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***Abstract*—This paper presents an equivalent circuit model to predict the transmission and the reflection characteristics of different Frequency Selective Surfaces (FSS). The effect of the substrate thickness and permittivity are discussed. A Gridded-Square-Loop (GSL) loaded with lumped-elements is employed to tune the reflection and the transmission characteristics of the FSS. The design method is validated for different polarizations and for different incident angles. In order to tune the response of the FSS, lumped elements (R,L,C) can be inserted in the metallic conductive grid of the FSS. The waveguide simulator method and full-wave simulations are employed to validate the derived analytical equations.**

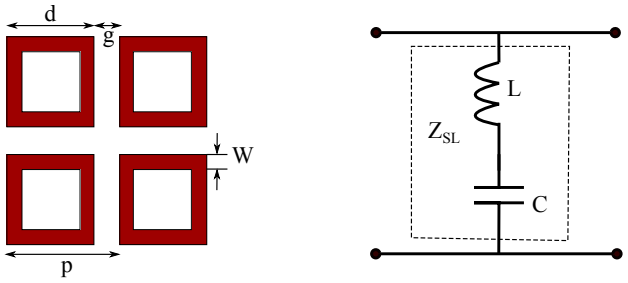
***Index Terms*—Analytical solution, frequency selective surfaces, lumped elements, waveguide simulator method.**

I. Introduction

Frequency Selective Surfaces (FSSs) consist of metallic grids or patches backed on a dielectric substrate. They are employed in diverse application fields, ranging from commercial to military sectors to provide band-pass or band-stop characteristics. In the framework of antenna systems, FSSs provide frequency windows in radomes or multiband frequency operations [1]. FSSs can be used for protection from harmful electromagnetic radiation especially in domestic environment, schools and offices.

Different techniques have been adopted for the analysis of frequency selective surfaces. Including the modal analysis [2] the Iterative Process [3] and the Equivalent Circuit Model [4]. The equivalent circuit model gives a fast and considerably accurate response in describing FSS transmission and reflection characteristics, however, is only valid for certain structures and it presents some limitations (i.e. no cross-polarization response can be estimated). A simple set of equations are employed to analyze the structures. They represents the particular geometry with resonant circuits consisting of inductances and capacitances.

In this paper, the equivalent circuit model will be used to calculate the band-pass and the band-stop characteristics of different FSS structures including the square-loop FSS and the gridded-square-loop FSS. The equivalent circuit model is than extended to allow the insertion of lumped elements in the grid. The following section introduces the equivalent circuit model to characterise square-loop FSS.



(a) Square-loop FSS (b) Equivalent circuit model

Fig. 1. Square-Loop (SL) frequency selective surface and its equivalent circuit model.

II. Square-Loop FSS

The square-loop FSS has been investigated in literature by several authors [5]–[7]. Fig. 1-a shows the unit cell of a square-loop FSS, where g is the gap between neighboring elements, W is the width of the loops d is the length of the square side and $p = (d + g)$ is the periodicity of the unit cell. The equivalent circuit of the square-loop is shown in Fig. 1-b and is a series LC circuit where L represents the inductance due to the the vertical metal conductor and C is the capacitance between the horizontal conductors.

The surface impedance of the square loop is given by:

$$Z_{fss} = j\left(X_L - \frac{1}{B_C}\right) \tag{1}$$

where X_L and B_C are derived as presented in [6]:

$$X_L = Z_0\left(\frac{d}{p}\right) F\left(p,\, 2w,\, \lambda\right) \tag{2}$$

$$B_C = \frac{4}{Z_0}\left(\frac{d}{p}\right) F\left(p,\, g,\, \lambda\right) \tag{3}$$

where $F_{TE,TM}\left(p,\, 2w,\, \lambda\right)$, $F_{TE,TM}\left(p,\, g,\, \lambda\right)$ are derived in [8], [9],

$$F_{TE}\left(p, w, \lambda\right) = \Upsilon\left[\ln\csc\left(\frac{\pi w}{2p}\right) + G\left(p, w, \lambda, \theta\right)\right], \tag{4}$$

$$F_{TM}\left(p, w, \lambda\right) = \Phi\left[\ln\csc\left(\frac{\pi w}{2p}\right) + G\left(p, w, \lambda, \varphi\right)\right], \tag{5}$$

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$$F_{TE}(p,g,\lambda) = \Psi\left[\ln\csc\left(\frac{\pi g}{2p}\right) + G(p,g,\lambda,\theta)\right], \quad (6)$$

$$F_{TM}(p,g,\lambda) = \Omega\left[\ln\csc\left(\frac{\pi g}{2p}\right) + G(p,g,\lambda,\varphi)\right], \quad (7)$$

where $\Upsilon = \frac{p*\cos\theta}{\lambda}$, $\Phi = \frac{p*\sec\varphi}{\lambda}$, $\Psi = \frac{p*\sec\theta}{\lambda}$ and $\Omega = \frac{p*\cos\varphi}{\lambda}$.

With θ and φ being the angles of incidence of a TE and a TM polarized wave respectively. The absolute value in dB of the transmission and the reflection coefficients is derived from the normalized surface impedance of the unit cell:

$$|T|_{dB} = 20\log_{10}\left[\frac{2Z_{fss}}{1+2Z_{fss}}\right], \quad (8)$$

$$|R|_{dB} = 20\log_{10}\left[\frac{-1}{1+2Z_{fss}}\right]. \quad (9)$$

TABLE I
ANALYZED SQUARE LOOP ARRAY CONFIGURATIONS

SL Array #	p mm	d mm	g mm	w mm	t mm	ϵ_r
1	10.0	8.75	1.25	0.625	0.0	1.0
2	(d+g)	21.0	-*	3.0	0.0	1.0
3	22.0	21.0	1.0	3.0	-*	2.2
4	32.25	31.0	1.25	3.0	0.25	-*

*Parameter has been varied

In order to verify the accuracy of the analytical equations, the reflection and the transmission coefficients are calculated and compared to the results obtained by Finite Integration Technique (FIT) [10]. Table I summarizes different array configurations (in terms of dimensions and dielectric characteristics) that are investigated here. For Array #1 of Table I, the substrate effect is not included. Fig. 2 shows the magnitude and the phase of the transmission and the reflection coefficients as a function of frequency calculated by the Analytical Equations (AE) and by the finite integration technique using CST microwave Studio®.

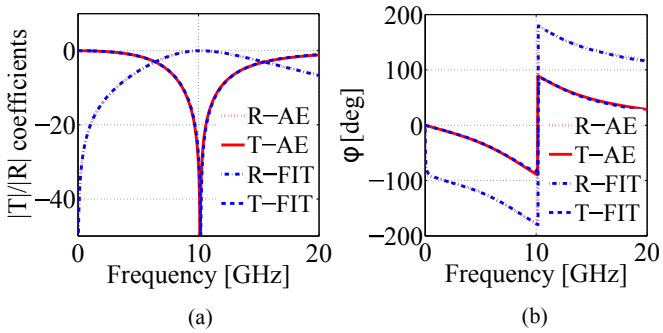

Fig. 2. Magnitude (a) and phase (b) of reflection and transmission coefficients as a function frequency for square loop Array #1 of Table I at normal incidence.

This figure indicates that the equivalent circuit model can accurately predict the reflection and the transmission bands.

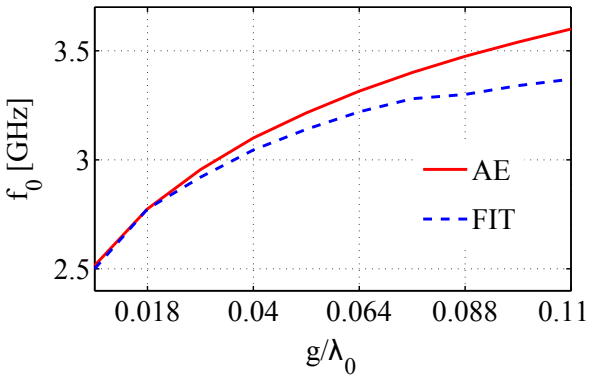

Fig. 3. Resonant frequency of transmission coefficient as a function of gap g variation for square loop Array #2 of Table I using the Analytical Equations (AE) compared to Finite Integration Technique Analysis (FIT) at normal incidence.

A. *Effect of the Gap Width g*

To investigate the model accuracy for different gap width g, the parameters of Array #2 (see Table I) are used to calculate the resonance frequency (f_0) and compared to the one obtained by the FIT. The results shown in Fig. 3 are plotted as a function of gap variation (g/λ_0). Here (λ_0) is calculated from the resonance frequency (f_0) calculated by FIT. It can be noticed that the equivalent circuit model diverges from the results obtained by FIT as the gap increases. This limitation is directly due to the assumption of treating the neighboring vertical conductors as one conductor which is twice the width of the single side (Eq. 2). However, this is only possible when the gap is reasonably small.

B. *Effect of the Dielectric Substrate Material*

The effect of dielectric substrate bonding the array elements was investigated in [11]. For thin dielectric layers the value of the capacitance varies as a function of dielectric thickness t. A value of the effective permittivity is calculated throughout an iterative procedure for thin dielectric substrates in [12]. The expression depends on both ϵ_r and t and the resulting effective permittivity can be included in the equivalent circuit model by multiplying the subsceptance B_C. The expression of the effective permittivity (ϵ_{eff}) is obtained as:

$$\epsilon_{eff} = \epsilon_r + (\epsilon_r - 1)\cdot\left[\frac{-1}{e^{x\cdot N}}\right] \quad (10)$$

where

$$x = 10\cdot t/p \quad (11)$$

t is the thickness of the substrate, p is the periodicity and N is an exponential factor that vary with the geometry of the unit cell. For square-loop arrays, N is around 1.8. When the width W is large compared with the global dimensions of the unit cell, N decreases accordingly (i.e. for metal patches N=1.3)

For the square-loop FSS the value of the susceptance is, therefore, modified as:

$$B_C = 4\cdot\epsilon_{eff}\cdot\frac{1}{Z_0}\left(\frac{d}{p}\right)F(p,\,g,\,\lambda) \quad (12)$$

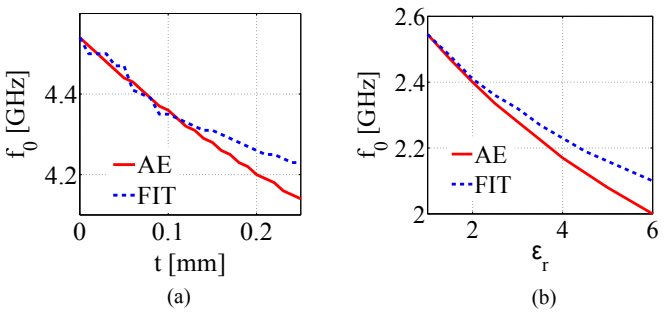

Fig. 4. Resonance Frequency as a function of substrate thickness variation t variation for square loop Array #3 of Table I using the analytical equations (AE) compared to finite integration technique (FIT) at normal incidence.

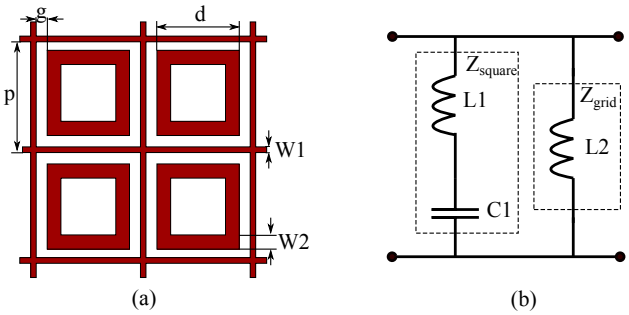

Fig. 5. Grided Square Loop configuration and its equivalent circuit model

To investigate the effect of the dielectric substrate, the resonant frequency is calculated by the equivalent circuit model and by the FIT, and the results are plotted as a function of thickness variation in Fig. 4-a and as a function of substrate permittivity in Fig. 4-b. It can be concluded from the figures that the equivalent circuit model approximates the results obtained by full wave analysis up to a very thin substrate (about 0.1 mm for Array #3). For the permittivity investigation, the parameters of Array #4 (Table I) are used to calculate the resonance frequency. It is also shown that the presented model holds for a low substrate permittivity ($\epsilon_r < 3$). In the following section, the gridded-square-loop FSS is investigated.

III. Gridded-Square-Loop Arrays

The Gridded-Square-Loop (GSL) configuration and its equivalent circuit model are shown in Fig. 5-a and Fig. 5-b respectively. p is the periodicity, d is the square side length, g is the gap between the inductive grid and the square loops, W_1 is the width of the grid and W_2 is the width of the squares. The presence of the grid is represented by an inductor inserted in parallel to the LC resonant circuit. The analytical equations are derived as follow:

$$Z_{fss} = \frac{Z_g \cdot Z_s}{Z_g + Z_s}. \tag{13}$$

where Z_g represents the grid impedance given by:

$$Z_g = jX_2 \tag{14}$$

and Z_s the square impedance derived as:

$$Z_s = j(X_1 - 1/B_1) \tag{15}$$

The values of X_1, X_2 and B_1:

$$X_1 = \omega L_1 = Z_0 \cdot 2 \cdot \frac{X_2 * X_3}{X_2 + X_3}, \tag{16}$$

$$X_2 = \omega L_2 = Z_0 \cdot F\left(p,\, w_1,\, \lambda\right), \tag{17}$$

$$X_3 = Z_0 \left(\frac{d}{p}\right) F\left(p,\, 2w_2,\, \lambda\right), \tag{18}$$

$$B_1 = \omega C_1 = \frac{2}{Z_0} \cdot \epsilon_{eff} \left(\frac{d}{p}\right) F\left(p,\, g,\, \lambda\right). \tag{19}$$

A. Model Verification Using Full Wave Analysis

The reflection and the transmission coefficients vs. frequency for a GSL unit-cell with $d = 12\,mm$, $g = 1.5\,mm$, $W1 = 3\,mm$ and $W2 = 1\,mm$ are calculated and compared with the simulation results obtained using CST microwave studio. Fig. 6 shows the amplitude of the transmission and the reflection coefficients of the simulated unit cell as a function of frequency for a TE incident wave at an incident angle of $\theta = 45°$. The figure shows that the equivalent circuit model can accurately predict the reflection and the transmission bands.

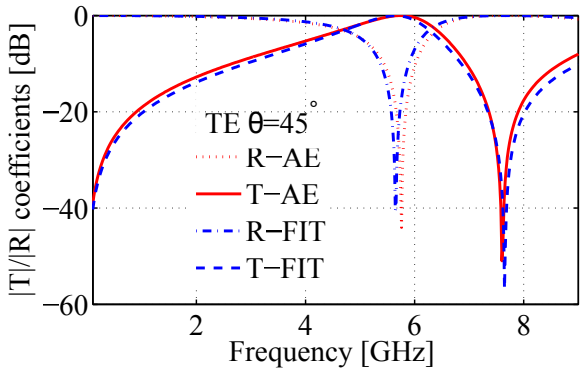

Fig. 6. Amplitude of the transmission and reflection coefficients vs. frequency at $\theta = 45°$ incidence, using Analytical Equations (AE) and Finite Integration Technique (FIT). $d = 12mm$, $g = 1.5mm$, $W1 = 3mm$ and $W2 = 1mm$.

Since the equivalent circuit model is accurate enough to be used as a starting point in the initial design phase. The equivalent circuit model is extended to include any combination of a lumped-element circuit in the next section.

IV. Tunable Gridded-Square-Loop FSS

The next step in the design, consists of loading lumped elements in the metal grid of the gridded square loop structure. Lumped elements can be added to the equivalent circuit model, therefore the behavior of the transmission characteristics can be predicted and controlled. Fig. 7 shows the loaded grided-square-loop configuration and its equivalent circuit model. It is indicated in the figure that the unit cell is loaded with two lumped elements for each polarisation. This is necessary

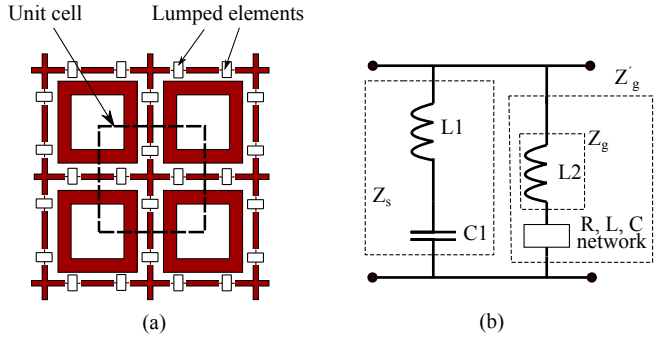

Fig. 7. Grided square loop configuration loaded with lumped elements and the suggested equivalent circuit model

in order to be coherent with image theory (lumped elements cannot be placed on the symmetry planes nor on the periodic boundaries). $Z_g^{'}$ is the partial impedance of the grid and Z_s is the impedance given by the inner square loops. 2 lumped elements are added to the grid for both polarizations (TE and TM) which returns a total of 4 elements per unit cell, each of them, having an equivalent impedance Z_{eq}.

A. FSS loaded with a Parallel RC Combination

In this subsection, the frequency response due to the parallel combination of a capacitor and a resistor inserted in the grid is investigated. The grid impedance $Z_g^{'}$ is given by:

$$Z_g^{'} = n \cdot \left[R_a - j \left(X_a - \frac{X_2}{n} \right) \right], \tag{20}$$

where:

$$R_a = \frac{R_{load}}{\left[\left(B_{load} \cdot R_{load} \right)^2 + 1 \right]}, \tag{21}$$

and:

$$X_a = R_a \cdot R_{load} \cdot B_{load}. \tag{22}$$

Applying the same procedures as before, the square impedance Z_s and the FSS impedance Z_{fss} are calculated using Eq. 15 and Eq. 13 respectively. The unit cell dimensions used for the calculation of the transmission and reflection coefficients are: $d = 31mm$, $g = 2mm$, $W1 = 1mm$ and $W2 = 4.5mm$, backed on a dielectric substrate with $\epsilon_r = 3$ and thickness $t = 125\mu m$.

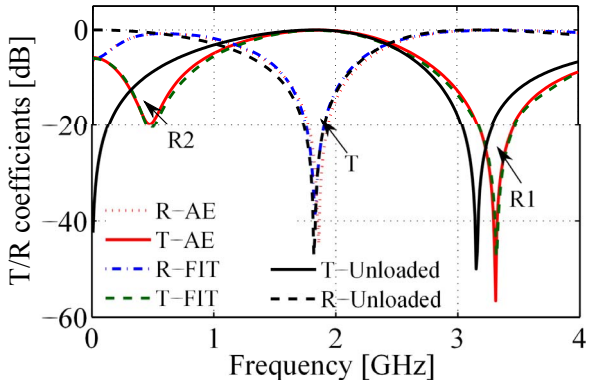

Fig. 8. Transmission and reflection coefficients of a unit cell of a GSL-FSS loaded with a capacitor ($C_{load} = 10$ pF) in parallel a resistor ($R_{load} = 100\Omega$). Calculated using Analytical Equations (AE) and compared with FIT.

Fig. 8 shows the transmission and the reflection coefficients as a function of frequency calculated by the analytical equations and by the FIT. It is shown in the figure that the analytical solution can accurately predict the transmission and the reflection bands of the FSS. It is again shown that a new reflection band R2 is created when the grid is loaded with the capacitor and the resistor.

V. Conclusion

In this paper an equivalent circuit model is presented to calculate the transmission and the reflection bands of different frequency selective surfaces including the square loop FSS and the gridded square loop FSS. It is shown that the model is only valid to characterize FSS backed on a thin and low permittivity substrates. The equivalent circuit model is then extended for the insertion of lumped elements in the grid. The derived equations are validated by full wave analysis simulations. The main advantage of the suggested lumped-elements FSS is that its frequency response can be tuned by only changing the load value without changing the FSS dimensions. One typical application is to use the grided square loop FSS loaded with rectifiers to harvest RF power. The impedance of the rectifier is modeled as a resistor in series with a capacitor. Using the equivalent impedance of the rectifier one can predict the transmission and the reflection bands which are consequently the RF harvesting bands.

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