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# Analysis of an Assemble-to-Order System with Different Review Periods

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**Abstract**

We consider a single item assembled from two components. One of the components has a long leadtime, high holding cost and short review period as compared to the other one. We assume that net stocks are reviewed periodically, customer demand is stochastic and unsatisfied demand is back ordered. We analyze the system under two different policies and show how to determine the policy parameters minimizing average holding and backorder costs. First, we consider a pure base stock policy, where orders for each component are placed such that the inventory position is raised up to a given base stock level. In contrast to this, only the orders for one component follow this logic while the other orders are synchronized in case of a balanced base stock policy. Through mathematical analysis, we come up with the exact long-run average cost function and we show the optimality conditions for both policies. In a numerical study the policies are compared and the results suggest that the balanced base stock policy works better than the pure base stock policy under low service levels and when there is a big difference in the holding costs of the components.

**Keywords:** inventory management, assemble-to-order system, base stock policy, stochastic demand, fixed replenishment intervals

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# 1 Introduction

In real-life supply chains, individual items have their own lot sizing and leadtime constraints based on contracts with suppliers or production process characteristics. Coordination of release decisions across multiple items is thereby not an easy task. In the existing literature, convenient assumptions are made, such as equal lot sizes for items (e.g. Svoronos and Zipkin (1988)), equal review periods (e.g. Clark and Scarf (1960)), nested lot sizes (e.g. Chen (2000)) and nested review periods (e.g. Van Houtum et al. (2007)). One of the consequences of these assumptions is that upstream items and long leadtime items should have larger lot sizes. Unfortunately, in practice, simple and cheap materials can have short leadtimes whereas complex and expensive materials usually have long leadtimes. On top of that the economic order quantity of complex and expensive items implies that such items should be ordered more frequently than cheap items if they have equal demand rates.

Whether an item is cheap or expensive is generally determined by the complexity and capital intensity of the production processes. Complex processes consisting of multiple transformation steps require longer leadtimes, due to waiting times between these transformation steps. On the other hand, capital-intense production is characterized by high utilization, which naturally translates into long leadtimes. Thus, in practice long-leadtime items are often more expensive than short-leadtime items. Typical examples of this situation can be found in High Volume Electronics, where key components like LED screens and IC's have leadtimes beyond ten weeks, whereas cheap plastic parts have leadtimes of less than one week. Similarly, in pharmaceutics industry active ingredients have leadtimes exceeding half a year, while packaging materials and documentation have leadtimes of several weeks. According to the lot sizing theory, long-leadtime items should have higher order frequencies. In capital goods industry, where typically products are assemble-to-order, long-leadtime expensive items (e.g. magnets for medical scanning equipment, lenses for lithography machines) are ordered daily or weekly, while short-leadtime metal and plastic parts may be ordered monthly on average. Similar leadtime and review period relations between components also exist in

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make-to-order and configure-to-order environments. Concisely, there is a need for control policies for assembly systems consisting of expensive, frequently ordered long-leadtime items and cheap, infrequently ordered short-leadtime items.

In this context, we consider a two component assemble-to-order system, where the inventory levels are reviewed periodically. One component has a high holding cost, long leadtime and short review period, whereas the other component has a relatively low holding cost, short leadtime and long review period. We further assume that leadtimes are deterministic and review periods are determined exogenously. Customer demand is stochastic and unsatisfied customer demand is backlogged. The objective is to minimize the expected cost per period consisting of holding and backordering costs by determining the optimal policy parameters. Since the form of the optimal policy is not known for this system, we explore the performance of two different heuristic inventory control policies and determine the cost optimal policy parameters minimizing holding and backorder costs.

The first policy considered is the pure base stock policy in which replenishment orders are placed to restore a fixed base stock level for each component. This policy is well studied in literature on assemble-to-order systems and widely applied in practice. Under a periodic review setting, pure base stock policies are shown to be optimal for serial systems with equal review intervals by Clark and Scarf (1960) and with nested review intervals by Van Houtum et al. (2007). Rosling (1989) shows that the results and methods for serial systems can be used for solving pure assembly systems. However, our problem does not fit into any of these cases due to the review period constraints. If we apply Rosling (1989)'s approach to our model, the equivalent serial network does not have the required nested review intervals property. As a consequence, Van Houtum et al. (2007)'s result cannot be applied to this model.

The second inventory control policy we consider is the so-called "balanced base stock policy". Here, we assume a base stock policy for the longest leadtime component. Then, all other components' base stock levels are coordinated with respect to the stock level of this pivot

component. Balanced base stock policy was first studied by Zhang (1995) for an assemble-to-order system with one end-item and equal replenishment intervals. The analytical results show that indeed the system behaves like a single stockpoint.

In assemble-to-order systems, there are two major challenging problems. The first one is the component allocation problem for the case of multiple end-items. As we study a single end-item model, this problem does not occur. The second problem is minimizing the expected number of backorders or item-based backorders under pure base stock policies. In general, this is computationally demanding because the process involves joint probabilities and optimization of nonseparable functions.

The literature on discrete-time assemble-to-order systems considers both of these issues. Hausman et al. (1998) study an assemble-to-order system with a decentralized base stock policy and normal distributed demand. They propose an equal fractile method for nonstockout probability and develop a heuristic for maximizing a lowerbound on the order fill rate. Zhang (1997) and Agrawal and Cohen (2001) study a similar system where the objective is to minimize total inventory investment subject to a service level constraint. Zhang (1997) defines a so-called fixed-priority allocation rule and concentrates on demand fulfillment rates. On the other hand, Agrawal and Cohen (2001) gain managerial insights on the problem when the component allocation is based on fair-shares rule. Akçay and Xu (2004) introduce a simple and order-based component allocation rule and compare it with the previously stated ones. De Kok (2003) defines a set of assemble-to-order systems named as "strongly ideal". Then, through rigorous analysis he finds exact expressions for the performance characteristics of such systems. Based on these expressions, he develops efficient approximation methods to solve large-scale assemble-to order systems.

In continuous-time framework, most research focuses on computing order-based back-orders, performance measures like order-fulfillment rates or finding bounds for item-based backorders. The most recent work in this setting includes Song (2002), Song and Yao (2002), Lu et al. (2005), Lu and Song (2005), and Hoen et al. (2010). All these papers assume

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assemble-to-order systems with Possion distributed customer demand and pure base stock policies. Finally, we refer the reader to the book chapter of Song and Zipkin (2004) for an extensive literature review on assemble-to-order systems.

To the best of our knowledge, this is the first paper on assemble-to-order systems with different review intervals for each component. We compute the exact expressions for expected cost and expected number of backorders, and we derive optimality conditions. Reinforcing the numerical results of De Kok (2003), we analytically prove the equivalence of nonstockout probability to newsboy fraction at optimality for both pure base stock policy and balanced base stock policy. Our numerical findings suggest that pure base stock policy is better for the majority of the cases but the balanced base stock policy outperforms the other one when service levels are low, demand is highly variable, and the difference between the holding costs of the components is large.

The remainder of the article is organized as follows. Firstly, we describe the detailed model assumptions and the related total cost function in Section 2. Secondly, we formulate and analyze the optimization models based on pure base stock policy and balanced base stock policy in Section 3. Next, In Section 4, we present numerical results to asses and compare the system performance under both replenishment policies. Finally, we summarize the main contributions of this study and give directions for further research in Section 5.

# 2 The Assemble-to-Order Model

We have a single item that is assembled from two components. One piece of each component is needed to produce one end item. The expensive component is stocked at stockpoint 1 and the cheap component is stocked at stockpoint 2. It is assumed that the inventories of components are replenished from suppliers with infinite capacity. Whenever customer demand occurs, the end item is assembled immediately if both components are available otherwise it is backordered.

Time is divided into periods of equal length and the planning horizon is infinite. We

want to make a clear distinction between a "period" and a "review period". Without loss of generality each period is assumed to have length 1 and periods are numbered as $\{0, 1, 2, \ldots\}$. A review period, on the other hand, is composed of multiple periods where at the beginning of a review period the stock levels are reviewed and orders are placed.

There are four main events that may occur during a period: (i) arrival of orders (if scheduled to this period), (ii) placing of orders (if the period is also the beginning of a review period), (iii) occurrence of demand, (iv) incurring costs. The first three events take place at the beginning of the period. We assume that customer demand occurs after ordering decisions are made. Holding and penalty costs are incurred at the end of each period.

We define $I_n(t)$ as the total on-hand inventory of component $n$ at the end of period $t$. The net stock of a component equals all on-hand inventory at this stockpoint minus the amount of backorders. $X_n(t)$ denotes the net stock of component $n$ at the end of period $t$. Also, we define the inventory position of a component as its net stock plus all material in transfer to that stockpoint. Let $IP_n(t)$ be the inventory position for component $n$ at the beginning of a period $t$ after ordering decision is made.

Component $n$ has a review period of length $R_n$ such that the inventory position of $n$ is reviewed and replenishments are made every $R_n$ periods. We assume that component 2's review period is an integer multiple of component 1's review period. Further, we define $r \in \mathbb{N}$ as the number of times that component 1 can be ordered per order of component 2. Thus, the relationship is $R_2 = rR_1$ and $R_2 \geq R_1$ by definition.

Customer demand in each period is independent and identically distributed with density function $f(.)$ and distribution function $F(.)$. $D[t, t+1)$ represents the demand during period $t$ with expected value $\mu$, variance $\sigma^2$, and coefficient of variation $c_v$. Cumulative demand occurring during a time interval between the beginning of period $t_1$ and till the beginning of period $t_2$ $(0 \leq t_1 < t_2)$ is denoted by $D[t_1, t_2)$. Further, we assume that $F(D[t, t+1) < 0) = 0$.

The leadtime $L_n$ between placing and arrival of an order for stockpoint $n$ is assumed to be deterministic and it is defined in periods. The relation between the leadtime of the

components is $L_2 < L_1$.

We further assume synchronization in the timing of order arrivals such that an order arrival at stockpoint 2 always coincides with an order arrival at stockpoint 1. Without loss of generality, we assume that stockpoint 1 places an order at the beginning of period zero. Thus, the ordering periods of stockpoint 1 are defined by the set $T_1 = \{kR_1 | k \in \mathbb{N}_0\}$. An order placed by stockpoint 1 at period $t$ ($t \in T_1$) will arrive at the beginning of period $t + L_1$ in which an order of stockpoint 2 will also arrive. Since $R_2$ is an integer multiple of $R_1$, at periods $kR_2 + L_1$ (where $k \in \mathbb{N}_0$) there will be an arrival of both components. So, the set of ordering periods for stockpoint 2 is $T_2 = \{kR_2 + L_1 - L_2 | k \in \mathbb{N}_0\}$. Note that, in the long-run the system state will not depend on the initial conditions, so there exists a specific period such that the inventory positions will never be above their base stock levels.

We assume linear inventory holding and backorder costs. Each end item backlogged at the end of a period is charged a penalty cost $p$. Each component in stock at stockpoint $n$ at the end of a period is charged a holding cost $h_n$. As we have already mentioned $h_1 \geq h_2$.

A representation of this assemble to order system can be seen in Figure 1.

Figure 1: Assemble to order problem with two stockpoints

For any variable or parameter $x$, we define the operators "$+$" and "$-$" as $x^+ = \max\{0, x\}$ and $x^- = \min\{0, x\} = \max\{0, -x\}$ such that $x = x^+ - x^-$. The backlog of the end item at the end of period $t$ is denoted by $B_0(t)$. It is equal to $\max\{[X_1(t)]^-, [X_2(t)]^-\}$ where $[X_1(t)]^-$ and $[X_2(t)]^-$ represents the shortage at the end of period $t$ for component 1 and 2 respectively. We summarize the model variables and parameters in Table 1.

Let $C(t)$ be the one period total costs incurred at the end of period $t$ which is simply the summation of holding costs for each component on stock and backlogging costs for each end item which cannot be assembled immediately. Then, $C(t)$ is equivalent to the following:

$$C(t) = h_1 I_1(t) + h_2 I_2(t) + p B_0(t). \tag{1}$$

| | |
|---|---|
| $D[t_1, t_2)$: | cumulative customer demand during the time interval $[t_1, t_2)$ |
| $f(.)$: | probability density function for demand per period |
| $F(.)$: | cumulative distribution function for demand per period |
| $\mu$: | expected value of demand per period |
| $\sigma$: | standard deviation of demand per period |
| $c_v$: | coefficient of variation of demand per period $(c_v = \sigma/\mu)$ |
| $R_n$: | review period of component $n$ |
| $r$: | the number of times that stockpoint 1 can order per order of stockpoint 2. $(r = R_2/R_1)$ |
| $L_n$: | leadtime for arrival of orders for component $n$ |
| $h_n$: | inventory holding cost per period for each unit of component $n$ |
| $p$: | penalty cost for each unit backlog |
| $S_n$: | base stock level for component $n$ |
| $X_n(t)$: | net stock of component $n$ at the end of period $t$ |
| $I_n(t)$: | on-hand inventory of component $n$ at the end of period $t$ |
| $IP_n(t)$: | inventory position of component $n$ at the beginning of period $t$ after the ordering decision is made |
| $B_0(t)$: | the amount of backlog of the item at the end of period $t$ |

Table 1: Summary of model parameters

The on-hand inventory is dependent on the net stock by $I_n(t) = X_n(t) + B_0(t)$ as follows from De Kok (2003). Be aware that, we can have backorders of the end item while having stock of one of the components at hand. So, the total cost per period becomes:

$$C(t) \quad = \quad h_1 X_1(t) + h_2 X_2(t) + (p + h_1 + h_2)B_0(t). \tag{2}$$

The objective is to minimize long-run expected system-wide cost per period under a given replenishment policy. In the following section, we analyze the expected cost per period under two different ordering policies. In order to facilitate the analysis, we define an order cycle as the time interval which starts with an arrival of orders to both stockpoints and ends with the period before the next delivery to both stockpoints again. Thus, the cycle has length of $R_2$ periods. Now, assume that a cycle starts with an order arrival to the stockpoints at the beginning of period $t_0$. During this cycle, stockpoint 2 will receive an order only once. The related order decision was taken at the beginning of period $t_0 - L_2$. On the other hand, stockpoint 1 will receive new components $r$ times. Then, Stockpoint 1 will receive orders at

the beginning of periods $t_0 + jR_1$ during the cycle where $j = 0, \ldots, r - 1$. These arrivals of orders are results of the ordering decisions taken at the beginning of periods $t_0 + jR_1 - L_1$. By renewal theory, in the long-run, expected average cost per period will be equivalent to expected average cycle cost divided by the cycle length. Therefore, the cost function to optimize is equal to:

$$\frac{1}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} E[C(t_0 + jR_1 + k)]. \tag{3}$$

# 3    Analysis for Different Policies

In this section, we analyze the assemble-to-order system as described above under two different heuristic policies. The first one is the pure base stock policy. Our motivation for choosing this policy is its ease of implementation, and its optimality in assembly systems and serial systems with stockpoints having different review periods. The other policy, is the balanced base stock policy which was first introduced by Zhang (1995). Under this policy, the system behaves like a single stockpoint and only one policy parameter has to be determined. While the optimization of the balanced base stock policy is easier, the application of the policy may be more difficult.

## 3.1    Pure Base Stock Policy

In this section, we assume a pure base stock policy which is characterized by two policy parameters $(S_1, S_2)$ such that at the beginning of each review period the inventory position of component $n$ is raised to the base stock level $S_n$:

$$IP_n(t) = S_n \text{ if } t \in T_n. \tag{4}$$

$G_P(S_1, S_2)$ symbolizes the long-run average cost function for the pure base stock policy where $S_1$ and $S_2$ represent the base stock levels. Then, the optimization problem $(P1)$ to be

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studied is given as:

$$(P1): \quad \text{Min} \quad G_P(S_1, S_2) \tag{5}$$

$$\text{s.t.} \quad S_1, S_2 \geq 0.$$

By using the cycle definition mentioned in Section 2, under pure base stock policy, the corresponding net stocks at the end of periods $t_0 + jR_1 + k$ for $j = 0, \ldots, r - 1$ and $k = 0, \ldots, R_1 - 1$ are

$$X_1(t_0 + jR_1 + k) = S_1 - D[t_0 + jR_1 - L_1, t_0 + jR_1 + k + 1), \tag{6}$$

$$X_2(t_0 + jR_1 + k) = S_2 - D[t_0 - L_2, t_0 + jR_1 + k + 1). \tag{7}$$

Then, the long-run average cost function given in equation (3) becomes:

$$G_P(S_1, S_2) = h_1 \left\{ S_1 - \mu \left[ L_1 + \frac{R_1 + 1}{2} \right] \right\} + h_2 \left\{ S_2 - \mu \left[ L_2 + \frac{R_2 + 1}{2} \right] \right\}$$

$$+ \frac{(p + h_1 + h_2)}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} E[B_0(t_0 + jR_1 + k)]. \tag{8}$$

Proposition 3.1 provides the analytical expression of the total expected backorders during the order cycle in equation (8). Apparently, this part is the most complicated part of the objective function. Let us define $j^*$ as the smallest integer that satisfies $t_0 + j^* R_1 - L_1 \geq t_0 - L_2$. (Please note that if $j^* \geq r$, the expression (10) is zero.) For the aggregate demand during $(L_2 + jR_1 + k + 1)$, $(k + L_1 + 1)$, and $(|L_1 - L_2 - jR_1|)$ periods, we define the convoluted distribution functions by $F_k^j$, $F_k$, and $F^j$ respectively. Similarly, $f_k^j$, $f_k$, and $f^j$ represent the density functions. We refer to the Appendix for the proofs of all propositions and theorems presented throughout the paper.

**Proposition 3.1.** *Under pure base stock policy, total expected backorders during a cycle is*

*given as:*

$$\sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} E[B_0(t_0 + jR_1 + k)] =$$

$$\sum_{j=0}^{j^*-1} \sum_{k=0}^{R_1-1} \left\{ E[X_2(t_0 + jR_1 + k)]^- + \int_0^\infty \int_0^{S_2+x} (1 - F^j(S_1 + x - y))f_k^j(y)dydx \right\} \quad (9)$$

$$+ \quad \sum_{j=j^*}^{r-1} \sum_{k=0}^{R_1-1} \left\{ E[X_1(t_0 + jR_1 + k)]^- + \int_0^\infty \int_0^{S_1+x} (1 - F^j(S_2 + x - y))f_k(y)dydx \right\}.(10)$$

Let, $S_1^P$ and $S_2^P$ be the optimal base stock levels for this assemble-to-order system under pure base stock policy. The properties of the objective function and optimality conditions for problem $(P1)$ are given by Theorem 3.2.

**Theorem 3.2.** *The cost function $G_P(S_1, S_2)$ is convex under pure base stock policy and the optimal base stock levels have to satisfy the following conditions:*

$$\frac{1}{R_2} \left\{ \left( \sum_{j=0}^{j^*-1} \sum_{k=0}^{R_1-1} \int_0^\infty \int_0^{S_2^P+x} f^j(S_1^P + x - y)f_k^j(y)dydx \right) \right.$$

$$\left. + \left( \sum_{j=j^*}^{r-1} \sum_{k=0}^{R_1-1} [1 - F_k(S_1^P)]F^j(S_2^P - S_1^P) \right) \right\} = \frac{h_1}{(p + h_1 + h_2)}, \quad (11)$$

$$\frac{1}{R_2} \left\{ \left( \sum_{j=0}^{j^*-1} \sum_{k=0}^{R_1-1} [1 - F_k^j(S_2^P)]F^j(S_1^P - S_2^P) \right) \right.$$

$$\left. + \left( \sum_{j=j^*}^{r-1} \sum_{k=0}^{R_1-1} \int_0^\infty \int_0^{S_1^P+x} f^j(S_2^P + x - y)f_k(y)dydx \right) \right\} = \frac{h_2}{(p + h_1 + h_2)}. \quad (12)$$

By Theorem 3.3, we show that at optimality the nonstockout probability is equal to the newsboy ratio similar to the serial and assembly systems. However, for this assemble-to-order system, this result does not lead to a recursive solution for finding the base stock levels.

**Theorem 3.3.** *Under optimal pure base stock policy the following holds:*

$$\frac{1}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} P\{B_0(t_0 + jR_1 + k) = 0\} = \frac{p}{p + h_1 + h_2}. \quad (13)$$

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## 3.2 Balanced Base Stock Policy

In this section, we assume a base stock policy for one component and we synchronize the inventory position of the other component. We define the review period plus the leadtime of the component as its uncertainty period. The uncertainty periods of stockpoint 1 and stockpoint 2 are denoted by $\Delta_1$ $(= R_1 + L_1)$ and $\Delta_2$ $(= R_2 + L_2)$ respectively. Moreover, we define the difference between the uncertainty periods of the components by $\Delta$ $(= \Delta_1 - \Delta_2)$.

When $\Delta \geq 0$, we are able to completely synchronize the net stock of component 2 with respect to the net stock of component 1 since component 2's uncertainty period is shorter. We explain this by using timing of orders. Firstly, we know that only one order of stockpoint 2 arrives during the order cycle that starts with period $t_0$ and this order is placed at the beginning of period $t_0 - L_2$. Secondly, the last order of the expensive item in a cycle is received at the beginning of period $t_0 + (r-1)R_1$. This order was placed at the beginning of period $t_0 + (r-1)R_1 - L_1$. If we compare these two ordering periods, we find the following relation:

$$t_0 + (r-1)R_1 - L_1 \quad = \quad t_0 + rR_1 - R_1 - L_1 = t_0 + R_2 - (R_1 + L_1) \tag{14}$$

$$\leq t_0 + R_2 - (R_2 + L_2) = t_0 - L_2. \tag{15}$$

So, just before component 2 is ordered at the beginning of period $t_0 - L_2$, we already know how much of component 1 will be available during the cycle. Consequently, the amount of order for component 2 can be made depending on the previous orders of component 1. This result enables us to avoid having excess stock of component 2.

When $\Delta < 0$, the uncertainty period of component 1 is shorter. So, it makes sense to completely synchronize the net stock of component 1 with respect to component 2. Again, consider the cycle where at the beginning of period $t_0$ the first orders of both components arrive. The ordering decision of these first arrivals take place at the beginning of periods $t_0 - L_1$ and $t_0 - L_2$ for item 1 and 2, respectively. Additionally, we know that by definition

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$L_1 > L_2$, so $t_0 - L_1 < t_0 - L_2$. This means that there is at least one order of stockpoint 1 that must be placed before we know what will be available for item 2 at the beginning of the cycle. As a result, synchronization fails.

We define the balanced base stock policy only for the case where $\Delta \geq 0$ as follows:

$$IP_1(t) = S_1, \text{ if } t \in T_1 \tag{16}$$

$$IP_2(t) = S_1 - D[t - \Delta, t), \text{ if } t \in T_2 \tag{17}$$

By this way, it is guaranteed that the net stocks of both components will be the same at the end of an order cycle. Let, $G_B(S_1)$ be the long-run average cost period under the balanced base stock policy. Then, the optimization problem $(P2)$ to be studied is

$$(P2): \quad \text{Min} \quad G_B(S_1) \tag{18}$$

$$\text{s.t.} \quad S_1 \geq 0.$$

In the long-run, the net stocks of stockpoint 1 during the cycle are the same as in the equations (6). The net stocks for stockpoint 2 at the end of periods $t_0 + jR_1 + k$ for $j = 0, \ldots, r - 1$ and $k = 0, \ldots, R_1 - 1$ are

$$X_2(t_0 + jR_1 + k) = S_1 - D[t_0 + (r-1)R_1 - L_1, t_0 + jR_1 + k + 1). \tag{19}$$

**Proposition 3.4.** *Under balanced base stock policy with $\Delta \geq 0$ the net stock of component 2 is always greater than or equal to the net stock of component 1.*

As a result of Proposition 3.4, the penalty cost will be charged with respect to the net

stock of component 1 only. Then, the cost per period is computed by:

$$G_B(S_1) = (h_1 + h_2) \left\{ S_1 - \mu(L_1 + \frac{1}{2}) \right\} - \mu(h_1 + (2 - r)h_2)\frac{R_1}{2} \tag{20}$$

$$+ \frac{(p + h_1 + h_2)}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} E[X_1(t_0 + jR_1 + k + 1)^-]. \tag{21}$$

Theorem 3.5 states necessary and sufficient conditions for optimal solution of problem $(P2)$.

**Theorem 3.5.** *The cost function $G_B(S_1)$ is convex and the corresponding optimal base stock level $S_1^B$ is found by:*

$$\frac{1}{R_1} \sum_{k=0}^{R_1-1} F_k(S_1^B) = \frac{p}{p + h_1 + h_2}. \tag{22}$$

The proof of this theorem can be done by following the same outline given for the proof of Theorem 3.2. Notice that equation (22) is also the nonstockout probability for this system and it is equivalent to newsboy ratio similar to a single stockpoint. This property makes it computationally easy to find the optimal base stock level, unlike the pure base stock policy case defined in Section 3.1.

# 4 Comparison of the Policies

In this section we compare the performance of the pure base stock policy and balanced base stock policy on the assemble-to-order model discussed in Sections 2 and 3. At first, we analytically show the relation between the optimal base stock levels found by the two policies by Theorem 4.1.

**Theorem 4.1.** *The optimal base stock levels of stockpoint 1 found by pure base stock policy and balanced base stock policy meet the following condition when $\Delta \geq 0$:*

$$S_1^B \leq S_1^P \tag{23}$$

We know that under balanced base stock policy component 2 never imposes a restriction

on the release decision to assemble the final product. However, the same service level requirement applies to both of the policies, which depends on the same nonstockout probability. To fulfill this requirement under balanced base stock policy, the base stock level of component 1 has to be lower so that the nonstockout probability always depends on the expensive component. This theorem implies that stockpoint 1 incurs lower holding costs under the balanced base stock policy whereas the penalty costs incurred due to the unavailability of component 1 will be lower for the pure base stock policy.

Furthermore, we have conducted numerical studies involving five different factors: holding costs, service level, coefficient of variation of demand, replenishment intervals, and leadtimes. Firstly, we fix $h_1 = 1$ and change the level of $h_2$. We vary the penalty cost according to type 1 service level criterion. This service level denotes the nonstockout probability per period, which is indeed equal to equations (13) and (22). Let, $\gamma$ be the target service level, then the corresponding penalty cost becomes: $p = \gamma(h_1 + h_2)/(1 - \gamma)$.

We take customer demand as Mixed Erlang distributed. There are two main reasons for choosing this distribution. Firstly, it is easy to approximate other distributions with a Mixed Erlang distribution by using two-moment fits (see Tijms, 1986). Secondly, there is an exact evaluation procedure to compute the backorders as described in Van Houtum (2006). The factors and their levels used in the numerical experiment are presented in the first two columns of Table 2. Additionally, we set the review periods as $R_1 = 1$, $R_2 = 4$ and the leadtime of component 2 as $L_2 = 1$.

We have solved 81 problem instances with Matlab. We used the built-in nonlinear optimization tool procedure "fmincon" to compute the optimal base stock levels of problem $(P1)$. Then we optimized problem $(P2)$ by finding the root of (22) for the balanced base stock policy. We compared the optimal cost and optimal base stock level of stockpoint 1 under different policies for each parameter setting. We present the results in terms of relative

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differences which are denoted by $\delta_G$ and $\delta_S$ as shown below:

$$\delta_G = \frac{G_P(S_1^P, S_2^P) - G_B(S_1^B)}{G_P(S_1^P, S_2^P)} \tag{24}$$

$$\delta_S = \frac{S_1^P - S_1^B}{S_1^P} \tag{25}$$

Table 2: Summary of the results of numerical experiment

| Factor | Level | $\%\delta_G$ | | | $\%\delta_S$ | | |
|--------|-------|------|------|------|------|------|------|
| | | Avg. | Max. | Min. | Avg. | Max. | Min. |
| $h_2$ | 0.1 | -0.06 | 1.50 | -1.95 | 0.56 | 2.19 | 0.00 |
| | 0.25 | -1.49 | 0.94 | -5.73 | 1.19 | 2.99 | 0.00 |
| | 0.5 | -4.33 | -0.11 | -11.58 | 2.52 | 5.48 | 0.68 |
| $\gamma$ | 0.9 | -1.36 | 1.5 | -9.24 | 1.64 | 5.48 | 0.00 |
| | 0.95 | -1.93 | 0.88 | -10.30 | 1.43 | 4.66 | 0.00 |
| | 0.99 | -2.57 | 0.70 | -11.58 | 1.21 | 3.54 | 0.00 |
| $c_v$ | 0.5 | -2.47 | 0.78 | -11.58 | 0.98 | 2.42 | 0.00 |
| | 1 | -1.74 | 1.25 | -9.86 | 1.46 | 4.10 | 0.00 |
| | 1.5 | -1.66 | 1.50 | -10.46 | 1.83 | 5.48 | 0.00 |
| $L_1$ | 4 | -0.06 | 0.70 | -0.50 | 0.42 | 2.19 | 0.00 |
| | 8 | -1.18 | 1.50 | -5.25 | 1.98 | 5.48 | 0.39 |
| | 16 | -4.63 | 0.13 | -11.58 | 1.88 | 5.22 | 0.35 |
| | Overall | -1.96 | 1.50 | -11.58 | 1.42 | 5.48 | 0.00 |

The results are summarized in Table 2. From the overall results, we see that there is not much cost difference between the two policies but pure base stock policies are on average slightly better. In cases with high holding cost at stockpoint 2, high service level, low coefficient of variation, and longer leadtime at stockpoint 1 the pure base stock policy performs better.

To understand these results better we conducted some experiments that show the marginal effects of the factors. We set a base case where $h_1 = 1$, $h_2 = 0.1$, $\gamma = 0.95$, $c_v = 1$, $R_1 = 1$, $R_2 = 4$, $L_1 = 8$, and $L_2 = 1$. For the following numerical results, we use these parameters unless they are specifically stated.

We start with analyzing the trade-off between the holding and backordering costs as

Figure 2: The effects of holding cost and service level on the performance of policies

shown in Figure 2. Clearly the the pure base stock policy performs better with service levels higher than 0.7 and with holding cost higher than 0.3. The balanced base stock policy, on the other hand, should be preferred for the cases where the service level is as low as 0.5 and the holding cost at stockpoint 2 is low compared to the holding cost of stockpoint 1. Such service levels may seem unrealistic but they can occur in practice if there is an agreed upon time window for fulfilling the customer demand. Here, the low service level makes holding cost as important as the penalty cost, as a result holding less stock on hand becomes favorable. Balanced base stock policy is better since it gives lower optimal base stock level for the expensive item. As the service level increases, together with the $h_2/h_1$ ratio, the pure base stock policy performs better since penalty cost is the most important term in the cost function. Moreover, the holding cost of both components have similar weights in the objective function. Finally, when $h_2$ is zero both policies give the same result because the system actually reduces to a single stockpoint and the policies converge.

Figure 3: The effect of difference between uncertainty periods on the performance of policies

Further, we demonstrate the effect of $\Delta$ on the optimal cost as shown in Figure 3. At the extreme case $\Delta = 0$, the base stock level of the cheap component will be fixed under balanced base stock policy, thus the policy will behave as a pure base stock policy. This explains $\delta_G$ being zero at that point. As the number of times stockpoint 1 can order during a cycle decreases (like when $r = 1$), we observe sharper differences on the optimal costs. If the difference between uncertainty periods is short, the balanced base stock policy outperforms the pure base stock policy. As absolute difference between uncertainty periods in terms of leadtimes and review periods increases, we need to apply more sophisticated policies to control the system. The pure base stock policy works better under high $\Delta$, by having two control parameters for the system instead of one.

Next, we explore the effect of coefficient of variation under extreme service levels and

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Figure 4: The effect of coefficient of variation on the performance of policies, $\gamma = 0.5$

Figure 5: The effect of coefficient of variation on the performance of policies, $\gamma = 0.95$

different holding costs in Figures 4 and 5. In cases where service level and $h_2/h_1$ is low, there is no regular pattern of results in terms of coefficient of variation in Figure 4. In all other low service level cases, the balanced base stock policy performs even better as the coefficient of variation increases. In contrast, under high service level, the gap between the optimal cost of both policies increases with lower $c_v$ levels as shown in Figure 5. This result suggests that synchronization of the net stocks is helpful under highly uncertain demand.

Table 3: The effect of $\gamma$, $c_v$ and $h_2/h_1$ on optimal base stock levels

| $\gamma$ | $c_v$ | $h_2/h_1$ | $S_1^B$ | $S_1^P$ | $S_2^P$ |
|---|---|---|---|---|---|
| 0.5 | 1 | 0.25 | 866.90 | 898.86 | 493.35 |
| | | 0.75 | 866.90 | 958.79 | 414.60 |
| 0.5 | 1.5 | 0.25 | 839.65 | 886.13 | 547.03 |
| | | 0.75 | 839.65 | 987.24 | 422.57 |
| 0.5 | 2.5 | 0.25 | 513.51 | 531.62 | 307.09 |
| | | 0.75 | 513.51 | 626.22 | 246.52 |
| 0.95 | 1 | 0.25 | 1443.46 | 1471.09 | 954.40 |
| | | 0.75 | 1443.46 | 1521.16 | 868.55 |
| 0.95 | 1.5 | 0.25 | 1738.10 | 1783.07 | 1220.02 |
| | | 0.75 | 1738.10 | 1858.04 | 1101.53 |
| 0.95 | 2.5 | 0.25 | 2439.19 | 2499.78 | 1876.32 |
| | | 0.75 | 2439.19 | 2670.13 | 1589.75 |

Furthermore, we analyze the behavior of optimal base stock levels in Table 3. As $h_2/h_1$ increases under pure base stock policy, the optimal base stock level of stockpoint 1 slightly increases and the base stock level of stockpoint 2 decreases to reduce the holding cost at stage 2. This result is related to the change in the weight of the optimality equations (11) and (12). Under given coefficient of variation and service level, optimal base stock level of stockpoint 1 found by the balanced base stock policy does not change due to different $h_2/h_1$ levels. This is because the optimality equation (22) depends on the $\frac{p}{p+h_1+h_2}$ ratio which is equal to the service level. As observed from the table, the high coefficient of variation results

in sharp changes in optimal base stock levels when service level changes. We explain this effect due to higher uncertainty conditions. As service level decreases holding cost becomes more important, under highly uncertain demand and it is advantageous to keep the net stocks as low as possible. On the contrary, when penalty cost is high, the optimal base stock levels are higher with high $c_v$.

# 5    Summary and Conclusions

We studied an assemble-to-order system with one final product that is composed of one long leadtime component with short review period and one short leadtime component with long review period. Such a system cannot be solved to optimality by existing methods in literature since the review periods are not nested. Instead, we assumed and compared two heuristic control policies: the so-called balanced base stock policy and pure base stock policy.

We derived the exact cost expressions for both policies and we provide the optimality conditions for the policy parameters. We also show the equivalence of newsboy ratio to the nonstockout probability at optimality for both policies. For both the balanced base stock policy and the pure base stock policy, we showed that the under the optimal policy parameters the final product nonstockout probability is equal to the Newsboy fractile. Moreover, we prove that the balanced base stock policy gives lower optimal base stock level for stockpoint 1 compared to the pure base stock policy. Here, we would like to mention that the analytical results of this model holds under general holding costs. The specific choice for the cost parameters is based on our observations from practice.

The numerical results show that the balanced base stock policy performs better under low service levels, low $h_2/h_1$ ratios, and high demand uncertainty. Conversely, the pure base stock policy is preferable under high service levels, high $h_2/h_1$ ratio, and large difference between uncertainty periods.

Computationally speaking, finding the optimal base stock level in balanced base stock policy is much easier since we directly use the newsboy formula given in equation (22). For

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the pure base stock policy we have to use a generic optimization method to determine the optimal solution which takes longer time. On the other hand, from a practical point of view, applying the pure base stock policy is easy for the companies. Since the base stock levels are fixed to a certain value.

Clearly, the model discussed in this paper is far from the true complexity in practice. Thus, we intend to extend our research to more complex systems. The balanced base stock policy can be applied to two-echelon ATO systems with multiple components as long as we preserve the ordering of sums of lead times and review periods indicated by the situation $\Delta \geq 0$. This leads to a single base stock level governing all subsequent decisions. The challenge is to develop the exact expressions for the component holding costs and final product penalty costs. Especially, the derivation of the exact backorder cost term will become complicated. Even for the current one-product two-component ATO system, we need to explore the case of $\Delta < 0$ and formulate policies that allow for exact analysis. Towards this end, we intend to derive optimal policies for this system based on stochastic dynamic programming. This research should reveal whether we can make steps towards multi-item multi-echelon inventory systems without restrictive conditions on lead times and order frequencies.

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# Appendix

Proof of Propostion 3.1.

*Proof.* The proof will be shown only for expression (9) which is $j < j^*$ case. For ease of notation, we define the following variables: $D_1(j,k) = D[t_0 + jR_1 - L_1, t_0 + jR_1 + k + 1)$, $D_2(j,k) = D[t_0 - L_2, t_0 + jR_1 + k + 1)$. The expectation of backorders at the end of period $t_0 + jR_1 + k$ is

$$
\begin{aligned}
E[B_0(t_0 + jR_1 + k)] &= \int_0^\infty P\{\ \max(X_1[t_0 + jR_1 + k]^-, X_2[t_0 + jR_1 + k]^-) > x\}dx \quad (26)\\
&= \int_0^\infty (1 - P\{[D_1(j,k) - S_1]^+ \le x, [D_2(j,k) - S_2]^+ \le x\})dx \quad (27)\\
&= \int_0^\infty (1 - P\{[D_1(j,k) \le S_1 + x, D_2(j,k) \le S_2 + x\})dx. \quad (28)
\end{aligned}
$$

Notice that $D_1(j,k)$ and $D_2(j,k)$ are dependent. When $j < j^*$, it holds that $t_0 - L_2 > t_0 + jR_1 - L_1$. Thus, $D_2(j,k) < D_1(j,k)$ for given $j$ and $k$. Now, we redefine $D_1(j,k)$ as $D_1(j,k) = D_2(j,k) + D_3(j,k)$ such that $D_3(j,k) = D[t_0 + jR_1 - L_1, t_0 - L_2)$. Then, the following holds:

$$P\{D_1(j,k) \le S_1 + x, D_2(j,k) \le S_2 + x\} \tag{29}$$

$$= P\{D_2(j,k) + D_3(j,k) \le S_1 + x, D_2(j,k) \le S_2 + x\} \tag{30}$$

$$= \int_0^{S_2+x} P\{D_3(j,k) \le S_1 + x - y, D_2(j,k) = y\}dy \tag{31}$$

$$= \int_0^{S_2+x} F^j(S_1 + x - y)f_k^j(y)dy. \tag{32}$$

By using the resulting equation (32), in equation (28) the expectation of backorder for $j < j^*$ becomes:

$$E[B_0(t_0 + jR_1 + k)] = \int_0^\infty \left\{ 1 - \int_0^{S_2+x} F^j(S_1 + x - y)f_k^j(y)dy \right\} dx \tag{33}$$

$$= \int_0^\infty \left\{ 1 - \int_0^{S_2+x} \left\{ 1 - P\{D_3(j,k) > S_1 + x - y\}f_k^j(y)\right\} dy \right\} dx \tag{34}$$

$$= \int_{S_2}^\infty \left\{ 1 - F_k^j(z) \right\} dz + \int_0^\infty \int_0^{S_2+x} P\{D_3(j,k) > S_1 + x - y\}f_k^j(y)dydx \tag{35}$$

$$= E[X_2(t_0 + jR_1 + k)]^- + \int_0^\infty \int_0^{S_2+x} (1 - F^j(S_1 + x - y))f_k^j(y)dydx. \tag{36}$$

The expression (10) can be proven by following the same steps. $\qquad\square$

The proof of Theorem 3.2

*Proof.* The proof of this theorem follows from standard calculus. First, take the first order partial derivatives of the cost function (8) with respect to $S_1$ and $S_2$. Then, by making these partial derivatives equal to zero, we get equations (11) and (12). To prove convexity, first derive the related Hessian matrix by taking second order partial derivatives. Then, it can be shown that the determinants of the Hessian are nonnegative. This result implies that the cost function is convex, and the first order derivatives provide the optimal base stock levels. $\qquad\square$

The proof of Theorem 3.3

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*Proof.* First, we write down the nonstockout probability in terms of conditional probabilities. For any $j$ and $k$:

$$P\{B_0(t_0 + jR_1 + k) = 0\} =$$

$$1 - P\{X_1(t_0 + jR_1 + k) < 0, X_1(t_0 + jR_1 + k) \leq X_2(t_0 + jR_1 + k)\}$$

$$-P\{X_2(t_0 + jR_1 + k) < 0, X_2(t_0 + jR_1 + k) < X_1(t_0 + jR_1 + k)\}. \tag{37}$$

Consider the expression $P\{X_1(t_0 + jR_1 + k) < 0, X_1(t_0 + jR_1 + k) \leq X_2(t_0 + jR_1 + k)\}$ in equation (37) when $j < j^*$. Then, we can rewrite $D_1(j, k)$ as $D_1(j, k) = D_2(j, k) + D_3(j, k)$ and the expression becomes:

$$P\{X_1(t_0 + jR_1 + k) < 0, X_1(t_0 + jR_1 + k) \leq X_2(t_0 + jR_1 + k)\}$$

$$= P\{D_1(j, k) > S_1, D_2(j, k) - D_1(j, k) \leq S_2 - S_1\} \tag{38}$$

$$= P\{D_2(j, k) + D_3(j, k) > S_1, D_3(j, k) \geq S_1 - S_2\} \tag{39}$$

$$= \int_0^\infty P\{D_2(j, k) + D_3(j, k) = S_1 + x, D_3(j, k) \geq S_1 - S_2\} dx \tag{40}$$

$$= \int_0^\infty \int_{S_1-S_2}^{S_1+x} f_k^j(S_1 + x - z) f^j(z) dz dx \tag{41}$$

$$= \int_0^\infty \int_0^{S_2+x} f_k^j(y) f^j(S_1 + x - y) dy dx. \tag{42}$$

Here again, under the same condition the other term

$P\{X_2(t_0 + jR_1 + k) < 0, X_2(t_0 + jR_1 + k) < X_1(t_0 + jR_1 + k)\}$ is equal to:

$$P\{X_2(t_0 + jR_1 + k) < 0, X_2(t_0 + jR_1 + k) < X_1(t_0 + jR_1 + k)\}$$

$$= P\{D_2(j, k) > S_2, D_1(j, k) - D_2(j, k) < S_1 - S_2\} \tag{43}$$

$$= [1 - F_k^j(S_2)]F^j(S_1 - S_2). \tag{44}$$

If we follow the same arguments for $j \geq j^*$ case, we get the following set of equations:

$$P\{X_1(t_0 + jR_1 + k) < 0, X_1(t_0 + jR_1 + k) \leq X_2(t_0 + jR_1 + k)\}$$

$$= [1 - F_k(S_1)]F^j(S_2 - S_1), \tag{45}$$

$$P\{X_2(t_0 + jR_1 + k) < 0, X_2(t_0 + jR_1 + k) < X_1(t_0 + jR_1 + k)\}$$

$$= \int_0^\infty \int_0^{S_1+x} f_k(y)f^j(S_2 + x - y)dydx. \tag{46}$$

If we plug the equations (42), (44), (45), and (46) in equation (37), total the nonstockout probability for the optimal base stock levels becomes:

$$\sum_{j=0}^{r-1}\sum_{k=0}^{R_1-1} P\{B_0(t_0 + jR_1 + k) = 0\} =$$

$$1 - \frac{1}{R_2}\left\{\left\{\sum_{j=0}^{j^*-1}\sum_{k=0}^{R_1-1}[1 - F_k^j(S_2^P)]F^j(S_1^P - S_2^P)\right\}\right.$$

$$+ \left\{\sum_{j=j^*}^{r-1}\sum_{k=0}^{R_1-1}\int_0^\infty\int_0^{S_1^P+x} f^j(S_2^P + x - y)f_k(y)dydx\right\}\right\}$$

$$- \frac{1}{R_2}\left\{\left\{\sum_{j=0}^{j^*-1}\sum_{k=0}^{R_1-1}\int_0^\infty\int_0^{S_2^P+x} f^j(S_1^P + x - y)f_k^j(y)dydx\right\}\right.$$

$$+ \left\{\sum_{j=j^*}^{r-1}\sum_{k=0}^{R_1-1}[1 - F_k(S_1^P)]F^j(S_2^P - S_1^P)\right\}\right\}. \tag{47}$$

We know that the optimal base stock levels satisfy the equations (11) and (12). Notice that the left hand side of the optimality conditions are contained in the equation (47). Then it holds that:

$$\sum_{j=0}^{r-1}\sum_{k=0}^{R_1-1} P\{B_0(t_0 + jR_1 + k) = 0\} =$$

$$= 1 - \frac{h_1}{(p + h_1 + h_2)} - \frac{h_2}{(p + h_1 + h_2)} = \frac{p}{(p + h_1 + h_2)}. \tag{48}$$

$\square$

The proof of Proposition 3.4

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*Proof.* For each $j = 0, \ldots, r-1$ and $k = 0, \ldots, R_1 - 1$ the following holds:

$$X_2(t_0 + jR_1 + k) - X_1(t_0 + jR_1 + k) = D[t_0 + jR_1 - L_1, t_0 + (r-1)R_1 - L_1) > 0. \quad (49)$$

□

The proof of Theorem 4.1

*Proof.* For ease of notation, we again use the following variables: $D_1(j, k) = D[t_0 + jR_1 - L_1, t_0 + jR_1 + k + 1)$, $D_2(j, k) = D[t_0 - L_2, t_0 + jR_1 + k + 1)$. From the equations (13) and (22) we have:

$$\frac{1}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} P\left\{D_1(j, k) \le S_1^P, D_2(j, k) \le S_2^P\right\} = \frac{1}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} P\left\{D_1(j, k) \le S_1^B\right\}. \quad (50)$$

Let's assume that $S_1^P < S_1^B$, then for any $j$ and $k$ it holds that

$$P\left\{D_1(j, k) \le S_1^P, D_2(j, k) \le S_2^P\right\} < P\left\{D_1(j, k) \le S_1^B, D_2(j, k) \le S_2^P\right\} \quad (51)$$

$$\le P\left\{D_1(j, k) \le S_1^B\right\}. \quad (52)$$

This results leads us to the following inequality:

$$\frac{1}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} P\left\{D_1(j, k) \le S_1^P, D_2(j, k) \le S_2^P\right\} < \frac{1}{R_2} \sum_{j=0}^{r-1} \sum_{k=0}^{R_1-1} P\left\{D_1(j, k) \le S_1^B\right\}. \quad (53)$$

Equation (53) contradicts with equation (50). Thus, the relation between the optimal base stock levels must be $S_1^P \ge S_1^B$. □

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