# Control and identification of bipedal humanoid robots : stability analysis and experiments 

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# Control and Identification of Bipedal Humanoid Robots 

Stability Analysis and Experiments

## disc

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# Control and Identification of Bipedal Humanoid Robots 

Stability Analysis and Experiments

## PROEFSCHRIFT

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## Summary

# Control and Identification of Bipedal Humanoid Robots 

Stability Analysis and Experiments

In the near future, humanoid robots will appear in society to assist people with a variety of tasks in industry, households, offices, hospitals, etc. A humanoid robot is a bipedal mechatronic system that resembles the appearance of a human to a high degree. This human-like appearance of humanoid robots is the key feature of these systems as it enables them to operate in environments which are shaped to human proportions, accomplish various tasks a human is capable of in their daily life and mingle among humans with limited social boundaries.
An obvious link between humans and humanoids is their ability to perform bipedal locomotion. Where this is a natural experience for humans, it is a difficult technical problem for humanoids. The level of robustness, versatility and energy efficiency of human mobility is remarkable and this level has not been achieved on humanoids despite four decades of scientific studies on bipedal walking. This indicates that the dynamics and control of bipedal locomotion is not yet sufficiently known. Therefore, the goal of this thesis is to increase the knowledge about modeling, identification, control and stability of bipedal systems to facilitate design and control of humanoid robots, prosthetic devices for disabled persons and human power augmenting exoskeletons.

The main challenge for humanoid robots is the necessity to remain balanced. Under normal operation and in the event of unforeseen disturbances the robot should not fall. It is difficult to remain balanced because a humanoid robot has relatively small feet through which it can generate limited forces on its relatively high positioned center of mass. This makes a humanoid robot a nonlinear mechanical system with hybrid, partly underactuated, phases and non-smooth ground contact characteristics. This thesis covers all of these topics and contributes a comparison between different bipedal models, a model parameter estimation framework, a dynamics analysis of bipedal locomotion, an asymptotically stable control law for walking, a gait parameter selection algorithm for walking speed and omnidirectional walking and experimental validation on a humanoid robot.

In this thesis, we focus on the modeling, analysis and control of bipeds with point as well as finite sized feet. We realize that humans and most humanoid robots have finite sized feet, but, by neglecting feet, we are forced to take underactuation into account, which is an important cause of imbalance and may occur in natural gaits. We intentionally model important dynamical aspects such as impact, underactuation and coupling between degrees of freedom of the multi-body dynamics, since we found during model validation experiments that these aspects are significantly important. This is also the reason why we do not focus on (linear) inverted pendulum models as these neglect important dynamics nor on the zero moment point as this concept only works when at least one foot of the robot is fully in contact with the ground and as such the robot is fully actuated.
We use these models to design asymptotically stable model based inverse dynamics controllers. These controllers asymptotically attract the full robot dynamics to a lower dimensional state space manifold as function of the unactuated degree of freedom of the system. We show that we can make this so called zero dynamics manifold asymptotically attractive. A thorough analysis of this zero dynamics shows that important gait parameters for balance of these underactuated bipeds are the foot placement location, torso lean angle and push off intensity. Moreover, once the robots state is on this manifold, we contribute a selection algorithm for these gait parameters in each step to achieve a desired walking speed and walking direction within a minimum number of steps, taking into account limits in the operation space of the humanoid.
Since these controllers are model based, we realized a model parameter estimation framework that is able to accurately identify the model parameters from experimental torque measurements using series elastic actuation. If torque measurements are not available, for example on position controlled humanoid robots, we show that we can estimate the model parameters of the center of mass model with solely joint angles and ground contact forces and of the full robot model with additional information about the joint velocities and accelerations.

Finally, we apply some of the aforementioned concepts on the adult-size humanoid robot TUlip. This robot has twelve degrees of freedom and finite sized feet. We contribute a model of TUlip in ROS Gazebo, estimate its model parameters and validate the model using experiments. TUlip is position controlled so we cannot apply all concepts, but we show how to let the real robot walk using traditional (linear) feedback control and feedforward dynamics compensation.

The main conclusion of this thesis is that advanced humanoid models are important to analyze the dynamics of bipedal locomotion and that extreme simplifications are not required to prove stability of model based controllers. These controllers require, however, accurate knowledge about the system, which can be acquired by careful design of identification experiments.

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## Chapter 1

## Introduction

### 1.1 Humanoid robots

Ever since the Czech Karel Čapek introduced the word robot in his play Rossum's Universal Robots people are imagining how the world would look like if robots were an element of their daily life. This can be concluded from the numerous, mostly science fiction, movies that are made, like Star Wars, A.I., RoboCop, the Terminator or the Transformers or from popular books from Isaac Asimov for example. Together with the increasing interest of people on the field of robotics, the development of robots started. A robot is usually seen as an electromechanical machine that can move around, operate a mechanical limb, sense and manipulate its environment, or exhibit intelligent behavior, especially behavior which mimics humans or other animals.
Originally, the behavior of robots was not of high intelligence, but nowadays robots have evolved to machines that are stronger than humans and can do tasks quicker and with higher accuracy. That's what makes the current field of robotics so incredibly fascinating and gives it its right of existence. Namely, robots can be used to assist or replace people in all sorts of work. They can take over monotonous, dangerous or unhealthy work from humans and assist where more hands, or more powerful hands are required.
Currently robots are used in various, mostly structured, environments. In factories there are many pick and place manipulators and welding robots. At industrial farms the cows are milked using milking robots. The first robots in households are appearing in the form of automated vacuum cleaners and lawn mowers and in hospitals surgical robots are used. Finally, in labs all over the world, an incredibly diverse amount of robots are being developed, which will find their way into society in the coming decades. In this robotic future, robots are intended to be used more often in industries, households, hospitals, offices, care facilities, disaster areas, etc. While current robots in society are mostly dedicated to one specific
task, it is foreseen that in the future, robots act as general purpose machines, that can perform different tasks, just like humans. As such, an interesting type of robot that perfectly features this development is the humanoid robot.
Humanoid robots are robots that kinematically resemble human beings as closely as possible and move in a human-like way. Typically, these robots have a torso with two legs and possibly a head and two arms. The most prominent feature of this type of robots is arguably their way of locomotion. Different ways of locomotion exist; robots have been build that drive, roll, swim, fly and walk respectively with wheels, tracks, fins, wings and legs. Interesting about the locomotion of humanoid robots is that they walk on two legs: they are bipedal. This type of locomotion gives certain advantages, but also introduces challenges in the areas of control and stability.
These advantages and challenges are addressed in this thesis, where modeling and identification of humanoid robots are studied and control and stability of bipedal locomotion are investigated.

### 1.2 Motivation

The motivation to study robots with legs is relatively straightforward, since wheeled or tracked robots cannot reach a large part of the earth's landmass. It is obvious to see that natural rough terrain is inaccessible to wheeled robots, but they neither can reach floors above ground level in buildings without an elevator. Legged robots are theoretically able to climb stairs, step over obstacles and use sparse footholds, which gives them an advantage over driving robots. In practice, also flying robots can reach all areas where a legged robot can come, but their energy cost of transport is in general orders of magnitude higher.
Commonly, bipedal humanoid robotics research is motivated by the fact that a robot with human-like proportions is arguably the most suitable choice to operate with humans and in environments specifically designed for humans. This is partly explained from a sociological and partly from a technical viewpoint. From a sociological aspect, it has been argued that cooperation and interaction of humans with machines provides more comfort for the human if the machine resembles and behaves like human beings. The technical aspect is quite obvious, humanoid robots with the same dimensions, kinematic structure and strength as humans can operate in environments with steps, stairs, doors, etc. where humans can easily operate. They are also able to use equipment, tools and devices such as pliers, drills and vacuum cleaners that are designed for human hands and feet. In contrast to dedicated robots, this makes them ideal general purpose robots.
Besides the sociological and technical advantages of humanoid robots, research in bipedal locomotion also increases insight in human walking. This insight is useful for the diagnosis and treatment of people that face problems with walking. The development of leg prostheses or rehabilitation devices benefits from the bipedal humanoid robotics research.

Finally, lower-body exoskeletons that are used by disabled as well as able-bodied people to augment their power requires understanding of human walking.

### 1.3 Problem statement

In the previous sections we have seen why robots are useful in general and why humanoid robots are perfectly suited as general purpose robots to assist people in their daily lives. Despite these strong motivations, there are only a few robots available on the market and operating in society. Let alone humanoid robots; currently there is no affordable humanoid robot available for the general public, except for toys.
The primary reason for the current absence of humanoid robots in society is still their limited capabilities. As stated before, an obvious link between humans and humanoids is their ability to perform bipedal locomotion. Where this is a natural experience for humans, it is a difficult technical problem for humanoids. The level of robustness, versatility and energy efficiency of human mobility is remarkable and this level has not been achieved on robots despite four decades of scientific studies on bipedal walking. This indicates that the dynamics and control of bipedal locomotion is not yet sufficiently known.
The main challenge for bipedal locomotion of humanoid robots is the necessity to remain balanced. Under normal operation and in the event of unforeseen disturbances the robot should not fall. It is difficult to remain balanced because a naturally dimensioned humanoid robot has relatively small feet through which it can generate limited, unilateral forces on its relatively high positioned center of mass. Due to these bounds, the feet of the humanoid robot easily tip or slip, even under mild disturbances. Tipping and slipping of the feet introduce additional degrees of freedom which are not directly controlled. This results in an underactuated system, i.e. a system with less actuators than degrees of freedom. When a humanoid robot tips, often the only solution to prevent a fall is to make a step. This step alters the contact situation of the biped with its environment, which changes the dynamics of the system, giving it a hybrid character. Moreover, steps often result in rigid collisions of the feet with the ground, which is a non-smooth phenomenon.
So, overall, a humanoid robot is a nonlinear mechanical system with hybrid, partly underactuated phases and non-smooth ground contact characteristics. Despite many years of research, there remain unsolved problems with the control and stability analysis of such systems. Although the separate aspects, such as hybrid phases, underactuation and nonsmooth dynamics have been studied individually and interesting results have been presented in each case, it is the combination of those in one system that deserves more attention. In the research field of humanoid robotics, often only one aspect is taken into account, but we believe that the linked occurrence of these aspects is the true reason that explains why the robustness, versatility and energy efficiency of state of the art humanoid robots is rather limited.

### 1.4 Goal and objectives

The advantages of humanoid robots motivate to research the ongoing challenges in humanoid robotics. As main challenge we pointed out the control and stability of humanoid robots that exhibit a combination of hybrid, underactuated and non-smooth dynamics. We believe that incorporation of these aspects in accurate mathematical models is required to analyze the stability of humanoid robots. Not only should the biped robot model contain all these important dynamical aspects, also reliable model parameters should be determined through identification experiments. The complete picture to improve the balance of humanoid robots and make them more robust, versatile and energy-efficient thus includes the topics modeling, identification, control and stability analysis.

Therefore, the goal of this thesis is the design of tools for modeling, identification, control and stability analysis of bipedal systems subject to a combination of hybrid, underactuated and non-smooth dynamics to facilitate design and control of humanoid robots, prosthetic devices for disabled persons and human power augmenting exoskeletons. Three objectives are formulated that represent these topics.
The first objective is accurate derivation and validation of humanoid robot models. In addition to existing work on bipedal modeling, the focus lies on the combination of the hybrid, underactuated and non-smooth dynamics of humanoid robots. The importance of these dynamical aspects is motivated by experiments, such that well considered modeling assumptions can be made, without compromising the model accuracy. The result is an experimentally motivated modeling framework that accurately describes the hybrid, underactuated and non-smooth dynamics of different types of bipeds with varying numbers of degrees of freedom and divergent contact situations.
The second objective is reliable model parameter identification for humanoid robots. The parameters involved in the derived models are reliably estimated using an identification framework suitable for humanoid robots. In addition to existing robotic identification literature, the focus lies on the additional sensors that are commonly available on humanoid robots, such as joint torque and ground contact force sensors. The result is a framework for model parameter estimation tailor-made for humanoid robots that is tested in experiments. The complete biped robot model including the estimated parameters is experimentally validated, which is rarely done in existing humanoid robotics literature.
The third objective is analysis of the dynamics of humanoid robots and controller design for stable bipedal locomotion. In addition to existing research on this topic, the focus lies on control and stability analysis of biped robots subject to a combination of hybrid, underactuated and non-smooth dynamics. These dynamical aspects are analyzed through the derived mathematical models to find structural properties that can be exploited to generate stable gaits. These gaits are controlled on the humanoid robot taking into account the hybrid, underactuated phases and discontinuities that may arise from impacts with the ground. Supplementary to existing control approaches for these types of bipeds, the aim is
to develop controllers for (three dimensional) humanoid robots with feet and double support phases in the gait. The performance of the controllers is benchmarked according to its ability to track desired gaits, which prescribe desired walking speeds and omnidirectional walking.

### 1.5 Contributions

As a result of the defined goals and objectives, this thesis contributes on the topics modeling, identification, stability and control of humanoid robots that exhibit a combination of hybrid, underactuated and non-smooth dynamics. It contributes a comparison between different bipedal models, model parameter estimation frameworks, analysis of the dynamics of the bipedal models, an asymptotically stable control law for walking, a gait parameter selection algorithm for walking speed and omnidirectional walking and experimental validation of a selection of these concepts on a humanoid robot.
The contribution regarding modeling is an experimental analysis of important dynamical aspects of humanoid robots, such as the difference between point and finite sized feet, the ground contact dynamics, the influence of non-massless limbs and coupling between different three dimensional directions in the biped. These important dynamical aspects are used to formulate model assumptions which facilitate the design of a modeling framework for humanoid robots that exhibit a combination of hybrid, underactuated and non-smooth dynamics. Different planar and three dimensional models are presented with varying complexity regarding the number of degrees of freedom and the type and number of possible contacts.
Regarding identification two different model parameter estimation procedures are contributed based on well-known regressor methods, but tailor made for humanoid robots. The identification procedures use sensors that are typically available on humanoid robots, such as joint torque or ground contact force sensors. The first approach assumes force controlled humanoid robots with joint torque sensing in the form of series elastic actuation. A parameter estimation algorithm is presented that can identify in a two-step procedure the model parameters of the elastic drive train and the rigid body model with only one set of experimental data. The second approach uses solely information from ground contact force sensors and joint encoders to estimate all model parameters of the limbs of the robot. This method does not require joint torque sensing or an actuator or drive train model. Both methods are regressor based, so as a final result, an algorithm is contributed that can transform any equation, which is linear in the model parameters into the regressor form.
On the field of control, inspired by existing work, a control law is presented for humanoid robots which exhibit combined hybrid, underactuated and non-smooth dynamics. The humanoid robots are allowed to have maximally one degree of underactuation. Moreover, a gait parameter selection algorithm is contributed that simultaneously regulates the foot
placement, torso lean angle and push off intensity to balance the humanoid robot and to follow a desired omnidirectional walking gait and desired walking speed. As shown in this thesis, this controller and gait parameter selection algorithm can be applied to planar as well as three dimensional robots with point or finite sized feet. The bipeds can exhibit single and double support phases and may be underactuated in certain phases of the gait. The contributed controller assumes that the joint torques of the humanoid robot can be accurately controlled. Since this is not possible on position controller humanoid robots without joint torque sensing, a different gait generation algorithm and controller is contributed for flat footed walking.
Finally, the control law and gait parameter selection algorithm are examined in numerical simulations. An analysis is contributed on the influence of gait parameters such as foot placement, torso lean angle and push off intensity on the walking speed of humanoid robots. Besides simulations, some concepts are implemented on a real humanoid robot. A model of this robot is derived and its model parameters are estimated through various experiments. The model is validated in experiments and the generated flat footed walking gait is shown to perform well on the robot.

### 1.6 Outline

The outline ${ }^{1}$ of this thesis is as follows. Chapter 2 contains an overview of literature available on the different topics that are treated in this thesis. Chapter 3 describes the hardware and software architecture of humanoid robot TUlip, which is used as test bed for selected topics of this thesis. Chapter 4 presents the experiments to determine important dynamical properties of humanoid robots and using these dynamical properties it presents modeling frameworks to model different types of humanoid robots. Chapter 5 introduces the two identification approaches for model parameter estimation. One approach is applicable to force controlled humanoid robots, the other to position controlled humanoid robots. Chapter 6 presents the asymptotically stable control law and gait parameter selection algorithm for the desired walking speed or omnidirectional walking. It also introduces a gait parameter selection algorithm and controller for flat footed walking of position controlled humanoid robots. In Chapter 7, the gait parameter selection algorithm is analyzed and the influence of the gait parameters on the walking speed is investigated. Chapter 8 presents the simulation model of TUlip, which is experimentally estimated and validated. Finally, the conclusions and recommendations are presented in Chapter 9.

[^0]
## Chapter 2

## Background

### 2.1 Introduction

In this chapter a literature overview of the modeling, identification, control and stability analysis of humanoid robots is given. Since this thesis studies bipedal locomotion of humanoid robots, the focus of this literature review is on robotic applications of two-legged walking or running. It does not treat human walking, which is mainly studied in the field of biomechanics, nor mono- or multi-legged ${ }^{1}$ walking.
Many bipedal humanoid robots are built in labs all over the world. Here, we introduce a few, which are, in the opinion of the author, the most advanced humanoid robots to date. Undoubtedly the most famous humanoid robot is Honda's Asimo as shown in Figure 2.ra. It is currently the most versatile humanoid robot build by an industrial company, able to perform walking, running, jumping, dancing and ball kicking. Also versatile are the robots in the HRP series from the National Institute of Advanced Industrial Science and Technology from which HRP-4C is shown in Figure 2.Ie. This robot is the most versatile humanoid build by an academic institution. Its versatility is comparable to that of Asimo. One of the first adult-sized humanoid robots capable of walking was built at the Waseda University. Its successor is Wabian 2, as shown in Figure 2.Ic. Another successful humanoid robot is Aldebaran's Nao, depicted in Figure 2.Ib. It is commercially available, which makes it the most popular humanoid robot, used in many research facilities. The most robust and powerful humanoid robot is Boston Dynamic's Petman as shown in Figure 2.Ig. It is also the only robot in this list which is hydraulically powered. Another interesting research robot is Rabbit from the Grenoble Institute of Technology, shown in Figure 2.rh, which is one of the first bipeds that used proven mathematical principles to walk with point feet.

[^1]

Figure 2.1: Advanced humanoid robots.

Finally, the humanoid robot with the most human-like way of walking is limit cycle walking robot Flame from the Delft University of Technology as shown in Figure 2.Id, which is also very energy efficient, but the most efficient robot is planar biped Ranger from Cornell University, pictured in Figure 2.If.
The principles behind these robots, and many more are reviewed in this chapter. In Section 2.2 commonly used terminology in the field of humanoid robots is explained. In Section 2.3, an overview of commonly used biped models is given. In Section 2.4, literature is presented about robotic parameter estimation and identification, in general and applied to humanoid robots. In Section 2.5 control and stability analysis approaches for humanoid robots are treated and finally in Section 2.6 concluding remarks are given.


Figure 2.2: Definition of the orthogonal body planes of humanoid robots.

### 2.2 Humanoid robot terminology

In this thesis a bipedal humanoid robot is defined as a mechatronic system with at least two kinematic structures representing its legs. These kinematic chains are connected in one point, which is called the hip. Optionally, another kinematic chain may be connected in the hip, representing the torso of the humanoid robot. The legs of the robot may contain knees and end in either point feet or finite sized feet. The foot that is in contact with the ground is called the stance foot; the foot that is progressing through the air is called the swing foot. The corresponding legs are respectively called the stance leg and swing leg.
Walking is defined as movement by putting forward each foot in turn, not having both feet of the ground at once. A certain manner of walking is indicated by a gait. Normally a gait consists of multiple phases, alternating between single support and double support. The single and double support phase indicate if the biped has respectively one or both feet in contact with the ground. Making contact with the ground happens through a collision of the foot with the walking surface. If during this collision an impulsive force is applied for a short period, it is called an impact. The convex hull of all contact points of the robot with the ground is called the support polygon.
A humanoid robot standing straight up can be divided into three orthogonal planes as shown in Figure 2.2. The plane that divides the left and right side of the robot is called the sagittal plane, the plane that divides the front and back side the coronal plane and the plane that divides the top and bottom side the transverse plane. Bipeds that live in the two dimensional world of the sagittal plane are called planar bipeds, in contrast to three
dimensional bipeds that live in three dimensions.
Finally, every humanoid robot has a certain number of degrees offreedom and a certain number of actuators. If the number of degrees of freedom is equal to the number of actuators, the biped is said to be fully actuated. If the number of degrees of freedom is larger than the number of actuators, the biped is underactuated. And if the number of degrees of freedom is smaller than the number of actuators, the biped is overactuated.

### 2.3 Modeling of humanoid robots

Modeling of robotic systems has been studied intensively [38, 99, 162, $163,168,158]$. Also humanoid robot models have been studied, although to a lesser degree [io, 83]. Interesting about humanoid robot models is that they exhibit a hybrid nature with ground contact. To progress, a humanoid robot must make steps, which indicates that portions of the system evolution are described by continuous dynamics, interrupted by discrete events when a step is completed and a new contact is made with the ground. These events can be as simple as a coordinate transformations, but may also incorporate ground impact dynamics resulting in a discontinuous change of the velocities. Humanoid robot models can be used on-line in control algorithms on the robot, for example to determine foot placement to prevent a fall. Or models can be used off-line to simulate the robot, and safely test control strategies before they are executed on the real robot. Several humanoid robot models with strongly varying properties and assumptions are available in literature.
The simplest possible model used to describe the dynamics of a planar biped is the Linear Inverted Pendulum Model (LIPM) [49, 89, 92, 94, I2I]. This model is used very often to control humanoid robots, see Figure 2.3a. The LIPM assumes a point foot and a single point mass that is kept at constant height by the extension of a telescoping leg. Many variations to this model have been proposed. In [IOI] it has been shown how the LIPM's stance leg can be extended by a foot to allow the use of ankle torques. In [I8] the model is additionally extended with a torso reaction mass to allow hip torques. Interaction between the swing foot and the ground at impact is, however, not considered. In [IO3] a human that is recovering balance is approximated by an inverted pendulum model with a rotational spring in the stance ankle and linear spring in the swing leg. A linear spring is also added in the Spring Loaded Inverted Pendulum (SLIP) model [47, I4I], which is often used to study running in the field of biomechanics [4, I47, I57]. The inverted pendulum model in [ $87, \mathrm{I}_{6} 9$ ] is kept at constant height only after ground impact, to approximate the corresponding energy loss. Finally, inverted pendulum models (IPMs) with constant leg length are used in [48, 91, 95, 96, 136, I88] and shown in Figure 2.3b.
The previous models assume that all the masses of the robot can be approximated by a single mass at the robot's center of mass (CoM) position. More complex, multi-body planar models are used as well in several publications [29, i86, 198]. These models consist of an


Figure 2.3: Models approximating a humanoid robot.
arbitrary number of masses and moments of inertia, as shown in Figure 2.3c. Although more difficult to analyze, these models arguably describe the dynamics of humanoid robots more accurately.
All the previously mentioned models describe the dynamics only in 2D. To describe the dynamics of the robot in 3D, often two separate 2D models are evaluated in the coronal and sagittal plane, neglecting coupling between these planes [92, ior]. In general, 3D full multi-body models [28, 79, 83, 88, ІІ8, I29] are more accurate and take these couplings into account, but might give less analytical insight at a higher computational cost.
Besides models for the multi-body dynamics of the humanoid robot, also ground contact models are important [84]. Much less research has been performed in this field. Most previously introduced models assume a rigid ground contact with sufficient friction to prevent slipping. Ground impact of the feet with the ground resulting in discontinuous velocities is mostly neglected. Two types of ground contact models can be found in literature, a compliant ground, modeled with a (nonlinear) spring-damper system [2I, II3, I39, I5O] and a rigid ground with impacts that discontinuously change the velocity [44, 83, 159].
The required model accuracy depends on the purpose of the model (simulation, control or analysis), but in literature, evaluation and experimental validation of humanoid robot models is lacking. The types of models and their underlying model assumptions vary significantly. Overall, from the discussed humanoid robot models, we can distinguish four major model assumptions that are commonly used in literature [io]: i) the dynamics of feet are often discarded and bipeds are modeled with point feet, 2) the ground contact dynamics is simplified and impacts that result in discontinuous velocities are not modeled, 3) the multi-body nature of bipeds is neglected and they are modeled as a single mass, 4) couplings between the coronal and sagittal planes are not taken into account, the biped model is composed of two perpendicular planar models.

### 2.4 Parameter estimation and identification

The humanoid robot models introduced in the previous section contain many model parameters. For an accurate description of the dynamics of a physical humanoid robot, to be used in simulation or control, these must be reliably identified. The presented models are all linear in the model parameters, which makes them suitable for regression based identification techniques. In this section, we first present literature about regression based parameter estimation on traditional robotic manipulators and, secondly, on humanoid robots. The traditional parameter estimation methods are subdivided into application to stiff robots and compliant robots.

### 2.4.1 Traditional robotic parameter estimation

The research field of regression based parametric robot identification was established long ago [5, 55, 56, 68, II4, 164, I72], but as new robotic systems are introduced, robotic identification techniques are evolving too [99, I75, I79, I9I]. Most of these methods derive a linear regression model from the equations of motion of the robotic system [97, 172]. To estimate the parameters using this model, information about the joint torques, velocities and accelerations is required, but these quantities are often not directly measurable. Estimation methods based on regression models from the energy and power equations do not rely on joint acceleration data [24,53,54,99]. Motor torques are often estimated from the motor current and converted to joint torques using a drive train model including friction [ 7, II, I72]. Joint velocities and accelerations are estimated from joint orientation measurements using central difference methods in combination with noise reduction filters [40, 184].
In contrast to identification of stiff robotic systems, model parameter estimation of flexible robots received little attention in the past [15I], but this increased in recent years when the advantages of compliant robots were discovered [2, 42, 86, I32, I33, 137, I49, 152]. One way to add compliance to a robotic system is through series elastic actuation as present on the first version of TUlip [72, 173, 197]. Nowadays, series elastic actuated systems are increasing in popularity, posing new challenges in modeling and system identification. Unfortunately, there is limited literature available on modeling and identification of robots with series elastic actuation. In [135] identification of a robotic system with elastic actuation is investigated, but the stiffness of the elastic drive train is estimated with static experiments only. The difference between unintentionally flexible and intentionally compliant robots is that, in series elastic actuated systems, the elastic element can be clearly distinguished, whereas in flexible systems the elasticity is commonly omnipresent due to the use of lightweight materials. Moreover, identification on flexible robots often requires measurements using expensive torque sensors, which is not necessary on series elastic actuated robots [197].
The presented methods all rely on the availability of a regression model. This regressor
model can be derived from any system equation that is linear in the model parameters [IIO]. Often, rewriting these system equations in the regressor form is not a trivial task, especially when the robot contains many degrees of freedom. Several methods have been developed to perform this derivation. In [i6, 51, 56, 98, 193] a symbolic method has been developed to determine the regressor form from the equations of motion of robotic systems with serial and closed loop kinematics. For the same equations of motions, a numerical method is presented in [52, 99]. In [III] symbolic and numerical methods are proposed for the energy formulation of robots. Finally, an algorithm that works on any system equation linear in the model parameters, is presented in [197], which relies on symbolic derivations performed in [97].
The final ingredient for reliable system identification is the design of so called persistently exciting experiments [8, 126]. These experiments guarantee proper excitation of all dynamics, which is required for reliable parameter estimation [99, 165, 172]. Different methods exist to find persistently exciting joint trajectories, most perform conditioning of the regressor form through optimization of the joint trajectories [24, 57, 172]. Parameterizing the joint trajectories as finite Fourier series or polynomials results in an efficient optimization problem [17, IO2].

### 2.4.2 Parameter estimation for humanoid robots

Estimation of the parameters of humanoid robot models has not been studied as intensively as traditional manipulators, despite the interesting additional sensing capabilities of many humanoid robots, such as joint torque sensing, ground contact force sensing or vision. In a few papers these sensors are used for identification. Dynamic identification of floating base systems, including humans and humanoid robots has been addressed in [II, I2, I3, I4]. It is shown that the full dynamics of a humanoid robot can be identified without joint torque data, but with contact force information. However, velocity and acceleration information about the joints and floating base are required and it appears difficult to find optimal persistently exciting motions for which the robot does not fall. In [II9] the same approach is used to show that if joint torque information is available, only partial contact force information is required. A completely different identification is performed in [135] where the modularity of their biped is used to separately estimate model parameters of different components, such as actuators, compliant drive trains and leg parts.
Besides identification of the full robot model, it is also interesting for humanoid robots to estimate the CoM model, because many control strategies focus on control of the CoM. Moreover, the equations that describe the CoM position are significantly simpler than the equations of motion of the full robot model. By definition, the CoM position does not rely on velocities or accelerations, which are often impossible to measure and estimates may be noisy or delayed. For the same reason, no information about joint torques, actuator dynamics nor friction characteristics is required to identify the CoM position. That is why early
work estimated the CoM position of humans by taking the double integral of the ground reaction forces [20, 108]. The problem with this method is that the integrals require unknown initial conditions. Another approach identifies a relation between the frequency response of the CoM and center of pressure (CoP), but this only works for periodic motions [26]. Finally, in [33, 34, 35, 36,58,59] the ground reaction forces are used in static experiments to estimate the CoM position of humans and humanoid robots. However, only a simple low dimensional planar model is used in multiple directions to estimate the parameters in 3D, neglecting coupling between these directions. On humans, a similar approach is used to determine their CoM [IOO, I 60 ]. In most of these studies the researchers are solely interested in the position of the CoM of the total robot, while also information about the masses and mass positions of the individual links can be retrieved from these experiments. Design of persistently exciting experiments is lacking in all of this literature.

### 2.5 Stability and balance of humanoid robots

Many strategies for the balance of humanoid robots have been proposed in recent years. Although these strategies show promising experimental results in labs, a general breakthrough for humanoid robots in society is still far away. The main problem that current humanoid robots face before they can be deployed in society is the lack of robustness in locomotion [I8]. Robustness is a measure for how well the biped performs against unforeseen disturbances such as pushes or uneven terrain. Often these disturbances need to be suppressed by an active controller. Control of humanoid robots can rigorously be divided into two groups. First we consider control of humanoid robots that remain fully actuated during their entire gait, thereafter we discuss control of humanoid robots which may have underactuated phases during the gait.

### 2.5.1 Fully actuated humanoid robots

For a humanoid robot to remain fully actuated it is necessary for its stance foot to remain fully in contact with the ground. In [180, I8I, 182] it is reasoned that there exist a point inside the support polygon where the net moment generated from the ground reaction forces is strictly perpendicular to the ground. This point represents the dynamic equivalence of the CoM and is named Zero Moment Point (ZMP). As long as it lies within the interior of the support polygon, the feet of the robot remain completely in contact with the ground. If it lies on an edge of the support polygon, the feet may start tipping. Since its derivation it has been used in a tremendous number of control approaches, because the design of walking trajectories for which the ZMP stays inside the support polygon guarantees that the humanoid remains fully actuated. This is convenient, because it simplifies control of humanoid robots to control of traditional robotic manipulators for which many control laws are available.

The amount of literature about ZMP based controllers and robots is immense. Many approaches use the natural evolution of the (linear) inverted pendulum, since for this motion the ZMP remains in the base of the pendulum [91, 92, 95, 96, I2I, 136]. Besides the inverted pendulum motions, also human like walking has been designed taking into account whole body trajectories using the ZMP [80, 88, IO9, 129, 192, 196]. Results of the ZMP based gait generation even extend to pushing objects and climbing stairs [70, 153]. Although this indicates that ZMP based approaches result in versatile motions, the trajectories are pre-computed and, hence, lack robustness. In [39, 65, 88, 89, 125, 128] this problem is examined and several adaptive gait generation schemes are presented.
Besides the ZMP, other ground reference points or stability indicators are introduced that determine if the humanoid robot remains fully actuated [I40]. Extensions to the ZMP are the Foot Rotation Indicator and Centroidal Moment Pivot [60, i69]. These points are also valid outside the support polygon and in that sense give information about the 'amount of instability' of the humanoid robot. Further, the balance of fully actuated humanoid robots can be accounted for using angular momentum control as in [48, 63, 76, 90, 154], computed torque as in [ 120,148 ] and ground contact force control as in [22, 46, 85, 118 ]. Finally, control of fully actuated bipeds has been performed by introduction of artificial constraints in [81, 82] and in [I43], where the control law is termed virtual model control.

### 2.5.2 Underactuated humanoid robots

Despite the abundant amount of literature about fully actuated humanoid robots, and in particular ZMP based controllers, the question remains what to do with large disturbances that inevitably let the humanoid robot topple, adding additional degrees of freedom and making the robot underactuated. The literature presented in the previous section does not answer this question, so robustness is only guaranteed for small disturbances for which full actuation can be retained. Besides this robustness argument, it has been argued through human walking studies that gaits with underactuated phases are more energy efficient [4I, 104].
A very elegant study of underactuated humanoid robots is the research on passive dynamic walking. A passive biped has no actuation at all, and hence, is inherently underactuated. It has been shown in [50, 6I, II5] that the passive compass biped exhibits stable limit cycles when walking down a gentle slope. This motivated researchers to build planar passive walkers with knees and a torso [73, I90] and three dimensional passive bipeds [3r] that can stably walk or even run [134]. In addition to fully passive walking down a gentle slope it has been studied how to add minimal actuation to be able to walk on flat ground [30, 72, 190]. Most of these robots use ad-hoc controllers and require manual tuning. A mathematically sound control law that is based on passive dynamic walking is called controlled symmetries [167], but this method can only be applied to fully actuated humanoid robots.
Another approach to increase the robustness of humanoid robots is the study of foot
placement. The importance of foot placement has already been established decades ago [73, I45, 147], but only recently these studies have been used to derive analytical modelbased foot placement controllers [146]. The main foot placement strategies are the Capture Point (CP) method [18, IOI, 142, I44, I69] and Foot Placement Estimator (FPE) [I88]. These methods are based on (linear) inverted pendulum models, but are extended to multi-body humanoid robot models in [170, 198, 20I, 202]. Finally, it has been noted that these stepping strategies accurately predict the foot placement of humans in [75, 77, IO3, II7, 176].
The last group of research on underactuated bipedal robots treats these systems in a hybrid framework. In [I86] tools are developed for the control and stability analysis of planar biped robots. An efficient method using virtual holonomic constraints is employed to find limit cycles in the biped's dynamics [27]. The stability of these limit cycles is analyzed through Poincaré return maps $[62,73]$, extended to incorporate impulsive effects induced by the ground impact dynamics [127]. The limit cycles are controlled on the humanoid robot using input-output linearizing controllers [66], extended to incorporate impulsive effects to generate the so called hybrid zero dynamics [187]. Initially, this work focused solely on planar biped walking, but in [28, 67, iI2, i66] the same approach has been extended to three dimensional underactuated humanoid robots and running. Instead of output regulated feedback control, the trajectories of such systems are also generated using optimization techniques and controlled using feedforward [I22, 156, I74].

### 2.6 Conclusion

In this chapter we presented a literature overview about humanoid robots regarding the different topics that are treated in this thesis. In the first parts we presented several of today's advanced humanoid robots and defined commonly used bipedal robotics terminology.
Subsequently, we introduced literature about modeling of humanoid robots. We showed various different models with diverse modeling assumptions. Very popular models, such as the (linear) inverted pendulum model, capture parts of the dynamics of humanoid robots, but may not be as accurate as full multi-body models. Due to the complexity of such models, often only planar models are investigated. We concluded that four main modeling assumptions are predominant and it is important to select the correct ones depending on the purpose of the model.
The second part of the literature overview discussed robotic system identification and robot model parameter estimation. We reviewed traditional techniques, developed for standard robotic manipulators, and can conclude that this field is relatively mature. This contrasts to identification of humanoid robots, for which the literature is quite scarce. Only a few methods were found that use typical sensory information on humanoid robots, such as joint torque sensors and ground contact force sensors.
Finally, control and stability of humanoid robots was addressed. An incredible amount
of literature is available that deals with control and stability analysis of fully actuated husmanoid robots. The literature is strongly clustered around the notion of ZMP. This is logical from a historical perspective, since on fully actuated humanoid robots, the same control approaches can be applied as on traditional robotic manipulators. Nevertheless, we believe that robustness of humanoid robots can only be improved if the problems around underactuation are solved, because large disturbances inevitably result in underactuated situations. Fortunately, research in this field has increased in recent years with promising results on planar as well as three dimensional point and finite sized footed robots.

## Chapter 3

## Humanoid robot TUlip

### 3.1 Introduction

In this chapter ${ }^{1}$ we introduce the experimental setup which is used to evaluate some concepts of this thesis. This setup is adult-size humanoid robot TUlip. The design of this robot started in 2006 as 3TU project under the umbrella of DutchRobotics, which is a collaboration of the Eindhoven University of Technology, the Delft University of Technology, the University of Twente and Philips [ 18,72 ]. In this collaboration, the three technical universities of the Netherlands and the Dutch industry have agreed to cooperate with the long term goal to develop a new generation of (humanoid) robots.
The first prototype of TUlip was completed in 2008 at the Delft University of Technology. In 2009, the Eindhoven University of Technology and the University of Twente received a copy of this prototype. The hardware of these first prototypes was exactly the same, but since TUlip's birth, each partner has adopted their own version of the robot. In this chapter we only focus on the hardware and software design of TUlip in Eindhoven. During the course of this research the robot has undergone two major hardware upgrades, so in this chapter three versions of humanoid robot TUlip from Eindhoven are described.
Besides research purposes, TUlip also participates in the annual RoboCup robot soccer tournaments. The robot takes part in the adult-size humanoid robot league, which is arguably the most complex league of the RoboCup, because it requires the difficult task of bipedal locomotion, combined with vision, localization, world modeling and strategy.
This chapter is structured as follows. In Section 3.2 the hardware of TUlip is discussed and in Section 3.3 its software architecture is presented. In Section 3.4 TUlip's performance at the past RoboCup tournaments is discussed.

[^2]
(a) Photo by David Joosten.

(b) Schematic drawing of kinematics.

Figure 3.1: First version of humanoid robot TUlip.

### 3.2 Hardware platform: TUlip

The hardware of TUlip in Eindhoven has had two major revisions, which results in three versions of the humanoid robot. All these versions are used as experimental setup for the different concepts of this thesis, so in the next sections, these versions are introduced chronologically.

### 3.2.1 Version 1

The first version of TUlip is an adult-size humanoid robot with seventeen degrees of freedom, as shown in Figure 3.I. Each leg has six degrees of freedom: three in each hip, one in each knee and two in each ankle. Consequently, TUlip is able to walk in every direction in a human like way. Each arm has one degree of freedom in the shoulder and the neck of the robot has three degrees of freedom. The robot is $124[\mathrm{~cm}]$ tall and weighs $19[\mathrm{~kg}]$.
The initial idea behind the design of TUlip is limit cycle walking [7I, 73, 74], which is the research on bipedal locomotion focused on design and experimental demonstrations of energy efficient and human-like walking gaits. This research is founded upon the theory of passive dynamic walking [II5, II6], which considers passive legged mechanisms capable of walking down a gentle slope without application of actuators or control. Since motions of such mechanisms are naturally stable under mild disturbances, no active control is needed. To be able to reject larger disturbances and to perform versatile, possibly aperiodic motions, actuators are implemented that can mimic the natural passive dynamics of the legs. To achieve this, actuators are required that can directly control the joint torque. In practice, this is realized on TUlip using custom designed series elastic actuation.


Figure 3.2: Drawing of series elastic actuation in humanoid robot TUlip [72, I73].

The series elastic actuation design is visualized in Figure 3.2. It uses a spring, placed between the load and the motor, i.e. in the steel cable connecting the motor with the joint. The rotational difference between the motor and the joint (measured with two optical incremental encoders) determines the spring expansion and thus the actuation torque. The joints actuated trough series elastic actuation are the pitch rotation of the knee, ankle and hip. The ankle roll joint is also equipped with springs, but it lacks an actuator, i.e. it is a fully passive joint. In addition to allowing accurate force control, the series elastic actuation also offers less unintentional damage to the environment in the case of collisions, the possibility to store energy, better shock tolerance and lower reflected inertia [I43, I78].
The actuators used in the actuated joints of the legs of the robot are 60 [W] Maxon DC motors of type RE30. Two actuators are placed in the upper leg to drive the knee and hip pitch rotations, one actuator is placed in the hip to drive the hip roll rotation and two actuators are placed in the torso to drive the hip yaw and ankle pitch rotations via a unidirectional Bowden cable and return spring. Transfer of torque in the drive trains is done via planetary gearboxes with reduction of ini:I. All the motors have optical incremental encoders of type Maxon HEDS 5540 with 2000 counts per revolution. The Maxon motors are powered by Elmo Whistle 5/60 and 20/ioo digital servo amplifiers that are PWM controlled by a PCio4 stack. In addition, there is a Mesa 4165 Anything-I/O PCB running the Mesa Hostmotı2 software on its on-board Xilinx Spartan-II 200k gate FPGA.
The hip pitch, knee pitch and ankle roll and pitch rotations of both legs have additional Scancon optical incremental encoders with 7500 pulses per revolution that measure the joint orientation. These are connected to a second 4 I 65 PCB running a modified version of the Hostmoti2 software on its FPGA. Each foot has four Tekscan Flexiforce pressure sensors to determine position of the center of pressure. The force sensors are interfaced
using a custom-designed ARM7 board, which linearizes the sensor signals and gives the pressure values to the central controller. Furthermore, the ARM7 board is equipped with a STMicroelectronics 3D accelerometer, which is used to precisely detect moments of impacts of the foot with the floor or with the ball. The feet electronics are interfaced to the main control system using a standard USB interface.
Both Mesa 4 I 65 PCB's and a 5VDC power supply are mounted on a PCio4-Plus stack of an EPIC format sized Diamond Poseidon single board computer with a iGHz Via Eden CPU. The Poseidon PCB contains 5I2MB SDRAM, a 4GB Flashdisk, digital and analog I/O's. All encoders are connected to a custom designed connection PCB, while the Whistle servo amps are mounted on 2 custom designed PCB's. The computer is powered by a Kokam 3-cell 6 Ah LiPo battery, the motors by a separate 8 cell 3.3 Ah LiPo battery. Uptime with this setup is about 30 minutes. A PCB with battery monitor IC's automatically switches off the power to prevent excessive discharge of the batteries.
Finally, TUlip is equipped with arms for aesthetic reasons and a head consisting of two miniature cameras as eyes. These cameras are of type Virtual Cogs VC2iCCi Camera Cog with a Virtual Cogs OV9653 optical sensor each. Both sensors are connected to a Virtual Cogs VCMX2ı2 single board CPU. An XSens Mti inertial measurement unit is mounted on the torso of the robot, which measures 3D orientation, acceleration, angular velocity and earth-magnetic field. The neck and arms are actuated by Robotis Dynamixel RX-28 servo motors. The body of TUlip and all its limbs are made of black anodized aluminum components, protected by body armor made of soft foam and a tough aramid composite material. This makes the robot robust to damage.

### 3.2.2 Version 2

The first version of TUlip suffered from a few problems in the legs. The main problem was the mechanical design of the series elastic actuation. To prevent loose cables, high pretension was required in the springs of the series elastic actuation. This resulted in high forces on the drive train, which induced high friction on the pulleys and motor and caused cables to easily break. Another problem was the placement of the ankle pitch actuator in the torso of the robot. The unidirectional Bowden cable that drove this joint suffered from high, nonlinear friction characteristics. The final problem was the lack of actuation in the ankle roll joint.
So, in 2010 it has been decided to perform a large modification on the TUlip robot. The series elastic actuation was removed from the robot. The drive train with cables and springs was replaced by thicker, stiff steel cables without springs. The Maxon motors in the knee and hip pitch joints of the robot were replaced by 90 [W] Maxon RE35 motors and the pulley radii were increased to allow for higher torques in these joints. Additionally, the placement of the ankle roll actuator was improved from its position in the torso to the lower leg. Finally, an actuator was placed in each foot of the robot to drive the ankle roll

(a) Photo by Ingrid Bussemakers.

(b) Schematic drawing of kinematics.

Figure 3.3: Second version of humanoid robot TUlip.
joint. This second version of TUlip is shown in Figure 3.3, the difference with Figure 3.1 can be seen in the lower leg and feet of the robot. The kinematic structure of the robot was not changed.
On top of the mechanical changes, also the cameras of the robot were changed to a Surveyor stereo vision system with on-board BlackFin processors to allow faster image processing.

### 3.2.3 Version 3

The second version of TUlip in Eindhoven was the first prototype of this robot which could walk autonomously, mainly because of the mechanical improvements as discussed in the previous section. The robot could walk forward, sideward and turn on a spot in a statically stable manner. To increase the speed of the robot, a large mechanical change was performed on its hip. In the kinematic design of the first and second version of TUlip the legs were placed relatively wide apart, which results in high torques required in the hip roll joint. Moreover, in this design, the hip did not constitute a ball joint, which seriously complicates the inverse kinematics of the legs.
Therefore, in 2012, the hip of TUlip was redesigned. In the new design, the legs are placed directly under the already existing hip yaw joint, so that a hip ball joint is formed and the legs are closer together. This new kinematic design is shown in Figure 3.4. Integrated in this new design are Scancon encoders on all hip joints, so that all joint orientations can be measured directly, instead of via the motor encoders ${ }^{2}$.

[^3]
(a) Photo by Bart van Overbeeke.

(b) Schematic drawing of kinematics.

Figure 3.4: Third version of humanoid robot TUlip.

In addition to the mechanical improvements, the computer of TUlip is replaced by a GHz dual core Diamond Neptune PCio4-Plus stack with I GB SDRAM and 8 GB FlashDisk storage to facilitate the increased computational requirements due to increased complexity of the walking algorithms.

### 3.3 Software architecture of TUlip

Figure 3.5 shows a simplified overview of the software components used during experiments on the humanoid robot TUlip. It aims at making the reader familiar with terminology and how the software components interact. TUlip has a computer inside its torso, which runs a controller program named TUlip Motion Controller (TUlipMC). Desired joint set-points can be specified directly, or computed from set-points in the task space using inverse kinematics. TUlipMC interpolates these set-points to generate joint reference trajectories. The desired joint angles are compared to the actual joint angles measured by the joint encoders, and PD-control plus feedforward dynamics compensation are used to compute the desired torques as is further discussed in Chapter 6. These are sent to TUlip's Hardware Abstraction Layer (TUlipHAL, not shown in Figure 3.5) which in turn communicates with the power amplifiers that control the current in the motors, and data acquisition boards to receive sensor inputs. Data used in TUlipMC is written to a shared memory using custom made program SHARED [23] such that it can be accessed from outside of TUlipMC. A logger then reads the data from SHARED and writes it to a log-file for generating plots off-line. Using a custom made program called Courier [23], the data can even directly be read at runtime from a remote computer to, for example, visualize the current


Figure 3.5: Architecture for experiments on the real robot.
and desired posture of the robot. Additionally, separate programs are running on TUlip's computer that read the data from the foot sensors and the inertial measurement unit and write it to $\log$ files for further analysis.

### 3.4 RoboCup

TUlip is developed primarily as a research object, but it also participates in the yearly RoboCup tournament [69, 194, 195, 199, 200]. Evolution took about four billion years to develop into an intelligent human creature, the goal of RoboCup is to achieve this in fifty years. Annually, humanoid robots participate in this soccer tournament with the goal to defeat the human world champion by the year 2050. Through this goal scientists are motivated to put much effort in the development of humanoid robots. Challenges are modeling, control and stability analysis of humanoid robots, which incorporate hybrid highly nonlinear dynamics, in all environments and under a diversity of tasks. Hence defeating the human world champion is not the only purpose of this tournament, it also leads to more breakthroughs in all kinds of scientific fields in the coming years. RoboCup is not only about soccer, there are also competitions in care and rescue robots, so already before fully autonomous humanoid robots will appear in society, parts of these robots and their functionalities will be integrated in care taking and rescue systems.
TUlip has participated in the RoboCup adult-size humanoid league since 2008. The main game in this competition is a penalty shoot-out [IO5]. Two robots of opposing teams are placed, respectively in the goal as defender and at the center of the field as offender. The
ball is placed behind the offending robot. It has to autonomously find the ball, walk to the ball and dribble it passed the mid-line before it may shoot at the goal. The defending goalkeeper robot has to stop the ball. Each team gets five trails and whichever team scores most, wins the match. In the first years, TUlip participated solely as goal keeper, in recent years it also participated as offender. For the last three years TUlip became fourth in the overall competition.

### 3.5 Conclusion

In this chapter we introduced the adult-size humanoid robot TUlip of the Eindhoven University of Technology, which is used as experimental setup for selected concepts in this thesis. The robot has an anthropomorphic design with two legs of each six degrees of freedom, two arms with one degree of freedom and a head with stereo-vision. TUlip can operate fully autonomously for approximately thirty minutes using its on-board computer and several battery packs. In Eindhoven, TUlip has undergone several hardware modifications to improve its mechanical design. The first version of TUlip was torque controlled using series elastic actuation. This was removed in later versions due to mechanical problems, such that the robot became stiff position controlled. TUlip's software architecture has a modular structure, which facilitates easy maintenance and connections with the actual robot as well as a simulated model. Finally, TUlip has participated in the adult-size humanoid league of the annual RoboCup tournaments since 2008 and became fourth in the last three years.

## Chapter 4

## Biped modeling

### 4.1 Introduction

In this chapter ${ }^{1}$ we introduce various humanoid robot models. As we have seen in Section 2.3 different humanoid robot models can be found in literature [47, 83, 94, ІІ8, I69, I86, 188]. These models are used for analysis purposes and in controller design. The types of models and their underlying model assumptions vary significantly. We distinguished four major model assumptions that are commonly used in literature [io]: i) the dynamics of feet are often discarded and bipeds are modeled with point feet [I2I], 2) the ground contact dynamics is simplified and impacts resulting in a sudden change of the velocities are not modeled [ 18 ], 3) the multi-body nature of bipeds is neglected and they are modeled as a single mass [92], 4) couplings between the coronal and sagittal planes are not taken into account, the biped model is composed of two perpendicular planar models [Ior]. Despite these simplifications, we did not find experimental evaluation of the models nor the consequences of these assumptions on the accuracy of the model with respect to reality. Therefore, in the first part of this chapter we perform an experimental study on TUlip to evaluate these model assumptions and investigate the influence of simplifications on the model accuracy.
Using the knowledge gained in this study, we introduce different biped model descriptions varying in complexity. The differences between the model descriptions lie in the types of phases that may occur in the models. A phase is in this context defined as a system dynamical regime with one set of contacts. As soon as the contact situation changes the model moves to a different phase. Contact situations can change if for example a foot hits or loses contact with the ground. The first distinction that we make between the model

[^4]descriptions is that phases can be ordered, in which the sequence of phases is fixed, or unordered, in which it is arbitrary. The second distinction is that in certain types of phases the model may be fully actuated ${ }^{2}$ whereas in others it is underactuated.

Combinations of these properties lead to four biped model descriptions: i) bipeds with unordered and underactuated phases, 2) bipeds with ordered and underactuated phases, 3) bipeds with ordered and fully actuated phases and 4) bipeds with unordered and fully actuated phases. The first description is the most general one, but also the most complex and most difficult to mathematically analyze. The other descriptions are derived from the first using assumptions on the order and actuation type of the phases, restricting their allowed dynamics evolution to simplify dynamical analysis. The different models are treated in descending order of complexity. In general we state that models with unordered phases are more complex than ordered ones and models with underactuated phases are more complex than models with solely fully actuated phases. The latter description is not treated in this thesis, because it is unrealistic for a fully actuated robot to exhibit unordered phases, as such robots can always be controlled to follow a fixed order of phases.
The models can be described by different coordinates. Mostly, we use a minimal set of relative coordinates with a fixed base in the stance foot [186], but sometimes it is more convenient to use non-minimal absolute coordinates and a free floating base [ıI8]. In the latter case, constraints on these coordinates are introduced to model the unilateral contact with the ground. The validity of these constraints must be checked constantly to determine phase changes of the model and to prevent inadmissible evolution of the dynamics. Important geometric quantities to check this validity are introduced.
The outline of this chapter is as follows. In Section 4.2 experiments are conducted on TUlip to verify commonly used modeling assumptions. In Section 4.3 biped models with unordered and possibly underactuated phases are introduced. In Section 4.4 we introduce models with ordered, but possible underactuated phases. In Section 4.5 models with ordered and solely fully actuated phases are derived. In Section 4.6 actuator and drive train models are presented. Finally, in Section 4.7, important geometric quantities for humanoid robots are introduced that hold for all models.

### 4.2 Important dynamics aspects of humanoid robots

As mentioned, we identified four main simplifications in biped models from literature: the robot is approximated with point feet and a single mass [92], ground impact is simplified [I8], and coupling between dynamics in the coronal and sagittal plane is discarded [Ior].

[^5]

Figure 4.1: The robot rotates around its ankle roll joints that imitate point feet (a,b,c). But the ankle joints never reach zero again, which indicates energy loss due to impact. In the experiment the robot also rotates around its ankle pitch joints because of the influence of the toes and heels ( $\mathrm{d}, \mathrm{e}$ ), resulting in an oscillation of the pitch orientation w.r.t. the vertical and perpendicular to the stepping direction (f).

The humanoid robot TUlip is used in two side-stepping experiments to investigate its real dynamics and evaluate these assumptions.

### 4.2.1 Experimental setup

The two experiments are on purpose kept as simple as possible to maximize insight on aspects that would otherwise be impossible to identify in a full 3D walking gait. The robot steps sideward instead of forward, because this makes the feet and CoM rotate in the same plane, better imitating the 2D models from Figure 2.3. Although the experiments are simple, they still include all important dynamic aspects that are present during a dynamic push recovery step and allow for in-depth analysis of the model assumptions from literature.
In the first experiment (snapshots above Figure 4.I), the robot initially stands on one straight leg in a similar configuration as the models in Figure 2.3. A point foot in the stepping direction may be imitated by a passive ankle roll joint. Ideally with a force controlled robot, a torque sensor allows regulating the ankle roll joint equal to zero torque,


Figure 4.2: Vertical forces on corners of the right (top) and left (bottom) foot are not equally distributed. Multiple impacts are visible during a single stance phase. At the end the sinusoidal shape indicates that pressure is also exchanged while both feet remain in contact with the ground.
creating a frictionless compliant joint. Instead, on our position controlled robot, we set the ankle roll controller gains equal to zero to create an unactuated joint. We investigate in Section 4.2.2 if the friction in the ankle roll joint resulting from the back-driven gears and motor is low enough to approximate a point foot. The other joints are all maintained at their initial position and are essentially 'locked', creating a rigid posture. The desired joint angle of right hip roll is kept equal to -0.3 [rad] and the left hip roll is kept equal to 0.3 [rad]. The desired joint angles of the other joints (including ankle pitch) are maintained at zero. The robot is then released without being pushed.
In the second experiment (snapshots above Figure 4.3), the left leg is rotated during the experiment to investigate the influence of the mass in the legs on the internal dynamics. The robot starts standing on its right leg, with an initial right hip roll angle of -0.4 [rad] and actuated right ankle roll joint at an angle of -0.08 [rad] tuned to make it stand stable without needing additional support. The left hip roll joint is then suddenly rotated from $0.0[\mathrm{rad}]$ to $0.3[\mathrm{rad}]$ in $0.5[\mathrm{~s}]$.

### 4.2.2 Evaluation of common model assumptions

The measurements of the joint encoders and the torso orientation from the IMU for the first experiment are also shown in graphs in Figure 4.I, and for the foot force sensors in Figure 4.2. The measurement data of the second experiment with moving leg is shown in Figure 4.3. We repeated the experiments three times, and no significant differences between the subsequent trials were observed. In this section the experimental data are analyzed and used to evaluate important model assumptions from literature, in Chapter 8


Figure 4.3: When the leg is quickly rotated, the action of the swing leg has an equal opposite reaction on the rest of the body. While the leg accelerates upward (a), the torso starts to rotate towards the swing leg around its right hip (b, d). The robot pushes the mass of the swing leg to the robot's left side, and as a reaction the rest of the robot's body is pushed to its right side. Therefore the robot rotates around its right ankle roll joint (c). For an instant the foot pressure shifts to the outer side of the foot (e-h).
these data are compared with simulation data.

## Point foot

When the robot is released, it starts to rotate around the $x$-axis towards the positive $y$ direction. If the right ankle roll joint would be locked or have significant friction, the robot's foot would rotate w.r.t. the ground, and the pressure under the robot's right foot would shift from the outer edge to the inner edge. However, in the experiment the pressure under the robot's right foot is almost constant in the first stance phase in the top of Figure 4.2. This
implies that the robot rotates around the right ankle roll joint while the foot remains flat on the ground. Hence by setting the ankle roll gains equal to zero, the joint friction and damping in the right ankle roll joint are low enough to approximate a compliant joint and successfully imitate a point foot in the stepping direction. We conclude that robots without stance ankle actuation and low friction can accurately be approximated by point footed models like in [29, 92, 169, 188, 198].

## Impact

The robot starts in a vertical orientation on its right leg, with right ankle roll angle equal to zero (Figure 4.ra). When the robot rotates to its left side, potential energy is converted into kinetic energy. If no energy would be lost at impact, the robot's motion would be symmetric around its double stance phase. All kinetic energy would be converted back into potential energy, and the robot would also reach an upright orientation on its left leg. But we do not see this in the experiment in the snapshots above Figure 4.I. After the left foot has adapted its orientation to the ground, the left ankle roll angle remains far from zero (Figure 4.Ib). Because we have earlier concluded that joint friction and damping are negligible, the only explanation for this behavior is that energy loss due to impact dynamics is significant, although being neglected in the popular LIPM [92, IOI]. This energy loss due to impact results in a damped oscillation of the roll orientation in Figure 4.Ic. The measured oscillation is not around zero, because the IMU cannot distinguish the accelerations due to gravity from the accelerations due to impact. This leads to an incorrect vertical z-axis, and a temporary negative offset to the roll angle that is defined relative to this axis.
Figure 4.2 also shows that instead of a single impact, multiple successive impacts occur during a single stance phase. This is caused by the coefficient of restitution of the rubber knobs at the corners of the feet, in combination with the considerably rigid posture of the robot. The sinusoidal shape at the end of Figure 4.2 shows that exchange of foot pressure between the left and right foot continues while they remain in contact with the ground. The foot sensors saturate at $\mathrm{I} 2 \mathrm{O}[\mathrm{N}]$.

## 3D dynamics

Due to calibration, the robot initially leans slightly forward when the right ankle pitch angle is zero. The ankle pitch gains are needed to balance the robot perpendicular to the stepping direction. However, the ankle pitch joints can still rotate because of backlash, causing the chattering behavior visible in Figure 4.id-e. Due to an asymmetric mass distribution, the robot initially rotates backward around the right ankle pitch joint (Figure 4.Id). When the left foot lands inclined on the ground at impact (Figure 4.re), the robot rotates further backward (Figure 4.If). The joint controller compensates for the resulting error and straightens the ankle. This makes the robot rotate forward, and the robot's inertia causes the right ankle pitch to overshoot zero, creating a new error. This effect results in a harmonic damped
pitch oscillation in Figure 4.If before the robot eventually comes to rest. An explanation is that the dynamics in the stepping direction is non-negligibly coupled with the dynamics perpendicular to the stepping direction. This coupling cannot be described by evaluating two decoupled 2D models as done in literature [92, IOI].

## Single mass

Most models approximate the multiple masses of a robot by a single mass at its CoM position [92, IOI, 169, I88]. When the masses are not separately taken into account, their internal dynamics is neglected. However, when for example the legs of the robot move in opposite direction, their motion may have a direct influence on the overall dynamics of the robot even when the CoM position and velocity do not change. In the following, we investigate the influence of the leg mass on the internal dynamics of the robot.
In the second experiment, the robot is in the same initial configuration as in the first experiment. It quickly rotates its left hip roll joint, as shown in Figure 4.3a. The acceleration and deceleration of the leg have an opposite effect on the rest of the robot. At the start of the leg rotation, the robot accelerates the leg by applying a torque in the left hip, so that the left leg starts rotating counterclockwise as can be seen in the snapshots above Figure 4.3. By Newton's third law, an equal opposite torque is applied to the torso. This makes the torso rotate in clockwise direction, towards the robot's left side around the right hip roll joint as can be seen in Figure 4.3 b and 4.3 d ). Due to this torso rotation, an equal opposite torque is applied on the robot's right leg, which results in a counterclockwise rotation around the robot's right ankle roll joint (Figure 4.3 c ). The ankle joint controller tries to keep the right ankle roll angle zero by applying a torque on the foot. Therefore the outer side of the robot's right foot is pushed harder against the ground. In the experiment, a decrease at the inner heel is visible in Figure 4.3 f . At the deceleration phase, the exact opposite effects happen. We can conclude from these observations that single mass models like [ 87,92 , IOI, 103, 169, I88] may not sufficiently describe the full dynamics of a humanoid robot.

### 4.3 Biped models with unordered and underactuated phases

Based on our observations in the previous section, we introduce here a general modeling framework for robotic systems with (unilateral) contacts. This framework includes all important aspects that we identified in Section 4.2. It is capable of modeling unilateral contacts, energy dissipating impacts that result in discontinuous velocities, ground friction, underactuation and any number of unordered phases. Both a planar, and three dimensional implementation of this modeling framework are presented. The general models
described in this section are solely used for simulation purposes and presented for completeness, simplified versions of these models are introduced in the next section and used for stability analysis and control.

### 4.3.1 Planar robot model with unilateral constraints

In this section a general planar biped model is introduced. First, the unconstrained dynamics of a robotic system is derived, after which constrains are added to model ground contact.

## Unconstrained dynamics

We model the dynamics of a planar bipedal robot with an arbitrary number of joints and either finite sized or point feet. A schematic kinematics representation of this general biped model is shown in Figure 4.4a. The biped consists of $N-1$ independently actuated revolute joints described by the relative coordinates ${ }^{3} \theta_{j}, j=2, \ldots, N$. It has $n$ point masses $m_{i}$ and moments of inertia $I_{i}, i=1, \ldots, n$. The biped is modeled as a floating base system, so the absolute coordinates $\varphi_{1}$ and $\varphi_{2}$ are used for the position and $\varphi_{3}$ for the orientation of the robot with respect to a fixed base coordinate frame attached to the world. Including these coordinates the system has $N_{d}=N+2$ degrees of freedom. The horizontal and vertical positions and orientation of each mass $m_{i}$ with respect to the base frame are represented by $x_{i}, z_{i}$ and $\psi_{i}$ respectively.
The equations of the unconstrained robot dynamics, i.e. the robot not being in contact with the ground, are derived as a chain of rigid bodies using the Lagrange-Euler method [i68]. They are presented here in standard form [106, 107]:

$$
\begin{equation*}
D_{e}(q) \ddot{q}_{e}+H_{e}\left(q_{e}, \dot{q}_{e}\right)=B_{e} \tau, \tag{4.I}
\end{equation*}
$$

where $q_{e}^{\top}=\left[\begin{array}{ll}\varphi_{e}^{\top} & \theta^{\top}\end{array}\right]=\left[\begin{array}{llllll}\varphi_{1} & \varphi_{2} & \varphi_{3} & \theta_{2} & \cdots & \theta_{N}\end{array}\right] \in \mathcal{Q}=\mathbb{T}^{N_{d}}$ is the state vector and $\mathbb{T}^{N_{d}}$ is the $N_{d}$-dimensional toroidal manifold, $D_{e} \in \mathbb{R}^{N_{d} \times N_{d}}$ is the symmetric positive definite inertia matrix, $H_{e}=C_{e}\left(q_{e}, \dot{q}_{e}\right) \dot{q}_{e}+G_{e}\left(q_{e}\right) \in \mathbb{R}^{N_{d}}$, with $C_{e} \dot{q} \in \mathbb{R}^{N_{d}}$ containing Coriolis and centrifugal terms and $G_{e} \in \mathbb{R}^{N_{d}}$ containing gravitational terms, and $B_{e} \in \mathbb{R}^{N_{d} \times N-1}$ maps the joint torques $\tau \in \mathbb{R}^{N-1}$ to generalized forces.

## Contact dynamics

When extremities of the biped makes contact with the ground it needs to satisfy Signorini's complementarity condition, which states that the contact points cannot penetrate the ground. We model the contacts as rigid and unilateral, there is no compliance between

[^6]
(a) General model of a planar biped with free floating base frame.

(b) General model of a planar biped with fixed base frame in the stance foot.

Figure 4.4: Schematics of a general planar biped model with $N$ links and $n$ point masses described using different sets of coordinates.
the robot and its environment, which can only push and not pull. This results in a complementarity condition between the normal contact distance $\kappa_{N i}$ and normal contact force $\lambda_{N i}$ at contact point $i \in \mathcal{I}$, where $\mathcal{I}$ is the set of $n_{c}$ contact points:

$$
\begin{equation*}
\kappa_{N i} \geq 0, \quad \lambda_{N i} \geq 0, \quad \kappa_{N i} \lambda_{N i}=0 \tag{4.2}
\end{equation*}
$$

When the robot makes contact with the ground, an impact occurs. Newton's impact law is used to model these impacts. This law relates the pre- and post-impact velocity of contact point $i \in \mathcal{I}$ with a restitution coefficient $0 \leq e_{N i} \leq 1$ :

$$
\begin{equation*}
\dot{\kappa}_{N i,+}=e_{N i} \dot{\kappa}_{N i,-}, \tag{4.3}
\end{equation*}
$$

where the - and + subscripts indicate the values of the variable just before and after the impact respectively, mathematically defined as [I86]:

$$
\begin{align*}
x_{-}(t) & =\lim _{\tau \uparrow t} x(\tau), \\
x_{+}(t) & =\lim _{\tau \downarrow t} x(\tau) . \tag{4.4}
\end{align*}
$$

When a robot foot is in contact with the ground, it might be subject to ground friction. This friction has a stick-slip characteristic that is modeled using Coulomb's friction law: at contact point $i \in \mathcal{I}$, the friction coefficient $\mu_{i}$ relates the friction force $\lambda_{T i}$ with the normal
contact force $\lambda_{N i}$ and the tangential contact velocity $\dot{\kappa}_{T i}$ :

$$
\lambda_{T i} \in-\mu_{i} \operatorname{Sign}\left(\dot{\kappa}_{T i}\right) \lambda_{N i}, \quad \operatorname{Sign}(x) \in \begin{cases}-1 & x<0  \tag{4.5}\\ {[-1,1]} & x=0 \\ 1 & x>0\end{cases}
$$

Note that the definition of the Sign function is set-valued as opposed to the normal use of $\operatorname{sign}(0)=0$. The resulting set-valued friction law (4.5) is an elegant way to describe the sticking phenomenon correctly [IO6]. A set-valued friction model allows for a nonzero friction force, even if the tangential contact velocity is zero. It is not possible to model this behavior without a set-valued friction law.
By combining the unconstrained equations of motion (4.I), Signorini's set valued contact law (4.2), Newton's impact law (4.3), and Coulomb's set valued friction law (4.5), we retrieve the complete model of the dynamics of the planar bipedal robot with ground contact and tangential friction:

$$
\begin{gather*}
D_{e}\left(q_{e}\right) \ddot{q}_{e}+H_{e}\left(q_{e}, \dot{q}_{e}\right)=B_{e} \tau+W_{N}^{\top}\left(q_{e}\right) \lambda_{N}+W_{T}^{\top}\left(q_{e}\right) \lambda_{T}, \\
\dot{\kappa}_{N} \geq 0, \quad \lambda_{N} \geq 0, \quad \dot{\kappa}_{N}^{\top} \lambda_{N}=0,  \tag{4.6}\\
\lambda_{T} \in-\mu \operatorname{diag}\left\{\operatorname{Sign}\left(\dot{\kappa}_{T}\right)\right\} \lambda_{N}, \\
\dot{\kappa}_{N,+}=e_{N} \dot{\kappa}_{N,-}, \quad \dot{\kappa}_{T,+}=e_{T} \dot{\kappa}_{T,-},
\end{gather*}
$$

where

$$
W_{N}^{\top}=\frac{\partial \kappa_{N}}{\partial q_{e}} \quad \text { and } \quad W_{T}^{\top}=\frac{\partial \kappa_{T}}{\partial q_{e}}
$$

are the Jacobians that map the vertical normal forces and horizontal friction forces to generalized forces, $\lambda_{N}=\operatorname{col}\left\{\lambda_{N i}\right\} \in \mathbb{R}^{n_{c}}$ and $\lambda_{T}=\operatorname{col}\left\{\lambda_{T i}\right\} \in \mathbb{R}^{n_{c}}$ are the contact forces implemented as Lagrange multipliers, $\kappa_{N}=\operatorname{col}\left\{\kappa_{N i}\right\} \in \mathbb{R}^{n_{c}}$ and $\kappa_{T}=\operatorname{col}\left\{\kappa_{T i}\right\} \in \mathbb{R}^{n_{c}}$ are the normal and tangential contact positions, $\mu=\operatorname{diag}\left\{\mu_{i}\right\} \in \mathbb{R}^{n_{c} \times n_{c}}, e_{N}=\operatorname{diag}\left\{e_{N i}\right\} \in$ $\mathbb{R}^{n_{c} \times n_{c}}$ and $e_{T}=\operatorname{diag}\left\{e_{T i}\right\} \in \mathbb{R}^{n_{c} \times n_{c}}$. Notice that the contact equations are only valid for $i \in \mathcal{I}$. Here, the contact equations are derived on the velocity level, in order to facilitate numerical integration, which is explained in the next section.

## Numerical implementation

Computing solutions for the set of equations (4.6) can be cumbersome. Fortunately, advanced numerical solvers have been developed for numerical integration of these types of problems, from which the time stepping method of Moreau [IO6, IO7, I24] is a particularly interesting one. This method uses a time-discretization of generalized positions $q_{e}$ and velocities $\dot{q}_{e}$. Forces acting on a system are taken into account integrally over every time step. This means that the time-stepping method does not make a distinction between impulsive and finite forces. The equations of motion are rewritten into measure differential equations [106, 107], which are not on the level of forces, but on the level of momenta. In this
way only increments in position and velocity are computed. The acceleration is not used, as it might become infinite due to the impacts. The time-stepping method of Moreau is basically a midpoint differential algebraic equation-integrator. If events occur due to impact or release during a time step, it is not necessary to switch to another mode in the system model, because all forces are taken into account through their momenta in an integral.
For the sake of numerical integration, the full set of equations of motion (4.6) needs to be written in equations rather than complementarity conditions. Using convex analysis and taking into account the notion of proximal point [106, IO7], we can write the contact conditions as (implicit) equations. Furthermore, the equations of motion should be written as measure differential equations as stated before:

$$
\begin{align*}
D_{e} d \dot{q}_{e}+H_{e} d t & =B_{e} \tau d t+W_{N} d \Lambda_{N}+W_{T} d \Lambda_{T}, \\
d \Lambda_{N} & =\operatorname{prox}_{C_{N}}\left(d \Lambda_{N}-r \xi_{N}\right),  \tag{4.7}\\
d \Lambda_{T} & =\operatorname{prox}_{C_{T}}\left(d \Lambda_{T}-r \xi_{T}\right),
\end{align*}
$$

with

$$
\begin{gathered}
C_{N}=\mathbb{R}^{+}, C_{T}=\left\{d \Lambda_{T} \mid-\mu d \Lambda_{N} \leq d \Lambda_{T} \leq \mu d \Lambda_{N}\right\} \\
\xi_{N}=\dot{\kappa}_{N,+}+e_{N} \dot{\kappa}_{N,-}, \quad \xi_{T}=\dot{\kappa}_{T,+}+e_{T} \dot{\kappa}_{T,-}, \\
\operatorname{prox}_{C}(z)=\underset{x \in C}{\arg \min }\|z-x\| .
\end{gathered}
$$

Here, $r>0$ is a tuning variable for the numerical iteration, while $d u, d t, d \Lambda_{N}=\lambda_{N} d t+$ $P_{N} d \eta$ and $d \Lambda_{T}=\lambda_{T} d t+P_{T} d \eta$ are the differential measures with $P_{N} d \eta$ and $P_{T} d \eta$ representing the so-called atomic parts, i.e. the impulsive part of the impact and friction forces with $d \eta=\delta\left(t_{i}\right) d t$, where $\delta\left(t_{i}\right)$ is the Dirac delta function at $t_{i}$. Notice that (4.7) is now written as measure differential equations with implicit contact equations. This system can be discretized and implemented in a numerical solver [IO6].

### 4.3.2 Three dimensional robot model with unilateral constraints

In the previous section, the biped model is described by a minimum set of relative coordinates that describe the orientation of a link with respect to the previous one. It may be beneficial to describe the humanoid robot with a set of non-minimal absolute coordinates, since this may result in a constant inertia matrix, which is not state nor time dependent [I77]. This is an advantage for numerical implementation of the forward dynamics, since the constant inertia matrix has to be inverted only once, instead of every time step when it is state dependent. Especially large systems with many coordinates, such as three dimensional humanoid robots, benefit from this, since numerically inverting large matrices is time consuming. Therefore, in this section we present a modeling framework for three dimensional humanoid robots described by absolute coordinates. First, the dynamics of the robot without ground contact is explained, after which constraints are added and the numerical implementation of the resulting system is derived.

## Contact-less dynamics

The dynamics of a three dimensional biped robot is modeled according to Newton's second law. The biped consists of $N_{d}$ independently actuated revolute joints and it has $n$ links with point mass $m_{i}$ and moments of inertia $I_{i}, i=1, \ldots, n$. The biped is modeled using absolute coordinates representing the position and orientation of each rigid body mass with coordinate $q_{i}^{\top}=\left[\begin{array}{llllll}x_{i} & y_{i} & z_{i} & \psi_{x i} & \psi_{y i} & \psi_{z i}\end{array}\right] \in \mathbb{R}^{3} \times \mathbb{T}^{3}$. The orientation is parameterized by Euler angles. The linear and angular velocity ${ }^{4}$ of each mass is represented by $v_{i}^{\top}=$ $\left[\begin{array}{cccccc}\dot{x}_{i} & \dot{y}_{i} & \dot{z}_{i} & \omega_{x i} & \omega_{y i} & \omega_{z i}\end{array}\right] \in \mathbb{R}^{6}$. Now, using Newton's second law, the dynamics of the complete system can be written as:

$$
\begin{equation*}
D \dot{v}=F_{e x t}+F_{c o n}, \tag{4.8}
\end{equation*}
$$

where $D=\operatorname{diag}\left\{D_{1}, D_{2}, \ldots, D_{n}\right\}$, with

$$
D_{i}=\left[\begin{array}{cc}
m_{i} \mathbb{I} & 0 \\
0 & I_{i}
\end{array}\right],
$$

where $m_{i}$ is the body's mass, $I_{i}$ is the moment of inertia tensor, $\mathbb{I}$ is the identity matrix, $v^{\top}=\left[\begin{array}{lll}v_{1}^{\top} & \cdots & v_{n}^{\top}\end{array}\right] \in \mathbb{R}^{6 n}, F_{\text {ext }}$ are externally applied forces such as gravity and actuator torques and $F_{\text {con }}$ are constrained forces that keep together the bodies that are connected through joints. These constraint forces are derived from equality constraints, $h_{c}(q)=0$, that represent the relative positions between bodies in directions where they cannot move due to the joints and $q^{\top}=\left[\begin{array}{lll}q_{1}^{\top} & \cdots & q_{n}^{\top}\end{array}\right] \in \mathbb{R}^{3 n} \times \mathbb{T}^{3 n}$. On velocity level, this can be written as:

$$
\begin{equation*}
J_{c}(q) v=0, \tag{4.9}
\end{equation*}
$$

where $J_{c}$ is the geometric Jacobian of the constrained directions [I68]. Similarly as in the previous section, these constraints are enforced through Lagrange multipliers $\lambda_{c}$ :

$$
\begin{equation*}
F_{c o n}=J_{c}^{\top}(q) \lambda_{c} . \tag{4.io}
\end{equation*}
$$

## Contact dynamics

Contact of parts of the robot with its environment are modeled again using the complementarity condition:

$$
\begin{equation*}
\kappa_{N i} \geq 0, \quad \lambda_{N i} \geq 0, \quad \kappa_{N i} \lambda_{N i}=0 \tag{4.II}
\end{equation*}
$$

where $\kappa_{N i}$ are the normal contact distances between bodies, $\lambda_{N i}$ are the normal contact forces at contact point $i \in \mathcal{I}$, where $\mathcal{I}$ is the set of $n_{c}$ contact points. Similarly as in the

[^7]previous section, a Coulomb friction model is implemented in contact point $i \in \mathcal{I}$ with friction coefficient $\mu_{i}$. The tangential friction force $\lambda_{T i}$ must remain in the friction cone:
\[

$$
\begin{equation*}
-\mu_{i} \lambda_{N i} \leq \lambda_{T i} \leq \mu_{i} \lambda_{N i} \tag{4.12}
\end{equation*}
$$

\]

The contact is sticking when $\lambda_{T i}$ is in the interior of the cone and sliding when it is on the edge. All these contact forces are externally applied, so they are collected in $F_{\text {ext }}$ :

$$
\begin{equation*}
F_{e x t}=W_{N}^{\top}(q) \lambda_{N}+W_{T}^{\top}(q) \lambda_{T}+f_{e x t}, \tag{4.I3}
\end{equation*}
$$

where $f_{e x t}$ contains gravitational forces and actuator torques and $W_{N}$ and $W_{T}$ again map the normal and friction forces of the contact points to the world coordinate frame. Now, all constraint forces are collected in $\lambda^{\top}=\left[\begin{array}{ccc}\lambda_{c}^{\top} & \lambda_{N}^{\top} & \lambda_{T}^{\top}\end{array}\right]$ and all constraint Jacobians in $J^{\top}(q)=\left[\begin{array}{lll}J_{c}^{\top} & W_{N}^{\top} & W_{T}^{\top}\end{array}\right]$. The total system is then given by:

$$
\begin{gather*}
D \dot{v}=J^{\top}(q) \lambda+f_{e x t}, \\
\lambda_{\min } \leq \lambda \leq \lambda_{\max }, \tag{4.14}
\end{gather*}
$$

where $\lambda_{\min }$ and $\lambda_{\max }$ are the minimal and maximal constraint forces according to the contact and friction models.

## Numerical implementation

A numerical implementation of this model is available in the open source software package Open Dynamics Engine (ODE). This package implements a discretized version of system (4.14), which in a rewritten form is given as:

$$
\begin{gather*}
J(q) D^{-1} J^{\top}(q) \lambda=-J(q)\left(\frac{v_{k}}{\Delta t}-D^{-1} f_{e x t}\right),  \tag{4.15}\\
\lambda_{\min } \leq \lambda \leq \lambda_{\max }
\end{gather*}
$$

where $v_{k}=v\left(t_{k}\right)$ is the velocity at time $t_{k}$ and $\Delta t$ is a fixed time step. In this formulation we also assumed that at time step $k+1$ the constraints are still valid which implies $J(q) v_{k+1}=$ 0 . This implies hard contacts and fully inelastic impacts.
Solving this discretized system can become problematic as the matrix $J D^{-1} J^{\top}$ may become singular. A numerical solution is implemented in the ODE, instead of an implementation with rigid contacts and fully inelastic impacts as in:

$$
\begin{equation*}
J(q) v=0 \tag{4.I6}
\end{equation*}
$$

the constraint equation is relaxed using the constraint force mixing method [43]:

$$
\begin{equation*}
J(q) v=K_{e} p_{e}-K_{c} \lambda, \tag{4•I7}
\end{equation*}
$$

where $K_{e}$ and $K_{c}$ are square diagonal matrices of stiffness and damping parameters and $p_{e}$ is the penetration depth of the contacts. This solution effectively makes the contacts
compliant and creates inelastic impacts. In addition, constraint force mixing stabilizes the solution against numerical drift [43]. The discretized system with constraint force mixing becomes:

$$
\begin{align*}
\left(J(q) D^{-1} J^{\top}(q)+\frac{1}{\Delta t} K_{c}\right) \lambda & =\frac{1}{\Delta t} K_{e} p_{e}-J(q)\left(\frac{v_{k}}{\Delta t}-D^{-1} f_{e x t}\right),  \tag{4.18}\\
\lambda_{\min } & \leq \lambda \leq \lambda_{\max } .
\end{align*}
$$

This is solved in the ODE using projected Gauss-Seidel with successive over-relaxation [43]. Moreover, the ODE approximates the friction cone of (4.I2) by a friction pyramid.

### 4.4 Biped models with ordered and underactuated phases

The models presented in the previous sections are rich and they may, given reliable model parameters, accurately describe reality. They are very useful for simulation, but less suitable for analysis of the bipedal dynamics, controller design or stability proofs due to their complexity. Therefore, in this section we present simplified biped models that comprise some interesting properties. These models still incorporate all important dynamical aspects as determined in Section 4.2 and they may possess underactuation, but the order of their phases is fixed, i.e. different contact situations happen subsequently and always in the same order. First a general planar biped model with point feet is introduced, after which a general planar and three dimensional biped model with finite sized feet are presented.

### 4.4.1 Planar biped with point feet

The general planar biped model of Section 4.3.I is represented using coordinates containing a floating base. This is required to model ground friction, but if we assume that the stick phenomenon of the ground friction is large enough to prevent the foot from sliding, the same model can be represented with a fixed base coordinate frame attached to the stance foot, as is pictured in Figure 4.4 b . The absolute orientation of the biped with respect to the fixed coordinate frame is again described by $\varphi$, which is simply an angle in this case. The actuated joints in the system are again presented by $\theta$.
We assume that the impact, which occurs when the swing leg end comes in contact with the ground, is instantaneous and fully inelastic. This means that the contact forces are impulsive and that, during the impact, there might be a discontinuity in the velocities. Furthermore, at the moment of impact, the configuration, although continuous, might not be differentiable. It is assumed that impulsive forces arise at the contact point between the leg and the ground, while no shocks occur at the actuators. We assume symmetric bipeds, so at the moment of impact, the swing and stance leg swap their role, so the stance foot lifts from the ground and the swing foot stays on ground without slipping or rebounding. This means that this model consists of only one phase.

Given these assumptions, the biped model (4.6) can be rewritten as a so called impulsive system [186, pp. 45-79]:

$$
\Sigma= \begin{cases}D(q) \ddot{q}+H(q, \dot{q})=B \tau, & x_{-} \notin \mathcal{S}  \tag{4.19a}\\ \dot{q}_{+}=\Delta\left(q_{-}\right) \dot{q}_{-}, & x_{-} \in \mathcal{S}\end{cases}
$$

where $x^{\top}=\left[\begin{array}{ll}q^{\top} & \dot{q}^{\top}\end{array}\right] \in \mathcal{X}=T \mathcal{Q}=\mathbb{T}^{N} \times \mathbb{R}^{N}, q^{\top}=\left[\begin{array}{ll}\varphi & \theta^{\top}\end{array}\right]=\left[\begin{array}{llll}\varphi & \theta_{2} & \cdots & \theta_{N}\end{array}\right] \in$ $\mathcal{Q}=\mathbb{T}^{N}$ is the state vector and $\mathbb{T}^{N}$ is the $N$-dimensional toroidal manifold, $D \in \mathbb{R}^{N \times N}$ is the symmetric positive definite inertia matrix, $H=C(q, \dot{q}) \dot{q}+G(q) \in \mathbb{R}^{N}$, with $C \dot{q} \in \mathbb{R}^{N}$ containing Coriolis and centrifugal terms and $G \in \mathbb{R}^{N}$ containing gravitational terms, $B \in \mathbb{R}^{N \times N-1}$ is the input matrix and $\tau \in \mathbb{R}^{N \times N-1}$ is the input vector. The impact map $\Delta: \mathcal{S} \rightarrow \mathcal{X}$ gives the state after impact $x_{+}(t)$ when the state just before impact $x_{-}(t)$ enters the set $\mathcal{S}=\left\{q \in \mathcal{Q} \mid \kappa_{N}(q)=0\right\}$, with $\kappa_{N}: \mathcal{Q} \rightarrow \mathbb{R}$ the differentiable level set representing the contact surface.
The equations of motion (4.19a) are the classical Euler-Lagrange equations [168], whereas the impact map (4.19b) is derived as in [186, pp. 55-57], based on conservation of momentum [83] on the floating base model (4-I):

$$
\begin{equation*}
D_{e}\left(q_{e-}\right) \dot{q}_{e+}-D_{e}\left(q_{e-}\right) \dot{q}_{e-}=J_{\kappa}^{\top}\left(q_{e-}\right) F_{r} \tag{4.20}
\end{equation*}
$$

with $J_{\kappa}=\frac{\partial \kappa\left(q_{e}\right)}{\partial q_{e}}$ and $\kappa^{\top}=\left[\begin{array}{ll}\kappa_{T} & \kappa_{N}\end{array}\right]$ is the position of the swing foot, and $F_{r} \in \mathbb{R}^{2}$ are the magnitudes of the impulsive reaction forces at the moment of strike of the swing foot with the ground. In the right-hand side of (4.20), the principle of virtual work is used to map the ground reaction forces to joint torques [168]. The momentum equation (4.20) represents $N+2$ equations with $N+4$ unknowns: $\dot{q}_{e+}$ and $F_{r}$. Two additional constraints come from the no slip and no rebound assumption because this implies that the velocity of the swing foot tip must vanish during the impact:

$$
\begin{equation*}
J_{\kappa} \dot{q}_{e+}=0 \tag{4.2I}
\end{equation*}
$$

Combining (4.20) with (4.2I) yields:

$$
\left[\begin{array}{cc}
D_{e} & J_{\kappa}^{\top}  \tag{4.22}\\
J_{\kappa} & 0
\end{array}\right]\left[\begin{array}{c}
\dot{q}_{e+} \\
F_{r}
\end{array}\right]=\left[\begin{array}{c}
D_{e} \dot{q}_{e-} \\
0
\end{array}\right]
$$

for which $\dot{q}_{e+}$ can easily be computed in the form (4.19b). This completes the general planar biped model with point feet.
In general, numerical implementation of this model is straightforward using an event based ordinary differential equation solver. The continuous differential equations (4.i9a) are numerically integrated until the state $x$ hits the surface $\mathcal{S}$. Then, the numerical integration is aborted and the impact equation (4.19b) is applied. The result is used as initial condition for the next numerical integration of the continuous part and the sequence repeats. It is important to note that during the simulation it must be checked that no contact assumptions are violated, such as the ground pulling on the robot or unnaturally high friction forces required to keep the stance foot in place.


Figure 4.5: Walking phases for the seven link biped model.

### 4.4.2 Planar biped with finite sized feet

Under the same assumptions as in the previous section, planar bipeds with finite sized feet can also be modeled as a chain of rigid bodies containing $N$ degrees of freedom, as schematically shown in Figure 4.4b. In this case, the chain is connected to the ground in the stance toe, where the base frame is placed. Again, one unactuated degree of freedom $\varphi$ describes the orientation of the stance foot with respect to this fixed base and $\theta$ describes the $N-1$ actuated joints.
When walking, the planar biped with feet exhibits, in contrast to the point footed model, different phases depending on its contact situation with the ground. We distinguish four subsequent phases that result in natural human-like walking as shown in Figure 4.5:
i. Fully actuated single support phase. This phase starts when the swing toe releases from the ground and ends when the stance heel releases from the ground. During the entire phase, the stance foot remains flat on the ground, which guarantees that the biped remains fully actuated. It has $N-1$ degrees of freedom and $N-1$ actuators. For clarity, this phase is subdivided into two sub-phases:
(a) Swing leg retraction phase. This phase starts when the swing toe releases from the ground and ends when the stance leg is perpendicular to the ground (i.e. mid-stance). During this phase, the swing leg is retracted to prevent scuffing.
(b) Swing leg extension phase. This phase starts at mid-stance and ends when the stance heel releases from the ground. In this phase the swing leg is fully extended to prepare for touch down.
2. Underactuated single support phase. This phase starts when the stance heel releases from the ground and ends when the swing heel impacts the ground. During this phase the stance heel remains off the ground such that the system is underactuated since it has $N$ degrees of freedom, but only $N-1$ actuators.

Table 4.1: Mathematical models for the different phases of a human-like walking gait

| $i$ | $\ddot{q}_{i}$ | $D_{i}$ | $H_{i}$ | $B_{i}$ | $\Delta_{i}$ | $W_{i}^{\top}$ | $\lambda_{i}$ | $w_{i}$ | $\Gamma_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\ddot{\theta}$ | $D_{22}$ | $C_{22} \theta+G_{2}$ | $B_{22}$ | $I_{N}$ | 0 | 0 | 0 | $\lambda_{N 3}$ |
| 2 | $\ddot{q}$ | $D$ | $C \dot{q}+G$ | $B$ | $\Delta_{2}$ | 0 | 0 | 0 | $\kappa_{N 3}$ |
| 3 | $\ddot{q}$ | $D$ | $C \dot{q}+G$ | $B$ | $\Delta_{3}$ | $\frac{\partial \kappa_{3}}{\partial q}$ | $Y_{3}\left(W_{3}^{\top} D_{3}^{-1}\left(H_{3}-B_{3} \tau\right)-w_{3}\right)$ | $\frac{\partial W_{3} \dot{q}_{3}}{\partial q_{3}}$ | $\kappa_{N 4}$ |
| 4 | $\ddot{\theta}$ | $D_{22}$ | $C_{22} \dot{\theta}+G_{2}$ | $B_{22}$ | $I_{N}$ | $\frac{\partial \kappa_{4}}{\partial \theta}$ | $Y_{4}\left(W_{4}^{\top} D_{4}^{-1}\left(H_{4}-B_{4} \tau\right)-w_{4}\right)$ | $\frac{\partial W_{4} \dot{\theta}}{\partial \theta}$ | $\lambda_{N 4}$ |

3. Touch down double support phase. This phase starts when the swing heel impacts the ground and ends when the swing toe impacts the ground. The swing heel remains in contact with the ground during the entire phase. During this phase, the system is overactuated since it has $N$ degrees of freedom, from which two are constrained, and $N-1$ actuators. At the end of this phase a coordinate transformation is performed to move the base frame to the other toe, such that the stance leg becomes swing leg and vice versa.
4. Push off double support phase. This phase starts when the stance toe impacts the ground and ends when the swing toe releases from the ground. The stance foot remains flat on the ground, so in this phase, the system is overactuated, because it has $N-1$ degrees of freedom, from which two are constrained, and $N-1$ actuators.

The general $N$-link model can be mathematically represented by:

$$
\Sigma_{i}= \begin{cases}D_{i}(q) \ddot{q}_{i}+H_{i}(q, \dot{q})=B_{i} \tau+W_{i}(q) \lambda_{i}, & x_{-} \notin \mathcal{S}_{i}  \tag{4.23a}\\ \dot{q}_{+}=\Delta_{i}\left(q_{-}\right) \dot{q}_{-}, & x_{-} \in \mathcal{S}_{i} \\ W_{i}^{\top}(q) \dot{q}=0, & \forall x,\end{cases}
$$

for $i=1,2,3,4$. All expressions are given in Table 4.I with $Y_{i}=\left(W_{i}^{\top} D_{i}^{-1} W_{i}\right)^{-1}$ and $D, C$ and $G$ as given in the previous section, decomposed in:

$$
D=\left[\begin{array}{ll}
D_{11} & D_{12} \\
D_{21} & D_{22}
\end{array}\right], \quad C=\left[\begin{array}{ll}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{array}\right], \quad G=\left[\begin{array}{l}
G_{1} \\
G_{2}
\end{array}\right], \quad \text { and } \quad B=\left[\begin{array}{l}
B_{11} \\
B_{22}
\end{array}\right],
$$

where the sub-matrices are $D_{11}, C_{11}, G_{1} \in \mathbb{R}, D_{12}^{\top}=D_{21}, C_{12}^{\top}, C_{21}, G_{2}, B_{11}^{\top} \in \mathbb{R}^{N-1}$ and $D_{22}, C_{22}, B_{22} \in \mathbb{R}^{N-1 \times N-1}$.
The impact maps $\Delta_{i}: \mathcal{S}_{i} \rightarrow \mathcal{X}$ give the state after impact $x_{+}$when the state just before impact $x_{-}$enters the set $\mathcal{S}_{i}=\left\{x \in \mathcal{X} \mid \Gamma_{i}(x)=0\right\}$, with $\Gamma_{i}: \mathcal{X} \rightarrow \mathbb{R}$ the level sets as described in Table 4.I. These functions represent in phase 2 and 3 the contact surface and in phase I and 4 the normal force in the contact point. If any of these vanish, the contact situation of the model changes and it switches to the subsequent phase. The impact maps are derived similarly as in Section 4.4.I, except that in phase 4, $J_{\kappa}$ and $F_{r}$ should also incorporate the impulsive reaction force acting on the swing heel when the swing toe hits the ground.

Finally, in phase 3 and 4, constraints are acting on the swing heel and toe. The constraints are defined on velocity level where $W_{3}, W_{4} \in \mathbb{R}^{N \times 2}$ are the Jacobians of the swing heel position $\kappa_{3}^{\top}=\left[\begin{array}{ll}\kappa_{T 3} & \kappa_{N 3}\end{array}\right]$ and swing toe position $\kappa_{4}^{\top}=\left[\begin{array}{ll}\kappa_{T 4} & \kappa_{N 4}\end{array}\right]$ respectively. These constraints are implemented using Lagrange multipliers $\lambda_{3}^{\top}=\left[\begin{array}{ll}\lambda_{T 3} & \lambda_{N 3}\end{array}\right]$ and $\lambda_{4}^{\top}=\left[\begin{array}{ll}\lambda_{T 4} & \lambda_{N 4}\end{array}\right]$, which can be computed from the time derivative of $(4.23 \mathrm{c})$ as derived in Table 4.I.
The numerical implementation of this model is the same as the previous one. It starts with phase I , the continuous dynamics (4.23a) is numerically integrated until the state $x$ hits the surface $\mathcal{S}_{1}$. Then the corresponding impact (4.23b) is computed and the result is used as initial condition for the numerical integration of the continuous dynamics of phase 2 . This is repeated until the end of phase 4 , after which the simulation switches to phase I and starts from the beginning. The existence and uniqueness of the forward dynamics of this system is guaranteed even under the constraints, there exist solely one $\ddot{q}$ for any input $\tau$ [I27]. The opposite is in general not true, there exist multiple inputs $\tau$ to achieve the same motion $\ddot{q}$.

### 4.4.3 Three dimensional biped with finite sized feet

The planar model derived in the previous section is quiet easily extended to three dimensions if we assume that it has maximally one degree of underactuation. This can be motivated by the fact that robots with feet tend to tip over an edge of the foot in contrast to a corner. Where a corner would imply at least two degrees of underactuation, an edge implies only one degree of underactuation. This means that the number and order of phases in a gait remain the same as in the planar case, see Figure 4.5. Under this assumption and the same ground contact assumptions of no slip and rebound, it suffices to describe the orientation of the three dimensional biped with respect to the ground with one coordinate $\varphi \in \mathbb{T}$. The $N-1$ other degrees of freedom are the actuated joints of the biped, described by $\theta \in \mathbb{T}^{N-1}$. In contrast to the planar case, these coordinates also represent rotations in the roll and yaw directions of the biped.
Using this set of coordinates, the dynamics of the biped can be described by the same model as in the planar case (4.23). The only difference lies in the constraints that need to be applied in phase 3 and 4 . To constrain the swing heel, respectively the swing toe to the ground, five constraints are required in each phase. For the swing heel and toe, these constraints must fix the translation and orientation in respectively three and two directions, such that the swing foot is allowed to rotate around the heel in phase three and toe in phase 4. The constraints are again implemented using the Lagrange multipliers $\lambda_{3}, \lambda_{4} \in \mathbb{R}^{5}$. The corresponding matrices $W_{3}, W_{4} \in \mathbb{R}^{N \times 5}$ are the Jacobians of the positions of the heel and toe corners of the swing foot respectively.

### 4.5 Biped models with ordered and fully actuated phases

The dynamic models presented in Sections 4.3 and 4.4 incorporate all important aspects for modeling humanoid robots as determined in Section 4.2. In this section we present an even simpler model that is often used in the humanoid robotics literature, but which does not incorporate an impact model which results in discontinuous velocities. This simplification is motivated by the apparent dynamical behavior of fully actuated humanoid robots with finite sized feet that move relatively slowly, and prevent hard impacts with the ground. These robots normally walk in a way where they keep their feet flat on the ground, unlike human-like walking. Although it has been stated that this way of walking is not robust, nor energy efficient [I8], it is fairly straightforward to apply classic robotic control to these systems, as they can simply be seen as fully actuated robotic manipulators with the stance foot as base and the swing foot as end effector. We present two types of this model, one with floating and one with fixed coordinates. Both models can be applied to planar and three dimensional bipeds.

### 4.5.1 Impact-less fully actuated model with floating base

The same assumptions as in Section 4.4 are adopted, with additional simplification that any impact dynamics is neglected. We assume that the stance foot remains always flat on the ground ${ }^{5}$ and that the ground friction is high enough to prevent the foot from sliding. Under these assumptions, we model a biped with $N$ joints, all actuated, using a set of floating coordinates. We denote the relative joint angles with $\theta \in \mathbb{T}^{N}$, and the position and orientation of the biped with respect to a fixed base coordinate frame with either $\varphi \in \mathbb{R}^{2} \times \mathbb{T}$ for the planar case, or $\varphi \in \mathbb{R}^{3} \times \mathbb{T}^{3}$ for the three dimensional case. The total number of coordinates is thus given by $N_{d}=N+3$ or $N_{d}=N+6$ for the planar or three dimensional case respectively. Using this coordinate set, the biped model can be represented by:

$$
\begin{align*}
D_{e}\left(q_{e}\right) \ddot{q}_{e}+H_{e}\left(q_{e}, \dot{q}_{e}\right) & =B_{e} \tau+W_{e}\left(q_{e}\right) \lambda_{e}, \\
W_{e}\left(q_{e}\right) \dot{q}_{e} & =0 \tag{4.24}
\end{align*}
$$

where $q_{e}^{\top}=\left[\begin{array}{ll}\varphi_{e}^{\top} & \theta^{\top}\end{array}\right] \in \mathcal{Q}=\mathbb{T}^{N_{d}}$ is the state vector, $D_{e} \in \mathbb{R}^{N_{d} \times N_{d}}$ is the symmetric positive definite inertia matrix, $H_{e}=C_{e}\left(q_{e}, \dot{q}_{e}\right) \dot{q}_{e}+G_{e}\left(q_{e}\right) \in \mathbb{R}^{N_{d}}$, with $C_{e} \dot{q} \in \mathbb{R}^{N_{d}}$ containing Coriolis and centrifugal terms and $G_{e} \in \mathbb{R}^{N_{d}}$ containing gravitational terms, and $B_{e} \in \mathbb{R}^{N_{d} \times N}$ maps the joint torques $\tau \in \mathbb{R}^{N}$ to generalized forces. Constraints on the system are required to prevent the feet of the biped to penetrate the ground. The number of constraints depends on the contact situation, this can either be single support with left or right foot, or double support. The constraints are enforced on the dynamics using the

[^8]Lagrange multipliers $\lambda_{e}=\left(W_{e}^{\top} D_{e}^{-1} W_{e}\right)^{-1}\left(W_{e}^{\top} D_{e}^{-1}\left(H_{e}-B_{e} \tau\right)-w_{e}\right)$, which can be subdivided into $\lambda_{E}^{\top}=\left[\begin{array}{ll}\lambda_{N}^{\top} & \lambda_{T}^{\top}\end{array}\right]$, representing the normal and tangential contact forces, with $w_{e}=\frac{\partial W_{e} \dot{q}_{e}}{\partial q_{e}}$.
Implementation of this model consist of numerically integrating the differential equation (4.24). The contact situation may vary, but the simulation does not have to be aborted, since there are no impacts. Similar as in the previous section the existence and uniqueness of solutions is only guaranteed for the forward dynamics.

### 4.5.2 Impact-less fully actuated model with fixed base

The same model without a double support phase can be presented with a set of relative coordinates and a fixed base, since the stance foot is assumed to stay flat on the ground. Using these coordinates, the biped model essentially reduces to a simple robotic manipulator with the stance foot as base and the swing foot as end effector. The dynamic equations of motion can easily be derived using the Euler-Lagrange method [i68]:

$$
\begin{equation*}
D(q) \ddot{q}+H(q, \dot{q})=B \tau \tag{4.25}
\end{equation*}
$$

where $q=\theta \in \mathbb{T}^{N}$ is the state vector, $D \in \mathbb{R}^{N \times N}$ is the symmetric positive definite inertia matrix, $H=C(q, \dot{q}) \dot{q}+G(q)$ with $C \dot{q} \in \mathbb{R}^{N}$ containing Coriolis and centrifugal terms and $G \in \mathbb{R}^{N}$ containing gravitational terms, and $B \in \mathbb{R}^{N \times N}$ maps the joint torques $\tau \in \mathbb{R}^{N}$ to generalized forces. In case that each joint has its own actuator, $B$ is simply the identity matrix of appropriate size.
As this model does not have a double support phase, numerical implementation consists of integrating (4.25) until the stance leg swaps. Then a coordinate transformation needs to be applied on the final state to find the initial condition for the dynamics defined for the new stance leg, after which the numerical integration can continue.

### 4.6 Actuator and drive train model

So far, we have introduced three different types of humanoid robot models. These models describe the multi-body dynamics of a biped under influence of external forces and moments, such as the gravitational force, ground contact forces and joint torques. While the gravitational and ground contact forces have been addressed, expressions for the joint torques have not been given. Therefore, in this section, we introduce actuator and drive train models that relate the joint torques to the system inputs. Two models are presented, the first model contains joint friction and flexibilities in the drive train to model the actuator and drive train of the first version of TUlip. For simplicity, the second model is an ideal actuator model which is used in Chapter 6 to develop controllers.

(a) Stiff drive train

(b) Series elastic actuated drive train

Figure 4.6: Schematic drawing of two different drive train concepts. The motor is indicated by $M$, the gearbox by $N$ and the load by $L$.

### 4.6.1 Model of a robotic system with series elastic actuation

In this section, we consider a general model that describes the dynamics of the actuators that commonly drive robotic systems and a drive train with friction and flexibilities. We model these flexibilities as linear elastic elements in series with the drive train as schematically shown in Figure 4.6, together with a schematic drawing of a stiff drive train. This type of actuation is called series elastic actuation and provides compliance in the drive train and allows for joint force control in robots. The model applies to the series elastic actuation of version I of humanoid robot TUlip [72, I73].
The model in Figure 4.6 represents an actuated link of a robotic system and consists of an actuator $M$, a load $L$, a gearbox $N$, and two pulleys with radii $r_{M}$ and $r_{L}$ interconnected by the elastic drive train with stiffness $k$. The torque delivered by the motor and torques acting after the gearbox and at the load are denoted by $\tau_{M}, \tau_{N}$ and $\tau_{L}$ respectively. The corresponding angular displacements are denoted by $\theta_{M}, \theta_{N}$ and $\theta_{L}$ respectively. In the rest of this section, equations of motion of the actuator with gearbox, the drive train, and the load are derived.

## Actuator and gearbox dynamics

The actuator comprises a geared electric DC motor. The dynamics of this motor and gearbox combination is subject to viscous and Coulomb friction. The mechanical equation of motion of the motor in joint $i$ of a robotic system can be represented by:

$$
\begin{equation*}
I_{M, i} \ddot{\theta}_{M, i}+F_{v, i} \dot{\theta}_{M, i}+F_{c, i} \operatorname{sign}\left(\dot{\theta}_{M, i}\right)+\frac{\tau_{N, i}}{N_{i}}=\tau_{M, i} \tag{4.26}
\end{equation*}
$$

where $I_{M, i}$ is the motor shaft inertia and $F_{v, i}$ and $F_{c, i}$ are the viscous and Coulomb friction coefficients respectively. A gearbox is used to increase the output torque from the motor by a factor $N_{i}$, equal to the reduction ratio. We do not take into account efficiency of the gearbox, because it is accounted for in the drive train friction. The angular velocity and
acceleration at the output of the gearbox are reduced with respect to the motor side with the same factor $N_{i}$ :

$$
\begin{align*}
& \dot{\theta}_{M, i}=N_{i} \dot{\theta}_{N, i}, \\
& \ddot{\theta}_{M, i}=N_{i} \ddot{\theta}_{N, i} . \tag{4.27}
\end{align*}
$$

## Drive train dynamics

The drive train converts the angular motion of the actuator into a linear displacement using a pulley with radius $r_{M, i}$. After the pulley, cables are connected in series with a spring with stiffness $k_{i}$. The stiffness of the cables can be neglected, if it is orders of magnitude higher than the spring stiffness. At the load side, the linear spring motion is converted to an angular one using a pulley with radius $r_{L, i}$. The difference in angular displacements at the actuator and the load sides determines the force exerted by the spring:

$$
\begin{equation*}
F_{s, i}=k_{i}\left(r_{M, i} \theta_{N, i}-r_{L, i} \theta_{L, i}\right) . \tag{4.28}
\end{equation*}
$$

This force acts on both sides of the drive train; it induces a load torque for the actuator and actuates the load:

$$
\begin{align*}
\tau_{N, i} & =k_{i} r_{M, i}\left(r_{M, i} \theta_{N, i}-r_{L, i} \theta_{L, i}\right),  \tag{4.29a}\\
\tau_{L, i} & =k_{i} r_{L, i}\left(r_{M, i} \theta_{N, i}-r_{L, i} \theta_{L, i}\right) . \tag{4.29b}
\end{align*}
$$

## Load dynamics

The different types of models for the load side dynamics are presented in Sections 4.3, 4.4 and 4.5. Here, we only show how to couple the drive train dynamics to the simplest version of these models, derivation for the other models is trivial. Consider model (4.25), which can be written per joint $i$ as:

$$
\begin{equation*}
\sum_{j=1}^{N} D_{i j} \ddot{\theta}_{L, j}+\sum_{j=1}^{N} C_{i j} \dot{\theta}_{L, j}+G_{i}=\tau_{L, i}, \tag{4.30}
\end{equation*}
$$

where $\theta_{L, i}$ and $\tau_{L, i}$ are the $i^{\text {th }}$ elements of the vector $\theta$ and $\tau$ respectively.

## Complete model

The complete model combines dynamics of the actuator, drive train and load. We present the model in two parts, one for the actuator side and another for the load side. Combining (4.26), (4.27) and (4.29a), the part of the model at the actuator side can be written as:

$$
\begin{equation*}
I_{M, i} \ddot{\theta}_{M, i}+F_{v, i} \dot{\theta}_{M, i}+F_{c, i} \operatorname{sign}\left(\dot{\theta}_{M, i}\right)+\frac{k_{i} r_{M, i}}{N_{i}}\left(\frac{r_{M, i}}{N_{i}} \theta_{M, i}-r_{L, i} \theta_{L, i}\right)=\tau_{M, i} . \tag{4.3I}
\end{equation*}
$$

In this model, we cannot directly apply the torque at the load side $\tau_{L, i}$, but only indirectly via the spring. Therefore, the input to the part of the model at the load side has to be calculated from (4.29b). By combining (4.29b) with (4.27) and (4.30), the part of the model at the load side is derived:

$$
\begin{equation*}
k_{i} r_{L, i}\left(\frac{r_{M, i}}{N_{i}} \theta_{M, i}-r_{L, i} \theta_{L, i}\right)=\sum_{j=1}^{n} D_{i j} \ddot{\theta}_{L, j}+\sum_{j=1}^{n} C_{i j} \dot{\theta}_{L, j}+G_{i} . \tag{4.32}
\end{equation*}
$$

In this complete model (4.3I) and (4.32), the system inputs $u$, which are later used to design controllers, are directly related to the motor torques per:

$$
\begin{equation*}
\tau_{M}=K_{M} u \tag{4.33}
\end{equation*}
$$

where $K_{M} \in \mathbb{R}^{N \times N}$ is a square constant diagonal matrix with motor constants.

### 4.6.2 Ideal actuator and drive train

We can derive an extremely simplified version of the actuator and drive train model as presented in the previous section by assuming that the drive train and links of the robot are stiff and that there is no friction present in the actuator. The relation of the system inputs $u$ to the joint torque is then simply given by:

$$
\begin{equation*}
\tau=K_{M} u \tag{4.34}
\end{equation*}
$$

where $\tau$ is the vector of joint torques as introduced in the biped models and $K_{M} \in \mathbb{R}^{N \times N}$ is again a square constant diagonal matrix with motor constants.

### 4.7 Important geometric quantities for humanoid robots

In the final section of this chapter, we introduce some important geometric quantities that are often encountered in humanoid robotics literature and that will be used throughout the rest of this thesis. Namely, these quantities are the center of mass (CoM), kinetic and potential energy, linear and angular momentum, the zero moment point (ZMP) and the center of pressure (CoP).

### 4.7.1 Center of mass

For all the models that we derived in this chapter, we can easily compute the positions of the centers of mass of each link with respect to the base frame. This position for link $i$ is defined as $p_{i}^{\top}(q)=\left[\begin{array}{lll}x_{i} & y_{i} & z_{i}\end{array}\right]$, where $x_{i}, y_{i}$ and $z_{i}$ are the positions along the corresponding coordinate axis. The total CoM of the robot is:

$$
p_{c o m}(q)=\left[\begin{array}{lll}
x_{c o m} & y_{c o m} & z_{c o m}
\end{array}\right]^{\top}=\frac{1}{M} \sum_{i=1}^{N} m_{i} p_{i}(q)
$$

where $M$ is the total mass of the robot. Often the CoM position is projected onto the ground to check if it lies within the support polygon. If it does in the static case, i.e. there are solely gravitational forces acting on the robot and no inertial ones, then the feet of the biped do not tip over and the biped is said to be statically stable.

### 4.7.2 Energy and momentum

For the models presented in this chapter, the kinetic energy is defined as:

$$
K(q, \dot{q})=\frac{1}{2} \dot{q}^{\top} D(q) \dot{q}=\frac{1}{2} \sum_{i=1}^{n} \dot{p}_{i}^{\top}(q, \dot{q}) m_{i} \dot{p}_{i}(q, \dot{q})+\omega_{i}^{\top}(q, \dot{q}) J_{i}(q) \omega_{i}(q, \dot{q}),
$$

where $J_{i}=R_{i}(q) I_{i} R_{i}^{\top}(q)$ and $\omega_{i}^{\top}=\left[\begin{array}{lll}\omega_{x i} & \omega_{y i} & \omega_{z i}\end{array}\right]$, respectively the inertia matrix and angular velocity of link $i$ with respect to the base coordinate frame. The potential energy is denoted by:

$$
\begin{equation*}
P(q)=g_{0} \sum_{i=1}^{n} m_{i} z_{i}(q) \tag{4.37}
\end{equation*}
$$

where $g_{0}$ is the gravitational constant. Further, the linear momentum can be computed per:

$$
\rho(q, \dot{q})=\left[\begin{array}{lll}
\rho_{x} & \rho_{y} & \rho_{z} \tag{4.38}
\end{array}\right]^{\top}=\sum_{i=1}^{n} m_{i} \dot{p}_{i}(q, \dot{q}),
$$

and the angular momentum via:

$$
\sigma(q, \dot{q})=\left[\begin{array}{lll}
\sigma_{x} & \sigma_{y} & \sigma_{z} \tag{4.39}
\end{array}\right]^{\top}=\sum_{i=1}^{n} p_{i}(q) \times m_{i} \dot{p}_{i}(q, \dot{q})+J_{i}(q) \omega_{i}(q, \dot{q}) .
$$

### 4.7.3 Zero moment point

The CoM position does not give information about tipping of the robot in the dynamic case, i.e. when inertial forces are acting on the robot. In this case, the dynamic equivalence of the ground projection of the CoM needs to be consulted, called the ZMP. It is defined as the location inside the support polygon where the net moment generated from the ground reaction forces is strictly perpendicular to the ground. The position of the ZMP can be derived using the robot's time derivatives of the linear momentum:

$$
\dot{\rho}(q, \dot{q}, \ddot{q})=\left[\begin{array}{ccc}
\dot{\rho}_{x} & \dot{\rho}_{y} & \dot{\rho}_{z} \tag{4.40}
\end{array}\right]^{\top}=\sum_{i=1}^{n} m_{i} \ddot{p}_{i}(q, \dot{q}, \ddot{q}),
$$

and angular momentum:

$$
\dot{\sigma}(q, \dot{q}, \ddot{q})=\left[\begin{array}{c}
\dot{\sigma}_{x}  \tag{4.4ㄷ}\\
\dot{\sigma}_{y} \\
\dot{\sigma}_{z}
\end{array}\right]=\sum_{i=1}^{n} \dot{p}_{i}(q, \dot{q}) \times m_{i} \ddot{p}_{i}(q, \dot{q}, \ddot{q})+J_{i}(q) \dot{\omega}_{i}(q, \dot{q}, \ddot{q})+\omega_{i}(q, \dot{q}) \times J_{i}(q) \omega_{i}(q, \dot{q}),
$$

where the angular velocity and acceleration of link $i$ are given by $\omega_{i}$ and $\dot{\omega}_{i}$ respectively. The ZMP position is now given by:

$$
p_{z m p}=\left[\begin{array}{l}
x_{z m p}  \tag{4.42}\\
y_{z m p} \\
z_{z m p}
\end{array}\right]=\frac{1}{M g_{0}+\dot{\rho}_{z}}\left[\begin{array}{c}
M g_{0} x_{c o m}-\dot{\sigma}_{y} \\
M g_{0} y_{c o m}+\dot{\sigma}_{x} \\
0
\end{array}\right]
$$

Similarly as with the ground projection of the CoM, the ZMP is used as a measure of the stability of a humanoid robot [ I 8 o ]. In contrast to the ground projection of the CoM, the ZMP can be used in the dynamic case, i.e. when the robot is under influence of inertial forces. If the ZMP remains inside the interior of the support polygon, the stance foot or feet remain flat on the ground and do not start to tip, and the robot is defined as dynamically stable. However, when the ZMP lies on the edge of the support polygon, the robot starts to tip [182].

### 4.7.4 Center of pressure

The CoP is the position based weighted average of all normal ground reaction forces acting inside the support polygon. We can use the Lagrange multipliers $\lambda_{N i}$, which represent the normal contact forces as introduced in the biped models, to compute the CoP as follows:

$$
p_{c o p}=\left[\begin{array}{lll}
x_{c o p} & y_{c o p} & z_{c o p} \tag{4.43}
\end{array}\right]^{\top}=\frac{1}{\Lambda} \sum_{i=1}^{n_{c}} \kappa_{N i} \lambda_{N i}
$$

where $\Lambda$ is the total normal force acting below the feet of the robot. Actually, the ZMP is the same point on the ground as the CoP [60, I40]. The only reason why we make a distinction is because we would like to distinguish the computation involved to arrive at this point. We call this point the ZMP if it is computed using joint angles, angular velocities and angular accelerations and we call it the CoP when it is computed using ground reaction forces.

### 4.8 Conclusion

In this chapter we have presented experimental studies that we performed on the humanoid robot TUlip to gain insight in the most important dynamical aspects that need to be modeled in order to accurately describe the motion of such a robotic system. From
those experiments, we can conclude that: i) point feet can successfully be imitated on humanoid robots with feet by turning off the ankle control, 2) collisions of the feet with the ground result in impacts that can be modeled with discontinuous velocities and energy loss, 3) there is significant coupling between different bodies in the humanoid robot, 4) dynamics in the coronal plane influences dynamics in the sagittal plane and vice versa. These dynamical aspects are often neglected in models presented in literature [I8, 92, IOI], but as we found in this experimental study, they should be included to accurately describe the dynamics of humanoid robots.

In the second part of this chapter, we introduced several models that include these dynamics aspects. The most complex model exhibits unordered phases, underactuation and ground friction and it is intended for simulation but less suitable for controller design and stability analysis. Therefore, we also present simpler models without ground friction for which the evolution of the dynamics is restricted to ordered phases. These models include planar bipeds with point feet, planar and three dimensional bipeds with finite sized feet and maximally one degree of underactuation, and fully actuated humanoid robots. Finally, we introduced actuator and drive train models and relevant humanoid robot geometric quantities such as the CoM, ZMP and CoP, which are often used as indicators for stability.

## Chapter 5

## Model parameter estimation

### 5.1 Introduction

The different bipedal models as presented in Chapter 4 contain different parameters that determine the evolution of their dynamics. These parameters constitute geometric ones, describing kinematic properties such as the lengths and positions of the centers of mass of each link in the biped, and inertial ones, describing dynamic properties such as the mass and moments of inertia of these links. To accurately describe the motion of a real humanoid robot by a model, accurate knowledge of the model parameters is very important. Some parameters can be determined directly using static measurements; others need to be estimated in dynamic experiments. Dynamic experiments become inevitable, especially when a robotic system is already assembled, since on an assembled system it is difficult to achieve static measurements of parameters of individual links.
Therefore, in this chapter ${ }^{1}$ we present different parameter estimation techniques tailor made for humanoid robots. We contribute two different approaches for different types of humanoid robots. In the first approach, we consider a force controlled humanoid robot with series elastic actuation, where we use the fact that we can measure the joint torques. In the second approach we use ground reaction force measurements on position controlled humanoid robots without joint torque sensing. Since both methods are regressor based techniques, we also present an algorithm that facilitates derivation of the regressor from any set of equations that is linear in the model parameters.
The outline of this chapter is as follows. In Section 5.2 the regressor derivation algorithm is explained. The parameter estimation technique for force controlled robots is presented in Section 5.3 and the one for position controlled robots in Section 5.4. Experimental iden-

[^9]tification of the model parameters of TUlip is performed in Chapter 8.

### 5.2 Regressor method for model parameter estimation

The methods that we propose in this chapter are regressor based. In the next sections we define the regressor, explain its derivation and describe how it can be used to estimate the model parameters.

### 5.2.1 Definition of the base regressor form

In this chapter we only focus on the models as presented in Chapter 4. The parts of these models containing the continuous dynamics have the same structure. They exhibit the interesting feature that they are so called linear in the model parameters, in the sense that terms containing model parameters can be separated from terms containing states of the system and inputs [168, pp. 270-27I]. This means that there exists a so called regressor matrix $R \in \mathbb{R}^{n \times m}$, a vector of $m$ unknown parameters $\vartheta \in \mathbb{R}^{m}$ and a vector of parameter independent terms $\zeta \in \mathbb{R}^{n}$ such that for example the equations of motion (4.25) can be rewritten in:

$$
\begin{equation*}
D(q) \ddot{q}+C(q, \dot{q}) \dot{q}+G(q)-B \tau=R(q, \dot{q}, \ddot{q}) \vartheta-\zeta=0, \tag{5.I}
\end{equation*}
$$

which is known as the regressor form [24, 54, 97, 183]. Normally in robotic systems, the number of parameters exceeds the number of equations of motion, so that the regressor matrix is not square. Thus, the unknown parameters cannot simply be computed from (5.1) using the inverse of the regressor matrix. Even if the regressor matrix were square, it would not be invertible, since its columns are in general linearly dependent, so that the matrix does not have full column rank. The regressor matrix can, however, be transformed into a matrix with linearly independent columns by combining multiple elements of $\theta$, which results in the base regressor form [55,56,97]. In the following, subscript ${ }_{0}$ is used to denote the base regressor form:

$$
\begin{equation*}
\zeta=\zeta_{0}=R_{0}(q, \dot{q}, \ddot{q}) \vartheta_{0}, \tag{5.2}
\end{equation*}
$$

where $R_{0} \in \mathbb{R}^{n \times p}$ is the base regressor matrix with linearly independent columns, $\vartheta_{0} \in \mathbb{R}^{p}$ is the base parameter set (BPS), and $p$ is the number of base parameters. The BPS is a minimal set of identifiable parameters that describes the system completely [55, 56]. It is in general not unique, since the elements of $\theta$ can be combined in multiple ways to derive (5.2). Identifiability means that the true value of a parameter can theoretically be determined with an infinite number of measurements.

The model equations as presented in Chapter 4 are not written in the linear parameter regressor form of (5.1) or (5.2). Rewriting these equations to the (base) regressor form is
not a trivial problem, since the model equations may contain numerous (trigonometric) terms and nonlinear couplings between the system coordinates. There exist methods that solve this problem for specific equations such as the equations of motion [16, 56, 98, 193] or the energy equations [III], but no method works in general, for example on the CoM or ZMP equations. Therefore, we developed an algorithm that is capable of automatically rewriting any set of model equations which is linear in the model parameters to the base regressor form. This algorithm is presented in the next section.

### 5.2.2 Derivation of the base regressor form

To facilitate derivation of the regressor form from the model equations, we develop an algorithm that can automatically separate mutually different terms from the equations that depend on the system states and their time-derivatives. After separation, this algorithm puts the terms into the appropriate matrices. The algorithm is schematically shown in Figure 5.I and utilizes the fact that one can convert fully expanded equations to text based strings. Within these strings, it is straightforward to locate mathematical operators, such as ' + ', ' - ', ' $\times$ ' and ' $/$ '. These operators, in fact, divide the given equations into different terms. Each term belongs to a certain group: a parameter independent term, a term that makes a product with unknown parameter in the model equation, or an unknown parameter. In this way, we can work ourselves through all equations and put every term into the corresponding matrix according to the regressor model (5.I): parameter independent terms constitute the vector $\zeta$, terms that make products with unknown parameters become elements of the matrix $R$, and unknown parameters make the vector $\vartheta$. Consequently, we build up the regressor form (5.I) in an automated way.
The algorithm presented in Figure 5.I delivers the regressor form, but it does not deliver the base regressor form in the general case. To facilitate automatic conversion of the regressor form (5.I) into the base regressor form, the reasoning proposed in [97] can be used. The regressor matrix $R$ contains terms that are (trigonometric) functions of joint variables and their time-derivatives. Some of these terms may also contain known system parameters (e.g. link lengths). A function of joint variables that appears in one or several terms of the regressor is called a fundamental function. The fundamental functions are stored in a vector $f \in \mathbb{R}^{k}$, where $k$ is the number of fundamental functions in $R$, whereas the known system parameters are stored in a vector $b_{i j} \in \mathbb{R}^{k}$ so that the following holds:

$$
\begin{equation*}
R_{i j}(q, \dot{q}, \ddot{q})=f^{\top}(q, \dot{q}, \ddot{q}) b_{i j} \tag{5.3}
\end{equation*}
$$

where, $i$ and $j$ represent a row and column index in the regressor matrix $R$. So for each element $R_{i j}$ in $R$ one can find the corresponding vector $b_{i j}$. All vectors $b_{i j}$ can be stored in

Start with fully expanded equations of motion:

$$
\begin{aligned}
& \overbrace{\left[\begin{array}{c}
\tau_{1} \\
\tau_{2} \\
\vdots
\end{array}\right]}^{B \tau}=\overbrace{\left[\begin{array}{c}
I_{1} \times \ddot{\theta}_{1}+m_{1} \times c_{1}^{2} \times \cos \left(\theta_{1}\right)-\ldots \\
\ldots \\
\cdots
\end{array}\right]}^{D(q) \ddot{q}+C(q, \dot{q} \dot{q}+G(q)} \\
& \left\{\left\{\begin{array}{c}
\tau_{1} \\
I_{1} \times \ddot{\theta}_{1} \\
m_{1} \times c_{1}^{2} \times \cos (\theta) \\
\cdots \\
\left\{\begin{array}{c}
\tau_{2} \\
\cdots
\end{array}\right\}
\end{array}\right\}\right. \\
& \left.\left\{\begin{array}{c}
\left\{\begin{array}{c}
\left\{\tau_{1}\right\} \\
\left\{I_{1} \ddot{\theta}_{1}\right\} \\
\left\{m_{1}\right. \\
c_{1}^{2} \\
\cos (\theta)
\end{array}\right\} \\
\{\cdots\}
\end{array}\right\}\right\} \\
& \underbrace{\left[\begin{array}{c}
\tau_{1} \\
\tau_{2} \\
\vdots
\end{array}\right]}_{\zeta}=\underbrace{\left[\begin{array}{ccc}
\ddot{\theta}_{1} & \cos (\theta) & \cdots \\
\vdots & \vdots & \ddots
\end{array}\right]}_{R(q, \dot{,}, \dot{q})} \underbrace{\left[\begin{array}{c}
I_{1} \\
m_{1} c_{1}^{2} \\
\vdots
\end{array}\right]}_{\vartheta}
\end{aligned}
$$

I. Separate equations between '+' and '-':
2. Separate expressions between ' $\times$ ' and '/':
3. Check if each expression is known or unknown and put in according matrix:

Figure 5.1: Example of the algorithm for derivation of a regressor form from the equations of motion. The unknown parameters are $I_{1}, m_{1}$ and $c_{1}$, the measurable states of the system are $\theta_{1}$ and $\ddot{\theta}_{1}$, and the inputs are $\tau_{1}$ and $\tau_{2}$.
one matrix $B \in \mathbb{R}^{l \times m}, l=n k$, as follows:

$$
B=\left[\begin{array}{ccc}
b_{11} & \cdots & b_{1 m}  \tag{5.4}\\
\vdots & \ddots & \vdots \\
b_{l 1} & \cdots & b_{l m}
\end{array}\right]
$$

This matrix $B$ can be transformed to the Echelon form $B_{E}$ [6] by performing a GaussJordan elimination [97]. The result is an upper triangular matrix of the following form:

$$
B_{E}=\left[\begin{array}{c}
B_{E u}  \tag{5.5}\\
0
\end{array}\right] .
$$

The matrix $B_{E}$ consists of columns that can be characterized according to three types. The first type has all elements equal to zero. This means that the corresponding column in the regressor $R$ is not needed for parameter identification and can be removed. The second type is a column in which all elements are zero except from only one element which is equal
to I . This means that the corresponding column in the regressor $R$ is required and linearly independent from the other columns. The last type is a column with zeros and known parameter elements. The corresponding column in regressor $R$ is linearly dependent on other columns and can be removed. The corresponding base parameter vector $\vartheta_{0}$ can be derived in the following way:

$$
\begin{equation*}
\vartheta_{0}=B_{E u} \vartheta \tag{5.6}
\end{equation*}
$$

This completes the derivation of the base regressor form (5.2) which contains the BPS. The complete algorithm is given in pseudo code by Algorithm 5.I. Input to this algorithm is an array of strings containing the fully expanded equations of motion. The outputs of this algorithm are an array of strings containing the base regressor and an array of strings containing unknown parameters. The algorithm is fast and can be applied to different systems. Besides the identification of TUlip, presented in Chapter 8, it has been successfully tested during identification of a Philips robotic arm containing seven degrees of freedom [I38]. The algorithm can be made more efficient without expanding the equations, but this requires additional bookkeeping of all expressions. We also tried symbolic manipulation of the equations by taking partial derivatives to the unknown parameters to obtain the regressor matrix as in [5I]. However, for this to work, one has to know the parameter combinations on forehand and they may not contain nonlinear expressions, such as the term $c_{1}^{2}$ in Figure 5.I. Another elegant approach, albeit more complex, for finding the regressor form is given in [IIO]. In the next section, it is described how the regressor form can be used to estimate the model parameters.

### 5.2.3 Parameter estimation using the base regressor form

As indicated before, in general, the number of base parameters is larger than the number of model equations $(p>n)$. Therefore, the base parameters cannot be estimated uniquely, since they appear as linear combinations in the equations and the inverse of the regressor matrix does not exist. Instead, we determine them by means of an algorithm for optimal fitting.
To improve the reliability of the parameter fit we collect system inputs and outputs at different samples. These samples contain different configurations of the robot, which can be obtained by either putting the robot in different configurations or by letting the robot move along trajectories consisting of several samples. With $r$ denoting the number of samples, we create the following compact regressor form:

$$
\begin{equation*}
y=H(q, \dot{q}, \ddot{q}) \vartheta_{0} \tag{5.7}
\end{equation*}
$$

with

$$
y^{\top}=\left[\begin{array}{lll}
\zeta_{1}^{\top} & \cdots & \zeta_{r}^{\top}
\end{array}\right] \quad \text { and } \quad H^{\top}=\left[\begin{array}{llll}
R_{0}^{\top}\left(q_{1}, \dot{q}_{1}, \ddot{q}_{1}\right) & \cdots & R_{0}^{\top}\left(q_{r}, \dot{q}_{r}, \ddot{q}_{r}\right)
\end{array}\right],
$$

```
Algorithm 5.I Regression algorithm
    for \(r=1 \rightarrow n\) do
        \(e \leftarrow \operatorname{eom}(r)\)
        pmop \(\leftarrow \operatorname{sort}([\operatorname{strfind}(e, "+"), \operatorname{strfind}(e, "-")])\)
        for \(i=1 \rightarrow \operatorname{len}(p m o p)-1\) do
            \(p m s \leftarrow e(p m o p(i)+1: \operatorname{pmop}(i+1)-1)\)
            \(m d o p \leftarrow \operatorname{sort}([\operatorname{strfind}(p m s, " \star\) " \(), \operatorname{strfind}(p m s, " / ")])\)
            for \(j \leftarrow 1\), len \((m d o p)-1\) do
                \(m d s \leftarrow \operatorname{pms}(\operatorname{mdop}(j)+1: \operatorname{mdop}(j+1)-1)\)
                if \(m d s\) is unknown parameter then
                    \(\vartheta_{i} \leftarrow \vartheta_{i} \times m d s\)
                else
                    \(r_{i} \leftarrow r_{i} \times m d s\)
                    if \(m d s\) is known parameter then
                    \(b_{i} \leftarrow b_{i} \times m d s\)
                    else
                    \(f_{i} \leftarrow f_{i} \times m d s\)
                    end if
                end if
            end for
            if \(\vartheta_{i}==1\) then
                \(\zeta_{0} \leftarrow \zeta_{0}+r_{i}\)
            else
                \(R(r\), end +1\() \leftarrow r_{i}\)
                \(\vartheta(\) end +1\() \leftarrow \vartheta_{i}\)
                \(f(\) end +r\() \leftarrow f_{i}\)
                \(B(\) end +I , end +r\() \leftarrow b_{i}\)
            end if
        end for
    end for
    Take together all duplicate terms in \(\vartheta\) and \(f\), properly adjusting \(R\) and \(B\).
    \(B_{E} \leftarrow\) Gauss-Jordan-Elimination \((B)\)
    for \(i \leftarrow 1\), len \(\left(B_{E}\right)\) do
        if column \(i\) in \(B\) contains zeros or known parameters then
            remove column \(i\) from \(R\)
        end if
    end for
    \(R_{0} \leftarrow R\)
    \(\vartheta_{0} \leftarrow B_{E} \times \vartheta\)
```

where $\zeta_{i}$ and $R_{0}\left(q_{i}, \dot{q}_{i}, \ddot{q}_{i}\right)$ is the regressor form of sample $i$. In this way all samples are incorporated in the estimation process and typically more equations than the number of base parameters are obtained $(r n>p)$. For the models of Chapter 4 the input $\tau$ appears in $y$ and not in $H$.
An exact solution of (5.7) exists if $\operatorname{rank}(H)=\operatorname{rank}\left(\left[\begin{array}{ll}H & y\end{array}\right]\right)$, but this is never the case because of noise, measurement errors and model inaccuracies. To average out these effects, $H$ should contain as many measurements as possible. The solution of (5.7) is thus at best a fit. Possible ways to find the base parameters are fitting the model to the experimental data using a maximum likelihood or least squares algorithm [24, 130, 13I, 172]. We assume that noise and errors on the measured signals are small compared to modeling errors, so that a deterministic parameter estimation framework suffices in contrast to a statistical framework with stochastic variables [17I]. In a deterministic framework, maximum likelihood fitting simplifies to least squares fitting and therefore the latter is used [172]. As a consequence, if the noise and measurement errors are significant, the individual estimated base parameters may be inconsistent, which means that they do not converge to the real parameter values even for many measurement data. However, the full model can still be consistent and that is where we are mainly interested in.
The least squares algorithm minimizes $\left\|H \vartheta_{0}-y\right\|^{2}$. The BPS ensures that $H$ has full column rank which means that there is a global minimum for $\left\|H \vartheta_{0}-y\right\|^{2}$, given by the Moore-Penrose pseudo-inverse:

$$
\begin{equation*}
\hat{\vartheta}_{0}=\left(H^{\top} H\right)^{-1} H^{\top} y, \tag{5.8}
\end{equation*}
$$

where $\hat{\vartheta}_{0}$ is an estimate for $\vartheta_{0}$. The dependence of $H$ on $q, \dot{q}$ and $\ddot{q}$ in (5.7) indicates that the quality of the estimation depends on the motion of the system during the identification experiment. It is for example clear that with a constant signal no information about dynamic parameters can be obtained. So, for reliable parameter estimation, the motion should excite all system dynamics [24, i26, I72]. Such motions are called persistently exciting [165, pp. 331-332]. The amount of persistent excitation is determined by the conditioning of $H$, represented by the condition number:

$$
\begin{equation*}
\kappa(H)=\left|\frac{s_{\max }(H)}{s_{\min }(H)}\right|, \tag{5.9}
\end{equation*}
$$

where $s_{\max }(H)$ and $s_{\text {min }}(H)$ are the maximum and minimum singular values of $H$ respectively. A lower condition number means that in (5.7) the unknown parameters $\theta_{0}$ are less sensitive to changes in $y$, which makes them more consistent. Designing persistently exciting trajectories can be difficult as we explain in the next sections, where we apply the regressor based estimation method to different humanoid robot models of Chapter 4.

### 5.3 Force controlled robots

In this section we contribute a model parameter estimation approach for force controlled robots. There are many ways in which force sensing can be made available on a robot. We restrict our approach to robots with series elastic actuation. This is a typical instrument to measure joint torques in robotic applications. It uses a flexible element in the drive train between the actuator and the load. If the stiffness of the elastic element is known and one can measure its elongation, it is straightforward to compute the force in the element, which relates to the torque applied by the actuator on the load. In the next section, we apply the regressor based estimation method derived in the Section 5.2 to the series elastic actuated model of Section 4.6.I.

### 5.3.1 Dynamic identification of robots with series elastic actuation

Consider the model as presented in Section 4.6.i. In this section, we transform the model to the regressor form using Algorithm 5.I after which we explain how to design persistently exciting trajectories for the parameter estimation.

## Identification procedure

The series elastic actuated model of Section 4.6.I consists of multiple parts. It contains a model for the actuator, drive train and load. Therefore also multiple regressor forms need to be determined, one for the part of the model at the actuator side and one for the part of the model at the load side.

First, application of the regression using Algorithm 5.I to the part of the system model at the actuator side (4.26) for joint $i$, gives the base regressor form:

$$
\begin{equation*}
\zeta_{M_{i}}=R_{M_{i}}(q, \dot{q}, \ddot{q}) \vartheta_{M_{i}}, \tag{5.10}
\end{equation*}
$$

with

$$
\zeta_{M_{i}}=\tau_{M, i}-I_{M, i} \ddot{\theta}_{M, i}, R_{M_{i}}=\left[\begin{array}{c}
\dot{\theta}_{M, i} \\
\operatorname{sign}\left(\dot{\theta}_{M, i}\right) \\
\frac{r_{M, i}}{N_{i}}\left(\frac{r_{M, i}}{N_{i}} \theta_{M, i}-r_{L, i} \theta_{L, i}\right)
\end{array}\right]^{\top}, \text { and } \vartheta_{M_{i}}=\left[\begin{array}{c}
F_{v, i} \\
F_{c, i} \\
k_{i}
\end{array}\right] .
$$

Here, we assume that the moment of inertia of the motor shaft $I_{M, i}$, the pulley radii $r_{M, i}$ and $r_{L, i}$ and the gearbox ratio $N_{i}$ are known. We also assume that we can compute the motor torque $\tau_{M, i}$ from the motor current using the motor torque constant supplied by the motor manufacturer. Finally, we can measure the motor angle $\theta_{M, i}$ using a motor encoder. As can be seen from (5.10), the regressor form of the part of the system model at the actuator side is the same as its base regressor form.

The second part of the estimation procedure is the derivation of the regressor form for the part of the model at the load side (4.30). Application of the regression method using Algorithm 5.I to this model results in the base regressor form:

$$
\begin{equation*}
\zeta_{L_{i}}=R_{L_{i}}(q, \dot{q}, \ddot{q}) \vartheta_{L_{i}}, \tag{5.Іі}
\end{equation*}
$$

where the individual terms in $\zeta_{L_{i}}, R_{L_{i}}$ and $\vartheta_{L_{i}}$ are too complicated to show here.
Now, identification experiments can be performed such that a sufficient amount of $r$ samples for $\tau_{M, i}, \theta_{M, i}$ and $\theta_{L, i}$ can be collected. These samples are used to stack the regressors for the part of the model at the actuator side:

$$
\begin{equation*}
y_{M}=H_{M}(q, \dot{q}, \ddot{q}) \vartheta_{M}, \tag{5.12}
\end{equation*}
$$

with

$$
y_{M}=\left[\begin{array}{c}
\zeta_{M, 1} \\
\vdots \\
\zeta_{M, r}
\end{array}\right], H_{M}=\left[\begin{array}{c}
R_{M, 1} \\
\vdots \\
R_{M, r}
\end{array}\right], \zeta_{M, j}=\left[\begin{array}{c}
\zeta_{M_{1}, j} \\
\vdots \\
\zeta_{M_{n}, j}
\end{array}\right], R_{M, j}=\left[\begin{array}{c}
R_{M_{1}, j} \\
\vdots \\
R_{M_{n}, j}
\end{array}\right] \text { and } \vartheta_{M}=\left[\begin{array}{c}
\vartheta_{M_{1}} \\
\vdots \\
\vartheta_{M_{n}}
\end{array}\right]
$$

where $\square_{j}$ means $\square$ evaluated with data from sample $j=1, \ldots, r$. We can use these stacked regressor forms for part of the model at the actuator side to estimate the friction parameters of the joint, $F_{v, i}$ and $F_{c, i}$, and the stiffness of the drive train $k_{i}$ :

$$
\begin{equation*}
\hat{\vartheta}_{M}=\left(H_{M}^{\top} H_{M}\right)^{-1} H_{M}^{\top} y_{M} . \tag{5.13}
\end{equation*}
$$

The load torque $\tau_{L, i}$ can then be computed using these parameters and the drive train model (4.29). The final step in the identification procedure is the estimation of the model parameters at the load side. Using the $r$ measured samples for $\theta_{L, i}$ and the computed samples for $\tau_{L, i}$ we can stack the regressors for the part of the model at the load side:

$$
\begin{equation*}
y_{L}=H_{L}(q, \dot{q}, \ddot{q}) \vartheta_{L}, \tag{5.14}
\end{equation*}
$$

where

$$
y_{L}=\left[\begin{array}{c}
\zeta_{L, 1} \\
\vdots \\
\zeta_{L, r}
\end{array}\right], H_{L}=\left[\begin{array}{c}
R_{L, 1} \\
\vdots \\
R_{L, r}
\end{array}\right], \zeta_{L, j}=\left[\begin{array}{c}
\zeta_{L_{1}, j} \\
\vdots \\
\zeta_{L_{n}, j}
\end{array}\right], R_{L, j}=\left[\begin{array}{c}
R_{L_{1}, j} \\
\vdots \\
R_{L_{n}, j}
\end{array}\right] \text { and } \vartheta_{L}=\left[\begin{array}{c}
\vartheta_{L_{1}} \\
\vdots \\
\vartheta_{L_{n}}
\end{array}\right] .
$$

We can use these stacked regressor forms for part of the model at the load side to estimate the model parameters at the load side:

$$
\begin{equation*}
\hat{\vartheta}_{L}=\left(H_{L}^{\top} H_{L}\right)^{-1} H_{L}^{\top} y_{L} \tag{5.15}
\end{equation*}
$$

This finalizes the identification procedure. Next we explain how sufficiently exciting trajectories can be found.

## Persistently exciting trajectories

In this section we explain how persistently exciting trajectories can be designed [8]. As mentioned earlier, the condition number of the stacked regressor matrix (5.9) is an indication of the amount of excitation of the trajectories. Minimizing this condition number increases the amount of excitation of the trajectories [24, 57, 172]. We choose to minimize the condition number of $H_{L}$, the stacked regressor matrix of the part of the model at the load side. The motion of the part of the model at the actuator side is always $N_{i}$ times 'faster' because of the gearbox, so we assume that persistently exciting trajectories for part of the model at the load side are also persistently exciting for part of the model at the actuator side.
Besides minimization of the condition number of $H_{L}$, the system trajectories in an identification experiment should also satisfy mechanical limits in the robot joints. Additionally, actuation capabilities should be taken into account by limiting velocities and accelerations in the joints. By taking into account these constraints on the trajectories, we consider the following optimization problem:

$$
\begin{gather*}
\min _{q_{r}} \kappa\left(H_{L}\left(q_{r}(t), \dot{q}_{r}(t), \ddot{q}_{r}(t)\right)\right), \quad t \in\left[0, t_{f}\right] \\
\underline{c}_{p} \leq q_{r}(t) \leq \bar{c}_{p}  \tag{5.16}\\
\underline{c}_{v} \leq \dot{q}_{r}(t) \leq \bar{c}_{v} \\
\underline{c}_{a} \leq \ddot{q}_{r}(t) \leq \bar{c}_{a},
\end{gather*}
$$

where $q_{r}$ are trajectories in the robot joints, $\dot{q}_{r}$ and $\ddot{q}_{r}$ are time-derivatives of these trajectories, $t_{f}$ is the duration of the motion and $c_{p}, c_{v}$ and $c_{a}$ are the boundary constraints on position, velocity and acceleration respectively with $\square$ and $\bar{\square}$ the lower and upper limits respectively. The presented minimization (5.16) is a constrained nonlinear optimization problem. There exist several optimization algorithms to find local minima of this problem. Solutions usually depend on the initial conditions, since the considered optimization problem is in general not convex. Finding joint trajectories that optimize (5.16) can be very time consuming for systems with many links. Therefore, postulating the trajectory in the form of a finitely parameterized function, such as a finite Fourier series or polynomial, is more pragmatic [IO2]. For instance, consider as arguments for optimization, the coefficients $a_{i}$, $i=0, \ldots, d$ of the polynomials of order $d$ :

$$
\begin{equation*}
q_{a}(t)=\sum_{i=0}^{d} a_{i} t^{i} . \tag{5.I7}
\end{equation*}
$$

The number of optimization arguments can be kept low to speed up the optimization process at the cost of getting farther from the optimum. This is not a problem, since we are interested to find feasible trajectories with a small condition number overall and not in
the optimum per se. The corresponding optimization problem becomes:

$$
\begin{align*}
& \min _{a_{i}} \kappa\left(H_{L}\left(q_{a}(t), \dot{q}_{a}(t), \ddot{q}_{a}(t)\right)\right), \quad t \in\left[0, t_{f}\right] \\
& \underline{c}_{p} \leq q_{a}(t) \leq \bar{c}_{p}  \tag{5.18}\\
& \underline{c}_{v} \leq \dot{q}_{a}(t) \leq \bar{c}_{v} \\
& \underline{c}_{a} \leq \ddot{q}_{a}(t) \leq \bar{c}_{a},
\end{align*}
$$

In our experimental case-study, parameterization using polynomial functions of degree 5 turns out to be successful, time-efficient, and fairly easy to implement. We designed the optimal polynomial trajectories of a duration of io s. In experiments, we realized these trajectories with a sampling frequency of I kHz .

### 5.4 Position controlled robots

In this section we consider position controlled humanoid robots, since many humanoid robots lack joint torque sensing. This makes system identification more complicated, because the dynamic equations of motion cannot be used directly. We develop a model parameter estimation approach tailored for humanoid robots without joint torque sensing. The approach uses joint orientation and ground contact force measurements, which are typically available on humanoid robots. Actuator models or joint friction characteristics are not required and it can be used for static as well as dynamic parameter estimation. In the next sections we explain how the regressor based method of Section 5.2 can be applied to the CoM and ZMP model to estimate their parameters.

### 5.4.1 Static parameter estimation using ground contact forces

Consider the CoM model as presented in Section 4•7.I. Here, we rewrite this model to the regressor form using Algorithm 5.I after which we explain how optimal static configurations can be chosen for the parameter estimation.

## Identification procedure

The ground projection of the CoM coincides with the CoP in the static case, so the idea of the identification algorithm is to measure the CoP for different static postures and fit this data to the CoM model $[34,36]$. The CoM model (4.35) is linear in the model parameters, and it can be easily rewritten in the regressor form:

$$
\begin{equation*}
p_{\text {com }}=R_{s}(q) \vartheta_{s} \tag{5.19}
\end{equation*}
$$

where $R_{s}$ is the regressor matrix and $\vartheta_{s}^{\top}=\left[\begin{array}{lll}\vartheta_{1} & \cdots & \vartheta_{n}\end{array}\right]$ is the unknown parameter vector consisting of four unknown parameters per joint $i$ :

$$
\vartheta_{i}=\left[\begin{array}{llll}
m_{i} & c_{i x} m_{i} & c_{i y} m_{i} & c_{i z} m_{i} \tag{5.20}
\end{array}\right] .
$$

Transforming the regressor form as written in (5.19) with (5.20) to the base regressor form can be done using Algorithm 5.I:

$$
\begin{equation*}
p_{c o m}=R_{c}(q) \vartheta_{c} . \tag{5.2I}
\end{equation*}
$$

where $\vartheta_{c}$ is the base parameter set (BPS) and $R_{c}$ is the base regressor matrix, which is of full column rank, i.e. all columns are linearly independent. This BPS is not unique, parameters may be grouped differently to improve the final identification results. For example, the link lengths, which we assume to be known, can be put in the regressor matrix as well as in the parameter vector. If put in the parameter vector, the regressor matrix only consists of trigonometric terms, so that it is better conditioned and the BPS can be solved easier.
For a standard three dimensional humanoid robot with six degrees of freedom in each leg, such as TUlip, the BPS consists of 27 parameters. This includes parameters in the unmeasurable $z$-direction of the CoM, but all of these parameters also appear in the $x$ - and $y$-direction, except for the vertical position of the center of mass of the feet. These parameters are only identifiable if the feet and ground are not parallel, which makes them hard to identify. However, these positions are small and assumed negligible, so the identifiable BPS consists of 25 parameters.
One static CoP measurement corresponds to two equations ( $x_{\text {com }}$ and $y_{c o m}$ ). There are 25 base parameters which can be derived from these equations, which means that a minimum of i3 measurements are required to solve (5.2I) for $\vartheta_{c}$. These measurements are collected in:

$$
\begin{equation*}
y_{c}=H_{c}(q) \vartheta_{c}, \tag{5.22}
\end{equation*}
$$

with

$$
y_{c}^{\top}=\left[\begin{array}{lll}
\zeta_{c, 1} & \cdots & \zeta_{c, r}
\end{array}\right] \text { and } H_{c}^{\top}=\left[\begin{array}{lll}
R_{c}^{\top}\left(q_{1}\right) & \cdots & R_{c}^{\top}\left(q_{r}\right)
\end{array}\right],
$$

where $\zeta_{c, j}=\left[\begin{array}{ll}x_{c o m, j} & y_{c o m, j}\end{array}\right]$ and $q_{j}$ are the joint angles, with $j=1, \ldots, r$ where $r$ is the number of measurements. For a sufficient amount of measurements the CoM model parameters can be estimated in a linear least squares sense:

$$
\begin{equation*}
\hat{\vartheta}_{c}=\left(H_{c}^{\top} H_{c}\right)^{-1} H_{c}^{\top} y_{c} . \tag{5.23}
\end{equation*}
$$

In the next section we explain how suitable static measurement postures can be found that give an optimal identification result.

## Static posture selection

In the previous section we have developed an identification procedure for estimation of the CoM model parameters. The sampled data that this procedure requires are static CoP measurements for different postures of the humanoid robot. The choice of suitable postures is


Figure 5.2: Schematic top view of robot feet, support polygon and stability margin.
important for reliable identification results. This is especially important for 3D models as these contain significantly more parameters than planar models. In this section we explain the procedure to find suitable postures.
All robot postures have to satisfy the following constraints:
I. All joint angles $q_{j}$ of all postures must lie within their lower $c_{l p}$ and upper $c_{u p}$ mechanical limits.
2. The robot must be stable, which means that the CoP should lie within the support polygon spanned by the feet. The support polygon is the area formed by a convex hull of the floor contact points as shown in Figure 5.2. The support polygon can be computed using the kinematic model of the humanoid robot and its foot dimensions, however, the CoM model parameters must be known. This sound contradictory, as we are trying to identify these parameters. Therefore in the posture selection procedure estimates are required for these parameters from e.g. CAD drawings. To cope with the errors in these parameters, a margin between the CoP and the support polygon is required. This margin is anyway advisable, since there are always errors in the joint controllers so that the desired posture is never exactly reached. Figure 5.2 shows the stability margin $d_{s}$.
3. Both feet must be on the ground and they may not touch each other, so the two foot polygons, formed by the four corner points of each foot, may not intersect.
4. For the sake of convenience during the experiments, the relative position of the feet must remain the same for a number of postures. Moving the feet relative to each other means that the robot must be lifted. To decrease the experiment time, the robot moves from one posture to another with $k$ fixed feet positions.

Each static posture can be defined either in the joint space where every joint angle is specified or in the task space where the position and orientation of the end effectors are specified. Both methods have merits and limitations. By defining the postures in joint space,
the joint limits are automatically satisfied, however, stability is not necessarily guaranteed. The opposite holds for defining the postures in task space. Either way, an optimization procedure is required, since analytical solutions cannot be found. We tried both ways and found that the optimization algorithm converges faster when the postures are defined in task space. We specify as design variables the CoM position $p_{\text {com }}$, torso orientation $\phi_{t}$ and swing foot position $p_{s}$ and orientation $\phi_{s}$. These variables constitute 9 degrees of freedom per posture, since both feet are constrained on the ground. The task space coordinates are mapped to the joint space using an inverse kinematics algorithm based on differential kinematics [163].
Initial values which are feasible within the constraints for the iterative optimization algorithm can be found as follows:
I. Choose a random position $\left(p_{s}\right)$ and orientation $\left(\phi_{s}\right)$ of the swing foot within predefined ranges.
2. Check if the polygons which define the perimeter of the feet do not intersect. If they do, go to step I .
3. Choose a random CoM position ( $p_{c o m}$ ) within the area which is a distance $d_{s}$ from the support polygon as shown in Figure 5.2. Also choose a random CoM height and torso orientation within a predefined range.
4. Solve the inverse kinematics and check if the configuration is feasible within the joint limits. If not, go to step I if no possible postures were found earlier for this foot placement or redo step 3 otherwise.
5. Store the feasible posture. If more postures with the same foot placement are desired, go to step 3, otherwise go to step i to find another foot placement, or stop if enough random postures are obtained.

Optimal postures correspond to a minimum in the condition number of the output data matrix $H_{c}$, such that we can formulate a nonlinear constrained optimization problem:

$$
\begin{gather*}
\min _{\chi} \kappa\left(H_{c}(\chi)\right), \\
c_{l p} \leq q_{j} \leq c_{u p},  \tag{5.24}\\
d_{e, j} \geq d_{s}, \\
d_{f, j} \geq 0,
\end{gather*}
$$

where $d_{e}$ is the distance from the CoP to the edge of the support polygon, $d_{f}$ is the smallest distance between the feet (see Figure 5.2) and $\chi$ are the design variables:

$$
\chi^{\top}=\left[\begin{array}{llllllllllll}
p_{c o m, 1}^{\top} & \cdots & p_{c o m, r}^{\top} & \phi_{t, 1}^{\top} & \cdots & \phi_{t, r}^{\top} & p_{s, 1}^{\top} & \cdots & p_{s, k}^{\top} & \phi_{s, 1}^{\top} & \cdots & \phi_{s, k}^{\top} \tag{5.25}
\end{array}\right] .
$$

Table 5.1: Optimized condition numbers for different initial values.

| Run | Initial | Optimum |
| ---: | ---: | ---: |
| I | 647.4 | 337.5 |
| 2 | $732 . \mathrm{I}$ | $405 . \mathrm{I}$ |
| 3 | 925.8 | 352.6 |
| 4 | $7 \mathrm{I} 4 . \mathrm{I}$ | 314.7 |
| 5 | 733.6 | 406.9 |
| 6 | $689 . \mathrm{I}$ | 394.4 |

For the identification experiment, we compute sets of 50 feasible initial values, where we choose ten different foot positions with, per foot position, five different CoM positions and torso orientations. These initial values are optimized using the interior point algorithm as implemented in the Matlab Optimization Toolbox. The optimization problem is not convex, so the optimized set of postures is a local minimum and dependent on the initial values. To show this, we repeated the optimization six times for different randomly chosen initial values. The initial and optimized condition number for each trial are presented in Table 5.I. As can be seen from this table the local minima vary depending on the initial values. We choose the three sets of postures with the lowest condition number for the experiments, for which the results are presented in Section 8.3.2.

### 5.4.2 Dynamic parameter estimation using ground contact forces

Consider the ZMP model as presented in Section 4.7.3. In this section, we derive the regressor form of this model using Algorithm 5.I after which we give the walking motions that we used for the model parameter estimation.

## Identification procedure

The ZMP coincides with the CoP, so in this estimation procedure we measure the CoP en fit the data to the ZMP model to estimate its model parameters. The ZMP model (4.42) is linear in the model parameters and can be written to regressor form using Algorithm 5.I:

$$
\begin{equation*}
p_{z m p}=R_{s}(q, \dot{q}, \ddot{q}) \vartheta_{s}, \tag{5.26}
\end{equation*}
$$

where $R_{s}$ is the regressor matrix and $\vartheta_{s}$ the parameter vector consisting of the following parameters for each link $i$ :

- The parameters as identified earlier using the CoM model. These are $m_{i}, c_{i x} m_{i}, c_{i y} m_{i}$ and $c_{i z} m_{i}$. However, these not only appear in the static part of the equations (4.42) but also in the linear momentum (4.40). The acceleration of a mass depends on the same parameters as the position of a mass, as it is a derivative of the position.
- The six elements of the symmetric inertia matrix $I_{i}, I_{i, x x}, I_{i, y y}, I_{i, z z}, I_{i, x y}, I_{i, y z}$ and $I_{i, x z}$. This inertia matrix is defined with respect to a link attached frame located at the center of mass of the link.
- Six cross terms of the link center of mass positions. These originate from the $\dot{p}_{i} \times$ $\left(m_{i} \ddot{p}_{i}\right)$ term in (4.4I). These six parameters are $c_{i x}^{2} m_{i}, c_{i y}^{2} m_{i}, c_{i z}^{2} m_{i}, c_{i x} c_{i y} m_{i}, c_{i y} c_{i z} m_{i}$ and $c_{i x} c_{i z} m_{i}$.

The parameter set thus contains i6 elements for each link. A small amount of these parameters are not identifiable. The dynamic parameters of the stance foot are not present in the equations because the stance foot is not moving. The $c_{i z}$ parameter of the stance foot is also not present as this foot is parallel to the ground, which makes this parameter unidentifiable from CoP data. From the stance foot only the first three elements of the static parameters (5.20) can be identified. Since the stance foot is parallel to the ground, the joint axis of the ankle $x$-joint is parallel to the base frame $x$-axis. The ankle link thus only moves around the base frame $x$-axis which means that the parameters $I_{i, y y}, I_{i, z z}, I_{i, y z}$, $c_{i y} c_{i z} m_{i}$ and $c_{i x}^{2} m_{i}$ of the stance ankle are not identifiable either. In total, this means that for a typical humanoid robot with 6 degrees of freedom in each leg, such as TUlip, $\vartheta_{s}$ contains igo parameters.
Transforming the regressor form (5.26) to the base regressor form using Algorithm 5.I results in:

$$
\begin{equation*}
p_{z m p}=R_{z}(q, \dot{q}, \ddot{q}) \vartheta_{z} . \tag{5.27}
\end{equation*}
$$

For a standard three dimensional humanoid robot the BPS $\vartheta_{z}$ consists of 85 parameters. One CoP measurement corresponds to two equations ( $x_{z m p}$ and $y_{z m p}$ ) so at least 43 measurements are required to solve $(5.27)$ for $\vartheta_{z}$. These measurements are collected in:

$$
\begin{equation*}
y_{z}=H_{z}(q, \dot{q}, \ddot{q}) \vartheta_{z}, \tag{5.28}
\end{equation*}
$$

with

$$
y_{z}^{\top}=\left[\begin{array}{lll}
\zeta_{z, 1} & \cdots & \zeta_{z, r}
\end{array}\right] \text { and } H_{z}^{\top}=\left[\begin{array}{llll}
R_{z}^{\top}\left(q_{1}, \dot{q}_{1}, \ddot{q}_{1}\right) & \cdots & R_{z}^{\top}\left(q_{r}, \dot{q}_{r}, \ddot{q}_{r}\right)
\end{array}\right],
$$

where $\zeta_{z, j}=\left[\begin{array}{ll}x_{z m p, j} & y_{z m p, j}\end{array}\right]$ and $q_{j}$ are the joint angles, with $j=1, \ldots, r$ where $r$ is the number of data points. For a sufficient amount of data points the ZMP model parameters can be estimated in a linear least squares sense:

$$
\begin{equation*}
\hat{\vartheta}_{z}=\left(H_{z}^{\top} H_{z}\right)^{-1} H_{z}^{\top} y_{z} . \tag{5.29}
\end{equation*}
$$

In the next section we explain how suitable dynamic motions can be designed for this estimation procedure.

## Walking gait selection

The identification algorithm that we derived in this section assumes that at least one foot remains fully in contact with the ground, otherwise, the ZMP model that we use is not
valid. So we must find flat footed motions for which the amount of persistent excitation is sufficient to estimate all parameters. In the static case we were able to use an optimization routine to find optimal stable postures for parameter estimation, however, in the dynamic case this would require a much longer investigation. Not only would the optimization problem be increasingly more complex, also a quite accurate ZMP model would be required, as the ZMP position must remain inside the foot for the robot not to tip over. Since we try to identify the ZMP model, it is not available during the persistently exciting trajectory design. In the static case we solved this paradox by guessing parameters from CAD data and by taking a rather large stability margin. In this dynamic case, we believe this is unfeasible since the number of parameters is much higher and the ZMP model is more sensitive to parameter mismatch. In fact, using optimization to find stable ZMP based humanoid robot gaits is an entire research field on its own.
So we do not consider an optimization algorithm to find persistently exciting motions for the dynamic parameter estimation, but we designed some extreme motions such as squatting, bending and kicking to excite as much of the dynamics as possible. We allowed the speeds of these motions to be increased during experiments to increase the excitation of the system without risking instability. Besides these specially designed motions, we also have some walking gaits available. These gaits are designed within the Tech United team to be used during the RoboCup tournaments. The design and control of these gaits will be explained in Section 6.3. The gaits that we use in this parameter estimation procedure are walking forward, side stepping and point turning. Results of the dynamic parameter estimation are given in Section 8.3.3.

### 5.5 Conclusion

In this chapter we detailed two model parameter estimation techniques tailor made for humanoid robots. One technique is designed for force controlled humanoid robots, whereas the other is intended for position controlled humanoid robots. The techniques use commonly available sensors in humanoid robots, such as joint torque sensing with series elastic actuation or ground contact force sensors. The first identification technique uses a twostep identification procedure to identify the model parameters of a series elastic actuation drive train and the rigid body model parameters with only one set of experimental data. The second identification technique estimates the static and dynamic rigid body model parameters using solely joint encoder and ground contact force experimental data.
Both methods are based on linear regression models, so we contributed an algorithm to automatically rewrite any dynamic model, which is linear in the model parameters, to its regressor form. Using this algorithm the models of Chapter 4 are rewritten to regressor forms. Besides this algorithm we also showed how to find persistently exciting input data, required for accurate and reliable parameter estimation.

Experimental and simulation results of these parameter estimation techniques are given in Chapter 8.

## Chapter 6

## Stability and control

### 6.1 Introduction

In this chapter ${ }^{1}$ we analyze the stability of the models presented in Chapter 4 and design controllers that guarantee stable bipedal locomotion. As indicated earlier, the models exhibit hybrid phases, underactuation and non-smooth dynamics, which seriously complicate the stability analysis and controller design. Nevertheless, it is possible to design controllers that guarantee stable bipedal locomotion for such bipeds.
In the first part of this chapter, the focus is on biped models with at most one degree of underactuation. It has been discovered in earlier work [25, 186] that this type of system exhibits an interesting feature when controlled via an input-output linearizing controller. Using this controller, output functions are asymptotically zeroed, in which case they represent virtual holonomic constraints that constrain the full system dynamics to a lower dimensional zero dynamics manifold. The resulting lower dimensional system appears Lagrangian for which a conserved quantity can be found along its solution. This conserved quantity is often named the zero dynamics energy, and can be exploited to control the gait of the biped. We contribute an algorithm that alters the output functions on a step by step basis using certain gait parameters such as step size, torso lean angle and push off intensity, to control the walking speed of the biped. First we present this controller for planar bipeds with point feet, after which we contribute an extension to planar and three dimensional bipeds with feet exhibiting a double support phase.

The proposed controller assumes that the joint torques of the humanoid robot can be accurately controlled. This is not always the case, as many humanoid robots are position controlled. Therefore, we contribute a gait design framework and position controller for

[^10]fully actuated humanoid robots in the second part of this chapter. This approach is based on the ZMP and involves an inverse dynamics controller for single as well as double support phases of the gait. During the double support phase, the position of the joints can be controlled simultaneously with the ground contact forces.
The outline of this chapter is as follows. In Section 6.2 the input-output linearizing controller and gait parameter selection algorithm for underactuated bipeds are presented. In Section 6.3 the gait design and control law for fully actuated robots, based on the ZMP, is explained.

### 6.2 Control of underactuated humanoid robots

The main difficulty around the stability of bipedal robots is the fact that the ground contact is unilateral. The ground can only push on the biped and not pull, i.e. bipeds are not attached to the ground. This makes them floating base systems and (parts of) the gait may be underactuated. As mentioned before, controlling underactuated parts of the gait is key in improving the robustness of humanoid robots and cannot be ignored. In this section we show that this problem can be solved using an input-output linearizing controller as presented in [186]. Such controller attracts the full system dynamics to a lower dimensional zero dynamics. This zero dynamics is Lagrangian by construction, and in this way the zero dynamics energy is conserved during a step.
First, we repeat the work of [25, 45, II2, I86] to prove the stability of walking for planar bipeds with point feet. As an addition to this method we contribute a gait parameter selection algorithm that selects gait parameters such as the step size to achieve a desired walking speed. This controller is inspired by the foot placement analysis of [198, 202].
In the second part of this section, we contribute an extension of the walking controller to planar and three dimensional bipeds with finite sized feet exhibiting multiple phases and double support during the walking gait.

### 6.2.1 Planar bipeds with point feet

The first derivation of a stable foot placement controller is performed on the general class of multi-mass planar bipeds with point feet as presented in [II2, I86].

## Model

Let's recapture the model of a planar biped robot with point feet stated as impulsive system in Section 4.4.I. The model is rewritten in the standard vector field notation:

$$
\Sigma= \begin{cases}\dot{x}=f(x)+g(x) u, & x_{-} \notin \mathcal{S},  \tag{6.га}\\ x_{+}=d\left(x_{-}\right), & x_{-} \in \mathcal{S}\end{cases}
$$

with

$$
\begin{gathered}
f(x)=\left[\begin{array}{c}
\dot{q} \\
D^{-1}(q)(-C(q, \dot{q}) \dot{q}-G(q))
\end{array}\right] \\
g(x)=\left[\begin{array}{c}
0 \\
D^{-1}(q) B
\end{array}\right] \text { and } d\left(x_{-}\right)=\left[\begin{array}{c}
q_{-} \\
\Delta\left(q_{-}\right) \dot{q}_{-}
\end{array}\right]
\end{gathered}
$$

where the expressions for the different matrices are given in Section 4.4.I. We assume an ideal actuator and drive train as in Section 4.6.2 and use the identity matrix for the motor constants ${ }^{2}$.

## Walking controller

The purpose of the walking controller is to control the biped to walk at a desired speed without falling and to assure that no contact constraints are violated. We propose to control the biped during all phases using an input-output linearizing controller. We use output functions with relative degree two to define a desired trajectory. The number of output functions is chosen to equal the number of inputs, i.e. $N-1$ output functions are defined. These output functions are defined with respect to a monotonic coordinate instead of time and controlled to zero to represent so called virtual holonomic constraints; the motion of $N-1$ coordinates in the system is constrained to the $N$-th coordinate. After application of a nonlinear input-output linearizing feedback controller and a linear controller that drives the outputs asymptotically to zero, the system dynamics is constrained to a two dimensional manifold, which is called the zero dynamics. The zero dynamics is used to find the step size and torso lean angle on a step by step basis.
The output functions that we use are defined as follows:

$$
\begin{equation*}
y=h(q, p)=h_{0}(q)-h_{d}(\phi, p)-h_{p}(\phi), \tag{6.2}
\end{equation*}
$$

where $h_{0} \in \mathbb{R}^{N-1}$ are quantities that need to be controlled and $h_{d} \in \mathbb{R}^{N-1}$ are the desired trajectories of $h_{0}$ as function of a monotonically increasing quantity $\phi(q) \in \mathbb{R}$ and gait parameters $p$ such as step size, torso lean angle and push off intensity that are used to alter and control the gait. In the rest of this section, the dependence of $h$ on $p$ is largely omitted, unless it may lead to confusion. Finally, $h_{p} \in \mathbb{R}^{N-1}$ is used to make sure that the output functions remain zero at the impact [186, pp. 138-I44]. It is designed to cope with discontinuities in the velocities by selecting $h_{p}$ from the class of polynomials and computing its coefficients just after impact such that $h_{+}=0, \dot{h}_{+}=0, h_{p-}=0$ and $\dot{h}_{p-}=0$. Assume now that the matrix of Lie derivatives $L_{g} L_{f} h(x)$ is invertible, then the controller:

$$
\begin{equation*}
u(x)=\left(L_{g} L_{f} h(x)\right)^{-1}\left(v-L_{f}^{2} h(x)\right), \tag{6.3}
\end{equation*}
$$

[^11]applied to (6.I) results in the second order systems $\ddot{y}=v$, which can easily be stabilized using a input $v$, such as:
\[

$$
\begin{equation*}
v=-K_{p} y-K_{d} L_{f} y=-K_{p} y-K_{d} \dot{y}, \tag{6.4}
\end{equation*}
$$

\]

where $K_{p}, K_{d} \in \mathbb{R}^{N-1 \times N-1}$ are positive definite (diagonal) gain matrices.
After application of controller (6.3), the output functions (6.2) vanish and the full system dynamics is virtually constrained. The resulting zero dynamics in the coordinate $\phi$ is two dimensional and can easily be found [r6I]. Using the output functions $h$ equal to zero, we can find expressions for the states of the system in the form:

$$
\begin{equation*}
q=T(\phi), \quad \dot{q}=\frac{d T}{d \phi} \dot{\phi}=T^{\prime}(\phi) \dot{\phi} \text { and } \ddot{q}=\frac{d T^{\prime}}{d \phi} \dot{\phi}^{2}+T^{\prime}(\phi) \ddot{\phi}=T^{\prime \prime}(\phi) \dot{\phi}^{2}+T^{\prime}(\phi) \ddot{\phi} \tag{6.5}
\end{equation*}
$$

where $T: \mathbb{R} \rightarrow \mathbb{R}^{N}$. Moreover, since (6.I) has underactuation degree one, we can always find a non-unique orthogonal $B^{\perp}$ with respect to $B$, such that $B^{\perp} B=0$. Now, substitute the expressions for $q, \dot{q}$ and $\ddot{q}$ of (6.5) in the equations of motion (4.19a) and pre-multiply with $B^{\perp}$ to find the zero dynamics in the form [i6I]:

$$
\begin{equation*}
\alpha(\phi) \ddot{\phi}+\beta(\phi) \dot{\phi}^{2}+\gamma(\phi)=0 \tag{6.6}
\end{equation*}
$$

where

$$
\begin{aligned}
\alpha(\phi) & =B^{\perp} D(T(\phi)) T^{\prime}(\phi) \\
\beta(\phi) & =B^{\perp}\left(C\left(T(\phi), T^{\prime}(\phi)\right) T^{\prime}(\phi)+D(T(\phi)) T^{\prime \prime}(\phi)\right) \\
\gamma(\phi) & =B^{\perp} G(T(\phi))
\end{aligned}
$$

As shown in [25, I86], this zero dynamics can be simplified using a more convenient coordinate choice. Namely, consider the angular momentum $\sigma=D_{1}(\theta) \dot{\theta}$, with $D_{1}$, the first row of the inertia matrix. Evaluated on the zero dynamics, this gives $\sigma=\alpha(\phi) \dot{\phi}$. Now, take as coordinates $\phi$ and $\sigma$, so that the zero dynamics is given by:

$$
\begin{equation*}
\dot{\phi}=\frac{1}{\alpha(\phi)} \sigma, \quad \text { and } \quad \dot{\sigma}=\alpha(\phi) \ddot{\phi}+\frac{d \alpha(\phi)}{d \phi} \dot{\phi}^{2}=-\gamma(\phi), \tag{6.7}
\end{equation*}
$$

which is much simpler than (6.6). Then, along the solution of (6.7) we can consider:

$$
\begin{equation*}
\frac{d \sigma}{d \phi}=-\frac{\alpha(\phi) \gamma(\phi)}{\sigma} \tag{6.8}
\end{equation*}
$$

which gives:

$$
\begin{equation*}
\frac{1}{2} \sigma^{2}-\frac{1}{2} \sigma_{+}^{2}=-\int_{\phi_{+}}^{\phi} \alpha(s) \gamma(s) d s \tag{6.9}
\end{equation*}
$$

where $\sigma_{+}$is the initial angular momentum. The expression (6.9) holds along the solution of the zero dynamics. This is very special and particularly useful, because it gives the
velocity $\dot{\phi}$ as function of $\phi$, and represents, as such, a conserved quantity. Namely, consider the following:

$$
\begin{equation*}
\mathcal{K}(\phi, \dot{\phi})=\frac{1}{2} \sigma^{2}(\phi, \dot{\phi}), \quad \mathcal{K}_{+}=\frac{1}{2} \sigma_{+}^{2}, \quad \text { and } \quad \mathcal{P}(\phi)=\int_{\phi_{+}}^{\phi} \alpha(s) \gamma(s) d s \tag{6.го}
\end{equation*}
$$

then (6.9) can be written as:

$$
\begin{equation*}
\mathcal{K}+\mathcal{P}=\mathcal{K}_{+}, \tag{6.iг}
\end{equation*}
$$

where $\mathcal{K}$ is defined, analogously to [I86], as the 'zero dynamics kinetic energy' and $\mathcal{P}$ as the 'zero dynamics potential energy'3. Naturally, the 'zero dynamics total energy' is defined as:

$$
\begin{equation*}
\mathcal{E}(\phi, \dot{\phi})=\mathcal{K}(\phi, \dot{\phi})+\mathcal{P}(\phi)=\frac{1}{2} \sigma^{2}+\int_{\phi_{+}}^{\phi} \alpha(s) \gamma(s) d s \tag{6.12}
\end{equation*}
$$

which is constant during a step as can be seen from its time derivative:

$$
\dot{\mathcal{E}}_{i}=\sigma \dot{\sigma}+\alpha \gamma \dot{\phi}=-\alpha \gamma \dot{\phi}+\alpha \gamma \dot{\phi}=0
$$

This indicates that the zero dynamics (6.6) is Lagrangian. Because of this result, it is important to remark that a biped can only complete a step when:

$$
\begin{equation*}
\mathcal{K}_{+} \geq \mathcal{P}_{\max }=\max _{\phi_{+} \leq \phi \leq \phi_{-}} \mathcal{P}(\phi) \tag{6.гз}
\end{equation*}
$$

otherwise in (6.II) the zero dynamics kinetic energy becomes negative, which is impossible per (6.io). This condition simply means that the robot should possess enough energy at the beginning of the step to overcome its dead center, i.e. the point where the biped's CoM lies exactly above its stance foot.

## Gait parameter selection algorithm

Using the fact that the zero dynamics energy is constant during the step $k$, gives:

$$
\begin{equation*}
\mathcal{E}_{+}[k]=\mathcal{E}_{-}[k]=\mathcal{K}_{-}[k]+\mathcal{P}_{-}[k], \tag{6.14}
\end{equation*}
$$

because $\mathcal{E}_{+}[k]$ and $\mathcal{E}_{-}[k]$ represent the total energy at the beginning and end of step $k$ respectively. Moreover, the zero dynamics potential energy at the beginning of a step is zero, which yields:

$$
\begin{equation*}
\mathcal{E}[k+1]=\mathcal{K}_{+}[k+1]=\delta[k] \mathcal{K}_{-}[k]=\delta[k]\left(\mathcal{E}[k]-\mathcal{P}_{-}[k]\right), \tag{6.15}
\end{equation*}
$$

with

$$
\delta=\eta^{2}, \quad \text { and } \quad \eta=\frac{\partial \phi}{\partial q} \Delta\left(T\left(\phi_{-}\right)\right) T^{\prime}\left(\phi_{-}\right) \sigma_{-}, \quad \text { since } \quad \dot{\phi}_{+}=\eta \dot{\phi}_{-}
$$

[^12]The impact map $\Delta$ is given in Section 4.4.I. Now assume that we keep the gait parameters $p$ constant, such that the output functions (6.2) are the same for each step and also the variables $\mathcal{P}_{-}[k]$ and $\delta[k]$ are the same step after step. Then, we can define the sequence which relates the energy of $\operatorname{step} n$ with the initial energies:

$$
\begin{align*}
\mathcal{E}[1] & =\delta[0]\left(\mathcal{E}[0]-\mathcal{P}_{-}[0]\right), \\
\mathcal{E}[2] & =\delta[0]\left(\mathcal{E}[1]-\mathcal{P}_{-}[1]\right), \\
& =\delta[0]\left(\delta[0]\left(\mathcal{E}[0]-\mathcal{P}_{-}[0]\right)-\mathcal{P}_{-}[0]\right), \\
& =\delta^{2}[0] \mathcal{E}[0]-\mathcal{P}_{-}[0]\left(\delta^{2}[0]+\delta[0]\right), \\
\mathcal{E}[3] & =\delta[0]\left(\mathcal{E}[2]-\mathcal{P}_{-}[2]\right), \\
& =\delta[0]\left(\delta^{2}[0] \mathcal{E}[0]-\mathcal{P}_{-}[0]\left(\delta^{2}[0]+\delta[0]\right)-\mathcal{P}_{-}[0]\right), \\
& =\delta^{3}[0] \mathcal{E}[0]-\mathcal{P}_{-}[0]\left(\delta^{3}[0]+\delta^{2}[0]+\delta[0]\right), \\
\mathcal{E}[n] & =\delta^{n}[0] \mathcal{E}[0]-\mathcal{P}_{-}[0] \sum_{i=1}^{n} \delta^{i}[0] . \tag{6.16}
\end{align*}
$$

As can be seen, for $0<\delta<1$ this sequence has a limit:

$$
\begin{equation*}
\mathcal{E}_{\infty}=\lim _{n \rightarrow \infty} \mathcal{E}[n]=\mathcal{P}_{-}[0] \frac{\delta[0]}{\delta[0]-1}, \tag{6.i7}
\end{equation*}
$$

which means that for given gait parameters $p$ and initial conditions $\phi_{+}$and $\dot{\phi}_{+}$, the limit case total energy can be computed a priori. This total energy corresponds to the walking speed $\dot{\phi}$ as function of $\phi$ via (6.9) and (6.II). We are particularly interested in the midstance walking speed $\dot{\phi}_{m}$, i.e. the biped's walking speed at its dead center configuration $\phi_{m}$. By definition, this is the configuration where $\mathcal{P}$ is maximal:

$$
\begin{equation*}
\mathcal{P}_{\text {max }}:=\mathcal{P}\left(\phi_{m}\right), \tag{6.18}
\end{equation*}
$$

and $\mathcal{K}$ is minimal:

$$
\begin{equation*}
\mathcal{K}_{m}:=\mathcal{K}\left(\phi_{m}, \dot{\phi}_{m}\right)=\mathcal{E}-\mathcal{P}_{\text {max }} . \tag{6.19}
\end{equation*}
$$

Our goal is now to control this mid-stance walking speed $\dot{\phi}_{m}$ on a step by step basis by intelligently selecting the gait parameters $p$ in each step.
The controller that we contribute to achieve this alters the output functions (6.2) using the gait parameters $p$ representing step size, torso lean angle and push off intensity on a step to step basis to achieve a desired mid-stance walking speed $\dot{\phi}_{d}$. Through the output functions, the zero dynamics (6.6) and also $\mathcal{P}_{-}(p)$ and $\delta(p)$ depend on $p$. Since there is an upper limit on the energy of the system (6.17), there exists a maximal mid-stance velocity $\dot{\phi}_{\max }$ such that $\dot{\phi}_{\max } \geq \dot{\phi}_{m}(p)$ for every parameter vector $p_{\min } \leq p \leq p_{\max }$. The zero dynamics kinetic energy corresponding to this maximal speed is given by:

$$
\begin{align*}
\mathcal{K}_{\max }(p) & =\max _{p_{\min } \leq p \leq p_{\max }} \mathcal{E}_{\infty}(p)-\mathcal{P}_{\max }(p),  \tag{6.20}\\
& =\max _{p_{\min } \leq p \leq p_{\max }} \mathcal{P}_{-}(p) \frac{\delta(p)}{\delta(p)-1}-\mathcal{P}_{\max }(p) . \tag{6.2I}
\end{align*}
$$

Thus, for all $\dot{\phi}_{m}[k]<\dot{\phi}_{\max }$ there exists a parameter set $p$ such that $\dot{\phi}_{m}[k+1]>\dot{\phi}_{m}[k]$. Likewise, there is a lower limit on the energy of the system, and hence, there exists a $\dot{\phi}_{\text {min }}$ such that $\dot{\phi}_{\text {min }} \leq \dot{\phi}_{m}(p)$ for every parameter vector $p_{\text {min }} \leq p \leq p_{\text {max }}$. The zero dynamics kinetic energy corresponding to this minimum speed is given by:

$$
\begin{equation*}
\mathcal{K}_{\min }(p)=\min _{p_{\min } \leq p \leq p_{\max }} \mathcal{P}_{-}(p) \frac{\delta(p)}{\delta(p)-1}-\mathcal{P}_{\max }(p) \tag{6.22}
\end{equation*}
$$

In general, for reasonable parameter sets, $\dot{\phi}_{\text {min }}=0$. Thus for all $\dot{\phi}_{m}[k]>\dot{\phi}_{\text {min }}$ there exists a parameter set $p$ such that $\dot{\phi}_{m}[k+1]<\dot{\phi}_{m}[k]$.
These two observations indicate that for any mid-stance velocity in step $k$ there exist parameters to accelerate and decelerate the biped. The set of parameters to reach the desired mid-stance velocity $\dot{\phi}_{\text {min }} \leq \dot{\phi}_{d} \leq \dot{\phi}_{\max }$ in a minimal number of steps is obviously the one that accelerates or decelerates the biped the most towards the desired velocity. The gait parameter selection algorithm that performs this is thus given by:

$$
\begin{equation*}
p[k+1]=\underset{p_{\min } \leq p \leq p_{\max }}{\arg \min }\left(\dot{\phi}_{m}(p)[k+1]-\dot{\phi}_{d}\right)^{2} . \tag{6.23}
\end{equation*}
$$

### 6.2.2 Planar bipeds with finite sized feet

In this section we contribute an extension of the controller presented in the previous section to the general class of multi-mass planar bipeds with finite sized feet, exhibiting double support. In the double support phases, both feet of the robot are in contact with the ground, resulting in a closed kinematic chain. This introduces physical constraints on the system, implemented as Lagrange multipliers, representing the ground reaction forces. These physical constraints need to be incorporated in the controller design. In contrast to [155], we do not use a coordinate transformation, but we use expressions for the ground reaction forces in the controller, which give the opportunity to control the ground reaction forces and, as such, the ZMP directly.

## Model

Consider again the model as presented in Section 4.4.2, rewritten in vector field notation:

$$
\Sigma_{i}= \begin{cases}\dot{x}=f_{i}(x)+g_{i}(x) u_{i}, & x_{-} \notin \mathcal{S}_{i}  \tag{6.24a}\\ x_{+}=d_{i}\left(x_{-}\right), & x_{-} \in \mathcal{S}_{i} \\ Z_{i}(x)=0, & \forall x\end{cases}
$$

where $f_{i}, g_{i}, d_{i}$ and $Z_{i}$ are defined in Table 6.I for the different phases $i=1,2,3,4$ of the model. We again use an ideal actuator and drive train with the identity matrix for the motor constants.

Table 6.1: Mathematical model with different phases $i$ as defined in Section 4.4.2 and visualized in Figure 4.5. The phases represent: i) fully actuated single support, 2) underactuated single support, 3) touch down double support and 4) push off double support.

| $i$ | $f_{i}(x)$ | $g_{i}(x)$ | $d_{i}\left(x_{-}\right)$ | $Z_{i}(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 I | $\left[\begin{array}{c}0 \\ \dot{\theta} \\ 0 \\ -D_{22}^{-1}(q) H_{2}(q, \dot{q})\end{array}\right]$ | $\left[\begin{array}{c}0 \\ 0 \\ 0 \\ D_{22}^{-1}(q)\end{array}\right]$ | $x_{-}$ | 0 |
| $\begin{aligned} & \Lambda_{2}^{2} \\ & \Lambda^{3} \end{aligned}$ | $\left.\begin{array}{c} {\left[\begin{array}{c} \dot{q} \\ -D^{-1}(q) H(q, \dot{q}) \end{array}\right]} \\ \dot{q} \\ D^{-1}(q)\left(W_{3}(q) \lambda_{3}-H(q, \dot{q})\right) \end{array}\right]$ | $\left[\begin{array}{c}0 \\ D^{-1}(q) B \\ 0 \\ D^{-1}(q) B\end{array}\right]$ | $\left[\begin{array}{c}q_{-} \\ \Delta_{2}\left(q_{-}\right) \dot{q}_{-} \\ q_{-} \\ \Delta_{3}\left(q_{-}\right) \dot{q}_{-}\end{array}\right]$ | 0 $W_{3}^{\top}(q) \dot{q}$ |
| $\Lambda_{4}$ | $\left[\begin{array}{c}0 \\ \dot{\theta} \\ 0 \\ D_{22}^{-1}(q)\left(W_{4}(q) \lambda_{4}-H_{2}(q, \dot{q})\right.\end{array}\right]$ | $\left[\begin{array}{c} 0 \\ 0 \\ 0 \\ D_{22}^{-1}(q) \end{array}\right]$ | $x_{-}$ | $W_{4}^{\top}(q) \dot{q}$ |

## Walking controller

The walking controller aims to make the biped walking at a desired speed without falling. The controller also needs to make sure that no model assumptions are violated and that the system goes through all phases subsequently. This means that the contact situation with the ground needs to be controlled. A possible way to control the contact situation is through the position of the ZMP. The controller that we present in this section is inspired on [186, pp. 34I-347]. It is able to directly control the ground contact forces and, as such, the position of ZMP point. Similarly as in the previous section an input-output linearizing controller is used. We design the output functions again with relative degree two and such that they result in a two dimensional zero dynamics in each phase. This means that $N-1$ constraints (physical and virtual) are required in each phase. Since the number of physical constraints differs per phase, also the number of output functions differs. In the double support phases we even have more inputs than output functions, which means that inputs appear in the zero dynamics. These inputs can be used to control the ground reaction forces and the position of the ZMP. We select the inputs that appear in the zero dynamics as follows:

$$
\begin{equation*}
B u_{i}=B_{f i} u_{f i}(x)+B_{g i} u_{g i}(x), \tag{6.25}
\end{equation*}
$$

where $u_{f i} \in \mathbb{R}^{M_{i}}$ are the inputs that appear in the zero dynamics and $u_{g i} \in \mathbb{R}^{N-M_{i}-1}$ are the inputs used for the feedback linearization, with $M_{i}$ the number of inputs that appear in the zero dynamics in phase $i$ and $B_{f i}$ and $B_{g i}$ permutation matrices on the inputs. Using this expression for $u_{i}$, we may rewrite (6.24a) to:

$$
\begin{equation*}
\dot{x}=f_{c i}(x)+g_{c i}(x) u_{g i}, \tag{6.26}
\end{equation*}
$$

Table 6.2: Feedback linearizing controller vector fields.

| $i$ | $f_{c i}(x)$ | $g_{c i}(x)$ | $M_{i}$ |
| :---: | :---: | :---: | :---: |
| I | $\left[\begin{array}{c}0 \\ \dot{\theta} \\ 0 \\ D_{22}^{-1}(q)\left(B_{f 1} u_{f 1}-H_{2}(q, \dot{q})\right)\end{array}\right]$ | $\left[\begin{array}{c}0 \\ 0 \\ 0 \\ D_{22}^{-1}(q) B_{g 1}\end{array}\right]$ | 1 |
| 2 | $\left[\begin{array}{c} \dot{q} \\ -D^{-1}(q) H(q, \dot{q}) \end{array}\right]$ | $\left[\begin{array}{c}0 \\ D^{-1}(q) B_{g 2}\end{array}\right]$ | 0 |
| 3 | $\left[\begin{array}{c}\dot{q} \\ D^{-1}(q)\left(B_{f 3} u_{f 3}+W_{3}(q) \lambda_{3}-H(q, \dot{q})\right)\end{array}\right]$ | $\left[\begin{array}{c}0 \\ D^{-1}(q) B_{g 3}\end{array}\right]$ | 2 |
| 4 | $\left[\begin{array}{c}0 \\ \dot{\theta} \\ 0 \\ D_{22}^{-1}(q)\left(B_{f 4} u_{f 4}+W_{4}(q) \lambda_{4}-H_{2}(q, \dot{q})\right.\end{array}\right]$ | $\left[\begin{array}{c}0 \\ 0 \\ 0 \\ D_{22}^{-1}(q) B_{g 4}\end{array}\right]$ | 3 |

with $f_{c i}, g_{c i}$ and $M_{i}$ as defined in Table 6.2.
The output functions that we use are defined as follows:

$$
\begin{equation*}
y_{i}=h_{i}(q, p)=h_{0 i}(q)-h_{d i}\left(\phi_{i}, p\right)-h_{p i}\left(\phi_{i}\right), \tag{6.27}
\end{equation*}
$$

where $h_{0 i} \in \mathbb{R}^{N-M_{i}-1}$ are quantities that need to be controlled, $h_{d i} \in \mathbb{R}^{N-M_{i}-1}$ are the desired trajectories of $h_{0 i}$ as function of a monotonically increasing quantity $\phi_{i}(q) \in \mathbb{R}$ and gait parameters $p$ and $h_{p i} \in \mathbb{R}^{N-M_{i}-1}$ again guarantees that the output functions remain zero at the impact, [186, pp. 138-144]. Assume again that the matrix of Lie derivatives $L_{g_{c i}} L_{f_{c i}} h_{i}(x)$ is invertible, then the controller:

$$
\begin{equation*}
u_{g i}(x)=\left(L_{g_{c i}} L_{f_{c i}} h_{i}(x)\right)^{-1}\left(v_{i}-L_{f_{c i}}^{2} h_{i}(x)\right), \tag{6.28}
\end{equation*}
$$

applied to (6.26) results in the second order systems $\ddot{y}_{i}=v_{i}$, which can easily be stabilized using a preferred input $v_{i}$, such as:

$$
\begin{equation*}
v_{i}=-K_{p i} y_{i}-K_{d i} L_{f_{c i}} y_{i}=-K_{p i} y_{i}-K_{d i} \dot{y}_{i}, \tag{6.29}
\end{equation*}
$$

where $K_{p i}, K_{d i} \in \mathbb{R}^{N-1 \times N-1}$ are positive definite (diagonal) gain matrices.
The controller (6.28) zeros the output functions such that the system is virtually constrained. The resulting zero dynamics in the coordinate $\phi_{i}$ is again two dimensional. Using the physical and virtual constraints, we can find expressions for the states of the system in the form:

$$
\begin{equation*}
q_{i}=T_{i}\left(\phi_{i}\right), \quad \dot{q}_{i}=\frac{d T_{i}}{d \phi_{i}} \dot{\phi}_{i}=T_{i}^{\prime}\left(\phi_{i}\right) \dot{\phi}_{i} \text { and } \ddot{q}_{i}=\frac{d T_{i}^{\prime}}{d \phi_{i}} \dot{\phi}_{i}^{2}+T_{i}^{\prime}\left(\phi_{i}\right) \ddot{\phi}_{i}=T_{i}^{\prime \prime}\left(\phi_{i}\right) \dot{\phi}_{i}^{2}+T_{i}^{\prime}\left(\phi_{i}\right) \ddot{\phi}_{i} . \tag{6.30}
\end{equation*}
$$

Table 6.3: Zero dynamics expressions (the dependence of $T_{i}$ on $\phi_{i}$ has been omitted).

|  | $i$ | $\alpha_{i}\left(\phi_{i}\right)$ | $\beta_{i}\left(\phi_{i}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | I | $A_{1}^{\perp} D_{22}\left(T_{1}\right) T_{12}^{\prime}$ | $A_{1}^{\perp}\left(C_{22}\left(T_{1}, T_{1}^{\prime}\right) T_{12}^{\prime}+D_{22}\left(T_{1}\right) T_{12}^{\prime \prime}\right)$ |  |
|  | 2 | $A_{2}^{\perp} D\left(T_{2}\right) T_{2}^{\prime}$ | $A_{2}^{\perp}\left(C\left(T_{2}, T_{2}^{\prime}\right) T_{2}^{\prime}+D\left(T_{2}\right) T_{2}^{\prime \prime}\right)$ |  |
|  | 3 | $A_{3}^{\perp} D\left(T_{3}\right) T_{3}^{\prime}$ | $A_{3}^{\perp}\left(D\left(T_{3}\right) T_{3}^{\prime \prime}-\left(Y _ { 3 } ( T _ { 3 } ) \left(W_{3}\left(T_{3}\right)^{\top} D^{-1}\left(T_{3}\right) C\left(T_{3}, T_{3}^{\prime}\right)+\right.\right.\right.$ |  |
|  | 4 | $A_{4}^{\perp} D_{22}\left(T_{4}\right) T_{42}^{\prime}$ | $\begin{array}{r} A_{3}^{\frac{1}{( }\left(D_{22}\left(T_{4}\right) T_{42}^{\prime \prime}-\left(Y _ { 4 } ( T _ { 4 } ) \left(W_{4}\left(T_{4}\right)^{\top} D_{22}^{-1}\left(T_{4}\right) C_{22}\left(T_{4}, T_{4}^{\prime}\right)+\right.\right.\right.} \\ \left.\left.\left.\Omega_{4}\left(T_{4}, T_{4}^{\prime}\right)\right)-C_{22}\left(T_{4}, T_{4}^{\prime}\right)\right) T_{42}^{\prime}\right) \end{array}$ |  |
| $i$ | $\gamma_{i}\left(\phi_{i}\right)$ |  |  |  |
| I | $\begin{array}{lr}\left.\left(T_{12}\right)-B_{f 1} u_{f 1}\right) & B_{g 1} \\ A_{2}^{\perp} G\left(T_{2}\right) & B_{g 2}\end{array}$ |  |  |  |
| 2 |  |  |  |  |
| 3 |  | $\frac{1}{3}\left(\mathbb{I}-Y_{3}\left(T_{3}\right) W_{3}^{\top}\right.$ | ( $\left.\left.{ }_{3}\right) D^{-1}\left(T_{3}\right)\right)\left(G\left(T_{3}\right)-B_{f 3} u_{f 3}\right)$ | $\left(\mathbb{I}-Y_{3}\left(T_{3}\right) W_{3}^{\top}\left(T_{3}\right) D^{-1}\left(T_{3}\right)\right) B_{g 3}$ |
| 4 |  | $\left(\mathbb{I}-Y_{4}\left(T_{4}\right) W_{4}^{\top}\right.$ | $\left.\left.{ }_{4}\right) D_{22}^{-1}\left(T_{4}\right)\right)\left(G_{2}\left(T_{4}\right)-B_{f 4} u_{f 4}\right)$ | $\left(\mathbb{I}-Y_{4}\left(T_{4}\right) W_{4}^{\top}\left(T_{4}\right) D_{22}^{-1}\left(T_{4}\right)\right) B_{g 4}$ |

Using these expressions and the same rational as in the previous section, we can find the zero dynamics in the different phases:

$$
\begin{equation*}
\alpha_{i}\left(\phi_{i}\right) \ddot{\phi}_{i}+\beta_{i}\left(\phi_{i}\right) \dot{\phi}_{i}^{2}+\gamma_{i}\left(\phi_{i}\right)=0 \tag{6.3I}
\end{equation*}
$$

where $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are given in Table 6.3, with $T_{i}^{\top}=\left[\begin{array}{cc}T_{i 1} & T_{i 2}^{\top}\end{array}\right], T_{i 1} \in \mathbb{R}$ and $T_{i 2} \in \mathbb{R}^{N-1}$. Because of the physical constraints in the double support phases, we could not find more convenient coordinates for this zero dynamics. However, in [16I] a similar expression as (6.9) is derived, that holds along the solution of the zero dynamics ${ }^{4}$ :

$$
\begin{equation*}
\dot{\phi}_{i}^{2}=\psi_{i}\left(\phi_{i+}, \phi_{i}\right)\left(\dot{\phi}_{i+}^{2}-2 \int_{\phi_{i+}}^{\phi_{i}} \psi_{i}\left(s, \phi_{i+}\right) \frac{\gamma_{i}(s)}{\alpha_{i}(s)} d s\right), \quad \text { with } \quad \psi_{i}\left(\phi_{a}, \phi_{b}\right)=e^{-2 \int_{\phi_{a}}^{\phi_{b}} \frac{\beta_{i}(\tau)}{\alpha_{i}(\tau)} d \tau} \tag{6.32}
\end{equation*}
$$

where $\phi_{i+}$ and $\dot{\phi}_{i+}$ are the initial conditions of phase $i$. We may again define the zero dynamics energies as:

$$
\mathcal{K}_{i}\left(\phi_{i}, \dot{\phi}_{i}\right)=\frac{1}{2 \psi\left(\phi_{i+}, \phi_{i}\right)} \dot{\phi}_{i}^{2}, \quad \mathcal{K}_{i+}=\frac{1}{2} \dot{\phi}_{i+}^{2}, \quad \text { and } \quad \mathcal{P}_{i}\left(\phi_{i}\right)=\int_{\phi_{i+}}^{\phi_{i}} \psi\left(s, \phi_{i+}\right) \frac{\gamma_{i}(s)}{\alpha_{i}(s)} d s
$$

${ }^{4}$ Expression (6.9) can also be derived from (6.32) if we consider $\beta=\frac{d \alpha(\phi)}{d \phi}$, then:

$$
\psi\left(\phi_{a}, \phi_{b}\right)=e^{-2 \int_{\phi_{a}}^{\phi_{b}} \frac{\beta(\tau)}{\alpha(\tau)} d \tau}=e^{-2 \ln \left(\alpha\left(\phi_{b}\right)\right)+2 \ln \left(\alpha\left(\phi_{a}\right)\right)}=\frac{\alpha^{2}\left(\phi_{a}\right)}{\alpha^{2}\left(\phi_{b}\right)}
$$

which substituted in (6.32) results in:

$$
\dot{\phi}^{2}=\frac{\alpha^{2}\left(\phi_{+}\right)}{\alpha^{2}(\phi)}\left(\dot{\phi}_{+}^{2}-2 \int_{\phi_{+}}^{\phi} \frac{\alpha^{2}(s)}{\alpha^{2}\left(\phi_{+}\right)} \frac{\gamma(s)}{\alpha(s)} d s\right)
$$

and after rearranging this gives (6.9).
such that (6.32) can be written as:

$$
\begin{equation*}
\mathcal{K}_{i}+\mathcal{P}_{i}=\mathcal{K}_{i+} . \tag{6.33}
\end{equation*}
$$

The left hand side of this expression is the zero dynamics total energy of phase $i$ :

$$
\begin{equation*}
\mathcal{E}_{i}\left(\phi_{i}, \dot{\phi}_{i}\right)=\mathcal{K}_{i}\left(\phi_{i}, \dot{\phi}_{i}\right)+\mathcal{P}_{i}\left(\phi_{i}\right)=\frac{1}{2 \psi\left(\phi_{i+}, \phi_{i}\right)} \dot{\phi}_{i}^{2}+\int_{\phi_{i+}}^{\phi_{i}} \psi\left(s, \phi_{i+}\right) \frac{\gamma_{i}(s)}{\alpha_{i}(s)} d s \tag{6.34}
\end{equation*}
$$

which is constant as can be seen from its time derivative:

$$
\begin{aligned}
\dot{\mathcal{E}}_{i} & =\frac{\beta\left(\phi_{i}\right)}{\alpha\left(\phi_{i}\right)} \frac{1}{\psi\left(\phi_{i+}, \phi_{i}\right)} \dot{\phi}_{i}^{3}+\frac{1}{\psi\left(\phi_{i+}, \phi_{i}\right)} \dot{\phi}_{i} \ddot{\phi}_{i}+\psi\left(\phi_{i}, \phi_{i+}\right) \frac{\gamma_{i}\left(\phi_{i}\right)}{\alpha_{i}\left(\phi_{i}\right)} \dot{\phi}_{i}, \\
& =\frac{\beta\left(\phi_{i}\right)}{\alpha\left(\phi_{i}\right)} \frac{1}{\psi\left(\phi_{i+}, \phi_{i}\right)} \dot{\phi}_{i}^{3}-\frac{1}{\psi\left(\phi_{i+}, \phi_{i}\right)} \dot{\phi}_{i}\left(\frac{\beta_{i}\left(\phi_{i}\right)}{\alpha_{i}\left(\phi_{i}\right)} \dot{\phi}_{i}^{2}+\frac{\gamma_{i}\left(\phi_{i}\right)}{\alpha_{i}\left(\phi_{i}\right)}\right)+\frac{1}{\psi\left(\phi_{i+}, \phi_{i}\right)} \frac{\gamma_{i}\left(\phi_{i}\right)}{\alpha_{i}\left(\phi_{i}\right)} \dot{\phi}_{i}=0 .
\end{aligned}
$$

Per phase, the zero dynamics is thus Lagrangian. Thus, $\mathcal{E}_{i+}=\mathcal{E}_{i-}$ and the total zero dynamics energy only alters at the phase changes from $i=1,2,3,4$ to $j=(i \bmod 4)+1$ :

$$
\begin{equation*}
\Delta \mathcal{E}_{i}=\mathcal{E}_{j+}-\mathcal{E}_{i-}=\mathcal{K}_{j+}+\mathcal{P}_{j+}-\mathcal{K}_{i-}-\mathcal{P}_{i-}, \tag{6.35}
\end{equation*}
$$

where

$$
\mathcal{K}_{j+}=\nu_{i} \mathcal{K}_{i-} \quad \text { and } \quad \mathcal{P}_{j+}=0
$$

with

$$
\nu_{i}=\eta_{i}^{2} \psi_{i}\left(\phi_{i-}, \phi_{i+}\right) \quad \text { and } \quad \eta_{i}=\frac{\partial \phi_{j}}{\partial q} \Delta_{i}\left(T_{i}\left(\phi_{i-}\right)\right) T_{i-}^{\prime}\left(\phi_{i-}\right), \quad \text { since } \quad \dot{\phi}_{j+}=\eta_{i} \dot{\phi}_{i-}
$$

The impact maps $\Delta_{i}$ are given in Table 6.I and $\Delta_{1}=\Delta_{4}=\mathbb{I}$, the identity matrix.

## Gait parameter selection algorithm

Now we can derive the energy evolution during the entire step $k$ containing all four subsequent phases:

$$
\begin{aligned}
& \mathcal{E}_{1}[k]=\mathcal{K}_{1+}[k]=\mathcal{K}_{1-}[k]+\mathcal{P}_{1-}[k], \\
& \mathcal{E}_{2}[k]=\mathcal{K}_{2+}[k]=\nu_{1}[k] \mathcal{K}_{1-}[k]=\nu_{1}[k]\left(\mathcal{E}_{1}[k]-\mathcal{P}_{1-}[k]\right), \\
& \mathcal{E}_{3}[k]=\mathcal{K}_{3+}[k]=\nu_{2}[k] \mathcal{K}_{2-}[k]=\nu_{2}[k]\left(\mathcal{E}_{2}[k]-\mathcal{P}_{2-}[k]\right), \\
& \mathcal{E}_{4}[k]=\mathcal{K}_{4+}[k]=\nu_{3}[k] \mathcal{K}_{3-}[k]=\nu_{3}[k]\left(\mathcal{E}_{3}[k]-\mathcal{P}_{3-}[k]\right) .
\end{aligned}
$$

Continuation of this sequence to the next step results in an expression of the energy evolution from step $k$ to step $k+1$ :

$$
\begin{align*}
\mathcal{E}_{1}[k+1] & =\mathcal{K}_{1+}[k+1]=\nu_{4}[k] \mathcal{K}_{4-}[k]=\nu_{4}[k]\left(\mathcal{E}_{4}[k]-\mathcal{P}_{4-}[k]\right), \\
& =\nu_{4}[k]\left(\nu_{3}[k]\left(\nu_{2}[k]\left(\nu_{1}[k]\left(\mathcal{E}_{1}[k]-\mathcal{P}_{1-}[k]\right)-\mathcal{P}_{2-}[k]\right)-\mathcal{P}_{3-}[k]\right)-\mathcal{P}_{4-}[k]\right), \\
& =\mathcal{E}_{1}[k] \prod_{i=1}^{4} \nu_{i}[k]-\sum_{i=1}^{4} \mathcal{P}_{i-}[k] \prod_{\varsigma=i}^{4} \nu_{\varsigma}[k] . \tag{6.36}
\end{align*}
$$

As can be seen, we must take into account that the variables $\nu_{i}[k]$ and $\mathcal{P}_{i-}[k]$ are not only different in each phase, but they may also differ per step, because they are dependent on the gait parameters $p$ like the step size, torso lean angle and push off intensity. Now, we want to analyze the behavior of the system on a step by step basis, for which we simplify the notation, such that it is no longer dependent on the different phases during a step:

$$
\begin{equation*}
\mathcal{E}[k+1]=\delta[k]\left(\mathcal{E}[k]-\mathcal{P}_{-}[k]\right), \tag{6.37}
\end{equation*}
$$

with

$$
\begin{gathered}
\mathcal{E}[k]=\mathcal{E}_{1}[k], \quad \delta[k]=\nu_{1}[k] \nu_{2}[k] \nu_{3}[k] \nu_{4}[k] \quad \text { and } \\
\mathcal{P}_{-}[k]=\mathcal{P}_{1-}[k]+\frac{1}{\nu_{1}[k]} \mathcal{P}_{2-}[k]+\frac{1}{\nu_{1}[k] \nu_{2}[k]} \mathcal{P}_{3-}[k]+\frac{1}{\nu_{1}[k] \nu_{2}[k] \nu_{3}[k]} \mathcal{P}_{4-}[k] .
\end{gathered}
$$

This is the same expression as (6.15) for robots with point feet and only one phase per step, except that the expressions for $\mathcal{E}, \mathcal{P}_{-}$and $\delta$ are much more complicated with feet. Hence, we can follow the same approach to find a gait parameter selection algorithm for step size, torso lean angle and push off intensity.

## ZMP controller

What remains, is to define the inputs $u_{f i}$. As mentioned before, these inputs are used to guarantee that the physical constraints are not violated and that all phases are executed subsequently. Inspired by [186, pp. 34I-347], this can be achieved by control of the ZMP, since the ZMP determines when a foot starts to rotate with respect to the ground. If the ZMP lies in the support polygon, the foot remains flat on the ground, if the ZMP moves to the edge of the support polygon, the foot may start to tip. Consider again the horizontal position of the ZMP (4.42) in phase $i=1,3,4$ :

$$
\begin{equation*}
x_{z m p i}=\frac{M g_{0} x_{c o m i}-\dot{\sigma}_{y i}}{M g_{0}+\dot{\rho}_{z i}}, \tag{6.38}
\end{equation*}
$$

which can be evaluated on the zero dynamics if we realize that:

$$
\begin{align*}
& \dot{\rho}_{z i}=\frac{\partial \rho_{z i}}{\partial q_{i}} \dot{q}_{i}+\frac{\partial \rho_{z i}}{\partial \dot{q}_{i}} \ddot{q}_{i}=\frac{\partial \rho_{z i}}{\partial q_{i}} T_{i}^{\prime} \dot{\phi}_{i}+\frac{\partial \rho_{z i}}{\partial \dot{q}_{i}}\left(T_{i}^{\prime \prime} \dot{\phi}_{i}^{2}-T_{i}^{\prime} \frac{1}{\alpha_{i}}\left(\beta_{i} \dot{\phi}_{i}^{2}+\gamma_{i}\right)\right),  \tag{6.39}\\
& \dot{\sigma}_{y i}=\frac{\partial \sigma_{y i}}{\partial q_{i}} \dot{q}_{i}+\frac{\partial \sigma_{y i}}{\partial \dot{q}_{i}} \ddot{q}_{i}=\frac{\partial \sigma_{y i}}{\partial q_{i}} T_{i}^{\prime} \dot{\phi}_{i}+\frac{\partial \sigma_{y i}}{\partial \dot{q}_{i}}\left(T_{i}^{\prime \prime} \dot{\phi}_{i}^{2}-T_{i}^{\prime} \frac{1}{\alpha_{i}}\left(\beta_{i} \dot{\phi}_{i}^{2}+\gamma_{i}\right)\right) . \tag{6.40}
\end{align*}
$$

Now, we can substitute the expressions for $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ from Table 6.3 into (6.39) and (6.40) and subsequently substitute these in (6.38) to evaluate the ZMP position on the zero dynamics, which has the general form:

$$
\begin{equation*}
x_{z m p i}\left(\phi_{i}, \dot{\phi}_{i}\right)=\frac{z_{1 i}\left(\phi_{i}, \dot{\phi}_{i}\right)-z_{2 i}\left(\phi_{i}\right) u_{f i}}{z_{3 i}\left(\phi_{i}, \dot{\phi}_{i}\right)+z_{4 i}\left(\phi_{i}\right) u_{f i}}, \tag{6.4I}
\end{equation*}
$$

where $z_{1 i}, z_{3 i} \in \mathbb{R}$ and $z_{2 i}, z_{4 i} \in \mathbb{R}^{M_{i}}$. The ZMP location is only one dimensional, so only one input is required to control it. Therefore, we decompose $u_{f i}^{\top}=\left[\begin{array}{ll}u_{f i a} & u_{f i b}^{\top}\end{array}\right]$, $z_{2 i}^{\top}=\left[\begin{array}{ll}z_{2 i a} & z_{2 i b}^{\top}\end{array}\right]$ and $z_{4 i}^{\top}=\left[\begin{array}{ll}z_{4 i a} & z_{4 i b}^{\top}\end{array}\right]$, with $u_{f i a}, z_{2 i a}, z_{4 i a} \in \mathbb{R}$ and $u_{f i b}, z_{2 i b}, z_{4 i b} \in \mathbb{R}^{M_{i}-1}$. The ZMP location can now be controlled using $u_{f i b}=0$ and:

$$
\begin{equation*}
u_{f i a}=\frac{z_{1 i}\left(\phi_{i}, \dot{\phi}_{i}\right)-z_{3 i a}\left(\phi_{i}\right) x_{z m p i, d}}{z_{2 i}\left(\phi_{i}, \dot{\phi}_{i}\right)+z_{4 i a}\left(\phi_{i}\right) x_{z m p i, d}} \tag{6.42}
\end{equation*}
$$

where $x_{z m p i, d}$ is the desired ZMP location. The choice to use only one input to control the ZMP location may cause a high torque in this input. In Section 6.3.2 a different approach is derived, which distributes the ZMP control among the available inputs. Note that the choice for $u_{f i}$ also affects the zero dynamics, since it needs to be substituted into the expressions in Table 6.3.

### 6.2.3 3D Bipeds with finite sized feet

In this section we explain how the walking controller can be extended to three dimensional bipeds with finite sized feet and maximally one degree of underactuation. As noted in Section 4.4.3, the model of this three dimensional biped is practically the same as the planar variant. This means that also the same controller can be applied as presented in the previous section. The only difference lies in the number of actuators $M_{i}$ that appear in the zero dynamics, because the number of physical constraints varies between the planar and three dimensional case. To make sure that the three dimensional biped also has a two dimensional zero dynamics in every phase, $M_{1}=1, M_{2}=0, M_{3}=5$ and $M_{4}=6$.
This also means that the controls $u_{f i}$ need to be redefined to make sure that the physical constraints are not violated and that all phases are executed subsequently. In this case the ZMP location is two dimensional (4.42), described by coordinates $x_{z m p}$ and $y_{z m p}$. Similarly as in the previous section, both can be evaluated on the zero dynamics, after which two inputs can be derived to control the ZMP location.

### 6.3 Control of fully actuated humanoid robots

Balance of humanoid robots consists of finding and controlling joint motions such that the robot can walk without falling. In this section we describe the gait design, trajectory generation and control of fully actuated humanoid robots. A humanoid robot with feet is fully actuated when its stance foot remains flat on the ground. As also indicated in Section 4.7.3, as long as the ZMP remains in the support polygon, the stance foot remains flat on the ground and the robot remains fully actuated. This condition can be used in gait design for humanoid robots with feet to quickly develop stable walking motions, although, it has been said that the resulting gaits are not energy efficient nor robust [I8].

Gaits of fully actuated humanoid robots with feet normally consist of single and double support periods. The model for the single support period of a gait is described in Section 4.5.2, whereas the model for the double support period is given in Section 4.5.r. The gaits designed in this section must take into account the constraints acting on the feet when they are both in contact with the ground. The actuator and drive train are again assumed to be ideal.

In this section, a convenient method for gait design based on the ZMP is explained and a controller is derived that is easily applicable to position controlled fully actuated humanoid robots with feet. This gait design and controller are used in Chapter 8 to let the humanoid robot TUlip walk.

### 6.3.1 Gait design

We design a ZMP based gait in two parts. First, preferred CoM and swing foot trajectories are designed in task space and secondly these task space trajectories are mapped to joint trajectories using an inverse kinematics algorithm. Two types of preferred task space trajectories are designed in this section. First we focus on statically stable gaits and then we derive a dynamically stable gait.

## Statically stable gait design

In this section we explain the design of a statically stable gait for fully actuated humanoid robots with feet. A statically stable gait is a gait designed such that the robot is always in static balance, i.e. the robot always has its CoM within the support polygon of its feet. This type of gait is stable in the absence of environmental disturbances as long as the humanoid robot tracks the trajectories relatively slowly such that any dynamic effect due to the motion of the links can be neglected.
Each statically stable gait consists of different phases, representing for example the initial, final, single support or double support phase. In each phase we prescribe the desired task space CoM position, torso orientation and swing foot position and orientation with respect to the stance foot. The orientations are parameterized by three angles using the roll-pitchyaw convention. One combination of a desired task space CoM position, torso orientation and swing foot position and orientation describes one posture of the robot.
Each phase $i \in \mathbb{N}$ has an initial posture $x_{0, i} \in \mathbb{R}^{6} \times \mathbb{T}^{6}$ and a final posture $x_{f, i} \in \mathbb{R}^{6} \times \mathbb{T}^{6}$ parameterized by coordinates that describe the desired task space positions and orientations. These initial and final postures are interconnected by smooth trajectories in task space described by a cosine velocity profile:

$$
\begin{equation*}
x_{d, i}(t)=\frac{x_{f, i}-x_{0, i}}{t_{f, i}}\left(\frac{t-\sin \left(\frac{2 \pi}{t_{f, i}} t\right)}{2 \pi} t_{f}\right)+x_{0, i}, \tag{6.43}
\end{equation*}
$$

where $t_{f, i} \in \mathbb{R}^{+}$is the duration of the phase. The total task space trajectory of the CoM and swing foot of phase $i$ is thus given by $x_{d, i} \in \mathbb{R}^{6} \times \mathbb{T}^{6}$. Finally, stitching multiple phases together yield the desired task space trajectories for the entire gait:

$$
x_{d}=\left[\begin{array}{lll}
x_{d, 1} & \ldots & x_{d, n} \tag{6.44}
\end{array}\right],
$$

where $n$ is the number of phases. Thus, each trajectory $x_{d}^{\top}=\left[\begin{array}{ll}p_{d}^{\top} & \phi_{d}^{\top}\end{array}\right]$ describes six position coordinates $p_{d} \in \mathbb{R}^{6}$ of the CoM and swing foot and six orientation angles $\phi_{d} \in \mathbb{T}^{6}$ of the torso and swing foot.

## Dynamically stable gait design

In this section we explain the design of a dynamically stable gait for fully actuated humanoid robots with feet. A dynamically stable gait is a gait designed such that the robot is always in dynamic balance, i.e. the robot always has its ZMP within the support polygon of its feet. We again prescribe the desired task space CoM position, torso orientation and swing foot position and orientation with respect to the stance foot.
The position of the CoM is prescribed by the evolution of a linear inverted pendulum [93] with its base in the stance ankle of the robot. The ZMP of a linear inverted pendulum that falls freely under influence of gravity lies in its base. So if this model is used for the CoM of the biped, this guarantees that the ZMP remains at the ankle of the robot. One step in this dynamically stable walking gait consists out of one phase described by the evolution of the linear inverted pendulum. The gait does not contain double support periods, but contains stacked single support periods alternating between the right and left ankle as base for the linear inverted pendulum. The desired evolution of the CoM in $x_{c o m, i}$ and $y_{c o m, i}$ direction in a horizontal plane at height $z_{c}$ in phase $i$ is:

$$
\begin{align*}
& x_{c o m, i}(t)=x_{c o m, i}\left(t_{0, i}\right) \cosh \left(\frac{t-t_{0, i}}{T_{c}}\right)+T_{c} \dot{x}_{c o m, i}\left(t_{0, i}\right) \sinh \left(\frac{t-t_{0, i}}{T_{c}}\right)  \tag{6.45}\\
& y_{c o m, i}(t)=y_{c o m, i}\left(t_{0, i}\right) \cosh \left(\frac{t-t_{0, i}}{T_{c}}\right)+T_{c} \dot{y}_{c o m, i}\left(t_{0, i}\right) \sinh \left(\frac{t-t_{0, i}}{T_{c}}\right) \tag{6.46}
\end{align*}
$$

where $t_{0, i}$ is the time at the beginning of phase $i$ and $T_{c}=\sqrt{\frac{z_{c}}{g_{0}}}$.
The swing foot position mirrors the CoM position to achieve a symmetric step. These foot and CoM positions during step $i$ are again stored in the vector $p_{d, i}^{\top}$. The desired orientations of the torso and swing foot are described by cosine velocity profiles, parameterized by three angles using the roll-pitch-yaw convention and stored in the vector $\phi_{d, i}^{\top}$. All these task space trajectories of step $i$ are again stored in $x_{d, i}^{\top}=\left[\begin{array}{ll}p_{d, i}^{\top} & \phi_{d, i}^{\top}\end{array}\right]$ and stitching multiple steps together yield the desired task space trajectories for the entire gait: $x_{d}=\left[\begin{array}{lll}x_{d, 1} & \ldots & x_{d, n}\end{array}\right]$, where $n$ is the number of steps in the gait.

## Inverse kinematics algorithm

The task space trajectories defined in the previous sections can be mapped to joint angles in order to control the robot in joint space. We use an inverse kinematics algorithm based on differential kinematics:

$$
\begin{equation*}
\dot{x}_{d}=J_{d}\left(q_{d}\right) \dot{q}_{d}, \tag{6.47}
\end{equation*}
$$

where $J_{d} \in \mathbb{R}^{N \times N}$ represents the geometric Jacobian of the CoM position, torso orientation and swing foot position and orientation with respect to the stance foot and $\dot{q}_{d} \in \mathbb{R}^{N}$ are the desired joint angular velocities, which are given by:

$$
\begin{equation*}
\dot{q}_{d}=J_{d}^{-1}\left(q_{d}\right) \dot{x}_{d} . \tag{6.48}
\end{equation*}
$$

The desired joint angles $q_{d} \in \mathbb{T}^{N}$ can be computed from:

$$
\begin{equation*}
q_{d}(t)=\int_{0}^{t} \dot{q}_{d}(\varsigma) d \varsigma+q_{d}(0) \tag{6.49}
\end{equation*}
$$

But, on a physical robot this must be implemented in discrete time and discrete integration may lead to drift. Therefore, we use an inverse kinematics algorithm presented in [163]. Hereto, we define the error between the desired and (possibly drifted) computed task space position as:

$$
\begin{equation*}
e_{p}=p_{d}-p_{e} \tag{6.50}
\end{equation*}
$$

where $p_{e}$ is the computed task space position of the CoM and swing foot using forward kinematics of the desired joint angles $q_{d}$. For the orientation we derive the desired roll-pitch-yaw rotation matrix $R_{d}=\left[\begin{array}{lll}n_{d} & s_{d} & a_{d}\end{array}\right]$ from the desired task space orientation $\phi_{d}$ where $n_{d}, s_{d}$ and $a_{d}$ are simply the columns of $R_{d}$. Similarly, the roll-pitch-yaw rotation matrix computed by forward kinematics of the desired joint angles $q_{d}$ is $R_{e}=\left[\begin{array}{lll}n_{e} & s_{e} & a_{e}\end{array}\right]$, which results in the task space orientation error:

$$
\begin{equation*}
e_{o}=\frac{1}{2}\left(n_{e} \times n_{d}+s_{e} \times s_{d}+a_{e} \times a_{d}\right) . \tag{6.5I}
\end{equation*}
$$

These errors (6.50) and (6.5I) are used in (6.48) to compensate for drift:

$$
\dot{q}_{d}=J_{d}^{-1}(q)\left[\begin{array}{c}
\dot{p}+K_{p} e_{p}  \tag{6.52}\\
L^{-1}\left(L^{\top} \omega_{d}+K_{o} e_{o}\right)
\end{array}\right],
$$

where

$$
\begin{equation*}
L=-\frac{1}{2}\left(S\left(n_{d}\right) S\left(n_{e}\right)+S\left(s_{d}\right) S\left(s_{e}\right)+S\left(a_{d}\right) S\left(a_{e}\right)\right) \tag{6.53}
\end{equation*}
$$

with $S(\cdot)$ the skew-symmetric matrix of its vector argument. The system (6.52) is asymptotically stable for the positive definite matrices $K_{p}=\operatorname{diag}\left(K_{p, 1}, \ldots, K_{p, 6}\right)$ and $K_{o}=$ $\operatorname{diag}\left(K_{o, 1}, \ldots, K_{o, 6}\right)$. Moreover, it can be shown that discrete time integration of (6.52) in (6.49) does not result in drift [163].

### 6.3.2 Control

The predefined joint trajectories need to be tracked on the humanoid robot such that it performs the desired gait. We propose a relatively simple position controller consisting of two parts: a local joint PD feedback controller $\tau_{f b}$ and a model based dynamics compensation feedforward controller $\tau_{f f}$ :

$$
\begin{equation*}
\tau=\tau_{f b}+\tau_{f f} \tag{6.54}
\end{equation*}
$$

First, expressions are given for the feedback controller. Then, the feedforward controller is explained. We must distinguish between the single and double support periods of the gait, since the controller must take into account the constraints that act on the feet of the robot in the double support period. In this period the positions and orientation of the feet are fixed, which in general results in an overactuated system since more actuators are available than degrees of freedom. We show that these actuators can be used to control the ground contact forces simultaneously with the joint positions.

## Linear local joint control

We use local PD control on each joint to track the desired reference trajectories and to make the system (4.25) and (4.24) robust against disturbances:

$$
\begin{equation*}
\tau_{f b}=K_{p} e+K_{d} \dot{e} \tag{6.55}
\end{equation*}
$$

where $\tau_{f b}$ are joint controller feedback torques, $e=q_{d}-q$ are the tracking errors and $K_{p}=\operatorname{diag}\left(K_{p, 1}, \ldots, K_{p, N}\right)$ and $K_{d}=\operatorname{diag}\left(K_{d, 1}, \ldots, K_{d, N}\right)$ are the controller gains. These gains must be tuned for maximal performance without destabilizing the system.

## Single support feedforward dynamics compensation

Due to the possibly limited bandwidth of the local PD controllers, there are always feedback tracking errors in the joint angles. As a solution, we propose a model based feedforward algorithm to improve the tracking performance. The feedforward torques for parts of the gait where the robot is in single support are computed from (4.25):

$$
\begin{equation*}
\tau_{f f}=D\left(q_{d}\right) \ddot{q}_{d}+C\left(q_{d}, \dot{q}_{d}\right) \dot{q}_{d}+G\left(q_{d}\right) \tag{6.56}
\end{equation*}
$$

which is relatively straightforward.

## Double support feedforward dynamics compensation

In double support, the situation is more complicated, because there exists a closed kinematic chain, which implies that the biped is overactuated. This means that there exist different combinations of actuator torques that realize the same desired motion. These
different combination, however, may result in a different distribution of the contact forces. In contrast to previous sections where we simply ignored some actuators, here we propose a method that optimally distributes the control effort over all actuators while following the desired joint reference trajectories. To prevent jumps when switching from the single to the double support phase and vice versa, the controller also makes sure that the actuator torques are smoothly transferred between these phases. This can be achieved by controlling the contact forces simultaneously with the joint trajectories. It must be assured that the joint torques are selected such that no forces act on the swing foot at the beginning and end of the double support phase, which guarantees that the contact situation before and after phase changes is the same.

For simplicity, we start with a planar model using a floating base and constraints as presented in Section 4.5.I. We assume that the friction is high enough to prevent the robot from sliding, so we are only interested in controlling the normal contact forces $\lambda_{N}^{\top}=$ $\left[\begin{array}{llll}\lambda_{2} & \lambda_{3} & \lambda_{4} & \lambda_{5}\end{array}\right]$. The corresponding vertical force and moment balance on the feet is shown in Figure 6.ia. As can be seen, two normal constraint forces are required per foot to fix their orientation and vertical position. These constraints can be positioned anywhere on the foot, as long as they do not intersect. In Figure 6.ra, the position of $\lambda_{2}$ and $\lambda_{5}$ has been chosen to be the start and end of the ZMP trajectory during the double support phase, because this simplifies the following analysis.
Consider $\alpha_{i}=\frac{t-t_{0, i}}{t_{f, i}-t_{0, i}}$, the normalized time in phase $i$. The vertical force and moment balance of the feet as depicted in Figure 6.ra should hold for the entire phase:

$$
\begin{equation*}
\sum_{j=1}^{5} \lambda_{j}=0 \quad \text { and } \quad \sum_{j=1}^{5} p_{j} \lambda_{j}=0, \quad 0 \leq \alpha_{i} \leq 1 \tag{6.57}
\end{equation*}
$$

where $p_{j}$ are the horizontal locations where $\lambda_{j}$ act. Herein, $\lambda_{1}=M g_{0}+\dot{\rho}_{z}\left(q_{d}, \dot{q}_{d}, \ddot{q}_{d}\right)$, the total ground reaction force of the robot which acts at the ZMP, thus $p_{1}=x_{z m p}\left(q_{d}, \dot{q}_{d}, \ddot{q}_{d}\right)$. The goal is now to find desired $\lambda_{j} \geq 0, j=2,3,4,5$ such that (6.57) holds and:

$$
\begin{cases}\lambda_{2}=-\lambda_{1}, \lambda_{3}=\lambda_{4}=\lambda_{5}=0, & \alpha_{i}=0,  \tag{6.58}\\ \lambda_{2} \geq 0, \lambda_{3} \geq 0, \lambda_{4} \geq 0, \lambda_{5} \geq 0, & 0<\alpha_{i}<1, \\ \lambda_{2}=\lambda_{3}=\lambda_{4}=0, \lambda_{5}=-\lambda_{1}, & \alpha_{i}=1,\end{cases}
$$

where the special positioning of $\lambda_{2}$ and $\lambda_{5}$ is used. Any expression that suffices these conditions guarantees a smooth transition between single and double support, because the contact situations at the moments of switching are the same. There exist many solutions to this problem, since (6.57) consist of only two linear equations with four unknowns. This can be solved using an optimization routine, but in order to have a low computational complexity, an analytical expression is preferred. An example of such expression is the solution to (6.57) in combination with:

$$
\begin{equation*}
\frac{\lambda_{2}}{\lambda_{2}+\lambda_{3}}=1-\alpha_{i}, \quad \text { and } \quad \frac{\lambda_{5}}{\lambda_{4}+\lambda_{5}}=\alpha_{i}, \tag{6.59}
\end{equation*}
$$



Figure 6.1: Normal force and moment balance on the constraint feet of the robot in double support. The ZMP trajectory is indicated with gray dashed line.
which results in expressions for $\lambda_{j}, j=2,3,4,5$ that suffice (6.57) and (6.58). Given these desired normal contact forces, the feedforward torques that simultaneously track the desired joint reference trajectories and normal contact forces can be computed using the Moore-Penrose pseudo-inverse:

$$
\begin{equation*}
\tau_{f f}=\left(A^{\top} A\right)^{-1} A^{\top} b \tag{6.60}
\end{equation*}
$$

with

$$
\begin{gathered}
A=\left(\mathbb{I}-W_{T} Y_{T} W_{T}^{\top} D_{e}^{-1}\right) B_{e}, \quad Y_{T}=\left(W_{T}^{\top} D_{e} W_{T}\right)^{-1} \quad \text { and } \\
b=D_{e} \ddot{q}_{d}+H_{e}-W_{N} \lambda_{N}-W_{T} Y_{T}\left(W_{T}^{\top} D_{e}^{-1}\left(H_{e}-W_{N} \lambda_{N}\right)-w_{e}\right),
\end{gathered}
$$

all evaluated at $q_{d}, \dot{q}_{d}$ and $\ddot{q}_{d}$.
For three dimensional robots a very similar approach can be employed, see Figure 6.rb. The differences are that in this case there are three equations in (6.57): one vertical force balance and two moment balances and six unknown desired normal contact forces. So instead of two, three additional expressions need to be found as in (6.59). Experimental results with this controller on the humanoid robot TUlip are presented in Chapter 8.

### 6.4 Conclusion

In this chapter stability of bipedal robots is analyzed and controllers are designed for stable bipedal locomotion. A controller design is presented for bipeds that exhibit hybrid phases, underactuation and non-smooth dynamics. Inspired by earlier work [I86], It is shown how these systems exhibit an interesting feature when they are controlled by input-output feedback linearization using output functions that, when controlled to zero, represent virtual holonomic constraints. This controller attracts the full system dynamics to a lower dimensional one, which appears Lagrangian and conserves its energy. This feature is exploited and used to design a gait parameter selection algorithm for foot placement, torso orientation and push off intensity to control the gait and walking speed of the biped. This algorithm is contributed for planar bipeds with point feet and planar and three dimensional
bipeds with finite sized feet, exhibiting a double support phase. The control structure works for all bipeds with maximal underactuation degree one.
Besides underactuated bipeds, also a traditional controller based on the ZMP is developed for fully actuated humanoid robots that are position controlled. This controller works in single as well as double support periods of the gait, simultaneously controlling the contact forces, which result from the double support constraints on the feet, and joint positions of the biped.

## Chapter 7

## Gait analysis

### 7.1 Introduction

In this chapter we analyze the influence on the walking gait, of gait parameters such as step size, torso lean angle and push off intensity as introduced in Chapter 6. More specifically, we investigate how the gait parameters influence the walking velocity of the biped step by step. We determine conditions on the gait parameters that accelerate and decelerate the biped and find parameter combinations for which the biped walks with constant speed as it performs a limit cycle.
In the first part of this chapter, we perform this gait analysis on the zero dynamics of the underactuated biped models presented in Section 4.4 and controlled by the controllers presented in Section 6.2. We use the fact that the zero dynamics is Lagrangian and exhibits a conserved quantity along its solution to compute the walking speed for various parameter combinations. We show that for some parameter combinations the biped maximally accelerates or decelerates and using this information we compute the minimal number of steps required to reach a specific desired walking speed from the biped's current walking speed. In the second part of this chapter, simulations are performed on the underactuated biped robot models controlled with the input-output linearizing controllers. We show how the gait parameter selection algorithm selects gait parameters to walk with varying desired walking speeds step after step. For the planar bipeds, different simulations of walking gaits are presented in which the bipeds accelerate, decelerate, perform limit cycles and come to stop. For the three dimensional walking biped model, an omnidirectional walking gait is presented.
The outline of this chapter is as follows. In Section 7.2 the influence of important gait parameters such as step size, torso lean angle and push off intensity on the walking speed is analyzed. In Section 7.3 simulation results are given.

### 7.2 Analysis of important gait parameters

The controllers that we proposed in Section 6.2 are analyzed in this section. They are applied to the different robot models as presented in Chapter 4. Since the resulting hybrid zero dynamics exhibits a conserved quantity, we can investigate the influence of certain gait parameters such as step size, torso lean angle and push off intensity on the walking gaits. In the first part of this section the mid-stance walking speed of planar bipeds with point feet is analyzed for different step sizes and torso lean angles. After that the influence of the push off intensity on the walking speed is analyzed for planar bipeds with finite sized feet and finally the influence of foot placement on the walking speed is investigated for three dimensional bipeds with finite sized feet.

### 7.2.1 Step size and torso lean angle for planar bipeds with point feet

In this section we analyze the hybrid zero dynamics of planar bipeds with point feet, to give insight into the controller that we proposed in Section 6.2.I. The walking gaits that these models perform are quite easy to design, since they live in a planar world. Walking is achieved if, in each step, the CoM of the robot passes its dead center and the robot moves its swing leg in front of its stance leg. The number of gait parameters that can be controlled is rather limited. In this section we focus on two gait parameters: step size and torso lean angle. We discovered through many output function trials that these parameters have a quite large, direct influence on the walking speed of the biped. So in this section we analyze the influence of step size and torso lean angle on the step-to-step walking speed of planar bipeds with point feet.

## Analysis of the three link biped

The first model that we analyze is the three link biped as shown in Figure 7.Ia. We model the biped as an impulsive system with ideal actuators as presented in Section 4.4.I. The biped is controlled using the input-output linearizing controller presented in Section 6.2.I that attracts the full system dynamics to a lower dimensional hybrid zero dynamics. The output functions are chosen as:

$$
h(\phi)=\left[\begin{array}{c}
-\phi-\alpha_{t}-\theta_{2}  \tag{7•I}\\
\pi-2 \phi-\theta_{2}-\theta_{3}
\end{array}\right]-h_{p}(\phi),
$$

which represent, when controlled to zero, two virtual holonomic constraints, driven by the unactuated degree of freedom $\phi=\varphi$. These output functions are controlled with the inputoutput linearizing controller (6.3) such that the swing leg symmetrically follows the stance leg and the torso is controlled to a desired torso lean angle with respect to the vertical $\alpha_{t}$. Impact is initiated when the step size $l_{s}=l_{1} \sin (\phi)+l_{3} \sin \left(\phi+\theta_{2}+\theta_{3}\right)$ has a desired value.


Figure 7.1: Schematics of three, five and seven link planar biped models.

These two parameters (step size and torso lean angle) are stored in the parameter vector $p$. They determine the evolution of a step because they directly influence the zero dynamics through the output functions.
Using this controller, the full system dynamics is constrained to a lower dimensional hybrid zero dynamics for which the evolution as function of $\phi$ can be computed a priori via (6.9). Now we are going to compare the biped's mid-stance velocity of step $k$ with its midstance velocity of step $k+1$. The mid-stance velocity is the velocity at $\phi=\phi_{m}$, i.e. the point where the biped's CoM passes its dead center and the potential energy is maximal. We do this for different step sizes and torso lean angles to find out the influence of these parameters on the walking speed of the biped. The results of this analysis are shown in Figure 7.2. The influence of the step size and torso lean angle on the mid-stance velocity $\dot{\phi}_{m}[k+1]$ of step $k+1$ is analyzed for six different mid-stance velocities $\dot{\phi}_{m}[k]$ of step $k$.
As can be seen in these plots, the higher the speed, the more freedom the biped has in the step size and torso lean angle to accomplish the next step. However, the more difficult it becomes to accelerate. Note that the torso lean angle is chosen in the interval $[-2,2][\mathrm{rad}]$, where a negative angle means leaning forward. The highest possible acceleration that can be achieved is when the CoM of the biped is placed as much forward as possible. Also interesting to see is that the highest acceleration does not necessarily occur at the smallest step size, which might have been expected, since the dissipation of energy due to impact is the lowest at small step sizes. However, with small step sizes the biped only falls for a short amount of time, so that it cannot reach a high velocity at impact. There appears to be
a trade-off between the energy dissipation at impact and fall duration when looking at the maximal acceleration.
Next, we introduce an example of realistic bounds on the step size and torso lean angle: $p_{\text {min }} \leq p \leq p_{\text {max }}$. These bounds are shown by the black dashed lines in Figure 7.2. As can be seen, these bounds limit the possibility of the biped to take steps or torso lean angles that guarantee any lower velocity in the next step. Only a limited set of velocities can be reached in one step. The maximal and minimal velocity that can be reached in one step with bounded step size and torso lean angle are computed via the gait parameter selection algorithm (6.23) and indicated by $\triangle$ and $\nabla$ respectively. In general, these maxima and minima lie at the border of the bounds as long as the edge of the feasible steps does not intersect the bounded area. This is only the case in Figure 7.2f.
Every point that lies on the edge of the feasible steps and in the bounded area guarantees that the biped comes to rest in the next step with its center of mass exactly above the stance foot. In [18] these points are called captured states. The minimal number of steps $N$ to come to this state is called $N$-step capturability. We can now compute this $N$-step capturability for any initial mid-stance velocity. A biped that decelerates maximally at each step comes to rest in a minimal number of steps. Hence, by subsequently taking a step size and torso lean angle at the maximal deceleration point in the bounded area, we can compute the $N$-step capturability. In Figure 7.3a the $N$-step capturability has been computed for a range of initial mid-stance velocities. As can be seen in this figure, the higher the velocity, the more steps are required to come to rest. The crosses correspond to the maximal deceleration of subsequent plots in Figure 7.2.
Finally, not only the capturability can be computed with the gait parameter selection algorithm, but we can compute the minimum number of steps to reach any desired mid-stance velocity for a range of initial mid-stance velocities. A biped that decelerates or accelerates maximally at each step reaches a desired velocity in a minimal number of steps. Hence, by subsequently taking a step size and torso lean angle at the maximal deceleration or acceleration point in the bounded area, we can compute the minimum number of steps. In Figure $7.3 b$ the minimum number of steps are plotted to reach a walking speed of $2[\mathrm{rad} / \mathrm{s}]$.

Figure 7.2 (on the next page): Influence of step size and torso lean angle on the walking speed of the planar three link biped. This influence is investigated for six different mid-stance velocities of step $k$ shown in the six different plots. The colors in these plots represent the mid-stance velocity of step $k+1$ for different combinations of step size and torso lean angle. The uncolored areas represent parameter combinations for which step $k+1$ is unsuccessful. The solid black line represents an isocline corresponding to the midstance velocity of step $k$, so that easily can be seen which combinations of step size and torso lean angle result in an acceleration or deceleration of the biped. The dashed black lines indicate an example for realistic bounds on the step size and torso lean angle. The maximal acceleration and deceleration within these bounds are indicated by $\triangle$ and $\nabla$.



Figure 7.3: Minimum number of steps required to reach the desired mid-stance velocity $\dot{\phi}_{d}$ from initial mid-stance velocity $\dot{\phi}_{m}[0]$ for the planar three link biped.

## Analysis of the five link biped

The same analysis can be performed on the five link planar biped model as shown in Figure 7.Ib. This model has knees, so foot scuffing can be prevented, but the walking gait is very similar to the three link model. The biped is modeled and controlled in the same way as in the previous section, with the output functions:

$$
h(\phi)=\left[\begin{array}{c}
\theta_{2}  \tag{7.2}\\
\phi+\theta_{3}-\alpha_{t} \\
\phi+\theta_{3}+\theta_{4}-\alpha_{4}(\phi) \\
\theta_{5}-\alpha_{5}(\phi)
\end{array}\right]-h_{p}(\phi),
$$

which represent, when controlled to zero, four virtual holonomic constraints, driven by the coordinate $\phi=\frac{1}{2}\left(\varphi+\theta_{2}\right)$, which represents the angle between the swing foot and hip

Figure 7.4 (on the next page): Influence of step size and torso lean angle on the walking speed of the planar five link biped. This influence is investigated for six different mid-stance velocities of step $k$ shown in the six different plots. The colors in these plots represent the mid-stance velocity of step $k+1$ for different combinations of step size and torso lean angle. The uncolored areas represent parameter combinations for which the step $k+1$ is not successful. The solid black line represents an isocline corresponding to the mid-stance velocity of step $k$, so that easily can be seen which combinations of step size and torso lean angle result in an acceleration or deceleration of the biped. The dashed black lines indicate an example for realistic bounds on the step size and torso lean angle. The maximal acceleration and deceleration within these bounds are indicated by $\triangle$ and $\nabla$.

if the upper and lower leg of the biped have the same length. These output functions are controlled with the input-output linearizing controller (6.3) such that the swing leg mimics the stance leg, where functions $\alpha_{4}$ and $\alpha_{5}$ guarantee that the swing foot does not scuff, and the torso is controlled in a desired torso lean angle with respect to the vertical $\alpha_{t}$. Impact is initiated when the step size has a desired value. These two parameters (step size and torso lean angle) are stored in the parameter vector $p$. They determine the evolution of a step because they directly influence the zero dynamics through the output functions.
Again, the mid-stance velocity of step $k$ is compared to the mid-stance velocity of step $k+1$. The results of this analysis are shown in Figure 7.4. The results are very similar to the results of the three link biped.

### 7.2.2 Step size and push off intensity for planar bipeds with finite sized feet

In this section we analyze the hybrid zero dynamics of planar bipeds with finite sized feet. The models of biped robots with feet are richer than point footed biped models and they may exhibit a larger variety of walking gaits, consisting of multiple phases and double support. In this section we analyze such gaits, but we restrict the analysis to natural walking gaits with a fixed order of phases as presented in Section 4.4.2. Besides an increase in support polygon area, an advantage of finite sized feet over point feet is the ability to push off using a stance ankle torque. We discovered in simulations that this push off has a large influence on the walking speed of the biped and therefore, in this section we analyze the influence of push off intensity and step size on the step-to-step walking speed of planar bipeds with finite sized feet.
The model that we use in this analysis is the seven link planar model as shown in Figure 7.Ic. This biped is modeled as an impulsive system with ideal actuators according to Section 4.4.2. It is controlled using the input-output linearizing controller as presented in Section 6.2.2, such that, again, the full system dynamics is attracted to a lower dimensional hybrid zero dynamics. For each phase in the walking gait, output functions are designed

Figure 7.5 (on the next page): Influence of step size and push off intensity on the walking speed of the planar seven link biped. This influence is investigated for six different mid-stance velocities of step $k$ shown in the six different plots. The colors in these plots represent the mid-stance velocity of step $k+1$ for different combinations of step size and push off intensity. The uncolored areas represent parameter combinations for which the step $k+1$ is not successful. The solid black line represents an isocline corresponding to the mid-stance velocity of step $k$, so that easily can be seen which combinations of parameters result in an acceleration or deceleration of the biped. The dashed black lines indicate an example for realistic bounds on the step size and push off intensity. The maximal acceleration and deceleration within these bounds are indicated by $\triangle$ and $\nabla$.

(a) $\dot{\phi}_{m}[k]=7.5[\mathrm{rad} / \mathrm{s}]$

(c) $\dot{\phi}_{m}[k]=4.5[\mathrm{rad} / \mathrm{s}]$

(e) $\dot{\phi}_{m}[k]=2.1[\mathrm{rad} / \mathrm{s}]$

(b) $\dot{\phi}_{m}[k]=5.9[\mathrm{rad} / \mathrm{s}]$

(d) $\dot{\phi}_{m}[k]=3.3[\mathrm{rad} / \mathrm{s}]$

(f) $\dot{\phi}_{m}[k]=0.6[\mathrm{rad} / \mathrm{s}]$
that, when controlled to zero, represent as virtual holonomic constraints. These are driven by $\phi_{i}$, which is chosen to be the horizontal position of the hip in every phase. The output functions are controlled using the input-output linearizing controller (6.28) and designed such that the biped steps symmetrically with its torso always oriented straight up, i.e. the angles of the stance and swing leg with the torso remain identical. Two parameters influence the evolution of a step through the output functions. These parameters are the step size, measured as the distance between the stance toe and swing heel and the push off intensity, measured in phase 4 as the angle of the swing foot with the ground. This angle determines the duration of the push off, its magnitude is determined by the ankle torque which is used to control the ZMP position as explained in Section 6.2.2. The combination of the change in ankle angle and the magnitude of the applied ankle torque is a measure for the amount of work put into the system, which is defined as the push off intensity.
The controller is used to attract the full system dynamics to a lower dimensional hybrid zero dynamics for which the evolution can be computed as a function of $\phi$ using (6.32). Note that in this case, one step covers four phases and its evolution consists of the subsequent evolution of those phases. We again compare the biped's mid-stance velocity of step $k$ with the mid-stance velocity of step $k+1$ for different parameter combinations of the step size and push off intensity. In Figure 7.5 the results of this analysis are plotted for six different initial mid-stance velocities $\dot{\phi}[k]$ of step $k$.
What immediately can be seen in these plots, is that the push off intensity has a very large influence on the walking speed of the biped. The longer an ankle torque is applied during the push off, the higher the speed of the biped in the next step. For bipeds with point feet we noticed that the step size has a larger influence on the walking speed than the torso lean angle. Here, we see that the push off intensity has an even larger influence on the walking speed which clearly indicates the advantage of finite sized feet.
It is also interesting to see that small step sizes with very large push off intensities are unsuccessful. This is purely a kinematic issue, since for small step sizes and large push off intensities the hip of the biped is pushed in front of the stance ankle before phase 4 ends. This kinematic configuration belongs to phase 2, which means that phase i will be skipped. This is not allowed as in Section 4.4.2 we assumed a fixed order of phases, and, hence, a step with this parameter combination is marked as unsuccessful. Technically these gaits may be successful, but they require a different phase order and they look rather unnaturally, so they are not taken into account in this analysis.
To further compare the point and finite sized footed bipeds we introduce realistic bounds on the step size and push off intensity as indicated by the dashed lines in Figure 7.5. The proposed gait parameter selection algorithm (6.23) is used to compute the maximal and minimal velocity that can be reached in one step. The corresponding step size and push off intensity are again indicated by $\triangle$ and $\nabla$ respectively. These values are used in the controller in subsequent steps to achieve a desired walking speed. For each step, the controller picks the parameter combination that results in the maximal or minimal walking speed


Figure 7.6: Minimum number of steps required to reach the desired mid-stance velocity $\dot{\phi}_{d}$ from initial mid-stance velocity $\dot{\phi}_{m}[0]$ for the planar seven link biped.
in the next step to accelerate or decelerate as much as possible. In this way the desired walking speed is reached in the minimum number of steps. In Figure 7.6 the minimum number of steps required to reach a desired walking speed is shown for a range of initial walking speeds. Comparing this figure with Figure 7.3, clearly shows that a biped robot with finite sized feet can accelerate much faster than a biped with point feet. Deceleration is not influenced, as a push off can only be used to accelerate.

### 7.2.3 Foot placement for 3D humanoids with finite sized feet

In this section we analyze the hybrid zero dynamics of three dimensional bipeds with finite sized feet. In contrast to planar bipeds, these bipeds can also move in the coronal plane. So in addition to multiple phases and double support, these bipeds exhibit motions in multiple directions, which gives them a very wide variety of possible gaits. Similarly as in the previous section, we restrict the analysis of these gaits to ones with a fixed order of phases as presented in Section 4.4.3.
We model a seven link three dimensional biped with finite sized feet as an impulsive system with ideal actuators using Section 4.4.3. In the sagittal plane this biped is the same as the planar biped with finite sized feet as shown in Figure 7.Ic, but in the coronal plane its hips are separated by a pelvis of nonzero width. Its feet are rectangular shaped with a finite sized length and width. The biped has roll and pitch degrees of freedom in both ankles, a pitch degree of freedom in both knees and roll, pitch and yaw degrees of freedom in both hips. It is controlled using the input-output linearizing controller as presented in Section 6.2.3. Output functions are designed that, when controlled to zero, represent vir-
tual holonomic constraints, driven by $\phi_{i}$, which is chosen to be the horizontal position of the stance hip with respect to the stance toe in the sagittal plane. The output functions are controlled on the robot using the input-output linearizing controller (6.28) and designed such that the biped steps symmetrically in the sagittal and coronal plane with its torso oriented straight up and facing in a desired walking direction. This means that the horizontal distances of the ankles to the torso are always equal and the robot's yaw orientation is controlled for turning in a desired walking direction. Designing output functions for three dimensional bipeds is significantly more challenging than for planar ones, because these bipeds can move in multiple directions in contrast to solely the sagittal plane. Here, as proof-of-concept, output functions in which the biped performs one step and no model assumptions are violated, are found on trail-and-error, but more sophisticated methods exist using optimization [28, II2].
In the following analysis, the influence of the sagittal and coronal foot placement location on the walking velocity is investigated. Once more, we compare the biped's mid-stance velocity of step $k$ with the mid-stance velocity of step $k+1$ for different parameter combinations of sagittal and coronal foot placement location. In Figure 7.7 the results of this analysis are plotted for six different initial mid-stance velocities $\dot{\phi}[k]$ of step $k$. The torso lean angle and push off intensity are kept at constant values.
In these figures, walking in a straight line happens when the coronal foot placement location is $0.15[\mathrm{~m}]$. The swing toe is pointing away from the stance foot for coronal foot placement locations greater than $0.15[\mathrm{~m}]$ and it is pointing towards the stance foot for coronal foot placement locations smaller than 0.15 [m]. In the latter case, the swing foot is also placed in front of the stance foot. As can be seen, walking in a straight line results in the smallest deceleration, the larger the turning angle, the lower the velocity in the next step. Also interesting to see is that stepping away from the stance foot is successful for a higher variety of foot placement locations. Stepping towards and in front of the stance foot appears more difficult.

Figure 7.7 (on the next page): Influence of sagittal and coronal foot placement location on the walking speed of the three dimensional seven link biped. This influence is investigated for six different mid-stance velocities of step $k$ shown in the six different plots. The colors in these plots represent the mid-stance velocity of step $k+1$ for different foot placement locations. The uncolored areas represent parameter combinations for which the step $k+1$ is not successful. The solid black line represents an isocline corresponding to the midstance velocity of step $k$, so that easily can be seen which foot placement location results in an acceleration or deceleration of the biped. The dashed black line indicates an example for realistic bounds on the foot placement location. The maximal acceleration and deceleration within these bounds are indicated by $\triangle$ and $\nabla$.


### 7.3 Simulations of underactuated humanoid robots

The controllers as presented in Section 6.2 cannot be used directly on the humanoid robot TUlip, because the most recent version of TUlip does not have torque sensing in its joints. The derived controllers generate torque signals that accurately need to be applied to the robot. The best option to apply the generated torques to the robot is using a closed loop joint torque feedback controller, for which joint torque sensing is required. Another option is an open loop feedforward control scheme, which requires an accurate actuator and drive train model. We have such models for TUlip, but the reliability is questionable, since these models do not include backlash, which is definitely present on the robot. Thus, the controllers are not applied to the real robot, but in numerical simulations. In this section the results of these simulations are presented for the same biped models as used in Section 7.2. First, simulations on the point foot models are discussed, after which simulation results on planar and 3D bipeds with finite sized feet are presented.

### 7.3.1 Walking speed for planar bipeds with point feet

The foot placement controller has been implemented in a simulation on the three link and five link planar biped model with point feet.

## Three link biped

In the first simulation, the biped is given an initial orientation and velocity, after which it is commanded to stop in one step, i.e. the desired mid-stance velocity $\phi_{d}=0$. Bounds on the step size and torso lean angle parameters are not taken into account in this simulation. Stopping a biped with point feet can only be done by making sure that the velocity of the CoM exactly vanishes when it moves above the stance foot. Due to the input-output linearizing controller and the Lagrangian characteristic of the resulting zero dynamics, we can compute the exact foot placement location to come to rest in the next step, using the controller presented in Section 6.2.I. The results of this simulation are shown in Figure 7.8a. It shows the energies of the zero dynamics during a step. As can be seen, the total zero dynamics energy is constant during a step and the zero dynamics kinetic energy vanishes, which means that the biped stops.
In the second simulation we control the biped to walk for five steps. At each step we define a desired mid-stance walking speed. This desired walking speed is slowly varying with small incremental steps to give the biped the opportunity to follow it. The gait parameter selection algorithm selects the step size and torso lean angle taking into account the same realistic bounds as in the previous section. The zero dynamics energies of this simulation are visualized in Figure 7.8 b . As can be seen from these figures, the biped exactly reaches the desired mid-stance walking speed at each step.


Figure 7.8: Energies of the zero dynamics for the three link planar biped for two different walking gait simulations.

## Five link biped

Next, we perform a simulation on the five link planar biped model. In this simulation we define a constant desired mid-stance walking speed which is higher than the one in the initial step, so that the biped has to accelerate. The gait parameter selection algorithm selects again the step size and torso lean angle taking into account reasonable bounds to achieve this motion. The results of this simulation are shown in Figure 7.9a. As can be seen in this figure, it takes the biped two steps to reach the desired walking speed. This is the minimum required number of steps in this situation, since the gait parameter selection algorithm guarantees the maximal acceleration of the biped per step. After the biped reached the constant desired walking speed it enters a limit cycle, since each next step is exactly the same as the current one. This can also be seen from the energy analysis in the previous section. If, for example, the biped steps on the solid black line in Figure 7.4e the mid-stance walking speed in the next step is the same as in the current step and, hence, the velocity plot of the next step is again Figure 7.4 e . This sequence repeats itself resulting in a limit cycle. On the limit cycle the total zero dynamics energy remains constant step after step, even in the presence of impacts, as can be seen in Figure 7.9a.
The final simulation on the five link planar biped with point feet is a continuously varying


Figure 7.9: Energies of the zero dynamics for the five link planar biped for two different walking gait simulations.
walking speed trajectory of one hundred steps, which includes periods of acceleration and deceleration. The zero dynamics energies for this simulation are shown in Figure 7.9b. We can conclude from this simulation and the previous ones, that decelerating is easier for these biped than accelerating, which corresponds to the energy analysis of the previous section. In this analysis we already saw that the area for decelerating is in general larger than for accelerating. As can be seen in this simulation, the biped is in some periods not able to follow the desired walking speed and takes multiple small steps to accelerate as quickly as possible. Only when the desired velocity decreases, the biped is capable of following the trajectory again.

### 7.3.2 Walking speed for planar bipeds with finite-sized feet

The seven link model as shown in Figure 7.Ic and mathematically represented in Table 6.I is implemented in a simulation. It is controlled using the controllers that are presented in Section 6.2.2 and Table 6.2. In the first simulation the biped is given an initial orientation and velocity and commanded to stop. This is similar as the first simulation of the three link biped with point feet. The gait parameter selection algorithm selects the step size, push off intensity and torso lean angle to achieve this. Bounds on these parameters are not taken into account. The results of this simulation are shown in Figure 7.Ioa. In this figure, again


Figure 7.10: Energies of the zero dynamics for the seven link planar biped with finite size feet for two different walking gait simulations.
the zero dynamics energies are plotted. As can be seen, the total zero dynamics energy is constant during each phase of the step. After each phase, the zero dynamics alters and, hence, the total zero dynamics energy changes, even if there is no impact at the end of the phase. The zero dynamics kinetic energy vanishes during the final phase, which means that the biped stops.
In a second simulation, we investigate the influence of the push off intensity on the acceleration of the biped. We define a constant desired velocity which is higher than the one in the initial step. For comparison reasons, the initial conditions and the desired velocity are the same as in the first simulation on the five link biped with point feet. The gait parameter selection algorithm selects the step size, torso lean angle and push off intensity to achieve this desired velocity in the minimum number of steps. The results for this simulation are shown in Figure 7.Iob. As can be seen in this figure, the biped is capable of achieving the desired walking speed in one step. Compared to Figure 7.9a this is faster, as the biped with point feet required two steps to achieve this velocity. The zero dynamics energies are different because the output functions are different, but the achieved walking speed is the same in both simulations. Also the inertial properties of the models are the same, which indicates that the push off of the biped with finite sized feet is responsible for the difference. Once again we can conclude that bipeds with feet can accelerate significantly faster than bipeds with point feet.


Figure 7.11: Omnidirectional walking of the seven link three dimensional biped with finite size feet.

### 7.3.3 Omnidirectional walking

The seven link three dimensional biped with finite sized feet as used in Section 7.2.3 is implemented in an event driven simulation. It is controlled using the controller as presented in Section 6.2.2. In this section, we show a proof of concept simulation of the biped's omnidirectional walking capability by letting it make a turning step of 30 degrees with a desired walking velocity in the sagittal plane. The biped is given an initial condition and the gait parameter selection algorithm selects the foot placement, torso lean orientation and push off intensity to achieve the desired walking speed. The desired yaw orientation of the biped is computed from the desired circular path. In Figure 7.II the results of this one step simulation are shown.
In this figure, it can be seen that the biped turns to the left. Also indicated in this figure is the ZMP position during the walking motion, which stays inside, or on the edge of the support polygon, indicating that no constraints have been violated.

### 7.4 Conclusion

In the first part of this chapter, the influence of the gait parameters step size, torso lean angle and push off intensity is analyzed on the walking speed of underactuated biped robot models. The biped models are controlled using output functions that, when controlled
to zero using an input-output feedback linearizing controller, represent virtual holonomic constraints. The resulting zero dynamics is Lagrangian and exhibits a conserved quantity that is used to compute the walking speed of the biped for different parameter combinations.
Interesting results include the inverse proportionality of the step size to the walking speed. The bigger the step size, the lower the walking speed in the next step. The opposite is true for the torso lean angle; the larger the torso lean angle, the higher the velocity in the next step. Finally, it appears that the push off intensity affects the walking speed significantly, which indicates the major advantage of finite sized feet over point feet; the higher the push off intensity the higher the walking speed in the next step.
These results are used in the second part of this chapter to show the capabilities of the gait parameter selection algorithm that selects the gait parameters to obtain a desired walking velocity. Using this controller, the minimal number of steps required to achieve a certain desired walking speed is computed. Again, the advantage of finite sized feet is shown, because with the possibility to push off, the biped is capable to accelerate faster. Finally, the gait parameter selection algorithm is used to control the walking speed of various biped robot models.

## Chapter 8

## Experimental validation

### 8.1 Introduction

In this chapter ${ }^{1}$ the humanoid robot TUlip, as introduced in Chapter 3, is modeled according to Chapter 4 and its model parameters are estimated using the techniques presented in Chapter 5. The reliability of the resulting estimated model is validated in experiments. In these experiments, the robot is controlled using controllers presented in Chapter 6.
The model that is concerned in this chapter is a simulation model of the latest version of humanoid robot TUlip. This simulation model is implemented in the ROS Gazebo simulator, because this simulation framework is computationally fast and incorporates all important dynamical aspects that have been identified in Chapter 4, such as three dimensional multi-body dynamics and discontinuous velocities due to ground impact. The numerical simulation model describes, as accurately as possible, the multi-body dynamics of the humanoid robot including ground contact, ground friction, joint friction and joint backlash. The simulation model of TUlip contains many geometric and inertial model parameters that are estimated using the concepts as presented in Chapter 5. The dynamic model parameters of the first version of TUlip are estimated using joint torque measurements with the series elastic actuation. The later versions of TUlip do not have joint torque sensing and their model parameters are estimated using ground contact force measurements. The reliability of the force sensors under TUlip's feet is unsatisfactory due to technical problems, so an external force measurement device is used for this estimation procedure. The dimensions of this device are limited, meaning that we can only show static identification results using ground contact measurements. Dynamic identification using ground contact measurements is, however, performed in the simulator to show the potential of this

[^13]method.
The quality of the estimated simulation model of TUlip is evaluated in walking experiments using statically stable and dynamically stable walking gaits. The latest version of humanoid robot TUlip does not have joint torque sensors and is therefore not able to directly control the torques in its joint. Thus in this chapter we can only experimentally validate the position controllers as developed in Chapter 6. These controllers are also used during RoboCup tournaments. Data from these experiments and data from the experiments presented in Chapter 4 are compared to the most accurate model of TUlip to evaluate the performance of the Gazebo simulator.

The outline of this chapter is as follows. In Section 8.2 the simulation model of TUlip in the ROS Gazebo simulator is presented. In Section 8.3 we describe how the model parameters are estimated using two different methods. Finally, in Section 8.4 simulation data are compared to experimental data during walking experiments.

### 8.2 Simulation model of humanoid robot TUlip

In this section, we discuss the simulation model of version 3 of TUlip as described in Section 3.2.3. The model is derived according to the techniques presented in Chapter 4. The presented model is specifically intended for simulation purposes and implemented in the Gazebo simulator, which enables elaborate modeling of ground contact, ground friction, joint friction and joint backlash. Gazebo has been chosen because it is an open source, fast simulator that uses the Open Dynamics Engine, which computes the physical dynamics as explained in Section 4.3.2. Gazebo is integrated in the Robot Operating System (ROS), which is a communication framework specifically made for robots that allows us to easily connect the TUlipMC software, which normally runs on the robot, to the simulator. Moreover, Gazebo features appealing visualizations (Figure 8.Ib), so that also vision can be simulated for RoboCup purposes [194]. A modeled ball and soccer field even allow for testing the ball tracking and localization algorithm, respectively. The model itself (including the ball and field) is available on-line to support other teams [9].
The model is written in the Unified Robot Description Format (URDF) [I89]. An URDF model is created that describes the kinematic chain of Figure 3.4 b and dynamic properties of the robot TUlip. Additionally, after each ankle pitch joint, an unactuated joint with low damping and joint limits of $\pm \mathrm{I}[\mathrm{deg}]$ models backlash. Equivalent data are retrieved from the simulation as are measured during experiments by the joint encoders, IMU and foot sensors on the robot ${ }^{2}$.

Figure 8.Ia gives the simplified software overview used during simulations. The simulation

[^14]

Figure 8.1: TUlip simulated in ROS Gazebo.
setup has to predict the outcome of the experimental setup. Therefore it should mimic the experimental software as closely as possible, reusing as many components as possible. This allows for better comparison when it is ensured that no differences can arise from different implementations. It additionally saves time when the software does not have to be written twice. Therefore TUlipMC can be instructed so that it compares the desired angles to joint angles from SHARED instead of joint encoders. It subsequently does not send the resulting torques to the motors, but also stores these in SHARED for use in the simulation.

### 8.3 Model parameter estimation

The models of TUlip, which are used in simulations and in controller design contain many model parameters. In this section we apply the identification techniques presented in Chapter 5 to TUlip, to estimate the model parameters. The dynamic identification procedure using series elastic actuation, as well as the static identification procedure using ground contact forces are applied to the robot. Due to technical problems with the calibration of the force sensors under TUlip's feet, it was not possible to apply the dynamic identification procedure using contact forces to the real robot, but simulation results are provided here to show the potential of this method.

### 8.3.1 Dynamic identification using series elastic actuation

The first identification experiment is performed on version a of TUlip described in Section 3.2.I. Since this version of the robot has joint torque sensing, we use the dynamic identification procedure for force controlled robots using series elastic actuation as presented in Section 5.3. First the measurement procedure and estimated parameters are given, after which these are validated and analyzed.

## Measurement procedure

In this experiment the robot hangs in a stand with its torso fixed in vertical orientation; we only identify model parameters of its legs. The legs of the robot are structurally an open kinematic chain as that of a robotic manipulator, so we model one leg according to Section 4.5 .2 and adapt it so that the torso is the fixed base of the manipulator and the foot is the end effector. As actuator and drive train models we use those described in Section 4.6.I. For the hip, knee and ankle pitch joints the elastic drive train model is used, because they are series elastically actuated, whereas for the hip yaw and roll joints no elastic drive train is modeled, since the gearbox output shaft is directly coupled with the link.
For the identification procedure, the model is rewritten in base regressor form using the theory presented in Section 5.2 and persistently exciting trajectories are computed with the optimization routine described in Section 5.3.1. The computed trajectories are realized by means of PD control on the robot and the trajectory tracking results are shown in Figure 8.2. The joint and motor angles are measured with the joint and motor encoders at a sampling rate of iooo $[\mathrm{Hz}]$. As can be seen, the signals are practically noise free, which fits the deterministic parameter estimation framework as explained in Section 5.3. The corresponding velocities and accelerations are determined through numerical differentiation of these angles with a first order forward difference method and a fourth order Butterworth filter with cutoff frequency of $50[\mathrm{~Hz}]$ applied in forward and reverse direction to suppress noise that may be induced with numerical differentiation.
The tracking behavior of TUlip using ordinary PD controllers is quite poor in some joints, because at this point, a model based controller capable of handling the series elastic actuation was not available. Despite using gains as high as possible without destabilizing the system, the tracking behavior could not be improved further. This poor tracking performance is mostly caused by the series elastic actuation, since the bandwidth of the PD controllers is kept low to avoid instabilities. Moreover, the ankle actuation, which is the poorest of all, uses so called Bowden cables that exhibit highly nonlinear friction phenomena, which cannot be compensated using a PD control strategy.
With the experimental data we can estimate the parameters of TUlip using the framework presented in Section 5.3.I. As stated before, the hip pitch (Figure 8.2c), knee pitch (Figure 8.2d) and ankle pitch (Figure 8.2e) drive trains contain series elastic actuation. So for these


Figure 8.2: Tracking behavior for each joint, upper plots: trajectories (dashed) and joint angles (solid); lower plots: input signals to the motor.

Table 8.1: Estimated parameters of TUlip.

| BPS <br> nr. | Parameter | Estimated <br> value | Standard <br> deviation |
| :--- | :--- | ---: | ---: |
| $\hat{\vartheta}_{M 1}$ | $B_{m 3}$ | 0.00008 | 0.0000045 |
| $\hat{\vartheta}_{M 2}$ | $F_{c 3}$ | 0.00456 | 0.0000745 |
| $\hat{\vartheta}_{M 3}$ | $k_{3}$ | 83008 | 131 |
| $\hat{\vartheta}_{M 4}$ | $B_{m 4}$ | 0.00008 | 0.000002 I |
| $\hat{\vartheta}_{M 5}$ | $F_{c 4}$ | 0.00268 | 0.0001168 |
| $\hat{\vartheta}_{M 6}$ | $k_{4}$ | 59966 | 90 |
| $\hat{\vartheta}_{M 7}$ | $B_{m 5}$ | 0.00004 | 0.0000227 |
| $\hat{\vartheta}_{M 8}$ | $F_{c 5}$ | 0.01178 | 0.0004170 |
| $\hat{\vartheta}_{M 9}$ | $k_{5}$ | 51394 | 801 |

three joints, we first have to identify the spring stiffness and motor friction properties using (5.10). These regressors are stacked in a 2000 by 9 matrix $H_{M}$, since there are 3 parameters per series elastically actuated joint. Also the 2000 by I vector $y_{M}$ is created and using a least squares estimate the stiffness and motor friction parameters are computed. Since the data are noise free, also the regressors are noise free and the least squares estimates are asymptotically unbiased [i7I]. These parameters are stated in SI units in Table 8.I. These values are the averages of the final 2000 data points, with corresponding standard deviation, assuming that the probability of each data point is equal.
The next step is to use the estimated spring stiffnesses of Table 8.I to calculate the load torques according to (4.29). These torques are used in a regressor of (4.32) and combined with the regressor of the hip yaw and roll joints of TUlip, that contain stiff drive trains (4.30). Also these regressors are stacked in a 2000 by 18 matrix $H_{L}$, since there are 18 base parameters, and a 2000 by I vector $y_{L}$. Using a least squares estimate, the model parameters at the load side $\hat{\vartheta}_{L}$ are computed. These estimated parameter combinations are listed in Table 8.2, again with their corresponding standard deviations, assuming that the probability of each data point is equal.

## Validation, statistical analysis and discussion

Looking at the values of the identified drive train parameters in Table 8.I, in particular, the order of magnitude of the stiffness values gives confidence. Namely, we know from the manufacturer that the stiffness of the full springs is 34 [ $\mathrm{N} / \mathrm{mm}$ ]. The springs used on TUlip are shortened, which increases the stiffness. Despite the fact that estimations of the stiffness match the expected values, we will further analyze the quality of the identified parameters by means of validation experiments. Input torques are calculated from the model (4.3I) with the identified parameters and compared to the measured torques from the experiment. These validations are shown in Figure 8.3.

Table 8.2: Estimated parameters of TUlip.

| BPS <br> nr. | Parameter <br> combination | Estimated <br> value | Standard <br> deviation |
| :--- | :--- | ---: | ---: |
| $\hat{\vartheta}_{L 1}$ | $c_{6 x}^{2} m_{6}+l_{11 z}^{2}\left(m_{5}+m_{4}+\frac{1}{\left.l_{9 y} c_{3 z} m_{3}\right)}\right.$ | 0.01942 | 0.15698 |
| $\hat{\vartheta}_{L 2}$ | $c_{6 x} m_{6}+l_{11 z}\left(m_{5}+m_{4}+\frac{1}{l_{9 y}} c_{3 z} m_{3}\right)$ | 0.90143 | 0.01490 |
| $\hat{\vartheta}_{L 3}$ | $m_{6}+m_{5}+m_{4}+\frac{1}{l_{9 y}} c_{3 z} m_{3}$ | I.3104I | 0.1125 I |
| $\hat{\vartheta}_{L 4}$ | $m_{5} c_{5 x}^{2}+l_{10 z}^{2} m_{4}+l_{102}^{2} \frac{1}{l_{99}} c_{3 z} m_{3}$ | 0.05399 | 0.1666 I |
| $\hat{\vartheta}_{L 5}$ | $m_{5} c_{5 x}+l_{10 z} m_{4}+l_{10 z} \frac{1}{l_{9 y}} c_{3 z} m_{3}$ | 0.7669 I | 0.01017 |
| $\hat{\vartheta}_{L 6}$ | $c_{4 x} m_{4}+l_{9 z}^{2} \frac{1}{l_{9 y} c_{3 z} m_{3}}$ | -0.25609 | 0.10250 |
| $\hat{\vartheta}_{L 7}$ | $c_{4 x} m_{4}+\frac{1}{l_{9 y} l_{9 z} c_{3 z} m_{3}}$ | -0.14063 | 0.02693 |
| $\hat{\vartheta}_{L 8}$ | $c_{3 z}^{2} m_{3}-l_{9 y} c_{3 z} m_{3}$ | -0.04169 | 0.0532 I |
| $\hat{\vartheta}_{L 9}$ | $m_{3}-\frac{1}{l_{9 y}} c_{3 z} m_{3}$ | I3.17218 | 0.53670 |
| $\hat{\vartheta}_{L 10}$ | $I_{6 y}$ | 0.04489 | 0.1654 I |
| $\hat{\vartheta}_{L 11}$ | $I_{5 z}$ | 0.0969 I | 0.17372 |
| $\hat{\vartheta}_{L 12}$ | $I_{4 z}$ | 0.14063 | 0.10245 |
| $\hat{\vartheta}_{L 13}$ | $I_{2 y}$ | 0.01008 | 0.03265 |
| $\hat{\vartheta}_{L 14}$ | $I_{3 y}$ | 0.28832 | 0.14656 |
| $\hat{\vartheta}_{L 15}$ | $B_{m 1}$ | 0.00004 | 0.00002 |
| $\hat{\vartheta}_{L 16}$ | $F_{c 1}$ | 0.0086 I | 0.00023 |
| $\hat{\vartheta}_{L 17}$ | $B_{m 2}$ | 0.00042 | 0.00004 |
| $\hat{\vartheta}_{L 18}$ | $F_{c 2}$ | 0.02167 | 0.00103 |

The agreement of the computed and measured torques is large. The scattering behavior starting around 7 seconds is due to stick slip behavior in the friction model. The stick-slip behavior in the friction model in (5.10) is not an accurate representation of reality. A more complex friction model including for example the Stribeck effect may be required to model the friction appropriately [IO6]. Replacing the sign function by a hyperbolic tangent did improve the results slightly. Despite this small error, from these results we might again conclude that the estimated parameters are of sufficient quality and useful for dynamic identification of parameters of the robot rigid dynamics.
From the values of the identified stiff drive train and load side parameters and their standard deviations in Table 8.2, it can be concluded that most parameters are identified confidently accurate. However, the moments of inertia in particular appear to be difficult to identify, because they only appear divided by the square of the gearbox ratio in the equations of motion. This makes them very small in comparison with other parameters and therefore difficult to identify. However, this also implies that they are of minor importance in the equations of motion and hence it is not important to identify them with high accuracy. The identified parameter values can be further validated by computing the torque with the estimated parameters and the measured joint angles using (4.30) and by comparing them with the measured torques. These validations are shown in Figure 8.4.


Figure 8.3: Validation of identified parameters for the flexible drive trains: measured torque (solid) and calculated torque (dashed).

From these figures we can say that in most joints, the identification gives proper results. As expected, the ankle joint seems to give the worst results. This is most likely caused by the fact that this joint is modeled inaccurately. Namely, in this joint significant nonlinear friction is present that is not appropriately taken into account in the model.
Overall, the experimental validation gives system identification results of sufficiently good quality, so that we can conclude that the identification algorithm is very well suitable to estimate the model parameters and identify robotic systems with series elastic actuation.


(e) Ankle pitch

Figure 8.4: Validation of identified parameters for the full model: measured torque (solid) and calculated torque (dashed).

### 8.3.2 Static identification using ground contact forces

The second identification experiment is performed on version 3 of TUlip (Section 3.2.3). In this version of the robot, the series elastic actuation has been removed, hence, it is impossible to measure the joint torque directly. Therefore, we use the static identification procedure for position controlled humanoid robots using ground contact forces as presented in Section 5.4. First, the measurement procedure and estimated parameters are given, after which these are validated and analyzed.

## Measurement procedure

In this experiment, the robot is put in many statically stable postures while measuring the ground contact forces underneath its feet. In this setup, the robot can be seen as a robotic manipulator with the stance foot as base and the swing foot as end effector. So, the robot is modeled as in Section 4.5.2 and from this model the CoM position is derived as in Section 4.7.I. This model for the CoM position is used in the identification procedure. First it is rewritten in base regressor form using the methods described in Section 5.2 after which optimal statically stable postures are determined using the optimization routine as explained in Section 5.4.I.
The projection of the CoM on the ground is in the static case equal to the CoP. The CoP can be determined by measuring the forces between the feet and the ground. We use Wii balance boards (WBB) to measure these forces, because they are more accurate than the force sensors in TUlip's feet. A WBB is a 50 cm by 30 cm board, originally designed by Nintendo. These boards are equipped with a force sensor on each corner, which means that it is possible to determine the CoP of the object that is put on the board. A WBB is an inexpensive measurement device. Because of the size of these balance boards, we place a WBB under each foot of TUlip. The joint angles are measured using the incremental joint encoders.

Now, the measurement procedure is as follows. First, the robot is lifted and brought to the first posture, such that the WBBs can be placed in a suitable position. The robot is put with his right foot on a marked fixed point (e.g. in a corner) on a WBB and the other WBB is positioned under its left foot. Then the relative position between the WBBs is measured; it is important that the WBBs are accurately placed in the same orientation. The position of the left foot of the robot is known through the joint encoders, so that the positions of the WBBs with respect to the robot's feet can be computed, and hence, the position of the CoP with respect to the robot is known.

The robot keeps the same foot positions for five different postures so that it can stay on the WBB without being lifted. The robot moves automatically between postures and may be supported during this movement to guarantee its stability. The robot stays in each posture for 60 seconds so that any initial motion has damped out. After five postures, the robot is


Figure 8.5: Examples of optimal identification postures on TUlip.
lifted and its foot positions are changed, so that the WBBs need to be repositioned.
A total of $m=108$ postures appeared to be stable and feasible on the real robot. Some postures fail because of tracking errors in the joint controllers, which unbalance the robot. A few postures have been measured twice to check the measurement consistency, which appears reliable. An example of four postures can be seen in Figure 8.5. For every posture, the average joint angles for the final 30 seconds are used in the regressor matrix as one data point. The first 30 seconds are omitted to guarantee that the posture is really static. With all these regressor matrices a 2 I 6 by 25 output data matrix $H$ is constructed, since every measurement consists of the CoP position in two directions and the model contains 25 base parameters. Also the contact forces are averaged per posture for the last 30 seconds so that the CoP can be computed via (4.43). The number of sensors is equal to $n_{c}=8$, as each WBB has four force sensors. With the CoP positions $p_{\text {cop }}$ for each posture, the output data $y$ is constructed in (5.22). Now, a least squares estimate of the BPS $\hat{\vartheta}_{c}$ can be found using (5.23). The data in the regressor matrix are again noise free so that asymptotically unbiased estimates can be expected [17I]. The obtained BPS estimates for a full CoM model of TUlip are given in Table 8.3.

## Validation, statistical analysis and discussion

To validate these results we compute the error between the measured and estimated CoP using the measured joint angles and estimated BPS:

$$
\begin{equation*}
e=y-\hat{y}=y-H \hat{\vartheta}_{c} . \tag{8.1}
\end{equation*}
$$

The norm of this error per posture is shown in Figure 8.6a. Overall, this figure shows good results, which means that we can estimate the CoM position accurately. The mean, standard deviation (assuming equal probability for each measurement) and maximum error for the full 3D CoM model are $4.2[\mathrm{~mm}], 3.6[\mathrm{~mm}]$ and $10.7[\mathrm{~mm}]$ respectively, which is similar as with the planar decoupled models of [34,36].
In contrast to [34, 36], we are not only interested in the results of the CoM position, but also in the reliability of the individual base parameters. Thus, Table 8.3 gives as precision measure the relative standard error [99, I23] of the $j^{\text {th }}$ base parameter estimate $\hat{\vartheta}_{c, j}$ :

$$
\begin{equation*}
\operatorname{se}\left(\hat{\vartheta}_{c, j}\right)_{r e l}=100 \cdot \frac{\sqrt{\hat{\sigma}^{2} C_{j j}}}{\left|\hat{\vartheta}_{c, j}\right|}, \tag{8.2}
\end{equation*}
$$

where $C_{j j}$ is diagonal element $j$ of $C=\left(H^{\top} H\right)^{-1}$ and:

$$
\begin{equation*}
\hat{\sigma}^{2}=\frac{\|e\|^{2}}{r-p}, \tag{8.3}
\end{equation*}
$$

where $r=2 m$ are the number of data points and $p$ is the number of base parameters. A practical consideration is given in [99] about the relative standard errors. A parameter is considered well identified if its relative standard error is less than 15 . If this is not the case then the parameter either has no significant effect on the model or it is not excited enough by the experimental data.
The results in Table 8.3 show that approximately one third of the parameters is estimated reasonably well. Fortunately, these parameters also constitute the ones with the most influence on the CoM position since their absolute values are the highest. All parameters have unit [ m ] and the regressor only consists of trigonometric functions which lie in the interval $[-1,1]$. So the influence of each base parameter on the CoM position is directly coupled with its absolute value. Unfortunately, the reliability of smaller parameters, although less interesting, is in some cases relatively low.
This can partly be explained by the small influence on the CoM model, but another reason is given in [99]. As a rule of thumb, the number of data points must be at least 500 times the number of parameters. With 25 base parameters, this is not practical in the current setup. A third reason may be that the optimized postures are still too similar, due to the small constrained optimization space. This would result in (almost) linearly dependent columns in the regressor. The correlation between columns $i$ and $j$ is [123]:

$$
\begin{equation*}
\rho_{i j}=\frac{C_{i j}}{\sqrt{C_{i i} C_{j j}}}, \tag{8.4}
\end{equation*}
$$

so that $\rho_{i j}=0$ means that parameters $i$ and $j$ are independent and $\rho_{i j}=1$ or $\rho_{i j}=-1$ means respectively a positive or negative linear relation.
The base parameters with the highest correlation are $\vartheta_{b, 12}$ with $\vartheta_{b, 13}$ and $\vartheta_{b, 15}$ with $\vartheta_{b, 16}$ ( $\rho_{12,13}=-0.9928$ and $\rho_{15,16}=-0.9882$ ) which explains their large relative standard errors

Table 8.3: Parameter estimation with experimental data.

| $\begin{aligned} & \text { BPS } \\ & \text { nr. } \end{aligned}$ | Full Model |  | Simplified Model |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Estimated [mm] | Rel.std. err. [\%] | Estimated [mm] | Rel.std. err. [\%] |
| $\vartheta_{c, 1}$ | -I0.96 | 13.0 | -II. 72 | II. 2 |
| $\vartheta_{c, 2}$ | -2.34 | 245.7 |  |  |
| $\vartheta_{c, 3}$ | 4.83 | 170.6 |  |  |
| $\vartheta_{c, 4}$ | I. 55 | 765.8 |  |  |
| $\vartheta_{c, 5}$ | -5.87 | 297.2 | -T. 34 | 129.4 |
| $\vartheta_{c, 6}$ | -14.07 | 47.3 |  |  |
| $\vartheta_{c, 7}$ | -30.91 | 26.9 |  |  |
| $\vartheta_{c, 8}$ | -3.28 | $243 \cdot 5$ |  |  |
| $\vartheta_{c, 9}$ | 5 I .35 | 30.2 | 3.60 | 32.9 |
| $\vartheta_{c, 10}$ | 76.39 | I. 9 | 77.4I | I. 8 |
| $\vartheta_{c, 11}$ | -8.80 | 67.2 |  |  |
| $\vartheta_{c, 12}$ | 44.15 | 104.6 | -II. 99 | 12.7 |
| $\vartheta_{c, 13}$ | -46.83 | 99.2 |  |  |
| $\vartheta_{c, 14}$ | 3.68 | 137.2 |  |  |
| $\vartheta_{c, 15}$ | -32.62 | 88.I | -3.02 | 32.8 |
| $\vartheta_{c, 16}$ | 26.54 | IIO.O |  |  |
| $\vartheta_{c, 17}$ | 130.33 | 2.2 | 135.69 | I. 2 |
| $\vartheta_{c, 18}$ | -5.78 | 65.7 |  |  |
| $\vartheta_{c, 19}$ | -270.08 | 0.8 | -271.53 | 0.6 |
| $\vartheta_{c, 20}$ | -255.75 | 2.2 | -256.39 | I.I |
| $\vartheta_{c, 21}$ | -36.09 | 23.9 | -35.78 | II. 3 |
| $\vartheta_{c, 22}$ | 0.68 | 50 I .3 |  |  |
| $\vartheta_{c, 23}$ | 39.34 | 9.1 | 48.15 | 4.5 |
| $\vartheta_{c, 24}$ | I6.45 | I6.5 | ${ }_{16.83}$ | II. 2 |
| $\vartheta_{c, 25}$ | -10.77 | 60.2 | -10.5I | 3 I .2 |

in Table 8.3. The base parameter $\vartheta_{b, 12}$ contains the $y$-position of the right hip, upper leg, lower leg and ankle mass, whereas $\vartheta_{b, 13}$ is the $y$-position of the right foot mass. The only difference in the corresponding columns of the regressor matrix between these parameters is a multiplication with $\cos \left(\theta_{6}\right)$, which is the right ankle roll joint. This joint has a small range $\left(\theta_{6} \in[-20,17][\mathrm{deg}]\right)$, so that $\cos \left(\theta_{6}\right) \approx 1$ for all possible postures, which explains the strong linear relationship between these two base parameters. The left ankle roll is similar which explains the correlation between parameters $\vartheta_{b, 15}$ and $\vartheta_{b, 16}$. These base parameters are separately identifiable in theory, but the difference in the corresponding columns in the regressor matrix is too small to estimate these base parameters independently in practice. All other correlations are smaller than 0.85 .
There also seems to be a problem, although to a lesser degree with all $x$-positions of the


Figure 8.6: Validation of statically (left) and dynamically (right) identified parameters.
masses of a leg. The base parameters $\vartheta_{b, 2}$ till $\vartheta_{b, 5}$ for the right leg and $\vartheta_{b, 6}$ till $\vartheta_{b, 9}$ for the left leg show large relative standard errors. The difficulty of estimation in these parameters is probably caused by the fact that the leg limbs on the robot are almost symmetrical in $x$ and $y$-direction, so the corresponding $c_{i x}$ and $c_{i y}$ parameters are probably very small. Possible ways to improve the estimation results include:

- Increased accuracy of CoP measurements lowers (8.3).
- Increased number of postures $m$ lowers (8.3).
- Improved optimized postures lowers the condition number of the matrix $H$ which lowers $C_{j j}$.

For the first option more accurate measuring instruments are required such as professional force plates or 6 axis force sensors mounted in the ankle of the robot, which we do not have available. Increasing the number of measurements is very time consuming. A better option would be to use a dynamic parameter estimation which results in more data. However, it is difficult to find optimal persistently exciting motions for which the robot does not fall. Improving the static postures to decrease the condition number of $H$ is difficult because of the joint limits and stability constraints. An option is to also include single support postures, although it is difficult to balance the robot on one foot.
Finally, another possible improvement is a simpler model. We derived a simplified model by taking into account symmetry in the robot links, and we use the same experimental data to estimate the following parameters:

- All mass values $m_{i}$.
- The 3D torso mass position, as it has the largest mass.
- The mass positions of the leg limbs only in $z$-direction, assuming symmetric legs in $x$ and $y$.
- The feet mass positions in $x$ - and $z$-direction, as the center of mass of a foot is in front of the ankle.

The estimates and relative standard errors (8.2) for this simplified model are also shown in Table 8.3. We can see that for most parameters, the relative standard errors decrease, which indicates that this model is more reliable. The norm of the errors (8.I) are shown in Figure 8.6a. As can be seen, the simplified model gives similar results as the full model. The mean, standard deviation (assuming equal probability for each measurement) and maximal error are $4.5[\mathrm{~mm}], 3.7[\mathrm{~mm}]$ and $12.9[\mathrm{~mm}]$ respectively. The maximal correlation (8.4) is 0.75 for the simplified model.

The advantage of the simplified model is that the CoM position can almost be computed as accurate as with the full model, without the problems that some base parameters cannot be estimated independently. A disadvantage is that the assumption is made beforehand that some parameters are zero, which cannot be confirmed by measurements. Overall, we can conclude that for both models, the estimation results look very reliable for the most important parameters, but some relatively small parameters appear difficult to estimate accurately.

### 8.3.3 Dynamic identification using ground reaction forces

In this section, the dynamic identification algorithm as presented in Section 5.4.2 is evaluated with simulations. It is hard to perform this algorithm on the current version of TUlip because it lacks accurate robot mounted ground force sensors. Using the balance boards as in the previous section is also difficult, since this would limit the number of possible motions. At least one foot of the robot should be fixed to the balance board, which excludes any walking motion.
The simulator as presented in Section 8.2 is used for the evaluation. We designed a dynamically stable curved walking gait and two different statically stable kicking motions according to Section 6.3.I. A fast kicking is performed with the right leg and a slow kicking motion is performed with the left leg. During these motions, the CoP is measured with respect to the stance ankle using simulated ground force sensors. From these measurements, the dynamic parameters of the robot are estimated according to the algorithm of Section 5.4.2. Using the estimated parameters and the ZMP model we can compute the estimated ZMP location and compare this to the actual ZMP location computed from the actual known parameters. The results of this analysis are shown in Figure 8.6b.

As can be seen from this figure, the estimation procedure works. The estimated ZMP overlaps the actual ZMP perfectly. A comparison between the parameters shows that all parameters are estimated within machine accuracy from the actual parameters.

### 8.4 Simulation model evaluation

The model parameters that we identified in the previous sections are also added to the simulation model of TUlip as derived in Section 8.2 and implemented in the Gazebo simulator. Parameters that are not identified, such as ground contact model parameters are guessed and tuned to fit simulations to experiments. In this section we try to find out how accurate this model describes reality. We compare multiple experiments with simulations in which the real and virtual robot run the same software and perform the same motions. First statically stable walking experiments are performed and compared to simulations, to investigate the accuracy of the static identification procedure. Secondly, the dynamic experiments performed in Chapters 4 are compared to simulations to determine the accuracy of the ground contact model and dynamically identified parameters. Finally, we perform dynamic walking experiments and compare these with simulations.

### 8.4.1 Statically stable walking experiments

Three types of statically stable walking gaits are performed on the real TUlip: forward walking, side stepping and point turning. These three gaits are compared to the same motions in simulation. The motions are designed using the framework as presented in Section 6.3.I. We prescribe the CoM position, torso orientation and swing foot position and orientation and use inverse kinematics to compute the desired joint trajectories. The task space motions are designed such that the CoM position always remains above the support polygon. Furthermore, the desired motions are relatively slow, so that TUlip is statically stable. The desired joint trajectories are controlled on the robot using the controller presented in Section 6.3.2. In this case we only use the PD controller with gravity compensation, because the motions are relatively slow and inertia compensation is not required.

## Forward walking

The first motion that we performed on TUlip and in the simulator is statically stable forward walking in a straight line. In Figure 8.7a the joint angles of the robot during this motion are shown. The solid lines represent the experimental data, the dashed lines the simulation data and the dotted lines the desired trajectories in the joints. As can be seen in this figure, both the virtual and the real robot follow the desired trajectories accurately. This indicates that the estimated model parameters form the static identification experiment are quite reliable. There are also some differences in the joint angle data. Especially the hip


Figure 8.7: Comparison of a simulation (dashed) with an experiment (solid) of a statically stable forward walking gait on humanoid robot TUlip.
pitch seems to move slightly different in some parts of the motion. This is caused by the joint friction model as implemented in Gazebo. Changing the viscous friction parameter increases the overlap at some points in the motion, but decreases it in others. We believe that the friction model does not entirely capture the real friction characteristics as present on the robot.
Next, the orientation of the torso of the robot with respect to the world is plotted in Figure 8.7 b using data from the IMU. The overlap between the simulation and experimental data appears weaker, but it must be noted that the range is relatively small. From this experiment, only conclusions can be drawn about the roll angle of the robot. Quantitatively, the simulation and experiment show a similar shape, but the real robot's inclination has a higher amplitude and a longer damped oscillation. Reasons for this difference are backlash in the hip and ankle roll joints, which is very difficult to model accurately, flexibilities in the robots links, which we assumed rigid, or compliance in the ground contact, which we modeled as a hard contact.


Figure 8.8: Comparison of a simulation (dashed) with an experiment (solid) of a statically stable side stepping gait on humanoid robot TUlip.

## Side stepping

The second motion that TUlip performed is statically stable side stepping in a straight line. This motion is perpendicular to forward walking, so especially the roll joints are active as can be seen in Figure 8.8a. Also these joints have the highest error between simulation and experiment. This is most likely caused by backlash in these joints, despite that we modeled a backlash effect. Modeling backlash in combination with friction is very hard and we expect that the backlash model that we implemented in Gazebo does not entirely describe the backlash characteristic of TUlip.
Also the orientation of the torso is shown in Figure 8.8b. The range in which the torso inclines is very small in every direction, so we cannot draw hard conclusions from this plot. The only interesting observation is the yaw angle, which seems to slowly deviate from zero. A possible explanation is that the stance foot is slightly spinning with respect to the ground, which would indicate a mismatch in the ground contact model and reality. In the


Figure 8.9: Comparison of a simulation (dashed) with an experiment (solid) of a statically stable point turning gait on humanoid robot TUlip.
next section this is further investigated ${ }^{3}$.

## Point turning

The final motion which is performed on TUlip is the point turning motion. Again, this motion involves different joints as the previous motions, as can be seen in Figure 8.9a. Especially the hip yaw is active, which also shows the largest error between simulation and reality. The mismatch of the model is likely caused by an error in the viscous friction estimation. It is not caused by a mismatch in the inertial parameters of the torso, which is the heaviest part of the robot, since it happens both when the leg is the stance leg and the

[^15]

Figure 8.10: Dynamic side step simulation (dashed) compared with experiment (solid) from Figure 4.I.
swing leg.
Looking at the orientation data for the torso in Figure 8.9b, we see a clear mismatch between the simulation and experimental yaw orientation of the robot. The robot is commanded to turn 15 degrees per step, so the first turning step in the experiment is incorrect. This is caused by the fact that the robot's stance foot slipped during the experiment which caused the robot to spin with respect to the ground. This effect was already presumed in the previous section, but here it is clearly visible, indicating that the ground contact model implemented in Gazebo not accurately describes all physical contact characteristics of TUlip.

### 8.4.2 Dynamic side-stepping experiments

The sidestepping experiment with static posture and moving leg that we used in Section 4.2 to determine important modeling aspects, are imitated in the simulator. The results are shown in graphs in Figure 8.10, 8.II and 8.I2 .
Overall the simulator accurately describes the experiments, and it includes the main aspects previously discussed. The energy loss due to impact is modeled, as the robot does not reach an upright position in Figure 8.rob. The feet can behave as a point contact in


Figure 8.11: Comparison of the ground reaction forces on corners of the feet between the dynamic side step simulation (right) and experiment (left) from Figure 4.2.
the stepping direction as shown in Figure 8.II. The simulation describes the oscillation perpendicular to the stepping direction in Figure 8.iof, because the model includes ankle pitch backlash and has feet. The effects that are caused by the moving leg mass on the rest of the robot in the simulation are qualitatively the same as in the experiments. The increase and decrease of the contact forces in Figure 8.12 is similar.
There are also some differences. The most important one is that the simulation's roll oscillation has a two times lower frequency compared to the experiment in Figure 8.Ioc. This could be caused by a higher energy loss at impact in the simulation. First of all, ground contact in simulation (Figure 8.II) is strongly dependent on factors such as the Gazebo constraint force parameters, the coefficient of restitution and the contact model, and therefore hard to match the experiment. Note that also the foot sensors in the experiment (Figure 8.II) might be inaccurate, as they are subjected to calibration, saturation, and a low sampling rate and resolution. Secondly, backlash in the hip roll joints is only present in the experiment, and causes a smaller angle between the legs at impact compared to simulation. This can be concluded from the final ankle roll rotations that are smaller at the end of the experiment in Figure 8.Ioa-b compared to the simulation. Backlash in the right ankle roll joint also creates initially a slightly higher rotational velocity than the simulation.
Model parameters were earlier estimated in identification experiments [197], but a small difference in mass distribution can already have a significant influence on the dynamics. Gazebo uses only a simple linear friction model, and after tuning the joint damping parameters, there remain differences between e.g. ankle roll angles in Figure 8.roa-b.


Figure 8.12: Dynamic leg swing simulation (dashed) compared with experiment (solid) from Figure 4.3.

### 8.4.3 Dynamic walking experiments

In the final model evaluation experiment TUlip performs a dynamic forward walking motion. The same motion is also performed in the simulator. The gait and controller that are used during this experiment and simulation are presented in Section 6.3.1 and 6.3.2 respectively. The motion is in this case significantly faster than in the statically stable walking gaits presented in the previous sections. Therefore, we expect that the difference between the experiment and simulation is larger. In Figure 8.I3a the joint angles in the experiment and in the simulation are plotted.
As can be seen in this plot, the robot takes six steps in I 6 seconds. This is more than five times faster than the statically stable walking experiments. As expected, the difference between the experiment and simulation is, however, larger. In general, the trajectories have the same shape, but the experimental data shows more oscillations, especially in the hip


Figure 8.13: Comparison of a simulation (dashed) with an experiment (solid) of a dynamically stable walking gait on humanoid robot TUlip.
and ankle roll joints, which is again caused by backlash. The joints that accurately overlap are the hip yaw and knee, hip and ankle pitch joints.
The IMU data does not provide additional information since the orientation angles are small. So in Figure 8.izb the foot sensor data is compared between the experiment and the simulation. The robot keeps its feet parallel to the ground, so we compare the total ground reaction force on each foot. As can be seen, the timings of the steps overlap nicely, which indicates that phase switches are modeled accurately. The amplitude of the forces is, however, lower in the experiments than in the simulations, but this is probable caused by saturation of the force sensors on TUlip.

### 8.5 Conclusion

In this chapter we contributed a simulation model of the humanoid robot TUlip, which includes multi-body dynamics, impulsive ground impact, ground friction, joint friction and backlash. Additionally, a head with a camera system and a soccer field with ball are modeled for RoboCup purposes.
The parameters of this model are estimated using identification experiments. Since TUlip has been altered in recent years, we were able to perform two types of identification experiments on the robot. The first version of the robot contained series elastic actuation in the drive trains, so that joint torque sensing was available. These sensors are used in dynamic identification experiments to estimate the drive train parameters, such as friction and stiffness, and the inertial parameters, such as link masses and center of mass positions. The current version of TUlip is purely position controlled and no joint torque sensing is available. Therefore, we used ground contact force measurements to estimate the inertial parameters of the robot with static identification experiments. It is not possible to compare the results of the two identification procedures, because the legs of the robot are significantly different in the two versions of the robot. Ground contact model parameters are not estimated, but tuned to fit the simulations to the experiments.
Finally, the derived simulation model and estimated parameters are validated in additional walking and side stepping experiments. We employed statically and dynamically stable walking gaits in experiments on TUlip and compared these with simulation results of the same motions on the model. The coherence of these results is satisfactory. Especially the statically stable motions are accurately predicted by the simulation, which indicates that the statically estimated model parameters are reliable. The analysis on the dynamically stable gait shows quantitatively the same behavior. The main differences in the statically as well as dynamically stable walking gaits are most likely caused by the ground contact model. On the one hand, the friction model does not contain spin friction, which definitely occurs on the robot, on the other hand, the impact model does not predict the amount of energy loss during impacts accurately. Another reason for mismatch between the experiments and simulations is the presence of severe backlash in some joints of the robot. We implemented a simple backlash model in the simulation, but this does not include all backlash characteristics as present on the real robot.

## Chapter 9

## Conclusions and recommendations

### 9.1 Conclusions and discussion

In this thesis, bipedal locomotion of humanoid robots is addressed. The main contributions can be concisely stated as follows. An experimental analysis of important dynamical aspects of humanoid robots is presented and used to emphasize the differences between several modeling frameworks commonly used for humanoid robots. Two regression based model parameter estimation techniques are contributed that explicitly use sensory information which is commonly available on humanoid robots, such as force and torque sensors in the joints and under the feet. The estimated model is experimentally validated on a humanoid robot. Based on existing work, a controller is used to asymptotically stabilize planar underactuated biped robots with point feet. This controller is extended to planar and three dimensional biped robots with finite sized feet and double support phases, but with maximally one degree of underactuation. For fully actuated humanoid robots, a ZMP based controller is proposed for flat-footed walking. A gait parameter selection algorithm is contributed, that selects on a step to step basis gait parameters such as step size, torso lean angle and push off intensity to regulate the biped's stability and walking speed. Finally, the influence of these gait parameters on the walking speed of the bipeds is analyzed.
In more detail, in Chapter 4 humanoid robot TUlip is experimentally analyzed to find important dynamical aspects of humanoid robots. It is concluded that four main aspects are important in the dynamics of humanoid robots: i) point feet can successfully be imitated on humanoid robots with feet by turning off the ankle control, 2) impacts of the feet with the ground can be modeled with discontinuous velocities and energy loss, 3) there is significant coupling between different bodies in the humanoid robot, 4) dynamics in the coronal plane influences dynamics in the sagittal plane and vice versa. These dynamical aspects are often simplified or neglected in models presented in literature, but as we found in this
experimental study, it is important to include them to accurately describe the dynamics of humanoid robots. So, in the remainder of Chapter 4, these important dynamical aspects are used to formulate model assumptions and to emphasize on the differences between several planar and three dimensional models with varying complexity regarding the number of degrees of freedom and the type and number of possible contacts.
In Chapter 5 two different model parameter estimation procedures are contributed, based on well-known linear regression techniques. The procedures are tailor made for humanoid robots because they use commonly available sensors in humanoid robots, such as joint torque sensing or ground contact force sensors. The first procedure assumes force controlled humanoid robots with joint torque sensing in the form of series elastic actuation. A parameter estimation algorithm is presented that can identify in a two-step procedure the model parameters of the elastic drive train and the rigid body model with only one set of experimental data. The second procedure uses solely information from ground contact force sensors and joint encoders to estimate all static and dynamic model parameters of the rigid body limbs of the robot. Both methods are regressor model based, so we contributed an algorithm to automatically rewrite any dynamic model, which is linear in the model parameters, to its regressor form. Besides this algorithm we also showed how to find persistently exciting trajectories, required for accurate and reliable parameter estimation.
In Chapter 6 the stability of underactuated humanoid robots is analyzed and a control structure is presented. This controller is based on existing work and uses input-output linearization to attract the full biped dynamics to a lower dimensional zero dynamics. As is known from literature, by construction, this zero dynamics exhibits a conserved quantity along its solution. This conserved quantity is exploited to develop a gait parameter selection algorithm for foot placement, torso orientation and push off intensity for planar bipeds with point feet. The input-output linearizing controller and gait parameter selection algorithm are extended to planar and three dimensional bipeds with finite sized feet that exhibit double support phases, but have maximally one degree of underactuation. The gait parameter selection algorithm can simultaneously regulate the balance of the humanoid robot and follow a desired omnidirectional walking gait and desired walking speed. Besides bipeds with underactuated phases, also a traditional controller based on the ZMP is developed for fully actuated humanoid robots that are position controlled. This controller works in single as well as double support periods of the gait, simultaneously controlling the contact forces, which result from the double support constraints on the feet, and joint positions of the biped.

In Chapter 7 an analysis of the dynamical properties of humanoid robots is contributed using the zero dynamics throughout a step. It is studied how stability of humanoid robots is influenced by the gait parameters foot placement, torso lean angle and push off intensity. This analysis is used to determine the walking speed and stability of bipedal locomotion for different combinations of the gait parameters. In the analysis, it is concluded that especially the step size and push off intensity have a large influence on the walking speed.

Furthermore, bipeds with finite sized feet can easier accelerate than bipeds with point feet. Based on this analysis, the gait parameter selection algorithm selects gait parameters to achieve a desired walking speed in the minimum number of steps. An implementation of this algorithm is shown by means of numerical simulations on planar and three dimensional walking humanoid robot models.
Finally, in Chapter 8 a simulation model of the humanoid robot TUlip is implemented in the ROS Gazebo simulator, which includes multi-body dynamics, impulsive ground impact, ground friction, joint friction and backlash. The parameters of this model are estimated using the two identification procedures as presented in Chapter 5. The resulting model with estimated model parameters is validated in additional walking and side stepping experiments. We employed statically and dynamically stable walking gaits as designed according to methods presented in Chapter 6 in experiments on TUlip and compared these with simulation results of the same motions on the model. The coherence of these results is satisfactory, especially for the statically stable gaits.

### 9.2 Recommendations

In this section we emphasize on a few, in our view important, recommendations for future work on humanoid robots. While working on this topic, we gained experience on bipedal locomotion that we would like to pass on to future generations of research by means of this section.

The first recommendation concerns the modeling of humanoid robots. Although different humanoid robot models are available in literature, there seems to lack a coherence among the models. Many model assumptions are unjustified or even contradictory and it is very difficult to critically judge the properties of many models. This results in a scattered research field, since many bipedal locomotion controllers are based on these models. We did not find a unified modeling approach which is directly suitable for system analysis, controller design and embedded implementation. In our opinion it would boost the humanoid robotics research if more coherence among the models and as a result among the different control procedures could be found.
Despite the many available models, humanoid robot model validation is scarce. In this thesis we performed a relatively small model evaluation and determined important dynamical aspects of humanoid robots, but we did not analyze the dynamics of the ground contact. In our view, especially, impact and environment contact modeling deserve more attention. The humanoid robot can only move by exerting forces on its environment, so these forces are relatively important, which indicates the importance of reliable impact and environment contact models.
Regarding the controller and gait parameter selection algorithm that we propose, a robustness study is required to judge its performance. The controller should be tested against
disturbances on the humanoid robot such as pushes and against model inaccuracies. We expect that the controller is robust against small disturbances or model inaccuracies. The input-output linearizing controller can reject these disturbances relatively fast, such that the trajectories do not diverge too far from the zero dynamics. Depending on the actuator limits, these disturbances may be rejected within one step, such that the foot placement is not influenced. It is difficult to predict the robustness of the foot placement controller against larger disturbances, especially, if more steps are required before the humanoid returns to the zero dynamics. This can be investigated in future work in simulations and experiments.
The walking controller that is presented in this thesis works in all situations where the biped has maximally one degree of underactuation. We do not offer solutions for situations where the biped has two or more degrees of underactuation. This is typically the case for three dimensional bipeds with point feet. A few results are available in literature that deal with this situation, but more research on this topic is required, since these situations occur in natural walking.
Regarding system identification and model parameter estimation, adaptive control for humanoid robots is heavily underexposed. To the best of our knowledge, there is no literature available for adaptive control on humanoid robots, while many model based controllers can seriously benefit from adaptive control. Not only can adaptive controllers solve problems with model inaccuracies, they are also crucial when humanoid robots start to handle tools, carry objects or walk over different types of terrain. The author is not familiar with the research of adaptive control on hybrid, non-smooth or underactuated systems, but it seems that these topics must be exploited in humanoid robotics research. In our opinion, adaptive control on humanoid robots or adaptive humanoid robot models are required before these robots can robustly operate in society.
Finally, an important missing feature in the humanoid robotics research field is benchmarking. Currently, there does not exist an adequate measure to compare different control approaches on humanoid robots. It is relatively unclear how existing approaches perform, which makes it very difficult to choose one for a humanoid robot. Implementation of most approaches is costly with respect to time, so it can take long before a satisfactory walking algorithm has been found for a humanoid robot. With a suitable benchmark, this problem can be solved.

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## Samenvatting

In de nabije toekomst zullen waarschijnlijk mensachtige robots in de maatschappij verschijnen om mensen te helpen met verschillende taken in industrieën, huishoudens, kantoren, ziekenhuizen, enz. Een mensachtige robot is een tweebenig mechatronisch systeem dat sterk lijkt op een mens. Deze menselijke gelijkenis is het belangrijkste kenmerk van mensachtige robots, want het stelt ze in staat om in omgevingen die gevormd zijn naar menselijke proporties te functioneren, alledaagse menselijke taken uit te voeren en zich onder mensen te begeven zonder sociale grenzen.
Een duidelijke connectie tussen mensen en mensachtige robots is hun tweebenige voortbeweging. Voor mensen voelt dit natuurlijk aan, maar voor mensachtige robots is dit een moeilijk technisch probleem. De robuustheid, veelzijdigheid en energie efficiëntie van menselijke mobiliteit is uitzonderlijk en nog niet geëvenaard met mensachtige robots ondanks vier decennia van onderzoek op tweebenig lopen. Dit geeft aan dat de dynamica en regeltechniek van tweebenige voortbeweging nog niet goed genoeg bekend is. Daarom is het doel van dit proefschrift om kennis over modeleren, identificeren, regelen en de stabiliteit van tweebenige systemen te vergroten zodat het ontwerp en de regeltechniek van mensachtige robots, prothetische apparaten voor personen met een lichamelijke beperking en kracht vergrotende exoskeletten verbeterd kunnen worden.
De belangrijkste uitdaging voor mensachtige robots is in balans te blijven. Onder normale omstandigheden en in het geval van onvoorziene verstoringen mag de robot niet vallen. Het is moeilijk om in balans te blijven omdat een mensachtige robot relatief kleine voeten heeft waarmee hij beperkte krachten uit kan oefenen op een relatief hoog gelegen massamiddelpunt. Dit maakt een mensachtige robot een niet-lineair mechanisch systeem met hybride, deels ondergeactueerde, fases en grondcontact met niet-gladde kenmerken. Dit proefschrift beschouwt deze onderwerpen en draagt verschillende tweebenige modellen, een schattingsmethodes voor de modelparameter, een analyse van de dynamica van tweebenige modellen, een asymptotische stabiele regelaar voor lopen, een tredparameter selectie algoritme voor loopsnelheid en omnidirectioneel lopen en experimentele validatie op een mensachtige robot bij.
In dit proefschrift worden tweebenige robots zowel zonder, als met voeten gemodelleerd,
geanalyseerd en geregeld. We beseffen dat mensen en mensachtige robots voeten hebben, maar door voeten weg te laten worden we gedwongen om onderactuatie, een belangrijke oorzaak van onbalans en aanwezig in een natuurlijke looppas, mee te nemen. In de modellen nemen we belangrijke dynamische aspecten mee zoals botsingen met de grond, onderactuatie en koppelingen tussen vrijheidsgraden in het model, omdat we tijdens experimenten ondervonden dat deze aspecten van significant belang zijn. Dat is ook de reden waarom we geen lineaire pendule modellen beschouwen omdat deze de belangrijke dynamische aspecten verwaarlozen, en ook niet kijken naar het nulmomentspunt omdat dit concept alleen werkt wanneer tenminste één voet van de robot volledig in contact is met de grond en de robot volledig geactueerd blijft.
De modellen gebruiken we om asymptotisch stabiele modelgebaseerde regelaars te ontwerpen. Deze regelaars trekken de volledige robotdynamica asymptotisch naar een toestandsvariëteit met een lagere dimensie, als functie van de niet-geactueerde vrijheidsgraad van het systeem. We laten zien dat we deze zogenaamde nuldynamica asymptotisch aantrekkelijk kunnen maken. In een diepgaande analyse van deze nuldynamica laten we zien dat de plaatsing van de voet, de leunhoek van de torso en de afzetintensiteit belangrijke tredparameters voor de balans van ondergeactueerde tweebenige robots zijn. Wanneer de toestand van de robot in de nuldynamica ligt, dragen we bovendien een selectiealgoritme bij dat per step de tredparameters vindt om in zo min mogelijk stappen een gewenste loopsnelheid en richting te krijgen, rekening houdend met eventuele limieten in het bereik van de robot.

Omdat deze regelaars modelgebaseerd zijn hebben we een schattingsmethode voor de modelparameters ontworpen, die in nauwkeurige identificatie voorziet aan de hand van experimenteel verkregen koppelmetingen met behulp van een in-serie elastische actuatie. We laten verder zien dat, wanneer koppelmetingen niet beschikbaar zijn, zoals op positiegeregelde mensachtige robots, we de parameters van het massamiddelpuntmodel kunnen schatten aan de hand van gewrichtshoeken en grond reactiekrachten en van het volledige model met gewrichtshoeken, snelheden en acceleraties en grond reactiekrachten.
Tot slot passen we enkele van de hiervoor beschreven concepten toe op de mensachtige robot TUlip. Deze robot heeft twaalf vrijheidsgraden en voeten. We dragen een model van TUlip bij in de ROS Gazebo simulator, schatten zijn modelparameters en valideren het model in experimenten. TUlip is positiegeregeld, dus we kunnen niet alle voorgenoemde concepten uitproberen, maar we laten wel zien hoe een robot kan lopen met traditionele (lineaire) regelaars en dynamische compensatie.
De belangrijkste conclusie van dit proefschrift is dat geavanceerde mensachtige modellen belangrijk zijn om de dynamica van tweebenige voortbeweging te analyseren en dat extreme aannames niet nodig zijn om toch stabiliteit te bewijzen met modelgebaseerde regelaars. Deze regelaars vereisen echter nauwkeurige kennis van het systeem, die verkregen kan worden met zorgvuldig ontworpen identificatie experimenten.

## Dankwoord

## Vind het wiel niet opnieuw uit!

Met dit motto in het achterhoofd ben ik vier jaar geleden begonnen aan een academische uitdaging. Wetenschappelijk onderzoek is vernieuwend en kennis wordt gedeeld. Vier jaar later is mijn aandeel hierin dit proefschrift, waarin ik heb geprobeerd zoveel mogelijk van mijn kennis over tweebenige robots te beschrijven. Deze kennis is natuurlijk niet zomaar uit de lucht komen vallen, maar aan de ene kant voortgeborduurd op resultaten van jarenlang onderzoek door vele voorgangers en aan de andere kant ontstaan tijdens samenwerking op technisch gebied en alledaagse gesprekken over het werk. Zonder deze laatste aspecten zou dit proefschrift er nooit zijn geweest, en daarom wil ik hiervoor een aantal mensen bedanken.
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## Curriculum vitae

Pieter van Zutven was born on April 4, I985 in Herpen, the Netherlands. He finished gymnasium at the Titus Brandsma Lyceum in Oss, the Netherlands in 2003. After that, he obtained his B.Sc. in Mechanical Engineering in 2006 from the Eindhoven University of Technology, Eindhoven, the Netherlands. For his master external internship he went to CINVESTAV (Centro de Investigacíon y de Estudios Avanzados del Instituto Politécnico Nacional) in Mexico City, Mexico, where he worked on a bilateral master slave teleoperation system. In 2009 he obtained his M.Sc. in Mechanical Engineering from the Eindhoven University of Technology. The title of his final project was 'Modeling, identification and stability of humanoid robots'.
In December 2009 he started as a Ph.D. candidate in the Dynamics and Control Group at the faculty of Mechanical Engineering of the Eindhoven University of Technology. The title of his Ph.D. project was 'Control and Identification of Bipedal Humanoid Robots'. The outcome of this project is presented in this thesis.
During this period, he was also team leader of the humanoid robotics group of the Eindhoven University of Technology. This group was part of Tech United, and participated with the humanoid robot TUlip in the annual RoboCup tournaments. For the RoboCup 2013 in Eindhoven, Pieter was local chair of the humanoid league and member of the technical and organizing committee.
Besides the humanoid activities he is a member of DISC (Dutch Institute of Systems and Control); in 2012 he successfully completed the DISC Ph.D. course program and obtained the DISC certificate. Pieter was a member of the program committees of the seventh workshop on humanoid soccer robots, held in Osaka, Japan in 2012 and of the fourth international conference on simulation, modeling and programming for autonomous robots, held in Bergamo, Italy in 2014 .


[^0]:    ${ }^{1}$ In the digital version of this dissertation, the snapshots above the figures can be clicked to open an Internet video of the experiment or simulation.

[^1]:    ${ }^{1}$ In the sense of more than two legs.

[^2]:    ${ }^{1}$ Parts of this chapter have been published in [194, 195, 199, 200].

[^3]:    ${ }^{2}$ The joint angle may differ from the motor angle due to the gearbox or other drive train characteristics such as flexibilities, friction and backlash.

[^4]:    ${ }^{1}$ Parts of this chapter have been published in [10, I96, I97, 198].

[^5]:    ${ }^{2}$ Note that a special case of full actuation is overactuation, in which the biped has more actuators than degrees of freedom. Although overactuation is technically not the same as full actuation, one can always not use a number of actuators to make up the difference.

[^6]:    ${ }^{3}$ Note that we start counting at 2.

[^7]:    ${ }^{4}$ Note that $v_{i} \neq \dot{q}_{i}$ because of the angular velocity representation, they are related via $v_{i}=J_{i}\left(q_{i}\right) \dot{q}_{i}$, where $J_{i}$ is the geometric Jacobian of the position and orientation of body $i$ as derived in [I68].

[^8]:    ${ }^{5}$ Conditions to guarantee that the stance foot remains flat on the ground are derived in Section 4.7.3.

[^9]:    ${ }^{1}$ Parts of this chapter have been published in [15, 197].

[^10]:    ${ }^{1}$ Parts of this chapter have been published in [194, 198, 20I, 202, 195].

[^11]:    ${ }^{2}$ The motor constants matrix acts as a gain on the input and is irrelevant for the analysis in this chapter.

[^12]:    ${ }^{3}$ Although $\mathcal{K}$ and $\mathcal{P}$ are defined as energies, they do not necessarily inherit properties from the classical definition of energy. For example, they may have unit different from Joule.

[^13]:    ${ }^{1}$ Parts of this chapter have been published in [IO, I5, I97].

[^14]:    ${ }^{2}$ The standard settings of ODE in the Gazebo simulator give unreliable results for the ground contact forces, as discussed in [Io].

[^15]:    ${ }^{3}$ It must also be noted that the yaw orientation of the robot is measured using a compass inside the IMU. Since the IMU is placed on the shoulder of the robot and high currents are running through its torso, this might influence the measurement. Although we performed some tests to check this hypothesis, we did not find any correlation.

