for all $t \in \mathcal{T}_s$. When t or v is fixed, the calculation of the ready time function is then relatively easier using the recursive Equation (3) as it involves the addition of two piecewise convex functions (in case of fixed t) or two piecewise linear functions (in case of fixed v). The new ready time functions can again be described by a set of segments resulting from the segments of the old ready time functions, travel time functions and time windows.

Figure 1 **Illustrations of the ready time functions for departure time $t = 0$ and traveling speed $v = v_{max}$**



4. Set Partitioning Formulation and Column Generation

This section provides the set partitioning formulation and branching rules for the PRP.

4.1. Set partitioning

We define Ω as the set of feasible paths. A path is feasible if it satisfies capacity and time window constraints. A path leaves and returns to the depot exactly once; multiple visits to customers are not allowed. A path is defined by the sequence of customers visited, the departure time at the depot t and the traveling speed v. For each path $p \in \Omega$, we let s_p be the departure time at the depot and v_p the traveling speed that lead to the cheapest path. Furthermore, we set $e_p = \delta^p_{n+1}(s_p, v_p)$, which represents the arrival time at the end depot. We let $c_p = c_d(e_p - s_p) + F_p(v_p)$ denote the path's cost comprising the path duration and fuel costs. We let σ_{ip} be a constant that measures the number of times node i is visited by the path p. Furthermore, if y_p is a binary variable that takes the value 1 if and only if the path p is included in the solution, the PRP is formulated as the following set partitioning problem:

$$\min \sum_{p \in \Omega} c_p y_p \tag{5}$$

subject to

$$\sum_{p\in\Omega} \sigma_{ip} y_p = 1 \qquad \forall i \in \mathcal{N} \tag{6}$$

$$y_p \in \{0,1\} \quad \forall p \in \Omega. \tag{7}$$

where the objective function (5) minimizes the cost of the chosen routes. Constraint (6) guarantees that each node is visited exactly once. We use column generation to solve the LP-relaxation of (5)–(7): starting with a small subset $\Omega' \subseteq \Omega$ of variables, we generate additional variables for the master problem (the LP-relaxation of (5)–(7)) by solving a pricing subproblem that searches for variables with negative reduced cost. The reduced cost of a variable (path) is defined as

$$\bar{c}_p = c_p - \sum_{i\in\mathcal{N}} \sigma_{ip}\pi_i = c_d(e_p - s_p) + F_p(v) - \sum_{i\in\mathcal{N}} \sigma_{ip}\pi_i. \tag{8}$$

where $\pi_i \geq 0$, $i \in \mathcal{N}_0$ is the dual variables associated with constraints (6).

4.2. Branching

The branch-and-bound tree is explored using a best bound strategy. The algorithm first branches on the number of vehicles $\sum_{j\in\mathcal{N}} x_{0j}$. It imposes two branches $\sum_{j\in\mathcal{N}} x_{0j} \geq \lceil \sum_{j\in\mathcal{N}} x_{0j} \rceil$ and $\sum_{j\in\mathcal{N}} x_{0j} \leq \lfloor \sum_{j\in\mathcal{N}} x_{0j} \rfloor$. If the number of vehicles is integer, the algorithm branches on the arc variables x_{ij}. It looks for pairs $(i,j), i,j \in \mathcal{N}_0$ such that $x^*_{ij} + x^*_{ji}$ is close to 0.5 (x^* is the current fractional solution expressed in the arc variables) and imposes two branches $x_{ij} + x_{ji} \leq \lfloor x^*_{ij} + x^*_{ji} \rfloor$ and $x_{ij} + x_{ji} \geq \lceil x^*_{ij} + x^*_{ji} \rceil$. If $x^*_{ij} + x^*_{ji}$ is integer for all pairs $(i,j), i,j \in \mathcal{N}$, then the algorithm looks for an arc $(i,j) \in \mathcal{A}$ for which x^*_{ij} is fractional and branches on that instead. Strong branching is used, that is, the impact of branching on several candidates is investigated every time a branching decision has to be made. For each branch candidate, we estimate the lower bound in the two child nodes by solving the associated LP-relaxation using a quick pricing heuristic. The branch that maximizes the lower bound in the weakest of the two child nodes is chosen. We consider 35 branch candidates in the first 20 nodes of the branch and bound tree, and 20 candidates in the rest.

5. The Pricing Problem

The pricing problem is solved by means of a tailored labeling algorithm which is a modification of the labeling algorithm used in the case of the ESPPRC. In order to speed up the algorithm, a bi-directional search is performed in which labels are extended both forward from the depot (i.e., node 0) to its successors, and backward from the depot (i.e., node n+1) to its predecessors. At the end of the algorithm, forward and backward labels are merged to construct complete routes. It has been observed (see, e.g., Righini and Salani (2006)) that the bi-directional search in practice can lead to substantially improved running times in algorithms for related resource constrained shortest path problems. In the rest of the paper, we denote $p(L)$ the partial path corresponding to the label L.

5.1. The Forward Labeling Algorithm

In the forward labeling algorithm, labels are extended from the depot (i.e., node 0) to its successors. For a label L_f, we define the following attributes:

i_{L_f}	Last node visited on the partial path $p(L_f)$.
c_{L_f}	Sum of the dual variables associated with the nodes visited by the partial path $p(L_f)$.
q_{L_f}	Total quantity delivered along the partial path $p(L_f)$.
$F_{L_f}(v)$	Fuel cost of the partial path $p(L_f)$ when traversed with speed v.
$\delta_{L_f}(t,v)$	Ready time at node i_{L_f}, for a departure time t at the depot and when reached through partial path $p(L_f)$ with speed v.
V_{L_f}	Set of nodes visited along the partial path $p(L_f)$.

Furthermore, we define $\overline{V}_{L_f}$ as V_{L_f} extended with nodes that cannot be visited by path $p(L_f)$. For simplicity of notation, we set $v_{min}^{L_f} = v_{min}^{p(L_f)}$, $A^0_{L_f} = A^0_{p(L_f)}$ and $A^1_{L_f} = A^1_{p(L_f)}$.

The main operation in the forward labeling algorithm is the extension of a label L'_f along an arc $(i(L'_f), j)$ to a node j to generate a new label L_f. The fuel consumption function $F_{L_f}(v)$ associated with the new label L_f can be extended as follows

$$F_{L_f}(v) = \left(c_0 + \frac{c_2}{v} + c_3 v^2\right) A^0_{L_f} + c_1 A^1_{L_f}, \tag{9}$$

where $A^0_{L_f}$ and $A^1_{L_f}$ are updated as

$$A^0_{L_f} = A^0_{L'_f} + d_{i_{L'_f},j} \quad \text{and} \quad A^1_{L_f} = A^1_{L'_f} + q_j A^0_{L_f} \tag{10}$$

Moreover, we update $v_{min}^{L_f}$ which can directly be taken from the ready time function, it corresponds to the lowest possible speed when departure time at the depot is $t=0$. Note that $v_{min}^{L_f} \geq v_{min}^{L'_f}$.

The ready time function $\delta_{L_f}(t,v)$ can be expressed as

$$\delta_{L_f}(t,v) = \max\left\{a_j + s_j, \delta_{L'_f}(t,v) + \tau_{i_{L'_f},j}(v) + s_j\right\}. \tag{11}$$

As discussed in section 3.2, the extension of the ready time function $\delta_{L_f}(t,v)$ using the recursive Formula (11) is not straightforward. For our purpose, we instead store and extend the two functions $\delta_{L_f}(0,v)$ and $\delta_{L_f}(t,v_{max})$. The extension of $\delta_{L_f}(0,v)$ amounts to constructing a new piecewise convex function from the piecewise convex function $\delta_{L'_f}(0,v)$ and the convex function $\tau_{i(L'_f),j}(v)$. Furthermore, the extension of $\delta_{L_f}(t,v_{max})$ amounts of constructing a piecewise convex linear function from a piecewise convex linear function $\delta_{L'_f}(t,v_{max})$ and the constant $\tau_{i(L'_f),j}(v_{max})$.

Additionally, we have:

$$V_{L_f} = V_{L'_f} \cup \{j\}\ , \quad c_{L_f} = c_{L'_f} + \pi_j \quad \text{and} \quad q_{L_f} = q_{L'_f} + q_j \tag{12}$$

where π_j is the dual variable associated with Constraints (6).

The extension of label L'_f to L_f is feasible if:

$$V_{L'_f} \cap \{j\} = \emptyset \quad \wedge \quad \delta_{L_f}(0, v_{max}) \leq \min\left\{t_m^f, b_j + s_j\right\} \quad \wedge \quad q_{L_f} \leq Q. \tag{13}$$

where t_m^f is the bounding upper bound used in the forward search (i.e., the middle of the planning horizon).

We set $\mathrm{dom}_\delta(L_f)$ to be the domain of function $\delta_{L_f}(t,v)$. We note that $\mathrm{dom}_\delta(L_f)$ is always of the form $[0,t] \times [v, v_{max}]$ for some $t \geq 0$ and $v \leq v_{max}$ because departure at time 0 with speed v_{max} is always feasible if the partial path is feasible. Furthermore, we denote $\mathrm{dom}_\delta^{t=t'}(L_f)$ and $\mathrm{dom}_\delta^{v=v'}(L_f)$ the domain of $\delta_{L'_f}(t,v)$ in case of a fixed departure time at the depot $t = t'$ and a fixed speed $v = v'$, respectively.

When $i_{L_f} = n+1$, the reduced cost of the path corresponding to L_f is

$$\bar{c}_{L_f} = \min_{(t,v) \in \mathrm{dom}_\delta(L_f)} \left\{c_d(\delta_{L_f}(t,v) - t) + F_{L_f}(v)\right\} - c_{L_f}. \tag{14}$$

In the labeling algorithm, for every label, all possible extensions are derived and stored. It ends when all labels are calculated. However, the number of labels can be very large. To reduce the number of labels, a *dominance test* is introduced. Let $E(L_f)$ denote the set of feasible extensions of the label L_f to node $n+1$. In other words, $E(L_f)$ is the set of all partial paths that can depart at node i_{L_f} at time $\delta_{L_f}(0, v_{max})$ or later and reach node $n+1$ without violating time windows, which has total demand less than $Q - q_{L_f}$ and which do not use nodes from V_{L_f}. If $L_f \in E(L_f)$, we denote $L_f \oplus L$ as the label resulting from extending L_f by L. In case of the forward labeling algorithm, dominance criteria are formulated as follows:

Definition 1. Label L_f^2 is dominated by label L_f^1 if:

1. $i_{L_f^1} = i_{L_f^2}$
2. $E(L_f^2) \subseteq E(L_f^1)$
3. $\bar{c}_{L_f^1 \oplus L} \leq \bar{c}_{L_f^2 \oplus L}, \quad \forall L_f \in E(L_f^2)$.

Definition 1 states that any feasible extension of label L_f^2 is also feasible for label L_f^1. Furthermore, extending L_f^1 should always result in a better route. However, it is not straightforward to verify the conditions of Definition 1 as it requires the computation and the evaluation of all feasible extensions of both labels L_f^1 and L_f^2. Consequently, sufficient dominance criteria that are computationally less expensive are desirable. Therefore, in Proposition 1, the sufficient conditions 1 to 8 are introduced.

PROPOSITION 1. *Label L_f^2 is dominated by label L_f^1 if:*

1. $i_{L_f^1} = i_{L_f^2}$
2. $c_{L_f^1} \geq c_{L_f^2}$
3. $q(L_f^1) \leq q(L_f^2)$
4. $\delta_{L_f^1}(t,v) \leq \delta_{L_f^2}(t,v), \quad \forall (t,v) \in \mathrm{dom}_\delta(L_f^2)$
5. $\mathrm{dom}_\delta(L_f^2) \subseteq \mathrm{dom}_\delta(L_f^1)$
6. $A^0_{L_f^1} \leq A^0_{L_f^2}$
7. $A^1_{L_f^1} \leq A^1_{L_f^2}$
8. $\overline{V}_{L_f^1} \subseteq \overline{V}_{L_f^2}$

Proof of Proposition 1: See appendix.

Conditions 3, 4 and 8 ensure that any feasible extension of L_f^2 is also feasible for L_f^1. Conditions 2, 4, 5, 6 and 7 ensure that for any $L \in E(L_f^2)$, the reduced cost of path $p(L_f^1 \oplus L)$ is less or equal to the reduced cost of path $p(L_f^2 \oplus L)$. We note that condition 5 is needed because if $\mathrm{dom}_\delta(L_f^2) \nsubseteq \mathrm{dom}_\delta(L_f^1)$, it might be possible to leave the depot at some time and with some speed $(t,v) \in \mathrm{dom}_\delta(L_f^2) \setminus \mathrm{dom}_\delta(L_f^1)$ for which an extension of L_f^2 can lead to a cheaper path.

Dominance as introduced in Proposition 1 has many weaknesses as it is not easy to test and probably will not be capable of discarding many labels. In fact, it is not straightforward to test condition 4 because of the complexity of the ready time functions. Moreover, conditions 6 and 7 require that the total distance $A^0_{L_f^1}$ and the total loaded distance $A^1_{L_f^1}$ of label L_f^1 are less or equal than these of label L_f^2 to ensure that, for any label $L \in E(L_f^2)$, path $p(L_f^1 \oplus L)$ consumes less fuel than path $p(L_f^2 \oplus L)$. It is basically possible that path $p(L_f^1 \oplus L)$ consumes less fuel than path $p(L_f^2 \oplus L)$ even if condition 5 or 6 (but not both) is not satisfied, and therefore, if all other conditions in Proposition 1 are satisfied, path $p(L_f^1 \oplus L)$ is cheaper than path $p(L_f^2 \oplus L)$. In this case, dominance fails to let L_f^1 dominate L_f^2. Furthermore, labels corresponding to paths with a very high fuel cost will probably result in unattractive routes with a high reduced cost, but dominance will fail to discard these labels if they have a high cost component (i.e., collected dual variables) as well. The same holds for labels corresponding to paths with very high duration cost. In other words, the weaknesses of the dominance test presented in Proposition 1 are stemming from the fact that the duration cost, the fuel cost and the cost related to the dual variables are treated separately.

Consider $L_f \oplus L$ as the label resulting from extending L_f by $L \in E(L_f)$. The corresponding fuel cost can be expressed as

$$F_{L_f \oplus L}(v) = F_{L_f}(v) + \left(c_0 + \frac{c_2}{v} + c_3 v^2\right) A_L^0 + c_1 q_L A^0_{L_f} + c_1 A_L^1. \tag{15}$$

We now consider two labels L_f^1 and L_f^2 that we want to compare, and a label $L \in E(L_f^2)$. We set $L_f^{1*} = L_f^1 \oplus L$ and $L_f^{2*} = L_f^2 \oplus L$. Knowing that $q_L \in [0, Q - q_{L_f^2}]$, we can show that $F_{L_f^{1*}}(v) - F_{L_f^{2*}} \leq 0$ if:

$$F_{L_f^1}(v) - F_{L_f^2}(v) \leq c_1 \left(Q - q_{L_f^2}\right) \min\left\{0, A^0_{L_f^2} - A^0_{L_f^1}\right\}.$$

Based on the discussion above, a new improved dominance criteria is given in Proposition 2.

PROPOSITION 2. *Label L_f^2 is dominated by label L_f^1 if:*

1. $i_{L_f^1} = i_{L_f^2}$
2. $F_{L_f^1}(v) - F_{L_f^2}(v) \leq c_1 \left(Q - q_{L_f^2}\right) \min\left\{0, A^0_{L_f^2} - A^0_{L_f^1}\right\} + c_{L_f^2} - c_{L_f^1}, \quad \forall v \in \mathrm{dom}_\delta^{t=0}(L_f^2)$
3. $q_{L_f^1} \leq q_{L_f^2}$
4. $\delta_{L_f^1}(t,v) \leq \delta_{L_f^2}(t,v), \quad \forall (t,v) \in \mathrm{dom}_\delta(L_f^2)$
5. $\mathrm{dom}_\delta(L_f^2) \subseteq \mathrm{dom}_\delta(L_f^1)$
6. $\overline{V}_{L_f^1} \subseteq \overline{V}_{L_f^2}$

Proof of Proposition 2: See appendix.

The dominance test of Proposition 2 is stronger than the one in Proposition 1. In fact, it is now possible to dominate labels even if conditions 5 or 6 of Proposition 1 are not satisfied. Moreover, labels corresponding to paths with very high fuel cost (and high cost component) can easily be dominated as fuel cost and cost corresponding to dual variables are jointly handled by condition 2. Additionally Proposition 2 consists of less conditions. However, it is still hard to test because of condition 4. Moreover, paths corresponding to labels with very high duration are still hard to dominate because condition 2 does not include paths' duration cost. Condition 4 is also too restrictive as it requires that, for all $(t,v) \in \mathrm{dom}_\delta(L_f^2)$, the ready time at the end node of path $p(L_f^1)$ must be less or equal to the ready time at the end node of path $p(L_f^2)$. Intuitively, it might be possible that label L_f^1 dominates label L_f^1 if, for any feasible speed, we can always depart early enough at the depot with path $p(L_f^1)$ and have the ready time at the end node of path $p(L_f^1)$ less or equal to the ready time at the end node of path $p(L_f^2)$. Moreover, it might also be possible to dominate label L_f^2 by label L_f^1 if, for any feasible departure time at the depot, we can always travel fast enough with path $p(L_F^1)$ and have the ready time at the end node of path $p(L_f^1)$ less or equal to the ready time at the end node of path $p(L_f^2)$. Departing earlier at the depot increases the duration cost, and traveling faster increases the fuel cost. However, this should not prevent the dominance of L_f^2 by L_f^1 if the overall reduced cost of the extension of path $p(L_f^1)$ is less or equal to the reduced cost of the extension of path $p(L_f^2)$.

Consider a label L, if for a feasible speed v, a vehicle travels x unit faster $(x > 0)$, the increase in the fuel cost is:

$$\phi_F^L(v) = F_L(v+x) - F_L(v) = \left(\frac{c_2}{v+x} - \frac{c_2}{v} - c_3 v^2 + c_3(v+x)^2\right) A_L^0 \tag{16}$$

The first derivative is calculated as:

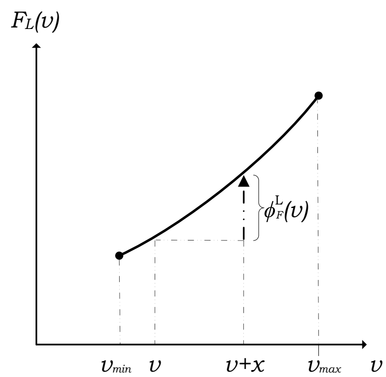
$$\frac{d\phi_F^L(v)}{dv} = \left(-\frac{c_2}{(v+x)^2} + \frac{c_2}{v^2} + 2c_3 x\right) A_L^0 \geq 0 \tag{17}$$

and the second derivative as

$$\frac{d^2\phi_F^L(v)}{dv^2} = 2c_2\left(\frac{1}{(v+x)^3} - \frac{1}{v^3}\right) A_L^0 \leq 0 \tag{18}$$

Hence, $\phi_F^L(v)$ is a convex and non-decreasing function in speed v. Therefore, for $x > 0$, the maximum increase in fuel cost is $\phi_F^L(v_{max} - x)$. Figure 2 illustrates $\phi_F^L(v)$ for a given $x > 0$.

Figure 2 Illustrations of $\phi_F^L(v)$



Based on the discussion above, an improved dominance test is provided in Proposition 3.

PROPOSITION 3. *Label L_f^2 is dominated by label L_f^1 if:*

1. $i_{L_f^1} = i_{L_f^2}$
2. $q_{L_f^1} \leq q_{L_f^2}$
3. $\overline{V}_{L_f^1} \subseteq \overline{V}_{L_f^2}$
4. (a) i. $\mathrm{dom}_\delta^{t=0}(L_f^2) \subseteq \mathrm{dom}_\delta^{t=0}(L_f^1)$

   ii. $\delta_{L_f^1}(0,v) \leq \delta_{L_f^2}(0,v), \quad \forall v \in \mathrm{dom}_\delta^{t=0}(L_f^2)$

   iii. $F_{L_f^1}(v) - F_{L_f^2}(v) \leq c_1\left(Q - q_{L_f^2}\right)\min\left\{0, A_{L_f^2}^0 - A_{L_f^1}^0\right\} - c_{L_f^2} + c_{L_f^1} - c_d\left(t_{max}^{L_f^2} - \underline{t}^{L_f^1}(v_{min})\right), \quad \forall v \in \mathrm{dom}_\delta^{t=0}(L_f^2)$ *(or)*

(b) i. $\mathrm{dom}_\delta^{v=v_{max}}(L_f^2) \subseteq \mathrm{dom}_\delta^{v=v_{max}}(L_f^1)$

ii. $\delta_{L_f^1}(t, v_{max}) \leq \delta_{L_f^2}(t, v_{max}), \quad \forall t \in \mathrm{dom}_\delta^{v=v_{max}}(L_f^2)$

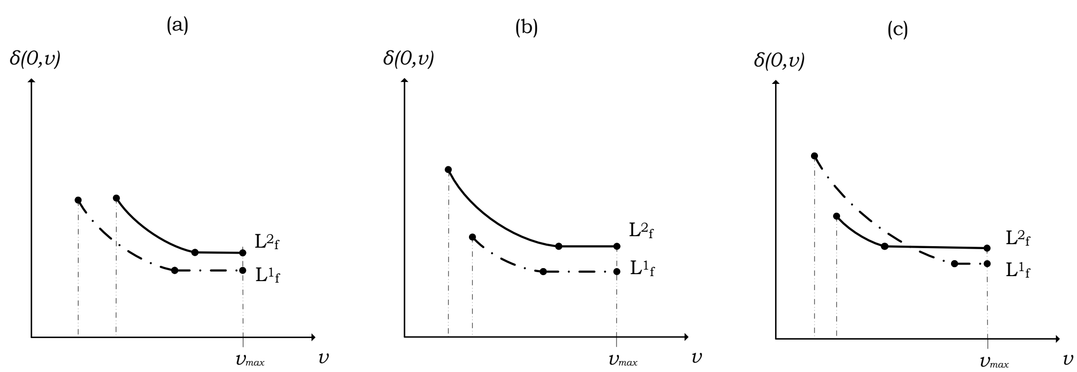
iii. $F_{L_f^1}(v_{max}) - F_{L_f^2}(v_{max}) \leq c_1 \left(Q - q_{L_f^2}\right) \min\left\{0, A^0_{L_f^2} - A^0_{L_f^1}\right\} - c_{L_f^2} + c_{L_f^1} - \phi_F^{L_f^2}\left(v_{min}^{L_f^2}\right)$

Proof of Proposition 3: See appendix.

In the dominance test presented in Proposition 3, conditions 4 and 5 of Proposition 2 are replaced by conditions that are relatively easier to verify. Note that either condition 4.a or 4.b is required to be satisfied for the dominance test to succeed. To dominate label L_f^2 by label L_f^1, condition 4.a requires that, for any speed $v \in \mathrm{dom}_\delta^{t=0}(L_2^f)$, we only need to ensure that the ready time at the end node of path $p(L_f^1)$ is less or equal to the ready time at the end node of path $p(L_f^2)$ when departure at the depot is $t=0$ (Figure 3.a). The increase in duration cost as a consequence of the earlier departure is in the worst case equal to $c_d\left(t_{max}^{L_f^2} - \underline{t}^{L_f^1}(v_{min}^{L_f^1})\right)$. Condition 4.b requires that, for any departure time at the depot, we need to ensure that the ready time at the end node of path $p(L_f^1)$ is less or equal to the ready time at the end node of path $p(L_f^2)$ when speed is $v = v_{max}$ (Figure 4.a). The increase in the fuel cost as result of traveling faster is in the worst case equal to $\phi_F^{L_f^2}\left(v_{max} - \left(v_{max} - v_{min}^{L_f^2}\right)\right) = \phi_F^{L_f^2}\left(v_{min}^{L_f^2}\right)$.

Obviously the dominance test of Proposition 3 will fail in case the ready time functions at time $t=0$ compare as in Figures 3.b and 3.c. In Figure 3.b $\mathrm{dom}_\delta^{t=0}(L_f^2) \nsubseteq \mathrm{dom}_\delta^{t=0}(L_f^1)$, and in Figure

Figure 3 **Drawbacks of Proposition 3**



Note. For a given departure time t at the depot

3.c, there exists at least a speed $v \in \mathrm{dom}_\delta^{t=0}(L_f^2)$ for which $\delta_{L_f^2}(0, v) \leq \delta_{L_f^1}(0, v)$. In order to handle these situations, we now define the interval I_v as

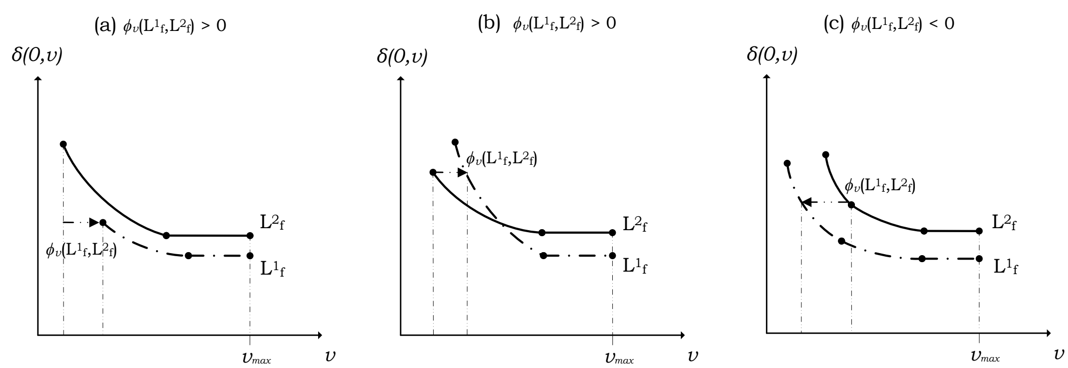
$$I_v = \left[v_{min}^{L_f^2} - v_{min}^{L_f^1}, +\infty\right)$$

and compute a value $\phi_v(L_f^1, L_f^2)$ when comparing labels L_f^1 and L_f^2 as

$$\phi_v(L_f^1, L_f^2) = \min\left\{x \in I_v : \delta_{L_f^1}(0, \min\{v+x, v_{max}\}) \leq \delta_{L_f^2}(0, v), \forall v \in \mathrm{dom}_\delta^{t=0}(L_f^2)\right\}. \tag{19}$$

The value of $\phi_v(L_f^1, L_f^2)$ can be either positive or negative. If $\phi_v(L_f^1, L_f^2)$ is positive, it measures how much faster we can travel with path $p(L_f^1)$, compared to path $p(L_f^2)$, to reach its last node at the same time or earlier than the last node of path $p(L_f^2)$, given that both paths $p(L_f^1)$ and $p(L_2^2)$ depart at the depot at time $t=0$. If $\phi_v(L_f^1, L_f^2)$ is negative, it measures how slower we can travel with path $p(L_f^1)$, compared to path $p(L_f^2)$, and still be able to reach its last node at the same time or earlier than the last node of path $p(L_f^2)$. In this case, the fuel consumption along path $p(L_f^1)$ is reduced. However, it is possible that customer time windows along feasible extensions of L_f^2 will be violated. Figures 4.a and 4.b illustrate cases where $\phi_v(L_f^1, L_f^2)$ is positive, and Figure 4.c illustrates a case where $\phi_v(L_f^1, L_f^2)$ is negative.

Figure 4 **Illustrations of** $\phi_v(L_f^1, L_f^2)$



For a given speed v, Proposition 3 does not handle such situations as in the Figures 5.b and 5.c. Figure 5.b depicts a case where $\mathrm{dom}_\delta^{v=v_{max}}(L_f^2) \nsubseteq \mathrm{dom}_\delta^{v=v_{max}}(L_f^2)$, and in Figure 5.c, there exists at least a departure time $t \in \mathrm{dom}_\delta^{v=v_{max}}(L_f^2)$ at the depot for which $\delta_{L_f^2}(t, v_{max}) \leq \delta_{L_f^1}(t, v_{max})$.

We now define the interval I_t as

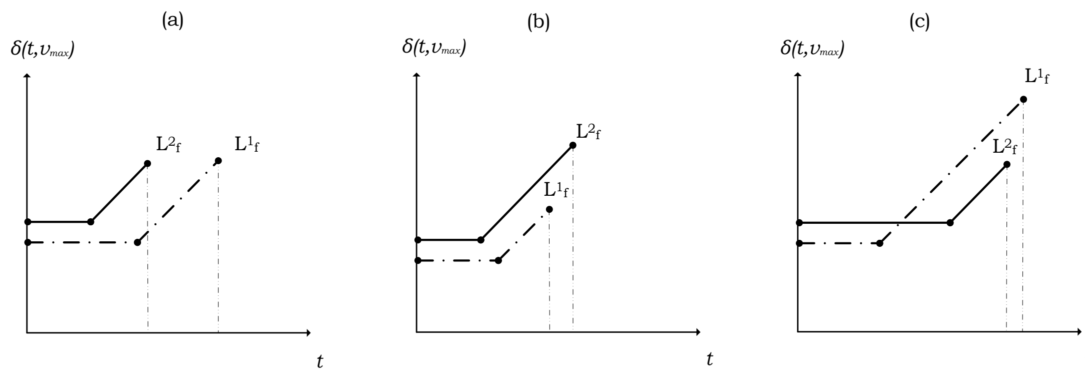
$$I_t = \left(-\infty, t_{max}^{L_f^1} - t_{max}^{L_f^2}\right]$$

and compute a value $\phi_t(L_f^1, L_f^2)$ labels L_f^1 and L_f^2 as

$$\phi_t(L_f^1, L_f^2) = \max\left\{x \in I_t : \delta_{L_f^1}(\max\{0, t+x\}, v_{max}) \leq \delta_{L_f^2}(t, v_{max}), \forall t \in \mathrm{dom}_\delta^{v=v_{max}}(L_f^2)\right\}. \tag{20}$$

The value of $\phi_t(L_f^1, L_f^2)$ can be either positive or negative. If $\phi_t(L_f^1, L_f^2)$ is positive, it measures how much later a vehicle on path $p(L_f^1)$ can depart from the depot, compared to path $p(L_f^2)$, and still

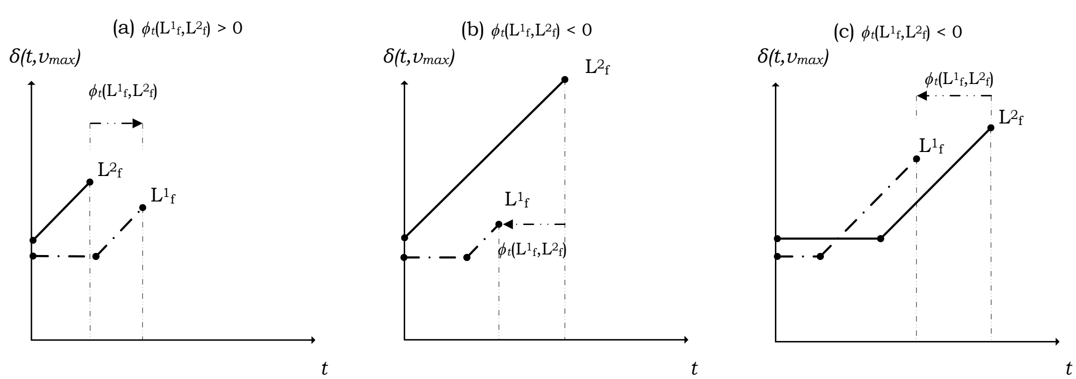
Figure 5 **Drawbacks of Dominance 3**



Note. For a given travel speed v

reach node $i(L_f^1)$ at the same time or earlier than path $p(L_f^2)$, no matter when in $\text{dom}_\delta(L_f^2)$ path $p(L_f^2)$ departs from the depot. If $\phi_t(L_f^1, L_f^2)$ is negative, it measures how much earlier a vehicle of path $p(L_f^1)$ must depart at the depot, compared to path $p(L_f^2)$, in order to ensure that node $i(L_f^1)$ is reached at the same time or earlier than when reached through path $p(L_f^2)$. Figure 6.a illustrates a case where $\phi_t(L_f^1, L_f^2)$ is positive, and Figures 6.b and 6.c illustrate cases where $\phi_t(L_f^1, L_f^2)$ is negative.

Figure 6 **Illustrations of $\phi_t(L_f^1, L_f^2)$**



Based on the discussion above, we now present the last improved dominance criteria in Proposition 4 below.

PROPOSITION 4. *Label L_f^2 is dominated by label L_f^1 if:*

1. $i_{L_f^1} = i_{L_f^2}$

2. $q_{L_f^1} \leq q_{L_f^2}$
3. $\overline{V}_{L_f^1} \subseteq \overline{V}_{L_f^2}$
4. $\delta_{L_f^1}(0, v_{max}) \leq \delta_{L_f^2}(0, v_{max})$
5. (a) $F_{L_f^1}(v) - F_{L_f^2}(v) \leq c_1 \left(Q - q_{L_f^2}\right) \min\left\{0, A^0_{L_f^2} - A^0_{L_f^1}\right\} - c_{L_f^2} + c_{L_f^1} - c_d \left(t_{max}^{L_f^2} - \underline{t}^{L_f^2}(v_{min})\right) - \phi_F^{L_f^2}\left(v_{max} - \phi_v(L_f^1, L_f^2)\right), \quad \forall v \in \mathrm{dom}_\delta^{t=0}(L_f^2)$ *(or)*
   (b) $F_{L_f^1}(v_{max}) - F_{L_f^2}(v_{max}) \leq c_1 \left(Q - q_{L_f^2}\right) \min\left\{0, A^0_{L_f^2} - A^0_{L_f^1}\right\} - c_{L_f^2} + c_{L_f^1} - \phi_F^{L_f^2}\left(v_{min}^{L_f^2}\right) + c_d \phi_t(L_f^1, L_f^2)$

Proof of Proposition 4: Condition 4 ensures that we can reach the end node $i_{L_f^1}$ as early using path $p(L_f^1)$ as using path $p(L_f^2)$, given that we depart at the depot early enough, and travel fast enough. Additionally, Conditions 2 and 3 guarantees that any feasible extension of label L_f^2 is also feasible for label L_f^1.

First we consider the case when Condition 5.(a) is satisfied. Let $L_f^{2^*} = L_f^2 \oplus L \wedge L_f^{1^*} = L_f^1 \oplus L$ and (t^*, v^*) optimal departure time and feasible speed for $L_f^{2^*}$.

Consider the traveling speed

$$v_1 = \min\left\{v^* + \phi_v(L_f^1, L_f^2), v_{max}\right\}$$

We have:

$$\delta_{L_f}^1(\underline{t}^{L_f^1}(v_1), v_1) \leq \delta_{L_f^1}(0, v^*) \leq \delta_{L_f}^2(0, v^*) \leq \delta_{L_f}^2(t^*, v^*)$$

Furthermore, we have

$$\begin{aligned}
\bar{c}_{L_f^{1*}} &\leq c_d \left(\delta_{L_f^{1*}}(\underline{t}^{L_f^1}(v_1), v_1) - \underline{t}^{L_f^1}(v_1)\right) + F_{L_f^{1*}}(v_1) - c_{L_f^1} - c_L \\
&\leq c_d \left(\delta_{L_f^{2*}}(t^*, v^*) - t^*\right) + F_{L_f^{2*}}(v^*) - c_{L_f^2} - c_L + F_{L_f^{1*}}(v_1) - F_{L_f^{2*}}(v^*) \\
&\quad + c_{L_f^2} - c_{L_f^1} + c_d \left(t^* - \underline{t}^{L_f^1}(v_1)\right) \\
&= \bar{c}_{L_f^{2*}} + F_{L_f^{1*}}(v_1) - F_{L_f^{2*}}(v_1) + F_{L_f^{1*}}(v_1) - F_{L_f^{2*}}(v^*) + c_{L_f^2} - c_{L_f^1} + c_d \left(t^* - \underline{t}^{L_f^1}(v_1)\right)
\end{aligned}$$

Using Condition 5.(a), we get:

$$\begin{aligned}
\bar{c}_{L_f^{1*}} &\leq \bar{c}_{L_f^{2*}} + F_{L_f^{2*}}(v_1) - F_{L_f^{2*}}(v^*) - \phi_F^{L_f^2}\left(v_{max} - \phi_v(L_f^1, L_f^2)\right) + c_d \left(t^* - \underline{t}^{L_f^1}(v_1)\right) \\
&\quad - c_d \left(t_{max}^{L_f^2}(v_{max}) - \underline{t}^{L_f^1}(v_{min})\right).
\end{aligned}$$

Note that

$$t^* \leq t_{max}^{L_f^{2*}} \leq t_{max}^{L_f^2} \quad \text{and} \quad \underline{t}^{L_f^{1^*}}(v_1) \geq \underline{t}^{L_f^1}(v_1) \geq \underline{t}^{L_f^1}(v_{min}^{L_f^1})$$

Furtheremore, we have

$$F_{L_f^{2*}}(v_1) - F_{L_f^{2*}}(v^*) \leq \phi_F^{L_f^2}\left(v_{max} - \phi_v(L_f^1, L_f^2)\right)$$

Therefore

$$\bar{c}_{L_f^{1*}} - \bar{c}_{L_f^{2*}} \leq 0$$

as desired. □

Second, we consider the case when Condition 5.(b) is satisfied, and let $t_1 = \max\left\{0, t^* + \phi_t(L_f^1, L_f^2)\right\}$ be a departure time at the depot.
We have $\delta_{L_f^1}(t_1, v_{max}) \leq \delta_{L_f}^2(t^*, v^*)$

Furthermore, we have

$$\begin{aligned}
\bar{c}_{L_f^{1*}} &\leq c_d\left(\delta_{L_f^{1*}}(t_1, v_{max}) - t_1\right) + F_{L_f^{1*}}(v_{max}) - c_{L_f^1} - c_L \\
&\leq c_d\left(\delta_{L_f^{2*}}(t^*, v^*) - t^*\right) + F_{L_f^{2*}}(v^*) - c_{L_f^2} - c_L + F_{L_f^{1*}}(v_{max}) - F_{L_f^{2*}}(v^*) \\
&+ c_{L_f^2} - c_{L_f^1} - c_d\phi_t(L_f^1, L_f^2) \\
&= \bar{c}_{L_f^{2*}} + F_{L_f^{1*}}(v_{max}) - F_{L_f^{2*}}(v_{max}) + F_{L_f^{2*}}(v_{max}) - F_{L_f^{2*}}(v^*) - c_{L_f^1} + c_{L_f^2} - c_d\phi_t(L_f^1, L_f^2) \\
&\leq \bar{c}_{L_f^{2*}} + F_{L_f}^{2^*}(v_{max}) - F_{L_f}^{2^*}(v_2) - \phi_F^{L_f^2}\left(v_{min}(0)\right).
\end{aligned}$$

This implies that

$$\bar{c}_{L_f^{1*}} - \bar{c}_{L_f^{2*}} \leq 0$$

□

as desired.

5.2. The Backward Labeling Algorithm

In the backward labeling algorithm, labels are extended from the depot (i.e., node $n+1$) to its predecessors. To a label L_b, we associate the following components:

i_{L_b}	First node visited on the partial path $p(L_b)$.
c_{L_b}	Sum of the dual variables associated with the nodes visited by the partial path $p(L)$.
q_{L_b}	Total quantity delivered along the partial path $p(L_b)$.
$F_{L_b}(v)$	Fuel cost of partial path $p(L_b)$ when traversed with speed v.
$\delta_{L_b}(t, v)$	Ready time at node $i(L_b)$ in order to arrive at node $n+1$ no later than time t when traveling with speed v.
V_{L_b}	Set of nodes visited along the partial path $p(L_b)$.

Furthermore, we define $\overline{V}_{L_b}$ as V_{L_b} extended with nodes that cannot be visited by path $p(L_b)$.

The set of feasible extensions $E(L_b)$ of L_b is the set of partial paths such that when departing at the depot (i.e., node 0) at time 0, reaching node i_{L_b} at some time $t \leq \delta_{L_b}(b_{n+1}, v_{max})$ without violating time windows. The basic operation in the backward labeling algorithm is the extension of a label L' along an arc $(j, i_{L'_b})$ to a node i to create a new label L_b. The ready time function associated with the new label L_b is computed as follows:

$$\delta_{L_b}(t,v) = \min\left\{b_j + s_j, \delta_{L'_b}(t,v) - \tau_{j,i_{L'_b}}(v)\right\}. \tag{21}$$

The fuel consumption function $F_{L_b}(v)$ associated with the new label L_b can be expressed as

$$F_{L_b}(v) = \left(c_0 + \frac{c_2}{v} + c_3 v^2\right) A^0_{L_b} + c_1 A^1_{L_b}, \tag{22}$$

where $A^0_{L_b}$ and $A^1_{L_b}$ are updated as

$$A^0_{L_b} = A^0_{L'_b} + d_{j,i_{L'_b}} \quad \text{and} \quad A^1_{L_b} = A^1_{L'_b} + q_{L_b} d_{j,i_{L'_b}} \tag{23}$$

Additionally, we have the followings:

$$V_{L_b} = V_{L'_b} \cup \{j\}, \quad c_L = c_{L'} + \pi_j \quad \text{and} \quad q_{L_b} = q_{L'_b} + q_j. \tag{24}$$

The extension of L'_b to L_b is feasible if:

$$V_{L'_b} \cap \{j\} = \emptyset \quad \wedge \quad \delta_{L_b}(b_{n+1}, v) \geq \max\{t^b_m, a_i + s_i\} \quad \wedge \quad q_{L_b} \leq Q, \tag{25}$$

where t^b_m is the bounding lower bound used in the forward search (i.e., the middle of the planning horizon).

As in the forward algorithm, we can show that for a label L_b, the ready time function $\delta_{L_b}(b_{n+1}, v)$ is a piecewise concave non-decreasing function, and the ready time function $\delta_{L_b}(t, v_{max})$ is a piecewise concave non-decreasing linear function. Dominance criteria for the backward algorithm are developed in a the same way as for the forward algorithm. In this section, we only present the improved dominance criteria.

We now define the interval I_v as

$$I_v = \left[v^{L^1_b}_{min} - v^{L^2_b}_{min}, +\infty\right)$$

and calculate $\phi_v(L^1_b, L^2_b)$ as

$$\phi_v(L^1_b, L^2_b) = \min\left\{x \in I_v : \delta_{L^1_b}(b_{n+1}, \min\{v + x, v_{max}\}) \geq \delta_{L^2_b}(b_{n+1}, v), \forall v \in \mathrm{dom}^{t=b_{n+1}}_{\delta}(L^2_b)\right\}. \tag{26}$$

Furthermore, we define the interval I_t as

$$I_t = \left(-\infty, t^{L^1_b}_{min} - t^{L^2_b}_{min}\right]$$

and compute the value $\phi_t(L^1_b, L^2_b)$ as

$$\phi_t(L^1_b, L^2_b) = \max\left\{x \in I_t : \delta_{L^1_b}(\min\{t + x, b_{n+1}\}, v_{max}) \geq \delta_{L^2_b}(t, v_{max}), \forall t \in \mathrm{dom}^{v=v_{max}}_{\delta}(L^2_b)\right\}. \tag{27}$$

The dominance criteria for the backward algorithm are introduced in Proposition 5.

PROPOSITION 5. *Label L_b^2 is dominated by label L_b^1 if:*

1. $i_{L_b^1} = i_{L_b^2}$
2. $q_{L_b^1} \leq q_{L_b^2}$
3. $\overline{V}_{L_b^1} \subseteq \overline{V}_{L_b^2}$
4. $\delta_{L_b^1}(b_{n+1}, v_{max}) \geq \delta_{L_b^2}(b_{n+1}, v_{max})$
5. (a) $F_{L_b^1}(v) - F_{L_b^2}(v) \leq c_1 d_{i_{L_b^1}, n+1}\left(q_{L_b^2} - q_{L_b^1}\right) - c_{L_b^2} + c_{L_b^1} - c_d\left(t_{min}^{L_b^2} - \overline{t}^{L_b^1}(v_{min})\right) - \phi_F^{L_b^2}(v_{max} - \phi_v(L_b^1, L_b^2)), \quad \forall v \in \mathrm{dom}_\delta^{t=b_{n+1}}(L_b^2)$ *(or)*
   (b) $F_{L_b^1}(v_{max}) - F_{L_b^2}(v_{max}) \leq c_1 d_{i(L_b^1), n+1}\left(q_{L_b^2} - q_{L_b^1}\right) - c_{L_b^2} + c_{L_b^1} - c_d \phi_t(L_b^1, L_b^2) - \phi_F^{L_b^2}\left(v_{min}^{L_b^2}\right), \quad \forall v \in \mathrm{dom}_\delta^{t=b_{n+1}}(L_b^2)$

Proof of Proposition 5: Similar to the proof of Proposition 4.

5.3. Merging Forward and Backward Labels

In the merging step, labels generated in the forward phase are merged with labels generated in the backward phase to construct complete routes with negative reduced cost.

A forward label L_f and a backward label L_b can be merged if the following conditions are satisfied:

- $V_{L_f} \cap V_{L_b} = \emptyset$
- $q_{L_f} + q_{L_b} \leq Q$
- $\delta_{L_f}(0, v_{max}) + \tau_{ij}(v_{max}) \leq \delta_{L_b}(b_{n+1}, v_{max})$

The resulting label $L = L_f \oplus L_b$ has the following attributes:

- $i_L = n+1$
- $c_L = c_{L_f} + c_{L_b}$
- $q_L = q_{L_f} + q_{L_b}$
- $V_L = V_{L_f} \cup V_{L_b}$
- $F_L = \left(c_0 + \frac{c_2}{v} + c_3 v^2\right)\left(A_{L_f}^0 + A_{L_b}^0 + d_{ij}\right) + c_1\left(A_{L_f}^1 + A_{L_b}^1 + q_L A_{L_f}^0\right)$

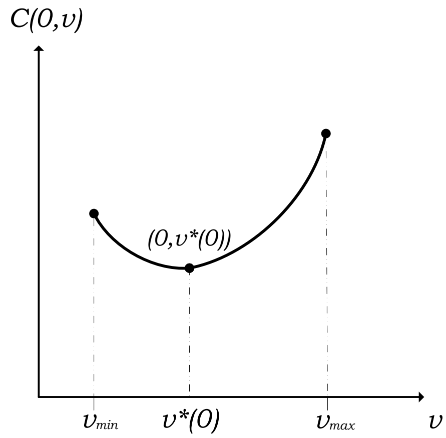
A cost of a route can be calculated as (see Equation 4)

$$C(t^*, v^*) = c_d\left(\delta_{i_k}^p(t^*, v^*) - t^*\right) + F_p(v^*)$$

where t^* is the optimal departure time at the depot and v^* is the optimal traveling speed. Figure 7 illustrates a cost function for a given departure time (i.e, $t = 0$).

As can be seen from Figure 7, an interval of the optimal speed is defined as $v^* \in [v^*(0), v_{max}]$ because the optimal speed will always be within the range of $[v^*(0), v_{max}]$ for any departure time

Figure 7 **An illustration of the cost function** $C(0, v)$



Note. For a given departure time t at the depot (i.e., $t = 0$)

Algorithm 1 Golden Search $(v^*(0), v_{max})$

initialize $\epsilon \leftarrow 0.03 \quad \wedge \quad \Phi \leftarrow 1.618$
$v_L \leftarrow v^*(0) \quad \wedge \quad v_U \leftarrow v_{max}$
$\Delta v \leftarrow (v_U - v_L)$
while $\Delta v \leq \epsilon$ **do**
  $\overline{v}_L \leftarrow v_L + (2 - \Phi)(v_U - v_L)$
  $C_L \leftarrow C(t^*(\overline{v}_L), \overline{v}_L)$
  $\overline{v}_U \leftarrow v_L + (\Phi - 1)(v_U - v_L)$
  $C_U \leftarrow C(t^*(\overline{v}_U), \overline{v}_U)$
  if $C_L \leq C_U$ **then**
    $v_U \leftarrow \overline{v}_U$
  else
    $v_L \leftarrow \overline{v}_L$
  end if
  $\Delta v \leftarrow (v_U - v_L)$
end while
return (t^*, v^*)

$t' > 0$. In order to calculate t^* and v^*, we implemented the *Golden Search* (GS) method by Kiefer (1953) as presented in Algorithm 1 below.

The GS method is used to find the minimum of our unimodal cost function by successively narrowing the range of values inside which the extremum is known to exist. The procedure is initialized with an interval of speed as input and returns t^* and v^*. The parameters ϵ and ϕ define the desired gap between the upper and lower bound of the speed interval and the golden ratio, respectively. In between lines 4 and 15, the algorithm updates the interval of the speed and calculates the corresponding cost functions. The algorithm iterates until the desired gap in the speed interval is achieved.

5.4. The Pricing Problem Heuristics

A solution time of the branch-and-price algorithm can be improved by using heuristics. More specifically, a heuristic searches for *easy-to-find* paths with negative reduced cost and add them to the master problem. When the heuristics fail to find any more paths with negative reduced cost, then an exact algorithm is run. Ideally, for every node in the branching tree, the exact algorithm is called only once to check that no more paths with negative reduced cost exist. In our branch-and-price framework, we use two different heuristic algorithms. First, a so-called k-node (i.e., $k = 5$) heuristic is run. It calculates the k-best dual values of each node and then generates a subgraph. In other words, k neighbor nodes of each and every node are selected based on their dual and the whole graph is trimmed until there is at most k path found with negative reduced cost for each node. The algorithm iterates two more times (i.e., for $2k$ and $4k$). Second, we implemented a truncated labeling heuristic in which only a limited number of labels (with the best cost) is kept and considered for a possible extension. The number of stored labels can be increased each time when the heuristic fails to find paths with negative reduced cost (e.g., we start with 250, then we increase the number of labels to 500, 1000 and finally to 2000 labels).

6. Computational Results

Our proposed branch-and-price algorithm is implemented in Java. All experiments are conducted on a server Intel(R) Core(TM)2 CPU, 2.6 GHz with 4 GB of memory. A linear programming relaxation of the master problem is solved by a standard linear programming solver, namely Gurobi Optimizer 5.6 (Gurobi 2013). For the experiments, we use Solomon's instances (Solomon 1987). Firstly, we adapted the well-known Solomon's data sets, which come in three sets R (Random), C(Clustered) and RC (Randomly Clustered) classified with respect to the geographical locations of the nodes. More specifically, Solomon's instances follow a naming convention of $GTn.c$ where G is the geographic distribution of the customers, T is the instance type which can be either 1 (tight time windows) or 2 (wide time windows). Moreover, n denotes the specific number of the instance, and c is the number of customers that needs to be served. To use a set of different speed levels on Solomon's data sets, the planning horizon is set as 24 hours and all time-related values are adjusted accordingly. Moreover, the maximum payload is set as 2000 kg and the demand of each customer is multiplied by 10. The complete data set used in this paper is available at www.smartlogisticslab.nl. We note that a time-limit of 2 hours is imposed on the solution time for all instances.

6.1. SDESPPRC vs. SDSPPRC

In this section, we compare two pricing problems (namely SDESPPRC and SDSPPRC) in Tables 2, 3 and 4 to report the instances for which we could at least solve the root node of the branch-and-bound tree. The first column of each table provides the name of the instance. The column "Root LB" presents the lower bound obtained in the root node. It is then followed by the columns denoted as "Best LB" and "UB" indicating the best lower and upper bounds found all over a branching tree, respectively. Furthermore, the column "Time" provides the CPU time (in seconds) spent to solve an instance, and in the last column "Tree" we provide the size of the branching trees.

In the case of instances with 25 customers, we solved 27 out of 29 instances with tight time windows of type 1 (i.e., R1xx, C10x, RC10x) to prove optimality with the exception of instance C102.25 for which only a lower bound is found when the pricing problem is the SDESPPRC. Moreover, we solved 24 out of 27 instances with wide time windows of type 2 (i.e., R2xx, C20x, RC20x) to prove optimality with the exception of instances R104.25, R208.25 and R203.25.

For instances with 50 customers, our proposed algorithm is able to solved eight out of 29 instances to optimality when the pricing problem is the SDESPPRC, and seven instances of the same set when the pricing problem is SDSPPRC. Moreover, within two hours, only a lower bound is found for some of the instances: seven instances with SDESPPRC and six instances with SDSPPRC. For type 2 instances, six out of 27 instances are solved to optimality and a lower bound is found for another three more instances when the pricing problem is SDESPPRC. Using the SDSPPRC, the algorithm is able to solve four instances to optimality and five more instances could only be solved for a lower bound.

In the case of instances with 100 customers, only three instances (C101, C105 and R101) are solved to optimality, and a lower bound is found only for four instances. Table 5 provides a summary of all solved instances for SDESPPRC and SDSPPRC. The first column "Nb. instances" presents the total number of solved instances with each type of pricing problem. The columns "Avg. root LB" and "Avg. best LB" show the average of the root lower bound and the average of the best lower bound of the instances for which both SDESRRPC and SDSRRPC are able to produce a lower bound. Moreover, the average CPU time (in seconds) spent over the instances for which an upper bound is found by both SDESRRPC and SDSRRPC, and the average trees, are reported in the columns "Avg. times" and "Avg. tree", respectively.

In total, we can solve about 82% of the instances with 25 customers, 32% of the instances with 50 customers, and 7% of the instances with 100 customers. Even though the analyzed problem is too complex, the proposed exact algorithm performs very well on Solomon's data sets with the help of tailored pricing algorithms.

Table 2 Instances with 25 customers.

	SDESPPRC (elementary paths)					SDSPPRC (non-elementary paths)				
Instance	**Root LB**	**Best LB**	**UB**	**Time(s)**	**Tree**	**Root LB**	**Best LB**	**UB**	**Time(s)**	**Tree**
r101	1442.47	1442.71	1442.71	12	4	1442.47	1442.71	1442.71	9	4
r102	1218.00	1218.00	1218.00	9	0	1218.00	1218.00	1218.00	14	0
r103	1101.05	1101.05	1101.05	61	0	1100.33	1101.05	1101.05	98	2
r104	1013.08	1016.95	1016.95	288	2	1013.08	1016.95	1016.95	745	2
r106	1075.45	1075.45	1075.45	43	0	1075.45	1075.45	1075.45	57	0
r107	1007.53	1015.84	1015.84	360	6	1002.33	1015.84	1015.84	3605	62
r108	945.81	945.81	945.81	893	0	944.73	945.81	945.81	1937	4
r109	1044.29	1048.48	1048.48	27	2	1024.72	1048.48	1048.48	599	118
r110	1007.68	1016.39	1016.39	320	6	988.71	1016.39	1016.39	4547	358
r111	1013.65	1017.15	1017.15	372	2	992.58	1017.15	1017.15	1147	50
r112	944.93	951.30	951.30	3213	22	918.22	932.72	–	–	82
c101	710.28	710.28	710.28	33	0	710.28	710.28	710.28	20	0
c102	696.06	696.06	–	–	2	–	–	–	–	0
c105	706.18	706.18	706.18	42	0	706.18	706.18	706.18	26	0
c106	793.59	798.86	798.86	33	4	793.59	798.86	798.86	24	4
c107	701.96	706.18	706.18	270	8	684.34	706.18	706.18	4296	358
c108	700.51	706.18	706.18	3386	64	673.76	690.21	–	–	106
c109	694.61	694.76	704.85	–	4	–	–	–	–	0
rc101	1019.87	1104.27	1104.27	107	72	1019.87	1104.27	1104.27	154	136
rc102	919.44	919.44	919.44	85	0	918.41	919.44	919.44	952	4
rc103	895.44	895.44	895.44	450	0	892.71	895.44	895.44	5781	16
rc104	869.07	869.07	869.07	5011	0	–	–	–	–	0
rc105	1015.79	1025.92	1025.92	31	2	1007.48	1025.92	1025.92	76	8
rc106	910.63	910.63	910.63	28	0	887.01	910.63	910.63	767	102
rc107	845.22	845.22	845.22	238	0	832.24	845.22	845.22	3054	22
rc108	841.99	841.99	841.99	1969	0	809.90	829.57	–	–	12
r201	1224.58	1225.30	1225.30	13	2	1224.58	1225.30	1225.30	12	2
r202	1152.63	1154.63	1154.63	57	2	1151.76	1154.63	1154.63	89	2
r203	1060.02	1060.02	1060.02	251	0	1059.35	1060.02	1060.02	888	6
r204	981.19	981.19	–	–	2	972.39	979.86	–	–	12
r205	1082.39	1082.39	1082.39	30	0	1061.43	1082.39	1082.39	345	42
r206	1024.80	1026.47	1026.47	284	2	984.23	1018.17	1050.47	–	162
r207	993.69	997.80	997.80	918	4	944.99	959.74	–	–	40
r208	931.34	931.34	–	–	2	897.36	897.36	–	–	2
r209	1021.90	1021.90	1021.90	40	0	1005.30	1021.90	1021.90	809	48
r210	1049.38	1067.71	1067.71	1093	26	1036.87	1067.71	1067.71	5952	230
r211	956.75	960.03	960.03	732	4	927.76	948.93	–	–	148
c201	1097.14	1141.61	1141.61	593	206	1097.14	1141.61	1141.61	569	240
c202	957.06	973.61	973.61	4086	50	950.77	973.61	973.61	4492	34
c203	820.07	820.07	–	–	2	–	–	–	–	0
c205	924.62	992.09	997.28	–	1352	924.62	992.03	999.49	–	1308
c206	894.14	930.84	930.84	1215	102	891.90	930.84	930.84	2056	256
c208	807.71	818.51	818.51	1635	4	806.89	818.51	818.51	1316	18
rc201	1048.60	1102.26	1102.26	32	14	1048.60	1102.26	1102.26	27	18
rc202	957.09	957.09	957.09	168	0	956.11	957.09	957.09	2266	2
rc203	914.97	914.97	914.97	763	0	913.99	914.97	914.97	5942	2
rc205	1002.44	1002.44	1002.44	28	0	995.86	1002.44	1002.44	89	4
rc206	922.75	922.75	922.75	36	0	873.34	922.75	922.75	675	104
rc207	876.69	876.69	876.69	154	0	860.68	876.69	876.69	3838	64
rc208	844.26	844.26	844.26	2052	0	–	–	–	–	0

Table 3 Instances with 50 customers.

	SDESPPRC (elementary paths)					SDSPPRC (non-elementary paths)				
Instance	**Root LB**	**Best LB**	**UB**	**Time(s)**	**Tree**	**Root LB**	**Best LB**	**UB**	**Time(s)**	**Tree**
r101	2503.55	2503.55	2503.55	17	0	2503.55	2503.55	2503.55	12	0
r102	2208.81	2220.77	2220.77	1884	24	2206.72	2220.77	2220.77	3448	46
r105	2091.09	2105.13	2105.13	264	22	2091.09	2105.13	2105.13	206	26
r106	1941.03	1941.08	–	–	4	1940.96	1940.96	–	–	2
r109	1887.91	1903.80	1903.80	–	40	1835.99	1860.27	–	–	62
r110	–	–	–	–	0	1761.22	1761.22	–	–	2
c101	1355.68	1355.68	1355.68	67	0	1355.68	1355.68	1355.68	71	0
c105	1333.97	1333.97	1333.97	390	0	1333.97	1333.97	1333.97	226	0
c106	1417.52	1417.52	1417.52	151	0	1417.49	1417.52	1417.52	100	2
c107	1326.56	1326.56	1326.56	1495	0	1318.08	1326.56	1326.56	5011	64
c108	1310.17	1310.17	1310.17	5983	0	–	–	–	–	0
rc101	2089.04	2189.29	2227.15	–	1544	2089.04	2196.39	2223.52	–	1512
rc102	1903.54	1976.23	–	–	50	1903.11	1923.89	–	–	4
rc105	1966.28	2048.23	–	–	436	1953.99	1998.94	–	–	218
rc106	1848.53	1883.09	–	–	204	1797.35	1819.97	–	–	198
rc107	1735.52	1743.66	–	–	6	–	–	–	–	0
r201	2224.64	2248.32	2248.32	1874	84	2220.06	2248.32	2248.32	737	66
r202	2046.50	2048.56	–	–	4	2044.10	2047.89	–	–	6
r205	1953.13	1967.46	1968.56	–	28	1893.73	1918.70	–	–	68
r210	–	–	–	–	0	1786.91	1791.73	–	–	8
c201	1876.60	1909.53	1909.53	5848	28	1876.60	1909.53	1909.53	3391	42
rc201	2074.93	2191.09	2191.09	3812	588	2074.93	2191.09	2191.09	3797	634
rc202	1863.66	1907.12	1907.12	6546	4	1860.24	1860.24	–	–	2
rc205	1835.79	1835.79	1835.79	839	0	1834.91	1835.79	1835.79	1382	4
rc206	1771.73	1792.34	1792.34	1629	2	1738.68	1757.09	–	–	68
rc207	1747.84	1747.84	1747.84	–	0	–	–	–	–	0

Table 4 Instances with 100 customers.

	SDESPPRC (elementary paths)					SDSPPRC (non-elementary paths)				
Instance	**Root LB**	**Best LB**	**UB**	**Time(s)**	**Tree**	**Root LB**	**Best LB**	**UB**	**Time(s)**	**Tree**
r101	4232.23	4233.28	4233.28	1434	2	–	–	4233.28	804	2
c101	2943.78	2943.78	2943.78	1059	0	2943.78	2943.78	2943.78	435	0
c105	2778.42	2778.42	2778.42	6113	0	2778.42	2778.42	2778.42	2572	0
c107	–	–	3434.73	–	0	–	–	–	–	–
r201	–	–	–	–	–	4232.23	4233.28	–	–	0
rc101	3936.97	3937.78	–	–	8	–	–	–	–	40
rc102	–	–	–	–	–	3936.97	3937.78	–	–	0
rc201	4062.57	4062.57	–	–	2	4062.57	4062.57	–	–	24

Table 5 Aggregate comparison between SDESRRPC and SDSPPRC

	Nb. instances	**Avg. Root LB**	**Avg. Best LB**	**Avg. Time (s)**	**Avg. Tree**
with SDESPPRC	62	958.31	967.64	349.10	34
with SDSPPRC	50	946.08	965.26	1803.40	80

6.2. Bi-directional vs. Mono-directional Labeling Algorithm

This section illustrates the gains of using a bi-directional search over the mono-directional search in Table 6. The best found lower and upper bounds are given in the columns "Best LB" and "UB", respectively. Moreover, the column "Time" provides the CPU time needed to solve an instance.

The performance of the bi-directional labeling algorithm is better than that of the mono-directional version. The mono-directional barely solves any instance with 50 customers or instance with 25 customers and wide time windows. This is mainly due to the fact that the number of labels that needs to be processed in the bi-directional labeling algorithm is considerably restricted compared with the mono-directional labeling algorithm.

6.3. An example solution

We provide a detailed solution of R206.25, which is solved to optimality with the bi-directional labeling algorithm in Table 7. The first and second columns of table provide the start and end time of each routes. The column "Speed" presents the traveling speed for each route. The incurred costs are shown separately in the columns of "Fuel cost" and "Driver cost" and as a total in the column of "Total cost". Moreover, the column "Route" provides the sequence of customers to be visited in each route.

7. Conclusions

We provided a branch-and-price algorithm and used this exact approach for the first time to minimize CO_2e emissions in the context of green road freight transportation, particularly on the PRP. To the best of our knowledge, this paper also appears to be the first one to accommodate speed and departure time optimization at the same methodology. Considering that speed-dependent travel times increases the complexity of the pricing problem, standard dominance tests are therefore not applicable to our pricing problem. We introduced a new, stronger dominance test. To help the reader appreciate the value of such a complicated case of the pricing problem, a step-by-step explanatory approach is followed to tailor the features of the pricing algorithm for the PRP. To fully evaluate the effectiveness of the algorithm, we have used our methodology on Solomon's data sets. Computational results showed that even some instances with up to 100 customers can be solved to optimality but several instances with only 25 customers remain unsolved.

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Appendix.

Table 6 **Bi-directional Labeling Algorithm vs. Mono-directional Labeling Algorithm (with SDESPPRC and 25 customers)**

	Bi-directional			Mono-directional		
Instance	**Best LB**	**UB**	**Time(s)**	**Best LB**	**UB**	**Time(s)**
r101	1442.71	1442.71	12	1442.71	1442.71	14
r102	1218.00	1218.00	9	1218.00	1218.00	16
r103	1101.05	1101.05	61	1101.05	1101.05	98
r104	1016.95	1016.95	288	1016.95	1016.95	3169
r106	1075.45	1075.45	43	1075.45	1075.45	121
r107	1015.84	1015.84	360	1015.84	1015.84	2406
r108	945.81	945.81	893	945.81	945.81	3320
r109	1048.48	1048.48	27	1048.48	1048.48	435
r110	1016.39	1016.39	320	1016.39	1016.39	2492
r111	1017.15	1017.15	372	1017.15	1017.15	3316
r112	951.30	951.30	3213	944.93	–	–
c101	710.28	710.28	33	710.28	710.28	21
c102	696.06	–	–	–	–	–
c105	706.18	706.18	42	706.18	706.18	57
c106	798.86	798.86	33	798.86	798.86	68
c107	706.18	706.18	270	702.76	706.18	–
c108	706.18	706.18	3386	–	–	–
c109	694.76	704.85	–	–	–	–
rc101	1104.27	1104.27	107	1104.27	1104.27	767
rc102	919.44	919.44	85	919.44	919.44	280
rc103	895.44	895.44	450	895.44	895.44	306
rc104	869.07	869.07	5011	–	–	–
rc105	1025.92	1025.92	31	1025.92	1025.92	656
rc106	910.63	910.63	28	910.63	910.63	114
rc107	845.22	845.22	238	845.22	845.22	4235
rc108	841.99	841.99	1969	841.99	841.99	3994
r201	1225.30	1225.30	13	1225.30	1225.30	23
r202	1154.63	1154.63	57	1154.63	1154.63	1042
r203	1060.02	1060.02	251	1060.02	1060.02	231
r204	981.19	–	–	–	–	–
r205	1082.39	1082.39	30	1082.39	1082.39	127
r206	1026.47	1026.47	284	1026.47	1026.47	4299
r207	997.80	997.80	918	996.66	1004.83	–
r208	931.34	–	–	–	-	–
r209	1021.90	1021.90	40	1021.90	1021.90	358
r210	1067.71	1067.71	1093	1057.45	-	–
r211	960.03	960.03	732	959.62	960.03	–
c201	1141.61	1141.61	593	1132.12	1146.52	–
c202	973.61	973.61	4086	–	–	–
c203	820.07	–	–	–	–	–
c205	992.09	997.28	–	937.10	-	–
c206	930.84	930.84	1215	894.14	-	–
c208	818.51	818.51	1635	–	–	–
rc201	1102.26	1102.26	32	1102.26	1102.26	334
rc202	957.09	957.09	168	957.09	957.09	984
rc203	914.97	914.97	763	–	–	–
rc205	1002.44	1002.44	28	1002.44	1002.44	81
rc206	922.75	922.75	36	922.75	922.75	323
rc207	876.69	876.69	154	876.69	876.69	1084
rc208	844.26	844.26	2052	–	–	–

Table 7 Solution of R206.25.

Start time (s)	End time (s)	Speed (km/h)	Fuel cost (€)	Driver cost (€)	Total cost (€)	Route
2,099.3	80,482.5	87.0	168.5	217.9	386.4	2, 15, 14, 16, 6, 18, 8, 17, 5, 13
7,688.3	71,603.4	90.0	168.2	177.7	345.9	7, 19, 11, 1, 3, 9, 20, 10
27,888.6	84,878.4	83.2	135.8	158.4	294.2	23, 22, 21, 4, 25, 24, 12

Proof of Proposition 1: Consider two labels L_f^1 and L_f^2 that satisfy the eight conditions in Proposition 1. We need to show that: (1) for any label L, $L \in E(L_f^2)$ implies that $L \in E(L_f^1)$, and (2) for any $L \in E(L_f^2)$, we have that $\bar{c}_{L_f^1 \oplus L} \leq \bar{c}_{L_f^2 \oplus L}$. Let $L_f^{1^*} = L_f^1 \oplus L$ and $L_f^{2^*} = L_f^2 \oplus L$.

To prove (1), consider $L \in E(L_f^2)$. Condition 3 ensures that $p(L_f^{1^*})$ does not vehicle capacity. $p(L_f^{1^*})$ is elementary due to condition 8, and conditions 4 and 5 ensure that $p(L_f^{1^*})$ does not violate time windows if the depot is left at any time $(t, v) \in \mathrm{dom}_\delta(L_f^2)$.

Regarding the second point, let (t^*, v^*) be the optimal departure time and traveling speed of path $p(L_f^{2^*})$. The reduced cost of the path $p(L_f^{2^*})$ is

$$\bar{c}_{L_f^{2*}} = c_d \left(\delta_{L_f^{2*}}(t^*, v^*) - t^* \right) + F_{L_f^{2*}}(v^*) - (c_{L_f^2} + c_L)$$

Due to condition 5, (t^*, v^*) are feasible departure time and traveling speed for path $p(L_f^{1^*})$, and induces an upper bound on the reduced cost of $p(L_f^{1^*})$:

$$\bar{c}_{L_f^{1*}} \leq c_d \left(\delta_{L_f^{1*}}(t^*, v^*) - t^* \right) + F_{L_f^{1*}}(v^*) - (c_{L_f^1} + c_L).$$

We get,

$$\bar{c}_{L_f^{1*}} - \bar{c}_{L_f^{2*}} \leq c_d \left(\delta_{L_f^{1*}}(t^*, v^*) - \delta_{L_f^{2*}}(t^*, v^*) \right) + F_{L_f^{1*}}(v^*) - F_{L_f^{2*}}(v^*) + c_{L_f^2} - c_{L_f^1}$$

We know that, due to conditions 2 and 4, $\delta_{L_f^{1*}}(t^*, v^*) - \delta_{L_f^{2*}}(t^*, v^*) \leq 0$ and $c_{L_f^2} - c_{L_f^1} \leq 0$. Moreover, we can show that

$$F_{L_f{}^{1*}}(v^*) - F_{L_f{}^{2*}}(v^*) = F_{L_f{}^1}(v) - F_{L_f{}^2}(v) + c_1 q_L \left(A^0_{L_f^1} - A^0_{L_f^2} \right)$$

Conditions 6 and 7 imply that $F_{L_f{}^1}(v) - F_{L_f{}^2}(v) \leq 0$ and $A^0_{L_f^1} - A^0_{L_f^2} \leq 0$, thus $F_{L_f^{1*}}(v) - F_{L_f^{2*}}(v) \leq 0$. Hence, $\bar{c}_{L_f^{1*}} - \bar{c}_{L_f^{2*}} \leq 0$ □

Proof of Proposition 2: Similar to Proposition 1, we can show that for any label L, $L \in E(L_f^2)$ implies that $L \in E(L_f^1)$. For simplicity, we again let $L_f^{1^*} = L_f^1 \oplus L$ and $L_f^{2^*} = L_f^2 \oplus L$. Next, $\bar{c}_{L_f^{1*}} \leq \bar{c}_{L_f^{2*}}$. Let (t^*, v^*) be the optimal departure time and traveling speed of path $p(L_f^{2^*})$. In the proof of Proposition 1, we have shown that

$$\bar{c}_{L_f^{1*}} - \bar{c}_{L_f^{2*}} \leq c_d \left(\delta_{L_f^{1*}}(t^*, v^*) - \delta_{L_f^{2*}}(t^*, v^*) \right) + F_{L_f^{1*}}(v^*) - F_{L_f^{2*}}(v^*) + c_{L_f^2} - c_{L_f^1}$$

and

$$F_{L_f{}^{1*}}(v^*) - F_{L_f{}^{2*}}(v^*) = F_{L_f{}^1}(v) - F_{L_f{}^2}(v) + c_1 q_L \left(A^0_{L_f^1} - A^0_{L_f^2} \right)$$

Moreover, we know that $q_L \in [0, Q - q_{L_f^2}]$, hence

$$F_{L_f{}^{1*}}(v^*) - F_{L_f{}^{2*}}(v^*) \leq F_{L_f{}^1}(v) - F_{L_f{}^2}(v) + c_1 \left(Q - q_{L_f^2} \right) \min \left\{ 0, A^0_{L_f^1} - A^0_{L_f^2} \right\}$$

Now, conditions 2 and 4 imply that $\bar{c}_{L_f^{1*}} - \bar{c}_{L_f^{2*}} \leq 0$ □

Proof of Proposition 3: Consider two labels L_f^1 and L_f^2 that satisfy the conditions in Proposition 3. To show that for any label feasible extension of label L_f^2 is also a feasible extension of label L_f^1, consider $L \in E(L_f^2)$ and let $L_f^{1^*} = L_f^1 \oplus L$ and $L_f^{2^*} = L_f^2 \oplus L$. Condition 2 ensures that $p(L_f^{1^*})$ does not vehicle capacity. $p(L_f^{1^*})$ is elementary due to condition 3. Moreover, $p(L_f^{1^*})$ is not violating any time windows because condition 4 ensures that we can reach node $v(L_f^1)$ as early using path $p(L_f^{1^*})$ as using path $p(L_f^{2^*})$, given that we depart at the depot at time 0 and travel fast enough, or travel at speed v_{max} and depart at the depot early enough. Next, we show that $\bar{c}_{L_f^{1*}} \leq \bar{c}_{L_f^{2*}}$.

First, we consider the case when the conditions in 4.(a) are satisfied. Let (t^*, v^*) be the optimal departure time and traveling speed for path $p(L_f^{2^*})$. We have

$$\delta_{L_f^1}(\underline{t}^{L^1_f}(v^*), v^*) = \delta_{L_f^1}(0, v^*) \leq \delta_{L_f^2}(0, v^*) \leq \delta_{L_f^2}(t^*, v^*)$$

$$\begin{aligned}
\bar{c}_{L_f^{1*}} &\leq c_d\left(\delta_{L_f^{1*}}(\underline{t}^{L^{1^*}_f}(v^*), v^*) - \underline{t}^{L^{1^*}_f}(v^*)\right) + F_{L_f{}^{1*}}(v^*) - c_{L_f^1} - c_L \\
&\leq c_d\left(\delta_{L_f^{2*}}(t^*, v^*) - t^*\right) + F_{L_f^{2*}}(v_2) - c_{L_f^2} - c_L + F_{L_f^{1*}}(v^*) - F_{L_f^{1*}}(v^*) - c_{L_f^1} \\
&+ c_d\left(t^* - \underline{t}^{L^{1^*}_f}(v^*)\right) + c_{L_f^2} \\
&= \bar{c}_{L_f^{2*}} + F_{L_f^{1*}}(v^*) - F_{L_f^{2*}}(v^*) - c_{L_f^1} + c_{L_f^2} + c_d\left(t^* - \underline{t}^{L^{1^*}_f}(v^*)\right) && \text{(due to condition 4.(a).iii)} \\
&\leq \bar{c}_{L_f^{2*}} + c_d\left(t^* - \underline{t}^{L^{1^*}_f}(v^*)\right) - c_d\left(t_{max}^{L_f^2} - \underline{t}^{L^1_f}(v_{min})\right) \\
&\leq \bar{c}_{L_f^{2*}} + c_d\left(\underline{t}^{L^1_f}(v_{min}) - \underline{t}^{L^{1^*}_f}(v^*)\right) && \text{(because } t^* \leq t_{max}^{L_f^{2*}} \leq t_{max}^{L_f^2}\text{)} \\
&\leq \bar{c}_{L_f^{2*}} && \text{(because } \underline{t}^{L^{1^*}_f}(v^*) \geq \underline{t}^{L^1_f}(v^*) \geq \underline{t}^{L^1_f}(v_{min})\text{)}
\end{aligned}$$

Second, we consider the case when the conditions in 4.(b) are satisfied. We have

$$\delta_{L_f^1}(t^*, v_{max}) \leq \delta_{L_f^2}(t^*, v_{max}) \leq \delta_{L_f^2}(t^*, v^*)$$

and therefore

$$\begin{aligned}
\bar{c}_{L_f^{1*}} &\leq c_d\left(\delta_{L_f^{1*}}(t^*, v_{max}) - t^*\right) + F_{L_f^{1*}}(v_{max}) - c_{L_f^1} - c_L \\
&\leq c_d\left(\delta_{L_f^{2*}}(t^*, v^*) - t^*\right) + F_{L_f^{2*}}(v_2) + F_{L_f^{1*}}(v_{max}) - F_{L_f^{2*}}(v^*) - c_{L_f^1} + c_{L_f^2} - c_{L_f^2} - c_L \\
&= \bar{c}_{L_f^{2*}} + F_{L_f^{1*}}(v_{max}) + F_{L_f^{2*}}(v_{max}) - F_{L_f^{2*}}(v_{max}) - F_{L_f^{2*}}(v^*) + c_{L_f^2} - c_{L_f^1} \\
&\leq \bar{c}_{L_f^{2*}} + F_{L_f^{2*}}(v_{max}) - F_{L_f^{2*}}(v^*) - \phi_F^{L_f^2}\left(v_{min}^{L_f^2}\right). \\
&\leq \bar{c}_{L_f^{2*}}
\end{aligned}$$

Note that the last inequality used the fact that $F_{L_f^{2*}}(v_{max}) - F_{L_f^{2*}}(v^*) \leq \phi_F^{L_f^2}\left(v_{min}^{L_f^2}\right)$ □

References

Baldacci, R, A Mingozzi, R Roberti. 2011. New route relaxation and pricing strategies for the vehicle routing problem. *Operations research* **59**(5) 1269–1283.

Barth, M, K Boriboonsomsin. 2008. Real-world CO_2 impacts of traffic congestion. *Transportation Research Record: Journal of the Transportation Research Board* **2058**(1) 163–171.

Barth, M, T Younglove, G Scora. 2005. Development of a Heavy-Duty Diesel Modal Emissions and Fuel Consumption Model. Tech. rep., UC Berkeley: California Partners for Advanced Transit and Highways (PATH). Available at: http://www.path.berkeley.edu/PATH/Publications/PDF/PRR/2005/PRR-2005-01.pdf (1.4.2014).

Bektaş, T, G Laporte. 2011. The pollution-routing problem. *Transportation Research Part B: Methodological* **45**(8) 1232–1250.

Bettinelli, A, A Ceselli, G Righini. 2011. A branch-and-cut-and-price algorithm for the multi-depot heterogeneous vehicle routing problem with time windows. *Transportation Research Part C: Emerging Technologies* **19**(5) 723–740.

Chabrier, A. 2006. Vehicle routing problem with elementary shortest path based column generation. *Computers & Operations Research* **33**(10) 2972–2990.

Cook, W, JL Rich. 1999. A parallel cutting-plane algorithm for the vehicle routing problem with time windows. Tech. rep., Computational and Applied Mathematics Department, Rice University, Houston, The United States.

Dabia, S, S Ropke, T Van Woensel, T De Kok. 2013. Branch and Price for the Time-Dependent Vehicle Routing Problem with Time Windows. *Transportation Science* **47**(3) 380–396.

Demir, E, T Bektaş, G Laporte. 2011. A comparative analysis of several vehicle emission models for road freight transportation. *Transportation Research Part D: Transport and Environment* **6**(5) 347–357.

Demir, E, T Bektaş, G Laporte. 2012. An adaptive large neighborhood search heuristic for the Pollution-Routing Problem. *European Journal of Operational Research* **223**(2) 346–359.

Demir, E, T Bektaş, G Laporte. 2014a. A review of recent research on green road freight transportation. *European Journal of Operational Research* **237**(3) 775–793.

Demir, E, T Bektaş, G Laporte. 2014b. The bi-objective Pollution-Routing Problem. *European Journal of Operational Research* **232**(3) 464–478.

Desaulniers, G, F Lessard, A Hadjar. 2008. Tabu search, partial elementarity, and generalized k-path inequalities for the vehicle routing problem with time windows. *Transportation Science* **42**(3) 387–404.

Desrochers, M, J Desrosiers, M Solomon. 1992. A new optimization algorithm for the vehicle routing problem with time windows. *Operations research* **40**(2) 342–354.

Feillet, D, P Dejax, M Gendreau, C Gueguen. 2004. An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems. *Networks* **44**(3) 216–229.

Figliozzi, M. 2010. Vehicle routing problem for emissions minimization. *Transportation Research Record: Journal of the Transportation Research Board* **2197**(1) 1–7.

Franceschetti, A, D Honhon, T Van Woensel, T Bektaş, G Laporte. 2013. The time-dependent pollution routing problem. *Transportation Research Part B: Methodological* **56** 265–293.

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Hickman, J, D Hassel, R Joumard, Z Samaras, S Sorenson. 1999. MEET-Methodology for calculating transport emissions and energy consumption. European Commission, DG VII. Tech. rep. Available at: http://www.transport-research.info/Upload/Documents/200310/meet.pdf (1.6.2014).

Irnich, S. 2008. Resource extension functions: Properties, inversion, and generalization to segments. *OR Spectrum* **30**(1) 113–148.

Irnich, S, D Villeneuve. 2006. The shortest-path problem with resource constraints and k-cycle elimination for k >3. *INFORMS Journal on Computing* **18**(3) 391–406.

Jepsen, M, B Petersen, S Spoorendonk, D Pisinger. 2008. Subset-row inequalities applied to the vehicle-routing problem with time windows. *Operations Research* **56**(2) 497–511.

Kiefer, J. 1953. Sequential minimax search for a maximum. *In proceedings of the American Mathematical Society*, vol. 4. 502–506.

Kohl, N, J Desrosiers, OBG Madsen, MM Solomon, F Soumis. 1999. 2-path cuts for the vehicle routing problem with time windows. *Transportation Science* **33**(1) 101–116.

Kramer, R, A Subramanian, T Vidal, LDAF Cabral. 2014. A matheuristic approach for the Pollution-Routing Problem. Tech. rep., Cornell University, New York. Available at: http://arxiv.org/abs/1404.4895 (1.6.2014).

Lübbecke, ME, J Desrosiers. 2005. Selected topics in column generation. *Operations Research* **53**(6) 1007–1023.

Righini, G, M Salani. 2006. Symmetry helps: bounded bi-directional dynamic programming for the elementary shortest path problem with resource constraints. *Discrete Optimization* **3**(3) 255–273.

Righini, G, M Salani. 2008. New dynamic programming algorithms for the resource constrained elementary shortest path problem. *Networks* **51**(3) 155–170.

Scora, G, M Barth. 2006. Comprehensive modal emission model (CMEM), version 3.01: User's guide. Tech. rep. Available at: http://www.cert.ucr.edu/cmem/docs/CMEM_User_Guide_v3.01d.pdf (1.4.2014).

Solomon, MM. 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research* **35**(2) 254–265.

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