Fall of viscous jet into a moving surface

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The first Anile-ECMI Prize for Mathematics in Industry was awarded at ECMI 2010 conference to Dr Andriy Hlod. The prize is given to a young researcher for an excellent PhD theses in industrial mathematics successfully submitted at a European university. The award is honoring Professor Angelo Marcello Anile (1948-2007) of Catania, Italy. The price is administered by Associazione Angelo Marcello Anile and ECMI. The following article is a summary of the PhD thesis of Andriy Hlod.

The editor

Fall of viscous jet onto a moving surface

Abstract

A fall of the thin jet of viscous fluid onto the moving surface is considered. The jet is described by the effects of elongational viscosity, inertia and gravity. For the model equations we derive the boundary conditions allowing us to show existence for all the parameters, and investigate uniqueness. For the jet fall we distinguish three flow regimes, which are characterized by the convexity of the jet shape, or by an equivalent characterization of the dominant effect in the momentum transfer through the jet cross-section.

Introduction

Processes, in which the thin curved liquid jets hit the moving surfaces are of a key importance in the productions of thermal isolation, glass wool, polymeric mats, aramid fibers, nonwovens. Understanding the jets behavior in these production processes provide a way to improve quality of the final products, optimize production etc. In all these situations experimental investigations of the jets are extremely expansive and give little insight. This makes an ample room for mathematical modeling and analysis of the curved liquid jets.

A configuration in which the jet hits a moving surface is observed when viscous fluid is allowed to fall from the nozzle onto the moving belt. In this case one can observe three distinguished situations.

The first one occurs for the high flow velocity at the nozzle. In this case the jet shape becomes concave resembling a ballistic trajectory, and the nozzle orientation becomes important for the overall jet shape; see Figure 1. In the second situation the main part of the jet becomes straight vertical; see Figure 2. In the third situation the jet has a convex shape touching the belt tangentially; see Figure 3. In all the three regimes we disregard possible bending and unsteady regions near the nozzle and/or the belt, which become smaller for the thinner jets. Each of the three jet flow regimes can be named according to the convexity of the jet shape i.e. concave, vertical, and convex.

Figure 1: *Inertial (concave) jet*

Figure 2: *Viscous-inertial (vertical) jet*

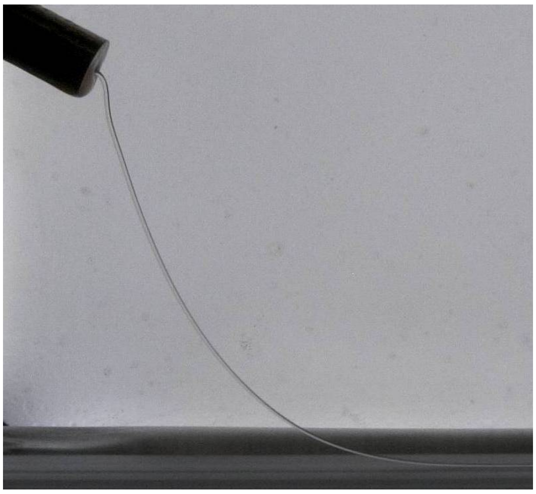

Figure 3: *Viscous (convex) jet*

Form the observations above natural questions arise why the three flow regimes occur, how to model the jet in each flow regime, and how to determine the parameter regions for each flow regime? Answering these questions will provide insight to the jets behavior in the modern industrial process mentioned above.

Model

To model the jet we make use of its slenderness, and include the effects of elongational viscosity, inertia, and gravity. The system describing the jet consists of the conservations of momentum and mass.

A key issue in modeling the jet is the boundary condition for the jet orientation. The observations above suggest that for the concave jet one should prescribe the jet orientation at the nozzle, and for the convex jet tangency with the belt. To understand how to prescribe the boundary conditions for the jet shape we consider the conservation of momentum for the dynamic jet

$$\boxed{\mathbf{r}_{tt} + 2v\mathbf{r}_{st} + \xi\mathbf{r}_{ss}} + (v_t + v_s v - 3\nu(v_s\mathcal{A})_s/\mathcal{A})\mathbf{r}_s = \mathbf{g}. \qquad (1)$$

Here, $\mathbf{r}$ is the position vector, v is the flow velocity in the jet, $\xi = v^2 - 3\nu v_s$ is the momentum transfer through the jet cross-section, $\mathcal{A}$ is the cross-sectional area of the jet, $\mathbf{g}$ is gravity, ν is the kinematic viscosity of the fluid, s is the arc-length, and t is time. The variable ξ is positive if inertia dominates in the momentum transfer through the jet cross-section, and negative if viscosity dominates. Moreover, for the steady jet $\xi(s)$ is a strictly increasing function.

The principle part of the equation (1) is of hyperbolic type for $\mathbf{r}$ provided that the jet is under tension $v_s > 0$. For hyperbolic equations in 1D following holds, the number of the boundary conditions at each jet end should be equal to the number of the characteristics pointing inside the domain. The later together with the monotonicity of ξ gives the three possibilities for the boundary conditions for the jet shape

- **Inertial jet** In the first case $\xi > 0$, and the momentum transfer due to inertia dominates everywhere in the jet. For the boundary conditions we prescribe the nozzle position and the nozzle orientation. The jet shape in this case becomes concave; see Figure 1.
- **Viscous-inertial jet** In the second case $\xi < 0$ at the nozzle and $\xi > 0$ at the belt, viscosity dominates at the nozzle and inertia dominates at the belt in the momentum transfer through the jet cross-section. In this case we can prescribe only one boundary condition, namely the nozzle position. Moreover, an additional condition for the jet orientation is prescribed at the point where $\xi = 0$, the jet should be aligned with the direction of gravity, making the jet **vertical**; see Figure 2.
- **Viscous jet** The third case is $\xi > 0$, and viscosity dominates in the momentum transfer through the jet cross-section everywhere in the jet. In this case we prescribe the nozzle position at the nozzle and tangency with the belt at the belt. The jet shape is **convex**; see Figure 3.

From the analysis above follows that the dominant effect in the momentum transfer provides an equivalent characterization for the three flow regimes. The inertial, viscous-inertial, and viscous jets correspond to the concave, vertical, and convex jets, respectively.

By demanding $\xi = 0$ at the nozzle and at the belt we obtain the boundaries between the inertial and viscous-inertial jet parameter regions, and viscous-inertial and viscous jet parameter regions respectively; see Figure 4.

The participation of the parameters in Figure 4 agrees the one obtained from the experiments.

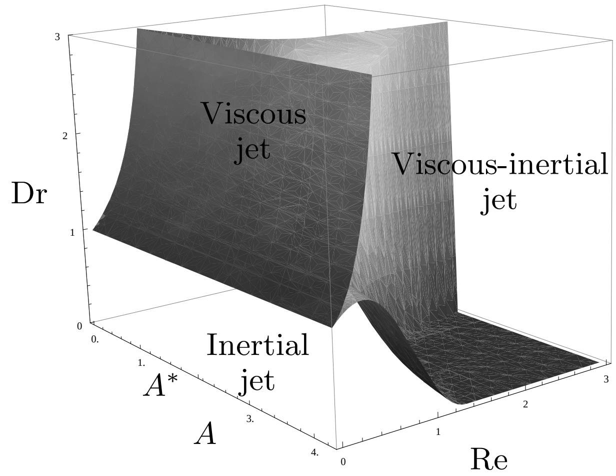

Figure 4: *Parameter regions for the three flow regimes in terms of the three dimensionless numbers* $Dr = v_{\text{belt}}/v_{\text{nozzle}}$, $A = 3g\nu/v_{\text{nozzle}}^3$, *and* $Re = v_{\text{nozzle}}L/(3\nu)$. *Here,* v_{belt} *is the belt velocity,* v_{nozzle} *is the flow velocity at the nozzle, g is the acceleration of gravity, and L is the falling height.*

Our steady jet model is analyzed and solved as follow. We partly solve the equations for the steady jet and transfer the ODE system into an equivalent algebraic equation that is more convenient for analysis. A jet solution exists for all physically admissible parameters and is unique for the viscous and viscous-inertial jets. If the nozzle does not point vertically downwards up to two inertial jets exist together with either viscous-inertial or viscous jet. This non-uniqueness result corresponds to the unsteady jet in the experiments.

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