Remote control and motion coordination of mobile robots

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The research reported in this thesis is part of the research program of the Dutch Institute of Systems and Control (DISC). The author has successfully completed the educational program of the Graduate School DISC.

This research was partially supported by the Mexican Council for Science and Technology (CONACYT) and the Mexican Ministry of Education (SEP).

A catalogue record is available from the Eindhoven University of Technology Library.

Remote Control and Motion Coordination of Mobile Robots./
by A. Alvarez-Aguirre. – Eindhoven : Technische Universiteit Eindhoven, 2011.
Proefschrift. – ISBN: 978-90-386-2570-6

This thesis was prepared with the pdfLaTeX documentation system.
Cover Design: Oranje Vormgevers, Eindhoven, The Netherlands.
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PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Eindhoven,
op gezag van de rector magnificus, prof.dr.ir. C.J. van Duijn,
voor een commissie aangewezen door het College voor Promoties
in het openbaar te verdedigen
op dinsdag 13 september 2011 om 16.00 uur

door

Alejandro Alvarez Aguirre

geboren te Naucalpan de Juarez, Mexico

Dit proefschrift is goedgekeurd door de promotor:

prof.dr. H. Nijmeijer

Copromotor:
dr.ir. N. van de Wouw

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Summary

Remote Control and Motion Coordination of Mobile Robots

As robots destined for personal and professional applications advance towards becoming part of our daily lives, the importance and complexity of the control algorithms which regulate them should not be underestimated. This thesis is related to two fields within robotics which are of major importance in the advancement of robotics in the scope above; namely, telerobotics and cooperative robotics. On the one hand, telerobotic systems support remote or dangerous tasks, whereas, on the other hand, the use of cooperative robotic systems supports distributed tasks and has several advantages with respect to the use of single-robot systems.

The use of robotic systems in remote tasks implies in many cases the physical separation of the controller and the robot. This separation is advantageous when carrying out a variety of remote or hazardous tasks, but at the same time constitutes one of the main drawbacks of this type of robotic systems. Namely, as information is being relayed from the controller to the robot and back over the communication network, a time-delay unavoidably appears in the overall control loop. Hence, controller designs which guarantee the stability and performance of the robot even in the presence of the aforementioned time-delay become necessary in order to ensure a safe and reliable completion of the assigned tasks.

On the other hand, using a group of robots to carry out a certain assignment, as compared to a single robot, provides several advantages such as an increased flexibility and the ability to complete distributed or more complex tasks. In order to successfully complete their collective task, the robots in the group generally need to coordinate their behavior by mutually exchanging information. When this information exchange takes place over a delay-inducing communication network, the consequences of the resulting time-delay must be taken into account. As a result, it is of great importance to design controllers which allow the group of robots to work together and complete their task in spite of the time-delay affecting their information exchange.

The two control problems explained previously are addressed in this thesis. Firstly, the remote tracking control problem for a unicycle-type mobile robot with network-induced communication delays is studied. The most important issue to consider is that the communication delay in the control loop most probably compromises the performance and stability of the robot. In order to tackle this problem, a state estimator with a predictor-like structure is proposed. The state estimator is based on the notion of anticipating synchronization and, when acting in conjunction with a tracking control law, the resulting control strategy stabilizes the system and mitigates the negative effects of the time-delay. By exploiting existing results on nonlinear cascaded systems with time-delay, the local uniform asymptotic stability of the closed-loop tracking error dynamics is guaranteed up to a maximum admissible time-delay. Ultimately, explicit expressions which illustrate the relationship between the allowable time-delay and the control parameters of the robot are provided.

Secondly, the remote motion coordination problem for a group of unicycle-type mobile robots with a delayed information exchange between the robots is considered. Specifically, remote master-slave and mutual motion coordination are studied. A controller design which allows the robots to maintain motion coordination even in the presence of a delayed information exchange is proposed. The ensuing global stability analysis, which also exploits existing results on nonlinear cascaded systems with time-delay, provides expressions which relate the control parameters of the robots and the allowable time-delay.

The thesis places equal emphasis on theoretical developments and experimental results. In order to do so, the proposed control strategies are experimentally validated using the Internet as the communication network and multi-robot platforms located in Eindhoven, The Netherlands, and Tokyo, Japan.

To summarize, this thesis addresses two related control problems. On the one hand, we consider the tracking control of a wheeled mobile robot over a communication network which induces a time-delay. On the other hand, we focus on the motion coordination of a group of these robots under the consideration that the information exchange between the robots takes place over a delay-inducing communication network.

1

INTRODUCTION

Abstract . This chapter begins with an introduction and an overview of the current developments in telerobotic and cooperative robotic systems, focused particularly on, but not limited to, wheeled mobile robots. This motivates a discussion on some of the challenges that arise when the information exchange of measurement data and control commands in a robotic system takes place over a delay-inducing communication network. In turn, this discussion leads to the formulation of the research objective and main contribution of this thesis.

1.1 Telerobotic and Cooperative Robotic Systems

In the last 50 years, advancements in different technological fields and the demand to lower production costs and manage workplace safety have led to a surprisingly quick development and the wide-scale adoption of what were once seen as futuristic robots. It is likely that, in the coming decades, robots in personal and professional applications will become part of our daily lives. As this revolution takes place, the tasks conferred to robotic systems and the control algorithms which regulate them will continue to become more decisive and complex as requirements for such systems now encompass flexibility, robustness, safety, and transparency, among others. Given the previous demands, this thesis touches upon two fields of robotics which have contributed and will continue to contribute to meet these requirements; namely, telerobotics and cooperative robotics.

On the one hand, *telerobotic systems*, that is, robotic systems which are controlled at a distance, have become significantly important as a way to support remote, dangerous or spatially distributed tasks. On the other hand, in the case

Figure 1.1 . Example of telerobotic systems. The da Vinci minimally invasive surgical robot (left) and the teODor remote handling robot from telerob GmbH (right). See photo credits in the footnote.

of *cooperative robotic systems*, it is widely agreed upon that the use of multi-robot systems, specifically of teams or groups of mobile robots which exhibit cooperative behavior, presents several advantages over the use of single-robot systems (Arai et al., 2002; Cao et al., 1997; Siciliano and Khatib, 2008).

The number of current and potential applications of telerobotic and cooperative robotic systems has significantly increased during the last two decades. Some of the applications of telerobotic systems include underwater robotics (Whitcomb, 2000; Yuh and West, 2001), space robotics (Biesiadecki et al., 2006; Hirzinger et al., 2004), robots for agriculture, forestry, construction and mining (Halme and Vainio, 1998; Ho et al., 2000; Vagenas et al., 1991), robots intended to carry out tasks in hazardous environments or participate in search and rescue missions (Murphy, 2004; Sanders, 2006; Yamauchi, 2004), and medical robots (Anvari et al., 2005; Ortmaier et al., 2007). For additional references regarding these applications see Siciliano and Khatib (2008). Two examples of telerobotic systems are shown in Figure 1.1[1].

Among the applications that one could think of for a group of cooperative robots are payload transportation (Wang et al., 2007), logistics (Adinandra et al., 2010; Guizzo, 2008), localization and sensing (Dunbar and Murray, 2006; Leonard et al., 2007), reconnaissance and surveillance (Casbeer et al., 2006; Grocholsky et al., 2005), pursuit and enclosure of a prey (Madden et al., 2010; Yamaguchi, 1999), automated highway systems (Naus et al., 2010; Swaroop and Hedrick, 1996), and

[1]Photo credit `http://www.davincisurgery.com` (left) and `http://www.telerob.de` (right).

Figure 1.2 . Example of cooperative robotic systems. Kiva Systems Drive Units for intelligent warehouse management (left) and a team of *swarm-bots* pulling a child (right). See photo credits in the footnote.

replenishment operations (Kyrkjebø et al., 2006, 2007). Note that these applications are not constrained to land-based robots, but also include unmanned aerial vehicles (UAVs), unmanned underwater vehicles (AUVs), and unmanned surface vehicle (USVs), among others. Two examples of cooperative robotic systems are shown in Figure 1.2[2], whereas the interested reader is referred to Arai et al. (2002) and Pettersen et al. (2006) for reviews of additional applications.

Having briefly defined telerobotic and cooperative robotic systems, Section 1.1.1 and Section 1.1.2 explain in greater detail some of their main features.

1.1.1 Telerobotic Systems

The notion of telerobotics first began to take shape during the 1940's and 1950's, making it one of the earliest applications of robotics. The developments in this field were motivated by stricter requirements for human safety in hazardous environments still present today; in particular when handling nuclear waste (see Hokayem and Spong, 2006; Siciliano and Khatib, 2008, for additional details). Nowadays, as noted in Siciliano and Khatib (2008), a wider definition of a telerobotic system considers a barrier between a so-called local site and a so-called remote site. Such barrier may be imposed, for instance, by a hazardous environment or an escalation to larger or smaller environments and results in the user not being able to reach the remote environment physically. As a consequence, telerobotic systems are generally split into a local (the user) and a remote site (the environment). In telerobotics, a number of possible control architectures exist depending on the overall requirements of the system. For instance, some control architectures

[2]Photo credit `http://www.gadgetreview.com` (left) and `http://lsro.epfl.ch` (right).

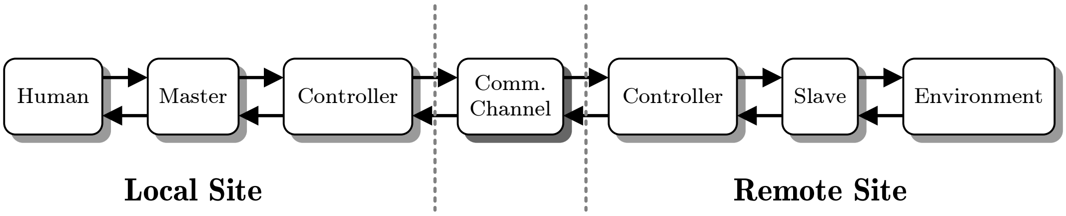


Figure 1.3 . Elements of a classic telerobotic system.

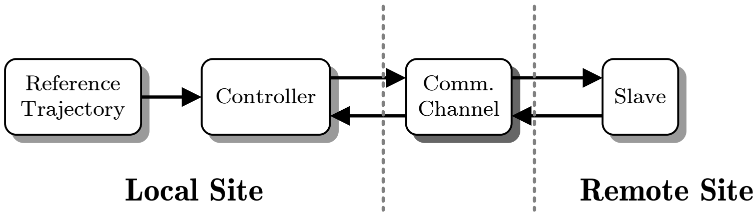


Figure 1.4 . Elements of the problem setting considered in this work.

consider a human operator in the local site. A schematic representation of a classical telerobotic system including a human-in-the-loop is shown in Figure 1.3. The operator is intended to perform control actions by means of the master device. These actions may range from low level direct commands to unsupervised *telecontrol* (remote control). Note that the local and remote sites are both equipped with a controller and that these controllers transmit information (such as position, velocity, and force measurements) over a communication channel which (physically) separates both sites. As a result, one of the main issues that has to be addressed during the design and analysis of a telerobotic system is the time-delay induced by the communication channel. This issue is particularly important since it is well known that time-delays can be detrimental to the stability and performance of a controlled system (Hale and Verduyn-Lunel, 1993; Kolmanovskii and Myshkis, 1992). The design of a control architecture which mitigates the negative effects of the network-induced delay in a remotely controlled robotic system constitutes one of the focal points of this thesis.

In this work, we will focus on the particular case in which the robotic system in the remote site is autonomously controlled from the local site without any human supervisory control. A schematic representation of this architecture is depicted in Figure 1.4. This remote control architecture may be useful when considering the remote operation of multiple robots (consider, for instance, the control of a group of mobile robots with minimal sensing and decision making capabilities from a remote command center) or in the context of Networked Control Systems. This type of systems will be introduced in Section 1.2.

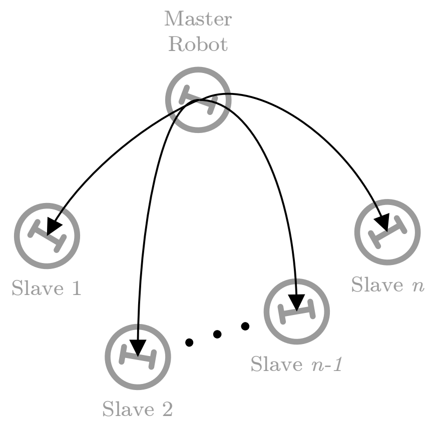


Figure 1.5 . Information flow between robots in master-slave motion coordination.

1.1.2 Cooperative Robotic Systems

The definition of cooperative behavior given in Cao et al. (1997) states, “given a certain task, a multi-robot system displays cooperative behavior if, due to some underlying cooperation mechanism, there is an increase in the total utility of the system”. As explained in Cao et al. (1997) and Siciliano and Khatib (2008), some of the reasons for favoring multi-robot systems may be that the task at hand is too complex for a single robot or that multi-robot systems are inherently distributed, well suited for parallelism, and usually possess greater flexibility and scalability.

In order to fully exploit the capabilities of multi-robot systems, one of the theoretical and technological issues which remains open for improvement is the development of appropriate group coordination and cooperative control strategies. In this respect, the leader-follower, the virtual structure, and the behavioral approaches are the most recurrent.

In leader-follower or master-slave motion coordination, one of the robots in the group acts as the leader or master, whereas the remaining robots are known as the followers or slaves. The master robot’s objective is to complete a certain task which would normally be related to the task of the group as a whole, such as guiding the slaves through a course with obstacles. Taking the master’s real motion as a basis, the slave robots generate their individual reference trajectories. This information flow is depicted in Figure 1.5 for a master robot with n slaves. One could think of a group of mobile robots in which one of the robots, the master, has greater capabilities than all the other robots. Even if the slave robots are only equipped with minimal sensing and decision-making capabilities, the group as a whole can still achieve quite complex tasks when being directed by a master robot.

It is this advantage however, that which at the same time constitutes two of the main drawbacks of this motion coordination strategy. In the first place, since the motion of the master robot is independent from the motion of the slave robots, the master has no knowledge about one or more of the slaves being unable to complete its task. This implies that master-slave motion coordination has very limited robustness against perturbations to the slave robots. The second pitfall is that any failure of the master robot translates into the complete group not being able to complete its task. Thus, relying mostly on the master robot poses a substantial risk.

Because there is no explicit feedback from the slave robots to the master, it is well known that master-slave motion coordination is in essence a problem of a tracking nature. In consequence, many classical control techniques such as feedback linearization (Desai et al., 2001; Fierro et al., 2001), dynamic feedback (Mariottini et al., 2007), backstepping (Li et al., 2005), and sliding mode control (Defoort et al., 2008; Sadowska, 2010) have been used to achieve this type of motion coordination (additional references may be found, for instance, in Kanjanawanishkul, 2010). The origins of master-slave motion coordination in the context of mobile robotics can be traced back to the seminal work of Das et al. (2002), Desai et al. (2001), and Fierro et al. (2001), where so-called separation-bearing and separation-separation controllers are proposed.

The majority of the latest work concerning master-slave motion coordination for mobile robots is devoted to estimating the translational and rotational velocities of the master robot, which are required to generate the reference trajectories of the slave robots. Most of the methods proposed in this respect are vision based and rely on some kind of observer or filter (Mariottini et al., 2007; Orqueda and Fierro, 2006; Vidal et al., 2003).

In mutual motion coordination, all the robots in the group generate their reference trajectory based on common reference known as the *virtual center*. In this case, the geometry which results from the desired motions of the robots is denoted as the virtual structure. As a result, mutual motion coordination is naturally suited to maintain a geometric formation which inherently possesses a certain robustness since, in most cases, the mobile robots are coupled to each other. The information flow between four mutually coordinated robots which are coupled and generate their reference trajectory based on a common virtual center is depicted in Figure 1.6.

The concept of a virtual structure was first introduced in Lewis and Tan (1997), where an important assumption is that all the robots possess global knowledge. The mobile robots are seen as particles which are intended to stay inside the virtual structure and the virtual structure looks to conform to the robots' positions and

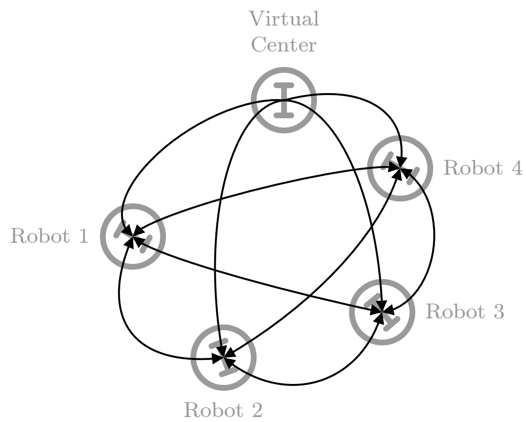


Figure 1.6 . Information flow between four coupled robots with mutually coordinated motions.

successively displaces. Additional works based on the virtual structure approach resulting in slight improvements may be found in Beard et al. (2000), Egerstedt and Hu (2001), and Young et al. (2001), among others.

More recent extensions to the virtual structure approach may be found in van den Broek et al. (2009), Kostić et al. (2010a), and Sadowska (2010). The definition of mutual motion coordination in these works is very closely related to the one provided in Nijmeijer and Rodriguez-Ángeles (2003) for mutual synchronization of robotic manipulators. In van den Broek et al. (2009), each of the robots in the group is equipped with a coordinating controller not only intended to track the robot's specific reference trajectory, which is based on the common virtual center, but also designed to provide a coupling mechanism between the robots in order to increase the group's ability to withstand perturbations. The coupling between the robots is based on all the robots exchanging their error information with each other, which results in a massive communication flow. This demanding communication requirement is substantially reduced in Sadowska (2010) by means of a decentralized control architecture. In this case, the robots are only allowed to communicate with other robots within a certain communication neighborhood, while still taking into account the group's overall behavior.

An additional approach to design cooperative robotic systems, which is not considered in this work, is the behavior-based approach. This approach was first introduced by Brooks (1986) and makes use of a set of so-called behaviors, or motion primitives, which are weighed in order to produce the robots' ultimate behavior. For instance, the control inputs to the robots would depend on the combination of the weights of a number of behaviors such as trajectory tracking, localization, collision avoidance, and others. The behavior-based formation control strategy was proposed in Balch and Arkin (1998) for a group of unicycle-type mobile robots by weighting several independent actions for the robots, ultimately resulting in a so-called motor schema-based control. A behavior-based approach has the advantage of being quite intuitive and allows the implementation of complex tasks by fragmenting them into a set of easier, more accessible actions. This feature, together with the fact that it focuses on a decentralized architecture makes it suitable for large groups of robots. The main drawback of this approach is that the resulting group's dynamics do not lend themselves to a straightforward mathematical analysis, making it extremely difficult to study the group's closed-loop stability and accurately predict its performance.

A comprehensive overview of additional approaches not considered in this work may be found in Kanjanawanishkul (2010) and the references therein. Some of the possibilities mentioned include the notion of string stability for line formations (Swaroop and Hedrick, 1996) and its generalization to formations in a planar mesh (Pant et al., 2002), optimization-based strategies relying on model predictive control (MPC) (Dunbar and Murray, 2006; Kanjanawanishkul, 2010), and a number of problems addressed by using algebraic graph theory, such as flocking and rendezvousing (Bullo et al., 2009; Jadbabaie et al., 2003; Olfati-Saber, 2006; Ren, 2008). Since the number of references related to the last approach are so vast, only some which could be considered *locus classicus* have been cited.

Considering master-slave and mutual synchronization of mechanical systems as defined in Nijmeijer and Rodriguez-Ángeles (2003), a clear resemblance appears between these ideas and the master-slave and mutual motion coordination approaches for mobile robots. In these approaches it is possible to employ an explicit mathematical model to predict the robots' motions, making it feasible to formally analyze the group's behavior (as opposed to the behavioral approach). This constitutes the main reason why the master-slave and mutual motion coordination strategies are the only ones considered throughout this thesis.

1.2 Networked Communication in Telerobotic and Cooperative Robotic Systems

A common element in telerobotic and cooperative robotic systems, especially when aiming at a practical implementation, is the use of a communication network to exchange important information. On the one hand, as explained already in Section 1.1.1, networked communication is necessary in telerobotic systems to transmit and receive measurement and control data between the local and remote sites (see Figure 1.3). On the other hand, in the case of cooperative robotic

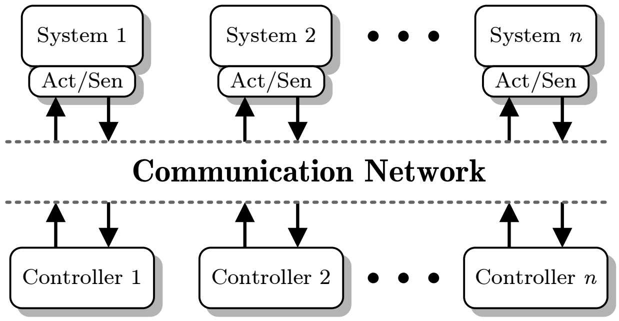


Figure 1.7 . Elements of a classic NCS.

systems, one would expect that the elements which conform the group exchange information between them (such as their current position or tracking error) in order to achieve a certain task. When considering a group of mobile robots, this information exchange almost certainly takes place over a communication network. As noted by Siciliano and Khatib (2008), when networked communication is used to coordinate and achieve cooperative behavior among the elements of a multi-robot system, one speaks of *networked robotics*.

Because of the time needed to transmit data over the network, the use of networked communication to exchange information in telerobotic and cooperative robotic systems results in time-delays. In addition, in the case of a cooperative robotic system with networked communication, the load placed on the communication channel becomes more demanding as the number of robots in the group increases. As a result, the time needed to transmit data over the network (that is, the magnitude of the time-delay) becomes larger. These network-induced delays are undesirable (albeit mostly unavoidable) because, as explained already in Section 1.1.1, they may degrade the performance of the system and even compromise its stability. Hence, when it comes to telerobotic and cooperative robotic systems with networked communication, the importance of designing control algorithms which are robust in the face of time-delays cannot be underscored.

In many cases, the issues that will arise when considering networked communication in telerobotic and cooperative robotic systems are typical of Networked Control Systems (NCSs), a key research field in control engineering (Antsaklis and Baillieul, 2007; Murray et al., 2003). Among these issues are (time-varying) network-induced delays, time-varying sampling intervals, packet losses, and other communication constraints. As defined in Posthumus-Cloosterman (2008), a NCS is a "system where the control loop, generally consisting of a continuous-time plant and a (discrete-time) controller, is closed over a communication channel". A schematic representation of a typical NCS is shown in Figure 1.7.

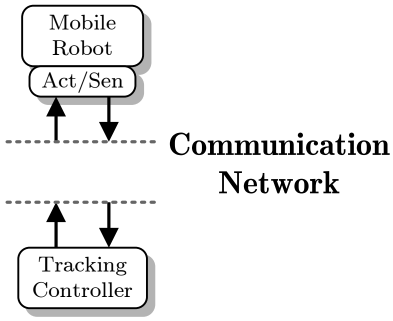


Figure 1.8 . Remote control system considered in this work.

Some of the reasons for advocating the use of NCSs are their flexibility, due to their inherent distributed nature, (Hespanha et al., 2007) and the benefit this may represent in terms of installation and maintenance costs (Tipsuwan and Chow, 2003). On the other hand, it is precisely the use of networked communication that which, at the same time, constitutes the main disadvantage of a NCS; namely, the unreliability (in terms of induced delays and information loss) of the communication channel. For an in-depth overview of these issues refer, among others, to Heemels and van de Wouw (2011), Heemels et al. (2010), Nešić and Liberzon (2009), Posthumus-Cloosterman (2008), and Tipsuwan and Chow (2003), and the references therein. It is worth noting that some of the typical applications of NCSs, such as remote surgery and automated highway systems, overlap those of telerobotic and cooperative robotic systems.

Given the importance of ensuring the stability and performance of telerobotic and cooperative robotic systems in the face of delay-inducing networked communication, Section 1.3 and Section 1.4 outline the control problems within these areas addressed in this thesis.

1.3 Remote Control of Mobile Robots

The negative effects of the network-induced delays when controlling a system over a communication network have already been highlighted in Section 1.2. In this work, we focus on the tracking control of mobile robots over a delay-inducing communication network. In other words, we consider that the system transmits its sensor measurements and receives its control commands using networked communication. We denote this problem as the *remote tracking control of mobile robots* and consider the schematic representation shown in Figure 1.8. In particular, we will propose a control strategy based on a state predictor and a tracking controller which ensures the stability and tracking performance of the robot in the face of a constant time-delay.

The problem of controlling a system over a communication network has been addressed in different fields of control engineering. In order to place the current work in the proper context and precisely formulate its contributions, an overview of closely related and relevant literature follows.

As explained before, much of the work on NCSs is devoted to the study of the effect of a wide range of network-induced impairments and uncertainties. On the other hand, in this thesis we only consider the effects of network-induced delays. Nevertheless, it is important to stress that the majority of the work in the field of NCSs focuses on robust stability and stabilization (refer, among others, to Cloosterman et al., 2009; Donkers et al., 2011; Garcia-Rivera and Barreiro, 2007; Nešić and Liberzon, 2009). In contrast, in the current work we consider the more complex problem of trajectory tracking control. In this respect, of the few works in the NCSs literature that address the tracking control problem, the vast majority focuses on linear systems (see for example van de Wouw et al., 2010a). Contrary to that, in this thesis we consider the remote tracking control of mobile robots with nonlinear dynamics. Although work on nonlinear NCSs exists, it focuses mainly on problems related to stabilization, rather than on more complex regulation tasks such as trajectory tracking and motion coordination (see, for instance, Carnevale et al., 2007; Heemels et al., 2010; Nešić and Teel, 2004a,b; van de Wouw et al., 2010b, for additional details). A distinctive feature of the work on NCSs mentioned above is that the sampled-data nature of the systems is explicitly taken into account, typically leading to switched uncertain discrete-time system models (Cloosterman et al., 2009, 2010; Donkers et al., 2011; Hetel et al., 2008) or hybrid system models (Dačić and Nešić, 2007; Heemels et al., 2010; Nešić and Teel, 2004a,b; Walsh et al., 2002). This constitutes a fundamental difference with the modeling, analysis, and controller design approach taken in this thesis, where the tracking control problem is studied on the basis of a continuous-time modeling perspective.

In the context of telerobotics, several techniques have been proposed to overcome the negative effects of a network-induced delay in a remotely controlled system. For an overview of these techniques, refer to Hokayem and Spong (2006) and Siciliano and Khatib (2008), and the references therein. Among the most common approaches are operation under delay by means of shared compliant control or the addition of local force loops (Hashtrudi-Zaad and Salcudean, 2002; Kim et al., 1992), the use of the scattering transformation (Anderson and Spong, 1989; Stramigioli et al., 2002), a passivity-based approach (Hannaford and Ryu, 2002; Ryu et al., 2005), and wave variable transformations (Munir and Book, 2002; Niemeyer and Slotine, 2004). Some works even consider the Internet as the communication channel, which introduces issues such as variability in the network-induced delay, packet dropout, and data retransmission (Munir and Book, 2003;

Oboe and Fiorini, 1998). Recall that in a classical telerobotic system, the local and remote sites are both equipped with a controller (see Figure 1.3), whereas in the current work we consider the more challenging scenario in which there is no controller on the remote site (see Figure 1.8). In other words, the mobile robot transmits its sensor measurements to the local site and receives its control commands from the local site over the communication channel. On the other hand, in the current work we do not focus on reflecting the interaction forces in the remote site (that is, the interaction forces between the robot and its surrounding environment) to the local site. This allows us to use the posture kinematic model of the mobile robots under consideration to design their remote tracking control strategies. Given that the posture kinematic model is the simplest representation capable of providing a global description of the state of the robot (Siciliano and Khatib, 2008), the resulting remote tracking control strategies require a minimal amount of information to be implemented (only the position and orientation of the robot are necessary, as opposed to the translational and rotational velocities and the system parameters required by the dynamic model).

It is worth noting that the remote control problem considered in this thesis is also related to the type of problems addressed by predictor-like control strategies such as the ones based on the classical Smith predictor (Smith, 1957) and its numerous extensions. Among these extensions are Smith-like predictors for nonlinear systems (Kravaris and Wright, 1989), for discrete nonlinear systems (Henson and Seborg, 1994), and for nonlinear systems with disturbances (Huang and Wang, 1992). Nonetheless, the applicability of these Smith-like predictors is restricted to certain classes of nonlinear systems and the majority of the work often focuses on mitigating the negative effects of input time-delays in industrial and chemical processes, with a limited number of applications to mechanical systems (for instance, in Smith and Hashtrudi-Zaad, 2006; Velasco-Villa et al., 2007). In contrast, as explained before, the current work focuses on a class of mechanical systems with rich nonlinear dynamics; namely, mobile robots.

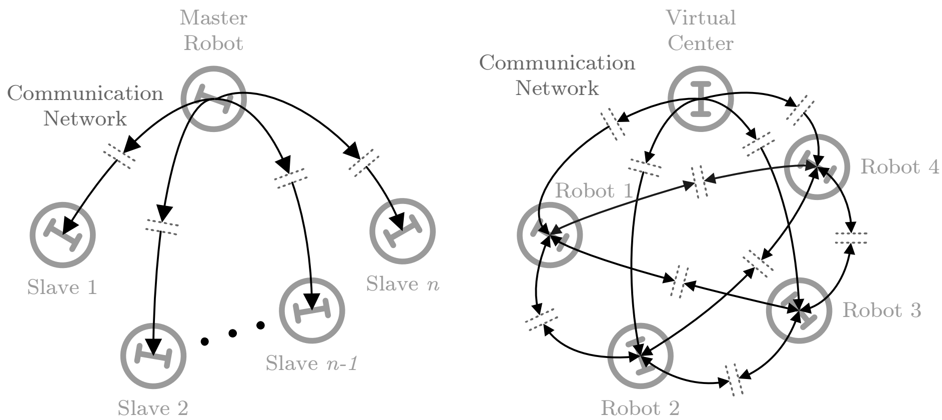
Recently, a number of predictor-based compensation techniques haven been proposed for a broader class of nonlinear systems in Karafyllis and Krstic (2010), Krstic (2009), and upcoming related works by the same authors. These control strategies are inspired on the ideas behind the original Smith predictor and combine a state predictor together with a feedback control law designed for the delay-free system. Even though the remote tracking control strategies presented in this thesis are based on a similar architecture and make use of a predictor-controller combination, the design procedure and characteristics of the state predictors in both cases are different (the main differences between both control strategies will be explained in greater detail in Section 4.5).

Finally, there is a wide array of control techniques and approaches that focus on the control of nonlinear systems with (input, output, or input and output) time-delays. Some of the methodologies for addressing these problems include intelligent control techniques such as neural networks (Hong et al., 1996; Tan and Keyser, 1994a,b) and fuzzy logic (Cao and Frank, 2000; Malki and Misir, 1996), delay decomposition and approximation techniques (Alvarez-Aguirre et al., 2008; Schoen, 1995), design of causal control laws for delayed systems (Marquez-Martinez and Moog, 2004), and finite spectrum assignment techniques Oguchi (2007); Oguchi et al. (2002). For an overview of recent results on the stability and control of nonlinear delayed systems refer to Gu and Niculescu (2003). The main difference is that the remote tracking controllers proposed in this work are based on tracking control laws which already exist and have proven merit in the high-performance tracking control of mobile robots, whereas the controllers designed in the aforementioned works are specifically (and sometimes exclusively) designed to accommodate time-delays.

1.4 Remote Motion Coordination of Mobile Robots

As explained in Section 1.2, it is not always practical to assume in a cooperative robotic system that an ideal communication channel is available for the robots to use. Motivated by this, in the current work we focus on the motion coordination of a group of mobile robots considering a communication network which induces a time-delay. We will denote this type of motion coordination as *remote motion coordination of mobile robots* and focus on both master-slave and mutual motion coordination. Furthermore, we consider the case of a constant network-induced delay, with the aim of describing a first step towards incorporating additional networked communication effects. The information flow between robots in remote master-slave and mutual motion coordination is depicted in Figure 1.9(a) and Figure 1.9(b), respectively.

It is worth noting that the vast majority of the work available on motion coordination does not take into account the properties of the communication network which the robots use to exchange information. In this respect, much more attention has been given to the cooperative aspect of motion coordination than to the communication aspect. For instance, we have already explained that the tracking control problem within the field of NCSs has received considerably less attention than the robust stability and stabilization problems. It then follows that, although some of the tools available in the NCSs literature would seem suitable to address the issues that arise when having networked communication in a cooperative robotic system, the problem, even more complex in nature than the tracking control problem, remains relatively unexplored within this field.



(a) Master-slave motion coordination using networked communication.

(b) Mutual motion coordination using networked communication.

Figure 1.9 . Information flow between robots in remote master-slave and mutual motion coordination. The dashed lines represent the time-delay which results from the use of networked communication.

On the other hand, there are a number of recent works which analyze the problem of reaching consensus or synchronization among multiple agents. These agents belong to a network with certain topology and interact with each other via a delayed coupling. As explained already in Section 1.1.2, the remote motion coordination problem considered in this work focuses on master-slave and mutual motion coordination. Recall that in these types of motion coordination it is necessary for the robots in the group to exchange information in order to reach certain coordination objectives. In other words, the robots in the group (agents) also interact with each other via a delayed coupling. It then follows that there is a relationship between reaching consensus and reaching motion coordination in a multi-agent system, and that this relationship is still in place when considering delay-inducing networked communication between the agents. A preliminary approach to further understand this relationship in the context of mobile robots (considering delay-free communication) can be found in Sadowska et al. (2011).

There are several important points to highlight regarding the works which address the consensus problem with communication delays. First, the majority of these works focus on agents with single or double integrator dynamics. For example, the consensus condition has been shown to be delay independent for first-order multi-agent systems (Blondel et al., 2005; Cao et al., 2009). On the other hand, frequency domain techniques have been proposed to study the consensus problem with communication delays for groups of higher-order agents (Lee and Spong, 2006; Münz et al., 2010; Tian and Liu, 2009). More recently, results have been obtained for nonlinear systems with relative degree one (Münz, 2010) and relative degree two (Münz et al., 2011). For a more in-depth review of the consensus problem with communication delays consult Münz (2010) and the references therein. Another recent work studies the delay independent synchronization of generic nonlinear systems which interact via delayed couplings (Steur and Nijmeijer, 2011). While the underlying assumption in our work is that the graph which represents the network's topology is strongly connected, the majority of the previous works explicitly attempt to relax this connectivity assumption, something which is not addressed in the current work. Nevertheless, it is important to emphasize that the controllers used to achieve consensus in the previous cases and the ones we will propose to achieve motion coordination are intrinsically different. Such difference lies in the fact that, when considering consensus problems, the control input of each agent is given only by the couplings between that particular agent and the other agents in the group. On the other hand, when considering motion coordination of mobile robots, the coupling between the robots in the group constitutes only a fraction of the total control input of each robot, with the remaining part of the input intended to drive the robots in order to achieve a certain task.

The work by Dong and Farrell (2008), Dong and Farrell (2009), and Dong (2011) is also closely related to the remote motion coordination problem studied in this thesis. These works focus on the formation control of a group of mobile robots with networked communication. In Dong and Farrell (2008) and Dong and Farrell (2009), the kinematic model of the robots is transformed to a reduced system in order to derive coordinating controllers which are robust in the face of communication delays. Contrary to that, the coordinating controllers in the current work are directly based on the kinematic model of the robots and on tracking control laws which already exist. The dynamic model of the robots is used in Dong (2011) to propose adaptive coordinating controllers which are robust against communication delays. This constitutes one of the main differences with the current work, in which we consider the posture kinematic model of the robots. As explained already in Section 1.3, this results in coordinating controllers which require a minimal amount of information to be implemented. In addition, the coordinating controllers proposed in Dong (2011) are specifically designed to let the robots reach a certain formation (as in rendezvousing), whereas in the current work we consider the case of letting the robots move along a reference trajectory while maintaining a formation (as in motion coordination).

It is worth noting that, of all the previous works, only Steur and Nijmeijer (2011) report experimental results (to be found in Neefs et al., 2010). In contrast, all the remote motion coordination strategies proposed in this thesis have been experimentally validated using a group of unicycle-type mobile robots.

Having defined the control problem addressed in this work, Section 1.5 formulates the research objective and outlines the main contributions of the thesis.

1.5 Research Objective and Main Contributions of the Thesis

Considering the problem settings outlined in the previous two sections, we have that this thesis focuses on the remote tracking control of a mobile robot over a delay-inducing communication network and on the motion coordination of a group of mobile robots under the consideration that the information exchange between the robots takes place over a communication network which is subject to a time-delay. In addition, the resulting remote tracking and motion coordination strategies will be validated by both numerical simulations and experimental results. The experiments are to be carried out in a multi-robot platform which will be introduced in Chapter 3 and is composed of two equivalent setups located at the Eindhoven University of Technology (TU/e) in the Netherlands and at Tokyo Metropolitan University (TMU) in Japan. These observations may be summarized in the following research objective:

Design and experimentally validate control strategies for the following two control problems:

1. *The remote tracking control of a mobile robot over a delay-inducing communication network.*
2. *The motion coordination of mobile robots communicating over a delay-inducing network.*

The fact that the aforementioned control strategies focus on wheeled mobile robots poses major challenges due to the fact that they are nonlinear systems with rich dynamics and may be subject to non-holonomic constraints. Furthermore, the presence of network-induced delays, even when constant, also constitutes an additional difficulty.

Considering the previous research objective, the main contributions of this work may be stated as follows:

- The design of controllers for robotic systems in which the robot and the controller are physically separated and linked via a two-channel, delay-inducing communication network must take into account the effects of the ensuing time-delay, since the stability and performance of the system may be compromised. In this thesis we address the problem of the remote tracking control of wheeled mobile robots; specifically, unicycle-type and omnidirectional robots. The proposed control architecture consists of a state estimator with a predictor-like structure and a modified state feedback tracking controller. Using the Lyapunov-Razumikhin approach and results for nonlinear delayed cascaded systems, we show that this predictor-controller combination guarantees the stability of the closed-loop system up to a maximum admissible delay. Moreover, the performance of the proposed remote tracking control strategy is assessed in both simulations and experiments.

- The state predictor used in the remote tracking control strategies presented in this thesis is inspired on the notion of anticipating synchronization in coupled chaotic systems. The application of this type of state predictor to mechanical systems (in particular to mobile robots) constitutes one of the main contributions of this work. Furthermore, the remote tracking control strategies which result from the application of this state predictor are not only intended to ensure the stability of the closed-loop system in the face of delays, but also to take a proactive approach towards mitigating the negative effects of the network-induced delay.

- The thesis presents a design approach to obtain coordinating controllers for unicycle and omnidirectional mobile robots. These controllers feature mutual couplings between the robots in order to ensure improved robustness against perturbations. The main focus is on assessing the effects of the delays induced by the communication network used to relay information between the robots. Firstly, we present results stating up to which maximal delay closed-loop stability (and hence motion coordination) can still be achieved. Secondly, the effect of delays on the coordination performance is assessed by means of simulations and experiments.

- We consider the experimental implementation and validation of the proposed control strategies to be an additional contribution of this work. The experiments are carried out using the multi-robot platforms present in the Netherlands and Japan, which communicate over the Internet.

- As an extension of the results regarding the remote tracking control of mobile robots, we also show that the predictor-controller combination can be successfully applied to the remote tracking control of a one-link robot. More specifically, we show that the stability and performance of the resulting closed-loop system is guaranteed up to a maximum allowable time-delay and validate the proposed control strategy by means of numerical simulations. This extension elucidates how the remote control architecture proposed in this work may be applied to a broader range of mechanical systems, such as robotic manipulators.

1.6 Structure of the Thesis

The thesis is organized as follows. Chapter 2 provides some basic preliminaries regarding the stability (in the sense of Lyapunov) of time-varying dynamical systems, retarded functional differential equations, and delay-free and delayed nonlinear cascaded systems. In addition, Chapter 2 recalls a trajectory tracking controller for the unicycle robot. These results will be used extensively in Chapters 4 to 6.

Chapter 3 contains a description of the multi-robot experimental platform used to implement the control strategies developed in Chapters 4 and 5.

In Chapter 4, a control strategy is proposed to solve the remote tracking control problem for a unicycle robot with network-induced delays. The control strategy compensates for the negative effects of the time-delay using a state predictor and ensures asymptotic stability up to a maximum admissible delay. Additionally, the proposed remote tracking controller is validated using the multi-robot platform described in Chapter 3.

Chapter 5 introduces the motion coordination strategies considered in this work; namely, the master-slave and mutual motion coordination strategies. These coordination strategies are studied under the assumption that a network-induced delay affects the communication channel which the robots use to exchange information. The subsequent stability analysis shows that, up to a certain admissible delay, the robots maintain motion coordination. Additionally, the coordinating controllers proposed for this purpose are validated in the experimental setup introduced in Chapter 3.

In Chapter 6, the remote control and motion coordination strategies introduced in Chapters 4 and 5 are applied to different dynamical systems besides the unicycle robot; namely omnidirectional mobile robots and a one-link robot. As with the unicycle robot, the proposed controllers ensure the stability of the resulting closed-loop system up to a maximum admissible delay.

Finally, Chapter 7 presents concluding remarks and recommendations for future research. For the sake of the readability of the main text, the proofs of the theorems proposed throughout this thesis are given in the appendices.

2

Preliminaries

Abstract. This chapter contains some of the mathematical definitions, stability concepts, and general results on cascaded systems and on the tracking control of a unicycle robot that will be used throughout this thesis.

2.1 Outline

This chapter recalls several concepts and definitions which will be useful when studying the stability properties of the different control strategies presented in this work. To begin with, Section 2.2 reviews some general mathematical notions. In Section 2.3, an overview of the stability of time-varying dynamical systems is provided and some fundamental concepts and results of Lyapunov stability theory for this type of systems are recalled. Stability results for a particular class of time-varying dynamical systems, namely time-varying cascaded systems, are presented in Section 2.4, whereas a number of stability results for retarded functional differential equations are given in Section 2.5. Finally, the posture kinematic model of a unicycle-type mobile robot and a suitable trajectory tracking controller for this system is presented in Section 2.6. The majority of the concepts and definitions presented in this chapter are taken from Khalil (2000), Lefeber (2000), and Siciliano and Khatib (2008), unless indicated otherwise. The reader is referred to these references for additional details and in-depth explanations.

2.2 General Mathematical Notions

The notation regarding vector and matrix norms in this work is as follows. The vector 1- and 2-norms of vector a are denoted as $\|a\|_1$ and $\|a\|_2$, respectively. The matrix sum norm, Frobenius norm, and induced matrix 1- and 2-norms of a matrix A are denoted as $\|A\|_{\text{sum}}$, $\|A\|_F$, $\|A\|_{i1}$, and $\|A\|_{i2}$, respectively. The minimum and maximum eigenvalues of a symmetric matrix A will be denoted as $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$. Throughout this thesis there are a number of results, especially theorems referred from other works, in which there is no distinction regarding the vector norm being used. This means that these results hold for any valid vector norm as long as their use is consistent. In these cases the vector norm will be denoted as $\|\cdot\|$.

The following theorem, known as Gershgorin's circle theorem, will be useful when checking the location of the eigenvalues of a square matrix.

Theorem 2.1. *(Skogestad and Postlethwaite, 2005, Appendix A.2.1). The eigenvalues of the $n \times n$ matrix A lie in the union of n circles in the complex plane, each with center a_{ii} (diagonal elements of matrix A) and radius $r_i = \sum_{j \neq i} |a_{ij}|$ (sum of the off-diagonal elements in row i).*

An interpretation of the theorem is that, if $a_{ii} > r_i$, all the eigenvalues of matrix A lie in the open right-half of the complex plane. Along the same lines, if $a_{ii} < -r_i$ all the eigenvalues lie in the open left-half of the complex plane.

Consider now the definition of class $\mathcal{K}, \mathcal{K}_\infty$, and $\mathcal{KL}$ comparison functions.

Definition 2.2. *(Khalil, 2000, Definition 4.2). A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to* ***class*** *$\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to* ***class*** *$\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.*

Definition 2.3. *(Khalil, 2000, Definition 4.3). A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to* ***class*** *$\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.*

The definition of a scalar persistently exciting (PE) signal is given next and will be useful when investigating the stability of a special type of linear time-varying systems.

Definition 2.4. *(Lefeber, 2000, Definition 2.3.5). A continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}$ is said to be persistently exciting if all of the following conditions hold:*

- *a constant $K > 0$ exists such that $|\psi(t)| \leq K$, $\forall t \geq 0$;*

- *a constant $L > 0$ exists such that $|\psi(t) - \psi(t')| \leq L|t - t'|$, $\forall t, t' \geq 0$;*
- *constants $\delta > 0$ and $\epsilon > 0$ exist such that*

$$\forall t \geq 0, \exists s : t - \delta \leq s \leq t \text{ such that } |\psi(s)| \geq \epsilon.$$

Similar to the definition of a scalar PE signal, the definition of a PE vector of continuous functions $\Psi(t) \in \mathbb{R}^n$, with $\Psi(t) = [\phi_1(t), \phi_2(t), \ldots, \phi_n(t)]^T$, will be useful when checking the stability of a particular type of linear time-varying systems.

Definition 2.5. *A vector $\Psi(t) : \mathbb{R}^+ \to \mathbb{R}^n$ of continuous functions $\phi_i : \mathbb{R}^+ \to \mathbb{R}$, for $i \in \{1, 2, \ldots, n\}$, is said to be persistently exciting if all of the following conditions hold:*

- *constants $K_i > 0$ exist such that $|\phi_i(t)| \leq K_i$ for all $t \geq 0$ and $i \in \{1, 2, \ldots, n\}$;*
- *constants $L_i > 0$ exist such that $|\phi_i(t) - \phi_i(t')| \leq L_i|t - t'|$ for all $t, t' \geq 0$ and $i \in \{1, 2, \ldots, n\}$;*
- *constants $\delta > 0$ and $\epsilon_i > 0$ exist such that*

$$\forall t \geq 0, \exists s : t - \delta \leq s \leq t \text{ such that } |\phi_i(s)| \geq \epsilon_i, \quad \forall i \in \{1, 2, \ldots, n\}. \tag{2.1}$$

Remark 2.6. *Note that condition (2.1) requires that, within the interval $[t - \delta, t]$, there exists a common time instant s at which the absolute value of every $\phi_i(s)$, $i \in \{1, 2, \ldots, n\}$, is equal to or greater than a certain $\epsilon_{\min} > 0$, where $\epsilon_{\min} = \min\{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}$.*

An interpretation of Definition 2.5 follows from the interpretation of a scalar PE signal made in Lefeber (2000). Assuming that we plot the graphs of all $|\phi_i(t)|$, for $i \in \{1, 2, \ldots, n\}$, and observe these graphs through a window of width $\delta > 0$. Then, no matter where we put this window on these graphs, always a time instant s in this window exists where all $\phi_i(s)$, $i \in \{1, 2, \ldots, n\}$, satisfy $|\phi_i(s)| \geq \epsilon_{\min} > 0$.

Finally, consider the following continuous functions which will be used in later results.

$$f_1(x) := \int_0^1 \cos(sx)ds = \begin{cases} \frac{\sin x}{x} & \text{for} \quad x \neq 0 \\ 1 & \text{for} \quad x = 0 \end{cases}, \tag{2.2a}$$

$$f_2(x) := \int_0^1 \sin(sx)ds = \begin{cases} \frac{1-\cos x}{x} & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}, \tag{2.2b}$$

and note that

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \text{and} \qquad \lim_{x\to 0} \frac{1-\cos x}{x} = 0.$$

For simplicity of notation, the expressions $\frac{\sin x}{x}$ and $\frac{1-\cos x}{x}$ will be used in this thesis, even though it would be more precise to use (2.2a) and (2.2b), respectively.

2.3 Stability of Time-Varying Dynamical Systems

This section presents a number of definitions regarding the stability of time-varying dynamical systems focusing mainly on Lyapunov stability of equilibria. Its contents are based on definitions provided in Khalil (2000) and Haddad and Chellaboina (2008), unless indicated otherwise.

Consider a non-autonomous nonlinear system described by

$$\dot{x} = f(t, x), \qquad x(t_0) = x_0, \qquad t \geq t_0, \tag{2.3}$$

where $x \in \mathcal{D}$, the space of states $\mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$, $f : [t_0, t_1) \times \mathcal{D} \to \mathbb{R}^n$ is such that $f(\cdot, \cdot)$ is jointly continuous in t and x, and for every $t \in [t_0, t_1)$, $f(t, 0) = 0$ and $f(t, \cdot)$ is locally Lipschitz in x uniformly in t for all t, in compact subsets of $[0, \infty)$. Note that under the above assumptions the solution $x(t)$, $t \geq t_0$, to (2.3) exists and is unique over the interval $[t_0, t_1)$.

The concept of stability is one of the most important ones when studying any dynamical system. The following definition introduces different stability notions for system (2.3).

Definition 2.7. *(Khalil, 2000, Definition 4.4). The equilibrium point $x = 0$ of (2.3) is*

- ***stable*** *if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon, t_0) > 0$ such that*

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon \qquad \forall t \geq t_0 \geq 0; \tag{2.4}$$

- ***uniformly stable*** *if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon) > 0$, independent of t_0, such that (2.4) is satisfied;*
- ***unstable*** *if it is not stable;*
- ***asymptotically stable*** *if it is stable and there is a positive contant $c = c(t_0)$ such that $x(t) \to 0$ as $t \to \infty$, for all $\|x(t_0)\| < c$;*

- ***locally uniformly asymptotically stable (LUAS)*** *if it is uniformly stable and there is a positive constant* c*, independent of* t_0*, such that for all* $\|x(t_0)\| < c$, $x(t) \to 0$ *as* $t \to \infty$*, uniformly in* t_0*; that is, for each* $\eta > 0$*, there is* $T = T(\eta) > 0$ *such that*

$$\|x(t)\| < \eta, \qquad \forall t \geq t_0 + T(\eta), \qquad \forall \|x(t_0)\| < c; \tag{2.5}$$

- ***globally uniformly asymptotically stable (GUAS)*** *if it is uniformly stable,* $\delta(\epsilon)$ *can be chosen to satisfy* $\lim_{\epsilon\to\infty} \delta(\epsilon) = \infty$*, and, for each pair of positive numbers* η *and* c*, there is* $T = T(\eta, c) > 0$ *such that*

$$\|x(t)\| < \eta, \qquad \forall t \geq t_0 + T(\eta, c), \qquad \forall \|x(t_0)\| < c. \tag{2.6}$$

The following lemma provides an equivalent definition of uniform stability and uniform asymptotic stability by making use of comparison functions.

Lemma 2.8. *(Khalil, 2000, Lemma 4.5). The equilibrium point* $x=0$ *of (2.3) is*

- ***uniformly stable*** *if and only if there exist a class* $\mathcal{K}$ *function* α *and a positive constant* c*, independent of* t_0*, such that*

$$\|x(t)\| \leq \alpha(\|x(t_0)\|), \qquad \forall t \geq t_0 \geq 0, \qquad \forall \|x(t_0)\| < c; \tag{2.7}$$

- ***locally uniformly asymptotically stable (LUAS)*** *if and only if there exist a class* $\mathcal{KL}$ *function* β *and a positive constant* c*, independent of* t_0*, such that*

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \qquad \forall t \geq t_0 \geq 0, \qquad \forall \|x(t_0)\| < c; \tag{2.8}$$

- ***globally uniformly asymptotically stable (GUAS)*** *if and only if inequality (2.8) is satisfied for any initial state* $x(t_0)$*.*

A special case of uniform asymptotic stability which receives its own designation is so-called exponential stability.

Definition 2.9. *(Khalil, 2000, Definition 4.5). The equilibrium point* $x = 0$ *of (2.3) is* ***exponentially stable*** *if there exist positive constants* $c, k,$ *and* λ *such that*

$$\|x(t)\| \leq k \|x(t_0)\| e^{-\lambda(t-t_0)}, \qquad \forall \|x(t_0)\| < c, \tag{2.9}$$

and ***globally exponentially stable*** *if (2.9) is satisfied for any initial state* $x(t_0)$*.*

The definitions of uniform asymptotic and exponential stability provided in Lemma 2.8 and Definition 2.9 may be characterized in terms of the existence of a so-called Lyapunov function. Next, sufficient conditions for the stability of the nonlinear time-varying system (2.3) are given in terms of a Lyapunov function $V(t,x)$. In order to do so, hereinafter we define

$$\dot{V}(t,x) := \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x),$$

for a given continuously differentiable function $V : [0,\infty) \times \mathcal{D} \to \mathbb{R}$. Additionally, $\phi(\tau; t, x)$ denotes the solution of system (2.3) at time τ which starts at (t,x).

The following theorems formulate sufficient conditions for uniform asymptotic and exponential stability in terms of the existence of a Lyapunov function exhibiting certain properties.

Theorem 2.10. *(Khalil, 2000, Theorem 4.9). Let $x = 0$ be an equilibrium point for (2.3) and $\mathcal{D} \subset \mathbb{R}^n$ be a domain containing $x = 0$. Let $V : [0,\infty) \times \mathcal{D} \to \mathbb{R}$ be a continuously differentiable function such that*

$$W_1(x) \leq V(t,x) \leq W_2(x), \tag{2.10a}$$

$$\dot{V}(t,x) \leq -W_3(x), \tag{2.10b}$$

$\forall t \geq 0$ and $\forall x \in \mathcal{D}$, where $W_1(x), W_2(x)$, and $W_3(x)$ are continuous positive definite functions on $\mathcal{D}$. Then $x = 0$ is uniformly asymptotically stable. If $\mathcal{D} = \mathbb{R}^n$ and $W_1(x)$ is radially unbounded, then $x = 0$ is globally uniformly asymptotically stable (GUAS).

Theorem 2.11. *(Khalil, 2000, Theorem 4.10). Let $x = 0$ be an equilibrium point for (2.3) and $\mathcal{D} \subset \mathbb{R}^n$ be a domain containing $x = 0$. Let $V : [0,\infty) \times \mathcal{D} \to \mathbb{R}$ be a continuously differentiable function such that*

$$k_1\|x\|^a \leq V(t,x) \leq k_2\|x\|^a, \tag{2.11a}$$

$$\dot{V}(t,x) \leq -k_3\|x\|^a, \tag{2.11b}$$

$\forall t \geq 0$ and $\forall x \in \mathcal{D}$, where k_1, k_2, k_3, and a are positive constants. Then $x = 0$ is exponentially stable. If the assumptions hold globally, then $x = 0$ is globally exponentially stable (GES).

The stability notions in Theorem 2.11 are stronger than the ones in Theorem 2.11. An intermediate notion situated between global exponential stability and global uniform asymptotic stability is global $\mathcal{K}$-exponential stability as defined in Sørdalen and Egeland (1995).

Definition 2.12. *(Sørdalen and Egeland, 1995, Definition 2). The equilibrium point $x = 0$ of (2.3) is said to be **globally $\mathcal{K}$-exponentially stable** if a class $\mathcal{K}$ function $\alpha(\cdot)$ and a constant $\beta > 0$ exist such that the following holds for all $x_0 \in \mathbb{R}^n$ and $t_0 \in [0, \infty)$:*

$$\|x(t)\| \leq \alpha(\|x_0\|)e^{-\beta(t-t_0)}, \qquad t \geq t_0 \geq 0. \tag{2.12}$$

The next definition introduces a linear time-varying (LTV) dynamical system.

Definition 2.13. *(Haddad and Chellaboina, 2008, Definition 2.3). Consider the dynamical system (2.3) with $\mathcal{D} = \mathbb{R}^n$. If $f(t, x) = A(t)x$, where $A : [t_0, t_1] \to \mathbb{R}^{n \times n}$ is piecewise continuous on $[t_0, t_1]$ and $x \in \mathbb{R}^n$, then (2.3) is called a **linear time-varying (LTV)** dynamical system.*

Based on Definition 2.13, consider the LTV system

$$\dot{x}(t) = A(t)x(t), \qquad x(t_0) = x_0, \qquad t \geq t_0, \tag{2.13a}$$

$$y = C(t)x, \tag{2.13b}$$

with $A : [0, \infty) \to \mathbb{R}^{n \times n}$ continuous. Recall that in the case of linear systems, global uniform asymptotic stability and global exponential stability are equivalent. The following theorem formalizes this fact for LTV systems.

Theorem 2.14. *(Ioannou and Sun, 1996, Theorem 3.4.6 v). The linear time-varying (LTV) system (2.13) is globally exponentially stable (GES) if and only if it is globally uniformly asymptotically stable (GUAS).*

As in the nonlinear case, the global exponential stability of an LTV system may be characterized in terms of the existence of a Lyapunov function $V(t, x)$ with certain properties. Moreover, the following converse theorem states that there exists a (time-varying) quadratic Lyapunov function for an LTV system when its origin is GES.

Theorem 2.15. *(Khalil, 2000, Theorem 4.12). Let $x = 0$ be the exponentially stable equilibrium point of (2.13). Suppose $A(t)$ is continuous and bounded. Let $Q(t)$ be a continuous, bounded, positive definite, symmetric matrix. Then, there exists a continuously differentiable, bounded, positive definite, symmetric matrix $P(t)$ which satisfies*

$$\dot{P}(t) + P(t)A(t) + A^T(t)P(t) = -Q(t). \tag{2.14}$$

Hence, $V(t, x) = x^T P(t)x$ is a Lyapunov function for the system which satisfies $\dot{V}(t, x) = -x^T Q(t)x$.

The next definition will be required in stability results for particular types of LTV systems presented later. Before the definition, recall that the system (2.13) is a bounded realization provided $A(t)$ and $C(t)$ are bounded. Let $\Phi(t, t_0)$ denote the state transition matrix for the system $\dot{x} = A(t)x$; then, the uniform complete observability of system (2.13) is defined as follows.

Definition 2.16. *(Silverman and Anderson, 1968). A bounded realization (2.13) is said to be* ***uniformly completely observable (UCO)*** *if* $\exists\delta > 0$ *such that*

$$G_O(t, t+\delta) \geq \alpha(\delta)I > 0, \qquad \forall t \geq 0, \tag{2.15}$$

where the ***observability Gramian*** *is defined as*

$$G_O(t, t+\delta) := \int_t^{t+\delta} \Phi^T(\tau, t)C^T(\tau)C(\tau)\Phi(\tau, t)d\tau. \tag{2.16}$$

The following theorem is based on Theorem 8.5 in Khalil (2000) and is useful when trying to determine the global exponential stability of an LTV system for which only a Lyapunov function with a negative semi-definite time derivative is available.

Theorem 2.17. *Consider the LTV system (2.13), where* $A(t)$ *is continuous for all* $t \geq 0$*. Suppose there exists a continuously differentiable, symmetric matrix* $P(t)$ *that satisfies, for* $c_1, c_2 > 0$*, the inequality*

$$0 < c_1 I \leq P(t) \leq c_2 I, \qquad \forall t \geq 0, \tag{2.17}$$

as well as the matrix differential equation

$$-\dot{P}(t) \geq P(t)A(t) + A^T(t)P(t) + C^T(t)C(t), \tag{2.18}$$

where $C(t)$ *is continuous in* t*. If the pair* $(A(t), C(t))$ *is uniformly completely observable (UCO), then, the origin of (2.13) is globally exponentially stable (GES).*

Proof. The proof straightforwardly follows from Theorem 8.5 and Example 8.11 in Khalil (2000). □

The next lemma characterizes the stability of a certain class of LTV systems and will be useful in the upcoming chapters.

Lemma 2.18. *(Jakubiak et al., 2002). Consider the LTV system*

$$\dot{x} = \begin{bmatrix} -c_1 & -c_2\psi(t) \\ c_3\psi(t) & 0 \end{bmatrix} x, \tag{2.19}$$

with $x \in \mathbb{R}^2$*. Given* $c_1 > 0, c_2c_3 > 0$ *and* $\psi(t) : \mathbb{R}^+ \to \mathbb{R}$ *persistently exciting,* $x = 0$ *is a globally exponentially stable equilibrium point of (2.19).*

The following theorem is based on Theorem 2 in Kern (1982) and Corollary 2.3.4 in Lefeber (2000). This result is useful when exploiting Theorem 2.17 to determine the global exponential stability of an LTV system which depends on a number of time-varying functions. These functions are denoted as $\phi_i(t)$, for $i \in \{1, 2, \ldots, n\}$, and together constitute the vector of continuous functions $\Psi(t) = [\phi_1(t), \ldots, \phi_n(t)]^T$.

Theorem 2.19. *Consider the following linear time-varying (LTV) system:*

$$\dot{x} = A(\Psi(t))x, \tag{2.20a}$$

$$y = Cx, \tag{2.20b}$$

where $\Psi(t) = [\phi_1(t), \ldots, \phi_n(t)]^T$ is persistently exciting according to Definition 2.5 and $A(\Psi)$ is Lipschitz continuous and bounded. Assume that, for $P = [p_1, \ldots, p_n]^T$, the pair $(A(P), C)$ is observable for P for which $p_i \neq 0$ for all $i \in \{1, 2, \ldots, n\}$. Then, system (2.20) is uniformly completely observable (UCO).

Proof. Since $\Psi(t)$ is PE, it is bounded. Using the fact that $A(\Psi)$ is bounded, we have that $A(\Psi(t))$ is bounded as well. This, in turn, implies that (2.20) is a bounded realization. Now, for any fixed $s \in [t-\delta, t]$, with $\delta > 0$, according to Definition 2.5, (2.20) may be rewritten as:

$$\dot{x} = A(\Psi(s))x + [A(\Psi(t)) - A(\Psi(s))]x, \tag{2.21a}$$

$$y = Cx, \tag{2.21b}$$

which results in the reduced system

$$\dot{x} = A(\Psi(s))x, \tag{2.22a}$$

$$y = Cx. \tag{2.22b}$$

Since s is fixed, the observability Gramian of (2.22) is given by

$$G_O(t-\delta, t) := \int_{t-\delta}^{t} e^{A^T(\Psi(s))(\tau-(t-\delta))} C^T C e^{A(\Psi(s))(\tau-(t-\delta))} d\tau. \tag{2.23}$$

Due to the fact that $\Psi(t)$ is PE, we can always chose $s \in [t-\delta, t]$ such that $\phi_i(s) \neq 0$ for all $i \in \{1, 2, \ldots, n\}$. Moreover, since the pair $(A(P), C)$ is observable for P for which $p_i \neq 0$ for all $i \in \{1, 2, \ldots, n\}$, we can always chose s such that the pair $(A(\Psi(s)), C)$ is observable. This implies that the observability Gramian $G_O(t-\delta, t)$ in (2.23) is non-singular. Considering the previous and due to the fact that $G_O(t-\delta, t)$ is positive semi-definite to begin with (due to its quadratic-like nature), we can conclude that the observability Gramian satisfies $G_O(t-\delta, t) \geq$

$\alpha(\delta)I_n > 0$, where $\alpha(\delta) > 0$ for all $\delta > 0$. Using the fact that $G_O(t-\delta, t)$ as in (2.23) satisfies $G_O(t-\delta,t) \geq \alpha(\delta)I_n > 0$ and the fact that $A(\Psi(t))$ is Lipschitz in t (since $A(\Psi)$ is Lipschitz in Ψ by assumption and $\Psi(t)$ is Lipschitz in t since it is PE), we can employ a similar line of reasoning as that in the proof of Theorems 1 and 2 in Kern (1982) to conclude that the LTV system (2.20) is UCO. □

2.4 Cascaded Systems

This section elaborates on the stability of cascaded systems and is based on the results presented in Panteley and Loría (1998), Lefeber (2000), and Lefeber et al. (2000). Consider the nonlinear time-varying system $\dot{x} = f(t,x)$, with $x = [x_1^T \, x_2^T]^T \in \mathbb{R}^{n+m}$, which can be written as the following nonlinear cascaded system:

$$\dot{x}_1 = f_1(t,x_1) + g(t,x_1,x_2)x_2, \tag{2.24a}$$
$$\dot{x}_2 = f_2(t,x_2), \tag{2.24b}$$

where $x_1 \in \mathbb{R}^n$ and $x_2 \in \mathbb{R}^m$, $f_1(t,x_1)$ is continuously differentiable in (t,x_1) and $f_2(t,x_2)$, $g(t,x_2,x_2)$ are continuous in their arguments and locally Lipschitz in x_2 and (x_1,x_2), respectively. The previous cascaded system is assumed to have an equilibrium point at $[x_1^T \, x_2^T]^T = 0$. In addition, $f_1(t,0) = 0$ and $f_2(t,0) = 0$ for all $t \geq t_0$.

The cascaded system (2.24) consists of a nonlinear system Σ_1 with state x_1 perturbed by the output of the nonlinear system Σ_2 with state x_2 by means of the interconnection gain $g(t,x_1,x_2)$, in which

$$\Sigma_1 : \dot{x}_1 = f_1(t,x_1), \qquad\qquad \Sigma_2 : \dot{x}_2 = f_2(t,x_2).$$

Note that if the x_2-dynamics converge to zero, the x_1-dynamics reduce to Σ_1. In this case, even if Σ_1 is asymptotically stable, the asymptotic stability of the cascaded system (2.24) cannot be ensured by merely requiring the stability of Σ_1 and Σ_2. The following theorem poses sufficient conditions for the global uniform asymptotic stability of the origin of the nonlinear cascaded system (2.24).

Theorem 2.20. *(Lefeber et al., 2000). The equilibrium point $[x_1^T \, x_2^T]^T = 0$ of the nonlinear cascaded system (2.24) is globally uniformly asymptotically stable (GUAS) if the following three assumptions hold:*

1. *System Σ_1: the system $\dot{x}_1 = f_1(t,x_1)$ is globally uniformly asymptotically stable (GUAS) and there exists a continuously differentiable function $V(t,x_1)$:*

$\mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ and positive definite functions $W_1(x_1)$ and $W_2(x_1)$ such that

$$\begin{aligned}
&(i) && W_1(x_1) \le V(t, x_1) \le W(x_1), && \forall t \ge t_0, \quad \forall x_1 \in \mathbb{R}^n; && (2.25a)\\
&(ii) && \dot{V}(t, x_1) \le 0, && \forall \|x_1\| \ge \eta; && (2.25b)\\
&(iii) && \left\| \frac{\partial V}{\partial x_1} \right\| \|x_1\| \le \zeta V(t, x_1), && \forall \|x_1\| \ge \eta; && (2.25c)
\end{aligned}$$

where $\eta, \zeta > 0$ are constants.

2. *Interconnection: the function $g(t, x_1, x_2)$ satisfies*

$$\|g(t, x_1, x_2)\| \le \alpha_1(\|x_2\|) + \alpha_2(\|x_2\|)\|x_1\|, \qquad \forall t \ge t_0, \tag{2.26}$$

where $\alpha_1(\cdot), \alpha_2(\cdot) : \mathbb{R}^+ \to \mathbb{R}$ are continuous functions.

3. *System Σ_2: the system $\dot{x}_2 = f_2(t, x_2)$ is globally uniformly asymptotically stable (GUAS) and satisfies*

$$\int_{t_0}^{\infty} \|x_2(t; t_0, x_2(t_0))\| dt \le \beta(\|x_2(t_0)\|), \qquad \forall t_0 \ge 0, \tag{2.27}$$

where the function $\beta(\cdot)$ is a class $\mathcal{K}$ function.

In the theorem, condition 1.(i) indicates that the Lyapunov function related to system Σ_1 should be bounded, condition 1.(ii) requires the derivative of this function to be negative semi-definite, and condition 1.(iii) restricts its growth, condition 2 imposes a growth condition on the interconnection gain, and condition 3 requires the integrability of the state of system Σ_2. Besides requiring that both Σ_1 and Σ_2 are GUAS, the three assumptions guarantee that the solutions of the cascaded system (2.24) remain bounded. As a result the cascaded system is GUAS.

A stronger notion regarding the stability of the cascaded system (2.24) can be obtained when both systems Σ_1 and Σ_2 are globally exponentially stable. This result is stated in the following lemma and is based on a previous result introduced by Panteley and Loría (1998).

Lemma 2.21. *(Aneke, 2003, Lemma 3.6.2). If in addition to the assumptions in Theorem 2.20 both systems Σ_1 and Σ_2 are globally exponentially stable (GES), then the cascaded system (2.24) is globally $\mathcal{K}$-exponentially stable.*

The following system is a particular case of the cascaded system (2.24):

$$\dot{x}_1 = A(t)x_1 + g(t, x_1, x_2)x_2, \tag{2.28a}$$

$$\dot{x}_2 = B(t)x_2, \tag{2.28b}$$

where $x_1 \in \mathbb{R}^n$ and $x_2 \in \mathbb{R}^m$, $A(t) \in \mathbb{R}^{n\times n}$ and $B(t) \in \mathbb{R}^{m\times m}$ are continuously differentiable and bounded. In this case, we have that

$$\Sigma_{1_{\mathrm{LTV}}} : \dot{x}_1 = A(t)x_1, \qquad \Sigma_{2_{\mathrm{LTV}}} : \dot{x}_2 = B(t)x_2,$$

resulting in the LTV systems $\Sigma_{1_{\mathrm{LTV}}}$ and $\Sigma_{2_{\mathrm{LTV}}}$ interconnected via a nonlinear coupling $g(t, x_1, x_2)$. The following corollary regarding the stability of the cascaded system (2.28) may be formulated along the same lines of Theorem 2.20 and Lemma 2.21.

Corollary 2.22. *The cascaded system (2.28) is globally $\mathcal{K}$-exponentially stable provided systems $\Sigma_{1_{\mathrm{LTV}}}$ and $\Sigma_{2_{\mathrm{LTV}}}$ are globally exponentially stable and the nonlinear interconnection $g(t, x_1, x_2)$ satisfies the second condition in Theorem 2.20.*

Proof. Recall the three conditions in Theorem 2.20 and note that if systems $\Sigma_{1_{\mathrm{LTV}}}$ and $\Sigma_{2_{\mathrm{LTV}}}$ are GES, they are necessarily GUAS due to Theorem 2.14. This means that the first parts of conditions 1 and 3 in Theorem 2.20 are satisfied. Additionally, from converse Lyapunov theory (see Theorem 2.15), the existence of a (time-varying) quadratic Lyapunov function $V(t, x_1)$ which satisfies (2.25) is guaranteed. Moreover, since system $\Sigma_{2_{\mathrm{LTV}}}$ is GES it satisfies condition (2.27). Since the requirement on the interconnection $g(t, x_1, x_2)$ is the same, all conditions in Theorem 2.20 are satisfied. Finally, due to the fact that systems $\Sigma_{1_{\mathrm{LTV}}}$ and $\Sigma_{2_{\mathrm{LTV}}}$ are GES, from Lemma 2.21 it follows that the cascaded system (2.28) is globally $\mathcal{K}$-exponentially stable. □

2.5 Stability of Retarded Functional Differential Equations

This section introduces some results concerning the stability of retarded functional differential equations. These results are required later on in order to study the stability of the remote tracking control and remote motion coordination strategies presented in Chapter 4, Chapter 5, and Chapter 6.

Consider the following retarded functional differential equation:

$$\dot{x}(t) = f(t, x_t), \tag{2.29}$$

where $f : \mathcal{D} \to \mathbb{R}^n$, $\mathcal{D} \subseteq (\mathbb{R} \times \mathcal{C}(n))$ and $\mathcal{C}(n) = \mathcal{C}([-\tau, 0], \mathbb{R}^n)$ is the (Banach) space of continuous functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$. This vector space is equipped with the norm $\|\cdot\|_c$, denoted as the continuous norm, which is defined for a function $\varphi \in \mathcal{C}([a, b], \mathbb{R}^n)$ as follows:

$$\|\varphi\|_c = \max_{a\le s\le b} \|\varphi(s)\|, \tag{2.30}$$

where $\|\cdot\|$ denotes any vector norm. In (2.29), $t \in \mathbb{R}$, $x(t) \in \mathbb{R}^n$, and $x_t \in \mathcal{C}(n)$ is defined as $x_t(s) = x(t+s)$, for $-\tau \leq s \leq 0$.

In this thesis we adopt notational conventions from Hale and Verduyn-Lunel (1993) and Sedova (2008a). We have that the function x is a solution of (2.29) on $[\sigma-\tau, \sigma+\beta)$ if there are $\sigma \in \mathbb{R}$ and $\beta > 0$ such that $x \in \mathcal{C}([\sigma-\tau, \sigma+\beta], \mathbb{R}^n)$, $(t, x_t) \in \mathcal{D}$, and $x(t)$ satisfies (2.29) for $t \in [\sigma, \sigma+\beta)$. Moreover, for given $\sigma \in \mathbb{R}$ and $\phi \in \mathcal{C}(n)$, $x(t;\sigma,\phi)$ is a solution of (2.29) with initial value ϕ at σ, or simply a solution through (σ, ϕ), if there is a $\beta > 0$ such that $x(t;\sigma,\phi)$ is a solution of (2.29) on $[\sigma-\tau, \sigma+\beta)$ and $x_\sigma(\sigma,\phi) = \phi$ (herein, the notation $x_t(\sigma,\phi)$ is used to denote the solution segment x_t starting at time $t-\tau$ and ending at time t corresponding to the initial value ϕ at σ). In particular, when $\sigma = t_0$, the solution of (2.29) is given by $x(t;t_0,\phi)$, and $x_{t_0} = \phi$ denotes the initial condition of the system. In addition, for any $\varphi \in \mathcal{C}(n)$, that is, any element of the Banach space, $\varphi(0) \in \mathbb{R}^n$ and $\varphi(-\tau) \in \mathbb{R}^n$ denote φ at the end and beginning of the interval $[-\tau, 0]$, respectively, and, generically, $\varphi(s) \in \mathbb{R}^n$ denotes φ at $s \in [-\tau, 0]$.

The functional f in (2.29) is assumed to be continuous on each set of the form $\mathbb{R}^+ \times \mathcal{C}_\rho(n)$, where $\rho > 0$, $\mathcal{C}_\rho(n) = \{\varphi \in \mathcal{C}(n) : \|\varphi\|_c < \rho\}$, bounded by some constant $M(\rho)$, and Lipschitz with some constant $L(\rho)$. As explained in Sedova (2008a), these assumptions ensure the existence and uniqueness of the solution $x(t;t_0,\phi)$ to system (2.29). We also assume that $f(t,0) = 0$, for all $t \in \mathbb{R}^+$, such that system (2.29) has a zero equilibrium state.

If $V : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is a continuous function and $x(s;t,\varphi)$ is the solution of (2.29) through (t,φ), with $\varphi \in \mathcal{C}(n)$, then the derivative of V along the solutions of (2.29) is defined as

$$\dot{V}(t,\varphi) = \lim_{h\to 0^+} \sup \frac{1}{h}[V(t+h, x(t+h;t,\varphi)) - V(t,\varphi(0))], \tag{2.31}$$

or equivalently,

$$\dot{V}(t,\varphi) = \frac{\partial V}{\partial t}(t,\varphi(0)) + \frac{\partial V}{\partial x}(t,\varphi(0))f(t,\varphi). \tag{2.32}$$

The following theorems, known as the Lyapunov-Razumikhin theorems, establish sufficient conditions to determine the stability of the origin of the retarded functional differential equation (2.29) in terms of the rate of change of a certain function.

Theorem 2.23. *(Hale and Verduyn-Lunel, 1993, Theorem 4.1). Suppose* $f : \mathbb{R} \times \mathcal{C} \to \mathbb{R}^n$ *takes* $\mathbb{R}\times$ *(bounded sets of* $\mathcal{C}$*) into bounded sets of* $\mathbb{R}^n$ *and consider the retarded functional differential equation (RFDE) (2.29). Suppose*

$u, v, w : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ are continuous, nondecreasing functions, $u(r)$ and $v(r)$ are positive for $r > 0$, $u(0) = v(0) = 0$, and v is strictly increasing. If there is a continuous function $V : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ such that

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|), \qquad t \in \mathbb{R}, x \in \mathbb{R}^n, \tag{2.33}$$

and the derivative of V along the solutions of (2.29) satisfies

$$\dot{V}(t, \varphi) \leq -w(\|\varphi(0)\|), \tag{2.34}$$

if

$$V(t + s, \varphi(s)) \leq V(t, \varphi(0)), \tag{2.35}$$

for $s \in [-\tau, 0]$, then the solution $x = 0$ of system (2.29) is uniformly stable.

Theorem 2.24. *(Hale and Verduyn-Lunel, 1993, Theorem 4.2). Suppose all conditions of Theorem 2.24 are satisfied and in addition $w(r) > 0$ if $r > 0$. If there is a continuous nondecreasing function $p(r) > r$ for $r > 0$ such that the conditions (2.34)-(2.35) are strengthened to*

$$\dot{V}(t, \varphi) \leq -w(\|\varphi(0)\|), \tag{2.36}$$

if

$$V(t + s, \varphi(s)) \leq p(V(t, \varphi(0))), \tag{2.37}$$

for $s \in [-\tau, 0]$, then the solution $x = 0$ of system (2.29) is uniformly asymptotically stable. If $u(s) \rightarrow \infty$ as $s \rightarrow \infty$, then the solution $x = 0$ is also a global attractor for the system (2.29).

As explained in Hale and Verduyn-Lunel (1993), the Lyapunov-Razumikhin approach exploits the idea that, if the solution of a RFDE begins in a ball and is to leave this ball at time t, it is only necessary to consider initial data satisfying $\|x(t+s)\| \leq \|x(t)\|$, for all $s \in [-\tau, 0]$, to study its stability. In addition, Münz et al. (2008) highlight the importance of noting that the Lyapunov-Razumikhin function in the theorem is not necessarily non-increasing along the system trajectories, meaning that it may actually increase within the delay interval.

A particular type of retarded functional differential equation of special interest in this work is the following nonlinear cascaded system[1]:

$$\dot{x}(t) = f_x(t, x_t) + g_{xy}(t, x_t, y_t), \tag{2.38a}$$

$$\dot{y}(t) = f_y(t, y_t), \tag{2.38b}$$

[1] Throughout this thesis the expression f_a (or more explicitly $f_a(t, a_t)$) will be used to denote the functional $f_a : \mathcal{D} \rightarrow \mathbb{R}^m$ in a retarded functional differential equation. This notation is not to be confused with a different usage of f_a often found in the literature, in which f_a is used as short notation for $\frac{\partial f}{\partial a}$.

where $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, $x_t \in \mathcal{C}(m)$, and $y_t \in \mathcal{C}(n)$. We assume that $f_y(t,0) = 0$, $f_x(t,0) = g(t,\varphi_x,0) = 0$ for all $t \in \mathbb{R}^+$ and $\varphi_x \in \mathcal{C}(m)$, such that the system has the zero equilibrium state. In the absence of the coupling term $g(t, x_t, y_t)$, system (2.38a) takes the following form:

$$x(t) = f_x(t, x_t), \tag{2.39}$$

denoted hereinafter as the x-dynamics without coupling.

The following theorems formulate sufficient conditions to establish the local and global uniform asymptotic stability of the nonlinear delayed cascaded system (2.38).

Theorem 2.25. *(Sedova, 2008a, Theorem 2). Consider the nonlinear delayed cascaded system (2.38) and let both the zero solution of the x-dynamics without coupling (2.39) and the y-dynamics in (2.38b) be locally uniformly asymptotically stable (LUAS). Then, the solution $[x^T\, y^T]^T = 0$ of system (2.38) is locally uniformly asymptotically stable (LUAS).*

Theorem 2.26. *(Sedova, 2008b, Theorem 4). Assume that for system (2.39) (the x-dynamics without coupling) there exists a function $V(t,x)$ of the Lyapunov-Razumikhin type which satisfies the following assumptions:*

1. *$V(t,x)$ is continuously differentiable, positive definite, and has the infinitesimal upper limit with $\|x\| \to 0$ and the infinitely great lower limit with $\|x\| \to \infty$;*
2. *due to system (2.39), the derivative of the function V (it represents the functional $\dot{V}(t,\varphi_x) = \frac{\partial V}{\partial t}(t,\varphi_x(0)) + \frac{\partial V}{\partial x}(t,\varphi_x(0)) f_x(t,\varphi_x)$ satisfies the estimate $\dot{V}(t,\varphi_x) \le 0$ for all $\varphi_x \in \Omega_t(V) = \{\varphi \in \mathcal{C}(m) : \max_{-\tau\le s\le 0} V(t+s,\varphi_x(s)) \le V(t,\varphi_x(0))\}$;*
3. *$|\dot{V}(t,\varphi_x)| \ge U(t,\varphi_x)$ for all $(t,\varphi_x) \in \mathbb{R}^+ \times \mathcal{C}(n)$, the functional $U(t,\varphi_x)$ is uniformly continuous and bounded in each set in the form $\mathbb{R}^+ \times K$ with a compact set $K \subset \mathcal{C}$;*
4. *the intersection of the sets $V^{-1}_{\max}(\infty, c) := \{\varphi_x \in \mathcal{C}(m) | \exists \varphi_n \to \varphi_x, t_n \to +\infty : \lim_{n\to\infty} \max_{-\tau\le s\le 0} V(t_n+s,\varphi_n(s)) = \lim_{n\to\infty} V(t_n,\varphi_n(0)) = c\}$ and $U^{-1}(\infty,0)$ is empty with $c \ne 0$;*

and that additionally the following conditions are satisfied:

5. *for all $x \in \mathbb{R}^n$ such that $\|x\| > \eta$, the inequality $\left\|\frac{\partial V}{\partial x}\right\| \cdot \|x\| \le c_1 V(t,x)$ holds, and, for all $x \in \mathbb{R}^n$ such that $\|x\| \le \eta$, the estimate $\left\|\frac{\partial V}{\partial x}\right\| \le c$ is valid with certain constants $\eta, c_1, c > 0$;*

6. *for* $\varphi_y \in \mathcal{C}(n)$ *and some continuous functions* $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$ *the functional* g *admits the following estimate:*

$$\|g(t, \varphi_x, \varphi_y)\| \leq (\alpha_1(\|\varphi_y\|_c) + \alpha_2(\|\varphi_y\|_c)\|\varphi_x(0)\|)\|\varphi_y\|_c;$$

7. *solutions of system (2.38b) admit the estimate* $\|y(t; t_0, \phi_y)\| \leq k_1\|\phi_y\|_c e^{-k_2 t}$ *with certain constants* $k_1, k_2 > 0$.

Then the zero solution of system (2.38) is globally uniformly asymptotically stable.

As explained in Sedova (2008b), several remarks are in place regarding Theorem 2.26. First, note that the second assumption on $V(t, x)$ is, in fact, the Lyapunov-Razumikhin condition for asymptotic stability. Although this condition usually requires the negative definiteness of the derivative of $V(t, x)$, the requirement in the assumption has been weakened in order to ease the construction of appropriate Lyapunov-Razumikhin functions. Nonetheless, in order to still guarantee asymptotic stability, additional conditions have been posed. These requirements appear in the third and fourth assumptions, and are satisfied by default by functions with a negative definite derivative. Second, from Remarks 2 and 3 in Sedova (2008b), we have that if the function $V(t, x)$ is quadratic, the bounds on its growth posed in the fifth condition are automatically satisfied and $\|\varphi_x(0)\|$ can be replaced by $\|\varphi_x\|_c$ in the estimate for the functional g in the sixth assumption. It is worth pointing out that the use of vector norms throughout the theorem should be consistent. This means that the vector norm used within the continuous norm $\|\cdot\|_c$ should be consistent with the rest of the vector norms used in the theorem (recall that the continuous norm has already been defined in (2.30)).

There definitely is a resemblance between Theorem 2.25 for nonlinear delayed cascaded systems and Theorem 2.20 for nonlinear delay-free cascaded systems. For instance, besides requiring some form of stability for the unperturbed dynamics of the first system, both of them pose growth conditions on the associated Lyapunov (-Razumikhin) function. In addition, the requirement for global uniform asymptotic stability of the second system plus the condition that its output is integrable in Theorem 2.20 corresponds to the global exponential stability requirement for the same system in Theorem 2.25. Finally, both theorems pose similar growth conditions on the interconnection, so that the solutions of the overall system remain bounded.

2.6 Tracking Control of Mobile Robots

A schematic representation of the unicycle robot considered in this work is shown in Figure 2.1. The robot is composed of two independently actuated wheels and

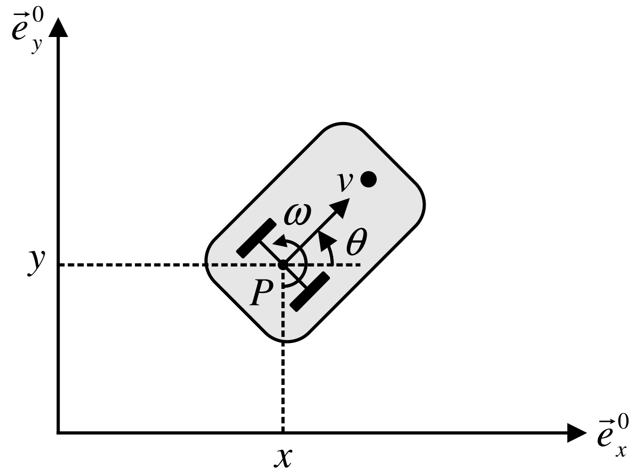


Figure 2.1 . Schematic representation of a unicycle-type mobile robot.

one or several passively orientable wheels (caster wheels). The position at time t of the point P (located at mid-distance from the driving wheels of the robot) with respect to the global coordinate frame $\underline{\vec{e}}^{\,0} = [\vec{e}_x^{\,0}\ \vec{e}_y^{\,0}]^T$ is denoted by the coordinates $(x(t), y(t))$, whereas the angle at time t between the heading direction of the robot robot and the $\vec{e}_x^{\,0}$-axis of the global coordinate frame is denoted by $\theta(t)$ (see Figure 2.1). The posture kinematic model of the unicycle robot is given by the following differential equations:

$$\dot{x}(t) = v(t)\cos\theta(t), \tag{2.40a}$$

$$\dot{y}(t) = v(t)\sin\theta(t), \tag{2.40b}$$

$$\dot{\theta}(t) = \omega(t), \tag{2.40c}$$

in which $v(t)$ and $\omega(t)$ constitute the translational and rotational velocities of the robot, respectively, and are regarded as its control inputs. The state of the system is denoted by $q(t) = [x(t)\, y(t)\, \theta(t)]^T$.

In the tracking control problem, the control objective of the unicycle robot is to track the reference position $(x_r(t), y_r(t))$ with an orientation $\theta_r(t)$. Feasible reference orientation and Cartesian velocities for the robot must satisfy the non-holonomic constraint which appears in the posture kinematic model of the unicycle, given by the expression $-\dot{x}_r(t)\sin\theta_r(t) + \dot{y}_r(t)\cos\theta_r(t) = 0$. Considering this constraint, the reference orientation and translational and rotational velocities of the robot are defined as follows:

$$\theta_r(t) = \arctan\left(\frac{\dot{y}_r(t)}{\dot{x}_r(t)}\right) + k\pi, \qquad k = 0, 1, \tag{2.41a}$$

$$v_r(t) = \sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)}, \tag{2.41b}$$

$$\omega_r(t) = \frac{\dot{x}_r(t)\ddot{y}_r(t) - \ddot{x}_r(t)\dot{y}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)} = \dot{\theta}_r(t), \tag{2.41c}$$

where the arctangent function in (2.41a) must consider the sign of each argument in the ratio $\dot{y}_r(t)/\dot{x}_r(t)$ in order to accurately determine to which quadrant the resulting angle belongs[2]. The two possible choices for k in (2.41a) allow for the same trajectory to be followed either forwards ($k=0$) or backwards ($k=1$).

As stated by Siciliano et al. (2009), in order for the tracking problem to be soluble, it is necessary that the reference position is admissible, that is, feasible, for the posture kinematic model of the unicycle. This means that $\theta_r(t)$ should satisfy (2.41a) and that there must exist reference translational and rotational velocities $v_r(t)$ and $\omega_r(t)$ which satisfy (2.41b) and (2.41c), respectively. Hence, the reference orientation and translational and rotational velocities satisfy the equations

$$\dot{x}_r(t) = v_r(t)\cos\theta_r(t), \tag{2.42a}$$

$$\dot{y}_r(t) = v_r(t)\sin\theta_r(t), \tag{2.42b}$$

$$\dot{\theta}_r(t) = \omega_r(t), \tag{2.42c}$$

with an associated reference state trajectory $q_r(t)=[x_r(t)\; y_r(t)\; \theta_r(t)]^T$.

In order to solve the tracking control problem for the unicycle, a set of tracking error coordinates which relate the reference trajectory $q_r(t)$ and the state of the unicycle $q(t)$ was first proposed by Kanayama et al. (1990). These error coordinates, shown in Figure 2.2, have an associated error state $q_e(t)=[x_e(t)\; y_e(t)\; \theta_e(t)]^T$ and are given by

$$\begin{bmatrix} x_e(t) \\ y_e(t) \\ \theta_e(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix}. \tag{2.43}$$

Differentiating the error coordinates (2.43) and exploiting the kinematics of the unicycle (2.40) and (2.42) yields the following open-loop error dynamics:

$$\dot{x}_e(t) = \omega(t)y_e(t) + v_r(t)\cos\theta_e(t) - v(t), \tag{2.44a}$$

$$\dot{y}_e(t) = -\omega(t)x_e(t) + v_r(t)\sin\theta_e(t), \tag{2.44b}$$

$$\dot{\theta}_e(t) = \omega_r(t) - \omega(t). \tag{2.44c}$$

[2]This variant of the classical arctangent function is known as the atan2 function in some programming languages, such as MATLAB. It is worth noting that all the arctangent functions which appear in this thesis have been implemented considering this variant.

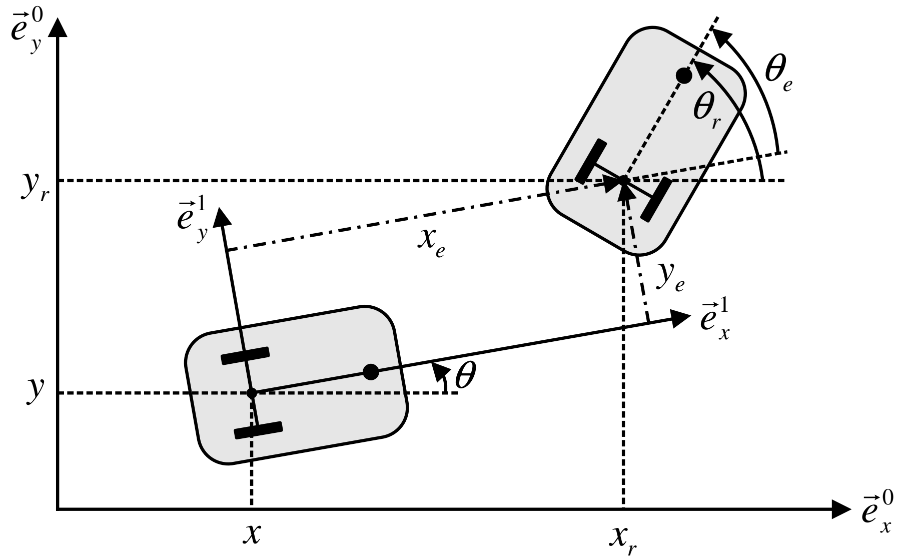


Figure 2.2 . Unicycle robot coordinates, reference trajectory coordinates, and tracking error coordinates.

The tracking control problem consists in finding an appropriate stabilizing control law $u(t)=[v(t)\,\omega(t)]^T$ such that the tracking error $q_e(t)$ converges to zero. Consider for the control input the following tracking controller first proposed in Panteley et al. (1998) and further studied in Lefeber et al. (2001):

$$v(t) = v_r(t) + c_x x_e(t) - c_y w_r(t) y_e(t), \qquad c_x > 0, \qquad c_y > -1, \tag{2.45a}$$

$$\omega(t) = \omega_r(t) + c_\theta \theta_e(t), \qquad c_\theta > 0, \tag{2.45b}$$

in which c_x, c_y, and c_θ constitute the feedback tracking gains and $v_r(t)$ and $\omega_r(t)$ the feedforward reference velocities, already defined in (2.41b) and (2.41c), respectively.

The error dynamics (2.44) together with the tracking control law (2.45) result in the following closed-loop error dynamics:

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{y}_e(t) \end{bmatrix} = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ y_e(t) \end{bmatrix} + \begin{bmatrix} c_\theta y_e(t)\theta_e(t) - v_r(t)(1-\cos\theta_e(t)) \\ -c_\theta x_e(t)\theta_e(t) + v_r(t)\sin\theta_e(t) \end{bmatrix}, \tag{2.46a}$$

$$\dot{\theta}_e(t) = -c_\theta \theta_e(t). \tag{2.46b}$$

Considering the following state definitions: $\xi_1(t) := [x_e(t)\,y_e(t)]^T$ and $\xi_2(t) := \theta_e(t)$, the closed-loop error dynamics (2.46) may be rewritten in the following form:

$$\dot{\xi}_1 = A(t)\xi_1 + g(t, \xi_1, \xi_2), \tag{2.47a}$$
$$\dot{\xi}_2 = -c_\theta \xi_2 \tag{2.47b}$$

with

$$A(t) = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix},$$
$$g(t, \xi_1, \xi_2) = \begin{bmatrix} c_\theta y_e(t)\theta_e(t) - v_r(t)(1-\cos\theta_e(t)) \\ -c_\theta x_e(t)\theta_e(t) + v_r(t)\sin\theta_e(t) \end{bmatrix}.$$

Note that the rearranged closed-loop error dynamics (2.47) form a cascaded system which is a particular case of the time-varying cascaded system (2.28). Remarkably, all the closed-loop error dynamics in this thesis which are related to the unicycle robot or a group of unicycles have a similar cascaded structure. As a result, the approach to study the stability of these error dynamics is, to a certain extent, similar in all cases.

The previous remark constitutes the main reason for including the following proposition, which formulates sufficient conditions under which $q_e(t)=0$ is a globally $\mathcal{K}$-exponentially stable equilibrium point of the closed-loop tracking error dynamics (2.46).

Proposition 2.27. *(Lefeber, 2000; Lefeber et al., 2001). Consider the posture kinematic model of a unicycle robot as given by (2.40). The reference Cartesian position of the robot is given by $(x_r(t), y_r(t))$, whereas its reference orientation $\theta_r(t)$ is given by (2.41a). Additionally, consider the tracking controller (2.45), with the feedforward terms $v_r(t)$ and $\omega_r(t)$ defined as in (2.41b) and (2.41c), respectively, and the feedback part based on the error $q_e(t)$ between the reference trajectory $q_r(t)$ and the state $q(t)$, as given in (2.43). If the following conditions are satisfied:*

- *the reference translational velocity $v_r(t) \neq 0$, $\forall t$, is bounded;*
- *the reference rotational velocity $\omega_r(t)$ is persistently exciting (PE);*
- *the tracking gains satisfy $c_x, c_\theta > 0, c_y > -1$,*

then, $q_e(t) = 0$ is a globally $\mathcal{K}$-exponentially stable equilibrium point of the closed-loop error dynamics (2.46).

The next chapter introduces the multi-robot platform used to experimentally validate the control strategies proposed in this thesis.

3

Experimental Platform

Abstract . This chapter introduces the experimental multi-robot platform used to implement and validate the control strategies developed in this thesis.

3.1 Experimental Platform Description

In this chapter, we introduce the multi-robot platform used in this thesis to experimentally validate the control strategies proposed in Chapter 4 and Chapter 5. There are two experimental setups available, one located at the Eindhoven University of Technology (TU/e) in the Netherlands and the other one at Tokyo Metropolitan University (TMU) in Japan. These setups may be used in a stand-alone fashion or, jointly, to conduct remote coordination experiments. The chapter begins with a brief description of the experimental setups and the elements that compose them and continues with an overview of the communication link set up between the setups so that they exchange information.

3.1.1 General Description

A multi-robot experimental platform was originally designed at TU/e to evaluate different formation control strategies for a group of unicycle-type mobile robots. All the necessary details regarding the design specifications, component choice, implementation, and calibration may be found in van den Broek (2008). Inspired by this design, a similar setup was subsequently implemented at TMU. Besides being used to evaluate the control strategies presented in this thesis, the setups

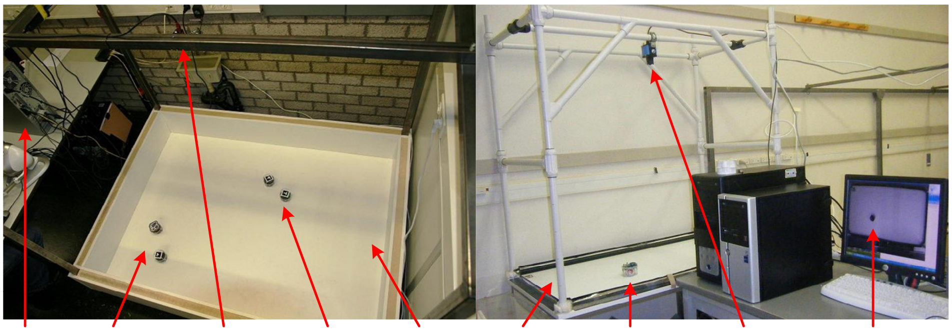


Figure 3.1 . Experimental setup at TU/e (left) and at TMU (right).

have proven to be very useful in testing low-level and high-level motion coordination strategies and collision avoidance, task allocation, and visual servoing algorithms, among others (see for example Adinandra et al., 2010; Kostić et al., 2009, 2010a,b).

Overview

Both experimental setups are shown in Figure 3.1. The setups have a driving arena in which the mobile robots execute their task. An overhead camera connected to a computer captures the location of the robots at regular intervals. This information is processed in the computer and the necessary (control) inputs for each robot are generated. Subsequently, these control inputs are transmitted to the corresponding robots in order to close the control loop.

Mobile Robot Platform

The mobile robot platform selected is the *e-puck* robot, shown in Figure 3.2. The e-puck is a differential-drive unicycle-type mobile robot developed at EPFL, Switzerland (refer to Mondada et al. (2009) for additional details). The wheels of an e-puck are driven by stepper motors which receive velocity control commands over a BlueTooth connection. All the data processing required to execute these commands is carried out in the robot's onboard processor. Establishing a BlueTooth connection between the e-puck and a computer allows the robot to be controlled using a variety of programming languages, whereas its processing capabilities open up the possibility to implement semi-decentralized and decentralized control algorithms.

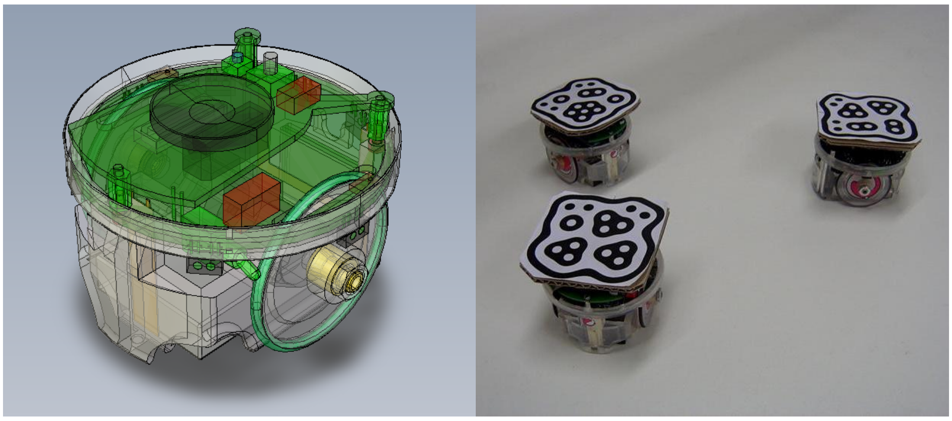


Figure 3.2 . CAD rendering of the e-puck mobile robot (left) and a team of e-pucks equipped with reacTIVision fiducial markers (right).

Vision System

The position and orientation of the robots is measured by means of static scene analysis by using an industrial FireWire camera. The camera in the setup at TU/e is an AVT Guppy F-080b (shown in the left-hand side of Figure 3.3), whereas the one used at TMU is an Imaging Source DMK-31BF03. Both cameras are equipped with a Computar T0412FICS-3 4 mm lens and, based on the height at which they are placed, the resulting driving arena is of 175×128 cm at TU/e and 100×50 cm at TMU.

Each robot is fitted with a unique fiducial marker of 7×7 cm, such as the one shown in the right-hand side of Figure 3.3. The markers are read by reacTIVision, a standalone application which is capable of determining the position and orientation of a large quantity of these markers at the same time (see Kaltenbrunner and Bencina, 2007, for additional details). In the setup at TU/e the data generated by reacTIVision is calibrated by means of a coordinate transformation, whereas at TMU it is calibrated by gridding the arena and determining the position of the robot with respect to the origin of the grid.

Software

The data stream generated by reacTIVision is composed of messages formatted with the TUIO protocol (see Kaltenbrunner et al. (2005) for additional details). The protocol uses UDP (User Datagram Protocol) port 3333 to relay messages to a specific client application. For this reason, incoming data from reacTIVision can

Figure 3.3 . Overhead FireWire camera used at TU/e (left) and a close-up view of a reacTIVision fiducial marker (right).

be managed by a number of programming languages, such as C++, Python, and Java, running on any operating system that supports them.

All the control algorithms presented in this thesis have been implemented in Python. The main reason behind this choice is the simplicity with which a network connection can be set up and configured in this language, which means a network connection between both setups can be easily established.

Sampling Rate and Bandwidth

Using the vision system results in a sampling rate of approximately 25Hz, which constrains the bandwidth of the overall setup. Nonetheless, this choice still allows the correct polling of the measurement data while ensuring an accurate control of the mobile robots.

3.1.2 Data Exchange over the Internet

Exchanging data between the experimental setups in the Netherlands and Japan is necessary in order to implement the remote control and motion coordination strategies proposed in this thesis. Because of its widespread availability and low cost, the Internet is chosen as the communication channel for this exchange.

Network Configuration

In order to establish a connection which guarantees a reliable and secure data exchange, the setup at TU/e accesses the TMU network via a Virtual Private

Network (VPN). Some of the advantages of using a VPN is that it offers secure access to the network without being a dedicated communication channel and that it bypasses the difficulties posed by closed ports and network security measures.

Socket Configuration

The data exchanged between the setups is transmitted as soon as it becomes available by means of non-blocking Transmission Control Protocol (TCP) sockets running on the Internet Protocol (IP). The low bandwidth of the system allows the use of the TCP protocol, which guarantees reliable data delivery. The correct serialization and deserialization of the data stream as required by Python is ensured by fixing the size of each transmitted packet and setting accordingly the reading buffer on the receiving end. Since data is exchanged bidirectionally in some of the control strategies, different processing threads are set up for transmitting and receiving data.

Although User Datagram Protocol (UDP) sockets allow data exchange at higher rates, they cannot guarantee reliable and integral data delivery. Because of this, the use of the more reliable TCP protocol is preferred since sampling the system faster has not been required so far. The reasoning behind this is that we assume that the unicycles move with low accelerations, since the controllers which will be designed for them are based on their posture kinematic model and their reference velocities are relatively slowly varying signals as well. Nonetheless, the use of UDP sockets clearly constitutes an option for data communication, especially when considering control systems requiring higher sampling rates, such as a robotic manipulator.

Data Payload

When implementing the control strategies presented in this thesis, the number of variables exchanged amounts to only a few per robot. For instance, the robots may broadcast their position and orientation or receive their control inputs or reference trajectories from a remote location. Since this information is bundled in each setup and then broadcast to the other setup, the demands placed on the communication network does not increase significantly as the number of robots in the setups grows.

Recall, however, that data is transmitted as soon as it becomes available. This means that the transmission rate is the same as the sampling rate of the setups. This might be unnecessarily taxing on the communication channel and, considering the current problem setting, it is highly probable that more efficient mechanisms to exchange information exist. Nevertheless, investigating such mechanisms lies beyond the scope of this work considering that the current approach has proven to be successful in practice.

Setup Synchronization

An additional aspect to consider is synchronizing the internal clocks of the computers at each setup. This is important in some of the experiments because having a global time scale allows a more accurate characterization of the performance of the system. Nonetheless, synchronizing the computer clocks with millisecond accuracy is not a trivial problem, especially when the computers are located so far apart (around 9000 km).

One possibility is to use the clock of Global Positioning System (GPS) devices as the reference for the computer clocks. Although this approach constitutes one of the most accurate options available, cost and implementation issues render this idea unfeasible in the current setting. Another option is using the Network Time Protocol (NTP) to set the computer clocks. In this case, the accuracy depends on several factors such as the operating system being used and the distance between the computers and the NTP server which provides the time reference. In Windows, the Windows Time Service allows to synchronize the computer clock with an NTP server. However, the Windows Time Service cannot maintain the system time more accurately than about a 1-2 second range (Microsoft, 2010). Ultimately, because of its simplicity and null cost, the setups (roughly) synchronize their clocks with the same NTP server using the Windows Time Service, resulting in (approximately) the aforementioned synchronization accuracy. Although not ideal, this does not represent a major inconvenience since, as will be shown later on, the time-delay induced by the communication network can be characterized quite accurately.

3.1.3 Round-Trip Delay Time

The round-trip delay between The Netherlands and Japan has been measured continuously during a 24 hour period in Botden (2011). The measurements were conducted between the computer of the experimental setup at TU/e and a (randomly chosen) server in Japan.

The *hrPing* (high resolution ping) command was used to measure the round-trip delay during the 24 hour interval. Throughout this time, 1200 *hrPing* requests were sent to the server in Japan during a two-minute interval. These two-minute bursts of *hrPing* requests were repeated every three minutes, resulting in a total of 288 bursts during the 24 hour period. The main reason for not measuring directly the round-trip delay with the computer of the setup at TMU, but rather with a

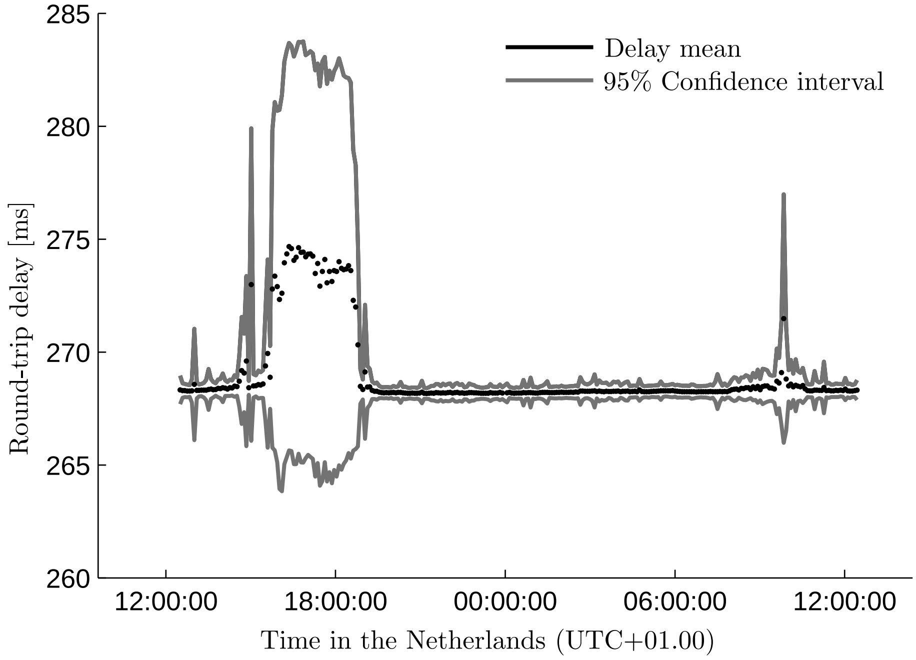


Figure 3.4 . Round-trip delay between The Netherlands and Japan measured during 24 hours.

randomly chosen server in Japan, is that the network at TMU does not accept ping requests.

The mean value of the round-trip delay for each burst is shown in Figure 3.4, together with the 95% confidence interval of the measurement. The overall mean value of the round-trip delay is around 268 ms. In the figure, the repercussions of network traffic variations throughout the day are clearly noticeable in the measurements.

In comparison, the round-trip delay between the computers of both setups has also been measured using time stamping. In this case, the measurements were conducted during different times of the day, for amounts of time ranging from 2 min to 10 min, and for a total time of around 60 min. In accordance with the measurements obtained using the high resolution ping, the mean delay value when using time stamping also resulted in approximately 268 ms (267.4917 ms from TU/e→TMU and 269.5307 ms from TMU→TU/e).

In conclusion, the proposed network and socket configuration allows the setups to reliably exchange data between each other with a delay which is fairly constant.

The next chapter addresses the problem of a mobile robot being controlled over a delay-inducing communication network.

4

Remote Tracking Control of a Mobile Robot

Abstract. In this chapter we address the tracking control problem for a unicycle-type mobile robot which is controlled over a two-channel, delay-inducing communication network. A control strategy capable of compensating for the negative effects of the time-delay is proposed and the local and global asymptotic stability of the closed-loop system is guaranteed up to a maximum admissible delay. The applicability of the proposed control strategy is demonstrated by means of experiments carried out between multi-robot platforms located in Eindhoven, The Netherlands, and Tokyo, Japan.

4.1 Introduction

The study of robotic systems controlled over a network has become important as a way to support the design of robotic systems that can perform remote, dangerous or distributed tasks. A schematic representation of the problem addressed in this chapter is depicted in Figure 4.1. In this case, the controller and the mobile robot are linked via a delay-inducing communication channel, which possibly compromises the performance and stability of the closed-loop system. As explained in Chapter 1, several techniques have been proposed so far in the context of teleoperated systems and Networked Control Systems (NCSs) to cope with network-induced delays; for example, the use of the scattering transformation, a passivity-based approach, wave variables formulation, queuing methodologies, delay compensation techniques, and robust control design, to name a few. A detailed description of such techniques may be found in some of the references

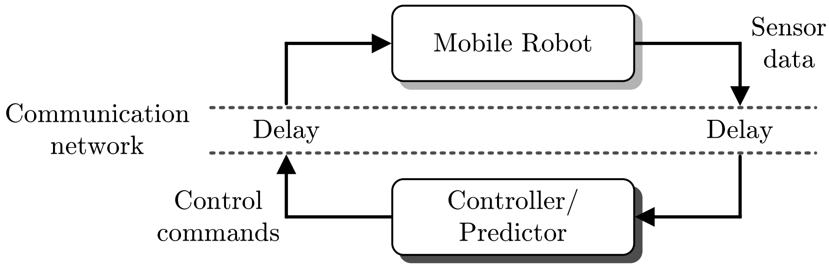


Figure 4.1 . Schematic representation of a mobile robot controlled over a delay-inducing communication network.

given in Chapter 1, whereas an overview of these techniques and many others can be found, for example, in Hokayem and Spong (2006), Heemels and van de Wouw (2011), and Tipsuwan and Chow (2003).

In this chapter, a control strategy which allows the remote tracking control of a unicycle-type mobile robot is proposed. The control scheme consists of a state predictor in combination with a tracking controller, which together compensate for the negative effects of the network-induced delay. The state predictor is inspired on the synchronization-based predictor introduced in Oguchi and Nijmeijer (2005a) and Oguchi and Nijmeijer (2005b), and the tracking control law is the one studied in Section 2.6. In Kojima et al. (2010), a similar state predictor is applied to a mobile robot subject to a communication delay, and sufficient conditions for the stability of the predictor error dynamics are derived. This thesis follows the alternative approach taken in Alvarez-Aguirre et al. (2010b) and Alvarez-Aguirre et al. (2011), and studies the stability of the entire closed-loop system, which consists of the mobile robot, the tracking controller, and the state predictor.

As explained before, we consider the case in which sensor data and control commands are communicated over a network inducing a delay. In the NCSs literature, researchers have been focusing on the study of robust stability in the face of uncertain, time-varying delays, while most of the work focuses on stabilization problems (often for linear systems). In the current work, we focus on the more complex control problems of tracking time-varying trajectories (this chapter) and achieving motion coordination (Chapter 5) for a class of nonlinear robotic systems. Motivated by the measurements of the network delays induced by the Internet link employed in this work (see Chapter 3), we consider the case of constant delays. Moreover, since these measurements also show that the magnitude of the network-induced delay ($\approx 270\,\text{ms}$) is much larger than the sampling period of the experimental setups introduced in Chapter 3 ($40\,\text{ms}$), we do not consider sample-and-hold effects in the system but rather carry out our complete analysis in continuous-time.

The contribution of this chapter is twofold. First, we propose a remote control strategy consisting of a tracking controller and a predictor which guarantees the local or global stability of the resulting closed-loop system for delays smaller than a certain upper bound. Second, the control strategy is experimentally validated using the multi-robot platform introduced in Chapter 3 using the Internet as the communication channel.

The remainder of this chapter is organized in the following way. The remote tracking control strategy for a mobile robot is introduced in Section 4.2, together with some ideas on how to overcome the main practical implementation issues apparent in the experimental study. The local and global stability of the resulting closed-loop error dynamics is studied in Section 4.3. Illustrative simulation and experimental results are included in Section 4.4. The chapter concludes with a discussion in Section 4.5.

4.2 Predictor-Based Remote Tracking Control of a Mobile Robot

In this section, we consider a mobile robot controlled over a network which induces time-delays, as depicted in Figure 4.2. The forward time-delay τ_f affects the robot's control input $u(t)$, resulting in the delayed control signal $u(t-\tau_f)$ being applied to the robot. The backward time-delay τ_b affects the robot's output $q(t)$ (in this case the state of the unicycle), resulting in the delayed output $q(t-\tau_b)$ being available for control purposes. The controller makes use of the reference trajectory $q_r(t)$ of the robot and the output of the state predictor $z(t)$ in order to produce the control signals for the unicycle. At the same time, the state predictor requires the control signal $u(t)$ and the correction term $\nu(t)$ to compute its output. The state predictor is a dynamical system which has very similar dynamics to those of the mobile robot. The output of the state predictor is intentionally delayed by $\tilde{\tau}_f$ and then by $\tilde{\tau}_b$, which are, respectively, the estimates of the forward and backward time-delays. This yields $z(t-\tilde{\tau})$, with $\tilde{\tau} = \tilde{\tau}_f + \tilde{\tau}_b$, which together with the delayed output of the robot $q(t-\tau_b)$ and the output of the state predictor $z(t)$ are required to compute the correction term $\nu(t)$. The objective of the closed-loop predictor-controller combination is to guarantee stability and ensure that the robot tracks (a delayed version) of the reference trajectory.

The origin of this type of predictor can be traced back to the appearance of the notion of anticipating synchronization in coupled chaotic systems, which was first noted by Voss (2000) for a scalar system. After the same behavior was observed in certain simple physical systems such as specific electronic circuits and lasers (see for instance Masoller, 2001; Sivaprakasam et al., 2001; Voss, 2002), it was studied for more general systems in Oguchi and Nijmeijer (2006). As a

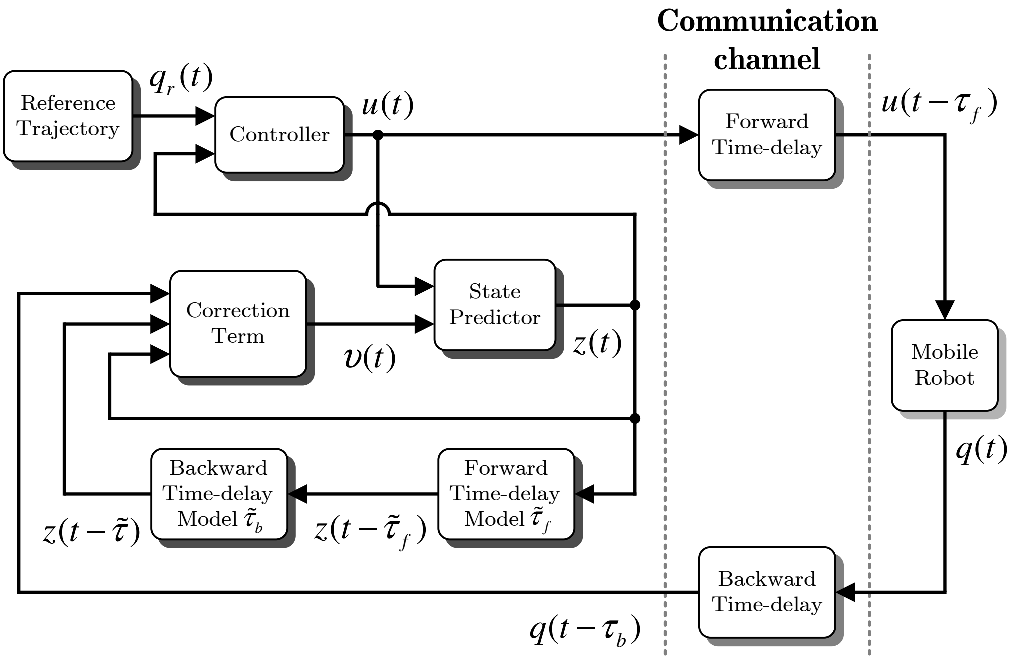


Figure 4.2 . Block diagram representation of the remote tracking control strategy.

result of this generalization, a synchronization-based state predictor for nonlinear systems with input time-delay was proposed in Oguchi and Nijmeijer (2005a). A similar predictor-controller combination was applied to a unicycle in Kojima et al. (2010), where a different tracking controller, correction term, and approach to study stability have been used. It is worth noting that, in that work, only sufficient conditions for the stability of the predictor error dynamics have been derived. As explained already, the remote control strategy presented in this chapter follows similar lines of reasoning as the work presented in Alvarez-Aguirre et al. (2010b) and Alvarez-Aguirre et al. (2011).

4.2.1 Experimental Implementation

One of the practical implementation issues of the proposed remote tracking controller is the availability of an accurate model of the network-induced delay. As noted in Hokayem and Spong (2006) and Michiels and Niculescu (2007), among others, predictor-like control strategies tend to be particularly sensitive to delay model mismatches, especially when dealing with nonlinear systems and a communication channel such as the Internet. Considering this, two different techniques have been used in order to implement the proposed delay remote tracking control strategy. Their objective it to provide an accurate estimate $\tilde{\tau}$ of the real delay τ.

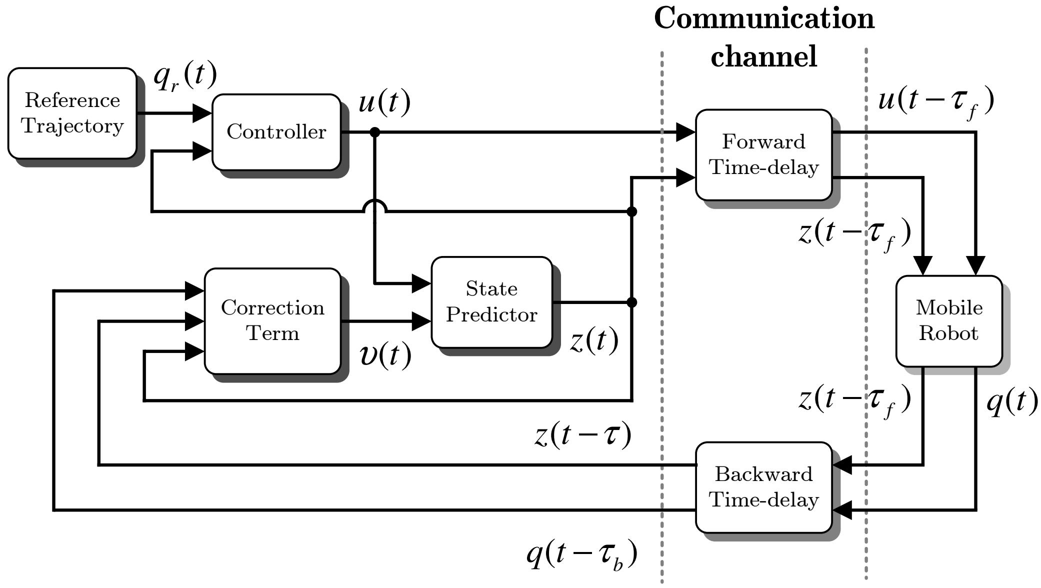


Figure 4.3 . Modified block diagram representation of the remote tracking control strategy considering signal bouncing.

Naturally, the most straightforward approach is to measure the communication delay. This has already been done in Section 3.1.3, where the round-trip delay between TU/e and TMU has been measured at approximately 268 ms using two slightly different techniques. Based on these measurements we can say that the round trip time-delay is fairly constant and reproducible, and that measuring it provides a (reasonably) reliable alternative for emulating the time-delay τ. The consistency in the measurements might be a consequence of using a VPN connection, which could be acting, to a certain degree, as a dedicated, albeit inexpensive, communication channel.

The second alternative is called *signal bouncing*, and is only suitable when considering a bilateral time-delay. In this case, the remote tracking control strategy is modified, resulting in the block diagram representation shown in Figure 4.3. The mobile robot in the remote location is assumed to have certain data reception and transmission capabilities, which is the case since it is already able to receive control commands and transmit its state. Instead of delaying the predicted state $z(t)$ by $\tilde{\tau}_f$ and then by $\tilde{\tau}_b$, it is sent to the mobile robot together with the control signal $u(t)$. This means that, at time t, $z(t-\tau_f)$ and $u(t-\tau_f)$ arrive at the mobile robot after being affected by the input time-delay τ_f. The control input $u(t-\tau_f)$ is applied to the mobile robot, producing the state $q(t)$, whereas the predicted state $z(t-\tau_f)$ is not processed at all and is sent by the robot over the network back to the controller side. Both signals exiting the mobile robot are subject to the output time-delay τ_b, resulting in $q(t-\tau_b)$ and $z(t-\tau)$ being available at time t for the computation of the correction term $\nu(t)$. As a result, by using the communication channel itself to delay the predicted state, it is no longer necessary to model the time-delay.

Considering the above we can say that, for the particular problem setting considered in this thesis, assuming the availability of an accurate estimate $\tilde{\tau}$ of the real delay τ is reasonable.

4.2.2 State Predictor and Controller Design

Consider a unicycle robot subject to a network-induced delay. In this case, the mobile robot is subject not only to an input (forward) time-delay τ_f, but also to an output (backward) time-delay τ_b, as denoted in Hokayem and Spong (2006). Throughout this thesis the forward and backward time-delays τ_f and τ_b, respectively, are assumed to be constant and known, with the round-trip time-delay defined as $\tau := \tau_b + \tau_f$. This assumption is motivated by the delay measurements conducted on the communication channel of the experimental platform used in this work (refer to Chapter 3). Given the posture kinematic model of a unicycle robot with state $q(t) = [x(t)\, y(t)\, \theta(t)]^T$ as in (2.40), if the mobile robot is subject to a network-induced input delay τ_f, its model becomes:

$$\dot{x}(t) = v(t-\tau_f)\cos\theta(t), \tag{4.1a}$$

$$\dot{y}(t) = v(t-\tau_f)\sin\theta(t), \tag{4.1b}$$

$$\dot{\theta}(t) = \omega(t-\tau_f). \tag{4.1c}$$

Additionally, since the time-delay affecting the system is bilateral, its state measurements are affected by an output time-delay τ_b, yielding the measured state $q(t-\tau_b) = [x(t-\tau_b)\, y(t-\tau_b)\, \theta(t-\tau_b)]^T$.

Even though it is subject to a network-induced delay, the unicycle robot is intended to track (a delayed version of) a reference trajectory with an associated state $q_r(t) = [x_r(t)\, y_r(t)\, \theta_r(t)]^T$. In order to improve the tracking performance of the robot, the following state predictor, with state $z(t) = [z_1(t)\, z_2(t)\, z_3(t)]^T$, is proposed:

$$\dot{z}_1(t) = v(t)\cos z_3(t) + \nu_x(t), \tag{4.2a}$$

$$\dot{z}_2(t) = v(t)\sin z_3(t) + \nu_y(t), \tag{4.2b}$$

$$\dot{z}_3(t) = \omega(t) + \nu_\theta(t), \tag{4.2c}$$

in which the robot kinematics can clearly be recognized and where $\nu(t) = [\nu_x(t)\ \nu_y(t)\ \nu_\theta(t)]^T$ constitutes a correction term based on the difference between the predicted and the measured states.

For the purpose of designing the correction term $\nu(t)$, a new set of error coordinates is introduced, namely, $p_e(t)$. This set of error coordinates is related to the difference between the delayed predicted state $z(t-\tilde{\tau})$ and the delayed system state $q(t-\tau_b)$ and is defined as:

$$p_e(t) = \begin{bmatrix} p_{1_e}(t) \\ p_{2_e}(t) \\ p_{3_e}(t) \end{bmatrix} = \begin{bmatrix} \cos z_3(t-\tilde{\tau}) & \sin z_3(t-\tilde{\tau}) & 0 \\ -\sin z_3(t-\tilde{\tau}) & \cos z_3(t-\tilde{\tau}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t-\tau_b) - z_1(t-\tilde{\tau}) \\ y(t-\tau_b) - z_2(t-\tilde{\tau}) \\ \theta(t-\tau_b) - z_3(t-\tilde{\tau}) \end{bmatrix}, \tag{4.3}$$

where $\tilde{\tau} := \tilde{\tau}_f + \tilde{\tau}_b$ represents the sum of the modeled input and output network-induced delays. Since the time-delays are assumed to be known or, in other words, modeled perfectly, we have that $\tilde{\tau}_f = \tau_f$ and $\tilde{\tau}_b = \tau_b$, which yields $\tilde{\tau} = \tau$.

The idea behind defining the prediction error based on the difference between the delayed predicted state and the delayed system state, that is, $q(t-\tau_b)-z(t-\tilde{\tau})$, is to compare these signals at the exact time instant when they are supposed to have the same values. This would mean that the predictor, as such, anticipates the state of the system by a time lapse equal to the forward time-delay τ_f.

Given the error coordinates (4.3), the correction term $\nu(t)$ is proposed as follows:

$$\nu_x(t) = k_x p_{1_e}(t) \cos z_3(t) - k_y p_{2_e}(t) \sin z_3(t), \tag{4.4a}$$
$$\nu_y(t) = k_x p_{1_e}(t) \sin z_3(t) + k_y p_{2_e}(t) \cos z_3(t), \tag{4.4b}$$
$$\nu_\theta(t) = k_\theta p_{3_e}(t), \tag{4.4c}$$

where k_x, k_y and k_θ are the correction gains.

Having constructed the state predictor, a new set of error coordinates denoted as $z_e(t)$ is defined. These error coordinates relate the difference between the predicted state $z(t)$ and the reference trajectory $q_r(t)$ and are given as follows:

$$z_e(t) = \begin{bmatrix} z_{1_e}(t) \\ z_{2_e}(t) \\ z_{3_e}(t) \end{bmatrix} = \begin{bmatrix} \cos z_3(t) & \sin z_3(t) & 0 \\ -\sin z_3(t) & \cos z_3(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - z_1(t) \\ y_r(t) - z_2(t) \\ \theta_r(t) - z_3(t) \end{bmatrix}. \tag{4.5}$$

The block diagram representation of the proposed control scheme, depicted in Figure 4.2, shows that the output of the state predictor constitutes the input of the tracking controller. The control law (2.45) will now make use of the predicted error coordinates $z_{1_e}(t), z_{2_e}(t)$, and $z_{3_e}(t)$ as in (4.5) (as opposed to the real error coordinates $x_e(t), y_e(t)$, and $\theta_e(t)$) and is given below:

$$v(t) = v_r(t) + c_x z_{1_e}(t) - c_y \omega_r(t) z_{2_e}(t), \qquad c_x > 0, \qquad c_y > -1, \tag{4.6a}$$

$$\omega(t) = \omega_r(t) + c_\theta z_{3_e}(t), \qquad c_\theta > 0. \tag{4.6b}$$

Due to the input time-delay, the control action applied to the mobile robot in (4.1) is given by

$$v(t-\tau_f) = v_r(t-\tau_f) + c_x z_{1_e}(t-\tau_f) - c_y \omega_r(t-\tau_f) z_{2_e}(t-\tau_f), \tag{4.7a}$$

$$\omega(t-\tau_f) = \omega_r(t-\tau_f) + c_\theta z_{3_e}(t-\tau_f). \tag{4.7b}$$

The resulting control input already hints at how the system should behave. Intuitively, the robot state $q(t)$ converges to the delayed reference state trajectory $q_r(t-\tau_f)$. This claim will be examined in greater detail in Section 4.3.

4.2.3 Closed-Loop Error Dynamics

The formulation of the closed-loop error dynamics is needed to study the stability properties of the remote control strategy. Exploiting the predictor (4.2), correction term (4.4), and control law (4.7), the following closed-loop error dynamics result:

$$\begin{aligned}
\dot{z}_{1_e}(t) &= -c_x z_{1_e}(t) + (1+c_y)\omega_r(t) z_{2_e}(t) - k_x p_{1_e}(t) + c_\theta z_{2_e}(t) z_{3_e}(t) \\
&\quad + k_\theta z_{2_e}(t) p_{3_e}(t) - v_r(t)(1-\cos z_{3_e}(t)), && (4.8a)\\
\dot{z}_{2_e}(t) &= -\omega_r(t) z_{1_e}(t) - k_y p_{2_e}(t) - c_\theta z_{1_e}(t) z_{3_e}(t) - k_\theta z_{1_e}(t) p_{3_e}(t) \\
&\quad + v_r(t)\sin z_{3_e}(t), && (4.8b)\\
\dot{z}_{3_e}(t) &= -c_\theta z_{3_e}(t) - k_\theta p_{3_e}(t), && (4.8c)\\
\dot{p}_{1_e}(t) &= \omega_r(t-\tau) p_{2_e}(t) - k_x p_{1_e}(t-\tau) + c_\theta p_{2_e}(t) z_{3_e}(t-\tau) + k_\theta p_{2_e}(t) p_{3_e}(t-\tau) \\
&\quad - (v_r(t-\tau) + c_x z_{1_e}(t-\tau) - c_y \omega_r(t-\tau) z_{2_e}(t-\tau))(1-\cos p_{3_e}(t)), && (4.8d)\\
\dot{p}_{2_e}(t) &= -\omega_r(t-\tau) p_{1_e}(t) - k_y p_{2_e}(t-\tau) - c_\theta p_{1_e}(t) z_{3_e}(t-\tau) - k_\theta p_{1_e}(t) p_{3_e}(t-\tau) \\
&\quad + (v_r(t-\tau) + c_x z_{1_e}(t-\tau) - c_y \omega_r(t-\tau) z_{2_e}(t-\tau))\sin p_{3_e}(t), && (4.8e)\\
\dot{p}_{3_e}(t) &= -k_\theta p_{3_e}(t-\tau). && (4.8f)
\end{aligned}$$

Considering the state definitions $\xi_1(t) := [z_{1_e}(t)\; z_{2_e}(t)\; p_{1_e}(t)\; p_{2_e}(t)]^T$ and $\xi_2(t) := [z_{3_e}(t)\; p_{3_e}(t)]^T$, the closed-loop error dynamics (4.8) may be represented as the following cascaded system:

$$\dot{\xi}_1(t) = A_1(t, t-\tau)\xi_1(t) + A_2\xi_1(t-\tau) + g(t, \xi_{1_t}, \xi_{2_t}), \tag{4.9a}$$

$$\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau), \tag{4.9b}$$

where ξ_{i_t}, $i = 1, 2$, is an element of the Banach space $\mathcal{C}(l_i) = C([-\tau, 0], \mathbb{R}^{l_i})$, with $l_1 = 4$ and $l_2 = 2$, defined by $\xi_{i_t}(s) := \xi_i(t+s)$ for $s \in [-\tau, 0]$. Note that by means of ξ_{i_t} it is possible to represent the state ξ_i of the system throughout the time interval $[t-\tau, t]$. The matrices in (4.9) are given by

$$A_1(t, t-\tau) = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) & -k_x & 0 \\ -\omega_r(t) & 0 & 0 & -k_y \\ 0 & 0 & 0 & \omega_r(t-\tau) \\ 0 & 0 & -\omega_r(t-\tau) & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -k_x & 0 \\ 0 & 0 & 0 & -k_y \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -c_\theta & -k_\theta \\ 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & -k_\theta \end{bmatrix},$$

$$g(t, \xi_{1_t}, \xi_{2_t}) = \begin{bmatrix} g_{11} & k_\theta z_{2_e}(t) \\ g_{21} & -k_\theta z_{1_e}(t) \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix} \xi_2(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ c_\theta p_{2_e}(t) & k_\theta p_{2_e}(t) \\ -c_\theta p_{1_e}(t) & -k_\theta p_{1_e}(t) \end{bmatrix} \xi_2(t-\tau),$$

with

$$\begin{aligned}
g_{11} &= c_\theta z_{2_e}(t) - v_r(t) \int_0^1 \sin(s z_{3_e}(t)) ds, \\
g_{21} &= -c_\theta z_{1_e}(t) + v_r(t) \int_0^1 \cos(s z_{3_e}(t)) ds, \\
g_{32} &= -(v_r(t-\tau) + c_x z_{1_e}(t-\tau) - c_y \omega_r(t-\tau) z_{2_e}(t-\tau)) \int_0^1 \sin(s p_{3_e}(t)) ds, \\
g_{42} &= (v_r(t-\tau) + c_x z_{1_e}(t-\tau) - c_y \omega_r(t-\tau) z_{2_e}(t-\tau)) \int_0^1 \cos(s p_{3_e}(t)) ds,
\end{aligned}$$

where the equalities in (2.2) have been used to define g_{11}, g_{21}, g_{32}, and g_{42}.

The upcoming section elaborates on the local and global stability of the closed-loop error dynamics (4.9).

4.3 Stability Analysis

In order to characterize the behavior of the predictor-tracking controller combination introduced in the previous section, theorems which pose sufficient conditions under which the closed-loop error dynamics (4.9) are either locally or globally uniformly asymptotically stable are presented in this section. Considering the remote control strategy and the control action applied to the robot, the control objectives of the complete system may be defined as follows:

- the state of the unicycle converges to the state of the reference trajectory delayed by τ_f, that is, $q(t) \to q_r(t - \tau_f)$;
- the predicted state anticipates the state of the unicycle by τ_f, that is, $z(t) \to q(t + \tau_f)$;
- the predicted state converges to the state of the reference trajectory, that is, $z(t) \to q_r(t)$.

The following control goal may be formulated based on the previous objectives:

Given the unicycle robot (4.1) subject to a network-induced delay $\tau = \tau_f + \tau_b$, the state estimator (4.2), (4.3), and (4.4), and the control law (4.5) and (4.6), the mobile robot should track a delayed version $q_r(t - \tau_f)$ of the reference trajectory.

Considering this control goal, it follows that in order to meet the control objectives it is sufficient to prove the stability of the equilibrium point $(z_e^T, p_e^T)^T = (z_{1_e}, z_{2_e}, z_{3_e}, p_{1_e}, p_{2_e}, p_{3_e})^T = 0$ of the closed-loop error dynamics (4.9).

4.3.1 Sufficient Conditions for Local Asymptotic Stability

The following theorem formulates sufficient conditions under which $(z_e^T, p_e^T)^T = 0$ is a LUAS equilibrium point of (4.9).

Theorem 4.1. *Consider the posture kinematic model of a unicycle robot subject to a constant input time-delay τ_f, as given by (4.1). The reference position of the robot is given by $(x_r(t), y_r(t))$, whereas its reference orientation $\theta_r(t)$ is given by (2.41a). Additionally, consider the tracking controller as given in (4.6), with the feedforward terms $v_r(t)$ and $\omega_r(t)$ defined as in (2.41b) and (2.41c), respectively, and the feedback part based on the error between the reference trajectory and the predicted state, as given in (4.5). Moreover, consider the state predictor (4.2), (4.3), (4.4), which uses state measurements delayed by a constant output time-delay τ_b. If the following conditions are satisfied:*

- *the reference translational velocity $v_r(t) \neq 0$, $\forall t$, is bounded;*
- *the reference rotational velocity $\omega_r(t)$ is persistently exciting (PE);*
- *the tracking gains satisfy $c_x, c_\theta > 0, c_y > -1$;*
- *the correction gains satisfy $k_x = k_y = k > 0, k_\theta > 0$;*
- *the time-delay is known, that is, $\tilde{\tau} = \tau = \tau_b + \tau_f$;*

- *the time-delay τ belongs to the interval $0 \leq \tau < \tau_{\max}$, with*

$$\tau_{\max} = \min\left\{\frac{1}{\sqrt{p}(\bar{\omega}_r + k)}, \frac{\pi}{2k_\theta}\right\}, \tag{4.10}$$

where $p > 1$ and $\bar{\omega}_r = \sup_{t\in\mathbb{R}} |\omega_r(t)|$,

then, $(z_e^T, p_e^T)^T = 0$ is a locally uniformly asymptotically stable (LUAS) equilibrium point of the closed-loop error dynamics (4.9). In other words, $z(t) \to q(t + \tau_f)$ as $t \to \infty$, that is, the predicted state anticipates the state of the system by τ_f, and $q(t) \to q_r(t - \tau_f)$ as $t \to \infty$, that is, the system tracks the reference trajectory delayed by τ_f.

Proof. For the sake of brevity, only a sketch of the proof is presented in this chapter. The complete proof is given in Appendix B.2.

Given that the closed-loop error dynamics (4.9) are arranged as a cascaded system of the same form as (2.38), Theorem 2.25 is used to establish the local uniform asymptotic stability of their equilibrium point $(z_e^T, p_e^T)^T = 0$. This results in the following conditions:

- the system $\dot{\xi}_1(t) = A_1(t, t-\tau)\xi_1(t) + A_2\xi_1(t-\tau)$, denoted as the ξ_1-dynamics without coupling, is locally uniformly asymptotically stable (LUAS);
- the system $\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t - \tau)$, denoted as the ξ_2-dynamics, is locally uniformly asymptotically stable (LUAS);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ vanishes when $\xi_{2_t} \to 0$, that is, $g(t, \xi_{1_t}, 0) = 0$.

The validity of these three conditions is then checked given the assumptions on the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, reference translational and rotational velocities $v_r(t)$ and $\omega_r(t)$, respectively, and maximum allowable time-delay $\tau_{\max}$, adopted in the theorem.

Regarding the first condition, note that the ξ_1-dynamics without coupling can be represented by a cascade itself. Using a similar reasoning as for the original cascaded system, it can be shown that the local uniform asymptotic stability of these dynamics may be concluded if the time-delay satisfies the following condition:

$$\tau < \frac{1}{\sqrt{p}(\bar{\omega}_r + k)}, \tag{4.11}$$

and the requirements for c_x, c_y, k_x and k_y stated in the theorem are met.

Regarding the second condition, the local uniform asymptotic stability of the ξ_2-dynamics is ensured for

$$\tau < \frac{\pi}{2k_\theta}, \tag{4.12}$$

provided c_θ and k_θ satisfy the conditions in the theorem. Satisfying condition (4.10) ensures that conditions (4.11) and (4.12) are met.

Regarding the third condition, it immediately follows that as $\xi_{2_t} \to 0$, the coupling term vanishes.

The local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (4.9) is then concluded for the requirements posed in the theorem, thereby completing the sketch of the proof. □

Remark 4.2. *One of the assumptions in Theorem 4.1 is that the reference trajectory is not known a priori. However, if the reference trajectory is available in advance and the value of the forward time-delay is known, it is possible to provide the controller with the reference $q_r(t + \tau_f)$. Since we expect the robot to track the delayed reference trajectory, it will actually track the desired reference trajectory, that is, $q_r(t)$.*

The relationship between the allowable time-delay τ and the control parameters for conditions (4.11) and (4.12) is shown in Figure 4.4. The left plot shows the maximum allowable time-delay satisfying (4.11), considering $p = 1$ and different values for the correction term gain k and the maximum reference rotational velocity $\bar{\omega}_r$. Depicted in the right plot is the maximum allowable time-delay satisfying (4.12), given $p = 1$ and different values for the correction term gain k_θ. Note that, for both conditions there exist choices for the correction gains such that it becomes possible to accommodate larger time-delays ($k \downarrow 0$ and $\bar{\omega}_r \to 0$ for (4.11) and $k_\theta \downarrow 0$ for (4.12)).

4.3.2 Sufficient Conditions for Global Asymptotic Stability

When considering a global stability result, the control objectives stated at the beginning of this section remain the same apart from the fact that we require these objectives to be attained for arbitrarily large initial conditions. In this light, the upcoming theorem formulates sufficient conditions under which $(z_e^T, p_e^T)^T = 0$ is a GUAS equilibrium point of (4.9). It is worth noting that the conditions stated in the theorem below do not explicitly place a bound on the allowable time-delay $\tau_{\max}$, but rather are only able to ensure the existence of a certain $\tau_{\max} > 0$ such that global uniform asymptotic stability can be guaranteed for any $\tau \in [0, \tau_{\max}]$. The absence of an explicit expression for $\tau_{\max}$ results in a qualitative

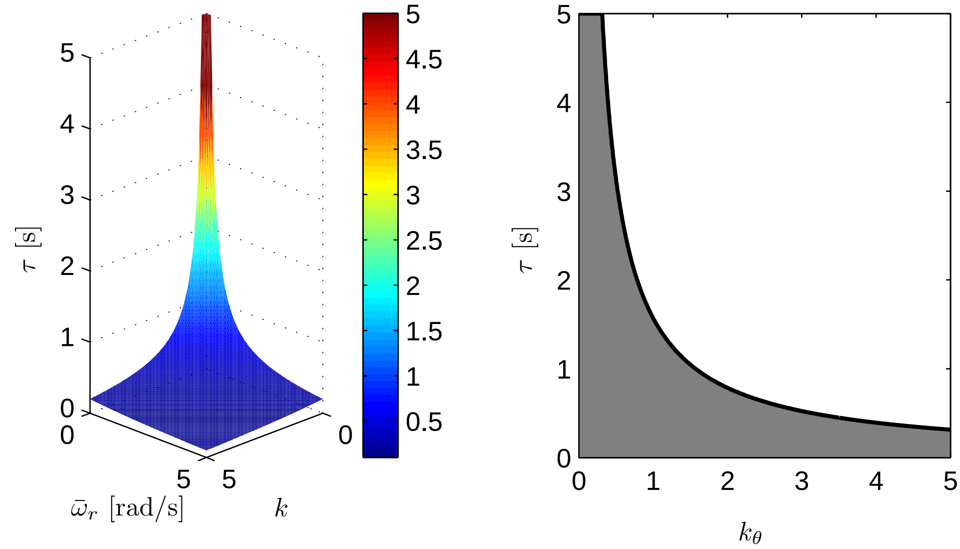


Figure 4.4 . Allowable time-delay τ for conditions (4.11) (left) and (4.12) (right). To better illustrate the relationship between the gains and the time-delay, the allowable delay has been cut off at 5 s.

characterization of global asymptotic stability in the next theorem, as opposed to local asymptotic stability which was characterized quantitatively by means of an explicit requirement on the allowable time-delay $\tau_{\max}$ in Theorem 4.1.

Theorem 4.3. *Consider the posture kinematic model of a unicycle robot subject to a constant input time-delay τ_f, as given by (4.1). The reference position of the robot is given by $(x_r(t), y_r(t))$, whereas its reference orientation $\theta_r(t)$ is given by (2.41a). Additionally, consider the tracking controller as given in (4.6), with the feedforward terms $v_r(t)$ and $\omega_r(t)$ defined as in (2.41b) and (2.41c), respectively, and the feedback part based on the error between the reference trajectory and the predicted state, as given in (4.5). Moreover, consider the state predictor (4.2), (4.3), and (4.4), which uses state measurements delayed by a constant output time-delay τ_b. Suppose that the following conditions are satisfied:*

- *the reference translational velocity $v_r(t) \neq 0$, $\forall t$, is bounded;*
- *the reference rotational velocity $\omega_r(t)$ is persistently exciting (PE);*
- *the time-delay is known, that is, $\tilde{\tau} = \tau = \tau_b + \tau_f$;*
- *the tracking gains satisfy $c_x, c_\theta > 0, c_y > -1$,*

then, for $k_x = k_y = k > 0$ sufficiently small and $k_\theta > 0$ there exists a $\tau_{\max} > 0$ for which the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (4.9) *is globally uniformly asymptotically stable (GUAS) for all* $0 \leq \tau \leq \tau_{\max}$*. In other words, for these values of* k_x, k_y*, and* k_θ*,* $z(t) \to q(t + \tau_f)$ *as* $t \to \infty$*, that is, the predicted state anticipates the state of the system by* τ_f*, and* $q(t) \to q_r(t - \tau_f)$ *as* $t \to \infty$*, that is, the system tracks the reference trajectory delayed by* τ_f*.*

Proof. For the sake of brevity, only a sketch of the proof is presented in this chapter. The complete proof is given in Appendix B.3.

Based on Theorem 2.26 and following a similar approach as in the proof of Theorem 4.1, the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (4.9) may be established if the following conditions are satisfied:

- the ξ_1-dynamics without coupling are globally exponentially stable (GES) with an explicit quadratic Lyapunov-Razumikhin function V_{ξ_1};
- the ξ_2-dynamics are globally exponentially stable (GES);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ admits the estimate

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c. \tag{4.13}$$

  for continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The exponential stability requirements on the ξ_1-dynamics without coupling and the ξ_2-dynamics are based on the assumptions in Theorem 2.26 and the remarks that follow the theorem (the formulation of these requirements is explained in greater detail in Appendix B.3). In addition, because of the stability requirement on the ξ_1-dynamics without coupling, the estimate on the coupling term may be rewritten as in (4.13) according to the remarks that follow Theorem 2.26.

The validity of the three conditions in the list above is checked given the assumptions on the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, reference translational and rotational velocities $v_r(t)$ and $\omega_r(t)$, respectively, and maximum allowable time-delay $\tau_{\max}$, adopted in the theorem.

The first step to check the condition on the ξ_1-dynamics without coupling is that their delay-free version is GES. Once this has been shown, it follows from Lyapunov converse theory (see Theorem 2.15) that there exists a strict Lyapunov function for the delay-free ξ_1-dynamics without coupling. Although such a strict Lyapunov function is shown to exist, it is not known explicitly. This function is then proposed as a candidate Lyapunov-Razumikhin function for the ξ_1-dynamics without coupling. Because the candidate Lyapunov-Razumikhin function is not known explicitly, the conditions on the allowable time-delay $\tau_{\max}$ which result from the ensuing stability analysis do not place an explicit bound on $\tau_{\max}$. This implies that the global asymptotic stability of the ξ_1-dynamics without coupling (and hence of the complete closed-loop error dynamics (4.9)) can only be characterized qualitatively.

Regarding the second condition, recall that the ξ_2-dynamics have already been shown to be LUAS in the proof of Theorem 4.1. Since these dynamics are given by a linear time-invariant (LTI) system, the global aspect follows directly and, in accordance with Definition 2.12, the system is also exponentially stable. This means that the ξ_2-dynamics are GES provided that the time-delay satisfies condition (4.12) and c_θ and k_θ satisfy the conditions in the theorem.

Regarding the condition on the coupling term, it can be shown using vector and matrix norms that the inequality (4.13) is satisfied with $\alpha_1(\|\varphi_{\xi_2}\|_c) = 4|\bar{v}_r|$ and $\alpha_2(\|\varphi_{\xi_2}\|_c) = 4(c_\theta + k_\theta + 2(c_x + |c_y|\bar{\omega}_r))$.

After checking the three conditions formulated at the beginning of the proof, we have that the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (4.9) is concluded for the requirements posed in the theorem. This completes the sketch of the proof. □

Although the theorem only poses qualitative conditions which ensure the global uniform asymptotic stability of the closed-loop error dynamics (4.9), it does shed some light on how the system will behave. To begin with, the conditions on the control parameters imply that there exist correction gains $k = k_x = k_y > 0$ and $k_\theta > 0$ for which the origin of the closed-loop error dynamics is GUAS for all $\tau \in [0, \tau_{\max}]$. In addition, the similarities between the local and global stability theorems would lead one to believe that the conditions posed in the local case might actually hold for a wider set of initial conditions, or that global stability can be ensured for a slightly smaller maximal time-delay than $\tau_{\max}$ as given in (4.10). The question remains, of course, how much wider would the set of initial conditions be, or how much smaller would the allowable time-delay be. Finally, it is worth noting that, since Theorem 4.3 has been formulated under the assumption that the reference trajectory is not known a priori, Remark 4.2 is also in place in the global stability analysis.

4.4 Simulation and Experimental Results

This section contains a number of simulations and experiments which illustrate the performance of the remote control strategy presented in this chapter. The experiments are carried out in the multi-robot platform introduced in Chapter 3 and allow for a robot located at the TU/e to be controlled from TMU and vice-versa. The results in this section encompass one numerical simulation and two experiments. In the case of the experiments, a mobile robot at TMU is controlled from TU/e. The reference trajectories are given as follows:

- Simulation: a circle with radius $r = 0.5\,\mathrm{m}$ centered at $(x_{r_c}, y_{r_c}) = (0.875, 0.65)\,\mathrm{m}$, a reference rotational velocity of $\omega_r = 0.5\,\mathrm{rad/s}$, and a reference translational velocity of $v_r = r\omega_r = 0.25\,\mathrm{m/s}$. Its parametric equations are given by

$$x_r(t) = x_{r_c} + r\sin\theta_r(t), \tag{4.14a}$$
$$y_r(t) = y_{r_c} - r\cos\theta_r(t); \tag{4.14b}$$

- Experiment 1: an eight curve with the following parametric equations:

$$x_r(t) = x_{r_c} + a\sin(bt), \tag{4.15a}$$
$$y_r(t) = y_{r_c} + c\sin(2bt), \tag{4.15b}$$

  where $(x_{r_c}, y_{r_c}) = (0.5, 0.25)\,\mathrm{m}$ denotes the center of the curve, $2a = 2b = 0.4\,\mathrm{m}$, its length and width, respectively, and $c = 0.2\,\mathrm{rad/s}$ constitutes its angular velocity. In this case, $\bar{\omega}_r = 1.16\,\mathrm{rad/s}$, which will be used to choose the correction gains $k = k_x = k_y$ such that (4.11) is satisfied.

- Experiment 2: sinusoid with the following parametric equations:

$$x_r(t) = x_{r_0} + v_{r_0} t\cos\theta_{r_0} - a\sin\theta_{r_0}\sin(\omega_{r_0} t), \tag{4.16a}$$
$$y_c(t) = y_{r_0} + v_{r_0} t\sin\theta_{r_0} + a\cos\theta_{r_0}\sin(\omega_{r_0} t), \tag{4.16b}$$

  where $(x_{r_0}, y_{r_0}) = (0.1, 0.25)\,\mathrm{m}$ denotes the origin of the sinusoid, $\theta_{r_0} = 0\,\mathrm{rad}$ represents the orientation of the curve, $v_{r_0} = 0.01\,\mathrm{m/s}$ and $\omega_{r_0} = 0.3\,\mathrm{rad/s}$ are the translational and rotational velocities of the sinusoid, respectively, and $a = 0.15\,\mathrm{m}$ its amplitude. For this reference trajectory $\bar{\omega}_r = 0.48\,\mathrm{rad/s}$, which will be used to choose the correction gains $k = k_x = k_y$.

The initial conditions for the system and the state predictor, $q(0)$ and $z(0)$, respectively, are given in Table 4.1, whereas the tracking gains of the controller, the correction gains of the predictor, the magnitude of the time-delay, and the duration of the simulation or experiment are provided in Table 4.2.

Several remarks are in place regarding the values contained in Table 4.2. First, given the selected values of the correction gains, the maximum allowable time-delay $\tau_{\max}$ according to condition (4.10) in Theorem 4.1 is 1.1 s for the simulation, 0.57 s for the first experiment, and 0.93 s for the second experiment. Recall that, in order to satisfy the conditions in Theorem 4.1, the network-induced delay τ should

Table 4.1 . Initial conditions of the unicycle and the state predictor used in each simulation/experiments.

	Initial conditions					
	System			Predictor		
	$x(0)$ [m]	$y(0)$ [m]	$\theta(0)$ [rad]	$z_1(0)$ [m]	$z_2(0)$ [m]	$z_3(0)$ [rad]
Simulation	1.1	0.3	$\frac{\pi}{3}$	0.0	0.0	0.0
Experiment 1	0.2307	0.2191	0.2851	0.0	0.0	0.0
Experiment 2	0.1796	0.5975	0.3916	0.0	0.0	0.0

Table 4.2 . Tracking gains, correction gains, communication delay, and duration of the simulation/experiment.

	Tracking gains			Correcting gains			Delay τ	Duration
	c_x	c_y	c_θ	k_x	k_y	k_θ	[ms]	[s]
Simulation	2.0	2.0	1.0	0.4	0.4	0.75	1000	60.0
Experiment 1	2.0	2.0	1.0	0.6	0.6	0.6	268	60.0
Experiment 2	2.0	2.0	1.0	0.6	0.6	0.6	268	120.0

belong to the interval $[0, \tau_{\max})$. Second, for the simulation, the communication delay is assumed to be $\tau = \tau_f + \tau_b = 0.5 + 0.5 = 1\,\text{s}$, which clearly belongs to the interval $[0, 1.1)$. In this case, the predictor-controller combination is implemented as in Figure 4.3. On the other hand, in the case of the experiments, the predictor-controller combination is implemented as in Figure 4.2 and the magnitude of the estimate of the communication delay $\tilde{\tau}$ is based on measurements. Because of this, only the magnitude of the round-trip time-delay is known, even though the specific values of the forward and backward delays remain unknown. Additionally, due to the sampling rate of the experimental platforms, which is 25 Hz, only delay models which are multiples of 40 ms can be implemented in software. This means that, in the experiments, the output of the predictor is delayed by $\tilde{\tau} = 280\,\text{ms}$ and not by $\tilde{\tau} = 268\,\text{ms}$. Motivated by the consistency of the communication delay measurements (see Chapter 3), we assume that $\tilde{\tau} = \tau$. As a result, $\tau \in [0, 0.57)$ for the first experiment and $\tau \in [0, 0.93)$ for the second experiment, which means that the condition on τ in Theorem 4.1 is satisfied for both experiments.

The plots in Figure 4.5, Figure 4.6, and Figure 4.7 show the reference, robot, and predictor trajectories in the global coordinate frame $\vec{\underline{e}}^0$ for the simulation and

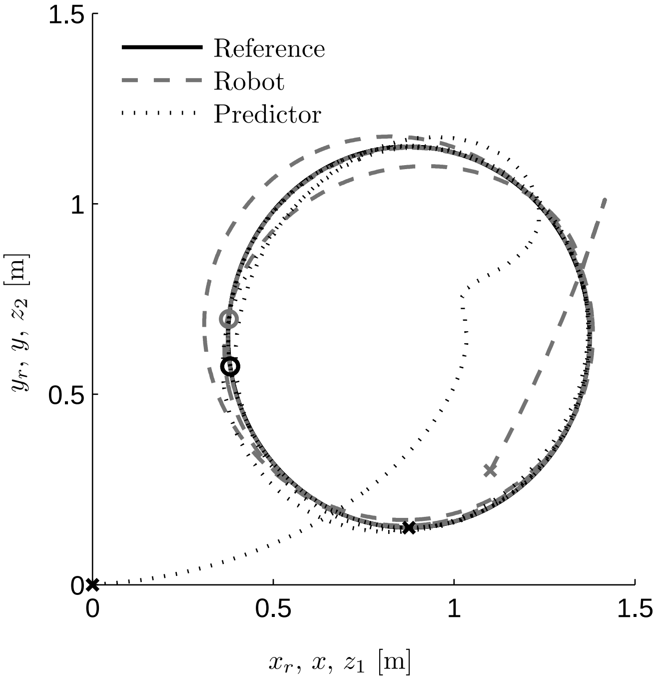


Figure 4.5 . Simulation. Remotely controlled unicycle (a perturbation is induced at $t = 30\,\mathrm{s}$).

both experiments, respectively. The initial and final positions of these trajectories are marked with a cross and a circle, respectively. Some of the crosses might not be visible because they overlap with the reference trajectory. In the simulation, the control inputs $v(t)$ and $\omega(t)$ of the robot are affected during 1.5 s starting at $t = 30\,\mathrm{s}$ by an additive perturbation of $0.2\,\mathrm{m/s}$ and $0.3\,\mathrm{rad/s}$, respectively. The purpose of perturbing the robot is to show that the proposed predictor-controller combination possesses certain robustness against perturbations. As expected, the trajectory of the robot lags the reference trajectory with a delay τ_f.

Further details are shown in Figure 4.8, Figure 4.10, and Figure 4.12 for the simulation and both experiments, respectively. In the figures, the plots in the first row show the tracking errors, defined as $e_x(t) = x_r(t - \tau_f) - x(t)$, $e_y(t) = y_r(t - \tau_f) - y(t)$, and $e_\theta(t) = \theta_r(t - \tau_f) - \theta(t)$, respectively. The plots in the second row depict the prediction error, given by $e_{p_1}(t) = x(t - \tau) - z_1(t - \tau)$, $e_{p_2}(t) = y(t - \tau) - z_2(t - \tau)$, and $e_{p_3}(t) = \theta(t - \tau) - z_3(t - \tau)$, respectively. The plots show that all the errors (practically) converge to zero and how all the errors in the simulation reflect the perturbation which affects the robot.

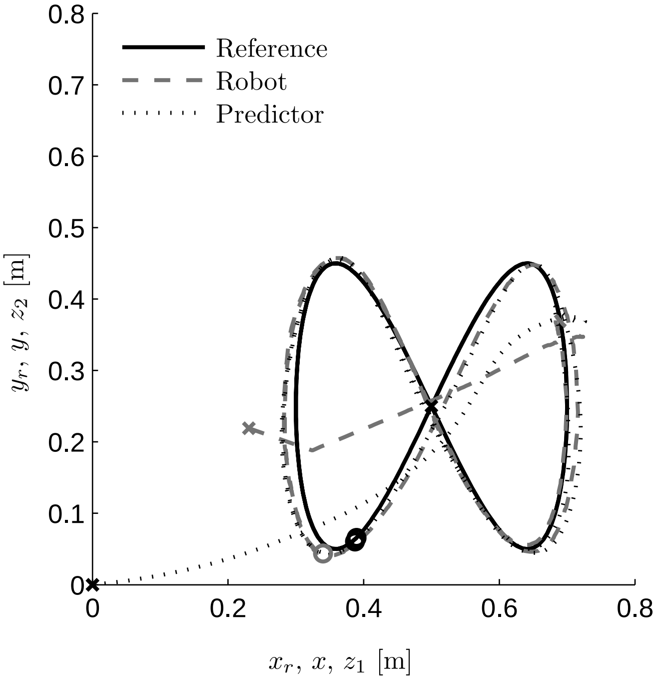


Figure 4.6 . First experiment. Unicycle at TMU remotely controlled from TU/e.

Finally, the translational and rotational velocities applied to the unicycle in the simulation and both experiments are shown in Figure 4.9, Figure 4.11, and Figure 4.13, respectively. In the case of the simulation, these control inputs reflect the perturbation that acts on the robot.

The simulation results validate the implementation of the predictor-controller combination as in Figure 4.3 and the conditions on the communication delay and the control parameters posed in Theorem 4.1 to guarantee local asymptotic stability. The results show that the robot lags its reference trajectory as expected and that the remote control strategy is able to recover from small, transient, additive perturbations. On the other hand, the experimental results validate the implementation of the predictor-controller combination as in Figure 4.2, as well as the conditions on the communication delay and the control parameters posed in Theorem 4.1 to guarantee local asymptotic stability. The results show that the error coordinates (practically) converge to zero even in the presence of a small delay model mismatch. In conclusion, the behavior of the remote control strategy is consistent with the local stability analysis (provided the conditions posed in Theorem 4.1 are satisfied) and the tracking performance of the robot can be ensured even in the presence of a network-induced delay.

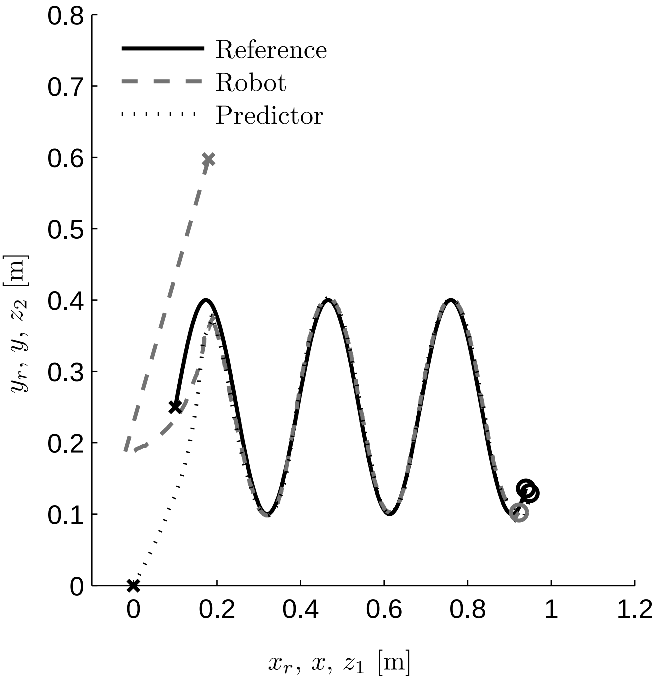


Figure 4.7 . Second experiment: Unicycle at TMU remotely controlled from TU/e.

4.5 Concluding Remarks

This chapter considers the tracking control problem of a unicycle robot controlled over a two-channel communication network which induces time-delays. A tracking controller and a state predictor, which together guarantee the tracking of a delayed reference trajectory, has been proposed. The tracking and prediction error dynamics have been shown to be LUAS with an explicit (quantitative) bound $\tau_{\max}$ on the allowable time-delay. This local stability analysis has been complemented with a global stability analysis which guarantees (qualitatively) the existence of a non-zero upper bound on $\tau_{\max}$. In addition, simulations and experiments validate the effectiveness of the proposed remote control approach and show that the predictor-controller combination can withstand small delay model mismatches and delay variations.

The local stability analysis shows that the choice of the tracking gains does not influence the magnitude of the allowable communication delay and that the magnitude of this delay is in fact related to the selected correction gains. As explained in Remark 4.2, it is possible to track the desired reference trajectory $q_r(t)$ at time t if the reference trajectory $q_r(t + \tau_f)$ is known a priori.

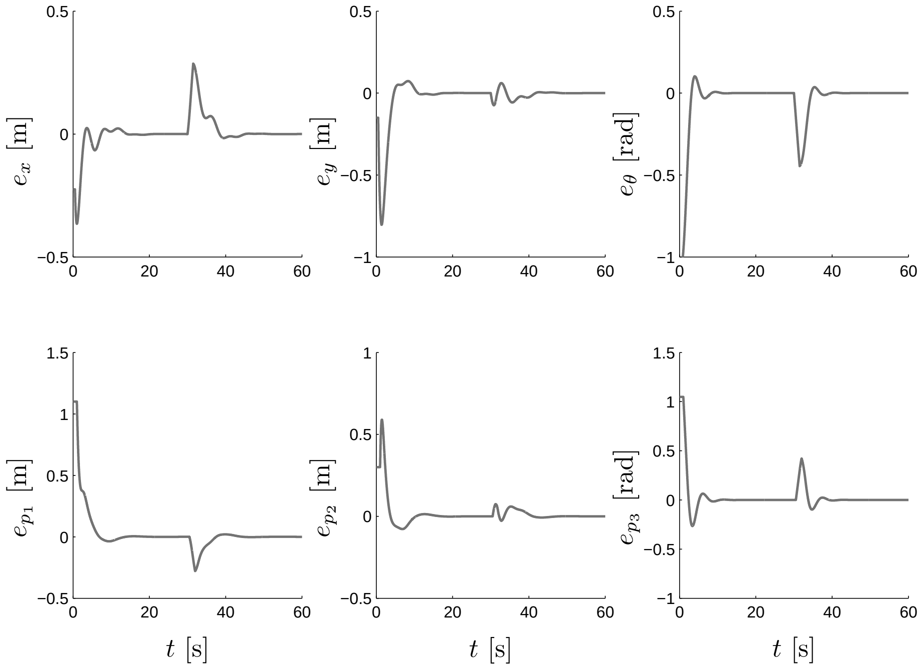


Figure 4.8 . Simulation. First row: predictor tracking error; second row: prediction error (a perturbation is induced at $t = 30\,\text{s}$).

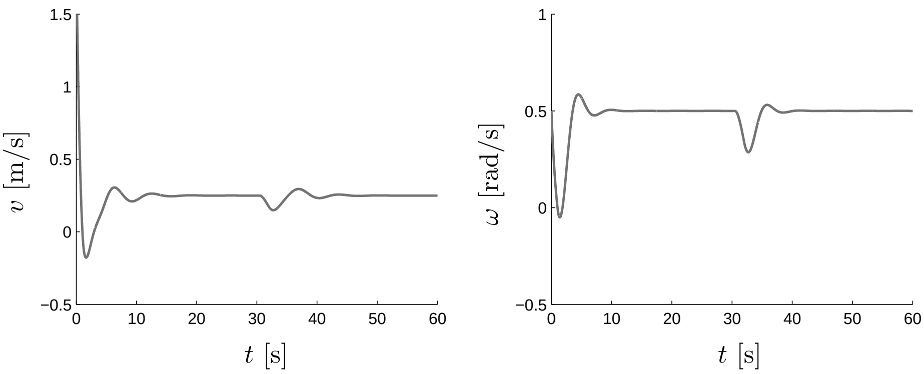


Figure 4.9 . Simulation. Left: translational control input; right: rotational control input (a perturbation is induced at $t = 30\,\text{s}$).

Compared to other control strategies which cope with communication delays, the main advantage of the current predictor-controller combination is that its implementation is straightforward and does not require any significant changes to a tracking controller which is already available. Moreover, deriving the predictor

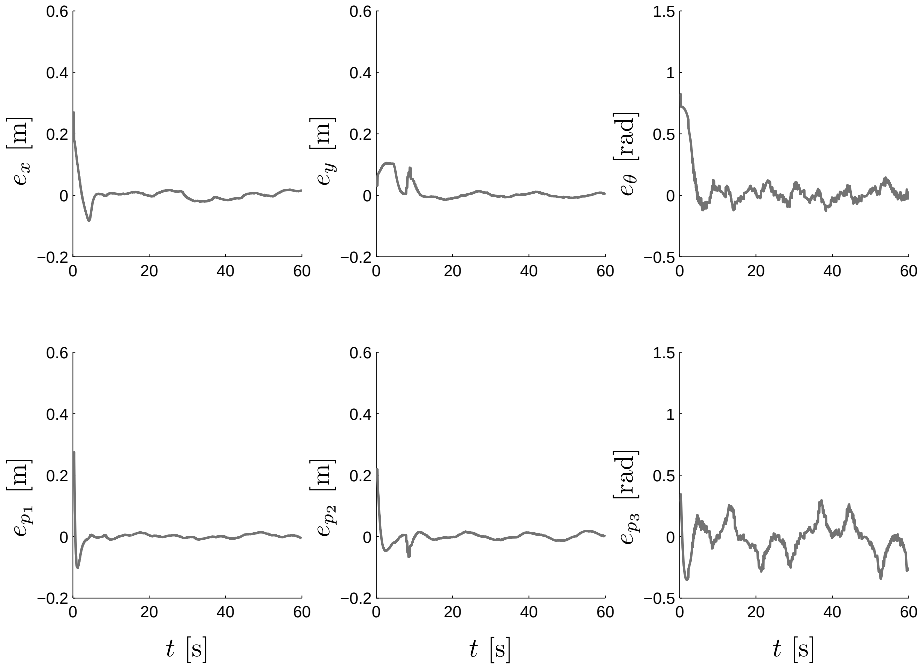


Figure 4.10 . First experiment. First row: predictor tracking error; second row: prediction error.

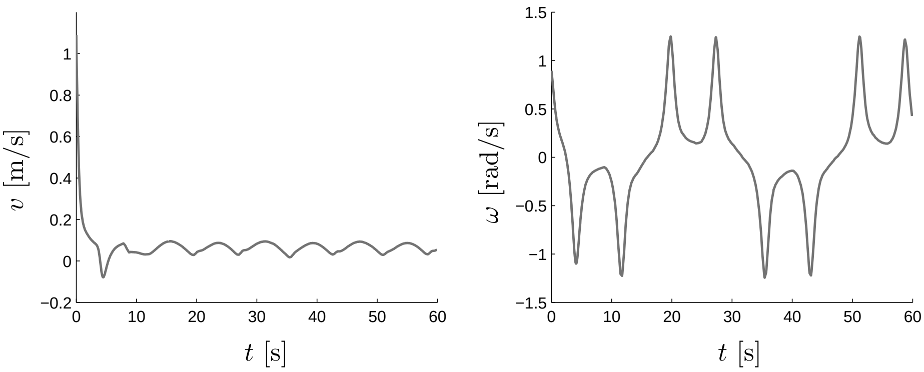


Figure 4.11 . First experiment. Left: translational control input; right: rotational control input.

dynamics is quite intuitive and follows from the system model. In addition, there is plenty of freedom in the choice of the correction term. The main drawback of the remote control strategy is that, in its current form, it is only suitable for constant time-delays. Issues such as time-varying delays, packet losses, communication

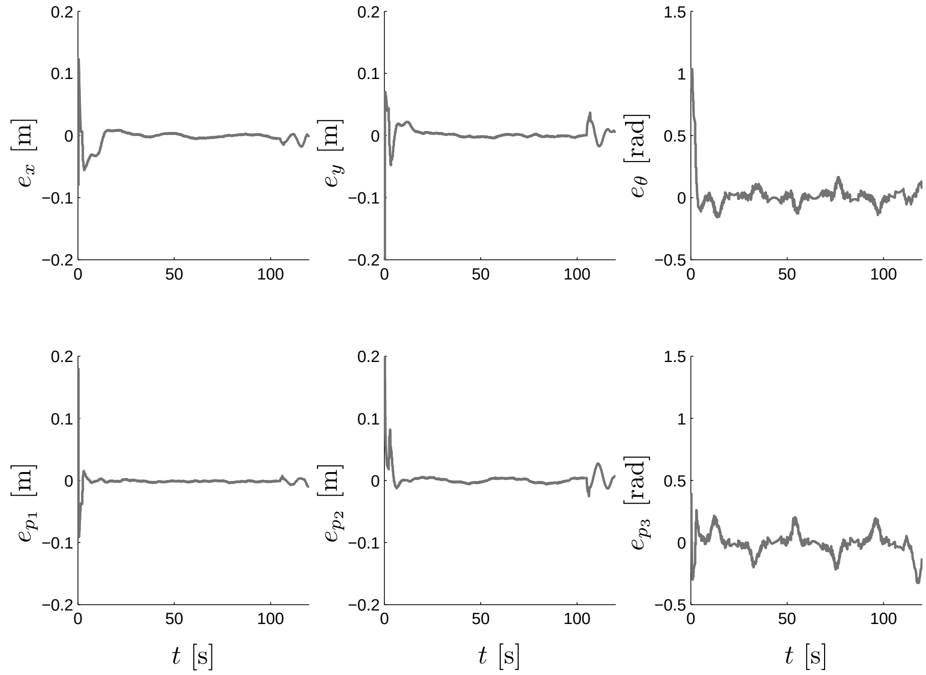


Figure 4.12 . Second experiment. First row: predictor tracking error; second row: prediction error.

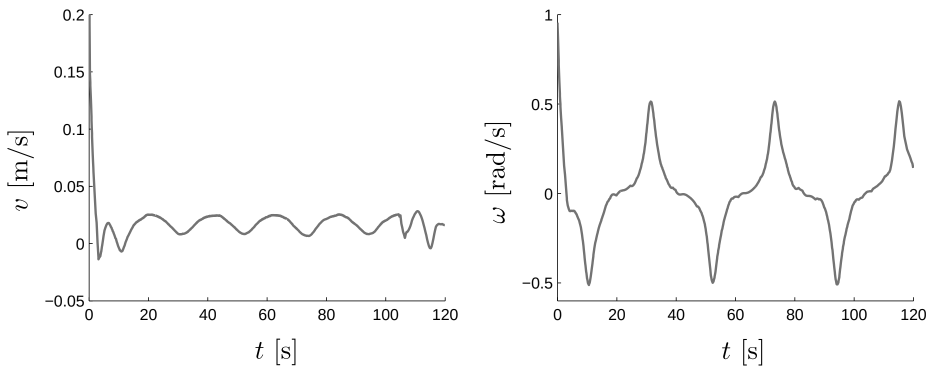


Figure 4.13 . Second experiment. Left: translational control input; right: rotational control input.

constraints, and quantization effects have yet to be considered in this framework. Although this certainly motivates future research, it (theoretically) limits, to some extent, the current application scope of the predictor-controller combination. Still, a challenging experimental case-study of controlling a mobile robot over the Internet between the Netherlands and Japan confirms the robustness of the proposed approach towards time-variation and uncertainty in the network delay.

A question that naturally arises is how the current state predictor inspired on synchronization differs from the well-known Smith predictor (Smith, 1957). Specifically, the application of the Smith predictor to nonlinear systems, first studied in Kravaris and Wright (1989), could be considered as an (alternative) option to achieve the remote tracking control of a unicycle. A distinguishing feature of the synchronization-based predictor is that it contains a mechanism that encourages the convergence of the delayed state of the system and the delayed predicted state by means of the correction term. In addition, this feature enables the predictor-controller combination to withstand (certain) perturbations, as shown by the simulation in Section 4.4.

On the other hand, as noted by (Michiels and Niculescu, 2007), most of the work regarding the Smith predictor focuses on its robustness and disturbance rejection characteristics. In its simplest implementation, such as in Kravaris and Wright (1989), the Smith predictor for nonlinear systems does not appear to provide a similar mechanism for convergence and disturbance rejection. Nevertheless, one should not carelessly jump into conclusions regarding the possible application of a Smith-predictor-based remote tracking control strategy to a unicycle robot, since a number of modifications and extensions which improve the performance of the Smith predictor in the context of nonlinear systems have been recently proposed (refer to Karafyllis and Krstic, 2010; Krstic, 2009, for an in-depth treatment of these ideas)). This modified Smith predictor requires the solution of the corresponding differential equation, which in the case of the unicycle robot can only be approximated numerically due to the system's non-holonomic constraint. In contrast, the predictor proposed in this thesis can be implemented in a similar way as an observer and has a lower computational cost than the aforementioned Smith-like predictor. Nevertheless, the remote control strategy resulting from the Smith-like predictor is capable of compensating for arbitrarily long delays, whereas the delay compensation strategy presented in this chapter has an upper bound on the maximum allowable time-delay. A detailed comparison between the modified Smith predictor and the predictor based on synchronization proposed in this thesis remains an interesting topic for future research.

The next chapter considers the occurrence of a communication-induced delay in a different setting; namely, within a group of mobile robots. In this case, the master-slave and mutual motion coordination strategies briefly introduced in Chapter 1 are explained in greater detail and studied under the assumption that the information exchange between the robots is subject to a communication delay.

5

Remote Motion Coordination of Mobile Robots

Abstract . This chapter considers the remote motion coordination of a group of unicycle robots. Suitable controllers to achieve delay-free master-slave and mutual motion coordination of a group of unicycle robots are presented at the beginning of the chapter. These two motion coordination strategies are then studied under the assumption that the communication channel which links the robots introduces a network-induced delay. A stability analysis shows that the group of robots can achieve master-slave or mutual motion coordination up to a maximum admissible delay. The performance of the proposed coordinating controllers is illustrated by means of simulations and experiments between the multi-robot platforms in Eindhoven, The Netherlands, and Tokyo, Japan.

5.1 Introduction

As explained already in Chapter 1, the use of a cooperative multi-robot system presents a number of advantages over the use of a single-robot system. The areas of application for this type of robotic systems is, at present, growing rapidly and such applications will most probably become commonplace in the future. In this context, the development of reliable coordination and cooperation strategies for teams or groups of mobile robots is particularly important.

This chapter focuses on different motion coordination methods for wheeled mobile robots and, more specifically, for unicycle-type mobile robots. In this respect, as explained in Chapter 1, the most prevalent approaches are the master-slave

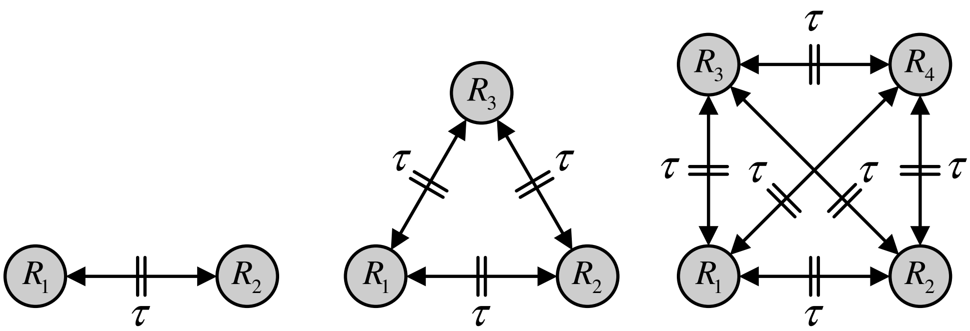


Figure 5.1 . Mutual motion coordination of two, three, and four mobile robots with delayed information exchange.

approach (also referred to as the leader-follower approach), the mutual approach (which allows the formation of a virtual structure), and the behavioral approach.

In this chapter, the coordinating controllers for master-slave and mutual motion coordination introduced in van den Broek et al. (2009) and Sadowska (2010) are recalled. These controllers are based on the trajectory tracking controller proposed by Panteley et al. (1998) and allow for the motion coordination of an arbitrary number of unicycles.

An important aspect to consider in these type of motion coordination strategies is the properties of the communication network which the robots use to exchange information. Nevertheless, most of the literature concerning motion coordination places more emphasis on the cooperative aspect than on the communication aspect. For this reason, we address the problem of the motion coordination of mobile robots considering a delay-inducing communication network. Motivated by our findings regarding the nature of the network-induced delay in the experimental platform being used in this work, it is possible to consider a constant communication delay between all the robots in the group (see Chapter 3 for additional details). In addition, because of the difficulty of the problem, we only consider the case when such delay is equal between all robots. Hereinafter, the problem being considered will be referred to as *remote motion coordination* of mobile robots. A schematic representation of the remote *mutual* motion coordination problem for two, three, and four robots is shown in Figure 5.1. In the figure, R_i denotes the i-th robot in the group, for $i \in \{1, 2 \ldots, n\}$, and the arrows represent the information takeing place between the robots which, as explained already, is delayed by τ. In this case, the network-induced delay is constant and equal for all robots.

It is worth noting that researchers have been focusing on the study of related problems, especially in the context of NCSs. In the NCSs literature, most of the work has focused on the study of robust stability in the face of uncertain, time-varying delays, and on stabilization problems, mostly for linear systems. While the trajectory tracking problem has received less attention, the problem of motion coordination has yet to be explored in detail. Motivated by this, we focus on the remote motion coordination of a group of n unicycle robots. It is worth noting that a preliminary approach to study some of the ideas contained in this chapter has already been reported in Alvarez-Aguirre et al. (2010a).

The contributions of this chapter may be summarized as follows. First, we propose a remote motion coordination strategy for a group of n unicycle robots. This strategy is based on existing coordinating controllers and guarantees the global uniform asymptotic stability of the complete group for delays smaller than a certain upper bound. In addition, we experimentally validate and study the performance of this motion coordination strategy using the multi-robot platform introduced in Chapter 3, which uses the Internet as the communication channel.

The outline for the remainder of this chapter is as follows. First, control schemes for delay-free master-slave and mutual motion coordination of a group of unicycle robots are introduced in Section 5.2 and Section 5.3, respectively. In Section 5.4, the remote master-slave motion coordination of a group of mobile robots is studied based on the definition of this type of motion coordination introduced in Section 5.2. A similar approach is taken in Section 5.5 where, inspired on the definition of mutual motion coordination presented in Section 5.3, sufficient conditions are derived to achieve this type of motion coordination when the communication exchange between the robots is subject to a time-delay. Both motion coordination strategies are tested in simulation and experiments in Section 5.6. In the experiments, two unicycle robots, one located at TU/e and the other one at TMU, are able to coordinate their motions in a master-slave and mutual fashion by exchanging information through the Internet. The concluding remarks of the chapter are given in Section 5.7.

5.2 Delay-Free Master-Slave Motion Coordination of Mobile Robots

This section concerns the master-slave motion coordination of a group of mobile robots. A control architecture to achieve this type of motion coordination in a group of $n+1$ mobile robots is shown in Figure 5.2. In the figure, u_m denotes the control inputs of the master robot, q_m its state, and q_{r_m} its reference trajectory; $u_{s,i}$ represents the control inputs of the i-th slave robot and $q_{s,i}$ its state, for $i \in \{1, 2, \dots, n\}$.

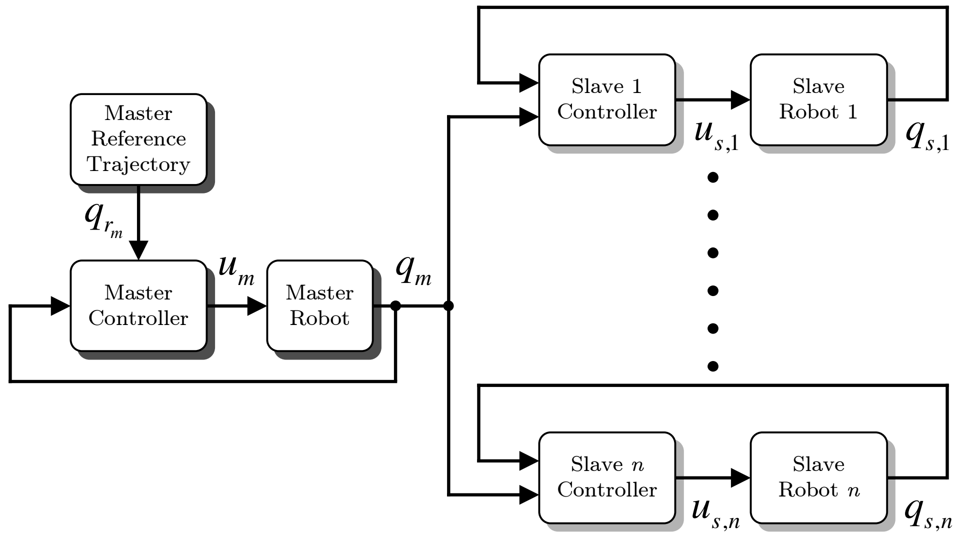


Figure 5.2 . Master-slave motion coordination of $n+1$ mobile robots.

As explained in Nijmeijer and Rodriguez-Ángeles (2003) for rigid robotic manipulators, in master-slave motion coordination one of the robots, denoted as the master, is independent from the other robots and dominates the group. In our case, the master robot prescribes a certain trajectory $q_m(t)=[x_m(t)\, y_m(t)\, \theta_m(t)]^T$ which, in general, depends on a given reference trajectory $q_{r_m}(t)=[x_{r_m}(t)\, y_{r_m}(t)\, \theta_{r_m}(t)]^T$ and control input $u_m(t)=[v_m(t)\, \omega_m(t)]^T$. The motions of the remaining n robots in the group, which are the non-dominant robots known as the slaves, are based on the trajectory prescribed by the master robot. In other words, the motion of the master robot is used as a starting point to derive the reference trajectories of the slaves. We consider two methods for constructing these reference trajectories.

The first method results in so-called *location oriented* reference trajectories. In this case, the reference displacement of each slave relative to the motion of the master is defined by possibly time-varying displacements $l_{x_{s,i}}(t)$ and $l_{y_{s,i}}(t)$ given with respect to the global coordinate frame $\underline{\vec{e}}^0$. If the displacements considered are constant, the slave reproduces the motion of the master in a different location within the workspace.

The second method results in so-called *formation oriented* reference trajectories. In this case, the relative displacements of the slaves with respect to the master are defined in terms of the robot-fixed coordinate frame of the master (denoted as $\underline{\vec{e}}^1$ in Figure 2.2). As a result, the master and slave robots maintain a geometrical formation defined by the distances between them in the frame fixed to the master.

The necessary mathematical expressions to construct the location oriented and formation oriented reference trajectories for the i-th slave robot in the group are provided in Appendix A. The resulting reference state trajectory in both cases is denoted by $q_{r_{s,i}}(t)=[x_{r_{s,i}}(t)\, y_{r_{s,i}}(t)\theta_{r_{s,i}}(t)]^T$. Due to the way $q_{r_{s,i}}(t)$ is derived, the reference trajectories of the slaves satisfy the non-holonomic constraint of the unicycle robot.

The i-th slave robot has a state $q_{s,i}(t) = [x_{s,i}(t)\, y_{s,i}(t)\, \theta_{s,i}(t)]^T$, whereas the state of its tracking error coordinates is $q_{e_{s,i}}(t)=[x_{e_{s,i}}(t)\, y_{e_{s,i}}(t)\, \theta_{e_{s,i}}(t)]^T$. These error coordinates are given as in (2.43), only adding subindex s, i in order to denote the i-th slave robot. In the same way, the tracking error dynamics of the i-th slave are given as in (2.44) (adding subindex s, i once more).

The following tracking controller $u_{s,i}(t) = [v_{s,i}(t)\, \omega_{s,i}(t)]^T$, equivalent to the one introduced for a single robot in (2.45), is proposed for the i-th slave robot:

$$v_{s,i}(t)=v_{r_{s,i}}(t)+c_{x_{s,i}}x_{e_{s,i}}(t)-c_{y_{s,i}}w_{r_{s,i}}(t)y_{e_{s,i}}(t), \quad c_{x_{s,i}} > 0,\ c_{y_{s,i}} > -1, \tag{5.1a}$$

$$\omega_{s,i}(t)=\omega_{r_{s,i}}(t)+c_{\theta_{s,i}}\theta_{e_{s,i}}(t), \quad c_{\theta_{s,i}} > 0, \tag{5.1b}$$

for $i \in \{1, 2, \ldots, n\}$.

Note that, because of the way that the reference trajectories of the slaves are derived, the master-slave motion coordination problem is in fact a problem of a tracking nature, since the objective of each slave robot is to track its respective reference trajectory. As mentioned above, the controller of the slave robots is the same as the original tracking controller (2.45). This implies that the controller of each slave ensures the global $\mathcal{K}$-exponential stability of its corresponding error dynamics as long as its reference rotational velocity is persistently exciting, its reference translational velocity is bounded and non-zero, and its tracking gains satisfy certain requirements. This follows directly from the stability analysis of the tracking controller (2.45).

5.3 Delay-Free Mutual Motion Coordination of Mobile Robots

A mutual motion coordination strategy for a group of n mobile robots is presented in this section. The corresponding control architecture is shown in Figure 5.3, where q_{vc} denotes the common virtual center (which is used to construct the reference trajectories of all the robots in the group), q_i denotes the state of the i-th robot, q_{e_i} its error coordinates, and u_i its control inputs, for $i \in \{1, 2 \ldots, n\}$.

Recall that mutual coordination of robotic manipulators was introduced in Ni-

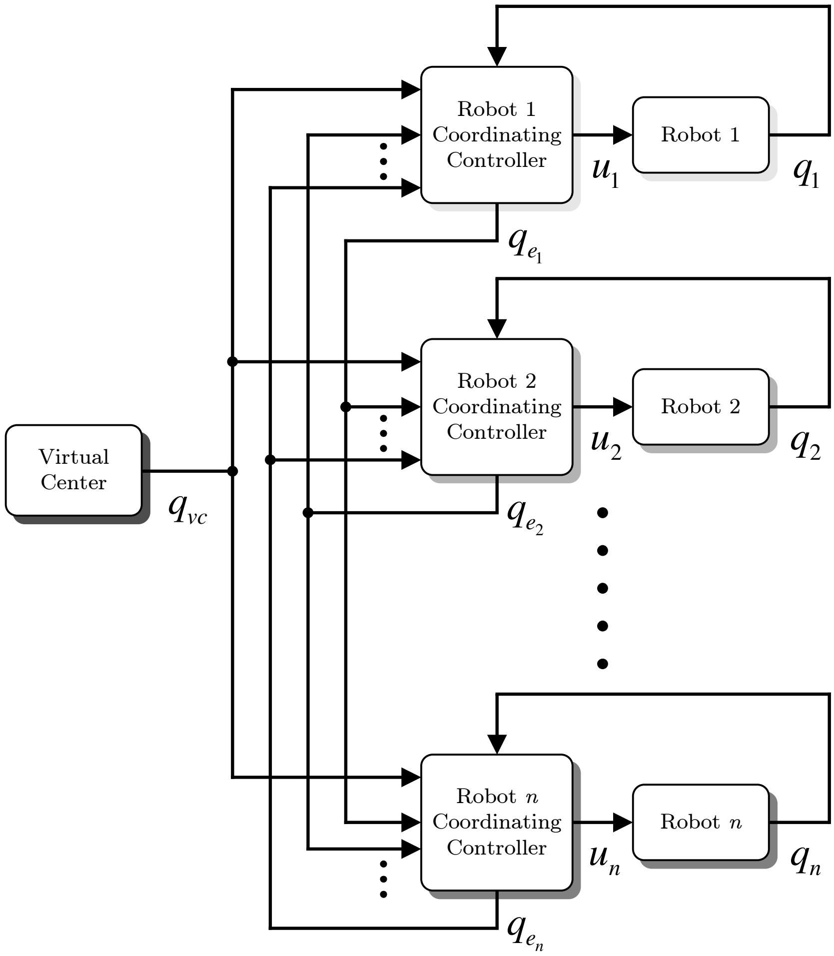


Figure 5.3 . Mutual motion coordination of n mobile robots.

jmeijer and Rodriguez-Ángeles (2003) and was applied to a group of two unicycle robots in van den Broek et al. (2009) by using the virtual structure approach. In this case, all the robots communicate their position and orientation errors to each other, forming a bidirectional all-to-all coupling. This means that, in this type of motion coordination, there is no hierarchy among the robots in the group, as opposed to master-slave motion coordination. As a result, the individual performance of each robot influences the overall performance of the group. This implies that a tradeoff exists between coordinated and individual behavior and is the reason why the group as a whole is better suited to withstand perturbations (this statement will be further clarified once the coordinating controller is introduced).

In mutual motion coordination, the reference trajectories for the n robots in the group are based on a common reference denoted as the virtual center. Equivalent to the master-slave case, two possibilities for constructing these reference trajectories are considered in this work; namely, a location oriented and a formation oriented approach. The main differences between these two methods have already been highlighted in Section 5.2, whereas the mathematical expressions required to produce the reference trajectories for a group of unicycles with mutually coordinated motions are available in Appendix A.

The reference trajectory for the i-th robot, for $i \in \{1, 2 \ldots, n\}$, is denoted by $q_{r_i}(t) = [x_{r_i}(t)\, y_{r_i}(t)\, \theta_{r_i}(t)]^T$, not withstanding if the reference is location oriented or formation oriented. Given the state of the i-th robot $q_i(t) = [x_i(t)\, y_i(t)\, \theta_i(t)]^T$, the error coordinates $q_{e_i}(t) = [x_{e_i}(t)\, y_{e_i}(t)\, \theta_{e_i}(t)]^T$ are defined as in (2.43), only adding subindex i in order to denote the i-th robot in the group. This means that the resulting error dynamics have the same form as (2.44) (adding subindex i).

These error dynamics are used to construct controllers $u_i(t) = [v_i(t)\, \omega_i(t)]^T$ that mutually coordinate the motions of the unicycles. Consider the tracking controller given in (2.45) as a starting point. As proposed in van den Broek et al. (2009), so-called mutual coupling terms are appended to the aforementioned control law. These coupling terms are introduced directly in the controller, resulting in the following mutual motion coordination control law for the i-th robot in the group:

$$\begin{aligned} v_i(t) &= v_{r_i}(t) + c_{x_i} x_{e_i}(t) - c_{y_i} w_{r_i}(t) y_{e_i}(t) + \sum_{j=1, i\neq j}^{n} k_{x_{i,j}} (x_{e_i}(t) - x_{e_j}(t)) \\ &\quad - \sum_{j=1, i\neq j}^{n} k_{y_{i,j}} \omega_{r_i}(t)(y_{e_i}(t) - y_{e_j}(t)), \end{aligned} \tag{5.2a}$$

$$\omega_i(t) = \omega_{r_i}(t) + c_{\theta_i} \theta_{e_i}(t) + \sum_{j=1, i\neq j}^{n} k_{\theta_{i,j}} \left(\theta_{e_i}(t) - \theta_{e_j}(t)\right). \tag{5.2b}$$

In this coordinating controller, the tracking gains are given by $c_{x_i}, c_{y_i} > 0$, and $c_{\theta_i} > -1$, whereas the gains $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} > 0$ determine the strength of the coupling between the i-th and j-th robots, for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$. Note that all the robots in the group exchange their error coordinates $q_{e_i}(t)$ with each other. This means that each robot receives the error coordinates of the other $n-1$ robots in order to determine its interaction with the group.

The reason for incorporating the coupling terms to the tracking control law (2.45) is for the controller of each robot to mediate between, on the one hand, the tracking of the individual reference trajectories and, on the other hand, ensuring that motion coordination is maintained. It should be clear, however, that such mediation is manifest only when one of the robots in the group is being perturbed. In this case, the magnitude of the coupling gain determines how the group copes with the perturbation, that is, whether the robots prioritize following their respective reference trajectory or maintaining coordinated motion. As a result, if none of the robots in the group is being perturbed, trajectory tracking and motion coordination can be guaranteed simultaneously in this setting.

The previous coordinating controller is the same as the one presented in Sadowska (2010) and is slightly different from the one introduced in van den Broek et al. (2009). The difference resides in the definition of the input rotational velocity $\omega_i(t)$; in van den Broek et al. (2009), the orientation error $\theta_{e_i}(t)$ and the differences between the orientation errors $\theta_{e_i}(t) - \theta_{e_j}(t)$ are all arguments of trigonometric functions (a sine in all cases). As stated in Jakubiak et al. (2002), this modification eases the construction of an orientation-error observer. Although this work only considers controllers which do not contain these sinusoidal terms, the approach to study their stability is (mostly) the same. However, as a consequence of the difference between them, with the current controller (that is, the one of Sadowska (2010)) a global stability analysis seems possible, whereas with the controller of van den Broek et al. (2009), only a local stability analysis is possible.

Given the open-loop error dynamics, the coordinating controller of the i-th robot, and the equalities defined in (2.2), the total closed-loop coordination error dynamics are given by

$$\begin{bmatrix}\dot{X}_e(t)\\ \dot{Y}_e(t)\end{bmatrix} = \begin{bmatrix}-C_x & \Omega_r(t)(I_{n\times n}+C_y)\\ -\Omega_r(t) & 0_{n\times n}\end{bmatrix}\begin{bmatrix}X_e(t)\\ Y_e(t)\end{bmatrix} + \begin{bmatrix}\bar{Y}_e(t)C_\theta V_r(t)\Theta_{e_{\sin}}\\ -\bar{X}_e(t)C_\theta + V_r(t)\Theta_{e_{\cos}}\end{bmatrix}\Theta_e(t), \tag{5.3a}$$

$$\dot{\Theta}_e(t) = -C_\theta \Theta_e(t), \tag{5.3b}$$

in which

$$\begin{aligned} X_e(t) &= \mathrm{col}(x_{e_1}(t),\dots,x_{e_n}(t)), & Y_e(t) &= \mathrm{col}(y_{e_1}(t),\dots,y_{e_n}(t)),\\ \bar{X}_e(t) &= \mathrm{diag}(x_{e_1}(t),\dots,x_{e_n}(t)), & \bar{Y}_e(t) &= \mathrm{diag}(y_{e_1}(t),\dots,y_{e_n}(t)),\\ \Omega_r(t) &= \mathrm{diag}(\omega_{r_1}(t),\dots,\omega_{r_n}(t)), & V_r(t) &= \mathrm{diag}(v_{r_1}(t),\dots,v_{r_n}(t)), \end{aligned}$$

$$\begin{aligned} \Theta_e(t) &= \mathrm{col}(\theta_{e_1}(t),\dots,\theta_{e_n}(t)),\\ \Theta_{e_{\sin}}(t) &= \mathrm{diag}\left(\int_0^1 \sin(s\theta_{e_1}(t))ds,\dots,\int_0^1 \sin(s\theta_{e_n}(t))ds\right),\\ \Theta_{e_{\cos}}(t) &= \mathrm{diag}\left(\int_0^1 \cos(s\theta_{e_1}(t))ds,\dots,\int_0^1 \cos(s\theta_{e_n}(t))ds\right), \end{aligned}$$

and

$$C_a = \begin{bmatrix} c_{a_1} + \sum k_{a_{1,j}} & -k_{a_{1,2}} & \cdots & -k_{a_{1,n}} \\ \vdots & \ddots & \ddots & \vdots \\ -k_{a_{n-1,1}} & \ddots & c_{a_{n-1}} + \sum k_{a_{n-1,j}} & -k_{a_{n-1,n}} \\ -k_{a_{n,1}} & -k_{a_{n,2}} & \cdots & c_{a_n} + \sum k_{a_{n,j}} \end{bmatrix},$$

where $\sum k_{a_{i,j}} = \sum_{j=1, j\neq i}^{n} k_{a_{i,j}}$ for $a \in \{x, y, \theta\}$.

Considering the following state definitions: $\xi_1(t) := [X_e^T(t)\, Y_e^T(t)]^T$ and $\xi_2(t) := \Theta_e^T(t)$, the closed-loop coordination error dynamics (5.3) may be represented as the following cascaded system:

$$\dot{\xi}_1 = A(t)\xi_1 + g(t, \xi_1, \xi_2)\xi_2, \tag{5.4a}$$

$$\dot{\xi}_2 = -C_\theta \xi_2, \tag{5.4b}$$

with

$$A(t) = \begin{bmatrix} -C_x & \Omega_r(t)(I_{n\times n} + C_y) \\ -\Omega_r(t) & 0_{n\times n} \end{bmatrix},$$

$$g(t, \xi_1, \xi_2) = \begin{bmatrix} \bar{Y}_e(t)C_\theta + V_r(t)\Theta_{e_{\sin}} \\ -\bar{X}_e(t)C_\theta + V_r(t)\Theta_{e_{\cos}} \end{bmatrix}.$$

Since the cascaded closed-loop error dynamics (5.4) are a particular case of the time-varying cascaded system (2.28), the approach to study their stability is similar to the one used for the tracking error dynamics in Section 2.6.

Given $Q_e(t) := [X_e^T(t)\, Y_e^T(t)\, \Theta_e^T(t)]^T$, the following proposition formulates sufficient conditions under which the equilibrium point $Q_e(t) = 0$ of the coordination error dynamics (5.3) is globally $\mathcal{K}$-exponentially stable.

Proposition 5.1. *Consider a group of n unicycle robots in which each robot has its own reference trajectory composed of its reference position $(x_{r_i}(t), y_{r_i}(t))$ and orientation $\theta_{r_i}(t)$, for $i \in \{1, 2 \ldots, n\}$. The reference trajectories of the robots are either location or formation oriented and are based on the position $(x_{vc}(t), y_{vc}(t))$ and orientation $\theta_{vc}(t)$ of a so-called virtual center and on possibly time-varying displacements $l_{x_i}(t)$ and $l_{y_i}(t)$ of robot i with respect to the virtual center. Suppose that all the robots in the group are equipped with a coordinating controller given as in (5.2), which contains feedforward terms $v_{r_i}(t)$ and $\omega_{r_i}(t)$ satisfying (2.41b) and (2.41c), respectively, a feedback part based on the error $q_{e_i}(t)$, and coupling terms. If the following conditions are satisfied:*

- *the reference translational velocities of the robots* $v_{r_i}(t) \neq 0$, $\forall t$ *and* $\forall i \in \{1, 2, \dots, n\}$, *are bounded;*
- *the reference rotational velocities of the robots* $\omega_{r_i}(t)$, $\forall i \in \{1, 2, \dots, n\}$, *are persistently exciting (PE);*
- *the tracking gains satisfy* $c_{x_i}, c_{\theta_i} > 0, c_{y_i} > -1$, $\forall i \in \{1, 2, \dots, n\}$, *whereas the coupling gains satisfy* $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} > 0$ *and are such that* $k_{x_{i,j}} = k_{x_{j,i}}$ *and* $k_{y_{i,j}} = k_{y_{j,i}}$, $\forall i, j \in \{1, 2, \dots, n\}$, $i \neq j$,

then, $Q_e(t) = 0$ *is a globally* $\mathcal{K}$*-exponentially stable equilibrium point of the closed-loop coordination error dynamics (5.3) and the group of* n *unicycles achieves mutual motion coordination.*

The proof of Proposition 5.1 is, in essence, the same as the proof of Theorem 5.1.2 in Sadowska (2010). Nevertheless, since there is a slight difference in the end result (we claim that the error dynamics (5.4) are globally $\mathcal{K}$-exponentially stable), we will at least present a sketch of the proof.

Proof. The proof follows from the application of Corollary 2.22. First, note that, as explained in Sadowska (2010), matrices C_x, C_y, and C_θ are positive definite (this can be shown by applying Gershgorin's circle theorem, that is, Theorem 2.1). In addition, from the conditions on $k_{x_{i,j}}$ and $k_{y_{i,j}}$ in Proposition 5.1, we have that matrices C_x and C_y are symmetric.

Consider now the ξ_1-dynamics without coupling $\dot{\xi}_1 = A(t)\xi_1$. In order to study their stability, the following candidate Lyapunov function has been proposed by Sadowska (2010):

$$V = \xi_1^T \frac{1}{2} \begin{bmatrix} I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} + C_y \end{bmatrix} \xi_1,$$

whose derivative along the solutions of (5.4) is given by

$$\dot{V} = -\xi_1^T \begin{bmatrix} C_x & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \xi_1 \leq 0.$$

Using Theorem 2.17 it is then possible to conclude the global exponential stability of the ξ_1-dynamics without coupling. First, given the above candidate Lyapunov function V and defining $C := [\hat{C}_x \, 0_{n\times n}]$ (where $\hat{C}_x^T \hat{C}_x = C_x$), we have that the following inequality

$$PA(t) + A^T(t) + C^T C \leq 0,$$

is satisfied for $P = \frac{1}{2}\begin{bmatrix} I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} + C_y \end{bmatrix}$, as required by Theorem 2.17. Then, with the aid of Theorem 2.19, we show that the pair $(A(t), C)$ is uniformly completely observable (UCO). As a result, the delay-free ξ_1-dynamics without coupling are GES (for additional details on this part, refer to the first part of the proof of Theorem 5.3, presented in Appendix C.2).

Regarding the ξ_2-dynamics, their global exponential stability follows from linear systems theory given that all eigenvalues of C_θ lie in the right-hand side of the complex plane. Finally, we can show that the growth of the coupling term term $g(t, \xi_1, \xi_2)$ is bounded by $2n\bar{V}_r + \|C_\theta\|_{\text{sum}} \|\xi_1\|_1$, where $\bar{V}_r := \max\limits_{i\in\{1,2,\ldots,n\}} \{\bar{v}_{r_i}\}$ and $\bar{v}_{r_i} := \sup\limits_{t\in\mathbb{R}} |v_{r_i}(t)|$. This completes the sketch of the proof. □

5.4 Remote Master-Slave Motion Coordination of Mobile Robots

The problem of achieving master-slave motion coordination in a group of $n + 1$ mobile robots has already been addressed in Section 5.2, where it has been concluded that this motion coordination problem is of a tracking nature.

In remote master-slave motion coordination we consider the case when the information of the master robot is available to the slaves after a certain time-delay τ_i, where subindex i denotes the i-th slave robot in the group. This delay is induced by the communication network which the master robot uses to relay its position and orientation measurements to the slave robots.

A control architecture which achieves master-slave motion coordination under these circumstances (denoted as remote master-slave motion coordination) is shown in Figure 5.4. In the figure, the slave robots receive the position and orientation information of the master after a certain time-delay τ_i, $i \in \{1, 2, \ldots, n\}$. In other words, the state of the master robot $q_m(t)$ is available to the i-th slave as $q_m(t - \tau_i)$, $i \in \{1, 2, \ldots, n\}$. With this information at hand, the slaves construct their respective reference trajectories using, for example, one of the methods described in Appendix A. Using the same tracking controller as the one presented in Section 5.2, the slaves should then be able to track these reference trajectories.

Since the reference trajectories of the slaves are formed with the delayed output of the master, the network-induced delay only affects these references. This means that the remote master-slave motion coordination problem remains a problem of a tracking nature.

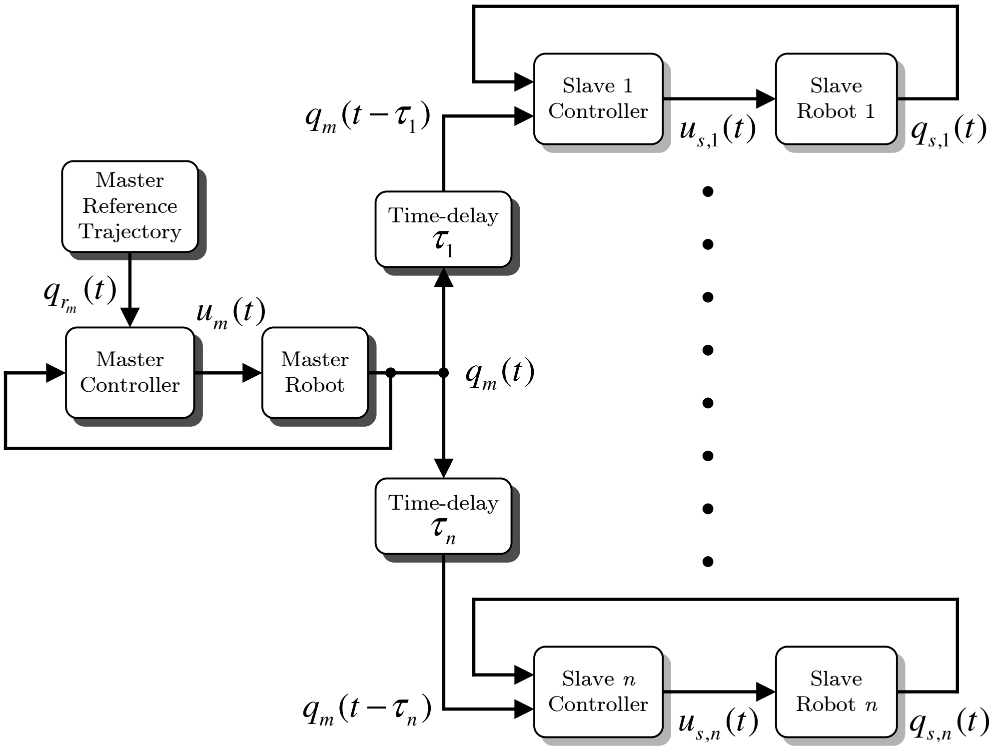


Figure 5.4 . Remote master-slave motion coordination of $n+1$ mobile robots. The i-th slave receives the output of the master, which is necessary to construct its own reference trajectory, after a time-delay τ_i.

A number of remarks regarding the nature of the network-induced delay are in place. To begin with, the time-delay τ_i may be different for each of the slaves. In addition, knowledge of the magnitude of these delays is not necessary to compute the reference trajectories of the slaves. Under some considerations, these delays may even be considered to be time-varying. The requirement in this case would be that the information from the master robot reaches the slaves in order. The key aspect to consider is that, eventually (and gradually), the state of the master robot becomes available to all the slaves and, as this happens, the slaves are able to derive their respective reference trajectories and track them.

An additional point to consider is the effect of the network-induced delay on the performance of the group. The definition of master-slave motion coordination given in Section 5.2 considers, for instance, the possibility of having the slaves reproduce exactly the same motion of the master in a different location within the workspace of the robots. The control objective in this case would be to have all the robots (including the master) tracking the same trajectory at the same time instant in different locations. Clearly, any network-induced delay in the information exchange from the master to the slaves would result in the slaves only being able to track the motion of the master after a certain time τ_i. As a consequence, the group would not be able to reach exact master-slave motion coordination but rather delayed coordination. In conclusion, it is possible to guarantee stability but not coordinated behavior in all cases.

5.5 Remote Mutual Motion Coordination of Mobile Robots

Recall the mutual motion coordination strategy presented in Section 5.3, where all the robots in a group of n mobile robots receive the trajectory of a virtual center which they use to produce their respective reference trajectories. In this type of motion coordination, the robots exchange their error coordinates, denoted as $q_{e_i}(t)$, $i \in \{1, 2, \ldots, n\}$, with each other. The error coordinates received from the remaining $n-1$ robots in the group are used to constitute so-called coupling terms, which in the event that any of the robots in the group is perturbed, help to mediate between having the robots follow their respective reference trajectories and ensuring that motion coordination is achieved.

In this section, we consider the case when the information exchange between the robots is subject to a network-induced delay. In the case of remote mutual motion coordination this delay, denoted by τ, is assumed to be constant and equal among all the robots in the group. Such a delay may be due to a number of reasons, some of which have already been explained in Chapter 1. Consider for instance the case when the characteristics of the communication channel which the robots use to exchange information are such that network-induced delays are unavoidable or the case when a considerable distance physically separates the robots. As explained already in Section 5.1, we refer to the resulting problem setting as remote mutual motion coordination. The control architecture which results when considering a delayed information exchange is shown in Figure 5.5, where all the robots in the group receive the common virtual center $q_{vc}(t)$ which they use to produce their respective reference trajectories $q_{r_i}(t)$, $i \in \{1, 2, \ldots, n\}$. In addition, each robot is equipped with a coordinating controller which produces the control inputs $u_i(t)$, $i \in \{1, 2, \ldots, n\}$. This controller requires the position and orientation measurements of the robot, that is, the state $q_i(t)$, the reference trajectory of the robot $q_{r_i}(t)$, which is based on the common virtual center, and the error coordinates of the remaining $n-1$ robots, which arrive after a time-delay τ, that is, $q_{e_j}(t-\tau)$, for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$.

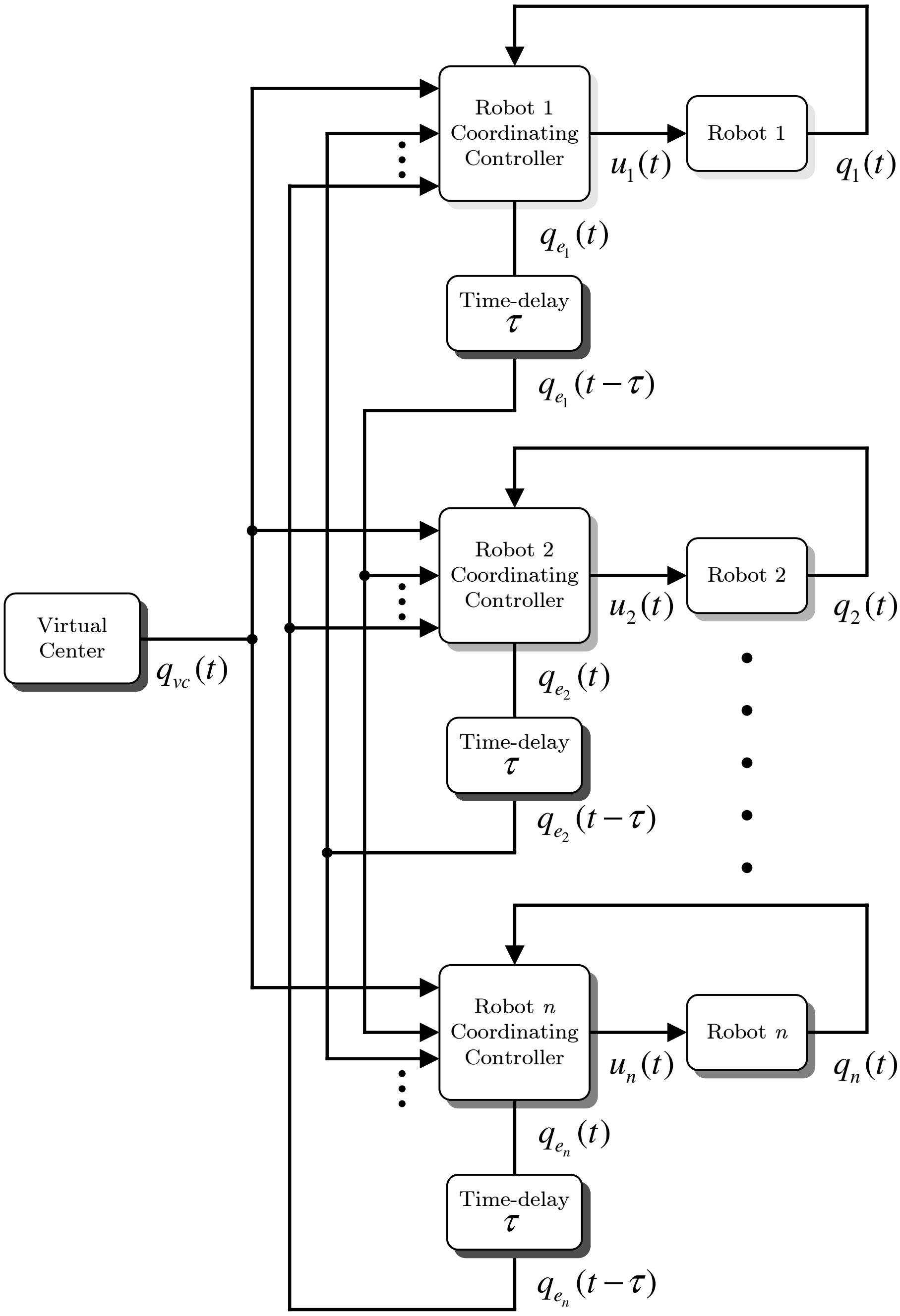


Figure 5.5 . Remote mutual motion coordination of n mobile robots. The error coordinates q_{e_i} that the robots exchange between each other are all subject to a constant, and equal, network-induced delay τ.

The requirement that all the robots have access to the virtual center trajectory without delay may be satisfied in a number of ways. One option is that the trajectory of the virtual center is known a priori and hence is locally available to all the robots. Another alternative is that the virtual center is generated in a remote location (consider for example a centralized command center) and is known a sufficient amount of time ahead such that $q_{r_i}(t)$ can be sent to the i-th robot so that the reference is available to the robot when required for coordination purposes. Note that the latter scenario would additionally require a precise synchronization of the clocks on the virtual center and all the robots.

Recall that the resulting reference trajectory for the i-th robot is denoted by $q_{r_i}(t)$ and that this reference is based on the common virtual center $q_{vc}(t)$ and can be computed using, for example, one of the methods described in Appendix A. Given the state of the robot, recall that its error coordinates $q_{e_i}(t)$ are given by

$$\begin{bmatrix} x_{e_i}(t) \\ y_{e_i}(t) \\ \theta_{e_i}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta_i(t) & \sin\theta_i(t) & 0 \\ -\sin\theta_i(t) & \cos\theta_i(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{r_i}(t) - x_i(t) \\ y_{r_i}(t) - y_i(t) \\ \theta_{r_i}(t) - \theta_i(t) \end{bmatrix}, \tag{5.5}$$

for $i \in \{1, 2, \ldots, n\}$. The ensuing error dynamics yield

$$\dot{x}_{e_i}(t) = \omega_i(t) y_{e_i}(t) + v_{r_i}(t)\cos\theta_{e_i}(t) - v_i(t), \tag{5.6a}$$

$$\dot{y}_{e_i}(t) = -\omega_i(t) x_{e_i}(t) + v_{r_i}(t)\sin\theta_{e_i}(t), \tag{5.6b}$$

$$\dot{\theta}_{e_i}(t) = \omega_{r_i}(t) - \omega_i(t), \tag{5.6c}$$

for $i \in \{1, 2, \ldots, n\}$, where $v_{r_i}(t)$ and $\omega_{r_i}(t)$ represent the reference translational and rotational velocities of the i-th robot, respectively. As explained in Section 2.6, these reference velocities must satisfy (2.41b) and (2.41c), respectively.

In order to achieve remote mutual motion coordination, the following coordinating controller $u_i(t) = [v_i(t)\, \omega_i(t)]^T$ is proposed for the i-th robot:

$$v_i(t) = v_{r_i}(t) + c_{x_i} x_{e_i}(t) - c_{y_i} w_{r_i}(t) y_{e_i}(t) + \sum_{j=1, j\neq i}^{n} k_{x_{i,j}} (x_{e_i}(t) - x_{e_j}(t-\tau))$$
$$- \sum_{j=1, j\neq i}^{n} k_{y_{i,j}} \omega_{r_i}(t)(y_{e_i}(t) - y_{e_j}(t-\tau)), \tag{5.7a}$$

$$\omega_i(t) = \omega_{r_i}(t) + c_{\theta_i}\theta_{e_i}(t) + \sum_{j=1, j\neq i}^{n} k_{\theta_{i,j}} \left(\theta_{e_i}(t) - \theta_{e_j}(t-\tau)\right), \tag{5.7b}$$

for $i \in \{1, 2, \ldots, n\}$. In (5.7), $c_{x_i}, c_{\theta_i} > 0$, and $c_{y_i} > -1$ denote the tracking gains of the controller and $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} > 0$ determine the coupling strength between the i-th and j-th robots, for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$.

The coordinating controller (5.7) is very similar to the one presented in Section 5.3 for delay-free mutual motion coordination. The difference between both controllers is that the one for remote mutual motion coordination considers a delayed information exchange. This means that, due to the network-induced delay, the coupling terms in (5.7) can only make use of the error coordinates of the other robots in the group once they become available after the time-delay τ, that is, $q_{e_j}(t-\tau)$ for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$. The following assumptions have been made on this delay.

Assumption 5.2. *The following holds for the communication delay τ:*

- *τ is constant and equal between all robots;*
- *τ is possibly unknown, but $q_{e_j}(t-\tau)$ for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$ is known for all t;*
- *$\tau \in [0, \tau_{\max}]$, where $\tau_{\max}$ is the maximum allowable time-delay for which the group of robots can achieve motion coordination.*

As discussed already, this assumption is motivated by the fact that the delay measurements presented in Chapter 3 (carried out between the experimental setups located in The Netherlands and Japan) are fairly constant.

In the current problem setting, all the robots in the group receive the information concerning the virtual center at the same time. This means that this information is readily available to all the robots and that they can use it to derive their respective reference trajectory. This motivates a related problem, which would be the case when the virtual center (and perhaps the reference trajectories themselves) are produced in a different location. Such circumstances would arise, for example, when considering a centralized and remote command center which assigns the reference trajectories for each individual robot. The main advantage of such configuration would be that it allows for optimal decisions to be taken based on global information about the group. Nonetheless, additional time-delays τ_{r_i} going from the command center that produces the trajectories to the i-th robot would have to be considered. The presence of these additional delays and the fact that the robots in the group do not receive their reference trajectories (or the virtual center) at the same time would require additional considerations in the stability analysis and perhaps a redefinition of what is to be considered as mutual motion coordination. We will refrain from such extensions in this thesis and focus on remote motion coordination as outlined at the beginning of this section.

Given the open-loop error dynamics (5.6) and the coordinating controller (5.7), the closed-loop error dynamics of the i-th robot are the following:

$$
\begin{aligned}
\dot{x}_{e_i}(t) = &-\left(c_{x_i} + \sum_{j=1,j\neq i}^{n} k_{x_{i,j}}\right) x_{e_i}(t) + \left(c_{y_i} + 1 + \sum_{j=1,j\neq i}^{n} k_{y_{i,j}}\right) \omega_{r_i}(t) y_{e_i}(t) \\
&+ \sum_{j=1,j\neq i}^{n} k_{x_{i,j}} x_{e_j}(t-\tau) - \sum_{j=1,j\neq i}^{n} k_{y_{i,j}} \omega_{r_i}(t) y_{e_j}(t-\tau) \\
&- v_{r_i}(t)\left(1 - \cos\theta_{e_i}(t)\right) + \left(c_{\theta_i} + \sum_{j=1,j\neq i}^{n} k_{\theta_{i,j}}\right) y_{e_i}(t)\theta_{e_i}(t) \\
&- y_{e_i}(t) \sum_{j=1,j\neq i}^{n} k_{\theta_{i,j}} \theta_{e_j}(t-\tau), \qquad (5.8a)
\end{aligned}
$$

$$
\begin{aligned}
\dot{y}_{e_i}(t) = &-\omega_{r_i}(t) x_{e_i}(t) + v_{r_i}(t)\sin\theta_{e_i}(t) - \left(c_{\theta_i} + \sum_{j=1,j\neq i}^{n} k_{\theta_{i,j}}\right) x_{e_i}(t)\theta_{e_i}(t) \\
&+ x_{e_i}(t) \sum_{j=1,j\neq i}^{n} k_{\theta_{i,j}} \theta_{e_j}(t-\tau), \qquad (5.8b)
\end{aligned}
$$

$$
\dot{\theta}_{e_i}(t) = -\left(c_{\theta_i} + \sum_{j=1,j\neq i}^{n} k_{\theta_{i,j}}\right) \theta_{e_i}(t) + \sum_{j=1,j\neq i}^{n} k_{\theta_{i,j}} \theta_{e_j}(t-\tau). \qquad (5.8c)
$$

Considering the complete group of n unicycles and the equalities defined in (2.2), the closed-loop error dynamics (5.8) may be rewritten in compact matrix form as follows:

$$
\begin{aligned}
\begin{bmatrix} \dot{X}_e(t) \\ \dot{Y}_e(t) \end{bmatrix} = &\begin{bmatrix} -C_x & \bar{\Omega}_r(t)(I_{n\times n} + C_y) \\ -\bar{\Omega}_r(t) & 0_{n\times n} \end{bmatrix} \begin{bmatrix} X_e(t) \\ Y_e(t) \end{bmatrix} + \begin{bmatrix} K_x & -\bar{\Omega}_r(t) K_y \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \begin{bmatrix} X_e(t-\tau) \\ Y_e(t-\tau) \end{bmatrix} \\
&+ \begin{bmatrix} \bar{Y}_e(t) C_\theta - V_r(t)\Theta_{e_{\sin}} \\ -\bar{X}_e(t) C_\theta + V_r(t)\Theta_{e_{\cos}} \end{bmatrix} \Theta_e(t) + \begin{bmatrix} -\bar{Y}_e(t) K_\theta \\ \bar{X}_e(t) K_\theta \end{bmatrix} \Theta_e(t-\tau), \qquad (5.9a) \\
\dot{\theta}_e(t) = &-C_\theta \Theta_e(t) + K_\theta \Theta_e(t-\tau), \qquad (5.9b)
\end{aligned}
$$

in which

$$
\begin{aligned}
X_e(t) &= \mathrm{col}(x_{e_1}(t), \ldots, x_{e_n}(t)), & Y_e(t) &= \mathrm{col}(y_{e_1}(t), \ldots, y_{e_n}(t)), \\
\Theta_e(t) &= \mathrm{col}(\theta_{e_1}(t), \ldots, \theta_{e_n}(t)), & \bar{X}_e(t) &= \mathrm{diag}(x_{e_1}(t), \ldots, x_{e_n}(t)), \\
\bar{Y}_e(t) &= \mathrm{diag}(y_{e_1}(t), \ldots, y_{e_n}(t)), & \bar{\Omega}_r(t) &= \mathrm{diag}(\omega_{r_1}(t), \ldots, \omega_{r_n}(t)), \\
V_r(t) &= \mathrm{diag}(v_{r_1}(t), \ldots, v_{r_n}(t)),
\end{aligned}
$$

$$\Theta_{e_{\sin}}(t) = \operatorname{diag}\left(\int_0^1 \sin(s\theta_{e_1}(t))ds, \dots, \int_0^1 \sin(s\theta_{e_n}(t))ds\right),$$
$$\Theta_{e_{\cos}}(t) = \operatorname{diag}\left(\int_0^1 \cos(s\theta_{e_1}(t))ds, \dots, \int_0^1 \cos(s\theta_{e_n}(t))ds\right),$$

and

$$C_a = \begin{bmatrix} c_{a_1} + \sum k_{a_{1,j}} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & c_{a_{n-1}} + \sum k_{a_{n-1,j}} & 0 \\ 0 & 0 & \dots & c_{a_n} + \sum k_{a_{n,j}} \end{bmatrix},$$

$$K_a = \begin{bmatrix} 0 & k_{a_{1,2}} & \dots & k_{a_{1,n}} \\ \vdots & \ddots & \ddots & \vdots \\ k_{a_{n-1,1}} & \ddots & 0 & k_{a_{n-1,n}} \\ k_{a_{n,1}} & k_{a_{n,2}} & \dots & 0 \end{bmatrix},$$

in which $\sum k_{a_{i,j}} = \sum_{j=1, j\neq i}^{n} k_{a_{i,j}}$, for $a \in \{x, y, \theta\}$.

Given the following state definitions: $\xi_1(t) := [X_e^T(t)\, Y_e^T(t)]^T$ and $\xi_2(t) := \Theta_e^T(t)$, the closed-loop error dynamics may be represented as the following cascaded system:

$$\dot{\xi}_1(t) = A_1(t)\xi_1(t) + A_2(t)\xi_1(t-\tau) + g(t, \xi_{1_t}, \xi_{2_t}), \tag{5.10a}$$
$$\dot{\xi}_2(t) = -C_\theta \xi_2(t) + K_\theta \xi_2(t-\tau), \tag{5.10b}$$

where ξ_{i_t}, $i = 1, 2$, is an element of the Banach space $\mathcal{C}(l_i) = C([-\tau, 0], \mathbb{R}^{l_i})$, with $l_1 = 2n$ and $l_2 = n$, defined by $\xi_{i_t}(s) := \xi_i(t+s)$ for $s \in [-\tau, 0]$ and

$$A_1(t) = \begin{bmatrix} -C_x & \bar{\Omega}_r(t)(I_{n\times n} + C_y) \\ -\bar{\Omega}_r(t) & 0_{n\times n} \end{bmatrix},$$
$$A_2(t) = \begin{bmatrix} K_x & -\bar{\Omega}_r(t)K_y \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix},$$
$$g(t, \xi_{1_t}, \xi_{2_t}) = \begin{bmatrix} \bar{Y}_e(t)C_\theta - V_r(t)\Theta_{e_{\sin}} \\ -\bar{X}_e(t)C_\theta + V_r(t)\Theta_{e_{\cos}} \end{bmatrix} \xi_2(t) + \begin{bmatrix} -\bar{Y}_e(t)K_\theta \\ \bar{X}_e(t)K_\theta \end{bmatrix} \xi_2(t-\tau).$$

The cascaded system (5.10) is a particular case of the nonlinear delayed cascaded system (2.38). This means that the results of Sedova (2008b) introduced in Theorem 2.26 in Chapter 2 will be particularly useful when studying its stability.

Considering the presence of the network-induced delay as a premise and the coupling between the robot as a design requirement, we now study the stability of the closed-loop error dynamics (5.10). This stability analysis will show that, for some bounded uncertainty interval for the constant (possibly unknown) time-delay and a given set of control parameters (that is, tracking and coupling gains and reference trajectories satisfying certain assumptions), the network-induced delay is not detrimental for mutual motion coordination. As a matter of fact, it is this capacity to attain motion coordination in the presence of a delay-inducing communication link which allows the use of a coordinating controller based on the one studied in Section 5.3 for delay-free mutual motion coordination.

Given the definition $Q_e(t) := [X_e^T(t)\, Y_e^T(t)\, \Theta_e^T(t)]^T$, the following theorem provides sufficient conditions to establish the global uniform asymptotic stability of the equilibrium point $Q_e(t) = 0$ of the closed-loop error dynamics (5.9).

Theorem 5.3. *Consider a group of n unicycle robots, in which each robot receives its own reference trajectory, composed of its reference position $(x_{r_i}(t), y_{r_i}(t))$ and orientation $\theta_{r_i}(t)$, $i \in \{1, 2, \ldots, n\}$. These reference trajectories are either location or formation oriented and are based on the position $(x_{vc}(t), y_{vc}(t))$ and orientation $\theta_{vc}(t)$ of a so-called virtual center, and on possibly time-varying displacements $l_{x_i}(t)$ and $l_{y_i}(t)$ relative to $x_{vc}(t)$ and $y_{vc}(t)$, respectively. Suppose that all the robots in the group are equipped with a coordinating controller as in (5.7), in which the feedforward terms $v_{r_i}(t)$ and $\omega_{r_i}(t)$ satisfy (2.41b) and (2.41c), respectively, and the network-induced delay τ satisfies Assumption 5.2. If the following conditions are satisfied:*

- *the reference translational velocities of the robots $v_{r_i}(t) \neq 0$, $\forall t$, and $\forall i \in \{1, 2, \ldots, n\}$, are bounded;*
- *the vector of reference rotational velocities $\Omega_r(t) = [\omega_{r_i}(t), \ldots, \omega_{r_n}(t)]^T$ is persistently exciting (PE) according to Definition 2.5;*
- *the tracking gains satisfy $c_{x_i}, c_{\theta_i} > 0, c_{y_i} > -1$, $\forall i \in \{1, 2, \ldots, n\}$, whereas the coupling gains satisfy $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} > 0$ and are such that $k_{x_{i,j}} = k_{x_{j,i}}$ and $k_{y_{i,j}} = k_{y_{j,i}}$, $\forall i, j \in \{1, 2, \ldots, n\}$, $i \neq j$,*

then, for $c_{\theta_i} > 0$ sufficiently large, there exists a $\tau_{\max} > 0$ for which $Q_e(t) = 0$ is a globally uniformly asymptotically stable (GUAS) equilibrium point of the closed-loop coordination error dynamics (5.9) for all $\tau \in [0, \tau_{\max}]$. This means that the group of n unicycles achieves remote mutual motion coordination under delayed communication.

Proof. Only a sketch of the proof is presented in this chapter, whereas the complete proof is given in Appendix C.2.

The global uniform asymptotic stability of the equilibrium point $Q_e(t) = 0$ of the cascaded system (5.9) is guaranteed provided the following conditions, which follow from the application of Theorem 2.26, are satisfied:

- the system $\dot{\xi}_1(t) = A_1(t)\xi_1(t) + A_2(t)\xi_1(t-\tau)$, denoted as the ξ_1-dynamics without coupling, is globally exponentially stable (GES) with a quadratic Lyapunov-Razumikhin function V_{ξ_1};
- the system $\dot{\xi}_2(t) = -C_\theta\xi_2(t) + K_\theta\xi_2(t-\tau)$, denoted as the ξ_2-dynamics, is globally exponentially stable (GES);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ admits the estimate

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c.$$

  for continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The proof goes along the same lines as the proof of Theorem 4.3. Hence, the exponential stability requirements on the ξ_1-dynamics without coupling and the ξ_2-dynamics are also based on the assumptions in Theorem 2.26 and the remarks that follow the theorem.

The three conditions are checked considering the conditions in the theorem on the tracking gains c_{x_i}, c_{y_i}, and c_{θ_i}, $i \in \{1, 2, \ldots, n\}$, coupling gains $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}}$, $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$, and reference translational and rotational velocities $v_{r_i}(t)$ and $\omega_{r_i}(t)$, $i \in \{1, 2, \ldots, n\}$, respectively.

Checking the condition on the ξ_1-dynamics without coupling is done in the same way as in the proof of Theorem 4.3. First, the delay-free version of the ξ_1-dynamics without coupling is determined to be GES. Consequently, from Lyapunov converse theory it follows that there exists a strict Lyapunov function V_{ξ_1} for these delay-free error dynamics. This function is then proposed as a candidate Lyapunov-Razumikhin function for the ξ_1-dynamics without coupling. Due to the fact that V_{ξ_1} is not known explicitly, the resulting conditions for the global asymptotic stability of the ξ_1-dynamics without coupling (and hence of the complete closed-loop error dynamics (5.9)) are only qualitative.

The ξ_2-dynamics, which are given by a delayed LTI system, are determined to be delay-independent using Lyapunov-Razumikhin stability criteria for LTI systems (see Proposition 5.3 of Gu et al., 2003, for additional details on the criterion used).

Using vector and matrix norms it is shown that the condition on the coupling term is satisfied with $\alpha_1(\|\varphi_{\xi_2}\|) = 2n\bar{V}_r$ and $\alpha_2(\|\varphi_{\xi_2}\|) = \|C_\theta\|_{\text{sum}} + \|K_\theta\|_{\text{sum}}$, where $\bar{V}_r := \max_{i\in\{1,2,\ldots,n\}} \{\bar{v}_{r_i}\}$ and $\bar{v}_{r_i} := \sup_{t\in\mathbb{R}} |v_{r_i}(t)|$, for $i \in \{1, 2, \ldots, n\}$.

Since the three conditions stated at the beginning of the proof have been checked, the equilibrium point $Q_e(t) = [X_e^T(t)\, Y_e^T(t)\, \Theta_e^T(t)]^T = 0$ of the closed-loop error dynamics (5.9) is concluded to be GUAS for the requirements posed in the theorem. This completes the sketch of the proof. □

Several remarks are in place regarding the previous theorem. Clearly, the most important one is that the theorem does not provide explicit bounds on the allowable time-delay $\tau \in [0, \tau_{\max}]$ such that the global uniform asymptotic stability of the closed-loop error dynamics (5.9) is guaranteed. This is due to the fact that a strictly decaying Lyapunov-Razumikhin function (that is, with a negative definite derivative) for the system $\dot{\xi}_1(t) = A_1(t)\xi_1(t) + A_2(t)\xi_1(t-\tau)$ (the unperturbed position error dynamics) is not available. In other words, although the global exponential stability of these error dynamics may be concluded, explicitly formulating the bounds on the allowable time-delay is not possible since a Lyapunov-Razumikhin function whose derivative contains all the elements of the state $\xi_1(t)$ is not known. Even if these bounds are not explicitly known, the proof presented in Appendix C.2 provides an expression which gives an idea about the nature of the allowable time-delay (refer to Section C.2.1 for additional details, in particular to (C.18) for the expression we refer to above).

It is worth noting that it might be possible to obtain explicit bounds for the allowable time-delay by making additional requirements on the reference rotational velocity $\omega_{r_i}(t)$. For instance, Sedova (2008a) studies the tracking problem considering a measurement delay and assumes a constant reference rotational velocity. Although this allows to formulate explicit bounds on the time-delay, it is not convenient at all from a practical point of view, due to the fact that the set of possible reference trajectories is greatly restricted. Since we are considering a group of mobile robots which are supposed to track a variety of reference trajectories in order to coordinate their motions and complete a certain task, making such an assumption does not represent an attractive option and consequently will not be pursued.

An additional remark which follows from the specific details in the proof is that there is a stark contrast between the stability result obtained for the unperturbed position error dynamics and the one obtained for system (5.10b), that is, the orientation error dynamics. Whereas the global exponential stability of the unperturbed position error dynamics depends on the magnitude of the time-delay τ, the global exponential stability of the orientation error dynamics is actually delay-independent. In consequence, exploring alternative approaches to study the stability of the unperturbed position error dynamics in order to obtain larger delay bounds remains an important issue for future work.

5.6 Simulation and Experimental Results

The following simulation and experiments demonstrate the applicability of the remote motion coordination strategies presented in this chapter. In the experiments, the multi-robot platform introduced in Chapter 3 is used to implement master-slave and mutual motion coordination between a robot located at TU/e and another one at TMU.

The results encompass one numerical simulation (remote mutual motion coordination of four robots) and two experiments (the first one for remote master-slave motion coordination and the second one for remote mutual motion coordination). In the case of the master-slave experiment, the master is located at TU/e and the slave is located at TMU. In mutual motion coordination, robot 1 is located at TMU and robot 2 is located at TU/e. As assumed in Section 5.5, in the remote mutual coordination simulation and experiment all the robots have access to the virtual center $q_{vc}(t)$ without delay. This means that copies of the virtual center trajectory are locally generated for all robots and that the starting time of these trajectories is determined from (almost) synchronized clocks.

The reference trajectory of the master robot in the first experiment and the virtual center in the simulation and second experiment are given as follows:

- Experiment 1 (remote master-slave motion coordination): a sinusoid with parametric equations given as in (4.16), with origin at $(x_{r_0}, y_{r_0}) = (0.3, 0.3)$ m, an orientation of $\theta_{r_0} = 0$ rad, translational and rotational velocities of $v_{r_0} = 0.01$ m/s and $\omega_{r_0} = 0.3$ rad/s, respectively, and an amplitude of $a = 0.15$ m.
- Simulation (remote mutual motion coordination): a circle with parametric equations given as in (4.14), with a radius of $r = 0.5$ m, centered at $(x_{r_c}, y_{r_c}) = (1.5, 1.5)$ m, a reference rotational velocity of $\omega_r = 0.5$ rad/s, and a reference translational velocity of $v_r = 0.25$ m/s.
- Experiment 2 (remote mutual motion coordination): an eight curve with parametric equations given as in (4.15), centered at $(x_{r_c}, y_{r_c}) = (0.85, 0.62)$ m, a length and width of $2a = 2b = 0.4$ m, and an angular velocity of $c = 0.2$ m/s.

Table 5.1 . Initial conditions, tracking gains, and displacements used in the simulation/experiments. The displacements are from the master or the virtual center, depending on the type of motion coordination, and are given with respect to the world-fixed coordinate frame of each robot.

	Initial conditions			Controller gains			Displacements		Delay
	x_0 [m]	y_0 [m]	θ_0 [rad]	c_x	c_y	c_θ	l_x [m]	l_y [m]	τ [ms]
First experiment: remote master-slave motion coordination.									
Master	0.1833	0.1124	1.1042	2.0	2.0	1.0	0.0	0.0	268
Slave	0.1028	0.2581	0.8591	5.5	13.0	3.0	−0.2	−0.25	268
Simulation: remote mutual motion coordination.									
Robot 1	1.4	0.6	$\frac{\pi}{4}$	1.0	1.0	1.0	1.0	0.5	750
Robot 2	1.8	1.4	$\frac{\pi}{3}$	1.0	1.0	1.0	1.0	−0.5	750
Robot 3	0.1	0.4	$\frac{\pi}{2}$	1.0	1.0	1.0	−1.0	−0.5	750
Robot 4	0.6	1.2	π	1.0	1.0	1.0	−1.0	0.5	750
Second experiment: remote mutual motion coordination.									
Robot 1	0.4746	0.3108	0.7413	4.0	8.0	1.0	0.0	0.0	268
Robot 2	0.7466	0.4381	0.8182	4.0	8.0	1.0	−0.35	−0.37	268

The initial conditions of the robots, the magnitude of their tracking gains, the desired displacements of each robot from either the master robot or the virtual center (depending on the motion coordination strategy being evaluated), and the magnitude of the network-induced delay are all given in Table 5.1 for the simulation and both experiments. In the mutual motion coordination case, the coupling gains are $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} = 0.5$ in the simulation and $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} = 1.0$ in the experiment, for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$, and $n = 2$ and $n = 4$, respectively.

The plots in Figure 5.6, Figure 5.7, and Figure 5.8 show the results of the remote master-slave experiment. The reference trajectory and path of the master and slave robots are shown in Figure 5.6 in their respective world-fixed coordinate frame ($\underline{\vec{e}}^m$ for the master at TU/e and $\underline{\vec{e}}^s$ for the slave at TMU). A cross and a circle highlight the initial and final positions of the reference trajectories and robot paths, respectively (this will also be the case for the remaining simulation and experiment). The plots show how the path of the slave robot converges to the path of the master robot (in this respect, recall that the reference of the slave equals the delayed trajectory of the master).

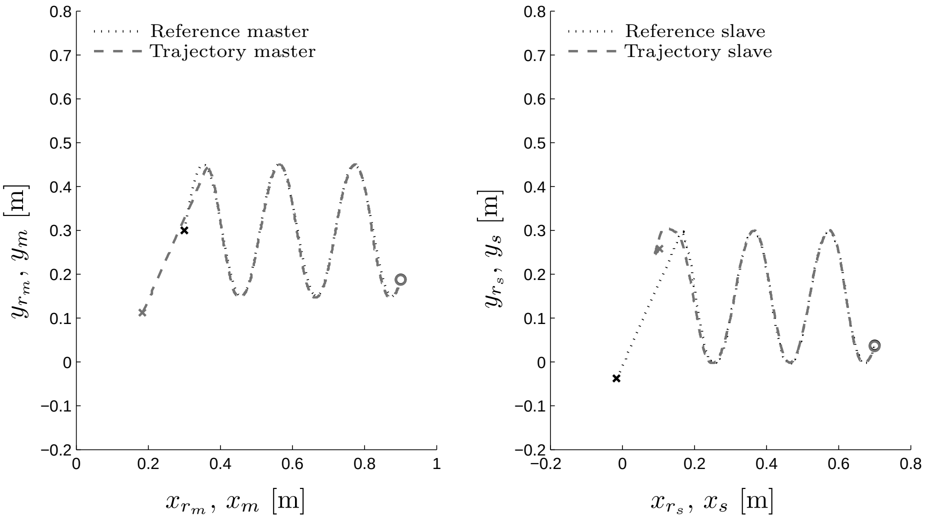


Figure 5.6 . First experiment: remote master-slave motion coordination of two unicycle robots located at TMU (slave) and TU/e (master). Reference and robot trajectories shown in the respective world-fixed coordinate frame of the master and slave robots, that is $\underline{\vec{e}}^m$ and $\underline{\vec{e}}^s$, respectively.

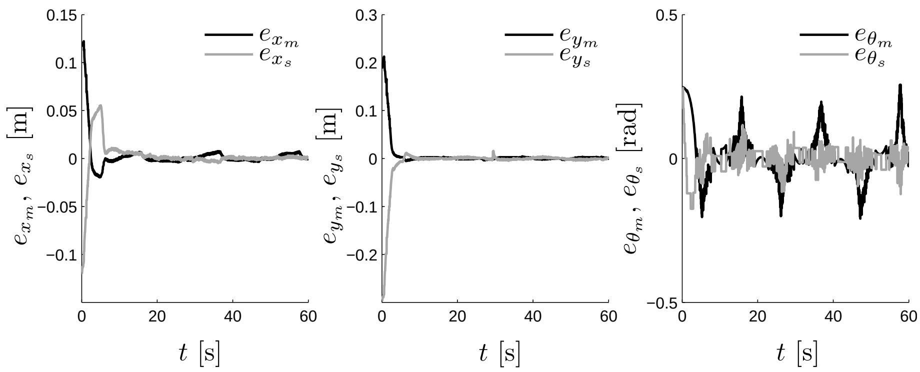


Figure 5.7 . First experiment. Tracking errors of the master and slave robots.

The plots in Figure 5.7 show the tracking errors of the master and slave robots, defined as $e_{x_m}(t) = x_{r_m}(t) - x_m(t)$, $e_{y_m}(t) = y_{r_m}(t) - y_m(t)$, and $e_{\theta_m}(t) = \theta_{r_m}(t) - \theta_m(t)$, for the master and $e_{x_s}(t) = x_{r_s}(t) - x_s(t)$, $e_{y_s}(t) = y_{r_s}(t) - y_s(t)$, and $e_{\theta_s}(t) = \theta_{r_s}(t) - \theta_s(t)$ for the slave. All errors converge to zero, which indicates that the unicycles achieve delayed master-slave motion coordination irrespective of the network-induced delay. The control inputs of each robot for the remote master-slave experiment are shown in Figure 5.8.

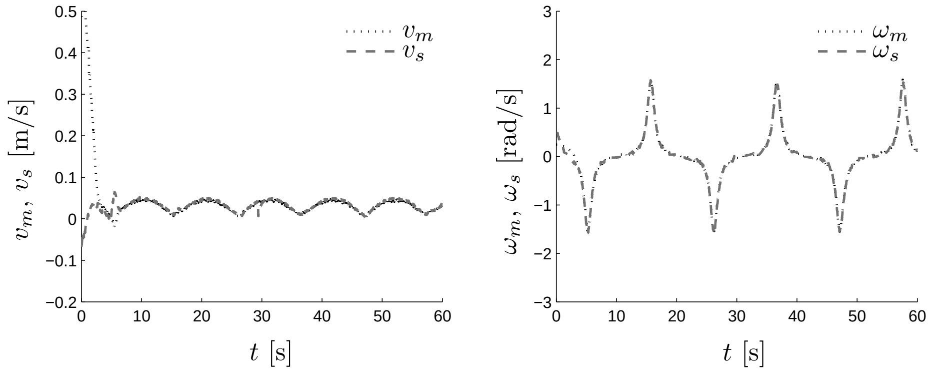


Figure 5.8 . First experiment. Left: translational control input of the master and slave; right: rotational control input of the master and slave.

The results of the mutual motion coordination simulation are shown in Figure 5.9, Figure 5.10, Figure 5.11, and Figure 5.12. The reference trajectory and path of each robot in the global coordinate frame are depicted in Figure 5.9. To highlight the behavior of the group in the face of perturbations, the translational and rotational velocities of the second robot are perturbed by 0.3 m/s and 0.6 rad/s, respectively, during 2 s at $t = 40$ s and of the fourth robot by 0.4 m/s and -0.2 rad/s during 3 s at $t = 80$ s. The plots show how the robots achieve mutual motion coordination, how the perturbations are reflected on all the other robots in the group, and how the couplings between the robots allow the group to cope with these disturbances.

The plots in Figure 5.10 show the tracking errors of each robot. These tracking errors are defined as $e_{x_i}(t) = x_{r_i}(t) - x_i(t)$, $e_{y_i}(t) = y_{r_i}(t) - y_i(t)$, and $e_{\theta_i}(t) = \theta_{r_i}(t) - \theta_i(t)$, for $i = 1, 2, 3, 4$. The coordination errors between the robots are shown in Figure 5.11. These errors are defined as $e_{x_i}(t) - e_{x_j}(\mathrm{t})$, $e_{y_i}(t) - e_{y_j}(\mathrm{t})$, and $e_{\theta_i}(t) - e_{\theta_j}(\mathrm{t})$, for $i, j \in \{1, 2, 3, 4\}$, $j \neq i$. The plots in both figures show that the tracking and coordination errors converge to zero and, as expected, all the robots in the group reflect the perturbations which affect second and fourth robots at $t = 40$ s and $t = 80$ s. The control actions of the i-th robot are shown in Figure 5.12 and also reflect the perturbations.

The results of the mutual motion coordination experiment are shown in Figure 5.13, Figure 5.14, and Figure 5.15. The reference trajectory and path of each robot in their respective world-fixed coordinate frame ($\underline{\vec{e}}^1$ for robot 1 at TMU and $\underline{\vec{e}}^2$ for robot 2 at TU/e) are depicted in Figure 5.13. In this experiment, the robot at TMU is perturbed by manually displacing it at approximately 14.6 s. The plots confirm that the robots achieve mutual motion coordination and that the perturbation on the robot at TMU is reflected on the robot at TU/e.

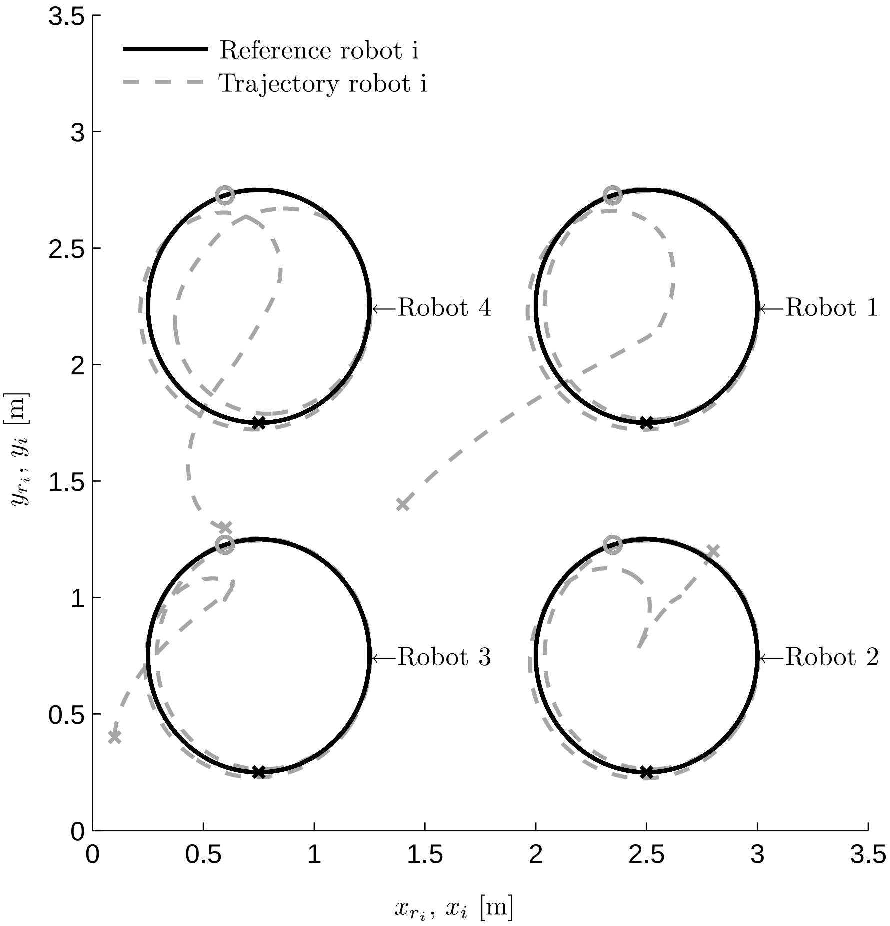


Figure 5.9 . Simulation: remote mutual motion coordination of four unicycle robots ($i = 1, 2, 3, 4$). The reference and robot trajectories are shown in the global coordinate frame $\underline{\vec{e}}^0$. The second and fourth robots are perturbed at $t = 40\,\mathrm{s}$ and $t = 80\,\mathrm{s}$, respectively.

The plots in Figure 5.14 show the tracking errors of each robot and the coordination errors between the robots as defined above for the simulation. These errors converge to zero and show the effects of the perturbation. The control inputs of the robots are depicted in Figure 5.15.

There is an important remark to be made regarding the effect of the perturbation on the robots when considering a delayed communication channel. Recall that, when considering a delay-free coupling between the robots and robot i is perturbed, the other $n-1$ robots immediately react to the perturbation because

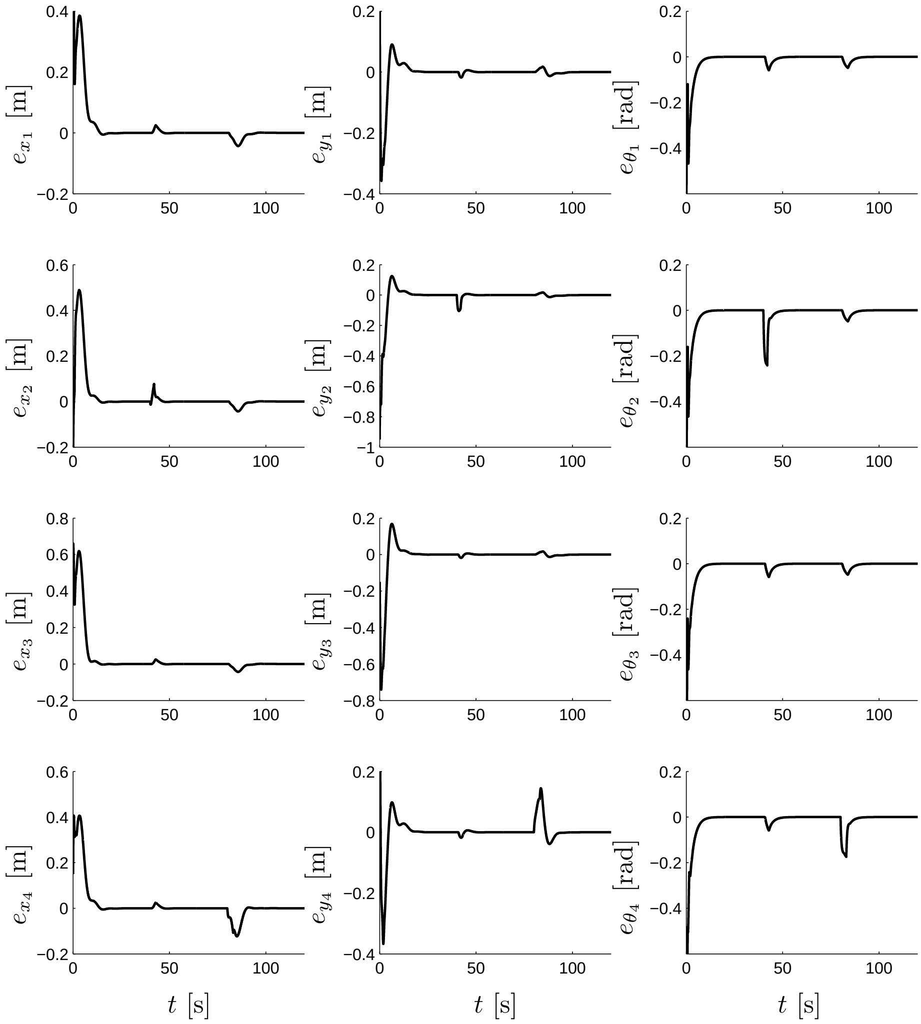


Figure 5.10 . Simulation. Tracking errors of each robot. The second and fourth robots are perturbed at $t = 40\,\mathrm{s}$ and $t = 80\,\mathrm{s}$, respectively.

of the coupling between them. This means that all of the robots contribute so that the i-th robot can overcome its perturbation as fast as possible. On the other hand, when the coupling between the robots is affected by a time-delay, the effects of a perturbation on robot i reach the remaining $n-1$ unperturbed robots after a certain time τ. This means that the disturbed robot copes with the perturbation by itself for some time before receiving the help from the other robots to overcome such perturbation. Naturally, as the network-induced delay becomes larger, the perturbed robot will have to cope with the perturbation by itself for a longer

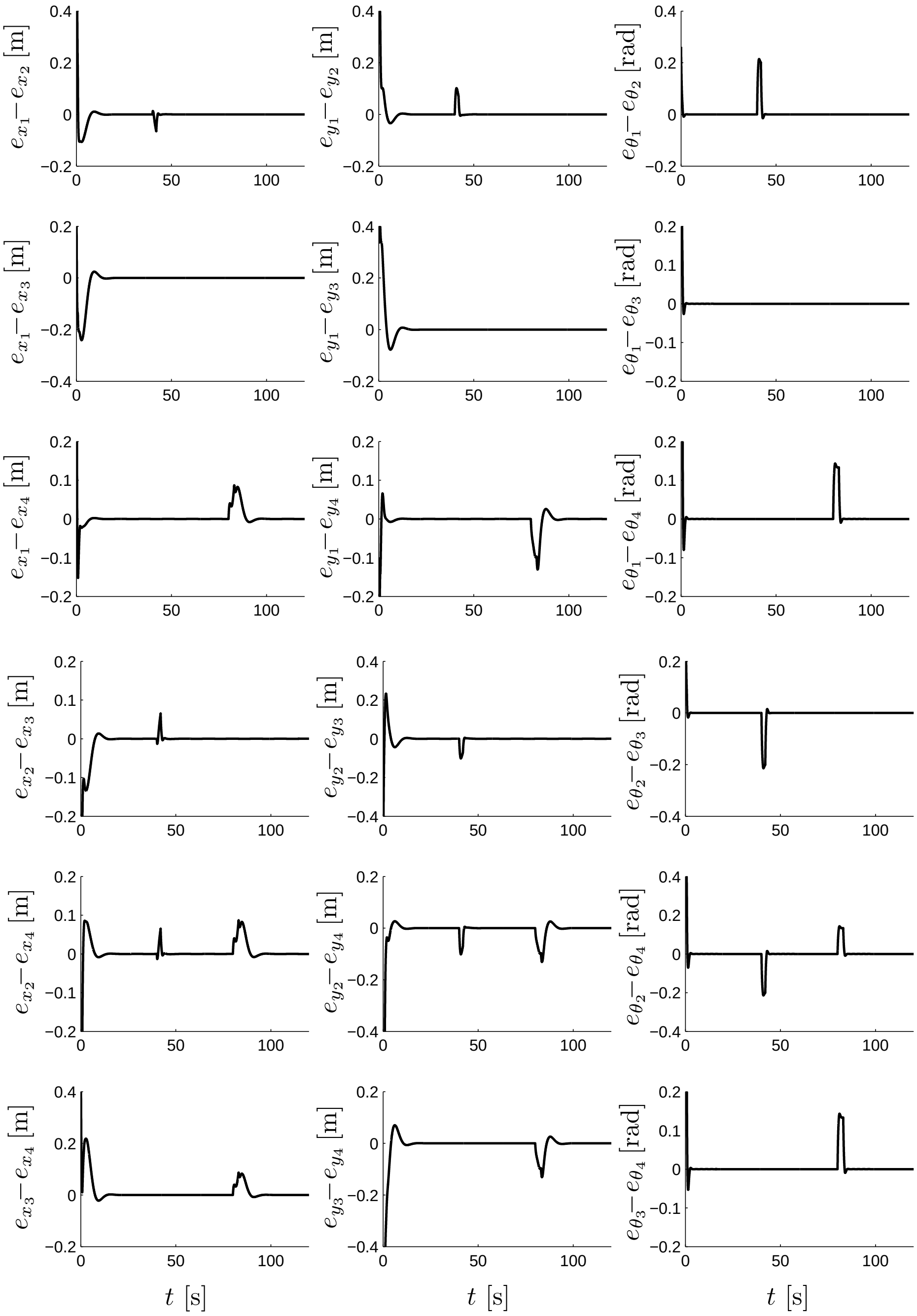


Figure 5.11 . Simulation. Coordination errors between the robots. The second and fourth robots are perturbed at $t = 40\,\mathrm{s}$ and $t = 80\,\mathrm{s}$, respectively.

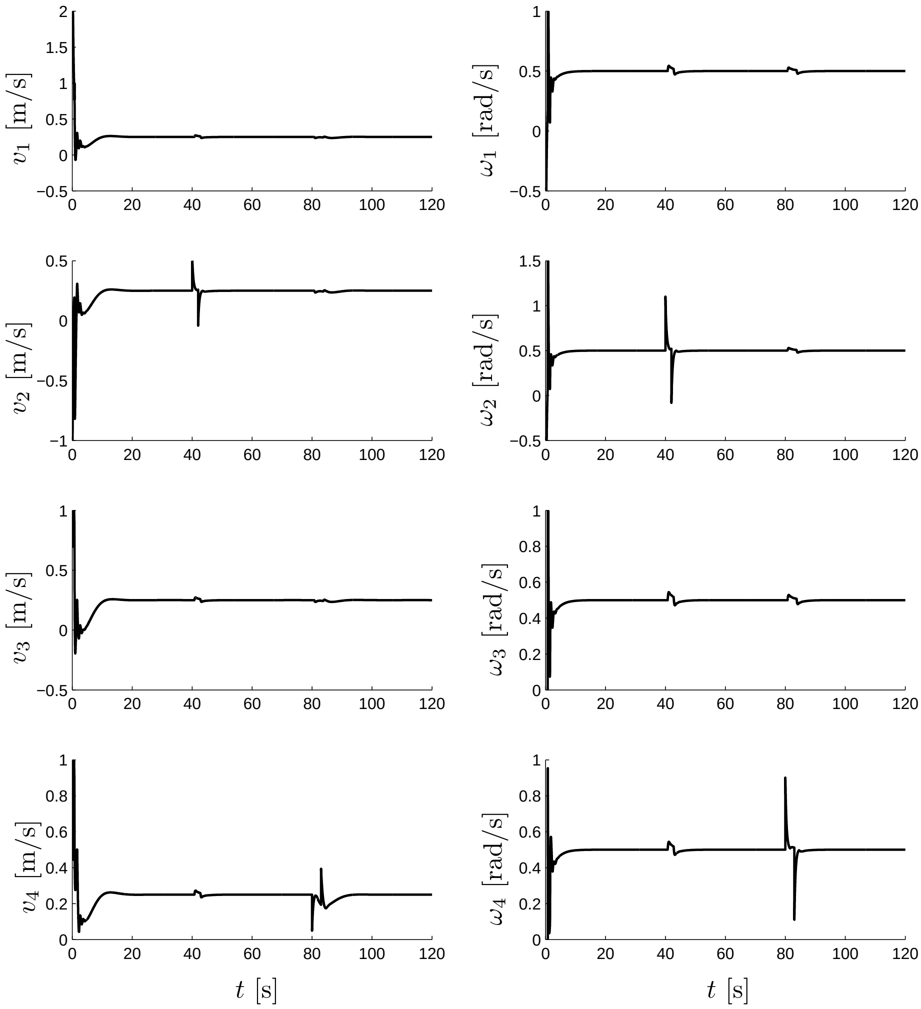


Figure 5.12 . Simulation. Left column: translational control input of each robot; right column: rotational control input of each robot. The second and fourth robots are perturbed at $t = 40\,\mathrm{s}$ and $t = 80\,\mathrm{s}$, respectively.

period of time. In other words, as the magnitude of the communication delay increases, the effectiveness of the coupling term in the face of perturbations diminishes.

To summarize, we have that the results of the first experiment validate the notion that attaining remote master-slave motion coordination is possible in the presence of a network-induced delay. In this case, the tracking errors converge

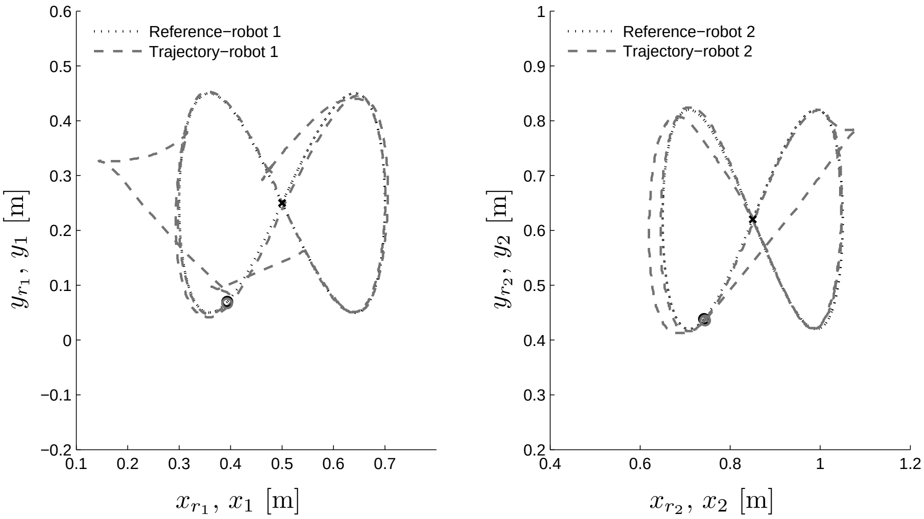


Figure 5.13 . Second experiment: remote mutual motion coordination of two unicycle robots located at TMU (robot 1) and TU/e (robot 2). Reference and robot trajectories shown in the respective world-fixed coordinate frame of the robots $\underline{\vec{e}}^i$, for $i = 1, 2$.

to zero even when considering a communication channel which induces a time-delay with minor variations. In the case of remote mutual motion coordination, the simulation and experiment validate the proposed coordinating controller. We have that the tracking errors and coordination errors (practically) converge to zero even in the presence of small, transient, additive perturbations and once more considering a communication delay with minor variations. Moreover, the results are in accordance with the presented stability analysis, which guarantees global uniform asymptotic stability for all $0 \leq \tau \leq \tau_{\max}$ up to a certain allowable time-delay $\tau_{\max} > 0$.

5.7 Concluding Remarks

This chapter begins by presenting two different types of motion coordination strategies for a group of unicycles. First, a master-slave motion coordination strategy, in which the motion of a pre-assigned master robot constitutes the base for defining the reference trajectories for a group of n slave robots, is presented in Section 5.2. Afterwards, a control strategy for mutual motion coordination is presented in Section 5.3. In this case, all the robots in the group base their reference trajectories on a common virtual center and motion coordination is maintained by coupling all the robots with each other.

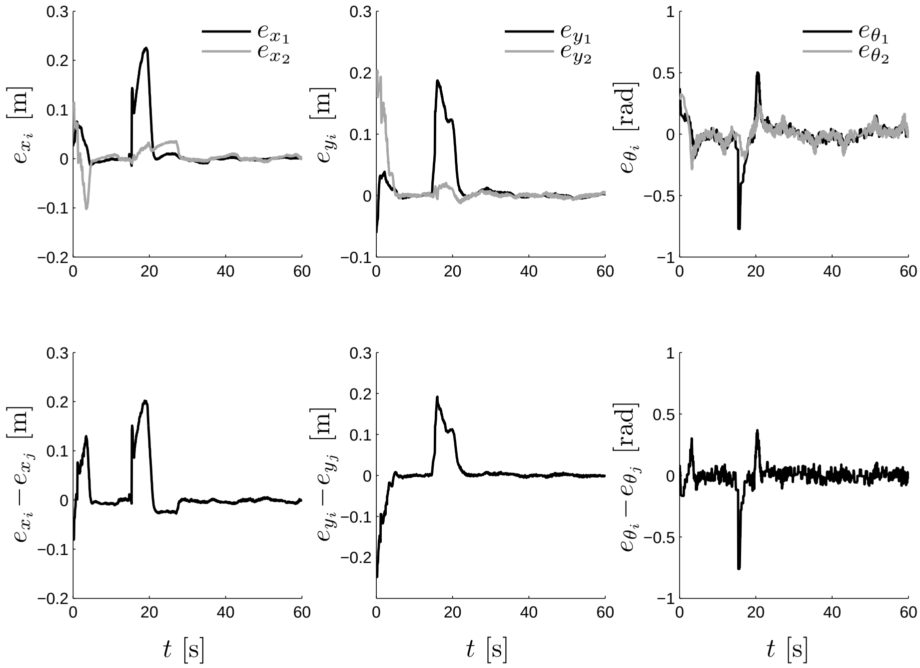


Figure 5.14 . Second experiment. First row: tracking errors of each robot; second row: coordination errors between the robots (a perturbation is induced at $t \approx 14.6\,\mathrm{s}$).

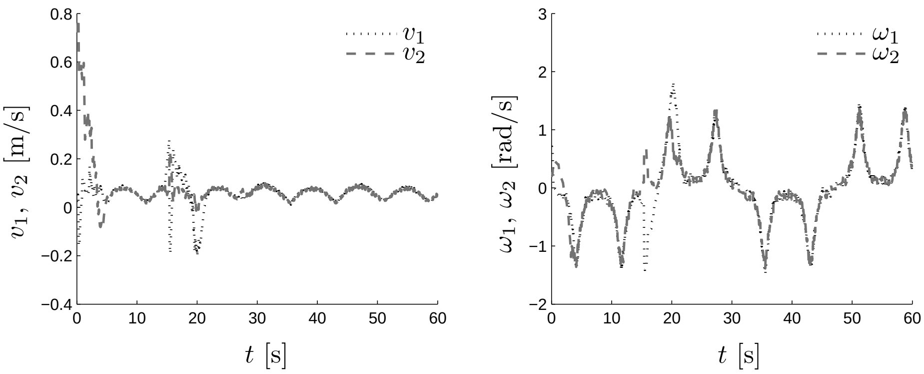


Figure 5.15 . Second experiment. Left: translational control input of each robot; right: rotational control input of each robot (a perturbation is induced at $t \approx 14.6\,\mathrm{s}$).

The main difference between both motion coordination strategies has already been highlighted by the experimental results in van den Broek et al. (2009) and is related to the fact that mutual motion coordination better copes with disturbances because of the couplings between the robots. In this sense, improved coordinated motion is ensured in the presence of perturbations.

The chapter focuses mainly on the problem of remote master-slave and mutual motion coordination of a group of unicycle robots. In this case, the motion coordination strategies are denoted as remote because the communication channel which the robots use to exchange information is subject to a network-induced delay.

Regarding remote master-slave motion coordination, we have determined that, because of the effects of the time-delay in the information exchange between the master and the slaves, it is not always possible to achieve this type of motion coordination using the coordinating controllers studied in Section 5.2. Nevertheless, as long as the measurements from the master become available to the slaves in order, it is possible to guarantee the stability of the slaves due to the fact that the problem is one of a tracking nature. In this sense, we have concluded that it is possible to guarantee stability but not always coordinated behavior. This means that, in the presence of a network-induced delay, the group might only be able to reach delayed master-slave motion coordination.

Regarding remote mutual motion coordination, we have established that it is possible to achieve this type of motion coordination using a controller very similar to the one introduced in Section 5.3, as long as the network-induced delay is constant and belongs to the interval $\tau \in [0, \tau_{\max}]$, with $\tau_{\max}$ small enough. Although it has not been possible to obtain explicit bounds for $\tau_{\max}$ because of the lack of a strict Lyapunov-Razumikhin function for a certain part of the closed-loop error dynamics, we have carried out a stability analysis which demonstrates the existence of such an interval for certain control parameters and a set of reference trajectories satisfying specific requirements. An additional advantage of the current approach is that the derivation of the coordinating controller is natural and based on an existing control law.

In contrast with some of the results available in the literature, the proposed remote motion coordination strategies have been validated experimentally using the Internet as the communication channel and considering a group of robots in two remote locations; namely, the Netherlands and Japan. The results for remote master-slave motion coordination show that attaining this type of motion coordination (at least a delayed version of it) is possible even when considering a network-induced delay subject to minor variations. Similarly, the simulation and experimental results for mutual motion coordination show that the coordinating controller is able to withstand small, transient, additive perturbations and a slightly time-varying communication delay.

The remote control and motion coordination strategies studied so far in this work only apply for a particular class of systems, namely, unicycle robots. In the next chapter we will apply the remote tracking control strategy introduced in Chapter 4 and the remote motion coordination strategies studied in this chapter to other mechanical systems, namely, an omnidirectional mobile robot and a one-link robot.

6

Application to Other Mechanical Systems

Abstract. In this chapter, the remote control and motion coordination strategies introduced in Chapter 4 and Chapter 5 are applied to dynamical systems other than the unicycle robot. In the first part of the chapter, a predictor-controller combination for an omnidirectional mobile robot and a remote mutual motion coordination strategy for a group of these robots are proposed, followed by their respective stability analyses and illustrative simulations. In the second part of the chapter a remote tracking control strategy for a one-link robot is presented, also followed by a stability analysis and simulations.

6.1 Introduction

The remote control and motion coordination strategies proposed in Chapter 4 and Chapter 5 have been applied so far to unicycle robots only. In this chapter we will advocate, however, that the principles behind these control strategies are valuable in a more general setting and can be used to develop similar control strategies for other mechanical systems; in particular, different types of mobile robots and robotic manipulators. With this possibility in mind, the current chapter focuses on the remote control and motion coordination of omnidirectional mobile robots and a one-link robot.

In the first part of the chapter a predictor-controller combination for the remote tracking control of an omnidirectional robot is proposed, and sufficient conditions to guarantee the global and local stability of the resulting closed-loop error dynamics are formulated. The second part of the chapter is devoted to the remote motion coordination of a group of omnidirectional robots. Since the remote master-slave motion coordination case has already been shown to be quite straightforward in Chapter 5, we will only focus on remote mutual motion coordination in the current chapter. Recall that, in this case, the information exchange between the robots is subject to a network-induced delay. In contrast with the global stability results obtained for a group of *unicycle* robots, where only a qualitative requirement for the allowable time-delay was obtained, when considering a group of *omnidirectional* robots it is possible to provide a quantitative upper bound for the network-induced delay up to which motion coordination can be achieved while still guaranteeing global asymptotic stability of the group's error dynamics.

It is worth noting that, in general, control problems addressed for unicycle robots are seldom extended to omnidirectional robots, since the latter are considered to be easier to control because their posture kinematic model is fully linearizable by static state feedback. For this reason, only a few examples exist in the literature which address the issues studied in this thesis. One exception is the work of Kanjanawanishkul (2010), in which delay-free motion coordination strategies for a group of omnidirectional robots are developed using model predictive control (MPC) techniques.

There are, however, a number of reasons which motivate the extension of the control architectures introduced in this thesis to omnidirectional robots. First, the feedback linearization controller is not applicable when this type of robot is controlled over a delay-inducing communication network. This means that addressing this type of issues would benefit from, for instance, the design of a predictor-controller combination such as the one introduced in Chapter 4 for a unicycle. In addition, a remote coordinating controller which uses the posture kinematic model of the robot and is based on a feedback tracking controller with coupling terms, such as the one introduced in Chapter 5 for a group of unicycles, has yet to be proposed for a group of omnidirectional robots. Finally, the number of applications which use this type of mobile robot has been on the rise in recent years, so it has become increasingly important to develop structured and unified control strategies for them.

In this chapter we also propose a predictor-controller combination for the remote tracking control of a one-link robot, accompanied by the local stability analysis of the resulting closed-loop error dynamics. The motivation behind proposing this control strategy is to provide additional insight into the applicability of this type of predictor-controller combination to more complex robotic manipulators.

The remainder of this chapter is structured in the following way. In Section 6.2, a predictor-controller combination for an omnidirectional mobile robot is designed based on the remote control strategy introduced in Chapter 4. In the same section, the remote mutual motion coordination strategy studied in Chapter 5 is applied to a group of omnidirectional mobile robots. The problem of the remote tracking control of a one-link robot is addressed in Section 6.3. The respective stability analyses and simulations of the proposed control strategies are included in each section, and the closing remarks of the chapter are given in Section 6.4.

6.2 Remote Tracking Control and Mutual Motion Coordination of Omnidirectional Robots

The remote control strategy proposed in Chapter 4 for a unicycle robot will be applied to an omnidirectional robot in this section. In addition, the possibility to achieve remote mutual motion coordination of a group of n omnidirectional robots is also studied in this section based on the ideas introduced in Chapter 5 for unicycle robots. In order to do so, we will first introduce the posture kinematic model of this type of robot.

6.2.1 Kinematic Model

An omnidirectional mobile robot has no fixed or steering wheels, but is rather equipped with at least three so-called Swedish or caster wheels (refer to Siciliano et al., 2009, for additional details on these types of wheels). A mobile robot with these characteristics is able to move in any direction within its workspace without the need to change its orientation.

The omnidirectional robot considered in this thesis is equipped with three Swedish wheels. Since this type of wheels have a series of rollers attached to their circumference they allow the movement of the robot in any direction without any restriction. Hence, the robot is not subject to any non-holonomic constraint. A schematic representation of the omnidirectional robot considered in this work is shown in Figure 6.1. Note that, in the figure, $\underline{\vec{e}}^1$ denotes a coordinate frame fixed to the robot and that there is a fixed angle of 120° between the axes on which the wheels of the robot are mounted. Considering that δ represents the angle between the axis on which the first wheel is mounted and $\vec{e}_x^1$, and since the angle between the axis on which the third wheel is mounted and $\vec{e}_x^1$ is 90°, it follows from the proposed wheel configuration that $\delta = 120° - 90° = 30°$.

Consider point P in Figure 6.1, which denotes the intersection of the axes on which the wheels of the robot are mounted. The position coordinates of this point (at time t) with respect to the global coordinate frame $\underline{\vec{e}}^0$ are denoted by the coordinates $(x(t), y(t))$, whereas the angle (at time t) between the heading

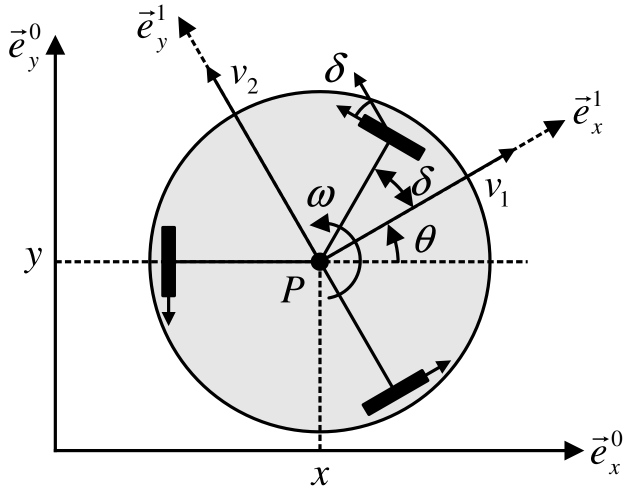


Figure 6.1 . Schematic representation of an omnidirectional mobile robot.

direction of the robot and $\vec{e}_x^{\,0}$ is denoted by $\theta(t)$. The resulting kinematic model, obtained as in Campion et al. (1996) and Canudas de Wit et al. (1996), is the following:

$$\dot{x}(t) = v_1(t)\cos\theta(t) - v_2(t)\sin\theta(t), \tag{6.1a}$$

$$\dot{y}(t) = v_1(t)\sin\theta(t) + v_2(t)\cos\theta(t), \tag{6.1b}$$

$$\dot{\theta}(t) = \omega(t). \tag{6.1c}$$

The control inputs of the robot are given by $v_1(t), v_2(t)$, and $\omega(t)$, where $\omega(t)$ denotes the robot's rotational velocity and $v_1(t)$ and $v_2(t)$ represent its translational velocity components aligned, respectively, with $\vec{e}_x^{\,1}$ and $\vec{e}_y^{\,1}$. The state of the robot is given by $q(t) = [x(t)\, y(t)\, \theta(t)]^T$.

6.2.2 Remote Tracking Control

We now consider the case when the robot is subject to a network-induced delay τ, composed of an input and an output time-delay τ_f and τ_b, respectively. Given the input time-delay τ_f, the kinematic model of the robot becomes:

$$\dot{x}(t) = v_1(t-\tau_f)\cos\theta(t) - v_2(t-\tau_f)\sin\theta(t), \tag{6.2a}$$

$$\dot{y}(t) = v_1(t-\tau_f)\sin\theta(t) + v_2(t-\tau_f)\cos\theta(t), \tag{6.2b}$$

$$\dot{\theta}(t) = \omega(t-\tau_f). \tag{6.2c}$$

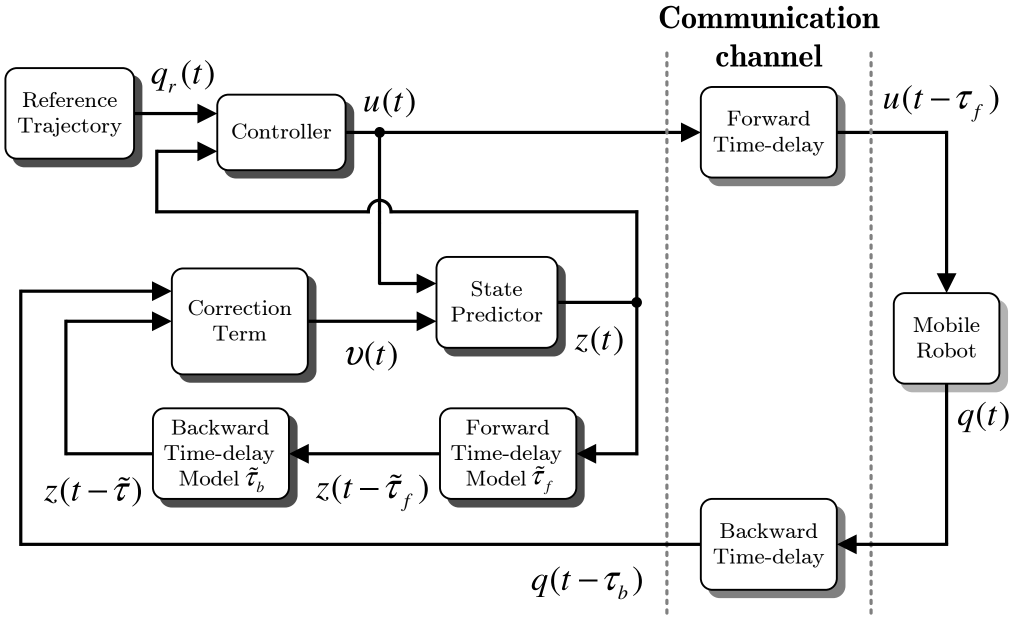


Figure 6.2 . Block diagram representation of the remote tracking control strategy for an omnidirectional mobile robot.

In addition, the output time-delay τ_b produces the state $q(t-\tau_b) = [x(t-\tau_b)\; y(t-\tau_b)\, \theta(t-\tau_b)]^T$.

The control objective of the robot is to track a prescribed reference position $(x_r(t), y_r(t))$ and orientation $\theta_r(t)$, which have an associated reference state trajectory $q_r(t) = [x_r(t)\, y_r(t)\, \theta_r(t)]^T$. Following the same approach as for the unicycle robot used in Chapter 4, the following state predictor with state $z(t) = [z_1(t)\, z_2(t)\, z_3(t)]^T$ is proposed:

$$\dot{z}_1(t) = v_1(t)\cos z_3(t) - v_2(t)\sin z_3(t) + \nu_x(t), \tag{6.3a}$$

$$\dot{z}_2(t) = v_1(t)\sin z_3(t) + v_2(t)\cos z_3(t) + \nu_y(t), \tag{6.3b}$$

$$\dot{z}_3(t) = \omega(t) + \nu_\theta(t), \tag{6.3c}$$

where $\nu(t) = [\nu_x(t)\, \nu_y(t)\, \nu_\theta(t)]^T$ denotes the correction term.

In order to define the correction term, consider first the prediction error, given by $p_e(t) = [p_{e_x}(t)\, p_{e_y}(t)\, p_{e_\theta}(t)]^T = [x(t-\tau_b) - z_1(t-\tilde{\tau}),\;\; y(t-\tau_b) - z_2(t-\tilde{\tau}),\; \theta(t-\tau_b) - z_3(t-\tilde{\tau})]^T$. The correction terms in (6.3) are then designed as follows:

$$\nu_x(t) = k_x p_{e_x}(t) = k_x(x(t-\tau_b) - z_1(t-\tilde{\tau})), \tag{6.4a}$$

$$\nu_y(t) = k_y p_{e_y}(t) = k_y(y(t-\tau_b) - z_2(t-\tilde{\tau})), \tag{6.4b}$$

$$\nu_\theta(t) = k_\theta p_{e_\theta}(t) = k_\theta(\theta(t-\tau_b) - z_3(t-\tilde{\tau})), \tag{6.4c}$$

where k_x, k_y, and k_θ are the correction term gains and $\tilde{\tau} := \tilde{\tau}_f + \tilde{\tau}_b$ represents the sum of the estimated input and output network-induced delays, which is assumed to be known (that is, $\tilde{\tau} = \tau = \tau_b + \tau_f$).

Note that the prediction error and correction terms of the omnidirectional robot are different from the ones proposed for the unicycle in (4.3). In addition, the delay-free output of the predictor, $z(t)$, is not required to compute the correction terms of the omnidirectional robot. This results in the remote tracking control strategy shown in Figure 6.2 for the omnidirectional robot.

Given the state predictor (6.3) and correction term (6.4), the tracking control law for the predictor-controller combination is defined as follows:

$$v_1(t) = \mu_x(t)\cos z_3(t) + \mu_y(t)\sin z_3(t), \tag{6.5a}$$
$$v_2(t) = -\mu_x(t)\sin z_3(t) + \mu_y(t)\cos z_3(t), \tag{6.5b}$$
$$\omega(t) = \mu_\theta(t), \tag{6.5c}$$

where the auxiliary controls $\mu(t) = [\mu_x(t)\,\mu_y(t)\,\mu_\theta(t)]^T$ are given by

$$\mu_x(t) = \dot{x}_r(t) + c_x z_{1_e}(t), \tag{6.6a}$$
$$\mu_y(t) = \dot{y}_r(t) + c_y z_{2_e}(t), \tag{6.6b}$$
$$\mu_\theta(t) = \dot{\theta}_r(t) + c_\theta z_{3_e}(t), \tag{6.6c}$$

in which $z_e(t) = [z_{1_e}(t)\, z_{2_e}(t)\, z_{3_e}(t)]^T = [x_r(t) - z_1(t)\, y_r(t) - z_2(t)\, \theta_r(t) - z_3(t)]^T$. The linearizing controller (6.5) in combination with the auxiliary control (6.6) constitute the regular tracking control law for an omnidirectional robot in which the output of the predictor is used instead of the output of the system.

The control action applied to the robot after the input delay τ_f is the following:

$$v_1(t-\tau_f) = \mu_x(t-\tau_f)\cos z_3(t-\tau_f) + \mu_y(t-\tau_f)\sin z_3(t-\tau_f), \tag{6.7a}$$
$$v_2(t-\tau_f) = -\mu_x(t-\tau_f)\sin z_3(t-\tau_f) + \mu_y(t-\tau_f)\cos z_3(t-\tau_f), \tag{6.7b}$$
$$\omega(t-\tau_f) = \mu_\theta(t-\tau_f). \tag{6.7c}$$

Closed-Loop Error Dynamics

Given the state predictor (6.3), correction term (6.4), and delayed tracking controller (6.7), the closed-loop error dynamics yield:

$$\dot{z}_{1_e}(t) = -c_x z_{1_e}(t) - k_x p_{x_e}(t), \tag{6.8a}$$

$$\dot{z}_{2_e}(t) = -c_y z_{2_e}(t) - k_y p_{y_e}(t), \tag{6.8b}$$
$$\dot{z}_{3_e}(t) = -c_\theta z_{3_e}(t) - k_\theta p_{\theta_e}(t), \tag{6.8c}$$
$$\begin{aligned}\dot{p}_{x_e}(t) = &-k_x p_{x_e}(t-\tau) - (\dot{x}_r(t-\tau) + c_x z_{1_e}(t-\tau))(1-\cos p_{\theta_e}(t)) \\ &- (\dot{y}_r(t-\tau) + c_y z_{2_e}(t-\tau))\sin p_{\theta_e}(t),\end{aligned} \tag{6.8d}$$
$$\begin{aligned}\dot{p}_{y_e}(t) = &-k_y p_{y_e}(t-\tau) - (\dot{y}_r(t-\tau) + c_y z_{2_e}(t-\tau))(1-\cos p_{\theta_e}(t)) \\ &+ (\dot{x}_r(t-\tau) + c_x z_{1_e}(t-\tau))\sin p_{\theta_e}(t),\end{aligned} \tag{6.8e}$$
$$\dot{p}_{\theta_e}(t) = -k_\theta p_{\theta_e}(t-\tau). \tag{6.8f}$$

Considering the state definitions $\xi_1(t) := [z_{1_e}(t)\, z_{2_e}(t)\, p_{x_e}(t)\, p_{y_e}(t)]^T$ and $\xi_2(t) := [z_{3_e}(t)\, p_{\theta_e}(t)]^T$, (6.8) may be rearranged in the following cascade:

$$\dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_1(t-\tau) + g(t, \xi_{1_t}, \xi_{2_t}), \tag{6.9a}$$
$$\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau), \tag{6.9b}$$

where ξ_{i_t}, $i = 1, 2$, is an element of the Banach space $\mathcal{C}(l_i) = C([-\tau, 0], \mathbb{R}^{l_i})$, for $l_1 = 4$ and $l_2 = 2$, defined by $\xi_{i_t}(s) := \xi_i(t+s)$ for $s \in [-\tau, 0]$. The matrices in (6.9) are given by

$$A_1 = \begin{bmatrix} -c_x & 0 & -k_x & 0 \\ 0 & -c_y & 0 & -k_y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -k_x & 0 \\ 0 & 0 & 0 & -k_y \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -c_\theta & -k_\theta \\ 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & -k_\theta \end{bmatrix},$$
$$g(t, \xi_{1_t}, \xi_{2_t}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix} \xi_2(t),$$

with

$$g_{32} = -(\dot{x}_r(t-\tau) + c_x z_{1_e}(t-\tau))\int_0^1 \sin(s p_{\theta_e}(t))ds - (\dot{y}_r(t-\tau) + c_y z_{2_e}(t-\tau))\int_0^1 \cos(s p_{\theta_e}(t))ds,$$
$$g_{42} = -(\dot{y}_r(t-\tau) + c_y z_{2_e}(t-\tau))\int_0^1 \sin(s p_{\theta_e}(t))ds + (\dot{x}_r(t-\tau) + c_x z_{1_e}(t-\tau))\int_0^1 \cos(s p_{\theta_e}(t))ds,$$

where the equalities in (2.2) have been used to define g_{32}, and g_{42}.

Stability Analysis

The control goal for the omnidirectional robot is the same as for the remotely controlled unicycle in Chapter 4; that is, the robot should track a delayed version

$q_r(t-\tau_f)$ of its reference trajectory. The following theorem formulates sufficient conditions under which $(z_e^T, p_e^T)^T = 0$ is a GUAS equilibrium point of (6.8), that is, under which the above control goal is achieved.

Theorem 6.1. *Consider an omnidirectional mobile robot subject to a constant input time-delay τ_f, whose posture kinematic model is given as in (6.2). The reference position and orientation of the robot are prescribed by $(x_r(t), y_r(t))$ and $\theta_r(t)$, respectively. Consider the tracking controller as given in (6.5) together with the auxiliary control (6.6). Additionally, consider the state predictor (6.3) with the correction term (6.4). If the following conditions are satisfied:*

- *the reference velocities $\dot{x}_r(t)$ and $\dot{y}_r(t)$ are bounded $\forall t$;*
- *the tracking gains satisfy $c_x = c_y = c > 0, c_\theta > 0$;*
- *the correction term gains satisfy $k_x = k_y = k > 0, k_\theta > 0$;*
- *the time-delay is constant and known, that is, $\tilde{\tau} = \tau = \tau_b + \tau_f$;*
- *the time-delay τ belongs to the interval $0 \leq \tau < \tau_{\max}$, with*

$$\tau_{\max} = \min\left\{ \frac{2\beta\left(c+3k-\sqrt{c^2-2ck+5k^2}\right)^{\frac{1}{2}}}{k\sqrt{p}\left(c+3k+\sqrt{c^2-2ck+5k^2}\right)^{\frac{3}{2}}\left(\sqrt{c^2+k^2}+k\right)}, \frac{\pi}{2k_\theta} \right\}, \tag{6.10}$$

  where $p > 1$ and $\beta = \min\left\{c\left(c+k\right), k^2\right\}$,

then $(z_e^T, p_e^T)^T = 0$ is a globally uniformly asymptotically stable (GUAS) equilibrium point of the closed-loop error dynamics (6.8). In other words, $z(t) \to q(t+\tau_f)$ as $t \to \infty$, that is, the predicted state anticipates the state of the system by τ_f, and $q(t) \to q_r(t-\tau_f)$ as $t \to \infty$, that is, the robot tracks the reference trajectory delayed by τ_f.

Proof. For the sake of brevity, only a sketch of the proof is presented in this chapter. The complete proof is given in Appendix D.1.

Based on Theorem 2.26 and following the same approach as in the proof of Theorem 4.3, the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the cascaded system (6.9) may be established if the following conditions are satisfied:

- the system $\dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_1(t-\tau)$, denoted as the ξ_1-dynamics without coupling, is globally exponentially stable (GES) with an explicit quadratic Lyapunov-Razumikhin function V_{ξ_1};

- the system $\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau)$, denoted as the ξ_2-dynamics, is globally exponentially stable (GES);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ admits the estimate

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c, \tag{6.11}$$

  for continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The validity of these three conditions is checked considering the requirements in the theorem for the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, and maximum allowable time-delay $\tau_{\max}$.

The stability of the ξ_1-dynamics without coupling is studied using a quadratic candidate Lyapunov-Razumikhin function. The global exponential stability of these error dynamics is ensured provided that the time-delay satisfies the following condition:

$$\tau < \frac{2\beta\left(c + 3k - \sqrt{c^2 - 2ck + 5k^2}\right)^{\frac{1}{2}}}{k\sqrt{p}\left(c + 3k + \sqrt{c^2 - 2ck + 5k^2}\right)^{\frac{3}{2}}\left(\sqrt{c^2 + k^2} + k\right)}. \tag{6.12}$$

The ξ_2-dynamics have already been shown to be GES in the proof of Theorem 4.3 considering the following bound on the time-delay:

$$\tau < \frac{\pi}{2k_\theta}. \tag{6.13}$$

Regarding the third condition, it can be shown that the inequality (6.11) can be satisfied by choosing $\alpha_1(\|\varphi_{\xi_2}\|) = 2\left(|\bar{\dot{x}}_r| + |\bar{\dot{y}}_r|\right)$ and $\alpha_2(\|\varphi_{\xi_2}\|) = 2(c_x + c_y)$, where $\bar{\dot{x}}_r := \sup_{t\in\mathbb{R}} |\dot{x}_r(t)|$ and $\bar{\dot{y}}_r := \sup_{t\in\mathbb{R}} |\dot{y}_r(t)|$.

The three conditions stated at the beginning of the proof have now been checked. As a result, the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (6.9) can be concluded given the requirements posed in the theorem. This completes the sketch of the proof. □

Theorem 6.1 has been formulated under the assumption that a preview of the reference trajectory is not available. However, as explained in Remark 4.2 for the unicycle robot, if the reference trajectory $q_r(t + \tau_f)$ is known at time t, it becomes possible for the omnidirectional robot to track its current reference trajectory, that is, $q_r(t)$.

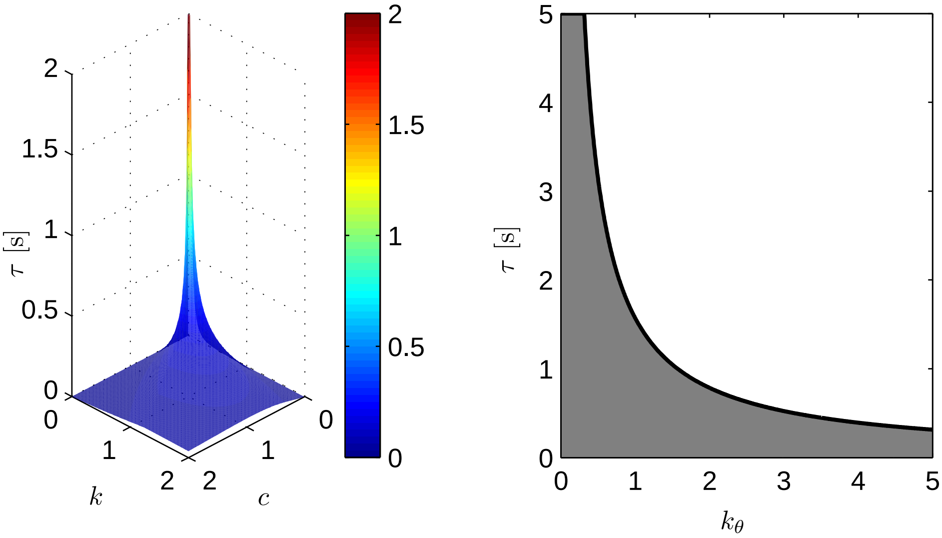


Figure 6.3 . Allowable time-delay τ for conditions (6.12) (left, cut off at 2 s) and (6.13) (right, cut off at 5 s).

The maximum allowable time-delay $\tau_{\max}$ satisfying (6.12) and (6.13) for $p = 1$ and different values of the control parameters is shown, respectively, in the left and right hand plots of Figure 6.3. As with the unicycle, a low-gain predictor design yields high robustness against delays.

The upper bound on the delay resulting from condition (6.12) may be quite conservative due to the fact that, when studying the global stability of (6.9), a strict Lyapunov-Razumikhin function is required for the ξ_1-dynamics without coupling (these dynamics are contained in (6.9a)). For this reason, we also formulate the following theorem, which yields a less strict bound on the delay by posing conditions under which $(z_e^T, p_e^T)^T = 0$ is a LUAS equilibrium point of (6.8).

Theorem 6.2. *Consider the same problem setting as in Theorem 6.1. If the following conditions are satisfied:*

- *the tracking gains satisfy* $c_x, c_y, c_\theta > 0$*;*
- *the correction term gains satisfy* $k_x, k_y, k_\theta > 0$*;*
- *the time-delay is constant and known, that is,* $\tilde{\tau} = \tau = \tau_b + \tau_f$*;*
- *the time-delay* τ *belongs to the interval* $0 \leq \tau < \tau_{\max}$*, with*

$$\tau_{\max} = \min\left\{\frac{\pi}{2k_x}, \frac{\pi}{2k_y}, \frac{\pi}{2k_\theta}\right\}, \tag{6.14}$$

then, $(z_e^T, p_e^T)^T = 0$ *is a locally uniformly asymptotically stable (LUAS) equilibrium point of the closed-loop error dynamics (6.8).*

Proof. For the sake of brevity, only a sketch of the proof is presented in this chapter. The complete proof is given in Appendix D.2.

The proof is based on Theorem 2.25 and is very similar to the proof of Theorem 4.1. We can formulate the following conditions in order to guarantee the local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (6.9):

- the ξ_1-dynamics without coupling are locally uniformly asymptotically stable (LUAS);
- the ξ_2-dynamics are locally uniformly asymptotically stable (LUAS);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ vanishes when $\xi_{2_t} \to 0$, that is, $g(t, \xi_{1_t}, 0) = 0$.

These conditions are checked given the requirements for the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, and maximum allowable time-delay, $\tau_{\max}$ posed in the theorem.

In order to check the first condition, the ξ_1-dynamics without coupling are represented as a cascaded system. The local uniform asymptotic stability of these dynamics is ensured provided the time-delay satisfies the following conditions:

$$\tau < \frac{\pi}{2k_x}, \qquad \tau < \frac{\pi}{2k_y}, \tag{6.15}$$

and the requirements for k_x and k_y stated in the theorem are met.

Regarding the second condition, the local uniform asymptotic stability of the ξ_2-dynamics has already been shown in Theorem 4.1 for

$$\tau < \frac{\pi}{2k_\theta}. \tag{6.16}$$

Finally, regarding the third condition, it immediately follows that as $\xi_{2_t} \to 0$, the coupling term vanishes.

Having checked the three conditions stated at the beginning of the proof, the local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (4.9) is concluded given the requirements posed in the theorem. This completes the sketch of the proof. □

Note that, contrary to the conditions posed in Theorem 6.1, the reference velocities $\dot{x}_r(t)$ and $\dot{y}_r(t)$ in Theorem 6.2 are not required to be bounded. Moreover, the conditions on the allowable time-delay $\tau_{\max}$ posed in Theorem 6.2 only depend on the correction gains k_x, k_y, and k_θ, as opposed to Theorem 6.1 where these conditions also depend on the tracking gains c_x, c_y, and c_θ and where the additional restriction that $c_x = c_y$ and $k_x = k_y$ is in place. In addition, the allowable time-delay $\tau_{\max}$ derived from (6.14) for different values of k_x, k_y, and k_θ (this condition is for local stability) has the same form as the one derived from (6.13) for different values of k_θ (this condition is for global stability). Recall that, when considering global stability, the allowable time-delay for different values of k_θ is depicted on the right-hand side of Figure 6.3, whereas the allowable time-delay for different values of k_x and k_y is shown on the left-hand side of the same figure. Considering the previous remark regarding the similarity between the allowable time-delay for different values of k_x, k_y, and k_θ for local stability and for different values of k_θ for global stability, a comparison of both plots in Figure 6.3 shows that the upper bounds on the time-delay for local stability are less conservative than the ones for global stability. This means that a tradeoff exists between using a global stability result with conservative delay bounds or a local stability result with less conservative delay bounds.

6.2.3 Remote Mutual Motion Coordination

We now consider the remote motion coordination problem for a group of n omnidirectional mobile robots. Recall that the posture kinematic model of this type of robot has already been given in (6.1), with $q_i(t) = [x_i(t)\, y_i(t)\, \theta_i(t)]^T$ denoting the state of i-th robot in the group, for $i \in \{1, 2, \ldots, n\}$.

The same trajectory generation approaches used for the mutual motion coordination of a group of unicycles may be used to construct the reference trajectories for the omnidirectional robots. Although the explicit mathematical expressions to do so are not provided in Appendix A, they can be easily derived using the same approach as for the unicycles. Not withstanding the trajectory generation approach used, the state of the reference trajectory for the i-th robot is given by $q_{r_i}(t) = [x_{r_i}(t)\, y_{r_i}(t)\, \theta_{r_i}(t)]^T$.

The tracking error of the i-th robot is given by the difference between its reference trajectory and its own state, that is, $q_{e_i}(t) = q_{r_i}(t) - q_i(t)$. In order to achieve mutual motion coordination as defined in Chapter 5, the robots in the group are supposed to exchange their tracking errors $q_{e_i}(t)$ with each other, meaning that each robot receives the tracking errors of the other $n-1$ robots in the group. In an ideal setting this information exchange would occur without any interference. Nevertheless, we consider the case when it is affected by a network-induced delay τ, just as with the unicycles in Chapter 5. Recall that the time-delay for all communication between the robots is assumed to be constant and equal, but not necessarily known.

Following the same approach as with the coordinating controller in Chapter 5, the following controller $u_i(t) = [v_{1_i}(t)\, v_{2_i}(t)\, \omega_i(t)]^T$ may be proposed to achieve remote mutual motion coordination in a group of n omnidirectional robots whose information exchange is subject to a time-delay τ:

$$v_{1_i}(t) = \mu_{x_i}(t)\cos\theta_i(t) + \mu_{y_i}(t)\sin\theta_i(t), \tag{6.17a}$$

$$v_{2_i}(t) = -\mu_{x_i}(t)\sin\theta_i(t) + \mu_{y_i}(t)\cos\theta_i(t), \tag{6.17b}$$

$$\omega_i(t) = \mu_{\theta_i}(t), \tag{6.17c}$$

where the auxiliary coordinating controller $\mu_i(t) = [\mu_{x_i}(t)\, \mu_{y_i}(t)\, \mu_{\theta_i}(t)]^T$ yields:

$$\mu_{x_i}(t) = \dot{x}_{r_i}(t) + c_{x_i}x_{e_i}(t) + \sum_{j=1, i\neq j}^{n} k_{x_{i,j}}(x_{e_i}(t) - x_{e_j}(t-\tau)), \tag{6.18a}$$

$$\mu_{y_i}(t) = \dot{y}_{r_i}(t) + c_{y_i}y_{e_i}(t) + \sum_{j=1, i\neq j}^{n} k_{y_{i,j}}(y_{e_i}(t) - y_{e_j}(t-\tau)), \tag{6.18b}$$

$$\mu_{\theta_i}(t) = \dot{\theta}_{r_i}(t) + c_{\theta_i}\theta_{e_i}(t) + \sum_{j=1, i\neq j}^{n} k_{\theta_{i,j}}(\theta_{e_i}(t) - \theta_{e_j}(t-\tau)). \tag{6.18c}$$

The tracking gains c_{x_i}, c_{y_i}, and c_{θ_i} and coupling gains $k_{x_{i,j}}, k_{y_{i,j}}$, and $k_{\theta_{i,j}}$ in (6.18) serve the same purpose as in the coordinating controller (5.7) for a group of unicycles. Namely, the tracking gains dictate the tracking behavior, whereas the coupling gains determine the strength of the coupling between the i-th and j-th robots and how the group will react to perturbations. Note that only the information sent over the network used in the coupling terms is subject to a time-delay τ.

Closed-Loop Error Dynamics

Given the definition of the tracking error $q_{e_i}(t)$ and the coordinating controller (6.17)-(6.18), the closed-loop error dynamics of the complete group are given as follows:

$$\begin{bmatrix}\dot{X}_e(t)\\ \dot{Y}_e(t)\\ \dot{\Theta}_e(t)\end{bmatrix} = -\begin{bmatrix}C_x & 0_{n\times n} & 0_{n\times n}\\ 0_{n\times n} & C_y & 0_{n\times n}\\ 0_{n\times n} & 0_{n\times n} & C_\theta\end{bmatrix}\begin{bmatrix}X_e(t)\\ Y_e(t)\\ \Theta_e(t)\end{bmatrix} + \begin{bmatrix}K_x & 0_{n\times n} & 0_{n\times n}\\ 0_{n\times n} & K_y & 0_{n\times n}\\ 0_{n\times n} & 0_{n\times n} & K_\theta\end{bmatrix}\begin{bmatrix}X_e(t-\tau)\\ Y_e(t-\tau)\\ \Theta_e(t-\tau)\end{bmatrix}, \tag{6.19}$$

in which

$$X_e(t) = \mathrm{col}(x_{e_1}(t), \ldots, x_{e_n}(t)), \qquad Y_e(t) = \mathrm{col}(y_{e_1}(t), \ldots, y_{e_n}(t)),$$
$$\Theta_e(t) = \mathrm{col}(\theta_{e_1}(t), \ldots, \theta_{e_n}(t)),$$

and

$$C_a = \begin{bmatrix} c_{a_1} + \sum k_{a_{1,j}} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & c_{a_{n-1}} + \sum k_{a_{n-1,j}} & 0 \\ 0 & 0 & \cdots & c_{a_n} + \sum k_{a_{n,j}} \end{bmatrix},$$

$$K_a = \begin{bmatrix} 0 & k_{a_{1,2}} & \cdots & k_{a_{1,n}} \\ \vdots & \ddots & \ddots & \vdots \\ k_{a_{n-1,1}} & \ddots & 0 & k_{a_{n-1,n}} \\ k_{a_{n,1}} & k_{a_{n,2}} & \cdots & 0 \end{bmatrix},$$

in which $\sum k_{a_{i,j}} = \sum_{j=1, j\neq i}^{n} k_{a_{i,j}}$, for $a \in \{x, y, \theta\}$.

Stability Analysis

The following theorem formulates sufficient conditions for the global exponential stability of the equilibrium point $Q_e(t) = [X_e^T(t)\, Y_e^T(t)\, \Theta_e^T(t)]^T = 0$ of the closed-loop error dynamics of the complete group as given in (6.19).

Theorem 6.3. *Consider a group of n omnidirectional mobile robots with their posture kinematic model given as in (6.1). Each robot in the group has its own reference trajectory, composed of its reference position $(x_{r_i}(t), y_{r_i}(t))$ and orientation $\theta_{r_i}(t)$, for $i \in \{1, 2, \ldots, n\}$ denoting the i-th robot in the group. Suppose that all the robots in the group are equipped with a coordinating controller given as in (6.17)-(6.18), in which the network-induced delay τ is constant, equal, and possibly unknown to all robots. If the following conditions are satisfied:*

- *the tracking gains satisfy $c_{x_i}, c_{\theta_i}, c_{y_i} > 0$, $\forall i \in \{1, 2, \ldots, n\}$;*
- *the coupling gains satisfy $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} > 0$ $\forall i, j \in \{1, 2, \ldots, n\}$, $i \neq j$,*

then, $Q_e = 0$ is a globally exponentially stable (GES) equilibrium point of the closed-loop error dynamics (6.19) for any $\tau \geq 0$. This results in the group of n omnidirectional robots achieving mutual motion coordination independently of the time-delay affecting their information exchange.

Proof. The closed-loop error dynamics (6.19) are composed of the following decoupled systems, whose stability may be studied individually:

$$\dot{X}_e(t) = -C_x X_e(t) + K_x X_e(t-\tau), \tag{6.20a}$$
$$\dot{Y}_e(t) = -C_y Y_e(t) + K_y Y_e(t-\tau), \tag{6.20b}$$
$$\dot{\Theta}_e(t) = -C_\theta \Theta_e(t) + K_\theta \Theta_e(t-\tau). \tag{6.20c}$$

Note that the individual X_e, Y_e, and Θ_e-dynamics have the same form as the ξ_2-dynamics (C.1b) studied in Section C.2.2. Recall also that the stability of the latter dynamics has already been determined to be delay independent by means of the Lyapunov-Razumikhin delay-independent stability criterion for linear systems (see Gu et al., 2003; Niculescu et al., 1998). Considering the previous, it follows that the equilibrium point $Q_e(t) = [X_e^T(t)\, Y_e^T(t)\, \Theta_e^T(t)]^T = 0$ of the closed-loop error dynamics of the complete group is globally exponentially stable for any $\tau \geq 0$, provided the tracking and correction gains satisfy the requirements posed in the theorem; namely, that the tracking gains satisfy $c_{x_i}, c_{\theta_i}, c_{y_i} > 0$ and the correction gains satisfy $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} > 0$ for all $i, j \in \{1, \ldots, n\}$, $i \neq j$. This completes the proof. □

The theorem states that mutual motion coordination in a group of omnidirectional robots is delay-independent when considering a delayed error exchange between the robots. This occurrence is quite remarkable but not at all unexpected since the omnidirectional robot is feedback linearizable and the coordinating controller (6.18) yields closed-loop error dynamics given by a delayed LTI system. Because of this fact, it is not necessary to make use of the results of Sedova (2008b) to analyze the stability of these dynamics (as was the case with the group of unicycles), but rather it is possible to make use of tools such as the Lyapunov-Razumikhin delay-independent stability criterion for linear systems (see Gu et al., 2003, for additional details).

The result implies that, from a stability perspective, the mutually coordinating controllers in the delay-free and delayed cases for a group of omnidirectional robots are essentially the same. The main difference that we expect between both cases is the reaction of the group to perturbations. Recall that the main contribution of the coupling terms is in helping maintain motion coordination when one of the robots in the group is perturbed. Nevertheless, as explained already in Chapter 5 for the unicycles, when a network-induced delay affects the error information exchange between the robots and one of the robots is perturbed, the remaining robots react to the perturbation after the time-delay. This means that the robot which is perturbed begins compensating for the perturbation by itself before the other

Table 6.1 . First simulation parameters; remote control of an omnidirectional robot.

Initial conditions						Gains					
System			Predictor			Tracking			Correction		
$x(0)$ [m]	$y(0)$ [m]	$\theta(0)$ [rad]	$z_1(0)$ [m]	$z_2(0)$ [m]	$z_3(0)$ [rad]	c_x	c_y	c_θ	k_x	k_y	k_θ
1.5	0.2	$\frac{\pi}{3}$	0.0	0.0	0.0	1.0	1.0	1.0	0.25	0.25	0.25

robots in the group react to it and make their contributions to overcome it. The simulation results in the next section will further illustrate this effect.

6.2.4 Simulation Results

The simulations in this section validate the stability results and illustrate the performance of the remote control and motion coordination strategies studied so far in this chapter. The first simulation illustrates the remote tracking control of an omnidirectional robot as proposed in Section 6.2. The initial conditions for the system $q(0)$ and the state predictor $z(0)$, together with the tracking and correction gains are provided in Table 6.1. The network-induced delay is assumed to be $\tau = \tau_f + \tau_b = 0.75 + 0.75 = 1.5\,\text{s}$ and the correction gains are selected such that the local stability criterion (6.14) posed in Theorem 6.2 is satisfied. Note that accommodating a time-delay of 1.5 s with the global stability criterion (6.10) posed in Theorem 6.1 would require smaller tracking and correction gains c_x, c_y and k_x, k_y, respectively. The reference trajectory is given by a number of line and arc segments which form a square starting at $(1.2, 1.4)\,\text{m}$, with a side-length of 0.8 m, and rounded edges (see Figure 6.4). It takes 60 s to trace a square and, since the simulation takes 120 s, the robot tracks the trajectory twice. The robot has a prescribed orientation of $\frac{\pi}{2}$ rad at all times.

The reference, robot, and predictor trajectories in the global coordinate frame $\underline{\vec{e}}^0$ are depicted in Figure 6.4, with their initial and final positions marked with a cross and a circle, respectively. A positive additive perturbation affects the inputs of the robot during 3 s starting at $t = 90\,\text{s}$. During this time, the robot's translational velocities $v_1(t)$ and $v_2(t)$ are affected by 0.1 m/s and 0.05 m/s, respectively, and its translational velocity $\omega(t)$ by 0.2 rad/s. Still, the plot clearly shows that the robot is capable of tracking the (delayed) reference trajectory.

The plots in Figure 6.5 further illustrate the performance of the predictor-controller combination. The plots in the first row depict the tracking errors, defined

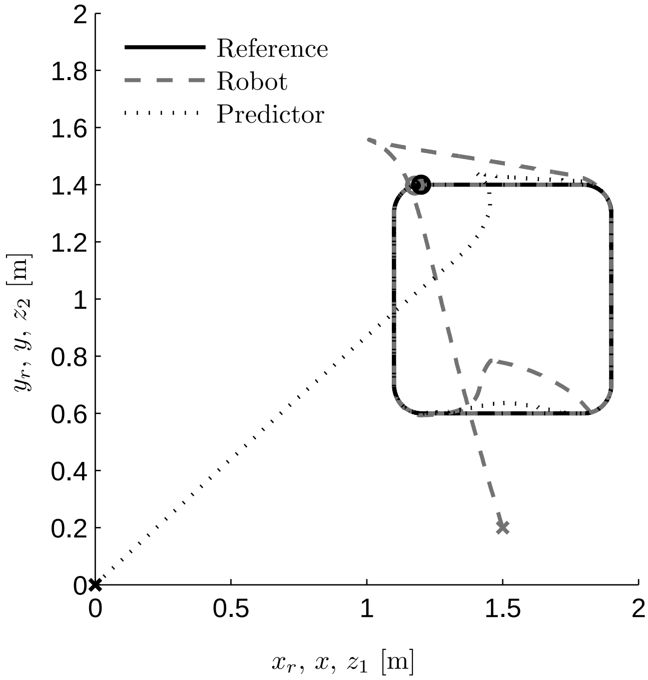


Figure 6.4 . First simulation: remote control of an omnidirectional robot. Reference, robot, and predictor trajectories in $\underline{\vec{e}}^0$.

as $x_e(t) = x_r(t-\tau_f) - x(t)$, $y_e(t) = y_r(t-\tau_f) - y(t)$, and $\theta_e(t) = \theta_r(t-\tau_f) - \theta(t)$, respectively. The plots in the second row show the prediction error $p_e(t)$, already defined in Section 6.2.2. Clearly, the tracking and prediction errors all converge to zero even in the presence of the (transient) perturbation.

The last plots for this simulation, which appear in Figure 6.6, show the control inputs of the robot and how they reflect the perturbation at 90 s. The considerable differences between the required velocities of the line and arc segments (which are joined to form the reference trajectory) are reflected as discontinuities in the resulting control inputs of the robot.

The second simulation illustrates the remote motion coordination of a group of four omnidirectional robots. The virtual center is the same square with rounded edges used in the first simulation and the reference orientation of the robots is prescribed as $\theta_{r_1}(t) = 0.0\,\text{rad/s}$, $\theta_{r_2}(t) = \frac{2\pi}{5}\,\text{rad/s}$, $\theta_{r_3}(t) = \frac{4\pi}{5}\,\text{rad/s}$, and $\theta_{r_4}(t) = \frac{6\pi}{5}\,\text{rad/s}$. The simulation takes 120 s and the initial conditions, controller gains, and displacements from the virtual center (given in terms of the global coordinate frame $\underline{\vec{e}}^0$) for each robot are given in Table 6.2. All the coupling gains are set to

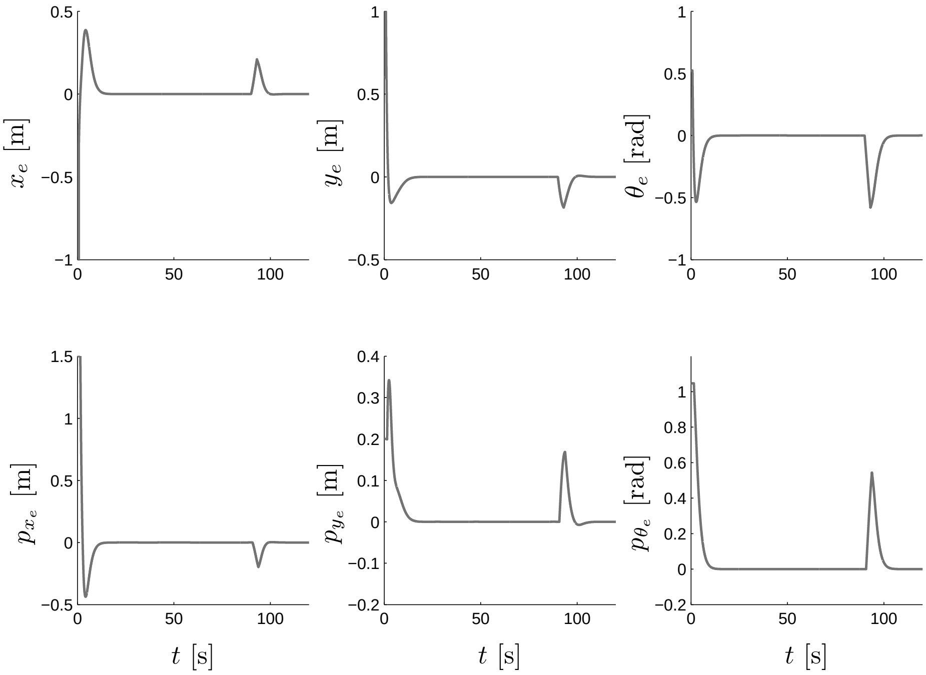


Figure 6.5 . First simulation. First row: tracking error; second row: prediction error.

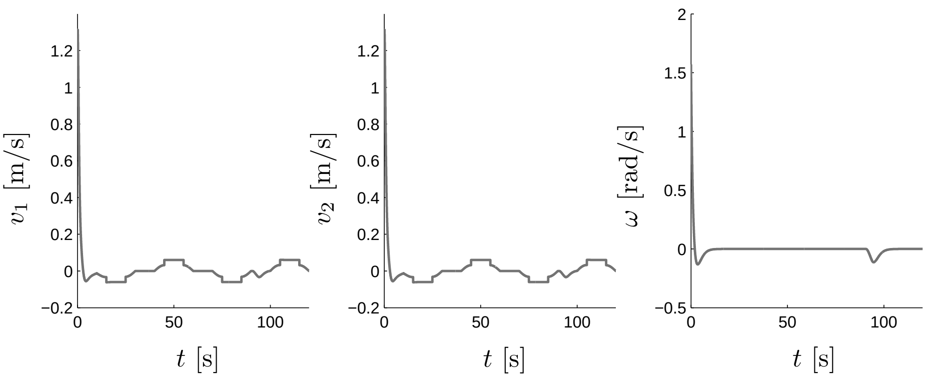


Figure 6.6 . First simulation. Control inputs.

0.75, that is $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}} = 0.75$, for $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$, and $i \neq j$. The network-induced delay $\tau = 1\,\text{s}$. The selected tracking and coupling gains are all positive, so that the conditions posed on them in Theorem 6.3 are satisfied and the delay-independent global stability of the remote mutual motion coordination strategy is ensured.

Table 6.2 . Second simulation parameters; remote motion coordination of a group of four omnidirectional robots.

	Initial conditions			Controller gains			Displacements in $\underline{\vec{e}}^0$		
	x_0 [m]	y_0 [m]	θ_0 [rad]	c_x	c_y	c_θ	l_x [m]	l_y [m]	l_θ [rad]
Robot 1	2.0	1.0	$\frac{\pi}{6}$	1.0	1.0	1.0	1.0	0.5	0.0
Robot 2	1.8	1.0	$-\frac{\pi}{4}$	1.0	1.0	1.0	1.0	−0.5	0.0
Robot 3	1.2	1.5	$\frac{\pi}{2}$	1.0	1.0	1.0	−1.0	−0.5	0.0
Robot 4	1.4	0.5	$\frac{\pi}{3}$	1.0	1.0	1.0	−1.0	0.5	0.0

The reference trajectory and path of each robot are shown in Figure 6.7 in the global coordinate frame $\underline{\vec{e}}^0$. At $t = 20\,$s the translational and rotational velocities of the second robot, $v_{1_2}(t)$, $v_{2_2}(t)$ and $\omega_2(t)$, are affected during 4 s by a positive additive perturbation of 1.0 m/s, 1.5 m/s, and 3.0 rad/s, respectively. The plots show how the perturbation directly affects the second robot and how this perturbation is reflected on the remaining robots, illustrating the tradeoff between following their respective reference trajectory and maintaining motion coordination. As explained already in Chapter 5, the larger the magnitude of the network-induced delay, the longer it will take for the unperturbed robots to reflect this perturbation. In other words, by the time the unperturbed robots begin to reflect the perturbation, the perturbed robot is already compensating for it. Since the network-induced delay considered is rather large (1 s), the perturbation reflected on the first, third, and fourth robot is smaller than the one directly affecting the second robot.

The plots in Figure 6.8 show the tracking errors of each robot, whereas the plots in Figure 6.9 depict the coordination errors between the robots, which are defined as $q_e(t) = q_{e_i}(t) - q_{e_j}(t) = [x_{e_i}(t) - x_{e_i}(t)\; y_{e_i}(t) - y_{e_j}(t)\; \theta_{e_i}(t) - \theta_{e_j}(t)]^T$, for $i, j \in \{1, 2, \ldots, n\}$, $i \neq j$. All the tracking and coordination errors converge to zero and temporarily reflect the perturbation which affects the second robot. The control inputs of each robot are depicted in Figure 6.10, where the perturbation is reflected on all the robots.

In summary, the previous simulation results validate the remote control and motion coordination strategies proposed for the omnidirectional mobile robot. The simulations yield the expected results according to the stability analyses carried out for both control strategies. Moreover, the simulations show that the proposed control strategies exhibit certain robustness against small, transient, additive perturbations in the control signals.

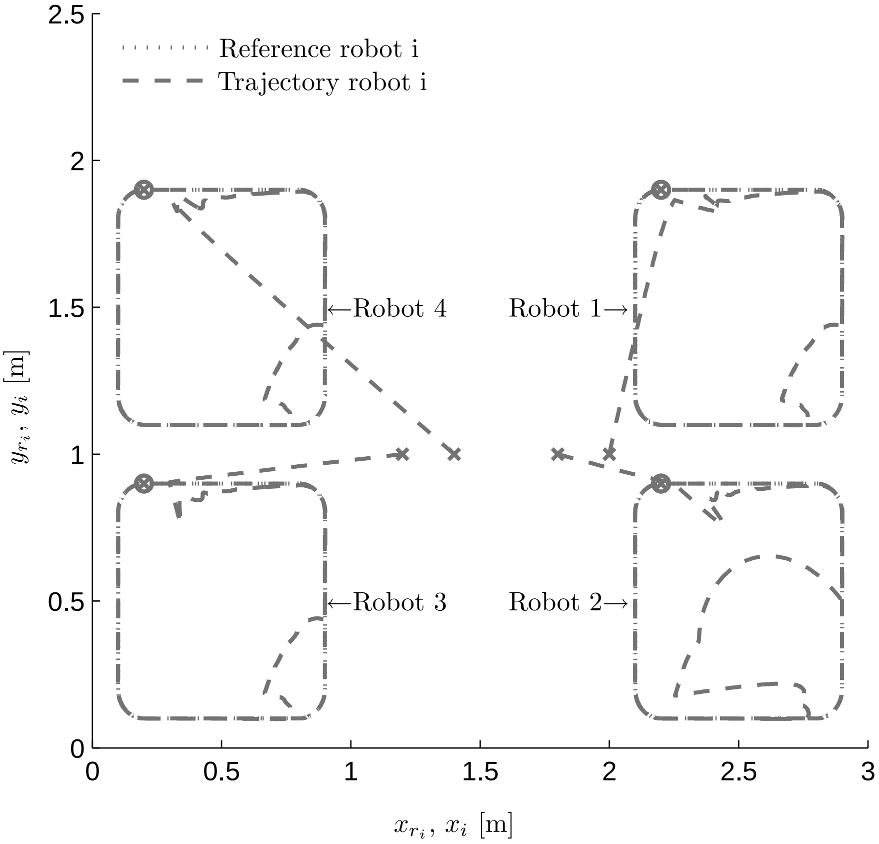


Figure 6.7 . Second simulation: remote mutual motion coordination of four omnidirectional robots ($i = 1, 2, 3, 4$) using a location oriented reference trajectory generation approach. The reference and robot trajectories are shown in the global coordinate frame $\underline{\vec{e}}^0$.

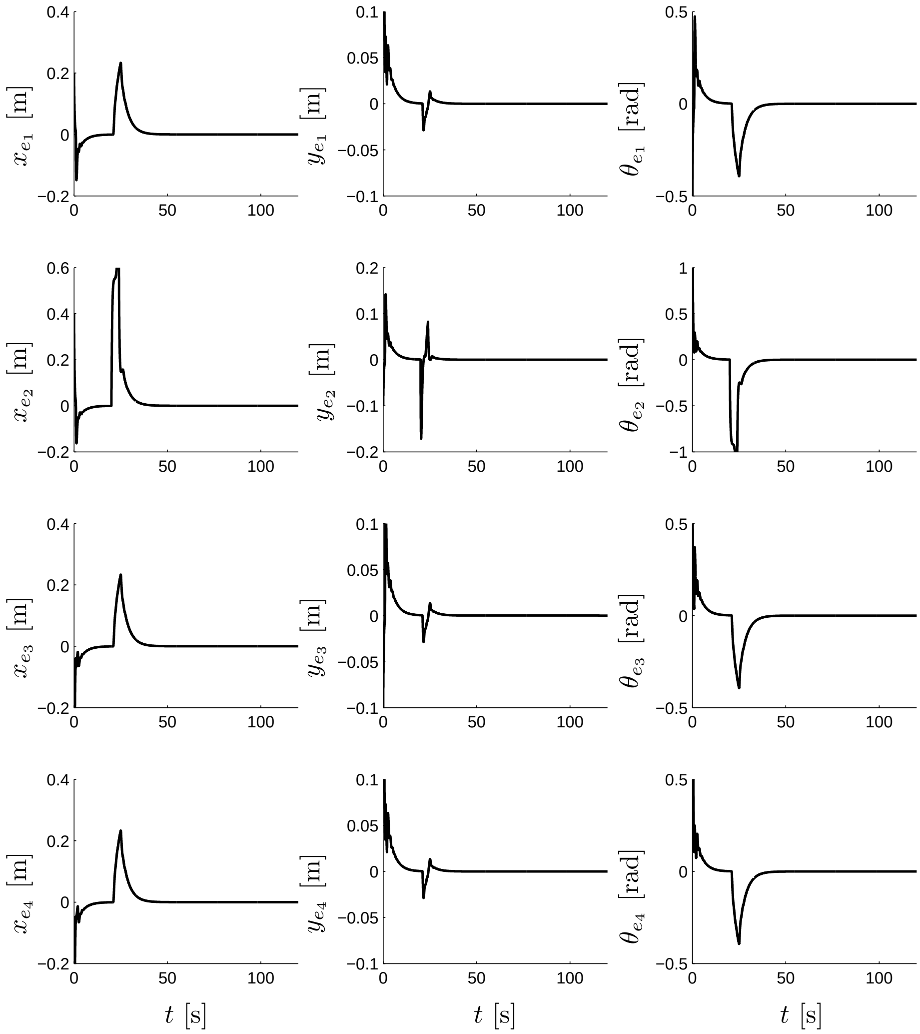


Figure 6.8 . Second simulation. Tracking errors of each robot. The second robot is perturbed at $t = 20\,\mathrm{s}$.

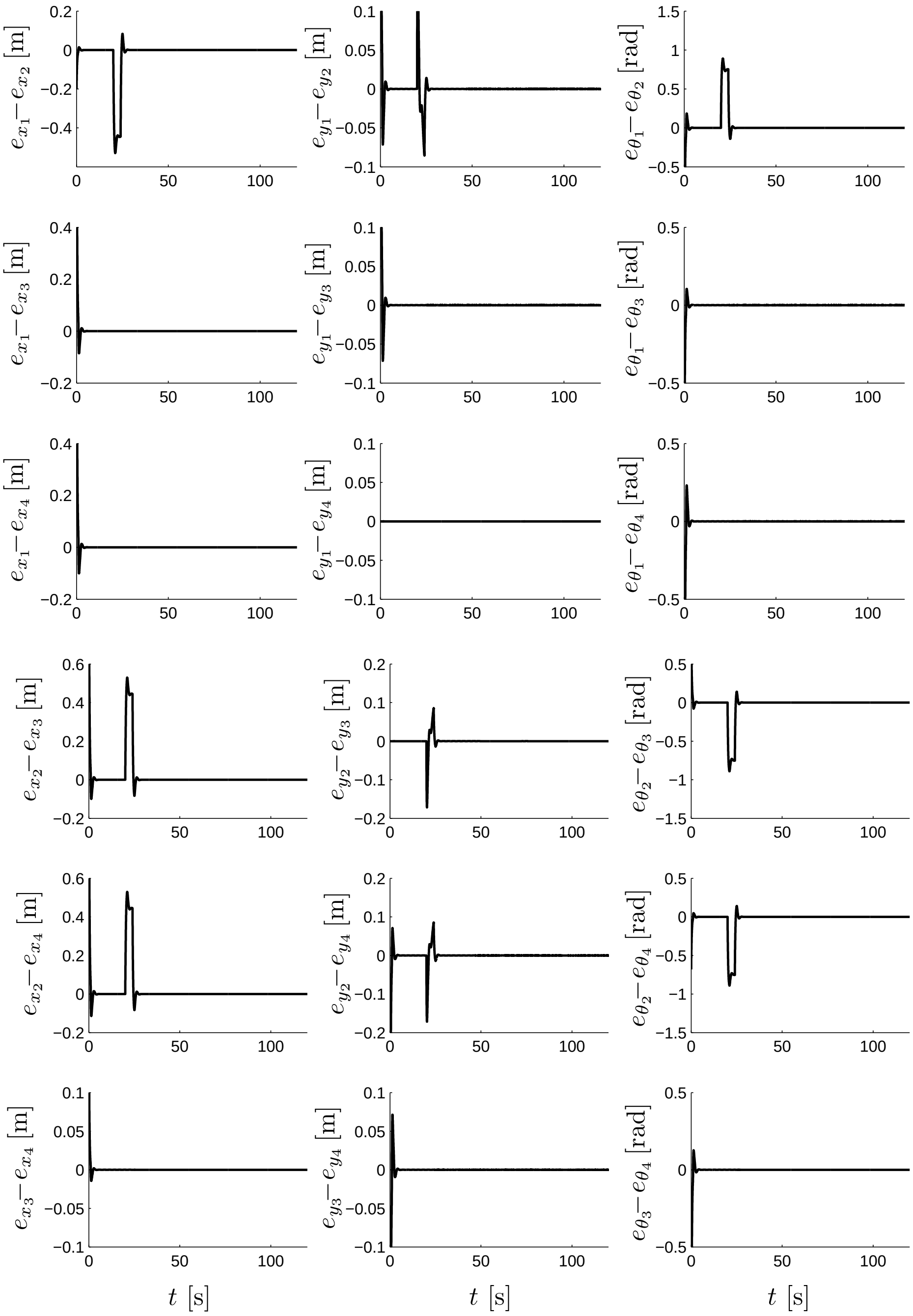


Figure 6.9 . Second simulation. Coordination errors between the robots. The second robot is perturbed at $t = 20\,\mathrm{s}$.

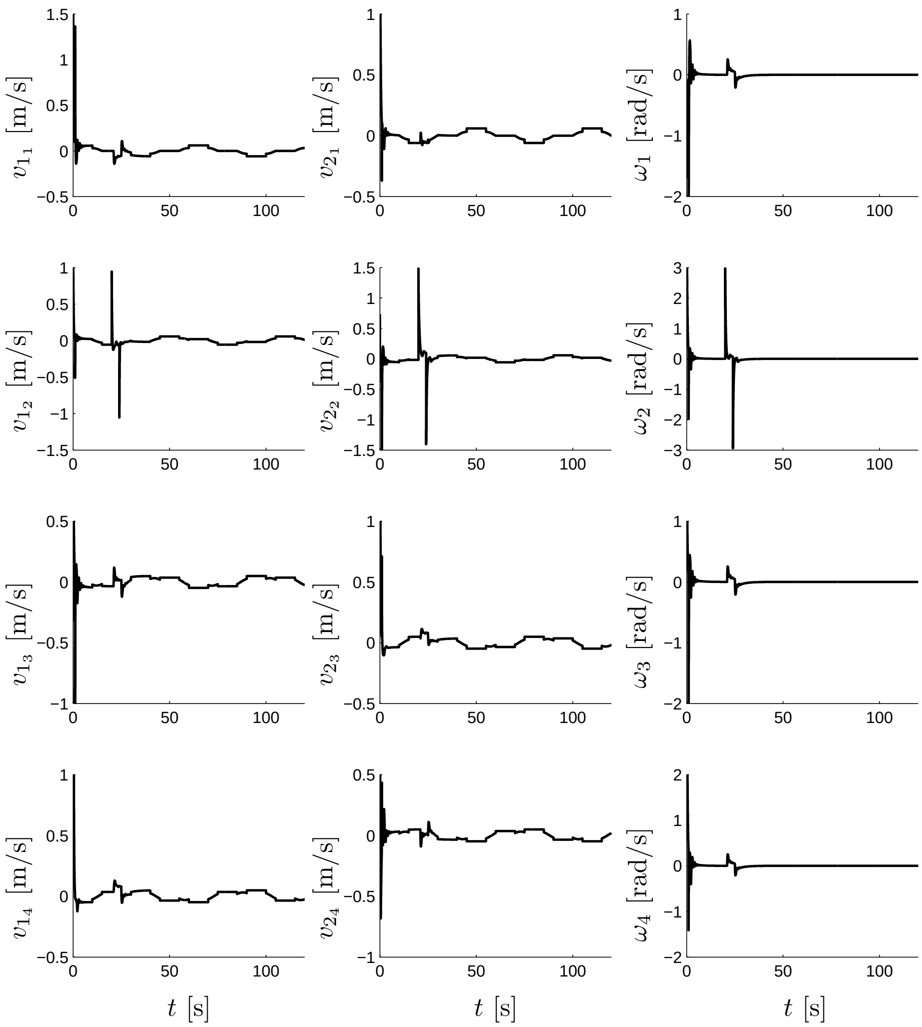


Figure 6.10 . Second simulation. Control inputs of each robot. The second robot is perturbed at $t = 20\,\text{s}$.

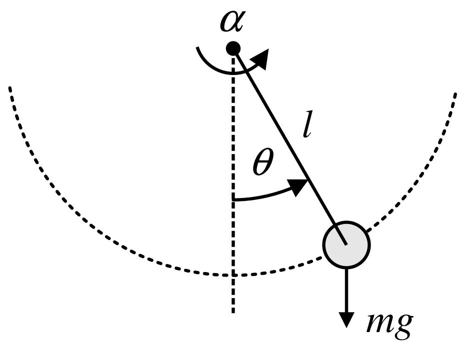


Figure 6.11 . Schematic representation of the one-link robot.

6.3 Remote Tracking Control of a One-Link Robot

The design of a predictor-controller combination for a one-link robot controlled over a delay-inducing communication network is investigated in this section. The purpose of this study is to motivate and make the first steps towards the application of such a remote control strategy to a broader class of mechanical systems; namely, robotic manipulators.

Consider a one-link robot with friction, such as the one shown in Figure 6.11. This robot has the following dynamic model:

$$ml^2\ddot{\theta}(t) + \kappa\dot{\theta}(t) + mgl\sin\theta(t) = \alpha(t), \tag{6.21}$$

where $\theta(t), \dot{\theta}(t)$, and $\ddot{\theta}(t)$ denote the angular position, velocity, and acceleration of the robot, respectively, and $\alpha(t)$ its input torque. The link has a length of l, a point mass m at its end, and its viscous friction coefficient is denoted by κ. The gravitational acceleration is denoted by g and the state of the system is defined as $q(t) = [\theta(t)\,\dot{\theta}(t)]^T$.

As with the unicycle and the omnidirectional robots, the one-link robot is assumed to be controlled over a two-channel network inducing a delay τ, comprised of an input and an output time-delay τ_f and τ_b, respectively. Rearranging (6.21) and including the input time-delay, the dynamic model of the robot becomes:

$$\ddot{\theta}(t) = -\frac{\kappa}{ml^2}\dot{\theta}(t) - \frac{g}{l}\sin\theta(t) + \frac{1}{ml^2}\alpha(t-\tau_f). \tag{6.22}$$

Additionally, the output time-delay τ_b produces the measured state $q(t-\tau_b) = [\theta(t-\tau_b)\,\dot{\theta}(t-\tau_b)]^T$.

The robot is intended to track a reference trajectory with a state $q_r(t) =$

$[\theta_r(t)\,\dot{\theta}_r(t)]^T$. For this purpose, the following state predictor is considered:

$$\ddot{z}(t) = -\frac{\kappa}{ml^2}\dot{z}(t) - \frac{g}{l}\sin z(t) + \frac{1}{ml^2}\alpha(t) + \nu_{\dot{\theta}}(t) + \nu_{\theta}(t), \tag{6.23}$$

with state $q_z(t) = [z(t)\,\dot{z}(t)]^T$ and correction term denoted by $\nu(t) = [\nu_{\dot{\theta}}(t)\,\nu_{\theta}(t)]^T$.

Given the prediction error $p_e(t) = \theta(t-\tau_b) - z(t-\tilde{\tau})$, the correction term are defined as follows:

$$\nu_{\theta}(t) = k_{\theta}(\theta(t-\tau_b) - z(t-\tilde{\tau})), \tag{6.24a}$$

$$\nu_{\dot{\theta}}(t) = k_{\dot{\theta}}(\dot{\theta}(t-\tau_b) - \dot{z}(t-\tilde{\tau})), \tag{6.24b}$$

where k_{θ} and $k_{\dot{\theta}}$ represent the correction term gains and $\tilde{\tau} := \tilde{\tau}_f + \tilde{\tau}_b$ represents the sum of the estimated input and output network-induced delays, which is assumed to be known (that is, $\tilde{\tau} = \tau = \tau_b + \tau_f$).

The tracking control law for the predictor-controller combination considering the state predictor (6.23) and correction term (6.24) is defined as follows:

$$\alpha(t) = ml^2\left(\ddot{\theta}_r(t) + k_d\dot{z}_e(t) + k_p z_e(t) + \frac{\kappa}{ml^2}\dot{z}(t) + \frac{g}{l}\sin z(t)\right), \tag{6.25}$$

where $z_e(t) = \theta_r(t) - z(t)$ and k_p and k_d denote the position and velocity tracking gains, respectively. The tracking controller (6.25) is a computed torque controller (see for example Kelly et al., 2005, for additional details on this type of controllers) which uses the output of the predictor instead of the output of the system.

The control action applied to the robot after the input delay τ_f yields:

$$\alpha(t-\tau_f) = ml^2\left(\ddot{\theta}_r(t-\tau_f) + k_d\dot{z}_e(t-\tau_f) + k_p z_e(t-\tau_f) + \frac{\kappa}{ml^2}\dot{z}(t-\tau_f) + \frac{g}{l}\sin z(t-\tau_f)\right). \tag{6.26}$$

Exploiting the state predictor (6.23), correction term (6.24), and delayed tracking controller (6.26), the closed-loop error dynamics are given as follows:

$$\ddot{z}_e(t) = -k_d\dot{z}_e(t) - k_p z_e(t) - k_{\dot{\theta}}\dot{p}_e(t) - k_{\theta}p_e(t), \tag{6.27a}$$

$$\ddot{p}_e(t) = -\frac{\kappa}{ml^2}\dot{p}_e(t) - \frac{g}{l}(\sin\theta(t-\tau_b) - \sin z(t-\tau)) - k_{\dot{\theta}}\dot{p}_e(t-\tau) - k_{\theta}p_e(t-\tau). \tag{6.27b}$$

Assuming that both $\theta(t)$ and $z(t)$ are small, it is possible to use the small angle approximation to rewrite the closed-loop error dynamics (6.27) in the following way:

$$\ddot{z}_e(t) = -k_d\dot{z}_e(t) - k_p z_e(t) - k_{\dot{\theta}}\dot{p}_e(t) - k_{\theta}p_e(t), \tag{6.28a}$$

$$\ddot{p}_e(t) = -\frac{\kappa}{ml^2}\dot{p}_e(t) - \frac{g}{l}p_e(t) - k_{\dot{\theta}}\dot{p}_e(t-\tau) - k_{\theta}p_e(t-\tau), \tag{6.28b}$$

which may be represented by the following delayed LTI system:

$$\begin{bmatrix} \dot{z}_e(t) \\ \dot{p}_e(t) \\ \ddot{z}_e(t) \\ \ddot{p}_e(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_p & -k_\theta & -k_d & -k_{\dot{\theta}} \\ 0 & -\frac{g}{l} & 0 & -\frac{\kappa}{ml^2} \end{bmatrix} \begin{bmatrix} z_e(t) \\ p_e(t) \\ \dot{z}_e(t) \\ \dot{p}_e(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -k_\theta & 0 & -k_{\dot{\theta}} \end{bmatrix} \begin{bmatrix} z_e(t-\tau) \\ p_e(t-\tau) \\ \dot{z}_e(t-\tau) \\ \dot{p}_e(t-\tau) \end{bmatrix} \tag{6.29}$$

As with the unicycle and omnidirectional robots, ensuring the stability of the linearized closed-loop error dynamics (6.29) guarantees that the robot will track a delayed version $q_r(t - \tau_f)$ of its reference trajectory. Note that this control goal assumes that a preview of the reference trajectory is not available. Hence, Remark 4.2 concerning the availability of $q_r(t + \tau_f)$ at time t is also in place.

There are a number of ways to study the stability of a delayed LTI system such as (6.29) (refer for example to Gu et al., 2003; Michiels and Niculescu, 2007; Niculescu et al., 1998, among others). In our case, we conducted an eigenvalue-based numerical test for a given set of system and control parameters (mass m, length l, and viscous friction coefficient κ of the one-link robot and tracking gains k_p and k_d) and for different values of the correction gains k_θ and $k_{\dot{\theta}}$. The test computes the rightmost eigenvalue for a certain combination of parameters and correction gains using DDE-BIFTOOL, a Matlab package for numerical bifurcation and stability analysis of delay differential equations (see Engelborghs et al., 2001). It is worth noting that the resulting stability condition is necessary and sufficient, and that this condition holds for a set of initial conditions which are close enough to the reference trajectory. In addition, since we are considering a delayed LTI system, the stability result obtained is of an exponential nature.

The results from the previous numerical test for a one-link robot with $m = 1.0\,\text{kg}$, $l = 1.0\,\text{m}$, $\kappa = 0.5$, $k_d = 20$ and $k_p = 10$ are shown in Figure 6.12. The plot shows how as $k_\theta \downarrow 0$ and $k_{\dot{\theta}} \downarrow 0$, the magnitude of the allowable time-delay increases. In other words, the same remark regarding how a low-gain predictor design yields high robustness against delays is in place.

An additional test was carried out considering $m = 0.5\,\text{kg}$, $l = 0.4\,\text{m}$, $\kappa = 0.3$ and the same values of k_p and k_d. In this case, the linearized closed-loop error dynamics (6.29) are stable for all $\tau \in [0, 2]\,\text{s}$ and $k_\theta, k_{\dot{\theta}} \in [0, 2]$. Nevertheless, as expected, additional tests with these parameters showed that as k_θ and $k_{\dot{\theta}}$ increase, the magnitude of the allowable time-delay decreased.

Note that the linearized closed-loop error dynamics (6.29) may also be represented as the following cascaded system:

$$\dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_2(t), \tag{6.30a}$$

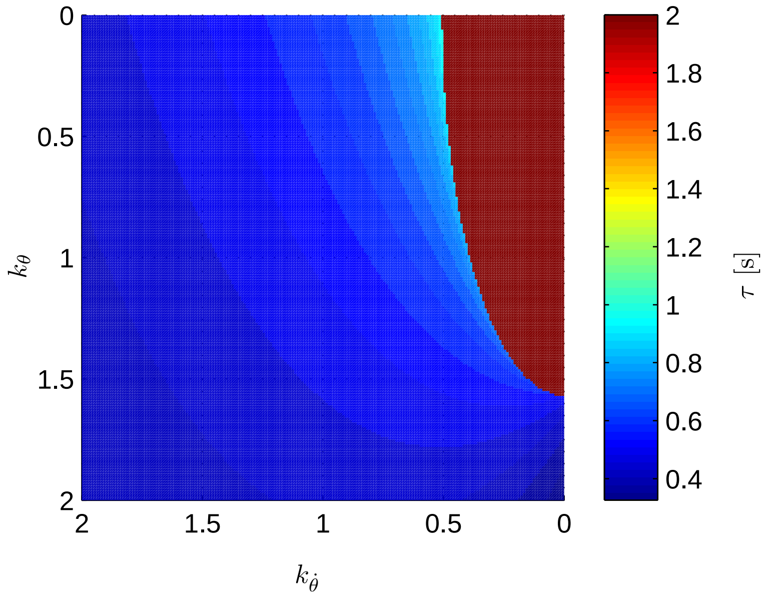


Figure 6.12 . Allowable time-delay τ for the one-link robot considering $m = 1.0\,\mathrm{kg}$, $l = 1.0\,\mathrm{m}$, $\kappa = 0.5$, and fixed tracking gains of $k_d = 20$ and $k_p = 10$. The magnitude of τ corresponds to the values in the color bar. To better illustrate the relationship between the correction gains $k_\theta, k_{\dot\theta}$ and the time-delay, the allowable delay has been cut off at 2 s.

$$\dot\xi_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau), \tag{6.30b}$$

where $\xi_1(t) := [z_e(t)\ \dot z_e(t)]^T$, $\xi_2(t) := [p_e(t)\ \dot p_e(t)]^T$, $\xi_{1_t} \in \mathcal{C}(2)$, $\xi_{2_t} \in \mathcal{C}(2)$, and

$$A_1 = \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix},\ \ A_2 = \begin{bmatrix} 0 & 0 \\ -k_\theta & -k_{\dot\theta} \end{bmatrix},\ \ B_1 = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & -\frac{\kappa}{ml^2} \end{bmatrix},\ \ B_2 = \begin{bmatrix} 0 & 0 \\ -k_\theta & -k_{\dot\theta} \end{bmatrix}.$$

The stability of the previous cascaded error dynamics may be studied using a similar approach as with the unicycle and the omnidirectional robots (in this case, based on Theorem 2.25). Following this approach it is possible to explicitly state the requirement on a certain allowable time-delay $\tau_{\max}$ for which the error dynamics are stable for all $\tau \in [0, \tau_{\max})$. Nevertheless, the results obtained using this approach turned out to be quite conservative (several orders of magnitude in fact). For this reason, we decided to present our stability results based on the numerical test explained previously.

Table 6.3 . Third simulation parameters; remote control of a one-link robot.

Initial conditions				Gains			
System		Predictor		Tracking		Correction	
$q(0)$ [rad]	$\dot{q}(0)$ [rad/s]	$z(0)$ [rad]	$\dot{z}(0)$ [rad/s]	k_p	k_d	k_θ	$k_{\dot{\theta}}$
$\frac{\pi}{3}$	0.0	0.0	0.0	10.0	20.0	0.2	0.2

6.3.1 Simulation Results

The results of the simulation are shown in Figure 6.13 and illustrate the predictor-controller combination applied to a one-link robot with a mass of $m = 1.0\,\text{kg}$, a length of $l = 1.0\,\text{m}$, and viscous friction coefficient of $\kappa = 0.5$. The initial conditions of the robot and the predictor, and the values of the tracking and correction gains appear in Table 6.3. The magnitude of the network-induced delay is $\tau = \tau_f + \tau_b = 0.3 + 0.3 = 0.6\,\text{s}$ and the simulation takes $120\,\text{s}$. The tracking and correction gains for this simulation are selected such that the closed-loop error dynamics (6.29) are exponentially stable. This is ensured by means of the eigenvalue based numerical test explained above and can be assessed for different values of the correction gains with the aid of Figure 6.12. The reference trajectory is a sine with an amplitude of 1 rad and an angular frequency of 0.5 rad/s, and the input torque is affected by a perturbation of 4 Nm at $t = 30\,\text{s}$ during 1 s.

The reference trajectory, robot, and predictor angular positions and velocities are depicted, respectively, in the first and second plots of the first row in Figure 6.13. The plots show how the trajectory of the robot converges to the reference trajectory delayed by $\tau_f = 0.3\,\text{s}$. The plots in the second and third rows depict the tracking errors, $q_e(t) = \theta_r(t - \tau_f) - \theta(t)$ and $\dot{q}_e(t) = \dot{\theta}_r(t - \tau) - \dot{\theta}(t)$, and prediction errors, $p_e(t)$ and $\dot{p}_e(t)$, respectively. These errors converge to zero and show how, after approximately 15 s, the system overcomes the perturbation.

The simulation results are in line with what was expected from the theoretical formulations and show the applicability of the remote control strategy for a one-link robot. Moreover, the results also show that the proposed control strategy is capable of overcoming small, transient, additive perturbations in the control signals.

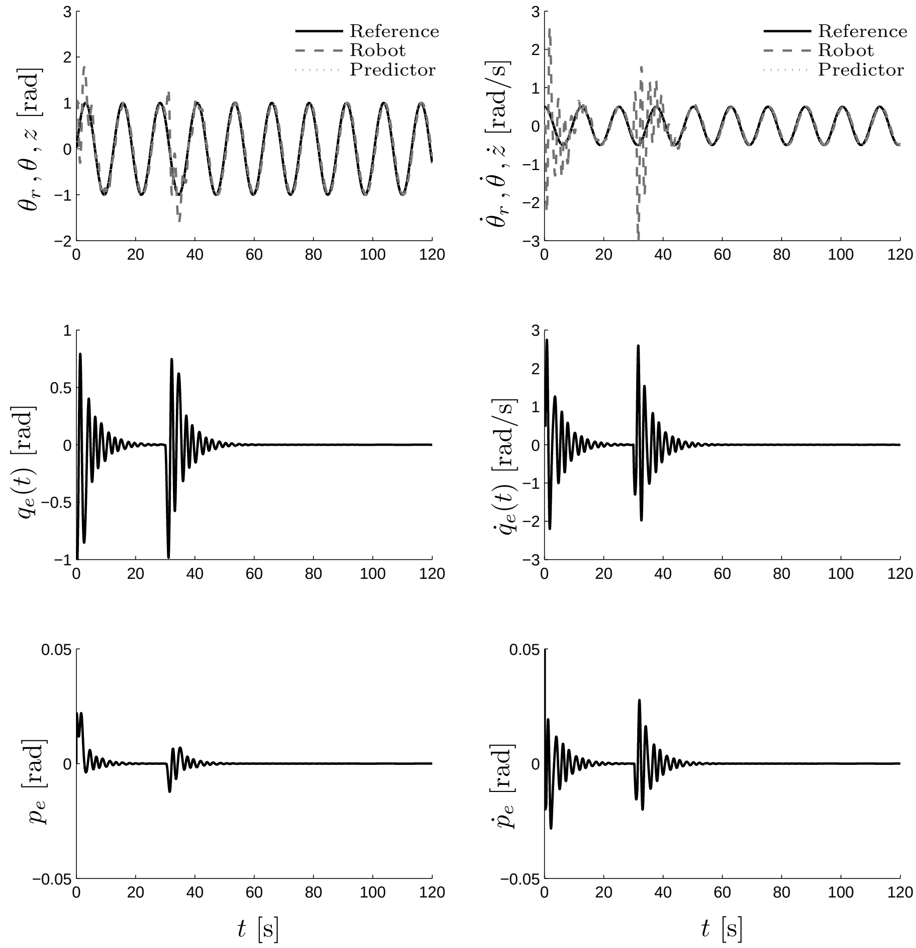


Figure 6.13 . Third simulation: remote control of a one-link robot. First row: angular position and velocity of the reference trajectory, robot, and predictor; second row: angular position and velocity tracking errors; third row: angular position and velocity prediction errors.

6.4 Concluding Remarks

In this chapter, the different remote control and motion coordination strategies studied in this thesis have been applied to other dynamical systems besides unicycle-type mobile robots.

In particular, predictor-controller combinations which allow for the remote tracking control of an omnidirectional robot and a one-link robot have been proposed. A stability analysis for each of the remote tracking control strategies has been presented and simulation results which validate the theoretical formulations of these analyses have been included. In the case of the omnidirectional robot, the resulting upper bound on the allowable network-induced delay based on a global stability analysis turned out to be quite conservative. For this reason, a local stability analysis which yields a less conservative upper bound on the delay has also been presented. As with the unicycle, if at time t the reference trajectory $q_r(t+\tau_f)$ is known, it becomes possible for the remote control strategies in this chapter to track the desired reference trajectory, that is, $q_r(t)$. Recall that, when considering the remote control or motion coordination of unicycles in this thesis, the resulting closed-loop error dynamics could always be rearranged in a cascade. Our initial assumption was that this occurrence was due to the fact that the controllers used in this work for the unicycle are based on a tracking control law which yields cascaded error dynamics. Nevertheless, since the predictor-controller combination for the omnidirectional robot and the one-link robot also yield error dynamics which can be rearranged as a cascaded system, it appears that this result is also related to the architecture of the remote control strategy. Specifically, it seems that the observer-like structure of the predictor is the main reason behind this occurrence. Whether the resulting cascaded system can be separated between the position and orientation error dynamics (as with the mobile robots) or between the $z_e(t)$ and $p_e(t)$ error dynamics (as with the one-link robotic) apparently depends on the particular system being studied.

An additional extension considered in this chapter concerns the remote mutual motion coordination of a group of omnidirectional robots. A global stability analysis of the resulting closed-loop error dynamics of the whole group indicates that this type of motion coordination is delay-independent for omnidirectional robots. A simulation with four omnidirectional robots and a network-induced delay of 1 s shows how the robots maintain mutual motion coordination in the face of transient perturbations.

In general, the results in this chapter confirm that the control strategies developed in this thesis may very well find application in a wide range of robotic systems.

7

Conclusions and Recommendations

Abstract. This chapter recapitulates the main contributions and results of this thesis. In addition, the chapter provides a number of recommendations for further research.

7.1 Conclusions

As the assignments conferred to robotic systems become more critical and complex, a number of scientific and technological challenges have to be tackled in order to fully realize the potential of this type of systems. In particular, in this thesis we propose control strategies which guarantee the stability and performance of a telerobotic and a cooperative robotic system in the face of delay-inducing networked communication. The problem is relevant both from a theoretical and a practical point of view, due to the fact that exchanging information through a communication network is a fundamental requirement in this type of robotic systems. The remainder of this section discusses in greater detail the main contributions and results of the remote tracking control and remote motion coordination strategies presented in this thesis.

Remote Tracking Control of a Mobile Robot

In Chapter 4 and Chapter 6, a predictor-controller combination which allows for the remote tracking control of different types of robotic systems is proposed. The state predictor in the predictor-controller combination, inspired on the notion of anticipating synchronization, anticipates the states of the system and enables the implementation of a remote tracking controller.

In Chapter 4, the predictor-controller combination is proposed for a unicycle-type mobile robot. The predictor-controller combination guarantees the tracking of a delayed reference trajectory. A local stability analysis shows that the tracking and prediction error dynamics are locally uniformly asymptotically stable up to a certain time-delay $\tau_{\max}$ (for which an explicit expression is provided). In addition, a global stability analysis shows that there exists a maximum delay $\tau_{\max} > 0$ such that the tracking and prediction error dynamics are globally uniformly asymptotically stable for delays up to $\tau_{\max}$. Experiments in which a unicycle robot located in Japan is controlled from the Netherlands using the Internet as the communication channel further validate the proposed remote control strategy.

In Chapter 6, the remote tracking control problem of an omnidirectional mobile robot is addressed. The closed-loop tracking and prediction error dynamics are determined to be globally uniformly asymptotically stable up to a certain time-delay $\tau_{\max} > 0$. Due to the fact that this bound appears to be rather conservative, a local stability analysis which yields a larger bound on the allowable time-delay has also been included. The problem of the remote tracking control of a one-link robot is also addressed in Chapter 6. In this case, the linearized error dynamics are shown to be stable for a given set of control parameters using an eigenvalue-based numerical test. Illustrative simulations further validate the proposed remote control strategies for the omnidirectional and the one-link robots.

The following can also be concluded for the remote tracking control strategy presented in Chapter 4 and Chapter 6:

- The remote tracking control strategies proposed in Chapter 4 and Chapter 6 assume that the reference trajectory for the system is not known a priori. As a consequence, the results in this thesis ensure tracking of delayed reference trajectories. However, if the reference trajectory is known in advance (at least for a time span of the forward network-induced delay), tracking non-delayed reference trajectories becomes possible.
- The closed-loop error dynamics which result from the tracking and prediction errors can be rearranged in as a cascaded system in all cases (that is, for the unicycle and the omnidirectional mobile robots and for the one-link robotic manipulator). In general, expressing these error dynamics as a cascaded system facilitates the ensuing stability analysis. In the case of the mobile robots, the systems conforming the cascade can be clearly divided between the error dynamics which correspond to the position errors and the

error dynamics which correspond to the orientation errors of the robots. An intuitive interpretation of the fact that the closed-loop error dynamics of the different systems can be rearranged as a cascaded system is that this is due to the structure of the predictor itself, which is similar to that of an an observer. Nevertheless, this claim has not been verified formally and the possibility that the closed-loop error dynamics of a broader class of systems can be rearranged as a cascaded system is something that remains to be seen.

- The stability analyses showed that the magnitude of the allowable communication delay is always related to the correction gains of the state predictor and not to the tracking gains. This shows a certain "separation" between tracking and estimation behavior.
- Even though the purpose of the remote tracking control strategy is to mitigate the negative effects of a constant time-delay, the experimental results show that the control strategy still works properly when considering a communication channel which induces (slightly) time-varying delays, such as the Internet link between the Netherlands and Japan introduced in Chapter 3.

Remote Motion Coordination of Mobile Robots

In Chapter 5 and Chapter 6, coordinating controllers which attain master-slave and mutual motion coordination under delayed communication for a group of mobile robots are proposed. Specifically, the problem of the remote master-slave and mutual motion coordination of a group of unicycle robots is addressed in Chapter 5. The proposed coordinating controller guarantees the global uniform asymptotic stability of the closed-loop error dynamics of the whole group. This stability result holds as long as the network-induced delay belongs to the interval $\tau \in [0, \tau_{\max}]$, with the maximum allowable time-delay $\tau_{\max} > 0$ small enough.

In Chapter 6, the same problem is addressed for a group of omnidirectional mobile robots. In this case, the coordinating controller guarantees the delay-independent global exponential stability of the closed-loop error dynamics of the whole group. In the same way as with the remote tracking control strategies, simulations and experiments in both chapters further validate the proposed remote coordinating controllers.

The following can also be concluded for the remote motion coordination strategy presented in Chapter 5 and Chapter 6:

- Recall that remote master-slave motion coordination has been shown to be a problem of a tracking nature in Chapter 5. In this type of motion coordination, the reference trajectories of the slaves are formed with the delayed output of the master. As a consequence, the communication delay only affects these reference trajectories. This implies that the closed-loop stability of the slaves can be guaranteed regardless of the delay induced by the communication network. However, performance is greatly affected by the networked-induced delay in the sense that the group might only be able to reach delayed master-slave motion coordination.

- A solution for the remote mutual motion coordination problem has been proposed in Chapter 5 for a group of unicycle robots and in Chapter 6 for a group of omnidirectional mobile robots. In the case of the unicycle robots, the robots reach remote mutual motion coordination as long as the (constant) network-induced delay is small enough. On the contrary, in the case of the omnidirectional mobile robots, this type of motion coordination has been shown to be delay-independent.
- As explained in Chapter 5, the coupling terms in the mutually coordinating controller determine how the group copes with perturbations. When considering a small communication delay, the couplings between the robots improve the robustness of the group against perturbations. However, as the magnitude of the communication delay increases, the ability of the group to cope with perturbations diminishes. Eventually, if the communication delay becomes too large, the group does not exhibit coordinated behavior any longer.

7.2 Recommendations

This final section enlists recommendations for further research. In the first part, recommendations regarding the remote tracking control of mobile robots and the related predictor-controller combination are presented. Afterwards, ideas for further research regarding the remote motion coordination of groups of mobile robots are discussed.

Remote Tracking Control of a Mobile Robot

Recommendations for future work regarding the remote tracking control strategy proposed in this thesis are given below:

- The stability analysis of the remote tracking control strategies proposed for a unicycle mobile robot in Chapter 4 and for an omnidirectional mobile robot and a one-link robot in Chapter 6 appeared to exhibit a certain degree of conservativeness in the upper bound on the maximum admissible communication delay. Using other techniques to study the stability of delayed systems might prove useful to obtain less conservative upper bounds.

- The remote tracking control strategy has been successfully validated in experiments using the Internet as communication medium. It has been shown that considering uncertain, though fairly constant delays suffices in these experiments. In other situations, for example when employing wireless communication links, one may consider other types of network-induced uncertainties as well. Such uncertainties would include time-varying communication delays, packet dropouts, and time-varying sampling intervals. Naturally, a first approach towards this goal would be to make use of the tools already available in the Networked Control Systems (NCSs) literature, such as in Carnevale et al. (2007), Heemels et al. (2010), Nešić and Teel (2004b), Nešić and Teel (2004a), van de Wouw et al. (2010b), van de Wouw et al. (2010a), and many others.
- A prerequisite to the previous recommendation might be to investigate a discrete-time implementation of the predictor-controller combination. Such implementation would allow considering the sample-and-hold effects in the system.
- A formal comparison between the predictor based on synchronization and the nonlinear version of the Smith predictor, together with its recent extensions, would provide a reliable measure of the applicability of the state predictor considered in this thesis.
- The remote tracking control strategy for a one-link robot proposed in Chapter 6 showed that the predictor-controller combination may be applied to a wider class of mechanical systems. Even though the ensuing stability analysis only considered the linearized system, investigating the applicability of the predictor-controller combination to other robotic systems such as mechanical manipulators appears quite appealing.
- The experimental setups in Japan and the Netherlands synchronize their clocks using the Windows Time Service (see Chapter 3). As a consequence, the internal clocks of the computers at each setup are roughly synchronized. Devising a method to synchronize these clocks more accurately would allow a better characterization of the performance of the remote control strategies.

Remote Motion Coordination of Mobile Robots

Below we present ideas for future developments regarding the remote motion coordination of a group of mobile robots:

- Recall from Chapter 5 that when considering the remote mutual motion coordination of unicycle robots it was not possible to provide an expression with explicit bounds for the allowable time-delay. Naturally, quantitative knowledge of the upper bound on the maximum admissible communication delay is desirable. Nonetheless, the lack of a strict Lyapunov function for the position error dynamics of the group obstructs the formulation of such an explicit delay bound. Nevertheless, there are certain assumptions which can be made on the reference rotational velocities of the robots which might allow to obtain an explicit bound on the allowable time-delay. Such assumptions may be found in Sedova (2008b), where constant reference rotational velocities have been considered.

- As with the remote tracking control strategy, considering additional uncertainties in the communication channel such as time-varying delays and packet dropouts would prove to be both challenging and fruitful. This would include, besides the usual uncertainties considered in NCSs, different communication delays between the elements of the group.

- The remote mutual motion coordination strategy in this thesis assumes that all the robots in the group receive the information from the virtual center at the same time. Exploring additional possibilities regarding the information flow between the virtual center and the robots in the group would provide the remote strategy with a more realistic architecture. One of these possibilities would include a centralized remote command center which assigns the reference trajectories for each individual robot. In this case, the reference trajectories of each robot would be generated at a different location. As a result, additional time-delays for the communication between the command center and the robots would have to be considered.

- A number of features can be added to the motion coordination strategies presented in this thesis in order to improve their performance in more realistic applications. Such features include, among others, inter-robot collision avoidance, obstacle avoidance, and considering a decentralized control architecture. These features are (partly) available in the literature. For instance, Kostić et al. (2009) and Kostić et al. (2010a) implement two different methods for collision avoidance and Sadowska (2010) proposes a decentralized control architecture in a similar setting as the one considered in this work.

- The experimental results in Chapter 5 include only two unicycle robots. Experiments with a larger number of robots would further illustrate the performance of the remote coordination strategies presented in this thesis.

- An important point to consider is how the principles behind the remote motion coordination strategies presented in this thesis translate to other robotic systems (such as manipulators) and to other systems in general (such as mechanical, electrical, and biological systems among others).

Even though the (remote tracking and coordinating) controllers designed in this thesis focus mainly on unicycle robots, we have also shown that it is possible to apply these control strategies to other (mechanical) systems. In this respect, we expect the control methodologies presented in this work to be applicable to, at least, a large class of linear time-invariant (LTI) systems and to certain classes of mechanical systems. In order to design a predictor-controller combination for a certain system, one of the assumptions that we have made so far is that a tracking control law for the delay-free version of the system is readily known. Moreover, we have assumed that the complete state of the system is measured and transmitted over the communication network. Because of the resemblance between the state predictor and a state observer, we foresee that an ‘observable-like’ condition on the system would be in place.

A

Location and Formation Oriented Reference Trajectories

Abstract . This appendix provides a detailed explanation of the different methods used throughout this thesis to derive the reference trajectories of the mobile robots which belong to a group and are supposed to coordinate their motions. In order to illustrate and compare the available options, a brief discussion has also been included.

A.1 Motivation

When considering a team of mobile robots, the main objective concerns the cooperation of the robots in order to successfully complete a task. In this sense, the main focus of this thesis is on ensuring that the robots in the group accurately track a specific reference trajectory. In other words, the main goal is to coordinate the motions of the robots, which could be useful for tasks as diverse as payload transportation, construction, and reconnaissance and surveillance.

For this purpose, the mobile robot motion coordination strategies presented in Chapter 5 considered, in both the master-slave and the mutual case, two different methods for deriving the reference trajectories of the mobile robots whose motions are to be coordinated. The first method results in a set of so-called *location oriented* reference trajectories, which are separated from each other by specific distances. A different approach consists in deriving the reference trajectories in such a way that the robots in the group maintain a specific geometrical shape while following a certain reference trajectory; these have been denoted as a set of *formation oriented* reference trajectories. Recall that in master-slave motion coordination the reference trajectories of all the slave robots connected to a certain master use as a starting point the trajectory tracked by the master robot. On the other hand, in mutual motion coordination the reference trajectories of all the robots belonging to a group are based on the reference trajectory of a common *virtual center*.

In this appendix, the location and formation oriented reference trajectories are derived for master-slave and mutual motion coordination of a group of mobile robots. In particular, the explicit expressions to derive the reference trajectories for a group of unicycle-type mobile robots are provided. In order to encourage further work and additional applications, the differences and similarities between a set of location oriented and formation oriented reference trajectories are discussed.

A.2 Location Oriented Reference Trajectories

As mentioned already, throughout this work the reference trajectory of each of the robots whose motion is to be coordinated is defined in terms of either the movement of the master robot or a common virtual center, depending on whether master-slave or mutual motion coordination is being implemented. Using the respective source of information as a starting point, possibly time-varying displacements are then used to design the specific trajectories for the robots in the group. Whenever such displacements are given with respect to the global coordinate frame $\underline{\vec{e}}^0 = [\vec{e}_x^{\,0} \; \vec{e}_y^{\,0}]^T$, the variables of interest to be defined are the distances in this frame between the coordinated robots and either the master robot or the virtual center, depending on the type of motion coordination being used. This definition gives way to what has been denoted as location oriented reference trajectories, which is shown in Figure A.1 for two robots, i and j, respectively. The figure highlights how by defining the displacement of each robot relative to the global coordinate frame, the distance between the robots in this frame and the direction of their position difference may be specified at will. For instance, in Figure A.1 the displacements considered are constant, which results in the robots mimicking the movement of their originating trajectory (either, the master robot or the virtual center) in a different location within their workspace.

In this section, the location oriented reference trajectories for master-slave and mutual motion coordination are derived for the unicycle robot. The posture kinematic model of the robot and its tracking control law, as given in Chapter 5, are used to obtain the mathematical expressions of its reference trajectories.

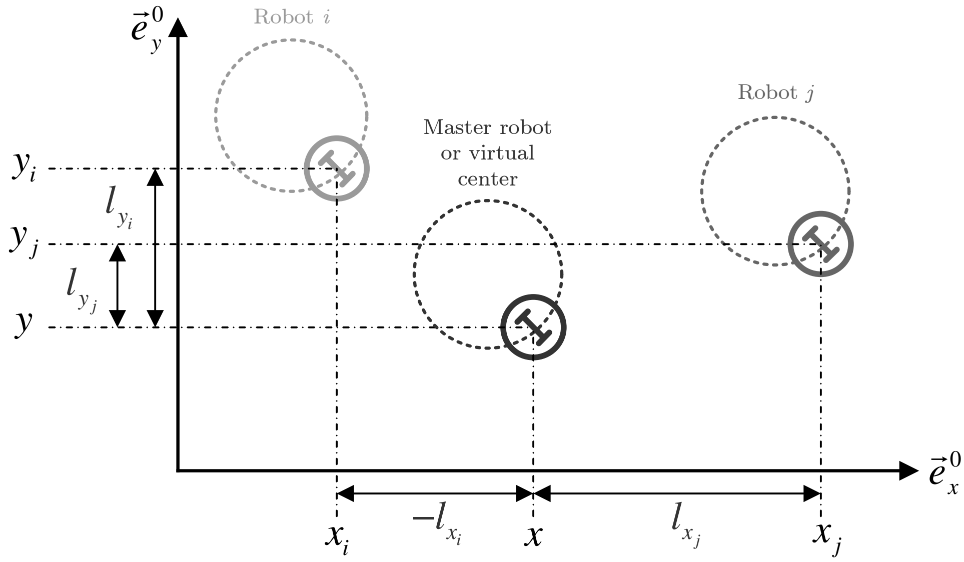


Figure A.1 . Location oriented reference trajectories for robots i and j, based on either the movement of the master robot or a common virtual center.

A.2.1 Unicycle Robot: Master-Slave Motion Coordination

Recall the master-slave motion coordination case, as defined in Chapter 5, in which n slave robots are required to coordinate their motions with the movements of a master robot. Consider that the reference position of the i-th slave robot, for $i \in \{1, 2, \ldots, n\}$, is given by possibly time-varying displacements $l_{x_{s,i}}(t)$ and $l_{y_{s,i}}(t)$ which define the distance between the slave and its master relative to the global coordinate frame $\underline{\vec{e}}^{0}$, resulting in the following reference positions:

$$\begin{bmatrix} x_{r_{s,i}}(t) \\ y_{r_{s,i}}(t) \end{bmatrix} \underline{\vec{e}}^{0} = \begin{bmatrix} x_m(t) + l_{x_{s,i}}(t) \\ y_m(t) + l_{y_{s,i}}(t) \end{bmatrix} \underline{\vec{e}}^{0}, \tag{A.1}$$

or, equivalently,

$$x_{r_{s,i}}(t) = x_m(t) + l_{x_{s,i}}(t), \tag{A.2a}$$

$$y_{r_{s,i}}(t) = y_m(t) + l_{y_{s,i}}(t), \tag{A.2b}$$

where $(x_m(t), y_m(t))$ represents the position (in $\underline{\vec{e}}^{0}$) tracked by the master.

The reference orientation and translational velocities of the i-th slave must satisfy the expression $-\dot{x}_{r_{s,i}}(t) \sin\theta_{r_{s,i}}(t) + \dot{y}_{r_{s,i}}(t) \cos\theta_{r_{s,i}}(t) = 0$, which is due to the non-holonomic constraint that appears in the posture kinematic model of the unicycle, as shown by Brockett (1983). Otherwise, unfeasible reference trajectories would result due to incompatibility with the robot's non-holonomic constraint. Considering this constraint and exploiting the kinematics of the unicycle, the reference orientation and translational and rotational velocities for the i-th slave are defined as

$$\theta_{r_{s,i}}(t) = \arctan\left(\frac{\dot{y}_{r_{s,i}}(t)}{\dot{x}_{r_{s,i}}(t)}\right) + k\pi, \qquad k = 0, 1, \tag{A.3a}$$

$$v_{r_{s,i}}(t) = \sqrt{\dot{x}^2_{r_{s,i}}(t) + \dot{y}^2_{r_{s,i}}(t)}, \tag{A.3b}$$

$$\omega_{r_{s,i}}(t) = \frac{\dot{x}_{r_{s,i}}(t)\ddot{y}_{r_{s,i}}(t) - \ddot{x}_{r_{s,i}}(t)\dot{y}_{r_{s,i}}(t)}{\dot{x}^2_{r_{s,i}}(t) + \dot{y}^2_{r_{s,i}}(t)} = \dot{\theta}_{r_{s,i}}(t). \tag{A.3c}$$

Recall that, in order for the tracking problem of the i-th robot to be soluble, it is necessary that the reference position $(x_{r_{s,i}}(t), y_{r_{s,i}}(t))$ is admissible for the posture kinematic model of the unicycle. This means that the reference orientation $\theta_{r_{s,i}}(t)$ and the translational and rotational velocities $v_{r_{s,i}}(t)$ and $\omega_{r_{s,i}}(t)$, respectively, must satisfy the equations

$$\dot{x}_{r_{s,i}}(t) = v_{r_{s,i}}(t)\cos\theta_{r_{s,i}}(t), \tag{A.4a}$$

$$\dot{y}_{r_{s,i}}(t) = v_{r_{s,i}}(t)\sin\theta_{r_{s,i}}(t), \tag{A.4b}$$

$$\dot{\theta}_{r_{s,i}}(t) = \omega_{r_{s,i}}(t), \tag{A.4c}$$

with an associated reference trajectory $q_{r_{s,i}}(t) = [x_{r_{s,i}}(t)\; y_{r_{s,i}}(t)\; \theta_{r_{s,i}}(t)]^T$.

The reference Cartesian velocities and accelerations of the i-th slave, required to compute its orientation and translational and rotational velocities as defined in (A.3), are obtained by differentiating its reference position (A.2) and exploiting the kinematics of the master robot, resulting in the following expressions:

$$\dot{x}_{r_{s,i}}(t) = v_m(t)\cos\theta_m(t) + \dot{l}_{x_{s,i}}(t), \tag{A.5a}$$

$$\dot{y}_{r_{s,i}}(t) = v_m(t)\sin\theta_m(t) + \dot{l}_{y_{s,i}}(t), \tag{A.5b}$$

$$\ddot{x}_{r_{s,i}}(t) = \dot{v}_m(t)\cos\theta_m(t) - v_m(t)\omega_m(t)\sin\theta_m(t) + \ddot{l}_{x_{s,i}}(t), \tag{A.5c}$$

$$\ddot{y}_{r_{s,i}}(t) = \dot{v}_m(t)\sin\theta_m(t) + v_m(t)\omega_m(t)\cos\theta_m(t) + \ddot{l}_{y_{s,i}}(t), \tag{A.5d}$$

where $v_m(t)$ and $\omega_m(t)$ denote the control signals of the master, that is, its translational and rotational velocities, respectively. Differentiating the translational velocity of the master yields its translational acceleration, $\dot{v}_m(t)$, which is required to estimate the reference accelerations of the slave (A.5c) and (A.5d).

In order to determine the values of the Cartesian velocities and accelerations of the slave as in (A.5), it is necessary to know the control signals $v_m(t)$ and $\omega_m(t)$ applied to the master robot. This means that the reference velocities and accelerations of the slave depend on the tracking control law being used by the master robot and, consequently, on the selected controller gains. For instance, consider that the tracking controller proposed by Lefeber et al. (2001) is applied to the master robot:

$$v_m(t) = v_{r_m}(t) + c_{x_m} x_{e_m}(t) - c_{y_m} w_{r_m}(t) y_{e_m}(t), \quad c_{x_m} > 0, \quad c_{y_m} > -1 \quad \text{(A.6a)}$$
$$\omega_m(t) = \omega_{r_m}(t) + c_{\theta_m} \theta_{e_m}(t), \qquad c_{\theta_m} > 0, \quad \text{(A.6b)}$$

in which $q_{e_m}(t) = [x_{e_m}(t)\, y_{e_m}(t)\, \theta_{e_m}(t)]^T$ represents the error coordinates of the master, and c_{x_m}, c_{y_m} and c_{θ_m} are the tracking control gains.

Given the tracking control law (A.6), the translational acceleration of the master as, required by (A.5c) and (A.5d), yields

$$\dot{v}_m(t) = \dot{v}_{r_m}(t) + c_{x_m} \dot{x}_{e_m}(t) - c_{y_m} \left(\dot{\omega}_{r_m}(t) y_{e_m}(t) + \omega_{r_m}(t) \dot{y}_{e_m}(t) \right), \quad \text{(A.7)}$$

where the tracking error dynamics $\dot{x}_{e_m}(t)$ and $\dot{y}_{e_m}(t)$ are given by

$$\dot{x}_{e_m}(t) = \omega_m(t) y_{e_m}(t) + v_{r_m}(t) \cos\theta_{e_m}(t) - v_m(t), \quad \text{(A.8a)}$$
$$\dot{y}_{e_m}(t) = -\omega_m(t) x_{e_m}(t) + v_{r_m}(t) \sin\theta_{e_m}(t). \quad \text{(A.8b)}$$

Computing the translational acceleration of the master as given in (A.7) requires its reference translational and rotational accelerations, defined as

$$\dot{v}_{r_m}(t) = \frac{\dot{x}_{r_m}(t)\ddot{x}_{r_m}(t) + \ddot{y}_{r_m}(t)\dot{y}_{r_m}(t)}{\sqrt{\dot{x}^2_{r_m}(t) + \dot{y}^2_{r_m}(t)}}, \quad \text{(A.9a)}$$
$$\dot{\omega}_{r_m}(t) = \frac{\dot{x}_{r_m}(t) \left(\dddot{y}_{r_m}(t) - 2\omega_{r_m}(t)\ddot{x}_{r_m}(t) \right) - \dot{y}_{r_m}(t) \left(\dddot{x}_{r_m}(t) + 2\omega_{r_m}(t)\ddot{y}_{r_m}(t) \right)}{\dot{x}^2_{r_m}(t) + \dot{y}^2_{r_m}(t)}. \quad \text{(A.9b)}$$

As a result, in order to compute (A.9a) and (A.9b), the reference Cartesian velocities, $\dot{x}_{r_m}(t)$ and $\dot{y}_{r_m}(t)$, accelerations, $\ddot{x}_{r_m}(t)$ and $\ddot{y}_{r_m}(t)$, and jerks, $\dddot{x}_{r_m}(t)$ and $\dddot{y}_{r_m}(t)$, of the master robot should be available.

A.2.2 Unicycle Robot: Mutual Motion Coordination

In mutual motion coordination as defined in Chapter 5, the members of a group of n mobile robots coordinate their motions by tracking their respective reference trajectories, based on a common virtual center, and exchanging their respective error signals. In this type of motion coordination, the distance between the i-th robot, for $i \in \{1, 2, \ldots, n\}$, and the virtual center is defined in terms of possibly time-varying displacements $l_{x_i}(t)$ and $l_{y_i}(t)$ given with respect to the global coordinate frame $\underline{\vec{e}}^0$. The resulting reference positions for robot the i-th are given by

$$x_{r_i}(t) = x_{vc}(t) + l_{x_i}(t), \tag{A.10a}$$
$$y_{r_i}(t) = y_{vc}(t) + l_{y_i}(t), \tag{A.10b}$$

where $(x_{vc}(t), y_{vc}(t))$ denotes the position of the virtual center.

The reference orientation and translational and rotational velocities for each of the robots in the group may be computed by using the expressions in (A.3), only substituting subindex s, i by i in order to refer to the i-th robot.

Differentiating the reference position (A.10) yields the reference Cartesian velocities and accelerations, which are necessary to compute the aforementioned reference orientation and velocities, and are given by

$$\dot{x}_{r_i}(t) = \dot{x}_{vc}(t) + \dot{l}_{x_i}(t), \tag{A.11a}$$
$$\dot{y}_{r_i}(t) = \dot{y}_{vc}(t) + \dot{l}_{y_i}(t), \tag{A.11b}$$
$$\ddot{x}_{r_i}(t) = \ddot{x}_{vc}(t) + \ddot{l}_{x_i}(t), \tag{A.11c}$$
$$\ddot{y}_{r_i}(t) = \ddot{y}_{vc}(t) + \ddot{l}_{y_i}(t). \tag{A.11d}$$

Knowledge of the velocities, $\dot{x}_{vc}(t)$ and $\dot{y}_{vc}(t)$, and accelerations, $\ddot{x}_{vc}(t)$ and $\ddot{y}_{vc}(t)$, of the virtual center is required in order to estimate the reference Cartesian velocities and accelerations of each robot as defined in (A.11). It is worth noting, however, that in mutual motion coordination it is not necessary to exploit the kinematic model of the system in order to compute these reference values, as opposed to the master-slave case where not only this model is required, but also the control inputs of the master robot. This difference results in less complicated expressions and more accessible computations for mutual motion coordination.

A.3 Formation Oriented References Trajectories

In the previous section, the reference trajectories of the mobile robots whose motions are to be coordinated have been specified with respect to the global coordinate frame $\underline{\vec{e}}^0$, resulting in so-called location oriented reference trajectories. An alternative approach is to define the reference trajectories of the robots with respect to the robot-fixed coordinate frame $\underline{\vec{e}}^{\mathrm{m/vc}}$ of either the master robot or the virtual center, depending on the type of motion coordination strategy implemented. As a result, the distance between the robots relative to the $\underline{\vec{e}}^{\mathrm{m/vc}}$

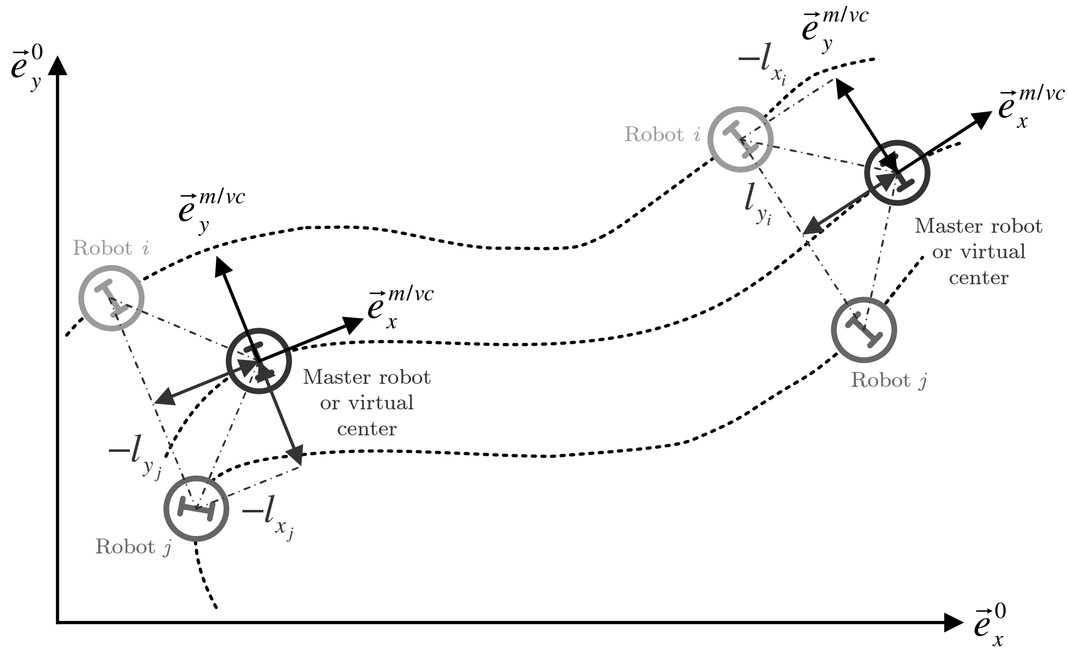


Figure A.2 . Formation oriented reference trajectories for robots i and j, based on either the movement of the master robot or a common virtual center.

coordinate frame and the direction of their position difference may be specified at will. The resulting reference trajectories have been denoted throughout this work as formation oriented reference trajectories.

In order to illustrate this approach, the formation oriented reference trajectories for two robots, i and j, are shown in Figure A.2. Coordinates $l_{x_i}(t), l_{y_i}(t), l_{x_j}(t)$, and $l_{y_j}(t)$ are considered constant in the figure, resulting in the robots platooning in a formation with an specific stationary geometry whose shape is defined by the distances between the members of the group in the $\underline{\vec{e}}^{\mathrm{m/vc}}$ coordinate frame. Since the robots are required to adapt their path in order to maintain a certain formation, their respective reference trajectories will be different and their desired orientation and velocities will not necessarily be equal. Note that in the location oriented approach the robots reproduce the same motions in different locations, whereas in the formation oriented case they track different trajectories in order to maintain a specific distance and orientation with respect to each other.

The remainder of this section explains how to derive the formation oriented reference trajectories for a group of unicycle robots for the master-slave and mutual motion coordination cases.

A.3.1 Unicycle Robot: Master-Slave Motion Coordination

Recall that in master-slave motion coordination, the time-varying displacements $l_{x_{s,i}}(t)$ and $l_{y_{s,i}}(t)$ define the distance from the i-th slave robot, for $i \in \{1, 2, \ldots, n\}$, to the position tracked by the master robot $(x_m(t), y_m(t))$. In a set of formation oriented reference trajectories these displacements are defined in terms of the robot-fixed coordinate frame of the master robot, resulting in the following reference position for the i-th slave:

$$\begin{bmatrix} x_{r_{s,i}}(t) \\ y_{r_{s,i}}(t) \end{bmatrix} \underline{\vec{e}}^{0} = \begin{bmatrix} x_m(t) \\ y_m(t) \end{bmatrix} \underline{\vec{e}}^{0} + \begin{bmatrix} l_{x_{s,i}}(t) \\ l_{y_{s,i}}(t) \end{bmatrix} \underline{\vec{e}}^{\mathrm{m}} \tag{A.12}$$

$$= \begin{bmatrix} x_m(t) \\ y_m(t) \end{bmatrix} \underline{\vec{e}}^{0} + \begin{bmatrix} l_{x_{s,i}}(t) \\ l_{y_{s,i}}(t) \end{bmatrix} \underline{A}^{10} \underline{\vec{e}}^{0}, \tag{A.13}$$

where

$$\underline{A}^{10} = \begin{bmatrix} \cos\theta_m(t) & -\sin\theta_m(t) \\ \sin\theta_m(t) & \cos\theta_m(t) \end{bmatrix}; \tag{A.14}$$

or, equivalently,

$$x_{r_{s,i}}(t) = x_m(t) + l_{x_{s,i}}(t)\cos\theta_m(t) - l_{y_{s,i}}(t)\sin\theta_m(t), \tag{A.15a}$$

$$y_{r_{s,i}}(t) = y_m(t) + l_{x_{s,i}}(t)\sin\theta_m(t) + l_{y_{s,i}}(t)\cos\theta_m(t). \tag{A.15b}$$

The reference orientation and translational and rotational velocities of the slaves are given as in (A.3) and require the Cartesian velocities and accelerations of the slave, which are given by

$$\dot{x}_{r_{s,i}}(t) = -\Delta_{1_m}(t)\sin\theta_m(t) + \Delta_{2_m}(t)\cos\theta_m(t), \tag{A.16a}$$

$$\dot{y}_{r_{s,i}}(t) = \Delta_{2_m}(t)\sin\theta_m(t) + \Delta_{1_m}(t)\cos\theta_m(t), \tag{A.16b}$$

$$\ddot{x}_{r_{s,i}}(t) = -\Delta_{3_m}(t)\sin\theta_m(t) + \Delta_{4_m}(t)\cos\theta_m(t), \tag{A.16c}$$

$$\ddot{y}_{r_{s,i}}(t) = \Delta_{4_m}(t)\sin\theta_m(t) + \Delta_{3_m}(t)\cos\theta_m(t), \tag{A.16d}$$

with $\Delta_{1_m}(t), \Delta_{2_m}(t), \Delta_{3_m}(t)$, and $\Delta_{4_m}(t)$ defined as

$$\Delta_{1_m}(t) = l_{x_{s,i}}(t)\omega_m(t) + \dot{l}_{y_{s,i}}(t), \tag{A.17a}$$

$$\Delta_{2_m}(t) = v_m(t) - l_{y_{s,i}}(t)\omega_m(t) + \dot{l}_{x_{s,i}}(t), \tag{A.17b}$$

$$\Delta_{3_m}(t) = v_m(t)\omega_m(t) - l_{y_{s,i}}(t)\omega_m^2(t) + l_{x_{s,i}}(t)\dot{\omega}_m(t) + 2\dot{l}_{x_{s,i}}(t)\omega_m(t) + \ddot{l}_{y_{s,i}}(t), \tag{A.17c}$$

$$\Delta_{4_m}(t) = \dot{v}_m(t) - l_{x_{s,i}}(t)\omega_m^2(t) - l_{y_{s,i}}(t)\dot{\omega}_m(t) - 2\dot{l}_{y_{s,i}}(t)\omega_m(t) + \ddot{l}_{x_{s,i}}(t). \tag{A.17d}$$

Note that some of the expressions in (A.17) require the control velocities of the master robot $v_m(t)$ and $\omega_m(t)$. This situation has already been addressed when generating location oriented reference trajectories in the master-slave motion coordination case by making use of the tracking controller (A.6).

In (A.17c) and (A.17d), the translational acceleration of the master robot $\dot{v}_m(t)$ is defined as in (A.7), and its rotational acceleration $\dot{\omega}_m(t)$ is given by

$$\dot{\omega}_m(t) = \dot{\omega}_{r_m}(t) - c_{\theta_m}\dot{\theta}_{e_m}(t)\cos\theta_{e_m}(t), \tag{A.18}$$

where the reference rotational acceleration $\dot{\omega}_{r_m}(t)$ is defined as in (A.9b) and the orientation tracking error dynamics $\dot{\theta}_{e_m}(t)$ yield

$$\dot{\theta}_{e_m}(t) = \omega_{r_m}(t) - \omega_m(t). \tag{A.19}$$

A.3.2 Unicycle Robot: Mutual Motion Coordination

In mutual motion coordination the time-varying displacements $l_{x_i}(t)$ and $l_{y_i}(t)$ denote the distances between the i-th robot, $i \in \{1, 2, \ldots, n\}$, and the virtual center, in the robot-fixed coordinate frame of the virtual center $\underline{\vec{e}}^{\text{vc}}$. The resulting reference position for the i-th robot is

$$x_{r_i}(t) = x_{vc}(t) + l_{x_i}(t)\cos\theta_{vc}(t) - l_{y_i}(t)\sin\theta_{vc}(t), \tag{A.20a}$$
$$y_{r_i}(t) = y_{vc}(t) + l_{x_i}(t)\sin\theta_{vc}(t) + l_{y_i}(t)\cos\theta_{vc}(t), \tag{A.20b}$$

where the reference orientation and translational and rotational velocities for each robot may be computed by substituting subindex s, i by i in (A.3).

The reference Cartesian velocities and accelerations, required to compute the aforementioned signals, are given by

$$\dot{x}_{r_i}(t) = \dot{x}_{vc}(t) - \Delta_{1_{vc}}(t)\sin\theta_{vc}(t) + \Delta_{2_{vc}}(t)\cos\theta_{vc}(t), \tag{A.21a}$$
$$\dot{y}_{r_i}(t) = \dot{y}_{vc}(t) + \Delta_{2_{vc}}(t)\sin\theta_{vc}(t) + \Delta_{1_{vc}}(t)\cos\theta_{vc}(t), \tag{A.21b}$$
$$\ddot{x}_{r_i}(t) = \ddot{x}_{vc}(t) - \Delta_{3_{vc}}(t)\sin\theta_{vc}(t) + \Delta_{4_{vc}}(t)\cos\theta_{vc}(t), \tag{A.21c}$$
$$\ddot{y}_{r_i}(t) = \ddot{y}_{vc}(t) + \Delta_{4_{vc}}(t)\sin\theta_{vc}(t) + \Delta_{3_{vc}}(t)\cos\theta_{vc}(t), \tag{A.21d}$$

with $\Delta_{1_{vc}}(t), \Delta_{2_{vc}}(t), \Delta_{3_{vc}}(t)$, and $\Delta_{4_{vc}}(t)$ defined as

$$\Delta_{1_{vc}}(t) = l_{x_i}(t)\omega_{vc}(t) + \dot{l}_{y_i}(t), \tag{A.22a}$$
$$\Delta_{2_{vc}}(t) = -l_{y_i}(t)\omega_{vc}(t) + \dot{l}_{x_i}(t), \tag{A.22b}$$
$$\Delta_{3_{vc}}(t) = -l_{y_i}(t)\omega_{vc}^2(t) + l_{x_i}(t)\dot{\omega}_{vc}(t) + 2\dot{l}_{x_i}(t)\omega_{vc}(t) + \ddot{l}_{y_i}(t), \tag{A.22c}$$
$$\Delta_{4_{vc}}(t) = -l_{x_i}(t)\omega_{vc}^2(t) - l_{y_i}(t)\dot{\omega}_{vc}(t) - 2\dot{l}_{y_i}(t)\omega_{vc}(t) + \ddot{l}_{x_i}(t), \tag{A.22d}$$

where the rotational acceleration of the virtual center, $\dot{\omega}_{vc}(t)$, is defined as follows:

$$\dot{\omega}_{vc}(t) = \frac{\dot{x}_{vc}(t)\left(\dddot{y}_{vc}(t) - 2\omega_{vc}(t)\ddot{x}_{vc}(t)\right) - \dot{y}_{vc}(t)\left(\dddot{x}_{vc}(t) + 2\omega_{vc}(t)\ddot{y}_{vc}(t)\right)}{\dot{x}_{vc}^2(t) + \dot{y}_{vc}^2(t)}. \quad (A.23)$$

Note that in order to compute (A.23), knowledge of the velocities, $\dot{x}_{vc}(t)$ and $\dot{y}_{vc}(t)$, accelerations, $\ddot{x}_{vc}(t)$ and $\ddot{y}_{vc}(t)$, and jerks, $\dddot{x}_{vc}(t)$ and $\dddot{y}_{vc}(t)$, of the virtual center is required.

A.4 Concluding Remarks

The objective of this appendix has been to facilitate the design of motion coordination tasks for a group of unicycle robots. This is an aspect which is often overlooked or left to higher level planning entities in the literature. Two different methods to construct the reference trajectories for a group of mobile robots have been introduced, resulting in so-called location oriented and formation oriented reference trajectories. The location oriented approach produces a set of trajectories distributed throughout the workspace of the robots, whereas the formation approach generates the reference trajectories of the robots in such a way that they are able to maintain a certain geometrical shape while following a prescribed reference trajectory. Both approaches have been cast in the master-slave and mutual motion coordination frameworks, and the necessary mathematical expressions to produce the reference trajectories for a group of unicycles have been included. Although the derivation of these mathematical expressions is rather straightforward, it shows that there are certain cases in which the amount of information required to produce the reference trajectory for a mobile robot could be more than one would expect. For instance, when deriving the reference trajectories for a group of slave robots in master-slave motion coordination considering a location oriented approach, knowing up to the third derivative of the reference position of the master is necessary. It then follows that the availability of these signals is something that should not be taken for granted. An additional aspect to take into account is the amount of information flowing between the robots in order to produce their respective reference trajectories. Depending on the number of robots composing the group, the type of motion coordination strategy, and the method being used to construct their reference trajectories, it is possible to estimate the demands placed on the communication channel and assess the feasibility of a certain task.

B

Remote Tracking Control of a Unicycle Robot: Proofs

B.1 Closed-Loop Error Dynamics

Recall the cascaded closed-loop error dynamics (4.9) repeated here for convenience:

$$\dot{\xi}_1(t) = A_1(t, t-\tau)\xi_1(t) + A_2\xi_1(t-\tau) + g(t, \xi_{1_t}, \xi_{2_t}), \tag{B.1a}$$

$$\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau), \tag{B.1b}$$

where $\xi_1(t) := [z_{1_e}(t)\, z_{2_e}(t)\, p_{1_e}(t)\, p_{2_e}(t)]^T$, $\xi_2(t) := [z_{3_e}(t)\, p_{3_e}(t)]^T$, $\xi_{1_t} \in \mathcal{C}(4)$, and $\xi_{2_t} \in \mathcal{C}(2)$. The matrices and the coupling term in (B.1) are given by

$$A_1(t, t-\tau) = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) & -k_x & 0 \\ -\omega_r(t) & 0 & 0 & -k_y \\ 0 & 0 & 0 & \omega_r(t-\tau) \\ 0 & 0 & -\omega_r(t-\tau) & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -k_x & 0 \\ 0 & 0 & 0 & -k_y \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -c_\theta & -k_\theta \\ 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & -k_\theta \end{bmatrix},$$

$$g(t, \xi_{1_t}, \xi_{2_t}) = \begin{bmatrix} g_{11} & k_\theta z_{2_e}(t) \\ g_{21} & -k_\theta z_{1_e}(t) \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix} \xi_2(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ c_\theta p_{2_e}(t) & k_\theta p_{2_e}(t) \\ -c_\theta p_{1_e}(t) & -k_\theta p_{1_e}(t) \end{bmatrix} \xi_2(t-\tau),$$

with g_{11}, g_{21}, g_{32}, and g_{42} already defined in Section 4.2.3.

B.2 Proof of Theorem 4.1: Remote Tracking Control of a Unicycle Robot (Local Uniform Asymptotic Stability)

Note that the cascaded system (B.1) is a particular case of the cascaded system (2.38) introduced in Section 2.5. Based on Theorem 2.25, the local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (B.1) may be established if the following conditions are satisfied:

- the system $\dot{\xi}_1(t) = A_1(t, t-\tau)\xi_1(t) + A_2\xi_1(t-\tau)$, denoted hereinafter as the ξ_1-dynamics without coupling, is locally uniformly asymptotically stable (LUAS);
- the system $\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau)$, denoted hereinafter as the ξ_2-dynamics, is locally uniformly asymptotically stable (LUAS);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ vanishes when $\xi_{2_t} \to 0$, that is, $g(t, \xi_{1_t}, 0) = 0$.

The validity of these three conditions is checked given the assumptions on the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, reference translational and rotational velocities $v_r(t)$ and $\omega_r(t)$, respectively, and maximum allowable time-delay $\tau_{\max}$, adopted in the theorem.

B.2.1 Requirement on the ξ_1-Dynamics Without Coupling

Recall the ξ_1-dynamics without coupling:

$$\dot{\xi}_1(t) = A_1(t, t-\tau)\xi_1(t) + A_2\xi_1(t-\tau), \tag{B.2}$$

where $A_1(t, t-\tau)$ and A_2 are defined below (B.1). Considering the following state definitions: $\eta_1(t) := [z_{1_e}(t)\, z_{2_e}(t)]^T$ and $\eta_2(t) := [p_{1_e}(t)\, p_{2_e}(t)]^T$, (B.2) may be rewritten as the following cascaded system:

$$\dot{\eta}_1(t) = \Delta_1(t)\eta_1(t) + \Delta_2\eta_2(t), \tag{B.3a}$$

$$\dot{\eta}_2(t) = \Delta_3(t-\tau)\eta_2(t) + \Delta_4\eta_2(t-\tau), \tag{B.3b}$$

where

$$\Delta_1(t) = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix}, \quad \Delta_2 = \Delta_4 = \begin{bmatrix} -k_x & 0 \\ 0 & -k_y \end{bmatrix}, \quad \Delta_3(t) = \begin{bmatrix} 0 & \omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix}.$$

The local uniform asymptotic stability of the cascaded system (B.3), and thus of the ξ_1-dynamics without coupling (B.2), can be concluded according to Theorem 2.25 if the following conditions are satisfied:

- the system $\dot{\eta}_1(t) = \Delta_1(t)\eta_1(t)$, denoted hereinafter as the η_1-dynamics without coupling, is locally uniformly asymptotically stable (LUAS);
- the system $\dot{\eta}_2(t) = \Delta_3(t-\tau)\eta_2(t) + \Delta_4\eta_2(t-\tau)$, denoted hereinafter as the η_2-dynamics, is locally uniformly asymptotically stable (LUAS).
- the coupling term $g_{\eta_1\eta_2}(t, \eta_{1_t}, \eta_{2_t})$ vanishes when $\eta_{2_t} \to 0$;

The previous conditions will be checked considering the requirements posed in Theorem 4.1.

Requirement on the η_1-Dynamics Without Coupling

The η_1-dynamics without coupling are given by the following expression:

$$\begin{bmatrix} \dot{z}_{1_e}(t) \\ \dot{z}_{2_e}(t) \end{bmatrix} = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} z_{1_e}(t) \\ z_{2_e}(t) \end{bmatrix}. \tag{B.4}$$

The error dynamics (B.4) have exactly the same form as the position tracking error dynamics of a mobile robot under the control law proposed by Jakubiak et al. (2002) and have already appeared in Section 2.6. According to Lemma 2.18, these dynamics are globally exponentially stable (GES) for the requirements on c_x, c_y, and $\omega_r(t)$ posed in Theorem 4.1. From Definition 2.14 it follows that these dynamics are also globally uniformly asymptotically stable (GUAS), which clearly satisfies what we require.

Requirement on the η_2-Dynamics

In order to establish the uniform asymptotic stability of the η_2-dynamics as given in (B.3b), let us first consider their delay-free version:

$$\dot{\eta}_2(t) = \Delta_0(t)\eta_2(t), \tag{B.5}$$

where $\Delta_0(t) = \Delta_3(t) + \Delta_4 = \begin{bmatrix} -k_x & \omega_r(t) \\ -\omega_r(t) & -k_y \end{bmatrix}$.

The following candidate Lyapunov function is proposed for (B.5):

$$V_{\eta_2} = \frac{1}{2}p_{1_e}^2 + \frac{1}{2}p_{2_e}^2 = \eta_2^T P_{\eta_2}\eta_2, \tag{B.6}$$

with $P_{\eta_2} = \frac{1}{2} I_2$. The derivative of the candidate Lyapunov function V_{η_2} is given by

$$\dot{V}_{\eta_2} = -k_x p_{1_e}^2 - k_y p_{2_e}^2 = -\eta_2^T Q_{\eta_2} \eta_2, \tag{B.7}$$

with $Q_{\eta_2} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$. Note that matrix P_{η_2} is positive definite, whereas matrix Q_{η_2} is positive definite for $k_x, k_y > 0$.

We will now use the Lyapunov-Razumikhin stability theorem (see Theorem 2.24) to show that the origin of the η_2-dynamics (B.3b) is LUAS. Using Newton-Leibniz's law, these dynamics may be written as the following distributed delay system:

$$\begin{aligned} \dot{\eta}_2(t) &= \Delta_3(t-\tau)\eta_2(t) + \Delta_4 \left(\eta_2(t) - \int_{t-\tau}^{t} \dot{\eta}_2(s) ds \right) \\ &= \tilde{\Delta}_0(t)\eta_2(t) - \Delta_4 \int_{t-\tau}^{t} \Delta_3(s-\tau)\eta_2(s) ds - \Delta_4^2 \int_{t-\tau}^{t} \eta_2(s-\tau) ds \end{aligned} \tag{B.8}$$

with $\tilde{\Delta}_0(t) := \Delta_3(t-\tau) + \Delta_4$. It is worth noting that the proposed Lyapunov function (B.6) is also a Lyapunov function for the system $\dot{\eta}_2(t) = \tilde{\Delta}_0 \eta_2(t)$, with both functions having the same decay rate, characterized by the matrix Q_{η_2}. For this reason, V_{η_2} as in (B.6) will be considered as a candidate Lyapunov-Razumikhin function for the η_2-dynamics (B.3b). Its derivative, given the distributed delay system (B.8), satisfies

$$\begin{aligned} \dot{V}_{\eta_2} &= -\eta_2^T Q_{\eta_2} \eta_2 - 2\eta_2^T P_{\eta_2} \Delta_4 \int_{-\tau}^{0} \Delta_3(t+s-\tau)\eta_2(t+s) ds - 2\eta_2^T P_{\eta_2} \Delta_4^2 \int_{-\tau}^{0} \eta_2(t+s-\tau) ds \\ &\leq -\eta_2^T Q_{\eta_2} \eta_2 + 2\|\eta_2\|_2 \lambda_{\max}(P_{\eta_2}) \left(\|\Delta_4\|_{i2} \sup_{t\in\mathbb{R}} \|\Delta_3(t)\|_{i2} \int_{-\tau}^{0} \|\eta_2(t+s)\|_2 ds \right. \\ &\quad \left. + \|\Delta_4^2\|_{i2} \int_{-\tau}^{0} \|\eta_2(t+s-\tau)\|_2 ds \right). \end{aligned} \tag{B.9}$$

The Lyapunov-Razumikhin stability theorem requires that $\dot{V}_{\eta_2}(t) < 0$ whenever

$$V_{\eta_2}(\eta_2(t+\delta)) \leq p V_{\eta_2}(\eta_2(t)), \tag{B.10}$$

for all t and $-2\tau \leq \delta \leq 0$ and some $p > 1$. This condition may be rewritten in terms of $\|\eta_2(t+\delta)\|_2$ and $\|\eta_2(t)\|_2$ as follows:

$$\|\eta_2(t+\delta)\|_2 \leq \sqrt{p \frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}} \|\eta_2(t)\|_2 \tag{B.11}$$

for all $-2\tau \leq \delta \leq 0$.

Using condition (B.11), the derivative (B.9) of the candidate Lyapunov-Razumikhin function is rewritten in the following way:

$$\dot{V}_{\eta_2} \leq \left(-\lambda_{\min}(Q_{\eta_2}) + 2\tau\lambda_{\max}(P_{\eta_2}) \sqrt{p\frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}} \left(\|\Delta_4\|_{i2} \sup_{t\in\mathbb{R}} \|\Delta_3(t)\|_{i2} + \|\Delta_4^2\|_{i2} \right) \right) \|\eta_2\|_2^2. \tag{B.12}$$

In order to conclude uniform asymptotic stability of the η_2-dynamics (B.3b) we require that $\dot{V}_{\eta_2} < 0$. Recall that the conditions in Theorem 2.25 state that $k := k_x = k_y$, which results in $\|\Delta_4\|_{i2} = k$ and $\|\Delta_4^2\|_{i2} = k^2$. Moreover, given $\bar{\omega}_r := \sup_{t\in\mathbb{R}} |\omega_r(t)|$, it follows that $\sup_{t\in\mathbb{R}} \|\Delta_3(t)\|_{i2} = \bar{\omega}_r$. In addition, note that $\lambda_{\max}(P_{\eta_2}) = \lambda_{\min}(P_{\eta_2}) = \frac{1}{2}$ and $\lambda_{\max}(Q_{\eta_2}) = \lambda_{\min}(Q_{\eta_2}) = k$. Considering the previous analysis, the derivative of the candidate Lyapunov-Razumikhin function V_{η_2} along the solutions of (B.3b) yields:

$$\dot{V}_{\eta_2} \leq -\left(k - \tau\sqrt{p}(k\bar{\omega}_r + k^2)\right) \|\eta_2\|_2^2. \tag{B.13}$$

Given (B.13) it is possible to pose requirements on the correction gains, reference rotational velocity, and allowable time-delay such that the condition for the local uniform asymptotic stability on the η_2-dynamics is met. Specifically, this type of stability may be concluded for (B.3b) if the following condition on the time-delay:

$$\tau < \frac{1}{\sqrt{p}(\bar{\omega}_r + k)}, \tag{B.14}$$

and the requirements for $\omega_r(t), k_x$, and k_y stated in Theorem 4.1 are all satisfied.

Requirement on the Coupling Term $g_{\eta_1\eta_2}(t, \eta_{1_t}, \eta_{2_t})$

Since the coupling term is only dependent on $\eta_2(t)$, $g_{\eta_1\eta_2}(t, \eta_{1_t}, \eta_{2_t}) \to 0$ as $\eta_{2_t} \to 0$, which means that the requirement on the coupling term is met.

In conclusion, the local uniform asymptotic stability of the ξ_1-dynamics without coupling (B.2) is guaranteed provided the time-delay satisfies condition (B.14) and the requirements for $\omega_r(t), c_x, c_y, k_x$, and k_y stated in Theorem 4.1 are met.

B.2.2 Requirement on the ξ_2-Dynamics

Recall the ξ_2-dynamics:

$$\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau), \tag{B.15}$$

where B_1 and B_2 have already been defined below (B.1). The ξ_2-dynamics may be rewritten as the following cascaded system:

$$\dot{z}_{3_e}(t) = -c_\theta z_{3_e}(t) - k_\theta p_{3_e}(t), \tag{B.16a}$$

$$\dot{p}_{3_e}(t) = -k_\theta p_{3_e}(t-\tau). \tag{B.16b}$$

In order to deduce the local uniform asymptotic stability of the cascaded system (B.16), and thus of the ξ_2-dynamics (B.15), the following is required:

- the system $\dot{z}_{3_e}(t) = -c_\theta z_{3_e}(t)$, denoted hereinafter as the z_{3_e}-dynamics without coupling, is locally uniformly asymptotically stable (LUAS);
- the system $\dot{p}_{3_e}(t)$, denoted hereinafter as the p_{3_e}-dynamics, is locally uniformly asymptotically stable (LUAS);
- the coupling term $g_{z_3 p_3}(t, z_{3_t}, p_{3_t})$ to vanish when $p_{3_t} \to 0$.

Requirement on the z_{3_e}-Dynamics Without Coupling

The exponential stability of $\dot{z}_{3_e}(t) = -c_\theta z_{3_e}(t)$ is ensured for $c_\theta > 0$. Given that the system is linear time-invariant (LTI), its (local) uniform asymptotic stability immediately follows.

Requirement on the p_{3_e}-Dynamics

The characteristic quasi-polynomial of (B.16b) is given by

$$s + k_\theta e^{-s\tau} = 0. \tag{B.17}$$

Substituting $s = \sigma + j\varpi$ into (B.17) and applying Euler's formula yields

$$(\sigma + j\varpi) + k_\theta e^{-\sigma\tau - j\varpi\tau} = (\sigma + j\varpi) + k_\theta e^{-\sigma\tau}(\cos\varpi\tau - j\sin\varpi\tau) = 0, \tag{B.18}$$

which is equivalent to the following set of equations:

$$\sigma + k_\theta e^{-\sigma\tau}\cos\varpi\tau = 0, \tag{B.19a}$$

$$\varpi - k_\theta e^{-\sigma\tau}\sin\varpi\tau = 0. \tag{B.19b}$$

When the system is marginally stable, at least one root is located on the imaginary axis (in this case $\sigma = 0$), resulting in the following condition for marginal stability:

$$k_\theta \cos \varpi\tau = 0, \tag{B.20a}$$

$$k_\theta \sin \varpi\tau = \varpi. \tag{B.20b}$$

For (B.20a) to hold it follows that $\varpi\tau = \frac{\pi}{2} + n\pi$ for $n = 0, \pm1, \pm2, \ldots$. Considering these values of $\varpi\tau$ and in order for (B.20b) to hold we have that $k_\theta = \pm\varpi$. From the previous equalities it follows that $k_\theta\tau = \frac{\pi}{2} + n\pi$ for $n = 0, \pm1, \pm2, \ldots$. Note that the characteristic polynomial of the delay-free version of system (B.17) is given by $s + k_\theta = 0$, which means that this delay-free system is exponentially stable for $k_\theta > 0$. Hence, invoking the continuity of eigenvalues argument for LTI systems, we have that the global exponential stability (and therefore the local uniform asymptotic stability) of the p_{3_e}-dynamics (B.16b) is ensured for

$$\tau < \frac{\pi}{2k_\theta}, \tag{B.21}$$

with $k_\theta > 0$.

Requirement on the Coupling Term $g_{z_3p_3}(t, z_{3_t}, p_{3_t})$

Note that the coupling term will vanish when $p_{3_t} \to 0$, as it is linear and only dependent on p_{3_t}. Thus, the condition on $g_{z_3p_3}(t, z_{3_t}, p_{3_t})$ is satisfied.

Given that the conditions on the z_{3_e}-dynamics without coupling, the p_{3_e}-dynamics, and the coupling term are met, we can conclude that the ξ_2-dynamics (B.15) are LUAS for the conditions in Theorem 4.1.

B.2.3 Requirement on the Coupling Term $g(t, \xi_{1_t}, \xi_{2_t})$

Given the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ as defined in (B.1) and from the definition of ξ_{2_t}, we have that as $\xi_{2_t} \to 0$, both $\xi_2(t) \to 0$ and $\xi_2(t - \tau) \to 0$. It then follows that, as $\xi_{2_t} \to 0$, the coupling term vanishes, which means that the requirement on the coupling term is satisfied.

In conclusion, the local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (B.1) may be established provided that the conditions in Theorem 4.1 for the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, constant p, and reference translational and rotational velocities, $v_r(t)$ and $\omega_r(t)$, respectively, are met, and guaranteeing that (B.14) and (B.21) are satisfied by means of condition (4.10) in the theorem. This means that the state estimator converges to the reference trajectory, since $z_e(t) \to 0$ as $t \to \infty$, or equivalently, $z(t) \to q_r(t)$ as $t \to \infty$. Additionally, the state estimator anticipates the system due to the fact that $p_e(t) \to 0$ as $t \to \infty$, that is, $z(t) \to q(t+\tau_f)$ as $t \to \infty$. It then follows that $q(t) \to q_r(t-\tau_f)$ as $t \to \infty$, which means that the unicycle robot subject to a communication delay τ tracks the reference trajectory delayed by τ_f. This completes the proof.

B.3 Proof of Theorem 4.3: Remote Tracking Control of a Unicycle Robot (Global Uniform Asymptotic Stability)

In order to prove the global uniform asymptotic stability of the cascaded system (B.1), a similar approach to the one used to prove its local stability will be followed. In this case, based on Theorem 2.26, the following conditions may be posed in order to establish the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the predictor's closed-loop error dynamics:

- the ξ_1-dynamics without coupling are globally exponentially stable (GES) with an explicit quadratic Lyapunov-Razumikhin function V_{ξ_1};
- the ξ_2-dynamics are globally exponentially stable (GES);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ admits the estimate

  $$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c, \tag{B.22}$$

  for continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The exponential stability requirements on the ξ_1-dynamics without coupling and the ξ_2-dynamics are based on the assumptions in Theorem 2.26 and the remarks that follow the theorem. In the case of the ξ_1-dynamics without coupling, the first four assumptions in Theorem 2.26 are satisfied if the system is GUAS characterized by a strict Lyapunov-Razumikhin function and the fifth assumption is fulfilled if the associated Lyapunov-Razumikhin function is quadratic. It then follows that requiring the global exponential stability of these dynamics characterized by a quadratic strict Lyapunov-Razumikhin function will satisfy all the assumptions related to them. Although this requirement may be too restrictive in general, it appears to be adequate for the problem currently being considered. Regarding the ξ_2-dynamics, the seventh assumption in Theorem 2.26 poses the requirement for the global exponential stability of these dynamics. It is also worth noting that, because of the stability requirement on the ξ_1-dynamics without coupling, the estimate on the coupling term may be rewritten as in (B.22) according to the remarks that follow Theorem 2.26.

The validity of the three conditions in the list above is checked given the assumptions on the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, reference translational and rotational velocities $v_r(t)$ and $\omega_r(t)$, respectively, and maximum allowable time-delay $\tau_{\max}$, adopted in the theorem.

B.3.1 Requirement on the ξ_1-Dynamics Without Coupling

Recall the ξ_1-dynamics without coupling:

$$\dot{\xi}_1(t) = A_1(t, t-\tau)\xi_1(t) + A_2\xi_1(t-\tau), \tag{B.23}$$

in which $A_1(t, t-\tau)$ and A_2 have already been defined in (B.1) and their cascaded representation introduced in (B.3) in Section B.2.1. This cascaded system without a time-delay is given by the following LTV system:

$$\dot{\eta}_1(t) = \Delta_1(t)\eta_1(t) + \Delta_2\eta_2(t), \tag{B.24a}$$

$$\dot{\eta}_2(t) = (\Delta_3(t) + \Delta_4)\eta_2(t). \tag{B.24b}$$

The delay-free η_1-dynamics without coupling in (B.24a) are given by

$$\begin{bmatrix} \dot{z}_{1_e}(t) \\ \dot{z}_{2_e}(t) \end{bmatrix} = \begin{bmatrix} -c_x & (1+c_y)\omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} z_{1_e}(t) \\ z_{2_e}(t) \end{bmatrix}, \tag{B.25}$$

which have exactly the same form as (B.4) and have already been determined to be GES.

From Lyapunov converse theory (see Theorem 2.15), we know that since the delay-free
η_1-dynamics without coupling are GES, there exists, for all t, a continuously differentiable, bounded, positive definite, symmetric matrix $P_{\eta_1}(t)$ that satisfies

$$-\dot{P}_{\eta_1}(t) = P_{\eta_1}(t)\Delta_1(t) + \Delta_1^T(t)P_{\eta_1}(t) + Q_{\eta_1}(t), \qquad \forall t, \tag{B.26}$$

where $Q_{\eta_1}(t)$ is continuous, bounded, positive definite, and symmetric, and $\Delta_1(t)$ is required to be continuous in t and bounded for all t. The requirement on $\Delta_1(t)$ is satisfied since $\omega_r(t)$ is required to be continuous and bounded, as it is PE (see Definition 2.4).

As a result, $V_{\eta_1} = \eta_1^T P_{\eta_1}(t)\eta_1$ is a Lyapunov function for the delay-free η_1-dynamics without coupling. The derivative of this function satisfies

$$\dot{V}_{\eta_1} = -\eta_1^T Q_{\eta_1}(t)\eta_1 \leq -\beta_1\|\eta_1\|_2^2, \tag{B.27}$$

where $\beta_1 := \inf_{t\in\mathbb{R}} \lambda_{\min}(Q_{\eta_1}(t))$.

On the other hand, the delay-free η_2-dynamics (B.24b) have already been determined to be GES for $k_x, k_y > 0$ in Section B.2.1, with the Lyapunov function V_{η_2} defined in (B.6).

Recapitulating, the delay-free η_1-dynamics without coupling admit a Lyapunov function $V_{\eta_1} = \eta_1^T P_{\eta_1}(t)\eta_1$, whereas the delay-free η_2-dynamics admit a Lyapunov function $V_{\eta_2} = \eta_2^T P_{\eta_2}\eta_2$. Considering this, we now propose

$$V_{\xi_1} = V_{\eta_1} + V_{\eta_2} = \eta_1^T P_{\eta_1}(t)\eta_1 + \eta_2^T P_{\eta_2}\eta_2 \tag{B.28}$$

as a candidate Lyapunov-Razumikhin function for the ξ_1-dynamics without coupling, that is, for (B.23).

We will now use the Lyapunov-Razumikhin stability theorem to show that the origin of (B.23) is GES and that (B.28) is a Lyapunov-Razumkhin function which satisfies the requirements stated in Theorem 4.3.

Recall that by using Newton-Leibniz's law, the η_2-dynamics (B.3b) may be written as the distributed delay system (B.8). Considering this, the derivative of the candidate Lyapunov-Razumikhin function (B.28) is given by

$$\begin{aligned}\dot{V}_{\xi_1} =& \eta_1^T Q_{\eta_1}(t)\eta_1 + 2\eta_1^T P_{\eta_1}(t)\Delta_2\eta_2 - \eta_2^T Q_{\eta_2}\eta_2 - 2\eta_2^T P_{\eta_2}\Delta_4 \int_{-\tau}^{0} \Delta_3(t+s-\tau)\eta_2(t+s)ds \\ & -2\eta_2^T P_{\eta_2}\Delta_4^2 \int_{-\tau}^{0} \eta_2(t+s-\tau)ds \\ \leq& -\beta_1\|\eta_1\|_2^2 + 2\sup_{t\in\mathbb{R}}(\|P_{\eta_1}(t)\|_{i2})\|\Delta_2\|_{i2}\|\eta_1\|_2\|\eta_2\|_2 - \eta_2^T Q_{\eta_2}\eta_2 + 2\|\eta_2\|_2\lambda_{\max}(P_{\eta_2}) \\ & \left(\|\Delta_4\|_{i2}\sup_{t\in\mathbb{R}}\|\Delta_3(t)\|_{i2}\int_{-\tau}^{0}\|\eta_2(t+s)\|_2 ds + \|\Delta_4^2\|_{i2}\int_{-\tau}^{0}\|\eta_2(t+s-\tau)\|_2 ds\right).\end{aligned} \tag{B.29}$$

The Lyapunov-Razumikhin stability theorem requires that $\dot{V}_{\xi_1}(t) < 0$ whenever

$$V_{\xi_1}(\xi_1(t+\delta)) \leq pV_{\xi_1}(\xi_1(t)), \tag{B.30}$$

for all t and $-2\tau \leq \delta \leq 0$ and some $p > 1$. This condition may be rewritten in terms of V_{η_1} and V_{η_2} as

$$V_{\eta_1}(\eta_1(t+\delta)) + V_{\eta_2}(\eta_2(t+\delta)) \leq p\left(V_{\eta_1}(\eta_1(t)) + V_{\eta_2}(\eta_2(t))\right), \tag{B.31}$$

for all t and $-2\tau \leq \delta \leq 0$ and some $p > 1$. It is possible to replace condition (B.31) by a condition given in terms of $\|\eta_1(t+\delta)\|_2$, $\|\eta_1(t)\|_2$, $\|\eta_2(t+\delta)\|_2$, and $\|\eta_2(t)\|_2$ as follows:

$$\|\eta_2(t+\delta)\|_2 \leq \left(p\frac{\sup_{t\in\mathbb{R}}(\lambda_{\max}(P_{\eta_1}(t)))}{\lambda_{\min}(P_{\eta_2})}\|\eta_1(t)\|_2^2 + p\frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}\|\eta_2(t)\|_2^2\right)^{1/2}. \tag{B.32}$$

Using the fact that $\|a\|_2 \leq \|a\|_1$, (B.32) may be replaced by the following condition:

$$\|\eta_2(t+\delta)\|_2 \leq \sqrt{p\frac{\sup_{t\in\mathbb{R}}\left(\lambda_{\max}(P_{\eta_1}(t))\right)}{\lambda_{\min}(P_{\eta_2})}}\|\eta_1(t)\|_2+\sqrt{p\frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}}\|\eta_2(t)\|_2. \quad \text{(B.33)}$$

Considering (B.29) and using condition (B.33), the derivative of the candidate Lyapunov-Razumikhin function becomes:

$$\begin{aligned}
\dot{V}_{\xi_1} \leq & -\beta_1\|\eta_1\|_2^2+2\sup_{t\in\mathbb{R}}(\|P_{\eta_1}(t)\|_{i2})\|\Delta_2\|_{i2}\|\eta_1\|_2\|\eta_2\|_2-\lambda_{\min}(Q_{\eta_2})\|\eta_2\|_2^2+2\tau\lambda_{\max}(P_{\eta_2}) \\
& \times\left(\|\Delta_4\|_{i2}\sup_{t\in\mathbb{R}}\|\Delta_3(t)\|_{i2}+\|\Delta_4^2\|_{i2}\right) \\
& \times\left(\sqrt{p\frac{\sup_{t\in\mathbb{R}}\left(\lambda_{\max}(P_{\eta_1}(t))\right)}{\lambda_{\min}(P_{\eta_2})}}\|\eta_1\|_2\|\eta_2\|_2+\sqrt{p\frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}}\|\eta_2\|_2^2\right) \\
= & -\beta_1\|\eta_1\|_2^2+2\|\eta_1\|_2\|\eta_2\|_2 \\
& \times\left(\sup_{t\in\mathbb{R}}(\|P_{\eta_1}(t)\|_{i2})\|\Delta_2\|_{i2}+\tau\lambda_{\max}(P_{\eta_2})\sqrt{p\frac{\sup_{t\in\mathbb{R}}\left(\lambda_{\max}(P_{\eta_1}(t))\right)}{\lambda_{\min}(P_{\eta_2})}}\right. \\
& \left.\times\left(\|\Delta_4\|_{i2}\sup_{t\in\mathbb{R}}\|\Delta_3(t)\|_{i2}+\|\Delta_4^2\|_{i2}\right)\right)-\left(\lambda_{\min}(Q_{\eta_2})-2\tau\lambda_{\max}(P_{\eta_2})\sqrt{p\frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}}\right. \\
& \left.\times\left(\|\Delta_4\|_{i2}\sup_{t\in\mathbb{R}}\|\Delta_3(t)\|_{i2}+\|\Delta_4^2\|_{i2}\right)\right)\|\eta_2\|_2^2. \qquad \text{(B.34)}
\end{aligned}$$

Given the following definitions:

$$\begin{aligned}
\beta_2 &:= \beta_{21}+\beta_{22}\tau \\
&= \underbrace{\lambda_{\min}(Q_{\eta_2})}_{\beta_{21}}\underbrace{-2\lambda_{\max}(P_{\eta_2})\sqrt{p\frac{\lambda_{\max}(P_{\eta_2})}{\lambda_{\min}(P_{\eta_2})}}\left(\|\Delta_4\|_{i2}\sup_{t\in\mathbb{R}}\|\Delta_3(t)\|_{i2}+\|\Delta_4^2\|_{i2}\right)}_{\beta_{22}}\tau,
\end{aligned} \quad \text{(B.35a)}$$

$$\begin{aligned}
\beta_3 &:= \beta_{31}+\beta_{32}\tau \\
&= \underbrace{2\sup_{t\in\mathbb{R}}(\|P_{\eta_1}(t)\|_{i2})\|\Delta_2\|_{i2}}_{\beta_{31}} \\
&\quad \underbrace{+2\lambda_{\max}(P_{\eta_2})\sqrt{p\frac{\sup_{t\in\mathbb{R}}\left(\lambda_{\max}(P_{\eta_1}(t))\right)}{\lambda_{\min}(P_{\eta_2})}}\left(\|\Delta_4\|_{i2}\sup_{t\in\mathbb{R}}\|\Delta_3(t)\|_{i2}+\|\Delta_4^2\|_{i2}\right)}_{\beta_{32}}\tau,
\end{aligned} \quad \text{(B.35b)}$$

the derivative of the candidate Lyapunov-Razumikhin function (B.28) satisfies

$$\dot{V}_{\xi_1} \leq -\beta_1\|\eta_1\|_2^2 - \beta_2\|\eta_2\|_2^2 + \beta_3\|\eta_1\|_2\|\eta_2\|_2. \tag{B.36}$$

Considering that for all $\gamma > 0$ the following holds:

$$\|\eta_1\|_2\|\eta_2\|_2 \leq \gamma\|\eta_1\|_2^2 + \frac{1}{\gamma}\|\eta_2\|_2^2,$$

we have that (B.36) may be rewritten as

$$\begin{aligned}\dot{V}_{\xi_1} &\leq -\beta_1\|\eta_1\|_2^2 - \beta_2\|\eta_2\|_2^2 + \beta_3\gamma\|\eta_1\|_2^2 + \beta_3\frac{1}{\gamma}\|\eta_2\|_2^2\\ &= -(\beta_1 - (\beta_{31}+\beta_{32}\tau)\gamma)\|\eta_1\|_2^2 - \left((\beta_{21}+\beta_{22}\tau) - (\beta_{31}+\beta_{32}\tau)\frac{1}{\gamma}\right)\|\eta_2\|_2^2. \end{aligned} \tag{B.37}$$

In order for $\dot{V}_{\xi_1} < 0$ to hold, the following inequalities must be satisfied:

$$\beta_1 - (\beta_{31} + \beta_{32}\tau)\gamma > 0, \tag{B.38a}$$

$$(\beta_{21} + \beta_{22}\tau) - (\beta_{31} + \beta_{32}\tau)\frac{1}{\gamma} > 0. \tag{B.38b}$$

Recall that the conditions in Theorem 4.3 state that $k := k_x = k_y$, so we have that $\|\Delta_2\|_{i2} = \|\Delta_4\|_{i2} = k$ and $\|\Delta_4^2\|_{i2} = k^2$. Moreover, given $\bar{\omega}_r := \sup_{t\in\mathbb{R}} |\omega_r(t)|$ it follows that $\sup_{t\in\mathbb{R}} \|\Delta_3(t)\|_{i2} = \bar{\omega}_r$. In addition, note that $\lambda_{\max}(P_{\eta_2}) = \lambda_{\min}(P_{\eta_2}) = \frac{1}{2}$ and $\lambda_{\max}(Q_{\eta_2}) = \lambda_{\min}(Q_{\eta_2}) = k$. Considering these facts, β_2 and β_3, as defined in (B.35a) and (B.35b), respectively, may be rewritten as follows:

$$\beta_2 = \underbrace{k}_{\beta_{21}} \underbrace{-\sqrt{p}k(\bar{\omega}_r + k)}_{\beta_{22}}\tau, \tag{B.39a}$$

$$\beta_3 = \underbrace{2k\sup_{t\in\mathbb{R}}\|P_{\eta_1}(t)\|_{i2}}_{\beta_{31}} \underbrace{+k(\bar{\omega}_r + k)\sqrt{2p\sup_{t\in\mathbb{R}}\|P_{\eta_1}(t)\|_{i2}}}_{\beta_{32}}\tau. \tag{B.39b}$$

The derivative (B.37) of the Lyapunov-Razumikhin function along the solutions of (B.23) may now be expressed as

$$\begin{aligned}\dot{V}_{\xi_1} \leq &-\left(\inf_{t\in\mathbb{R}}\lambda_{\min}(Q_{\eta_1}(t)) - 2\gamma k\sup_{t\in\mathbb{R}}\|P_{\eta_1}(t)\|_{i2} - \tau\gamma k(\bar{\omega}_r + k)\sqrt{2p\sup_{t\in\mathbb{R}}\|P_{\eta_1}(t)\|_{i2}}\right)\|\eta_1\|_2^2\\ &-\left(k - \tau\sqrt{p}k(\bar{\omega}_r + k) - \frac{2}{\gamma}k\sup_{t\in\mathbb{R}}\|P_{\eta_1}(t)\|_{i2} - \frac{\tau}{\gamma}k(\bar{\omega}_r + k)\sqrt{2p\sup_{t\in\mathbb{R}}\|P_{\eta_1}(t)\|_{i2}}\right)\|\eta_2\|_2^2,\end{aligned} \tag{B.40}$$

in which $\beta_1 = \inf_{t\in\mathbb{R}} \lambda_{\min}(Q_{\eta_1}(t))$ has been used.

From (B.40), we know that it is possible to pose requirements on the tracking gains, correction gains, reference rotational velocity, and allowable time-delay such that the condition for global exponential stability of the ξ_1-dynamics without coupling is met.

The global exponential stability of (B.23) may be concluded from (B.40) in terms of the time-delay by considering (B.38), resulting in the following requirements for the time-delay τ:

$$\tau < \frac{\beta_1 - \gamma\beta_{31}}{\beta_{32}\gamma}, \tag{B.41}$$

and

$$\tau < \frac{\beta_{21}\gamma - \beta_{31}}{\beta_{32} - \beta_{22}\gamma}, \tag{B.42}$$

which may be rewritten as follows by taking (B.39a) and (B.39b) into account:

$$\tau < \frac{\inf_{t\in\mathbb{R}} \lambda_{\min}(Q_{\eta_1}(t)) - 2\gamma k \sup_{t\in\mathbb{R}} \|P_{\eta_1}(t)\|_{i2}}{\gamma k(\bar{\omega}_r + k)\sqrt{2p\sup_{t\in\mathbb{R}} \|P_{\eta_1}(t)\|_{i2}}}, \tag{B.43}$$

and

$$\tau < \frac{\gamma - 2\sup_{t\in\mathbb{R}} \|P_{\eta_1}(t)\|_{i2}}{(\bar{\omega}_r + k)(\gamma\sqrt{p} + \sqrt{2p\sup_{t\in\mathbb{R}} \|P_{\eta_1}(t)\|_{i2}})}. \tag{B.44}$$

Given the fact that $\sup_{t\in\mathbb{R}} \|P_{\eta_1}(t)\|_{i2}$ is bounded and $\inf_{t\in\mathbb{R}} \lambda_{\min}(Q_{\eta_1}(t)) > 0$, the previous conditions imply that there exist $k > 0$ sufficiently small and $\gamma > 0$ sufficiently large so that there exists $\tau_{\max} > 0$ such that (B.43) and (B.44) are satisfied for all $\tau \in [0, \tau_{\max}]$. Hence, for all $0 \leq \tau \leq \tau_{\max}$, there exist $k > 0$ sufficiently small and $\gamma > 0$ sufficiently large for which $\xi_1 = 0$ is a GES equilibrium point of (B.23).

In this sense, there seem to be two conditions which can always be fulfilled. Namely, first chose γ large enough to satisfy condition (B.44). Then, chose k small enough to satisfy condition (B.43). Note that as $\gamma \to \infty$, it is necessary for $k \downarrow 0$.

Considering the previous remarks, it is possible to conclude that $k > 0$ can always be chosen small enough such that the condition on the ξ_1-dynamics without coupling is satisfied; namely, that these dynamics are GES.

B.3.2 Requirement on the ξ_2-Dynamics

To begin with, recall that the ξ_2-dynamics are required to be GES. On the other hand, note that the local uniform asymptotic stability of these dynamics has already been established in Section B.2.2. Since the system is LTI, the global aspect follows directly and, in accordance with Definition 2.12, the system is also exponentially stable. In conclusion, the global exponential stability of the ξ_2-dynamics is ensured for

$$\tau < \frac{\pi}{2k_\theta}, \tag{B.45}$$

provided c_θ and k_θ satisfy the conditions in Theorem 4.3.

B.3.3 Requirement on the Coupling Term $g(t, \xi_{1_t}, \xi_{2_t})$

We are required to show that there exist continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$ such that the coupling terms satisfies the following bound:

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c, \tag{B.46}$$

for any $\varphi_{\xi_1} \in \mathcal{C}(4)$ and $\varphi_{\xi_2} \in \mathcal{C}(2)$.

We can rewrite the coupling term as defined in (B.46) in terms of φ_{ξ_1} and φ_{ξ_2} as follows:

$$g(t, \varphi_{\xi_1}, \varphi_{\xi_2}) = g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\xi_2(t) + g_2(\varphi_{\xi_1}(0))\xi_2(t-\tau), \tag{B.47}$$

with

$$g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0)) = \begin{bmatrix} g_{11} & k_\theta z_{2_e}(t) \\ g_{21} & -k_\theta z_{1_e}(t) \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix}, \quad g_2(\varphi_{\xi_1}(0)) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ c_\theta p_{2_e}(t) & k_\theta p_{2_e}(t) \\ -c_\theta p_{1_e}(t) & -k_\theta p_{1_e}(t) \end{bmatrix},$$

and g_{11}, g_{21}, g_{32}, and g_{42} already defined below (B.1).

Considering (B.46) and (B.47), the following holds:

$$\begin{aligned} \|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 &= \|g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\varphi_{\xi_2}(0) + g_2(\varphi_{\xi_1}(0))\varphi_{\xi_2}(-\tau)\|_1 \\ &\leq \|g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\varphi_{\xi_2}(0)\|_1 + \|g_2(\varphi_{\xi_1}(0))\varphi_{\xi_2}(-\tau)\|_1 \\ &\leq \|g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\|_{i1}\|\varphi_{\xi_2}(0)\|_1 + \|g_2(\varphi_{\xi_1}(0))\|_{i1}\|\varphi_{\xi_2}(-\tau)\|_1 \\ &\leq (\|g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\|_{i1} + \|g_2(\varphi_{\xi_1}(0))\|_{i1})\,\|\varphi_{\xi_2}\|_c \\ &\leq (\|g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\|_{\text{sum}} + \|g_2(\varphi_{\xi_1}(0))\|_{\text{sum}})\,\|\varphi_{\xi_2}\|_c, \end{aligned} \tag{B.48}$$

where we have used the fact that $\|A\|_{i1} \leq \|A\|_{\text{sum}}$ in the last inequality.

From (B.48) it follows that, in order to satisfy the requirement on the coupling term, it suffices to show that there exist continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$

such that the following inequality is satisfied:

$$\|g_1(t,t-\tau,\varphi_{\xi_1},\varphi_{\xi_2}(0))\|_{\text{sum}}+\|g_2(\varphi_{\xi_1}(0))\|_{\text{sum}} \leq \alpha_1(\|\varphi_{\xi_2}\|_c)+\alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c. \tag{B.49}$$

Considering the coupling term expressed in terms of particular solutions of (B.1), the following condition results from (B.49) and the definition of the sum matrix norm and the triangle inequality:

$$\begin{aligned}
&\|g_1(t,t-\tau,\xi_{1_t},\xi_{2_t}(0))\|_{\text{sum}}+\|g_2(\xi_{1_t}(0))\|_{\text{sum}}\\
&\leq |c_\theta z_{2_e}(t)|+\left|v_r(t)\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right|+|k_\theta z_{2_e}(t)|+|c_\theta z_{1_e}(t)|+\left|v_r(t)\frac{\sin z_{3_e}(t)}{z_{3_e}(t)}\right|+|k_\theta z_{1_e}(t)|\\
&\quad+\left|v_r(t-\tau)\frac{1-\cos p_{3_e}(t)}{p_{3_e}(t)}\right|+\left|c_x z_{1_e}(t-\tau)\frac{1-\cos p_{3_e}(t)}{p_{3_e}(t)}\right|+|c_\theta p_{2_e}(t)|+|k_\theta p_{2_e}(t)|\\
&\quad+\left|v_r(t-\tau)\frac{\sin p_{3_e}(t)}{p_{3_e}(t)}\right|+\left|c_x z_{1_e}(t-\tau)\frac{\sin p_{3_e}(t)}{p_{3_e}(t)}\right|+|c_\theta p_{1_e}(t)|+|k_\theta p_{1_e}(t)|\\
&\quad+\left|c_y\omega_r(t-\tau)z_{2_e}(t-\tau)\frac{1-\cos p_{3_e}(t)}{p_{3_e}(t)}\right|+\left|c_y\omega_r(t-\tau)z_{2_e}(t-\tau)\frac{\sin p_{3_e}(t)}{p_{3_e}(t)}\right|.
\end{aligned} \tag{B.50}$$

Given the equalities defined in (2.2) note that the functions $\frac{\sin x}{x}$ and $\frac{1-\cos x}{x}$ are continuous and always ≤ 1 for all x, which allows to rewrite inequality (B.50) as follows:

$$\begin{aligned}
&\|g_1(t,t-\tau,\xi_{1_t},\xi_{2_t}(0))\|_{\text{sum}}+\|g_2(\xi_{1_t}(0))\|_{\text{sum}}\\
&\leq (|c_\theta|+|k_\theta|)\left(|z_{1_e}(t)|+|z_{2_e}(t)|+|p_{1_e}(t)|+|p_{2_e}(t)|\right)+2\,|v_r(t)|+2\,|v_r(t-\tau)|\\
&\quad+2\,|c_x z_{1_e}(t-\tau)|+2\,|c_y\omega_r(t-\tau)z_{2_e}(t-\tau)|.
\end{aligned} \tag{B.51}$$

The theorem requires $c_x, c_y, c_\theta, k_\theta > 0$ and recall that $\bar{\omega}_r = \sup_{t\in\mathbb{R}}|\omega_r(t)|$. Since $v_r(t)$ is bounded, we may define $\bar{v}_r := \sup_{t\in\mathbb{R}}|v_r(t)|$. Inequality (B.51) may now be expressed as follows:

$$\begin{aligned}
&\|g_1(t,t-\tau,\xi_{1_t},\xi_{2_t}(0))\|_{\text{sum}}+\|g_2(\xi_{1_t}(0))\|_{\text{sum}}\\
&\leq (c_\theta+k_\theta)\left(|z_{1_e}(t)|+|z_{2_e}(t)|+|p_{1_e}(t)|+|p_{2_e}(t)|\right)+4\,|\bar{v}_r|+2c_x\,|z_{1_e}(t-\tau)|\\
&\quad+2|c_y|\bar{\omega}_r\,|z_{2_e}(t-\tau)|\\
&\leq 4\,|\bar{v}_r|+(c_\theta+k_\theta)\left(|z_{1_e}(t)|+|z_{2_e}(t)|+|p_{1_e}(t)|+|p_{2_e}(t)|\right)\\
&\quad+2(c_x+|c_y|\bar{\omega}_r)\left(|z_{1_e}(t-\tau)|+|z_{2_e}(t-\tau)|+|p_{1_e}(t-\tau)|+|p_{2_e}(t-\tau)|\right)\\
&\leq 4\,|\bar{v}_r|+(c_\theta+k_\theta)\|\xi_1(t)\|_1+2(c_x+|c_y|\bar{\omega}_r)\|\xi_1(t-\tau)\|_1\\
&\leq 4\,|\bar{v}_r|+(c_\theta+k_\theta+2(c_x+|c_y|\bar{\omega}_r))\|\xi_{1_t}\|_c.
\end{aligned} \tag{B.52}$$

We may now express inequality (B.52) in terms of φ_{ξ_1} and φ_{ξ_2}, that is, for any elements of the Banach spaces $\mathcal{C}(4)$ and $\mathcal{C}(2)$, respectively, as follows:

$$\|g_1(t, t-\tau, \varphi_{\xi_1}, \varphi_{\xi_2}(0))\|_{\text{sum}} + \|g_2(\varphi_{\xi_1}(0))\|_{\text{sum}} \leq 4\,|\bar{v}_r| + (c_\theta + k_\theta + 2(c_x + |c_y|\bar{\omega}_r))\|\varphi_{\xi_1}\|_c. \tag{B.53}$$

From (B.53) we have that inequality (B.49) can be satisfied with $\alpha_1(\|\varphi_{\xi_2}\|_c) = 4|\bar{v}_r|$ and $\alpha_2(\|\varphi_{\xi_2}\|_c) = 4(c_\theta + k_\theta + 2(c_x + |c_y|\bar{\omega}_r))$, which means that the requirement on the coupling term is met.

After checking the three conditions formulated at the beginning of the proof we have that the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (B.1) is concluded, provided that the conditions in Theorem 4.3 for the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, constant p, and reference translational and rotational velocities, $v_r(t)$ and $\omega_r(t)$, respectively, are met. The conditions on the control parameters imply that there exist correction gains $k = k_x = k_y > 0$ sufficiently small, $k_\theta > 0$, and a constant $\gamma > 0$ sufficiently large such that the conditions on τ posed in (B.43), (B.44), and (B.45) are satisfied, and for which the origin of the closed-loop error dynamics (B.1) is GUAS.

As a result, the state estimator converges to the reference trajectory, since $z_e(t) \to 0$ as $t \to \infty$, or equivalently, $z(t) \to q_r(t)$ as $t \to \infty$. Additionally, the state estimator anticipates the system due to the fact that $p_e(t) \to 0$ as $t \to \infty$, that is, $z(t) \to q(t+\tau_f)$ as $t \to \infty$. It then follows that $q(t) \to q_r(t-\tau_f)$ as $t \to \infty$, which means that the unicycle robot subject to a network-induced delay τ tracks the reference trajectory delayed by τ_f. This completes the proof.

C

Remote Motion Coordination of Unicycle Robots: Proofs

C.1 Closed-Loop Error Dynamics

Recall the cascaded closed-loop error dynamics (5.10), repeated here for convenience:

$$\dot{\xi}_1(t) = A_1(t)\xi_1(t) + A_2(t)\xi_1(t-\tau) + g(t, \xi_{1_t}, \xi_{2_t}), \tag{C.1a}$$

$$\dot{\xi}_2(t) = -C_\theta \xi_2(t) + K_\theta \xi_2(t-\tau), \tag{C.1b}$$

where $\xi_1(t) := [X_e^T(t)\, Y_e^T(t)]^T$, $\xi_2(t) := \Theta_e^T(t)$, $\xi_{1_t} \in \mathcal{C}(2n)$, and $\xi_{2_t} \in \mathcal{C}(n)$. The matrices and the coupling term in (C.1) are given by

$$\begin{aligned}
A_1(t) &= \begin{bmatrix} -C_x & \bar{\Omega}_r(t)(I_{n\times n} + C_y) \\ -\bar{\Omega}_r(t) & 0_{n\times n} \end{bmatrix}, \\
A_2(t) &= \begin{bmatrix} K_x & -\bar{\Omega}_r(t)K_y \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}, \\
g(t, \xi_{1_t}, \xi_{2_t}) &= \begin{bmatrix} \bar{Y}_e(t)C_\theta - V_r(t)\Theta_{e_{\sin}} \\ -\bar{X}_e(t)C_\theta + V_r(t)\Theta_{e_{\cos}} \end{bmatrix} \xi_2(t) + \begin{bmatrix} -\bar{Y}_e(t)K_\theta \\ \bar{X}_e(t)K_\theta \end{bmatrix} \xi_2(t-\tau),
\end{aligned}$$

where all the elements in $A_1(t), A_2(t)$, and $g(t, \xi_{1_t}, \xi_{2_t})$ have already been defined below (5.9) in Section 4.2.3. In addition, we define the vector of reference rotational velocities of the robots as $\Omega_r(t) := \{\omega_{r_i}(t), \ldots, \omega_{r_n}(t)\}$, for all $i \in \{1, 2, \ldots, n\}$.

C.2 Proof of Theorem 5.3: Remote Motion Coordination of Unicycle Robots

The global uniform asymptotic stability of the equilibrium point $Q_e(t) = 0$ of the cascaded system (C.1) is guaranteed provided the following conditions, which follow from the application of Theorem 2.26, are satisfied:

- the system $\dot{\xi}_1(t) = A_1(t)\xi_1(t) + A_2(t)\xi_1(t-\tau)$, denoted hereinafter as the ξ_1-dynamics without coupling, is globally exponentially stable (GES) with a quadratic Lyapunov-Razumikhin function V_{ξ_1};
- the system $\dot{\xi}_2(t) = -C_\theta\xi_2(t) + K_\theta\xi_2(t-\tau)$, denoted hereinafter as the ξ_2-dynamics, is globally exponentially stable (GES);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ admits the estimate

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c,$$

  for continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

These three conditions are checked considering the conditions in the theorem on the tracking gains c_{x_i}, c_{y_i}, and c_{θ_i}, $i \in \{1, 2, \ldots, n\}$, coupling gains $k_{x_{i,j}}, k_{y_{i,j}}, k_{\theta_{i,j}}$, $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$, and reference translational and rotational velocities $v_{r_i}(t)$ and $\omega_{r_i}(t)$, $i \in \{1, 2, \ldots, n\}$, respectively.

C.2.1 Requirement on the ξ_1-Dynamics Without Coupling

The ξ_1-dynamics without coupling are given by

$$\dot{\xi}_1(t) = A_1(t)\xi_1(t) + A_2(t)\xi_1(t-\tau), \tag{C.2}$$

whereas their delay-free version is given as follows:

$$\dot{\xi}_1(t) = A_0(t)\xi_1(t), \tag{C.3}$$

with $A_0(t) = A_1(t) + A_2(t) = \begin{bmatrix} -\tilde{C}_x & \bar{\Omega}_r(t)(I_{n\times n} + \tilde{C}_y) \\ -\bar{\Omega}_r(t) & 0_{n\times n} \end{bmatrix}$, where $\tilde{C}_x := C_x - K_x$ and $\tilde{C}_y := C_y - K_y$.

Before studying the stability of the delayed system (C.2), we will first study the delay-free system (C.3). In order to do so, consider the following candidate Lyapunov function first proposed by Sadowska (2010):

$$V = \xi_1^T P \xi_1, \tag{C.4}$$

with $P = \frac{1}{2}\begin{bmatrix} I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n}+\tilde{C}_y \end{bmatrix}$.

It is worth noting that the matrices $\tilde{C}_x$ and $\tilde{C}_y$ are positive definite (that is, $\tilde{C}_x, \tilde{C}_y > 0$) provided the requirements on the position tracking and coupling gains in the theorem are satisfied, that is, $c_{x_i} > 0, c_{y_i} > -1$, and $k_{x_{i,j}} = k_{x_{j,i}}, k_{y_{i,j}} = k_{y_{j,i}} > 0$ for $i, j \in \{1, 2, \ldots, n\}$, $j \neq i$. This has already been shown in Sadowska (2010) using Gershgorin's circle theorem (see Theorem 2.1). In turn, the fact that $\tilde{C}_y > 0$ implies that P in (C.4) is positive definite.

The time-derivative of V as in (C.4) along the solutions of (C.3) is given by

$$\dot{V} = -\xi_1^T Q \xi_1, \tag{C.5}$$

with $Q = \begin{bmatrix} \tilde{C}_x & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}$, which is negative semi-definite as long as $\tilde{C}_x > 0$.

We will now make use of Theorem 2.17 to conclude the global exponential stability of (C.3). To begin with, given the candidate Lyapunov function V, condition (2.18) is rewritten as follows:

$$PA(t) + A^T(t) + C^T C \leq 0, \tag{C.6}$$

which, clearly, is satisfied for $C := [\hat{C}_x \, 0_{n\times n}]$, where $\hat{C}_x^T \hat{C}_x = \tilde{C}_x$.

It only remains to be shown that the pair $(A_0(t), C)$ is uniformly completely observable (UCO) in order to conclude the global exponential stability of (C.3). This can be done with the aid of Theorem 2.19. First, note that under the conditions of Theorem 5.3, the vector of reference rotational velocities $\Omega_r(t)$ is persistently exciting, as required in Theorem 2.19. This means that $\omega_{r_i}(t)$ is bounded and Lipschitz in t for all $i \in \{1, 2, \ldots, n\}$, which implies that $A(t)$ in (C.3) is also bounded and Lipschitz in t. We now have to show that the pair $(A_0(\Omega_r), C)$ is observable provided every ω_{r_i} in Ω_r is different from zero, that is, $\omega_{r_i} \neq 0$ for $i \in \{1, 2, \ldots, n\}$. The observability matrix $\mathcal{O}$ of the pair $(A_0(\Omega_r), C)$ is given as follows:

$$\begin{aligned} \mathcal{O} &= \begin{bmatrix} C \\ CA_0(\Omega_r) \end{bmatrix} \\ &= \begin{bmatrix} \hat{C}_x & 0_{n\times n} \\ -\hat{C}_x\tilde{C}_x & \hat{C}_x\Omega_r(I_{n\times n}+\tilde{C}_y) \end{bmatrix}, \end{aligned} \tag{C.7}$$

whose rank is n due to the fact that $\hat{C}_x, \tilde{C}_x$, and $I_{n\times n} + \tilde{C}_y$ are positive definite and the fact that all $\omega_{r_i} \neq 0$ for all $i \in \{1, 2, \ldots, n\}$, which implies that Ω_r is full rank. Since all the requirements in Theorem 2.19 have been satisfied, we can ensure that the pair $(A_0(t), C)$ is UCO. This shows, using Theorem 2.17, that the delay-free ξ_1-dynamics without coupling (C.3) are GES.

Now, in order to study the stability of (C.2), which includes the delay, recall that according to Lyapunov converse theory (see Theorem 2.15), since system (C.3) is GES, there exists a continuously differentiable, bounded, positive definite, symmetric matrix $P_{\xi_1}(t)$ which satisfies the following equation:

$$-\dot{P}_{\xi_1}(t) = P_{\xi_1}(t)A_0(t) + A_0^T(t)P_{\xi_1}(t) + Q_{\xi_1}(t), \tag{C.8}$$

where $Q_{\xi_1}(t)$ is continuous, bounded, positive definite, and symmetric, and $A_0(t)$ is required to be continuous in t and bounded for all t. It then follows that the requirements on $A_0(t)$ are satisfied, since all $\omega_{r_i}(t)$ contained in $\Omega_r(t)$ are continuous and bounded due to the fact that $\Omega_r(t)$ is persistently exciting in accordance to Definition 2.5.

As a result, $V_{\xi_1} = \xi_1^T P_{\xi_1}(t)\xi_1$ is a strict Lyapunov function for system (C.3), with

$$\dot{V}_{\xi_1} = -\xi_1^T Q_{\xi_1}(t)\xi_1 \leq -\beta_1 \|\xi_1\|_2^2, \tag{C.9}$$

where $\beta_1 := \inf_{t\in\mathbb{R}} \lambda_{\min}(Q_{\xi_1}(t))$.

Considering the previous developments, we now propose the following candidate Lyapunov-Razumikhin function for (C.2):

$$V_{\xi_1} = \xi_1^T P_{\xi_1}(t)\xi_1. \tag{C.10}$$

Using the Lyapunov-Razumikhin stability theorem (see Theorem 2.24), we will show that the origin of system (C.2) is GUAS. In order to do so, (C.2) is first rewritten as the following distributed delay system:

$$\begin{aligned}\dot{\xi}_1(t) &= A_1(t)\xi_1(t) + A_2(t)\left(\xi_1(t) - \int_{t-\tau}^{t} \dot{\xi}_1(s)ds\right)\\ &= A_0(t)\xi_1(t) - A_2(t)\int_{t-\tau}^{t} A_1(s)\xi_1(s)ds - A_2(t)\int_{t-\tau}^{t} A_2(s)\xi_1(s-\tau)ds. \end{aligned}\tag{C.11}$$

Considering (C.11), the derivative of the candidate Lyapunov-Razumikhin function (C.10) is the following:

$$\begin{aligned}\dot{V}_{\xi_1} = &-\xi_1^T Q_{\xi_1}(t)\xi_1 - \left(\int_{t-\tau}^{t} A_1(s)\xi_1(s)ds\right)^T A_2^T(t)P_{\xi_1}(t)\xi_1\\ &- \left(\int_{t-\tau}^{t} A_2(s)\xi_1(s-\tau)ds\right)^T A_2^T(t)P_{\xi_1}(t)\xi_1 - \xi_1^T P_{\xi_1}(t)A_2(t)\int_{t-\tau}^{t} A_1(s)\xi_1(s)ds\\ &- \xi_1^T P_{\xi_1}(t)A_2(t)\int_{t-\tau}^{t} A_2(s)\xi_1(s-\tau)ds\end{aligned}$$

$$\leq -\beta_1 \|\xi_1\|_2^2 + 2 \sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2} \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \|\xi_1\|_2$$
$$\times \left(\sup_{t\in\mathbb{R}} \|A_1(t)\|_{i2} \int_{-\tau}^{0} \|\xi_1(t+s)\|_2 ds + \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \int_{-\tau}^{0} \|\xi_1(t+s-\tau)\|_2 ds \right). \tag{C.12}$$

Note that $\sup_{t\in\mathbb{R}} \|A_1(t)\|_{i2}$ and $\sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2}$ exist, since the elements of $A_1(t)$ and $A_2(t)$, including $\bar{\Omega}_r(t)$, are bounded.

The Lyapunov-Razumikhin stability theorem requires that $\dot{V}_{\xi_1}(t) < 0$ whenever

$$V_{\xi_1}(\xi_1(t+\delta)) \leq pV_{\xi_1}(\xi_1(t)), \tag{C.13}$$

for all t and $-2\tau \leq \delta \leq 0$ and some $p > 1$. The following (stricter) condition, given in terms of $\|\xi_1(t+\delta)\|_2$ and $\|\xi_1(t)\|_2$, may replace condition (C.13):

$$\|\xi_1(t+\delta)\|_2 \leq \sqrt{p \frac{\sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}{\inf_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}} \|\xi_1(t)\|_2, \tag{C.14}$$

for all t and $-2\tau \leq \delta \leq 0$ and some $p > 1$.

Considering (C.12) and using condition (C.14), the derivative of the candidate Lyapunov-Razumikhin function can be upper-bounded as follows:

$$\dot{V}_{\xi_1} \leq -\left(\beta_1 - 2\tau \sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2} \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \sqrt{p \frac{\sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}{\inf_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}} \times \left(\sup_{t\in\mathbb{R}} \|A_1(t)\|_{i2} + \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \right) \right) \|\xi_1\|_2^2. \tag{C.15}$$

Given the following definition:

$$\beta_2 := 2 \sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2} \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \sqrt{p \frac{\sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}{\inf_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}} \left(\sup_{t\in\mathbb{R}} \|A_1(t)\|_{i2} + \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \right), \tag{C.16}$$

we require that $\beta_1 - \tau\beta_2 > 0$ in order for $\dot{V}_{\xi_1} < 0$. Given this requirement, the global exponential stability of (C.2) may be concluded from (C.15) if the time-delay satisfies the following bound:

$$\tau < \frac{\beta_1}{\beta_2}, \tag{C.17}$$

which may be rewritten as follows:

$$\tau < \frac{\inf_{t\in\mathbb{R}} \lambda_{\min}(Q_{\xi_1}(t))}{2 \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2} \sqrt{p} \frac{\left(\sup_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}\right)^{3/2}}{\sqrt{\inf_{t\in\mathbb{R}} \|P_{\xi_1}(t)\|_{i2}}} \left(\sup_{t\in\mathbb{R}} \|A_1(t)\|_{i2} + \sup_{t\in\mathbb{R}} \|A_2(t)\|_{i2}\right)} =: \beta_3. \tag{C.18}$$

Hence, for a set of given tracking and coupling gains $c_{x_i}, k_{x_{i,j}}, k_{y_{i,j}} > 0$ and $c_{y_i} > -1$, for $i, j \in \{1, 2, \dots, n\}$, $j \neq i$, it is always possible to find a $\tau_{\max} < \beta_3$ such that (C.2) is GES for all $\tau \in [0, \tau_{\max}]$.

C.2.2 Requirement on the ξ_2-Dynamics

Recall the ξ_2-dynamics already defined in (C.1b):

$$\dot{\xi}_2(t) = -C_\theta \xi_2(t) + K_\theta \xi_2(t-\tau). \tag{C.19}$$

From Lyapunov-Razumikhin stability criteria and, as explained in Proposition 5.3 of Gu et al. (2003) and in Niculescu et al. (1998), the delay independent stability of system (C.19) may be established if there exist a scalar $\rho > 0$ and a real, positive definite, symmetric matrix $P_{\xi_2} > 0$ such that the following inequality is satisfied:

$$-P_{\xi_2} C_\theta - C_\theta^T P_{\xi_2} + \frac{1}{\rho} P_{\xi_2} K_\theta P_{\xi_2}^{-1} K_\theta^T P_{\xi_2} + \rho P_{\xi_2} < 0. \tag{C.20}$$

Assuming matrix P_{ξ_2} is diagonal and given the definitions of C_θ and K_θ, (C.20) is given as follows:

$$\begin{aligned}
&- PC_\theta - C_\theta^T P + \frac{1}{\rho} P K_\theta P^{-1} K_\theta^T P + \rho P \\
&= - \begin{bmatrix} 2\left(c_{\theta_1} + \sum k_{\theta_{1,j}}\right) p_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & 2\left(c_{\theta_{n-1}} + \sum k_{\theta_{n-1,j}}\right) p_{n-1} & 0 \\ 0 & 0 & \dots & 2\left(c_{\theta_n} + \sum k_{\theta_{n,j}}\right) p_n \end{bmatrix} \\
&+ \frac{1}{\rho} \begin{bmatrix} p_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & p_{n-1} & 0 \\ 0 & 0 & \dots & p_n \end{bmatrix} \begin{bmatrix} 0 & k_{\theta_{1,2}} & \dots & k_{\theta_{1,n}} \\ \vdots & \ddots & \ddots & \vdots \\ k_{\theta_{n-1,1}} & \ddots & 0 & k_{\theta_{n-1,n}} \\ k_{\theta_{n,1}} & k_{\theta_{n,2}} & \dots & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{p_1} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & \frac{1}{p_{n-1}} & 0 \\ 0 & 0 & \dots & \frac{1}{p_n} \end{bmatrix} \\
&\times \begin{bmatrix} 0 & k_{\theta_{2,1}} & \dots & k_{\theta_{n,1}} \\ \vdots & \ddots & \ddots & \vdots \\ k_{\theta_{1,n-1}} & \ddots & 0 & k_{\theta_{n,n-1}} \\ k_{\theta_{1,n}} & k_{\theta_{2,n}} & \dots & 0 \end{bmatrix} \begin{bmatrix} p_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & p_{n-1} & 0 \\ 0 & 0 & \dots & p_n \end{bmatrix} + \rho \begin{bmatrix} p_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ddots & p_{n-1} & 0 \\ 0 & 0 & \dots & p_n \end{bmatrix} < 0,
\end{aligned}$$

where

$$\sum k_{\theta_{i,j}} = \sum_{j=1, j\neq i}^{n} k_{\theta_{i,j}}.$$

After a number of simplifications the previous inequality becomes

$$-PC_\theta - C_\theta^T P + \frac{1}{\rho} PK_\theta P^{-1} K_\theta^T P + \rho P \tag{C.21}$$

$$= \frac{1}{\rho}\begin{bmatrix} a_{1,1} & p_1p_2 \sum \frac{k_{\theta_{1,l}} k_{\theta_{2,l}}}{p_l} & \cdots & p_1p_n \sum \frac{k_{\theta_{1,l}} k_{\theta_{n,l}}}{p_l} \\ \vdots & \ddots & \ddots & \vdots \\ p_{n-1}p_1 \sum \frac{k_{\theta_{1,l}} k_{\theta_{n-1,l}}}{p_l} & \ddots & a_{n-1,n-1} & p_{n-1}p_n \sum \frac{k_{\theta_{n,l}} k_{\theta_{n-1,l}}}{p_l} \\ p_np_1 \sum \frac{k_{\theta_{1,l}} k_{\theta_{n,l}}}{p_l} & p_np_2 \sum \frac{k_{\theta_{2,l}} k_{\theta_{n,l}}}{p_l} & \cdots & a_{n,n} \end{bmatrix} < 0, \tag{C.22}$$

where

$$a_{i,i} = -2\rho\left(c_{\theta_i} + \sum k_{\theta_{i,j}}\right) p_i + p_i^2 \sum \frac{k^2_{\theta_{i,j}}}{p_j} + \rho^2 p_i, \quad \sum \frac{k^2_{\theta_{i,j}}}{p_j} = \sum_{j=1, j\neq i}^{n} \frac{k^2_{\theta_{i,j}}}{p_j},$$

$$\sum \frac{k_{\theta_{i,l}} k_{\theta_{j,l}}}{p_l} = \sum_{l=1, l\neq i,j}^{n} \frac{k_{\theta_{i,l}} k_{\theta_{j,l}}}{p_l}, \qquad j \neq i.$$

We can ensure that the inequality (C.20) is satisfied if the matrix on the right-hand side of (C.22) is negative definite. Using Gershgorin's circle theorem (see Theorem 2.1) results in the following condition for the negative definiteness of the right-hand side of (C.22):

$$a_{i,i} < -r_i, \tag{C.23}$$

with $r_i = p_i \sum p_j \sum \frac{k_{\theta_{i,l}} k_{\theta_{j,l}}}{p_l}$. After some manipulations, condition (C.23) may be rewritten as follows:

$$c_{\theta_i} + \sum k_{\theta_{i,j}} > \frac{1}{2\rho}\left(p_i \sum \frac{k^2_{\theta_{i,j}}}{p_j} + \sum p_j \sum \frac{k_{\theta_{i,l}} k_{\theta_{j,l}}}{p_l}\right) + \frac{\rho}{2}, \tag{C.24}$$

where $\sum p_j = \sum_{j=1, j\neq i}^{n} p_j$. From (C.24) it follows that it is always possible to find $c_{\theta_i} > 0$ sufficiently large and $\rho > 0$ such that inequality (C.23), and hence inequality (C.20), are satisfied. As a result, in remote mutual motion coordination of unicycle robots the orientation error dynamics are GES for a network-induced delay τ of arbitrary magnitude.

C.2.3 Requirement on the Coupling Term $g(t, \xi_{1_t}, \xi_{2_t})$

We are required to show that the coupling term satisfies the following bound:

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \leq (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c, \tag{C.25}$$

for any $\varphi_{\xi_1} \in \mathcal{C}(2n)$ and $\varphi_{\xi_2} \in \mathcal{C}(n)$ and continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The coupling term as defined in (C.1), given in terms of φ_{ξ_1} and φ_{ξ_2}, yields

$$g(t, \varphi_{\xi_1}, \varphi_{\xi_2}) = g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\varphi_{\xi_2}(0) + g_2(\varphi_{\xi_1}(0))\varphi_{\xi_2}(-\tau), \tag{C.26}$$

with

$$g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0)) = \begin{bmatrix} \bar{Y}_e(t)C_\theta - V_r(t)\Theta_{e_{\sin}} \\ -\bar{X}_e(t)C_\theta + V_r(t)\Theta_{e_{\cos}} \end{bmatrix}, \quad g_2(\varphi_{\xi_1}(0)) = \begin{bmatrix} -\bar{Y}_e(t)K_\theta \\ \bar{X}_e(t)K_\theta \end{bmatrix},$$

and $C_\theta, K_\theta, \bar{X}_e(t), \bar{Y}_e(t), \Theta_{e_{\sin}}, \Theta_{e_{\cos}}$, and $V_r(t)$ already defined below (5.9) in Section 5.5.

The following holds given (C.25) and (C.26):

$$\begin{aligned}
\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 &= \|g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\varphi_{\xi_2}(0) + g_2(\varphi_{\xi_1}(0))\varphi_{\xi_2}(-\tau)\|_1 \\
&\le \|g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\varphi_{\xi_2}(0)\|_1 + \|g_2(\varphi_{\xi_1}(0))\varphi_{\xi_2}(-\tau)\|_1 \\
&\le \|g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\|_{i1}\|\varphi_{\xi_2}(0)\|_1 + \|g_2(\varphi_{\xi_1}(0))\|_{i1}\|\varphi_{\xi_2}(-\tau)\|_1 \\
&\le \|g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\|_{\mathrm{sum}}\|\varphi_{\xi_2}(0)\|_1 + \|g_2(\varphi_{\xi_1}(0))\|_{\mathrm{sum}}\|\varphi_{\xi_2}(-\tau)\|_1 \\
&\le \left(\|g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\|_{\mathrm{sum}} + \|g_2(\varphi_{\xi_1}(0))\|_{\mathrm{sum}}\right)\|\varphi_{\xi_2}\|_c, \quad \text{(C.27)}
\end{aligned}$$

where we have used the fact that $\|A\|_{i1} \le \|A\|_{\mathrm{sum}}$ in the last two inequalities.

In order to satisfy the requirement on the coupling term, it follows from (C.27) that it suffices to show that there exist continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$ such that the following inequality is satisfied:

$$\|g_1(t, \varphi_{\xi_1}(0), \varphi_{\xi_2}(0))\|_{\mathrm{sum}} + \|g_2(\varphi_{\xi_1}(0))\|_{\mathrm{sum}} \le \alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c. \tag{C.28}$$

Consider now the coupling term expressed in terms of particular solutions of (C.1). Then, the following condition results from (C.28) and the definition of the sum matrix norm and the triangle inequality:

$$\begin{aligned}
&\|g_1(t, \xi_{1_t}(0), \xi_{2_t}(0))\|_{\mathrm{sum}} + \|g_2(\xi_{1_t}(0))\|_{\mathrm{sum}} \\
&= \left\|\bar{Y}_e(t)C_\theta - V_r(t)\Theta_{e_{\sin}}\right\|_{\mathrm{sum}} + \left\|-\bar{X}_e(t)C_\theta + V_r(t)\Theta_{e_{\cos}}\right\|_{\mathrm{sum}} + \left\|\bar{Y}_e(t)K_\theta\right\|_{\mathrm{sum}} \\
&\quad + \left\|\bar{X}_e(t)K_\theta\right\|_{\mathrm{sum}} \\
&\le \left\|\bar{Y}_e(t)C_\theta\right\|_{\mathrm{sum}} + \|V_r(t)\Theta_{e_{\sin}}\|_{\mathrm{sum}} + \left\|\bar{X}_e(t)C_\theta\right\|_{\mathrm{sum}} + \|V_r(t)\Theta_{e_{\cos}}\|_{\mathrm{sum}} \\
&\quad + \left\|\bar{Y}_e(t)K_\theta\right\|_{\mathrm{sum}} + \left\|\bar{X}_e(t)K_\theta\right\|_{\mathrm{sum}}.
\end{aligned} \tag{C.29}$$

Given the equalities defined in (2.2), note that the functions $\frac{\sin x}{x}$ and $\frac{1-\cos x}{x}$ (contained in $\Theta_{e_{\sin}}$ and $\Theta_{e_{\cos}}$ in (C.29)) are continuous and always ≤ 1 for all x, which allows to rewrite inequality (C.29) as follows:

$$\begin{aligned}
&\|g_1(t,\xi_{1_t}(0),\xi_{2_t}(0))\|_{\text{sum}} + \|g_2(\xi_{1_t}(0))\|_{\text{sum}} \\
&\leq 2\,\|V_r(t)\|_{\text{sum}} + \left\|\bar{X}_e(t)C_\theta\right\|_{\text{sum}} + \left\|\bar{X}_e(t)K_\theta\right\|_{\text{sum}} + \left\|\bar{Y}_e(t)C_\theta\right\|_{\text{sum}} + \left\|\bar{Y}_e(t)K_\theta\right\|_{\text{sum}} \\
&\leq 2\,\|V_r(t)\|_{\text{sum}} + (\|C_\theta\|_{\text{sum}} + \|K_\theta\|_{\text{sum}})\left(\left\|\bar{X}_e(t)\right\|_{\text{sum}} + \left\|\bar{Y}_e(t)\right\|_{\text{sum}}\right). \qquad \text{(C.30)}
\end{aligned}$$

Recall that $v_{r_i}(t)$ for $i \in \{1,2,\ldots,n\}$ is bounded. Defining $\bar{v}_{r_i} := \sup_{t\in\mathbb{R}} |v_{r_i}(t)|$, the following holds:

$$\|V_r(t)\|_{\text{sum}} = \sum_{i=1}^{n} |v_{r_i}(t)| \leq \sum_{i=1}^{n} \bar{v}_{r_i} \leq n\bar{V}_r, \qquad \text{(C.31)}$$

where $\bar{V}_r := \max_{i\in\{1,2,\ldots,n\}} \{\bar{v}_{r_i}\}$.

Given the previous inequality and considering that $\|\bar{X}_e\|_{\text{sum}} + \|\bar{Y}_e\|_{\text{sum}} = \|\xi_1(t)\|_1$, (C.30) may be rewritten in the following way:

$$\begin{aligned}
\|g_1(t,\xi_{1_t}(0),\xi_{2_t}(0))\|_{\text{sum}} + \|g_2(\xi_{1_t}(0))\|_{\text{sum}} &\leq 2n\bar{V}_r + (\|C_\theta\|_{\text{sum}} + \|K_\theta\|_{\text{sum}})\,\|\xi_1(t)\|_1 \\
&\leq 2n\bar{V}_r + (\|C_\theta\|_{\text{sum}} + \|K_\theta\|_{\text{sum}})\,\|\xi_{1_t}\|_c. \qquad \text{(C.32)}
\end{aligned}$$

Note that inequality (C.32) may be expressed in terms of φ_{ξ_1} and φ_{ξ_2}, that is, for any elements of the Banach spaces $\mathcal{C}(2n)$ and $\mathcal{C}(n)$, respectively, as follows:

$$\|g_1(t,\varphi_{\xi_1}(0),\varphi_{\xi_2}(0))\|_{\text{sum}} + \|g_2(\varphi_{\xi_1}(0))\|_{\text{sum}} \leq 2n\bar{V}_r + (\|C_\theta\|_{\text{sum}} + \|K_\theta\|_{\text{sum}})\,\|\varphi_{\xi_1}\|_c. \qquad \text{(C.33)}$$

As a result, inequality (C.28) can be satisfied with $\alpha_1(\|\varphi_{\xi_2}\|) = 2n\bar{V}_r$ and $\alpha_2(\|\varphi_{\xi_2}\|) = \|C_\theta\|_{\text{sum}} + \|K_\theta\|_{\text{sum}}$, which means that the requirement on the coupling term is met.

The three conditions stated at the beginning of the proof have now been checked. Consequently, the equilibrium point $Q_e(t) = [X_e^T(t)\,Y_e^T(t)\,\Theta_e^T(t)]^T = 0$ of the closed-loop error dynamics (C.1) is concluded to be GUAS for $\tau \in [0,\tau_{\max}]$ and some $\tau_{\max} > 0$, given tracking gains c_{x_i}, c_{y_i}, and c_{θ_i}, coupling gains $k_{x_{i,j}}, k_{y_{i,j}}$, and $k_{\theta_{i,j}}$, and reference rotational velocities $\omega_{r_i}(t)$, for all $i,j \in \{1,2,\ldots,n\}$, $j \neq i$, meeting the requirements posed in the theorem. This completes the proof.

D

Extensions: Proofs

D.1 Proof of Theorem 6.1: Remote Tracking Control of an Omnidirectional Robot (Global Uniform Asymptotic Stability)

Recall the cascaded closed-loop error dynamics (6.9) which result from the predictor-controller combination applied to an omnidirectional robot:

$$\dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_1(t-\tau) + g(t, \xi_{1_t}, \xi_{2_t}), \tag{D.1a}$$

$$\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau), \tag{D.1b}$$

where $\xi_1(t) := [z_{1_e}(t)\, z_{2_e}(t)\, p_{x_e}(t)\, p_{y_e}(t)]^T$, $\xi_2(t) := [z_{3_e}(t)\, p_{\theta_e}(t)]^T$, $\xi_{1_t} \in \mathcal{C}(4)$, $\xi_{2_t} \in \mathcal{C}(2)$, and

$$A_1 = \begin{bmatrix} -c_x & 0 & -k_x & 0 \\ 0 & -c_y & 0 & -k_y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -k_x & 0 \\ 0 & 0 & 0 & -k_y \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -c_\theta & -k_\theta \\ 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & -k_\theta \end{bmatrix},$$

$$g(t, \xi_{1_t}, \xi_{2_t}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix} \xi_2(t),$$

with g_{32} and g_{42} already defined below (6.9) in Section 6.2.

According to Theorem 2.26, the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the cascaded system (D.1) may be established if the following conditions are satisfied:

- the system $\dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_1(t-\tau)$, denoted hereinafter as the ξ_1-dynamics without coupling, is globally exponentially stable (GES) with an explicit quadratic Lyapunov-Razumikhin function V_{ξ_1};
- the system $\dot{\xi}_2(t) = B_1\xi_2(t) + B_2\xi_2(t-\tau)$, denoted hereinafter as the ξ_2-dynamics, is globally exponentially stable (GES);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ admits the estimate

  $$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \le (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c,$$

  for continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The validity of these conditions is assessed considering the requirements in the theorem for the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, and maximum allowable time-delay $\tau_{\max}$.

D.1.1 Requirement on the ξ_1-Dynamics Without Coupling

From (D.1a), the ξ_1-dynamics without coupling are given as follows:

$$\dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_1(t-\tau), \tag{D.2}$$

whereas their delay-free version is given by

$$\dot{\xi}_1(t) = A_0\xi_1(t), \tag{D.3}$$

where $A_0 := A_1 + A_2$.

The following candidate Lyapunov function is proposed for (D.3):

$$V_{\xi_1} = \xi_1^T P_{\xi_1} \xi_1, \tag{D.4}$$

with

$$P = \frac{1}{2}\begin{bmatrix} c_x + k_x & 0 & -k_x & 0 \\ 0 & c_y + k_y & 0 & -k_y \\ -k_x & 0 & 2k_x & 0 \\ 0 & -k_y & 0 & 2k_y \end{bmatrix},$$

which is positive definite for $c_x, c_y, k_x, k_y > 0$.

The derivative of (D.4) is given by

$$\dot{V}_{\xi_1} = -\xi_1^T Q_{\xi_1} \xi_1, \tag{D.5}$$

with,

$$Q = \begin{bmatrix} c_x(c_x+k_x) & 0 & 0 & 0 \\ 0 & c_y(c_y+k_y) & 0 & 0 \\ 0 & 0 & k_x^2 & 0 \\ 0 & 0 & 0 & k_y^2 \end{bmatrix},$$

which is negative definite for $c_x, c_y, k_x, k_y > 0$.

The global exponential stability of (D.2) will now be shown using Lyapunov-Razumikhin's stability theorem (confer to Theorem 2.24). In order to do so, (D.2) is rewritten as the following delay distributed system:

$$\dot{\xi}_1(t) = A_0\xi_1(t) - A_2A_1\int_{t-\tau}^{t}\xi_1(s)ds - A_2^2\int_{t-\tau}^{t}\xi_1(s-\tau)ds. \tag{D.6}$$

We now propose (D.4) as a candidate Lyapunov-Razumikhin function for the ξ_1-dynamics without coupling (D.2). Given the delay distributed system (D.6), the derivative of the candidate Lyapunov-Razumikhin function is given by

$$\begin{aligned} \dot{V}_{\xi_1} &= -\xi_1^T Q_{\xi_1}\xi_1 - \left(A_2A_1\int_{t-\tau}^{t}\xi_1(s)ds\right)^T P_{\xi_1}\xi_1 - \left(A_2^2\int_{t-\tau}^{t}\xi_1(s-\tau)ds\right)^T P_{\xi_1}\xi_1 \\ &\quad - \xi_1^T P_{\xi_1}A_2A_1\int_{t-\tau}^{t}\xi_1(s)ds - \xi_1^T P_{\xi_1}A_2^2\int_{t-\tau}^{t}\xi_1(s-\tau)ds \\ &\leq -\lambda_{\min}\left(Q_{\xi_1}\right)\|\xi_1\|_2^2 + 2\lambda_{\max}(P_{\xi_1})\|A_2\|_{i2}\left(\|A_1\|_{i2}\int_{-\tau}^{0}\|\xi_1(t+s)\|_2 ds\right. \\ &\quad \left. +\|A_2\|_{i2}\int_{-\tau}^{0}\|\xi_1(t+s-\tau)\|_2 ds\right)\|\xi_1\|_2, \end{aligned} \tag{D.7}$$

where $\lambda_{\max}(P_{\xi_1})$ and $\lambda_{\min}(Q_{\xi_1})$ denote the maximum and minimum eigenvalues of matrices P_{ξ_1} and Q_{ξ_1}, respectively.

One of the requirements of Lyapunov-Razumikhin's theorem is that $\dot{V}_{\xi_1}(t) < 0$ whenever

$$V_{\xi_1}(\xi_1(t+\delta)) \leq pV_{\xi_1}(\xi_1(t)), \tag{D.8}$$

for all t and $-2\tau \leq \delta \leq 0$ and some $p > 1$, which may be rewritten in terms of $\|\xi_1(t+\delta)\|_2$ and $\|\xi_1(t)\|_2$ as follows:

$$\|\xi_1(t+\delta)\|_2 \leq \sqrt{p\frac{\lambda_{\max}(P_{\xi_1})}{\lambda_{\min}(P_{\xi_1})}}\|\xi_1(t)\|_2 \tag{D.9}$$

for all $-2\tau \leq \delta \leq 0$.

Using condition (D.9), the derivative (D.7) of the candidate Lyapunov-Razumikhin function is given by

$$\dot{V}_{\xi_1} \leq \left(-\lambda_{\min}\left(Q_{\xi_1}\right) + 2\tau \|A_2\|_{i2} \sqrt{p} \frac{\lambda_{\max}(P_{\xi_1})^{\frac{3}{2}}}{\lambda_{\min}(P_{\xi_1})^{\frac{1}{2}}} \left(\|A_1\|_{i2} + \|A_2\|_{i2}\right) \right) \|\xi_1\|_2^2. \tag{D.10}$$

We require that $\dot{V}_{\xi_1} < 0$ in order to conclude the global exponential stability of (D.2). Recall that from Theorem 6.1 we know that $c := c_x = c_y$ and $k := k_x = k_y$, which results in the following:

$$\|A_1\|_{i2} = \sqrt{c^2 + k^2}, \qquad \|A_2\|_{i2} = k,$$

$$\lambda_{\max}(P_{\xi_1}) = \frac{c + 3k + \sqrt{c^2 - 2ck + 5k^2}}{4}, \quad \lambda_{\min}(P_{\xi_1}) = \frac{c + 3k - \sqrt{c^2 - 2ck + 5k^2}}{4},$$

$$\lambda_{\min}\left(Q_{\xi_1}\right) = \min\left\{c\left(c+k\right), k^2\right\} =: \beta.$$

Considering the previous definitions, the derivative of the candidate Lyapunov-Razumikhin function as in (D.10) may be rewritten as follows:

$$\dot{V}_{\xi_1} \leq \left(-\beta + \frac{1}{2}\tau k \sqrt{p} \frac{\left(c + 3k + \sqrt{c^2 - 2ck + 5k^2}\right)^{\frac{3}{2}}}{\left(c + 3k - \sqrt{c^2 - 2ck + 5k^2}\right)^{\frac{1}{2}}} \left(\sqrt{c^2 + k^2} + k\right) \right) \|\xi_1\|_2^2. \tag{D.11}$$

Given (D.11), it is possible to conclude the global exponential stability of (D.2) if the allowable time-delay satisfies the following condition:

$$\tau < \frac{2\beta \left(c + 3k - \sqrt{c^2 - 2ck + 5k^2}\right)^{\frac{1}{2}}}{k\sqrt{p}\left(c + 3k + \sqrt{c^2 - 2ck + 5k^2}\right)^{\frac{3}{2}} \left(\sqrt{c^2 + k^2} + k\right)}, \tag{D.12}$$

and the requirements for the tracking and correction gains stated in Theorem 6.1 are satisfied. Note that the right-hand side of (D.12) is positive for $c > 0$ and $k > 0$. As a result, for $0 \leq \tau < \tau_{\max}$ with $\tau_{\max}$ as in (6.10), the inequality (D.12) is satisfied.

D.1.2 Requirement on the ξ_2-Dynamics

The ξ_2-dynamics have already been determined to be GES in Section B.2.2 if c_θ and k_θ satisfy the conditions in Theorem 6.1 and the time-delay τ satisfies the following bound:

$$\tau < \frac{\pi}{2k_\theta}. \tag{D.13}$$

D.1.3 Requirement on the Coupling Term $g(t, \xi_{1_t}, \xi_{2_t})$

Recall that the coupling term must satisfy the following bound:

$$\|g(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 \le (\alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c)\|\varphi_{\xi_2}\|_c, \tag{D.14}$$

for any $\varphi_{\xi_1} \in \mathcal{C}(4)$ and $\varphi_{\xi_2} \in \mathcal{C}(2)$ and continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$.

The coupling term, as defined in (6.9), may be expressed in terms of φ_{ξ_1} and φ_{ξ_2} as follows:

$$g(t, \varphi_{\xi_1}, \varphi_{\xi_2}) = g_1(t - \tau, \varphi_{\xi_1}(-\tau), \varphi_{\xi_2}(0))\varphi_{\xi_2}(0), \tag{D.15}$$

with $g_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix}$, and g_{32} and g_{42} already defined below (6.9).

Given (D.14) and (D.15), we have that the following holds:

$$\begin{aligned} \|g_1(t, \varphi_{\xi_1}, \varphi_{\xi_2})\|_1 &= \|g_1(t - \tau, \varphi_{\xi_1}(-\tau), \varphi_{\xi_2}(0))\varphi_{\xi_2}(0)\|_1 \\ &\le \|g_1(t - \tau, \varphi_{\xi_1}(-\tau), \varphi_{\xi_2}(0))\|_{i1}\|\varphi_{\xi_2}(0)\|_1 \\ &\le \|g_1(t - \tau, \varphi_{\xi_1}(-\tau), \varphi_{\xi_2}(0))\|_{\text{sum}}\|\varphi_{\xi_2}(0)\|_1 \\ &\le \|g_1(t - \tau, \varphi_{\xi_1}(-\tau), \varphi_{\xi_2}(0))\|_{\text{sum}}\|\varphi_{\xi_2}\|_c. \end{aligned} \tag{D.16}$$

From (D.16) it follows that, in order to satisfy the requirement (D.14), it suffices to show that there exist continuous functions $\alpha_1, \alpha_2 : \mathbb{R}^+ \to \mathbb{R}^+$ such that the following bound is satisfied:

$$\|g_1(t - \tau, \varphi_{\xi_1}(-\tau), \varphi_{\xi_2}(0))\|_{\text{sum}} \le \alpha_1(\|\varphi_{\xi_2}\|_c) + \alpha_2(\|\varphi_{\xi_2}\|_c)\|\varphi_{\xi_1}\|_c. \tag{D.17}$$

Consider now the coupling term given in terms of particular solutions of (D.1). From (D.17) and the definition of the sum matrix norm, the equalities in (2.2), and the triangle inequality, we have the following:

$$\begin{aligned} &\|g_1(t - \tau, \xi_{1_t}(-\tau), \xi_{2_t}(0))\|_{\text{sum}} \\ &= \left| -(\dot{x}_r(t-\tau) + c_x z_{1_e}(t-\tau))\left(\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right) - (\dot{y}_r(t-\tau) + c_y z_{2_e}(t-\tau))\frac{\sin z_{3_e}(t)}{z_{3_e}(t)} \right| \\ &\quad + \left| -(\dot{y}_r(t-\tau) + c_y z_{2_e}(t-\tau))\left(\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right) + (\dot{x}_r(t-\tau) + c_x z_{1_e}(t-\tau))\frac{\sin z_{3_e}(t)}{z_{3_e}(t)} \right| \\ &\le \left| \dot{x}_r(t-\tau)\left(\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right) \right| + \left| c_x z_{1_e}(t-\tau)\left(\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right) \right| + \left| \dot{y}_r(t-\tau)\frac{\sin z_{3_e}(t)}{z_{3_e}(t)} \right| \end{aligned}$$

$$+\left|c_y z_{2_e}(t-\tau)\frac{\sin z_{3_e}(t)}{z_{3_e}(t)}\right|+\left|\dot{y}_r(t-\tau)\left(\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right)\right|+\left|c_y z_{2_e}(t-\tau)\left(\frac{1-\cos z_{3_e}(t)}{z_{3_e}(t)}\right)\right|$$
$$+\left|\dot{x}_r(t-\tau)\frac{\sin z_{3_e}(t)}{z_{3_e}(t)}\right|+\left|c_x z_{1_e}(t-\tau)\frac{\sin z_{3_e}(t)}{z_{3_e}(t)}\right|. \tag{D.18}$$

Due to the fact that the functions $\frac{\sin x}{x}$ and $\frac{1-\cos x}{x}$ are continuous and always ≤ 1 for all x, inequality (D.18) becomes:

$$\|g_1(t-\tau,\xi_{1_t}(-\tau),\xi_{2_t}(0))\|_{\text{sum}} \leq 2\left(|\dot{x}_r(t-\tau)|+c_x\,|z_{1_e}(t-\tau)|+|\dot{y}_r(t-\tau)|+c_y\,|z_{2_e}(t-\tau)|\right). \tag{D.19}$$

Since $\dot{x}_r(t)$ and $\dot{y}_r(t)$ are bounded, we may define $\bar{\dot{x}}_r := \sup_{t\in\mathbb{R}}|\dot{x}_r(t)|$ and $\bar{\dot{y}}_r := \sup_{t\in\mathbb{R}}|\dot{y}_r(t)|$ so that inequality (D.19) becomes:

$$\begin{aligned}\|g_1(t-\tau,\xi_{1_t}(-\tau),\xi_{2_t}(0))\|_{\text{sum}} &\leq 2\left(|\bar{\dot{x}}_r|+|\bar{\dot{y}}_r|+c_x\,|z_{1_e}(t-\tau)|+c_y\,|z_{2_e}(t-\tau)|\right)\\ &\leq 2\left(|\bar{\dot{x}}_r|+|\bar{\dot{y}}_r|+(c_x+c_y)\|\xi_1(t-\tau)\|_1\right)\\ &\leq 2\left(|\bar{\dot{x}}_r|+|\bar{\dot{y}}_r|\right)+2(c_x+c_y)\|\xi_{1_t}\|_c.\end{aligned} \tag{D.20}$$

Consider now inequality (D.20) expressed in terms of φ_{ξ_1} and φ_{ξ_2}

$$\|g_1(t-\tau,\varphi_{\xi_1}(-\tau),\varphi_{\xi_2}(0))\|_{\text{sum}} \leq 2\left(|\bar{\dot{x}}_r|+|\bar{\dot{y}}_r|\right)+2(c_x+c_y)\|\varphi_{\xi_1}\|_c, \tag{D.21}$$

and note that from (D.21) we can satisfy inequality (D.17) by choosing $\alpha_1(\|\varphi_{\xi_2}\|) = 2\left(|\bar{\dot{x}}_r|+|\bar{\dot{y}}_r|\right)$ and $\alpha_2(\|\varphi_{\xi_2}\|) = 2(c_x+c_y)$. With this, the requirement on the coupling term is met.

The three conditions stated at the beginning of the proof have now been checked. As a result, the global uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (D.1) can be concluded given the conditions posed in Theorem 6.1 for the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, constant p, reference velocities, $\dot{x}_r(t)$ and $\dot{y}_r(t)$, and time-delay τ. This implies that the state predictor converges to the reference trajectory, that is $z(t) \to q_r(t)$ as $t \to \infty$, and that the state predictor anticipates the system by τ_f, that is, $z(t) \to q(t+\tau_f)$ as $t \to \infty$. From the previous assertions it follows that the omnidirectional robot subject to a communication delay τ tracks the reference trajectory delayed by τ_f, that is, $q(t) \to q_r(t-\tau_f)$ as $t \to \infty$. This completes the proof.

D.2 Proof of Theorem 6.2: Remote Tracking Control of an Omnidirectional Robot (Local Uniform Asymptotic Stability)

Based on Theorem 2.25, we can formulate the following conditions in order to guarantee the local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the predictor closed-loop error dynamics:

- the ξ_1-dynamics without coupling are locally uniformly asymptotically stable (LUAS);
- the ξ_2-dynamics are locally uniformly asymptotically stable (LUAS);
- the coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ vanishes when $\xi_{2_t} \to 0$, that is, $g(t, \xi_{1_t}, 0) = 0$.

These conditions are checked given the requirements on the tracking gains c_x, c_y, and c_θ, correction gains k_x, k_y, and k_θ, and maximum allowable time-delay, $\tau_{\max}$ posed in Theorem 6.2.

D.2.1 Requirement on the ξ_1-Dynamics Without Coupling

The following state definitions: $\xi_1(t) := [\eta_1(t)\, \eta_2(t)]^T$, $\eta_1(t) := [z_{1_e}(t)\, z_{2_e}(t)]^T$, and $\eta_2(t) := [p_{x_e}(t)\, p_{y_e}(t)]^T$, allow rewriting (D.2) as the following cascade:

$$\dot{\eta}_1(t) = \Delta_1 \eta_1(t) + \Delta_2 \eta_2(t), \tag{D.22a}$$

$$\dot{\eta}_2(t) = \Delta_3 \eta_2(t - \tau), \tag{D.22b}$$

where $\Delta_1 = \begin{bmatrix} -c_x & 0 \\ 0 & -c_y \end{bmatrix}$ and $\Delta_2 = \Delta_3 = \begin{bmatrix} -k_x & 0 \\ 0 & -k_y \end{bmatrix}$.

The locally uniform asymptotic stability of the ξ_1-dynamics can be concluded by studying the stability of the cascaded system (D.22). In particular, the global uniform asymptotic stability of (D.22) is guaranteed provided the following conditions are satisfied:

- the system $\dot{\eta}_1(t) = \Delta_1 \eta_1(t)$, denoted hereinafter as the η_1-dynamics without coupling, is locally uniformly asymptotically stable (LUAS);
- the system $\dot{\eta}_2(t) = \Delta_3 \eta_2(t - \tau)$, denoted hereinafter as the η_2-dynamics, is locally uniformly asymptotically stable (LUAS);
- the coupling term $g_{\eta_1\eta_2}(t, \eta_{1_t}, \eta_{2_t})$ vanishes when $\eta_{2_t} \to 0$, that is, $g_{\eta_1\eta_2}(t, \eta_{1_t}, 0) = 0$.

The previous requirements are assessed considering the conditions on the control parameters posed in Theorem 6.2.

Requirement on the η_1-Dynamics Without Coupling

From linear systems theory it follows that the η_1-dynamics without coupling, given by

$$\dot{\eta}_1(t) = \Delta_1 \xi_1(t), \tag{D.23}$$

are GES provided $c_x, c_y > 0$. This result clearly satisfies the local uniform asymptotic stability requirement.

Requirement on the η_2-Dynamics

The η_2-dynamics, already defined in (D.22b), may be rewritten as:

$$p_{x_e}(t) = -k_x p_{x_e}(t-\tau), \tag{D.24a}$$

$$p_{y_e}(t) = -k_y p_{y_e}(t-\tau), \tag{D.24b}$$

which can be regarded as two scalar decoupled systems. The global exponential stability of this type of delayed scalar system has already been derived in Section B.2.2 and can be guaranteed for $k_x, k_y > 0$ and

$$\tau < \frac{\pi}{2k_x}, \qquad \tau < \frac{\pi}{2k_y}. \tag{D.25}$$

As a result, the local uniform asymptotic stability requirement is satisfied for the previous result.

Requirement on the Coupling Term $g_{\eta_1\eta_2}(t, \eta_{1_t}, \eta_{2_t})$

The coupling term $g_{\eta_1\eta_2}(t, \eta_{1_t}, \eta_{2_t})$ is linear and clearly vanishes when $\eta_{2_t} \to 0$.

D.2.2 Requirement on the ξ_2-Dynamics

The ξ_2-dynamics (D.1b) have already been determined to be GES in Section D.1.2 for $c_\theta > 0$ and $k_\theta > 0$ provided the time-delay τ satisfies the following condition:

$$\tau < \frac{\pi}{2k_\theta}. \tag{D.26}$$

This result clearly satisfies the local uniform asymptotic stability requirement.

D.2.3 Requirement on the Coupling Term $g(t, \xi_{1_t}, \xi_{2_t})$

The coupling term $g(t, \xi_{1_t}, \xi_{2_t})$ as given in (D.1a) clearly vanishes when $\xi_{2_t} \to 0$.

The local uniform asymptotic stability of the equilibrium point $(z_e^T, p_e^T)^T = 0$ of the closed-loop error dynamics (D.1) can be concluded, since the three conditions stated at the beginning of the proof haven been checked for the requirements posed in Theorem 6.2. As a result, the omnidirectional robot subject to a communication delay τ (satisfying the condition in the theorem) tracks the reference trajectory delayed by τ_f, that is, $q(t) \to q_r(t - \tau_f)$ as $t \to \infty$. This completes the proof.

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Samenvatting

Robots ontworpen voor het gebruik in zowel professionele als huiselijke omgeving worden in toenemende mate een onderdeel van ons dagelijks leven. De complexiteit en benodigde daadkracht van de regelalgoritmen voor deze robots mag niet worden onderschat. Dit proefschrift is gerelateerd aan een tweetal disciplines binnen de robotica, namelijk tele-robotica en coöperatieve robotica. Beide disciplines staan voor een aantal grote uitdagingen om te kunnen voldoen aan de verwachtingen van toekomstige robotische systemen. Aan de ene kant zijn tele-robotische systemen in staat op afstand te opereren en (voor de mens) gevaarlijke taken te verrichten, terwijl aan de andere kant coöperatieve robotische systemen in staat zijn gedistribueerde taken te verrichten en verschillende voordelen te bieden ten opzichte van het gebruik van een individuele robot.

Het gebruik van een robot om op afstand taken uit te voeren impliceert in het algemeen de fysieke separatie van de robot en de regelaar. Deze separatie biedt voordelen wanneer op afstand gevaarlijke taken moeten worden uitgevoerd, maar representeert tegelijkertijd ook een belangrijk nadeel. Namelijk, de informatiestromen tussen de robot en de regelaar zullen typisch plaatsvinden over een communicatienetwerk, welke een tijdsvertraging in de regellus introduceert. Als gevolg hiervan wordt het systematische ontwerp van regelaars, welke stabiliteit en de gewenste prestatie induceren ondanks dergelijke tijdsvertragingen, essentieel om een veilig en betrouwbaar functioneren van de robot te garanderen.

Het gebruik van een groep robots teneinde een bepaalde taak uit te voeren, in vergelijking tot het gebruik van een enkele robot, biedt verschillende voordelen, zoals additionele flexibiliteit en het vermogen gedistribueerde en complexe taken uit te voeren. Teneinde gezamenlijke taken succesvol te verrichten dienen de robots hun gedrag te coördineren door het onderling uitwisselen van informatie. Wanneer deze communicatie plaatsheeft over een communicatienetwerk dient rekening gehouden te worden met de gevolgen van resulterende tijdsvertragingen. Het is daarom met name belangrijk regelaars voor de robots te ontwerpen welke samenwerken om een gemeenschappelijke taak te verrichten ondanks de tijdsvertraging die de communicatie tussen de robots beïnvloedt.

De twee regelproblemen hierboven beschreven zijn het onderwerp van studie in dit proefschrift.

Ten eerste wordt het volgprobleem voor op afstand geregelde eenwielerrobots beschouwd. Het belangrijkste aspect van deze studie is het feit dat de tijdsvertraging geïnduceerd door het netwerk, welke gebruikt wordt voor de communicatie tussen robot en regelaar, de stabiliteit en prestatie van het geregelde systeem beïnvloedt. Teneinde problemen veroorzaakt door deze tijdsvertraging aan te pakken, wordt in dit proefschrift een toestand schatter voorgesteld met de structuur van een voorspeller. Deze toestand schatter is gebaseerd op de notie van anticiperende synchronisatie. Wanneer gebruikt samen met een volgregelaar, onderdrukt deze regelstrategie de nadelige effecten van de tijdsvertraging en stabiliseert deze het resulterende gesloten-lus systeem. Door gebruik te maken van bestaande resultaten voor niet-lineaire cascadesystemen met tijdsvertraging, kan de locale uniforme asymptotische stabiliteit van de gesloten-lus foutdynamica gegarandeerd worden tot een bepaalde maximale tijdsvertraging. Tenslotte worden expliciete uitdrukkingen voor de relatie tussen de maximaal toegestane tijdsvertraging en de parameters van de regelaar gepresenteerd.

Ten tweede behandelt dit proefschrift de bewegingscoördinatie van een groep van mobiele robots, waarbij de communicatie tussen de robots onderhevig is aan tijdsvertraging. Meer specifiek worden 'master-slave' synchronisatie en wederzijdse synchronisatie van groepen van eenwielerrobots beschouwd. Een regelaarontwerp wordt voorgesteld, welke bewegingscoördinatie van de robots garandeert zelfs in de aanwezigheid van communicatie-geïnduceerde tijdsvertragingen. De gerelateerde stabiliteitsanalyse, welke ook gebruik maakt van bestaande resultaten voor niet-lineaire cascadesystemen, voorziet in uitdrukkingen welke de regelaarparameters en de maximaal toelaatbare tijdsvertraging relateren.

Het proefschrift legt evenveel nadruk op de theoretische ontwikkeling van regelalgoritmen, als op de experimentele validatie van de ontworpen regelaars. De ontwikkelde regelstrategiën zijn experimenteel gevalideerd gebruikmakend van multi-robot systemen in Eindhoven, Nederland, en Tokyo, Japan, waarbij de communicatie via het Internet verloopt.

Resumerend, dit proefschrift bestudeert een tweetal gerelateerde regelproblemen. Ten eerste wordt het volgprobleem voor een op afstand (over een vertraagd communicatienetwerk) geregelde eenwielerrobot beschouwd. Ten tweede wordt de bewegingscoördinatie van een groep van mobiele robots beschouwd, waarbij de informatie-uitwisseling tussen de robots plaatsheeft over een (vertraagd) communicatienetwerk.

Resumen

Conforme los robots destinados para aplicaciones personales y profesionales comienzan a convertirse en parte de nuestra vida diaria, la importancia y complejidad de los algoritmos de control que los regulan no debe ser subestimada. El trabajo de investigación presentado en esta tesis está relacionado con dos áreas de estudio dentro de la robótica, la telerobótica y la robótica cooperativa, que son primordiales para el avance de esta disciplina en el contexto mencionado anteriormente. Por un lado, los sistemas telerobóticos facilitan la realización de tareas remotas o riesgosas, mientras que, por el otro lado, el uso de sistemas robóticos cooperativos facilita la realización de tareas distribuidas y proporciona varias ventajas con respecto al uso de un solo robot.

La utilización de sistemas robóticos para llevar a cabo tareas remotas implica, en la mayoría de los casos, la separación física entre el controlador y el robot. Dicha separación resulta favorable cuando se realizan tareas de alto riesgo o a distancia, pero constituye a la vez una de las principales desventajas de este tipo de sistemas robóticos. A saber, el uso de un canal de comunicaciones para intercambiar información entre el controlador y el robot conlleva la aparición de un retardo de tiempo en el lazo de control. Es por ello que resulta de primordial importancia diseñar controladores capaces de garantizar la estabilidad y el buen desempeño del robot aun considerando el retardo de tiempo. Lo anterior con la finalidad de asegurar que las tareas asignadas al sistema se lleven a cabo de manera confiable y segura.

Por el otro lado, la utilización de un grupo de robots para llevar a cabo ciertas tareas posee varias ventajas en comparación con el uso de un solo robot, tales como un incremento en la flexibilidad del sistema y la habilidad de completar tareas de naturaleza distribuida o de mayor complejidad. Para completar de manera exitosa su asignación, los robots que componen el grupo deben coordinar su comportamiento. Lo anterior generalmente se consigue mediante el intercambio mutuo de información entre los robots. Cuando dicho intercambio se lleva a cabo a través de un canal de comunicaciones que induce un retardo de tiempo, las consecuencias de dicho retardo deben ser consideradas al momento de controlar al sistema. Con-

secuentemente, resulta indispensable diseñar controladores que permitan al grupo de robots trabajar en conjunto y completar la tarea asignada a pesar del retardo que afecta su intercambio mutuo de información.

Los dos problemas de control planteados previamente son abordados en esta tesis. En primer lugar, se estudia el problema del control a distancia de un robot móvil del tipo uniciclo, el cual se ve afectado por un retardo de tiempo en el canal de comunicaciones que utiliza para transmitir su estado y recibir señales de control. El aspecto más importante a considerar es que dicho retardo deteriora el desempeño del robot y probablemente compromete la estabilidad del sistema. Para abordar el problema se propone un estimador de estados con una estructura similar a la de un predictor. Dicho estimador está inspirado en el concepto de sincronización anticipada. Cuando el predictor de estado actúa en conjunto con una ley de control para seguimiento, la estrategia de control que resulta es capaz de estabilizar al sistema y compensar los efectos negativos ocasionados por el retardo. Haciendo uso de resultados para sistemas no lineales con retardo en cascada, se concluye que la dinámica del error en lazo cerrado es local uniforme asintóticamente estable para un cierto rango de valores admisibles del retardo. Del análisis de estabilidad se obtienen expresiones que explícitamente ilustran la relación entre el retardo de tiempo y los parámetros de control del sistema.

En segundo lugar, se considera el problema de la coordinación a distancia de un grupo de robots móviles del tipo uniciclo, en donde el intercambio de información entre los robots está sujeto a un retardo de tiempo. Específicamente, se considera la coordinación remota del tipo maestro-esclavo y mutua. Los controladores propuestos permiten a los robots mantener la sincronía a pesar de que el intercambio de información entre ellos se lleva a cabo de manera retardada. Se realiza un análisis de estabilidad global el cual considera al grupo entero. De dicho análisis se derivan una serie de expresiones que relacionan los parámetros de control de los robots con el máximo retardo admisible para el cual el grupo se mantiene coordinado.

En este trabajo los desarrollos teóricos y los resultados experimentales reciben el mismo énfasis. Por tal motivo, las estrategias de control propuestas se validan de manera experimental. Para tal fin, se utiliza el Internet como canal de comunicaciones y las plataformas experimentales para múltiples robots disponibles en Eindhoven, Holanda, y Tokio, Japón.

Resumiendo, en esta tesis se abordan dos problemas de control relacionados. Por un lado, se considera el control para seguimiento de un robot móvil utilizando un canal de comunicaciones que induce un retardo de tiempo en el lazo de control. Por el otro lado, se estudia el problema de la coordinación de un grupo de estos robots considerando que el intercambio de información entre ellos está sujeto a un retardo de tiempo.

Acknowledgments

Completing my Ph.D. would not have been possible or as enjoyable (should I say bearable?) without the contributions of a very large number of people. To them, I would like to express my gratitude.

First and foremost, I would like to thank my promotor Henk Nijmeijer, for his constant support, encouragement, confidence, guidance, and (uncannily long-lasting) patience. It has been quite a long journey from our first e-mail exchanges and, all of this time, I have felt grateful and privileged for the opportunity of working with you.

I would also like to thank my copromotor Nathan van de Wouw. I am so grateful and happy for having had the opportunity of working with you; it has been such a great pleasure and learning experience. I must say that, though it probably feels like a very long time to you, working together for only one year feels like to little.

I am most grateful to the members of my promotion committee, prof.dr.ir. Wim Michiels, dr. Toshiki Oguchi, prof.dr. Kees van Hee, and prof.dr. Claudio de Persis, for their willing and timely participation, for thoroughly reading my manuscript, and for their valuable feedback.

Without the financial support of the Mexican Council for Science and Technology (CONACYT) and the Mexican Ministry of Education (SEP) even beginning (let alone completing) my Ph.D. would have been impossible. I would also like to thank those involved in the TSR project for their support and understanding during the last year.

Throughout this time I have had a number of stimulating and very productive conversations (sometimes virtual) related to my work. Among the people who took part in these and to whom I would like to express my gratitude are Dragan Kostić, Jonatán Peña, Sisdarmanto Adinandra, Anna Sadowska, Natalia Sedova, and Jurjen Caarls.

Successfully carrying out experiments would not have been possible without the help of a number of students at Tokyo Metropolitan University. Thank you

Kotarou Kojima, Katsumata Hisashi, and Shingo Watanabe for your patience and enthusiasm while conducting experiments, most of the times late at night.

I would like to thank my current and former roommates (in order of disappearance): Dennis, Wouter, Ewout, Arjan, Eloísa, Rob, and Pieter, who helped me integrate to the group, created such a nice working environment, and, through their example and advice, helped me in more ways than they can imagine. Also, I would like to thank the members of the D&C, CST, and H&NS groups, and my RoboCup teammates. Being too many to list, please know that thanks to you I have enjoyed my time at the university greatly.

To Tano, Danny, Gonzalo, and Álvaro, my fellow Ph.D. friends scattered here and there: thank you for sharing the ups and downs of your own experiences while letting me share mine. Also, a special word of thanks to Luis, who visited at such a convenient time, and to Erik and Anna who so kindly opened their home to us.

Sorting out and navigating through many of the practical aspects of my stay in the Netherlands would have been much more difficult and, at times, not even possible without the help of Lia Neervoort and Petra Aspers at the secretariat, and of Thea Weijers and Hetty van Neerven at DPO. Thank you for your helpfulness and availability throughout this time.

I am particularly grateful to those who have made my day-to-day life in the Netherlands such a joy; my friends Jonatán, Nandra, Eloísa, Miguel, Roel, Juan Francisco, Zhenyu, Alper, and Carlos, thank you for so many interesting (and sometimes not so interesting) conversations and breaks from work; my current and former housemates: Stefan, Menno, Maurice, Benoit, Thomas, Jody, Natalia, and, naturally, Alma; and the members of the Eindhoven ward of the Church of Jesus Christ of Latter Day Saints: the Stewart clan (Aidan and Marlene), familia Cadillo (Patricia, Paula, and Jerson), familie Veugelers (Willy, Gladys, Emma and Matias), families Daams, van Empel, Knudsen, Vennerholm, Witte, and many others, together with Harry van de Sande (de leukste ooit) and Lahai Foday.

None of this would have been possible without the everlasting love and unfailing support of my family: my parents Elvia and Jaime, to whom I owe everything, my brother Sergio and his wonderful family, my parents-in-law Manuel and Leticia, and my sisters-in-law Mónica and Mariana. You are what is most dear to me, I love you all.

Then, there's the one person who has gone through it all together with me. Alma, for all that we are and for all that we'll ever be, thank you.

Alejandro Alvarez-Aguirre,
August, 2011.

CURRICULM VITAE

Alejandro Alvarez Aguirre was born on April 29th, 1981 in Naucalpan de Juarez, Mexico. In 2003 he obtained a bachelor's degree in Electronics and Communications Engineering from the Monterrey Institute of Technology (ITESM-CEM) in the State of Mexico, Mexico. He continued his education at the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV-IPN) in Mexico City, where in 2006 he obtained a master's degree in Electrical Engineering. His master's thesis was entitled "Time-Delay Compensation for Mobile Robots" and was carried out at the Mechatronics Research Group of CINVESTAV-IPN.

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