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Competitive Solutions for Cooperating Logistics Providers

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Abstract

This paper discusses solutions for gain sharing in consortia of logistic providers where joint planning of truckload deliveries enables the reduction of empty kilometres. The highly competitive nature of freight transport markets necessitates solutions that distinguish among the logistics providers based on their characteristics, even in situations with two players only. We introduce desirable properties in these situations and propose a solution that satisfies such properties. By comparing the existing solutions against the introduced properties we demonstrate the advantages of our proposed solution.

1 Introduction

Road freight constitutes the most dominant form of transportation. However, the industry suffers from significant inefficiencies. In 2012 more than 24% of all the distance driven by commercial vehicles in European Union were empty (Eurostat, 2012). This paper is motivated by a project initiated by two major European logistics providers to create a consortium for cooperative planning of truckload delivery requirements of joining companies in order to, among others, reduce the costs of *empty kilometres*. Cruijssen et al. (2007) discuss the other benefits that logistics providers could achieve by cooperation. We address the important issue of sharing the gains obtained from joint planning.

Cooperative truckload delivery (CTLD) situations comprise a number of logistics providers, their resources (e.g. depots, trucks, drivers, equipments, etc.), and their delivery requirements. A delivery requirement can simply be considered as an order for picking up cargo at some location and transporting it to another location. But it may actually involve delivery time windows, special equipments and personnel, and other practical constraints. The delivery requirements must be fulfilled by vehicles in feasible trips. The feasibility of a trip depends on the number and type of deliveries fulfilled in a trip, specific depots and equipment that must be employed, and other details.

The optimal delivery plans of individual companies in most cases include a significant amount of unavoidable repositioning movements, i.e. empty kilometres, among the depots and various pick-up/delivery locations. By taking advantage of the synergy in aggregated networks of depots and delivery requirements, cooperating companies can decrease their overall empty kilometres. As the cooperating companies are usually in direct competition with each other, it is absolutely critical for them to understand how the cooperation would benefit them as well as their competitors. Thus the existence of formal models that unambiguously determine allocations of gains and justifies their fairness and/or competitiveness in these situations are imperative to success of such consortia.

There are many simple ways to divide the savings among the logistics providers. Such simple ways often divide the savings proportional to some measure defined harmoniously for all players, e.g. number and/or amount of exchanged deliveries, additional costs incurred, empty kilometres avoided, or contributions to total savings. However, despite their practical appeal, simple solutions often fail to produce outcomes which are desirable in terms of fairness/competitiveness in these situations. But what constitutes as a desirable allocation in these situations? In this paper, we introduce a set of formal properties that have the ability to capture the notions of fairness and/or competitiveness with regard to allocations in CTLD situations.

The gain sharing problems in the literature are often approached via the well-known solutions developed in *cooperative game theory*. By abstracting a cooperative situation into a cooperative game, usually consisting of the player set and the amount of gains attainable by different groups of players, cooperative game theory studies allocations that satisfy collections of logically desirable properties expressed in relation to such abstraction. In some situations, however, the properties expressible in relation to the associated cooperative games are insufficient to capture all desirable requirements of the allocations. Therefore, by disregarding the information contained in the underlying cooperative situation, indispensable properties in some cooperative situations would be impossible to formalize. The cooperative organizations of logistics providers assessed in this paper are instances of such cooperative situations. In this paper, we allow solutions to draw upon the cooperative situations to determine the allocations of savings.

The highly competitive nature of logistics markets as well as the limited number of participants necessitate solutions that are capable of incorporating the notion of competitiveness to distinguish among the logistics providers. Such a requirement implores solutions that could potentially distinguish among the logistics providers who are identical in terms of their contribution to the obtained savings. A typical example of the latter is situations with only two players. Most of the well-known solutions in cooperative game theory, e.g. Shapley value and nucleolus, are incapable of differentiating among the allocations in two-player situations. Nevertheless, a number of papers in the OR/OM literature, e.g. Frisk et al. (2010), introduce alternative solutions to tackle the latter drawback. However, we show that the available solutions in the literature do not satisfy the properties that are desirable in CTLD situations introduced in this paper.

Part of the desirable properties of solutions in CTLD situations can be expressed in relation to the cooperative games associated with those situations. The *nonemptiness property* demands at least one allocation in every situation. The *uniqueness property* distinguishes solutions that upon nonemptiness, yield a single allocation in every situation. This property is needed so that no further negotiations would be required to choose among the multiple possible allocations. Finally, the *least unstability property* prescribes allocations that minimize the incentives of sub-coalitions to organize cooperation within themselves. In addition to these rather standard properties, we introduce two new properties which are specific to CTLD situations.

The first desirable property defined specifically for CTLD situations is the *independence of irrelevant deliveries property* which expresses that the allocated savings to the players must be insensitive to the parts of their networks which could not have any possible contribution to the savings obtained by cooperation. In this regard, this property defines a boundary for the relevant scope of operation for every logistics provider such that anything beyond this scope should be ignored in allocation of savings. If a solution for CTLD situations does not satisfy this property, then the logistics providers would have the incentives to inflate their shared delivery requirements in a cooperative organization and smaller companies with high contribution to the saving could be discriminated against the larger companies which may not have significant contributions.

The last property introduced in this paper addresses the ability of solutions to consider the *competitive positions* of cooperating logistics providers. Although there is no standard measure of competitive positions in logistics markets, we draw upon the notation of *average cost of fulfilment* to define one. The average cost of fulfilment of a set of delivery requirements is the minimum cost of fulfilment divided by the size of full kilometres involved. The motivation for using the average cost of fulfilment as a measure of competitiveness is its pricing implications. Suppose that companies have to announce a fixed price for a unit distance of their delivery services. The average cost of fulfilment then represents the lowest unit price at which a logistics provider neither makes profit nor incurs loss. Thus if the average cost of fulfilment of a logistics provider i is lower than that of j, i would be able to announce a lower unit price for its delivery services while making a profit. We take this as an indication that prior to cooperation i is in a better competitive position. An allocation of savings to players could alter the average costs of fulfilment after cooperation. Under certain conditions, the *restricted competitiveness property* requires that solutions equalize the ratio of average costs of fulfilment of players before and after cooperation. In this regard, the appropriate solutions in CTLD situations preserve the competitive positions of logistics providers.

We propose a solution that satisfies the listed properties in all CTLD situations. In doing so, we first introduce the *essential deliveries* of the players as the subsets of delivery requirements of each player which are necessary and sufficient in bringing about their contribution to cost savings in the grand coalition. When the optimal delivery plan of the grand coalition is unique, the essential deliveries correspond to the deliveries whose fulfilment trips in the grand coalition involve some other players. Therefore the deliveries which are not essential can be fulfilled independently by the players owning them as efficiently as in the grand coalition. A preliminary version of our proposed solution equalizes the average cost of fulfilments of essential deliveries of the players before and after cooperation. Although this solution can be easily implemented in such situations, it does not necessarily produce stable outcomes. Nevertheless, our final solution obtains a unique point in the core (Gillies, 1959), or when the latter is empty, in the least-core (Maschler et al., 1979) which has the shortest distance to the aforementioned allocation.

The rest of this paper is organized as following. In Section 2 we discuss the literature on allocation problems with special focus on logistics and transportation context. In Section 3 we model a general class of truckload delivery problems and in Section 4 we introduce the cooperative version of such situations. The desirable properties in CTLD situations are defined in Section 5. In Section 6 we develop our proposed solution for CTLD situations. Specifically, Section 6.2 outlines the formula for the proposed solution and shows that it satisfies all the listed properties. Section 7 discusses some of the known solutions in the literature in line with the properties introduced in this paper. Section 8 concludes the paper.

2 Literature Review

There is an extensive literature on allocation problems arising in cooperative operations. Tijs and Driessen (1986) provide a structured view of general cost allocation methods along with references to early adoption of such methods in practice. The literature on cooperative logistics operations, on the other hand, is relatively recent. This is mainly due to the industry's shrinking margins and advances in information technology which motivate and facilitate cooperation.

In order to deal with the allocation problems in logistics and transportation context, many authors have proposed the adoption of well-known solutions of cooperative game theory. Krajewska et al. (2007) discuss the implementation of the *Shapley value* (Shapley, 1953b) as the solution in cooperative organizations of logistics providers. Özener and Ergun (2008) study cooperative truckload delivery situations where all logistics providers have available depots at every location and show that the *core* (Gillies, 1959) of the games associated with these situations are always non-empty and dual solutions provide allocations in their core. Hezarkhani et al. (2013) further delineate the possibilities and impossibilities for a complete characterization of the core of these games via dual solutions. In cooperative vehicle routing situations, where the core could be empty or it may include many elements, Göthe-Lundgren et al. (1996) and Engevall et al. (2004) elaborate on the implementation of the *nucleolus* (Schmeidler, 1969) as the solution of choice.

Several papers in the recent literature investigate the solutions that incorporate some proportional measures defined on specific features of the underlying situations to divide the savings/costs among the logistics providers. Frisk et al. (2010) propose a solution that draw upon the stand-alone costs of individual companies. Their suggested solution, i.e. the equal profit method (EPM), chooses allocations in the core, or in the *least-core* (Maschler et al., 1979) when the core is empty, such that the spread of ratios of allocated savings to stand-alone costs over all players in minimized. A similar method is proposed independently by Drechsel and Kimms (2010). Multiple extensions of this solution have been proposed ever since. Audy et al. (2010) extend the EPM by including additional constraints that ensure a minimum allocation of savings for all logistics providers. Liu et al. (2010) directly incorporate the marginal contributions of players as weights into the EPM formulation. Finally, Dai and Chen (2012) draw attention to allocations in the core with the property that the greatest deviation from the Shapley value is minimum. What seems to be lacking in this stream of research is the formal definition of the situation-specific properties that are expected from the allocations to satisfy. Vanovermeire et al. (2013) and Lozano et al. (2013) discuss the different outcomes of various solutions in transportation contexts via numerical examples.

Another stream of research investigate the special structures in cooperative games associated with simplified delivery problems. Hamers (1997) analyses the cooperative Chinese postman games and Hamers et al. (1999) discuss the cost allocation problem in these situations. Granot et al. (1999) investigate the special classes of single-depot delivery games whose cores are always non-empty. Platz and Hamers (2013) characterize the graphs whose induced multi-depot Chinese postman games have non-empty cores. We refer the reader to Curiel (2008) for an overview of cooperative games associated with logistics/transportation situations. Nevertheless, the complexity of games associated with combinatorial situations have given rise to new research frontiers that seek *reasonable* theoretical compromises in finding *good* solutions (Caprara and Letchford, 2010).

3 Truckload Delivery Situations

Truckload delivery situations reflect the key features of centralized road freight sector. Let V be a set of nodes corresponding to spatial locations and $w : V \times V \to \mathbb{R}^+$ be a distance function which satisfies triangular inequalities. A set of m delivery requirements $\boldsymbol{D} = \{d^1, ..., d^m\}$ is given. A *delivery requirement* $d^k \in \boldsymbol{D}$ is determined by its pickup location, $a(d^k) \in V$, and its delivery location, $b(d^k) \in V$. The *fulfilment* of the delivery requirement d^k corresponds to a single traverse of the arc $[a(d^k), b(d^k)]$ for k, i.e. two delivery requirements with identical pickup and delivery locations correspond to two non-identical fulfilments. We assume that the distance between the pickup and delivery locations of every trip is positive. A non-empty set of available depots $\boldsymbol{O} = \{o^1, ..., o^h\} \subseteq V$ stations vehicles that fulfil the delivery requirements.

Delivery requirements must be fulfilled in trips. A *trip* is a sequence of deliveries that starts and ends at a particular depot. Formally, a trip l is a tuple (o^l, D^l, σ^l) where $o^l \in \boldsymbol{O}$ is the origin/destination, D^l is a subset of deliveries in $\boldsymbol{D}$ that are fulfilled in l, and σ^l is an ordering of deliveries in D^l which represents the sequence of fulfilments in trip l. Let $\mathcal{L}$ be the set of all such trips. In order to incorporate the practical constraints that could render some trips infeasible–e.g. delivery time windows, number of possible trips per day, traffic network–we introduce the *feasible trip set* $L \subseteq \mathcal{L}$. A *truckload delivery situation* is characterized by a tuple

$$\Lambda = (V, w, \boldsymbol{D}, \boldsymbol{O}, L).$$

We assume that cost and distance are linearly proportional and without loss of generality normalize the proportion to one. The cost of the feasible trip l, $D^l \neq \varnothing$, is comprised of two parts. The first part is the cost associated with the distance travelled between the pickup and delivery locations. The *full kilometres cost* of a trip is independent of both the choice of the trip's depot and the sequence of fulfilments:

$$c_F^l = \sum_{d^k \in D^l} w[a(d^k), b(d^k)] \tag{1}$$

The second part of a trip's cost, i.e. *empty kilometre cost*, is the cost associated with the distance travelled from/to the depot and among different fulfilments:

$$c_E^l = w[o, a(\sigma_1^l)] + \sum_{j=1}^{|D^l|-1} w[b(\sigma_j^l), a(\sigma_{j+1}^l)] + w[b(\sigma_{|D^l|}^l, o)] \tag{2}$$

where the shorthand notation σ_j^l represents the delivery requirement which is fulfilled after all the $j-1$ deliveries preceding it in σ^l are fulfilled. By $|D^l|$ we denote the number of deliveries in D^l. The *cost of trip* l is defined by $c^l = c_F^l + c_E^l$.

Let $L(O, D) = \{l \in L | o^l \in O, D^l \subseteq D\}$ be the set of feasible trips from depots in $O \subseteq \boldsymbol{O}$, $O \neq \varnothing$, to satisfy deliveries in $D \subseteq \boldsymbol{D}$. A fulfilment plan P, hereafter a *plan*, from O to D is a collection of trips in $L(O, D)$ that fulfils all the deliveries in D. The deliveries fulfilled in the collection of trips partition the set of delivery requirements, i.e. $\bigcup_{l \in P} D^l = D$ and $D^l \cap D^k = \varnothing$ for all $k, l \in P$ with $l \neq k$. The cost of a plan P is the total cost of its trips, i.e. D^ls are disjoint and $c(P) = \sum_{l \in P} c^l$. Accordingly, $c(P)$ is decomposable into full and empty parts:

$$c(P) = c_F(P) + c_E(P), \tag{3}$$

where $c_F(P) = \sum_{l \in P} c_F^l$ and $c_E(P) = \sum_{l \in P} c_E^l$ are the total costs of full and empty kilometres of P respectively.

Let $\mathcal{P}(O, D)$ denote the set of all possible fulfilment plans from O to D. We call $P \in \mathcal{P}(O, D)$ an *optimal plan* from O to D if

$$c(P) \leq c(P') \qquad \text{for all } P' \in \mathcal{P}(O, D). \tag{4}$$

The set of all optimal plans from O to D is denoted by $\mathcal{P}^*(O, D)$. If there are multiple optimal plans from O to D, their costs are the same. We denote the *minimum cost of delivery* from O to D with $c^*(O, D)$, and the full kilometres cost of D, which is independent of the choice of depots, with $c_F(D)$.

We provide some observations which will be used in the rest of the paper.

Lemma 1. *Let Λ be a TLD situation. Then,*

(i) $c^*(O, D) \geq c^*(O, D')$ *for all* $\varnothing \neq O \subseteq \boldsymbol{O}$ *and* $D' \subset D \subseteq \boldsymbol{D}$,

(ii) $c^*(O, D) \leq c^*(O', D)$ *for all* $O' \subset O \subseteq \boldsymbol{O}$ *and* $D \subseteq \boldsymbol{D}$,

(iii) $c^*(O, D) + c^*(O, D') \geq c^*(O, D \cup D')$ *for all* $\varnothing \neq O \subseteq \boldsymbol{O}$ *and* $D, D' \subseteq \boldsymbol{D}$ *with* $D \cap D' = \varnothing$.

Proof. (i) Let $D' \subset D \subseteq \boldsymbol{D}$ and define $D'' = D \setminus D'$. Let $P \in \mathcal{P}^*(O, D)$ be an optimal plan from O to D. For every $l \in P$ construct l' where $D^{l'} = D^l \setminus D''$ and $\sigma^{l'}$ keeps the precedence

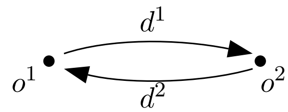

Figure 1: A TLD situation

ordering of D^l in σ^l. As the triangular inequalities hold we have $c^{l'} \leq c^l$ for every $l \in P$. Note that the plan P' obtained in this manner is a feasible plan from O to D'. Since the cost of a plan is the total sum of the costs of its trips we have $c(P') \leq c(P) = c^*(O,D)$. Considering that the optimal plan from O to D' is at most as costly as $c(P')$ we have $c^*(O,D') \leq c^*(O,D)$.

(ii) Let $O' \subset O \subseteq \boldsymbol{O}$ and suppose $P' \in \mathcal{P}^*(O',D)$ is an optimal plan from O' to D. Note that P' is a feasible plan from O to D as well since for any trip $l \in P'$ we have $o^l \in O$. By definition of optimal plans it must be that $c^*(O,D) \leq c^*(O',D)$.

(iii) Let $D, D' \subseteq \boldsymbol{D}$ be such that $D \cap D' = \varnothing$. Let $P \in \mathcal{P}^*(O,D)$ and $P' \in \mathcal{P}^*(O,D')$. Note that $P \cap P' = \varnothing$ and $P \cup P'$ is a feasible plan from O to $D \cup D'$. Furthermore,

$$c(P \cup P') = \sum_{l \in P \cup P'} c^l = \sum_{l \in P} c^l + \sum_{l \in P'} c^l = c^*(O,D) + c^*(O,D').$$

By definition of optimal plans we have $c^*(O,D) + c^*(O,D') \geq c^*(O,D \cup D')$. $\square$

Part (i) of Lemma 1 shows that shrinking the set of delivery requirements cannot increase the optimal cost. A reverse effect is shown in part (ii) for the depots, that is, augmenting the set of depots cannot increase the optimal cost. Finally, part (iii) demonstrates the subadditive effect with regard to the optimal costs that results from aggregated planning of delivery requirements.

We define the *average cost of fulfilment* from $O \neq \varnothing$ to $D \neq \varnothing$ as

$$z(O,D) = \frac{c^*(O,D)}{c_F(D)}. \tag{5}$$

The average cost of fulfilment $z(O,D)$ represents the average cost for fulfilling a unit distance of delivery.[1] If one were supposed to determine a fixed price for every unit distance of delivery services, the average cost of fulfilment would represent the minimum price at which no loss is incurred. By assumption, the full kilometre cost of a non-empty delivery set is strictly positive. For $D = \varnothing$, we let $z(O,\varnothing) = 0$.

Example 1. *Figure 1 depicts a TLD situation with two locations, two depots and two delivery requirements. The distance between the two locations is one and the trip which fulfils both deliveries is feasible. We have* $c^*(O,D) = 2$ *and* $z(O,D) = 1$. $\triangle$

[1] The equivalence of cost and distance assumed in this paper is for simplifying the notation. When this assumption is relaxed, the denominator in equation (5) must be replaced with the total distance of full kilometres in D.

The last lemma in this section establishes the connection between the separability of optimal delivery plans and the additivity of their costs.

Lemma 2. *Let Λ be a TLD situation. Let $\varnothing \neq O' \subseteq O \subseteq \boldsymbol{O}$ and $D' \subseteq D \subseteq \boldsymbol{D}$. We have $c^*(O', D') + c^*(O, D \setminus D') = c^*(O, D)$ if and only if for every $P' \in \mathcal{P}^*(O', D')$ and $P \in \mathcal{P}^*(O, D \setminus D')$, it holds that $P \cup P' \in \mathcal{P}^*(O, D)$.*

Proof. (if) Let $P' \in \mathcal{P}^*(O', D')$ and $P \in \mathcal{P}^*(O, D \setminus D')$ and assume that $P \cup P' \in \mathcal{P}^*(O, D)$. By definition of optimal costs we have

$$c^*(O, D) = \sum_{l \in P' \cup P} c^l = \sum_{l \in P'} c^l + \sum_{l \in P} c^l = c^*(O', D') + c^*(O, D \setminus D').$$

(only if) Assume that $c^*(O', D') + c^*(O, D \setminus D') = c^*(O, D)$. Let $P' \in \mathcal{P}(O', D')$ and $P \in \mathcal{P}(O, D \setminus D')$. Note that $P \cup P'$ is a feasible delivery plan from O to D. We have

$$c(P' \cup P) = \sum_{l \in P' \cup P} c^l = c^*(O', D') + c^*(O, D \setminus D') = c^*(O, D)$$

where the last equality follows by assumption. Hence $P' \cup P \in \mathcal{P}^*(O, D)$. $\square$

4 CTLD Situations and Games

This section introduces the cooperative versions of truckload delivery situations wherein a number of logistics service providers, hereafter *players*, have the option to jointly plan their fulfilment plans. While these situations reflect the key features of decentralized road freight markets, their associated games formalize the underlying gain-sharing problems.

4.1 CTLD situations

Consider a non-empty set $N = \{1, ..., n\}$ of players. Each player $i \in N$ possesses a non-empty set of depots $O_i = \{o_i^1, ..., o_i^{h_i}\}$ and a set of delivery requirements $D_i = \{d_i^1, ..., d_i^{m_i}\}$.

A *coalition* is a subset of players. Let $O_S = \cup_{i \in S} O_i$ and $D_S = \cup_{i \in S} D_i$ denote the combined set of depots and delivery requirements of players in coalition $S \subseteq N$. As above, denote the set of feasible trips for the grand coalition N with L. The set $L(O_S, D_S) = \{l \in L | o^l \in O_S, D^l \subseteq D_S\}$ contains all feasible trips for coalition $S \subseteq N$. We assume that the set of feasible trips of any player is rich enough to enable that player to fulfil its own deliveries individually. The following formalizes this requirement.

Assumption 1. *For all $i \in N$, it holds that $\cup_{l \in L(O_i, D_i)} D^l = D_i$.*

A *cooperative truckload delivery (CTLD) situation* is a tuple

$$\Gamma = (N, V, w, (O_i)_{i \in N}, (D_i)_{i \in N}, L)$$

with all elements being as described previously. We denote the set of all possible CTLD situations with $\mathcal{T}$.

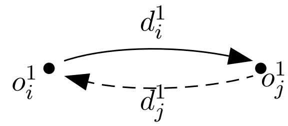

Figure 2: A CTLD situation

By joint planning of its fulfilments, a coalition in a CTLD situation could reduce the cost of its empty kilometres and thus obtain savings. The savings generated by a coalition can be due to utilization of a larger pool of depots for constructing trips or combining fulfilments together more efficiently, or both.

The cost savings obtained by a coalition $S \subseteq N$ in Γ is

$$\sum_{i \in S} c^*(O_i, D_i) - c^*(O_S, D_S).$$

Recall that $c^*(O_S, D_S)$ is the cost of an optimal plan from O_S to D_S in the TLD situation $\Lambda_\Gamma = (V, w, O_N, D_N, L)$.

Example 2. *Figure 2 depicts a CTLD situation Γ with two players i and j each having a single depot o_i^1 and o_j^1, and a single delivery requirement d_i^1 and d_j^2 respectively. The pick-up location of each player's delivery requirement is its depot and the delivery location coincides with the other player's depot. Assuming that the distance between the two locations is one kilometre, we have $c^*(O_i, D_i) = c^*(O_j, D_j) = c^*(O_N, D_N) = 2$. The cost saving obtained by the grand coalition is 2.* $\triangle$

4.2 CTLD games

A cooperative game is a tuple (N, v) consisting of a player set N and a coalition function v which assigns to every coalition $S \subseteq N$ a value $v(S)$ with $v(\varnothing) = 0$. A cooperative game (N, v) is superadditive if for all $S, T \subseteq N$ such that $S \cap T = \varnothing$ it holds that $v(S \cup T) \geq v(S) + v(T)$. If a game is superadditive, then the savings obtained in the grand coalition N is never less than the total savings obtained by any other partitioning of players into coalitions.

The cooperative CTLD game associated with situation $\Gamma \in \mathcal{T}$ with player set N is the pair (N, v^Γ) where for every $S \subseteq N$:

$$v^\Gamma(S) = \sum_{i \in S} c^*(O_i, D_i) - c^*(O_S, D_S). \tag{6}$$

The following theorem states that the games associated with CTLD situations are supperadditive.

Theorem 1. *For every CTLD situation $\Gamma \in \mathcal{T}$, the associated game (N, v^Γ) is superadditive.*

Proof. Let Γ be a CTLD situation and let $S, T \subseteq N$ such that $S \cap T = \varnothing$. Then,

$$\begin{aligned} v^{\Gamma}(S \cup T) &= \sum_{i \in S \cup T} c^*(O_i, D_i) - c^*(O_{S \cup T}, D_{S \cup T}) \\ &\geq \sum_{i \in S \cup T} c^*(O_i, D_i) - c^*(O_{S \cup T}, D_S) - c^*(O_{S \cup T}, D_T) \\ &\geq \sum_{i \in S \cup T} c^*(O_i, D_i) - c^*(O_S, D_S) - c(O_T, D_T) \\ &= \sum_{i \in S} c^*(O_i, D_i) - c^*(O_S, D_S) + \sum_{i \in T} c^*(O_i, D_i) - c(O_T, D_T) \\ &= v^{\Gamma}(S) + v^{\Gamma}(T). \end{aligned}$$

The first inequality follows from part (iii) of Lemma 1 and the second inequality from part (ii) of the same lemma. Thus, $v^{\Gamma}(S \cup T) \geq v^{\Gamma}(S) + v^{\Gamma}(T)$. $\square$

A game (N, v) is zero-normalized if $v(\{i\}) = 0$ for all $i \in N$. As the coalition functions of CTLD games yield the savings comparing the individual and aggregated costs, they are zero-normalized. We discuss other features of CTLD games in Section 7.

5 CTLD Solutions and Their Properties

A *CTLD allocation* is a point in $|N|$-dimensional Euclidean space denoted by $\alpha = (\alpha_i)_{i \in N}$ with α_i being the allocation to player i. A *CTLD solution* is a set-valued function, A, which for every CTLD situation $\Gamma \in \mathcal{T}$ determines a set of allocations for the players in Γ.

This definition of solution is innocuously different than the standard definition of solutions in cooperative game theory as our definition draws upon the situation rather than the game. Note that different situations can correspond to the same cooperative game. This definition allows us to utilize information other than the savings obtained by different coalitions to devise allocations. The rest of this section is devoted to introducing desirable properties of CTLD solutions.

5.1 General properties for CTLD solutions

We start with properties which can also be expressed in relation to the cooperative games associated with CTLD situation. Perhaps the most intuitive desirable property of solutions in any cooperative situation is the efficiency property which requires that all the savings obtained in the grand coalition be completely divided among the players.

Property 1. *A CTLD solution A satisfies the **efficiency property** if for all $\Gamma \in \mathcal{T}$ and every $\alpha \in A(\Gamma)$ it holds that $\sum_{i \in N} \alpha_i = v^{\Gamma}(N)$.*

The nonemptiness property defined below reflects the ability of a solution to produce at least one allocation in every given CTLD situation.

Property 2. *A CTLD solution A satisfies the **nonemptiness property** (NE) if for all $\Gamma \in \mathcal{T}$, $A(\Gamma)$ is non-empty.*

A solution would be inconclusive if it yields more than one allocation in situations wherein it could suggest any allocation at all. The uniqueness property addresses this issue.

Property 3. *A CTLD solution A satisfies the **uniqueness property** (UQ) if for all $\Gamma \in \mathcal{T}$ such that $A(\Gamma) \neq \varnothing$, it holds that $|A(\Gamma)| = 1$.*

The notion of stability is a critical concept in many cooperative situations, including CTLD situations. Given a CTLD situation Γ and $\epsilon \in \mathbb{R}$, we call an allocation α *ϵ-stable* if (a) α is efficient, i.e. $\sum_{i \in N} \alpha_i = v^\Gamma(N)$, and (b) for all $S \subset N$, it holds that $\sum_{i \in S} \alpha_i + \epsilon \geq v^\Gamma(S)$. The set of all ϵ-stable allocations of a situation comprises the *ϵ-core* of its associated cooperative game (Shapley and Shubik, 1966). An ϵ-stable allocation provides sufficient incentives for all players not to break apart from the grand coalition if the cost of reorganizing cooperation in a sub-coalition is larger than ϵ. Ideally, a 0-stable (stable) allocation provides sufficient incentives for all players to remain in the grand coalition even if separating from the grand coalition is free. We will give an example in Section 7 to show that it may be impossible to find stable allocations in CTLD situations. The following property reflects the need for solutions that either produce stable allocations, or, when the latter cannot be achieved, obtain allocations that are as stable as possible.

Property 4. *A CTLD solution A satisfies the **least unstability property** (LU) if for all $\Gamma \in \mathcal{T}$ and every $\alpha \in A(\Gamma)$, α is ϵ^*-stable where*

$$\epsilon^* = \min \left\{ \epsilon \geq 0 \,\middle|\, \sum_{i \in S} \alpha_i + \epsilon \geq v^\Gamma(S) \text{ for all } S \subset N, \sum_{i \in N} \alpha_i = v^\Gamma(N) \right\}. \tag{7}$$

5.2 Specific properties for CTLD solutions

The two properties introduced in this section are specific to CTLD situations and address issues concerning the competitive positions of the players and the scope beyond which the network of deliveries of a player should be ignored by the solution. We start by introducing two special classes of delivery requirements in CTLD situations.

Definition 1. *Let Γ be a CTLD situation. $D \subseteq D_i$ is a **separable delivery set** (SDS) of i if*

$$c^*(O_i, D) + c^*(O_N, D_N \setminus D) = c^*(O_N, D_N). \tag{8}$$

Let $SDS_i(\Gamma)$ be the set of separable delivery sets of i.

The stand-alone cost of fulfilling a separable delivery set of a player is additive to the cost of fulfilling the remaining deliveries of the grand coalition. Therefore, a player can individually fulfil a separable delivery set of itself without disrupting the optimality of delivery plans of the grand coalition.

Example 3. *Figure 3 depicts a CTLD situation Γ with two players. It is easy to see that player i can individually fulfil the delivery requirement $\{d_i^1\}$. Also, i can take out either*

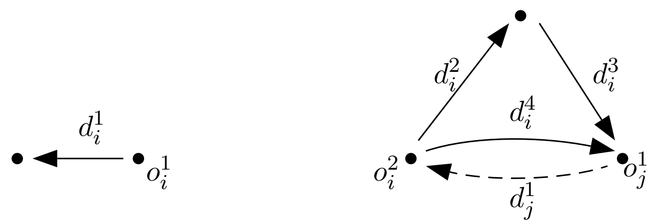

Figure 3: Separable and irrelevant deliveries

$\{d_i^2, d_i^3\}$ or $\{d_i^4\}$ (but not both!) from the grand coalition's delivery requirements and fulfil them separately such that total optimal cost does not increase. Thus, we have

$$SDS_i(\Gamma) = \left\{ \{d_i^1\}, \{d_i^2, d_i^3\}, \{d_i^4\}, \{d_i^1, d_i^2, d_i^3\}, \{d_i^1, d_i^4\} \right\}.$$

$\triangle$

The previous example shows two different types of separable delivery sets in CTLD situations. While some separable delivery sets of a player can be substituted with each other, there might exists separable delivery sets which are separable in all scenarios. Therefore, among the separable delivery sets of a player i, we distinguish delivery sets which do not have any possible internal or external relevance to the rest of the network of deliveries in any coalition with irrelevant deliveries.

Definition 2. *Let Γ be a CTLD situation. $D \subseteq D_i$ is an **irrelevant delivery set** (IDS) of i if for all $D' \subseteq D$, all $S \subseteq N$ with $i \in S$, and all $D'' \subseteq D_S \setminus D$ it holds that*

$$c^*(O_i, D') + c^*(O_S, D'') = c^*(O_S, D' \cup D''). \tag{9}$$

Let $IDS_i(\Gamma)$ be the set of irrelevant delivery sets of i.

The cost of fulfilling any subsets of irrelevant deliveries of a player is additive to any subset of the set of remaining deliveries in any coalition that includes that player, so the player can fulfil such deliveries separately in any possible combination with other deliveries. In Example 3, $\{d_i^1\}$ is the only irrelevant delivery set of i. Note that neither $\{d_i^2, d_i^3\}$ nor $\{d_i^4\}$ would remain separable if i removes the other set and plan its delivery by himself. This example clarifies that every irrelevant delivery set is also separable, but the reverse does not hold necessarily.

We are ready to present the first property in this section which specifies a scope of consideration for CTLD situations where the delivery requirements beyond this scope should be ignored in the calculation of allocations. We define the *independence of irrelevant deliveries* property as the insensitivity of a solution to the exclusion of irrelevant deliveries of the players. Given $D_i' \subseteq D_i$, let $\Gamma \setminus D_i'$ be a CTLD situation that coincides with Γ except for the delivery set of i which is replaced by $D_i \setminus D_i'$.

Property 5. *A CTLD solution A satisfies the **independence of irrelevant deliveries property** (IID) if for all $\Gamma \in \mathcal{T}$, any $i \in N$, and every $D \in IDS_i(\Gamma)$ it holds that $A(\Gamma) = A(\Gamma \setminus D)$.*

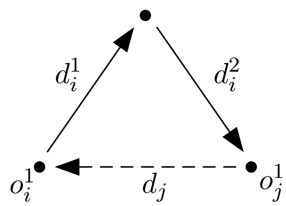

Figure 4: A CTLD situation where i and j have different competitive positions

The final property in this section addresses the competitive aspect of solutions in CTLD situations. Considering the limited number of players in consortia of logistics providers and the competitive nature of transportation markets, a key requirement for solutions in CTLD situations is their ability to maintain the competitive positions of the players in dividing the savings obtained by cooperation.

Recall from Section 3 that the average cost of fulfilment represents the lowest price that i can charge for every unit distance of its delivery services. In this regard, the average cost of fulfilment provides a basis for calculating unit delivery prices in logistics markets. However, it can also be utilized as a measure of comparison among the players. This idea is motivated by the observation that a lower average cost of fulfilment of a logistics player compared to that of another logistics player allows the former to charge a lower unit price for its delivery services while remaining profitable. Therefore, if for two players i and j it holds that $z(O_i, D_i) < z(O_j, D_j)$, it can be stated that prior to cooperation, i is in a better competitive position than j. The definition of average cost of fulfilment can be naturally extended to incorporate the savings allocated to the players after the cooperation. Given an allocation α and player $i \in N$, $D_i \neq \varnothing$, define the average cost of fulfilment of player i under α as

$$z_i^{\alpha}(O_i, D_i) = \frac{c^*(O_i, D_i) - \alpha_i}{c_F(D_i)} \tag{10}$$

Note that the cost of full kilometres of any non-empty set of delivery requirements is strictly positive. The notion of competitiveness in this paper is motivated by the observation that non-competitive allocations eliminate the advantage of a player over other players in terms of its competitive position before and after cooperation. We elaborate with the help of an example.

Example 4. *Figure 4 represents a CTLD situation with two logistics players i and j. Assuming that the distance between any two locations is 1, we get* $z(O_i, D_i) = 1.5$ *and* $z(O_j, D_j) = 2$*. The cooperation in this case results in 2 units of savings, i.e.* $v^{\Gamma}(N) = 2$*. Observe that the equal allocation* $\alpha = (1, 1)$ *results in having* $z_i^{\alpha}(O_i, D_i) = z_j^{\alpha}(O_j, D_j) = 1$*. Thus, the equal allocation eliminates i's advantage over j with regard to their competitive positions prior to the cooperation.* $\triangle$

We are now ready to introduce a competitiveness property defined over a restricted set of CTLD situations. Let $\mathcal{T}_2^{\circ}$ be the set of 2-player situations such that for all $j \in N$, $SDS_j(\Gamma) = \varnothing$.

Property 6. *A CTLD solution satisfies the* ***restricted competitiveness property*** *(RC) if for every* $\Gamma \in \mathcal{T}_2^\circ$ *and any* $\alpha \in A(\Gamma)$ *it holds that*

$$z_i^\alpha(O_i, D_i) z(O_j, D_j) = z_j^\alpha(O_j, D_j) z(O_i, D_i). \tag{11}$$

If the set of delivery requirements of each player is non-empty, then (11) boils down to $z(O_i, D_i)/z_i^\alpha(O_i, D_i) = z(O_j, D_j)/z_j^\alpha(O_j, D_j)$. In this respect, the RC property prescribes allocations that *preserve the competitive positions* of the players, that is, an allocation satisfying the RC property equalizes the ratio of average costs of fulfilments of the players before and after cooperation. Note that the latter can also be expressed in terms of the stand-alone costs of the players, i.e. for players with non-empty delivery sets (11) is simplified to $\alpha_i/c^*(O_i, D_i) = \alpha_j/c^*(O_j, D_j)$. In Example 4, the allocation $\alpha = (1.2, 0.8)$ preserves the competitive positions of i and j before and after the cooperation, resulting in $z_i^\alpha(O_i, D_i) = 0.9$ and $z_j^\alpha(O_j, D_j) = 1.2$.

6 A Solution for CTLD Situations

In this section we introduce a CTLD solution that satisfies all the properties mentioned above. Our solution draws upon essential delivery sets, i.e. particular subsets of deliveries of players which are necessary and sufficient in creating the contribution of the player to the grand coalition.

6.1 Essential delivery sets

Let $S \setminus i$ denotes the coalition S excluding player i.

Definition 3. *Let* Γ *be a CTLD situation.* $D \subseteq D_i$ *is an* ***essential delivery set*** *(EDS) of* i *if*

$$c^*(O_i, D_i \setminus D) + c^*(O_N, D_{N \setminus i} \cup D) = c^*(O_N, D_N). \tag{12}$$

and for every $D' \subset D$, $D \neq \varnothing$:

$$c^*(O_i, D_i \setminus D') + c^*(O_N, D_{N \setminus i} \cup D') > c^*(O_N, D_N) \tag{13}$$

Let $EDS_i(\Gamma)$ *be the set of all essential delivery sets of* i.

An essential delivery set of a player i meets two conditions. First, the complement of this set comprises a separable delivery set of i, that is, an essential delivery set is *sufficient* for creating the cost savings of players in the grand coalition. Second, one cannot expand the complement of this set to obtain a larger separable delivery set for i. In fact, an essential delivery set is *necessary* for creating the players' contribution to the grand coalition in the sense that the situation obtained by excluding its complement delivery set contains no separable delivery sets. The following lemma formalizes this.

Lemma 3. *Let* Γ *be a CTLD situation,* $D \in EDS_i(\Gamma)$*, and* $\Gamma' = \Gamma \setminus (D_i \setminus D)$*. We have* $SDS_i(\Gamma') = \varnothing$.

Proof. It suffices to show that for any $D^* \subseteq D$ it holds that

$$c^*(O_i, D^*) + c^*(O_N, D_{N\setminus i} \cup (D \setminus D^*)) > c^*(O_N, D_{N\setminus i} \cup D). \tag{14}$$

For any $D^* \subseteq D$ we have

$$\begin{aligned}
& c^*(O_i, D^*) + c^*(O_N, D_{N\setminus i} \cup (D \setminus D^*)) \\
= \quad & c^*(O_i, D^*) + c^*(O_N, D_{N\setminus i} \cup (D \setminus D^*)) + c^*(O_i, D_i \setminus D) - c^*(O_i, D_i \setminus D) \\
\geq \quad & c^*(O_i, D_i \setminus (D \setminus D^*)) + c^*(O_N, D_{N\setminus i} \cup (D \setminus D^*)) - c^*(O_i, D_i \setminus D) \\
> \quad & c^*(O_N, D_N) - c^*(O_i, D_i \setminus D) \\
= \quad & c^*(O_N, D_{N\setminus i} \cup D).
\end{aligned}$$

where the first inequality follows from part (i) of Lemma 1, the second inequality follows from the second condition of EDS in (13) for $D' = D \setminus D^*$, and the last equality follows from the first condition of EDS in (12). Therefore (14) holds for any $D^* \subseteq D$. □

In Example 3 (Figure 3), player i has two sets of essential delivery sets $\{d_i^2, d_i^3\}$ and $\{d_i^4\}$. This demonstrates that a player in a CTLD situation might have multiple essential delivery sets.

The next lemma elaborates on the relation between essential and irrelevant delivery sets.

Lemma 4. *Let Γ be a CTLD situation, $i \in N$, and $D_i^r \in IDS_i(\Gamma)$. Then*

(i) *for every $D \in EDS_i(\Gamma)$, $D \cap D_i^r = \varnothing$,*

(ii) *for every $j \in N$, $EDS_j(\Gamma) = EDS_j(\Gamma \setminus D_i^r)$.*

Proof. (i) Suppose the contrary that $D \cap D_i^r \neq \varnothing$. Let $D' = D \cap D_i^r$, hence $D' \subseteq D$. By definition of irrelevant deliveries of i in Γ it must be that

$$c^*(O_i, D') + c^*(O_N, D_{N\setminus i} \cup (D \setminus D')) = c^*(O_N, D_{N\setminus i} \cup D). \tag{15}$$

The latter implies that D' is a separable delivery set of i in $\Gamma \setminus (D_i \setminus D)$ which contradicts Lemma 3. Thus it must be that $D \cap D_i^r = \varnothing$.

(ii) First we show that for any $j \in N$ and every $D' \subseteq D_N \setminus D_i^r$ such that $D' \neq \varnothing$ it holds that

$$\begin{aligned}
& c^*(O_j, D_j \setminus D') + c^*(O_N, D_{N\setminus j} \cup D') - c^*(O_N, D_N) = \\
& c^*(O_j, D_j \setminus D') + c^*(O_N, (D_{N\setminus j} \setminus D_i^r) \cup D') - c^*(O_N, D_N \setminus D_i^r).
\end{aligned} \tag{16}$$

Since $D_i^r \in IDS_i(\Gamma)$, we have

$$c^*(O_i, D_i^r) + c^*(O_N, D_N \setminus D_i^r) = c^*(O_N, D_N), \tag{17}$$

and, since $(D_{N\setminus j} \setminus D_i^r) \cup D' \subset D_N \setminus D_i^r$, we have

$$c^*(O_i, D_i^r) + c^*(O_N, (D_{N\setminus j} \setminus D_i^r) \cup D') = c^*(O_N, D_{N\setminus j} \cup D'). \tag{18}$$

By (17) and (18) we get

$$c^*(O_N, D_{N\setminus j} \cup D') - c^*(O_N, D_N) = c^*(O_N, (D_{N\setminus j} \setminus D_i^r) \cup D') - c^*(O_N, D_N \setminus D_i^r) \tag{19}$$

Adding $c^*(O_j, D_j \setminus D')$ to both sides of the equation (19) obtains (16).

Suppose that $D \in EDS_j(\Gamma)$. For $j \neq i$ it holds that $D \subseteq D_j$, and for $j = i$ part (i) of this lemma indicates that $D \subseteq D_i \setminus D_i^r$. Therefore for $j \in N$ we have $D \subseteq D_N \setminus D_i^r$. By definition of EDS it must be that

$$c^*(O_j, D_j \setminus D) + c^*(O_N, D_{N\setminus j} \cup D) = c^*(O_N, D_N) \tag{20}$$

and for every $D'' \subset D$, $D'' \neq \varnothing$,

$$c^*(O_j, D_j \setminus D'') + c^*(O_N, D_{N\setminus j} \cup D'') > c^*(O_N, D_N). \tag{21}$$

However, due to (16), equality in (20) implies that

$$c^*(O_j, D_j \setminus D) + c^*(O_N, (D_{N\setminus j} \setminus D_i^r) \cup D) = c^*(O_N, D_N \setminus D_i^r) \tag{22}$$

and, due to (16), inequality in (21) implies that

$$c^*(O_j, D_j \setminus D'') + c^*(O_N, (D_{N\setminus j} \setminus D_i^r) \cup D'') > c^*(O_N, D_N \setminus D_i^r) \tag{23}$$

for every $D'' \subset D$, $D'' \neq \varnothing$. The conditions in (22) and (23) indicate that $D \in EDS_j(\Gamma \setminus D_i^r)$.

Suppose that $D \in EDS_j(\Gamma \setminus D_i^r)$. For $j \neq i$ it holds that $D \subseteq D_j$, and for $j = i$ it is the case that $D \subseteq D_i \setminus D_i^r$. Therefore for $j \in N$ we have $D \subseteq D_N \setminus D_i^r$. By definition of EDS equality (22) as well as inequality (23) for every $D'' \subset D$, $D'' \neq \varnothing$, hold. As direct results of (16), equality (20) as well as inequality (21) for every $D'' \subset D$, $D'' \neq \varnothing$, must also hold. We conclude that $D \in EDS_j(\Gamma)$. □

Part (i) of Lemma 4 asserts that the essential delivery sets never include any irrelevant deliveries. Part (ii) shows that the exclusion of irrelevant deliveries of any player from the entire delivery set of the grand coalition does not affect the sets of essential delivery sets of all players.

The last lemma in this section shows that for finding essential delivery sets, it is sufficient to compare the optimal individual plans and those of the grand coalition. Given an optimal plan of a player i and an optimal plan for the grand coalition, an essential delivery set of i comprises the delivery requirements whose fulfilment in the grand coalition involve other players, i.e. they are either fulfilled from depots of other players or in trips which contain delivery requirements of players other than i.

Lemma 5. *Let Γ be a CTLD situation and $D \in EDS_i(\Gamma)$. There exists $P \in \mathcal{P}^*(O_N, D_N)$ such that $D = \bigcup_{l \in P \setminus L(O_i, D_i)} D^l$.*

Proof. Let $P' \in \mathcal{P}^*(O_i, D_i \setminus D)$, $P'' \in \mathcal{P}^*(O_N, D_{N\setminus i} \cup D)$, and $P = P' \cup P''$. By first condition of EDS in (12) we have $c^*(O_i, D_i \setminus D) + c^*(O_N, D_{N\setminus i} \cup D) = c^*(O_N, D_N)$. By Lemma 2, we have $P \in \mathcal{P}^*(O_N, D_N)$. Note that $P' \subseteq L(O_i, D_i)$. To complete the proof it suffices to show that $P'' \cap L(O_i, D_i) = \varnothing$. Suppose the contrary. Then there must exist trip l such that $l \in P'' \cap L(O_i, D_i)$ which requires that $D^l \subset D \subseteq D_i$. As the cost of the delivery plan P'' is the sum of its individual trips, it must be that

$$c^l + c^*(O_N, D_{N\setminus i} \cup (D \setminus D^l)) = c^*(O_N, D_{N\setminus i} \cup D).$$

Since $l \in L(O_i, D_i)$, we have $c^l = c^*(O_i, D^l)$. Then it must be that

$$c^*(O_i, D^l) + c^*(O_N, D_{N\setminus i} \cup (D \setminus D^l)) = c^*(O_N, D_{N\setminus i} \cup D).$$

The latter implies that D^l is a separable delivery set of i in $(\Gamma \setminus D_i) \cup D$ which contradicts Lemma 3. Thus, given D, any optimal fulfilment plan of the form $P = P' \cup P''$ has the feature that $D = \bigcup_{l \in P \setminus L(O_i, D_i)} D^l$. □

Given the favourable features of essential delivery sets, we concentrate on them when determining players allocations in CTLD situations. However, as seen in Example 3, the essential delivery sets of a player can be multiple. In these cases, we focus on essential delivery sets which have the lowest costs when fulfilled individually. In this way, we make sure that players fulfil the bulkiest parts of their delivery requirements by themselves. We introduce the *minimal essential delivery sets* as the essential delivery sets with minimum stand-alone cost.

Definition 4. *Let Γ be a CTLD situation. $D \subseteq D_i$ is a* ***minimal essential delivery set*** *(MEDS) of i if $D \in EDS_i(\Gamma)$ and*

$$c^*(O_i, D) \leq c^*(O_i, D') \qquad \text{for all } D' \in EDS_i(\Gamma). \tag{24}$$

Let $MEDS_i(\Gamma)$ be the set of minimal essential delivery sets of i.

Note that even if the set of minimal essential delivery sets has multiple elements, the stand-alone costs of fulfilment for all of them are equal.

6.2 The proposed solution

Our proposed CTLD solution is introduced in two steps. In the first step, we introduce a proportional CTLD solution, A^P, which incorporates the notions of competitiveness and scope defined in the previous section. In the second step, we use the latter proportional allocation to construct a least-unstable solution, A^E.

Fix Γ, let $D_i^m \in MEDS_i(\Gamma)$, and define $A^P(\Gamma) = \{\alpha^P(\Gamma)\}$ such that

$$\alpha_i^P(\Gamma) = \begin{cases} \dfrac{c^*(O_i, D_i^m)}{\sum_{j\in N} c^*(O_j, D_j^m)} v^\Gamma(N) & \text{if } \displaystyle\sum_{j\in N} c^*(O_j, D_j^m) \neq 0 \\ \dfrac{1}{n} v^\Gamma(N) & \text{otherwise} \end{cases} \tag{25}$$

When there exists at least one player with a non-empty essential delivery set, the solution A^P obtains a unique efficient allocation that divides the savings obtained in the grand coalition of CTLD situation Γ among players with non-empty essential delivery sets proportional to the stand-alone cost of their minimal essential deliveries. If the set of essential delivery set of every player is empty, then A^P allocates the savings among the players equally. Example 5 in Section 7 discusses a CTLD situation where the latter is the case.

The CTLD solution A^P completely preserves the competitive positions of the players with regard to their minimal essential delivery sets. This means that for every pair of players $i, j \in N$ with non-empty essential delivery sets we have $\alpha_i^P(\Gamma)/c^*(O_i, D_i^m) = \alpha_j^P(\Gamma)/c^*(O_j, D_j^m)$ which implies that

$$\frac{z_i(O_i, D_i^m)}{z_i^{\alpha^P(\Gamma)}(O_i, D_i^m)} = \frac{z_j(O_j, D_j^m)}{z_j^{\alpha^P(\Gamma)}(O_j, D_j^m)}.$$

The solution A^P does not necessarily obtain an ϵ^*-stable allocation. In order to achieve this, we present the stable CTLD solution A^E. Define $A^E(\Gamma)$ such that

$$A^E(\Gamma) = \arg\min_{\alpha} \sum_{i \in N} (\alpha_i^P(\Gamma) - \alpha_i)^2 \tag{26}$$

$$s.t. \quad \sum_{i \in S} \alpha_i + \epsilon^* \geq v^\Gamma(S) \qquad \forall S \subset N \tag{27}$$

$$\sum_{i \in N} \alpha_i = v^\Gamma(N) \tag{28}$$

where ϵ^* is defined in (7). Below we show that A^E always produces a single ϵ^*-stable allocation with the minimum Euclidean distance from the proportional allocation $\alpha^P(\Gamma)$.

Theorem 2. *The solution A^E satisfies the NE, UQ, and LU.*

Proof. Let $\Gamma \in \mathcal{T}$. We proceed in order.

NE: By Assumption 1, v^Γ is well-defined. From (25) it is clear that $\alpha^P(\Gamma)$ always exists since either $\sum_{j \in N} c^*(O_j, D_j^m) \neq 0$ or $\sum_{j \in N} c^*(O_j, D_j^m) = 0$. Also, definition of ϵ^* guarantees the existence of α that satisfy (27) and (28) (Maschler et al., 1979). Therefore $A^E(\Gamma) \neq \emptyset$. We conclude that A^E satisfies the nonemptiness property.

UQ: The allocations contained in $A^E(\Gamma)$ minimize the Euclidean distance between $\alpha^P(\Gamma)$ and the set of ϵ^*-stable allocations defined via (27) and (28). Note that the region defined via (27) and (28), which is essentially an ϵ-core, is a compact convex polyhedron (Maschler et al., 1979). The convex projection theorem (Davidson and Donsig, 2010) asserts that there exists a unique point in every non-empty closed and convex set having the minimum Euclidean distance from any given point. Therefore, $|A^E(\Gamma)| = 1$ which implies that A^E satisfies the uniqueness property.

LU: The unique allocation obtained by $A^E(\Gamma)$ satisfies the constraints in (27) and (28). By definition of ϵ^*-stability in (7), this allocation is an ϵ^*-stable allocation as well. It follows that the solution A^E satisfies the LU property. $\square$

Since the allocation A^E satisfies the UQ property, in any CTLD situation Γ it results in a single allocation. We denote this single allocation with $\alpha^E(\Gamma)$, i.e. $A^E(\Gamma) = \{\alpha^E(\Gamma)\}$.

Before providing the results regarding the ability of A^E to satisfy IID and RC properties, we show that the coalition functions in CTLD situations remain intact if irrelevant delivery sets of the players are excluded from the situation.

Lemma 6. *Let Γ be a CTLD situations, i be a player in N and $D \in IDS_i(\Gamma)$. We have $v^\Gamma(S) = v^{\Gamma\setminus D}(S)$ for every $S \subseteq N$.*

Proof. From the definition of irrelevant delivery sets we know that

$$c^*(O_i, D) + c^*(O_i, D_i \setminus D) = c^*(O_i, D_i)$$

and for $S \subseteq N$, $i \in S$,

$$c^*(O_i, D) + c^*(O_S, D_S \setminus D) = c^*(O_S, D_S).$$

By definition of v^Γ it then follows that

$$\begin{aligned} v^\Gamma(S) &= \sum_{j\in S\setminus i} c^*(O_j, D_j) + c^*(O_i, D_i) - c^*(O_S, D_S) \\ &= \sum_{j\in S\setminus i} c^*(O_j, D_j) + c^*(O_i, D_i \setminus D) - c^*(O_S, D_S \setminus D) \\ &= v^{\Gamma\setminus D}(S). \end{aligned}$$

□

In light of the previous lemma, it can be inferred that the CTLD game associated with a CTLD situation remains the same after excluding the irrelevant delivery sets of players.

Next theorem asserts that A^E also satisfies the remaining two properties defined specifically for CTLD situations.

Theorem 3. *The solution A^E satisfies the IID and RC properties.*

Proof. Let Γ be given.

IID: Let $D_i^r \in IDS_i(\Gamma)$. By Lemma 6, we have $v^\Gamma(S) = v^{\Gamma\setminus D_i^r}(S)$ for all $S \subseteq N$. Thus the constraints in (27) and (28) would not be affected by exclusion of the irrelevant deliveries. It remains to show that $\alpha_j^P(\Gamma) = \alpha_j^P(\Gamma \setminus D_i^r)$ for every $j \in N$. By part (ii) of Lemma 4 we know that $EDS_j(\Gamma) = EDS_j(\Gamma \setminus D_i^r)$ for all $j \in N$. Consequently, $MEDS_j(\Gamma) = MEDS_j(\Gamma \setminus D_i^r)$ for all $j \in N$. From the definition of α^P in (25) in conjunction with Lemma 6 it immediately follows that $\alpha_j^P(\Gamma) = \alpha_j^P(\Gamma \setminus D_i^r)$ for every $j \in N$. Thus, $A^E(\Gamma) = A^E(\Gamma \setminus D_i^r)$ which proves that A^E satisfies the IID property.

RC: Suppose $|N| = 2$. In this case the constraints in (27) and (28) are reduced to $\alpha_i + \alpha_j = v^\Gamma(N)$ and $\alpha_i, \alpha_j \geq 0$. The allocation $\alpha^P(\Gamma)$ is within this region. Hence, $\alpha^E(\Gamma) = \alpha^P(\Gamma)$ which results in the objective function value of zero in (26). Next suppose that $SDS_j = \varnothing$ for $j \in N$. In this case we have $MEDS_j(\Gamma) = EDS_j(\Gamma) = \{D_j\}$ for $j \in N$. By definition of $\alpha^P(\Gamma)$

we get $\alpha_i^P(\Gamma)/c^*(O_i, D_i) = \alpha^P(\Gamma)/c^*(O_j, D_j)$ which implies that (11) holds. Therefore the solution A^E satisfies the RC property. $\square$

The solution A^E incorporates the notions of scope and competitiveness to produce allocations for CTLD situations. When the proportional solution A^P is within the set of ϵ^*-stable allocations, defined via (27) and (28), A^E coincides with A^P. Otherwise, when the allocation obtained by A^P is not an ϵ^*-stable allocation, A^E draws upon a simple mechanism to single out an allocation in relation to the A^P.

7 Adopting Existing Solutions for CTLD Situations

This section discusses the adoption of some of the existing solutions for CTLD situations and compares them with regard to the properties introduced earlier in this paper. Table 1 at the end of this section exhibits the summary of the results.

One of best-known solutions in cooperative game theory is the *core*. The core of a cooperative game (N, v), i.e. $\mathcal{C}(N, v)$, contains all allocations that are stable. In this regard, the core in itself is an ϵ-core with $\epsilon = 0$. The core of a CTLD situation can be defined accordingly as a mapping that assign to every CTLD situation the core of its associated game. In this manner, one can extend the core as a CTLD solution. Formally,

$$A^{\mathcal{C}}(\Gamma) = \mathcal{C}(N, v^\Gamma) = \left\{ \alpha \,\middle|\, \sum_{i \in S} \alpha_i \geq v^\Gamma(S) \text{ for all } S \subset N, \text{ and } \sum_{i \in N} \alpha_i = v^\Gamma(N) \right\}.$$

The following example shows that the core of CTLD situations does not satisfy the NE property.

Example 5. *Consider the CTLD situation Γ depicted in Figure 5. There are three players each having a depot and a delivery requirement. The distance between the pickup and delivery locations for all delivery requirements is two and the distance from the depots to any pickup/delivery point is one. The set of feasible trips includes all trips which fulfil no more than two delivery requirements, i.e. $L = \{l \in \mathcal{L} \| D^l| \leq 2\}$ (only two deliveries can be fulfilled sequentially during a day). For $S \subseteq N$, we have $c^*(O_S, D_S) = 4$ if $|S| = 1$, $c^*(O_S, D_S) = 6$ if $|S| = 2$, and $c^*(O_N, D_N) = 10$. This results in $v^\Gamma(S) = 0$ when $|S| = 1$, $v^\Gamma(S) = 2$ when $|S| = 2$, and $v^\Gamma(N) = 2$. No stable allocation can be found in this setting thus the core of Γ is empty.* $\triangle$

Upon existence, the core of a CTLD situation could as well contain an infinite number of allocations. For example, in two-player CTLD situations with $v^\Gamma(N) > 0$, any efficient allocation that gives non-negative allocations to the players is in the core. Therefore, the core of CTLD situations does not satisfy UQ property. On the other hand, the definition of the RC property in combination with the efficiency property necessitates a unique allocation in every two-player situation. Consequently, the core does not satisfy the RC property either. Since the core solely draws upon the CTLD game, it follows from Lemma 6 that it satisfies the IID property.

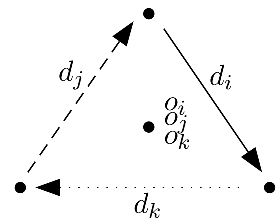

Figure 5: A CTLD situation with empty core

The intuitive appeal of the stability concept on one side, and the possibility of having empty cores on the other side motivates alternative solutions that mainly address the stability issue. The *least-core* of the game (Maschler et al., 1979), i.e. $\mathcal{LC}(N, v)$, is the intersection of all non-empty ϵ-cores of (N, v). Accordingly, the least core of CTLD solutions is defined as a mapping that assigns to every situation the least-core of its associated game:

$$A^{\mathcal{LC}}(\Gamma) = \left\{ \alpha \left| \sum_{i \in S} \alpha_i + \epsilon^{\min} \geq v^\Gamma(S) \text{ for all } S \subset N, \text{ and } \sum_{i \in N} \alpha_i = v^\Gamma(N) \right. \right\}.$$

where

$$\epsilon^{\min} = \min \left\{ \epsilon \left| \sum_{i \in S} \alpha_i + \epsilon \geq v^\Gamma(S) \text{ for all } S \subset N, \sum_{i \in N} \alpha_i = v^\Gamma(N) \right. \right\}. \tag{29}$$

Considering the definition ϵ-core, one can always find values of ϵ such that the corresponding ϵ-core is non-empty. Consequently, the least-core, unlike the core, satisfies the NE property. However, the least-core does not necessarily results in a unique allocation (see Example 7 below) so it does not satisfy the UQ property. Since the least-core solely draws upon the CTLD game, it follows from Lemma 6 that it satisfies the IID property. It is straightforward to verify that in zero-normalized two-player games, the least-core contains only the allocation which divides the savings equally between the players which imply that it does not satisfy the RC property. Note that the definition of least-unstability implies that the set of ϵ^*-stable allocations for a CTLD situation always contains the corresponding $\epsilon^{\min}$-stable allocations. This is due to the fact that $\epsilon^{\min}$ can take negative values while ϵ^* is always non-negative. Therefore, when the core of a CTLD situation is not empty, the least-unstable allocations are within the core and when the latter is empty, the set of least-unstable allocations coincide with the least-core. Thus leas-core satisfies the LU property.

The *nucleolus* is another well-studied solution for cooperative games. As in the case of the core, the nucleolus in CTLD solutions can be defined as a mapping that assigns to every situation Γ the nucleolus of its corresponding game, $\eta(N, v^\Gamma)$. For a CTLD situation Γ and an allocation α, define the vector of excesses $\theta(\alpha)$ as a vector in $\mathbb{R}^{2^n}$ whose components are the numbers $v^\Gamma(S) - \sum_{i \in S} \alpha_i$ arranged non-increasingly. For two vectors $x, x' \in \mathbb{R}^m$, the lexicographical order $x \leq_{lex} x'$ implies that either $x = x'$, or there is $1 \leq t \leq m$ such that $x_i = x'_i$ for $1 \leq j < t$ and $x_t < x'_t$. Define the *imputation set* $M(\Gamma) = \{\alpha | \sum_{i \in N} \alpha_i = v^\Gamma(N) \text{ and } \alpha_i \geq 0 \text{ for all } i \in N\}$. The nucleolus of a CTLD situation Γ, i.e. $A^\eta(\Gamma)$, is the set of imputations

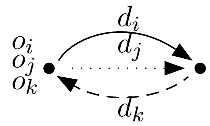

Figure 6: A CTLD situation with three players

whose associated vectors of excesses are lexicographically minimal:

$$A^{\eta}(\Gamma) = \eta(N, v^{\Gamma}) = \{\alpha \,|\theta(\alpha) \leq_{lex} \theta(\alpha') \text{ for all } \alpha, \alpha' \in M(\Gamma)\}.$$

The nucleolus selects the allocations which lexicographically minimize the vector of objections for all coalitions of players where the objection is defined as the difference between the savings obtainable by that coalition and the given allocation. For every cooperative game, the nucleolus always exists, is unique, and is contained in the least-core (Schmeidler, 1969). Therefore, the nucleolus of CTLD situations satisfies the NE, UQ, and LU properties. Since the nucleolus solely draws upon the CTLD game, it follows from Lemma 6 that it satisfies the IID property. However, it does not satisfy the RC property as in two-player situations it always results in the allocation that divides the savings equally between the two players (Aumann and Maschler, 1985).

The *Shapley value* is a classic solution in cooperative game theory. The Shapley value of a game $\Phi(N, v)$ is a single-valued solution that allocates the savings to players based on their average contributions to all coalitions. By extending the notion of Shapley value to CTLD situations, we obtain the solution A^{Φ} wherein for situation Γ we have $A^{\Phi}(\Gamma) = \{\alpha^{\Phi}(\Gamma)\} = \{\Phi(N, v^{\Gamma})\}$, that is for all $i \in N$:

$$\alpha_i^{\Phi}(\Gamma) = \sum_{S \subseteq N \setminus i} \frac{|S|!(n - |S| - 1)!}{n!} \left[v^{\Gamma}(S \cup i) - v^{\Gamma}(S)\right].$$

For every CTLD situation, the corresponding Shapley value always exists and is unique. Since the Shapley value solely draws upon the CTLD game, it follows from Lemma 6 that it satisfies the IID property. Moreover, it always divides the savings equally in two-player situations (Aumann and Maschler, 1985) thus it does not satisfy the RC property. The following example shows that the Shapley value does not satisfy the least-unstability property.

Example 6. *Consider the CTLD situation Γ depicted in Figure 6. There are three players each having a depot at the same location and a delivery requirement. The distance between the pickup and delivery locations for all delivery requirements is one. The stand-alone cost for all players is 2. Although cooperation between players i and j does not create any savings, either of them can cooperate with k to generate 2 units of savings. This results in $v^{\Gamma}(S) = 0$ when $|S| = 1$, $v^{\Gamma}(\{i, j\}) = 0$, $v^{\Gamma}(\{i, k\}) = v^{\Gamma}(\{j, k\}) = 2$, and $v^{\Gamma}(N) = 2$. The allocation $\alpha = (\alpha_i, \alpha_j, \alpha_k) = (0, 0, 2)$ is the unique stable allocation. The Shapley value of the situation is $\alpha^{\Phi}(\Gamma) = (1/3, 1/3, 4/3)$. Thus the Shapley value is not the least-unstable allocation.* △

Weighted Shapley values (Shapley, 1953a) extend the Shapley value by allowing unequal

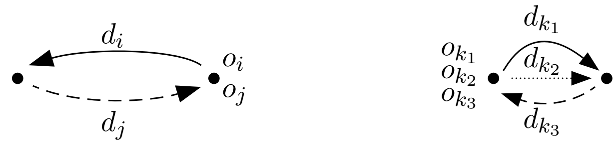

Figure 7: A CTLD situation with five players

allocations of savings in two player situations based on exogenously given weights. Note that the Shapley value by itself is a weighted Shapley value where all players have equal weights. The exogenous weights reflect the different bargaining powers of the players which justifies discrimination among their allocations irrespective of the characteristics of the situation. In CTLD situations encountered by the authors, no player has ex-ante a higher bargaining power over the others.

The *alternative cost avoided* (ACA) method introduced by Tijs and Driessen (1986) is a solution which draws upon the stand-alone costs. We can adopt the ACA as a solution for CTLD situations. Let $\Gamma \in \mathcal{T}$ and define $m_i = c^*(O_N, D_N) - c^*(O_{N\setminus i}, D_{N\setminus i})$. The allocation of savings[2] obtained by solution A^{ACA} in situation Γ is $A^{ACA}(\Gamma) = \{\alpha^{ACA}(\Gamma)\}$ where for all $i \in N$:

$$\alpha_i^{ACA}(\Gamma) = [c^*(O_i, D_i) - m_i]\left[1 - \frac{\sum_{j\in N} m_j - c^*(O_N, D_N)}{\sum_{j\in N}(m_j - c^*(O_j, D_j))}\right].$$

This solution does not satisfy the NE property: in the situation depicted in Figure 5 (Example 5) we have $m_j - c^*(O_j, D_j) = 0$ for all $j \in N$ so $A^{ACA}(\Gamma)$ is not well-defined for the situation in this example due to division by zero. Upon existence A^{ACA} obtains a single allocation. Definition of alternate cost avoided implies that the cost of irrelevant deliveries of a player i is additive both in $c^*(O_i, D_i)$ and m_i so it would cancel out in $c^*(O_i, D_i) - m_i$. Therefore exclusion of irrelevant deliveries would not affect A^{ACA} and consequently it satisfies the IID property. It is straightforward to verify that in situations with two players only, the ACA solution always results in equal division of savings. Thus it does not satisfy the RC property. Finally, the next example shows that the ACA solution does not satisfy the LU property.

Example 7. *Consider the CTLD situation in Figure 7. Players k_1, k_2, and k_3 each have a depot at the same location and a delivery with equal stand-alone costs of 2 (the distance between the two points at right is 1). Players i and j each also have a depot at the same location with a single delivery whose stand-alone costs is 4 (the distance between the two points at left is 2). The deliveries and depots in the left and right sides of the figure are distant enough so that no optimal trip can be constructed by combining the corresponding components. The stand-alone costs are $c^*(O_i, D_i) = c^*(O_j, D_j) = 4$ and $c^*(O_{k_1}, D_{k_1}) = c^*(O_{k_2}, D_{k_2}) = c^*(O_{k_3}, D_{k_3}) = 2$. The saving obtained by coalition $\{i, j\}$ is 4, the saving obtained by coalitions $\{k_1, k_2\}$, $\{k_1, k_3\}$, $\{k_2, k_3\}$, and $\{k_1, k_2, k_3\}$ are 0, 2, 2, and 2 respectively. The savings obtained by the grand coalition is 6. The core of the associated game is non-empty and can*

[2]Note that ACA is originally defined over cooperative cost games. We have modified the original ACA formula to describe corresponding allocations of savings rather than costs.

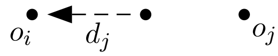

Figure 8: A CTLD where i has zero stand-alone cost

be completely characterized by the allocations $\alpha = (\alpha_i, \alpha_j, \alpha_{k_1}, \alpha_{k_2}, \alpha_{k_3}) = (\theta, 4-\theta, 0, 0, 2)$ *with* $0 \le \theta \le 4$. *With respect to ACA solution, we have* $m_i = m_j = m_{k_3} = 0$ *and* $m_{k_1} = m_{k_2} = 2$. *Consequently, we get* $\alpha^{ACA} = (2.4, 2.4, 0, 0, 1.2)$. *So* α^{ACA} *is not the least-unstable allocation.* $\triangle$

The *proportional solution* introduced by Ortmann (2000) incorporates proportionality as a measure of fairness in division of savings. In order to provide the formula for this solution in CTLD situations, let Γ^S be the situation obtained by ignoring the players not in S. Note that Γ^N is the complete situation. The proportional solution *a la* Ortmann is defined via

$$A^{\psi}(\Gamma) = \left\{ \left(c^*(O_i, D_i) - \psi_i(\Gamma^N) \right)_{i \in N} \right\}$$

where for all $i \in N$ and $S \subseteq N$, $\psi_i(\Gamma^S)$ is obtained recursively from[3]

$$\psi_i(\Gamma^S) = c^*(O_S, D_S) \left[1 + \sum_{j \in S \setminus i} \frac{\psi_j(\Gamma^{S \setminus i})}{\psi_i(\Gamma^{S \setminus j})} \right]^{-1}$$

with $\psi_i(\Gamma^i) = c^*(O_i, D_i)$. With the above definition, this solution only exists in situations where for every $i \in N$ it holds that $c^*(O_i, D_i) \neq 0$. This is not always the case in CTLD situations as shown in the next example.

Example 8. *Consider the CTLD situation depicted in Figure 8. The player i has a depot but no deliveries and player j has a depot and a delivery which is close to i's depot. The distance between the adjacent points is 1. Through cooperation the players can obtain 2 units of savings, i.e.* $v^{\Gamma}(N) = 2$, *we have* $c^*(O_i, D_i) = 0$. $\triangle$

The example above shows that the proportional solution does not satisfy the NE property. Upon existence the proportional solution provides a single allocation (Ortmann, 2000). In CTLD situations with two players, the proportional solution divides savings in proportion to stand-alone costs of the players. As the result, this solution satisfies the RC property. However, considering the entire stand-alone cost players hinders this solution from satisfying the IID property. Looking back at Example 6, observe that the proportional solution A^{ψ} obtains the allocation $\alpha^{\psi} = (2/5, 2/5, 6/5)$. Given that the only stable allocation in this situation is $\alpha = (0, 0, 2)$, we conclude that this solution does not satisfy the LU property.

The equal profit-sharing method (EPM) proposed by Frisk et al. (2010) addresses the concerns over stability as well as competitiveness in transportation situations. The EPM solution chooses allocations in the core, or in the least-core when the core is empty, which

[3] Note that the original solution is extended to incorporate proportionality with regard to costs, instead of savings.

Solution		RC	IID	NE	UQ	LU
$A^{\mathcal{C}}$	Core	×	✓	×	×	✓
$A^{\mathcal{LC}}$	Least-core	×	✓	✓	×	✓
A^{η}	Nucleolus	×	✓	✓	✓	✓
A^{Φ}	Shapley value	×	✓	✓	✓	×
A^{ACA}	ACA	×	✓	×	✓	×
A^{ψ}	Proportional	✓	×	×	✓	×
A^{EPM}	EPM	✓	×	×	×	✓
A^{E}	Proposed	✓	✓	✓	✓	✓

RC: restricted competitiveness, IID: independence of irrelevant deliveries,
NE: non-emptiness, UQ: uniqueness, LU: least unstability

Table 1: Comparing CTLD solutions

minimize the maximum difference between all pairwise ratios of allocation to stand-alone cost. In CTLD situations, the EPM solution is defined as A^{EPM} where for a situation Γ:

$$A^{EPM}(\Gamma) = \arg\min_{\alpha} f \tag{30}$$

$$s.t. \quad \frac{\alpha_i}{c^*(O_i, D_i)} - \frac{\alpha_j}{c^*(O_j, D_j)} \le f \qquad \forall i, j \in N \tag{31}$$

$$\sum_{i \in S} \alpha_i + \epsilon^{\min} \ge v^{\Gamma}(S) \qquad \forall S \subset N \tag{32}$$

$$\sum_{i \in N} \alpha_i = v^{\Gamma}(N) \tag{33}$$

with $\epsilon^{\min}$ being defined in (29). In CTLD situations with two players, the EPM allocates savings proportional to stand-alone costs of the players. Thus A^{EPM} satisfies the RC property. However, this solution does not satisfy the IID property as it considers the entire stand-alone costs of the players. Moreover, EPM is not defined for the case where stand-alone cost of a player is zero (as in Figure 8), thus it does not satisfy the NE property either. We return to Example 7 in order to show that EPM can produce more than a single allocation in some situations. With regard to later example, first note that $\epsilon^{\min} = 0$. Now consider the family of allocations $(\alpha_i, \alpha_j, \alpha_{k_1}, \alpha_{k_2}, \alpha_{k_3}) = (\theta, 4 - \theta, 0, 0, 2)$ with $0 \le \theta \le 4$. Note that the largest difference between pairwise ratios of allocation to stand-alone cost in this family is 1. In fact, all allocations in this family are optimal solutions to the program defined by (30)–(33). We conclude that A^{EPM} does not satisfy the UQ property.

8 Final Remarks and Conclusions

In this paper we proposed a solution for cooperative truckload delivery situations. The proposed solution satisfies a series of properties which reflect the requirements for fairness and/or competitiveness in these situations. The solution always exists, gives a unique allocation, and is situated within the core or, if the latter is empty, within the least-core (Theorem 2). The proposed solution is insensitive to the deliveries which could not play any role in cooperation and satisfies a minimal requirement for competitiveness of allocations (Theorem 3). When the stability constraints on allocations permits, the obtained allocation preserves the competitive positions of all logistics providers with regard to their minimal essential deliveries, i.e. the subsets of delivery requirements of players which are necessary and sufficient in creating their contributions to the grand coalition and have the minimum stand-alone costs (Definition 4). The preservation of competitive positions implies that the ratio of average cost of fulfilments of the players remain the same before and after cooperation.

Our allocation is proposed with special attention to implementability considerations. The preliminary allocation introduced in this paper, i.e. A^P, which draws upon minimal essential delivery sets of players, can be calculated by comparing the individual optimal delivery plans of the logistics providers in stand-alone mode versus those in the grand coalition. This is due to the fact that the essential delivery sets are detectable from the latter comparison (as implied by Lemma 5). In practical instances where the multiplicity of optimal delivery plans are improbable, the essential delivery sets of the players can be obtained by detecting the deliveries whose fulfilment in the grand coalition involve other players, that is, the trips including those deliveries are either initiated from another player's depot or they include deliveries of others. When the essential delivery sets are multiple, finding the minimal sets among them requires comparison among their stand-alone delivery costs.

To the best of our knowledge, this paper is the first to formally incorporate an endogenous measure of competitiveness in logistics markets. This is done by considering the lowest possible price that a logistics provider is able to charge for a unit-distance of its delivery services within a specified scope without incurring loss. Such a measure reflects the internal efficiency of the logistics providers' operations. Consequently, our solution takes advantage of information contained in a situation in addition to the savings generated in all possible coalitions to calculate the allocation.

We argue that in cooperative operations management, investigating the desirable properties of solutions and their formal definition ex-ante obtains more meaningful results than using generic solutions. Without having a solid ground for comparing among different solutions, one cannot objectively evaluate the performance of various available solutions. The definition of desirable properties in CTLD situations in this paper allows us to conduct such comparison. The results clarify the advantages of our proposed solution over some of the existing solutions in the literature which are more suited to be adopted in these situations.

As a practical advantage of our proposed solution, it can be extended to handle more realistic cooperative truckload delivery situations. The restrictions for delivery time windows, requirements for trip lengths, availability of personnel and shifts can all be easily included in the basic model which obtains costs. In spite of possible challenges in solving the optimization problem, as long as one can identify the best joint plans for fulfilling the delivery requirements in the grand coalition, our approach in obtaining competitive allocations remains valid for cooperating logistics players.

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