An efficient approach for eigenmode analysis of distributive mixing by the mapping method

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Introduction

Mapping matrix can be used to describe advective mixing and all information about transport properties is contained in its eigenmodes. Eigenmode decomposition of the mapping matrix allows to obtain a physical interpretation of the mixing process.

Objective

Investigation of the truncated eigenmode decomposition of the mapping matrix. Analysis of dominant eigenvectors and coefficients in the eigenmode decomposition.

Methods

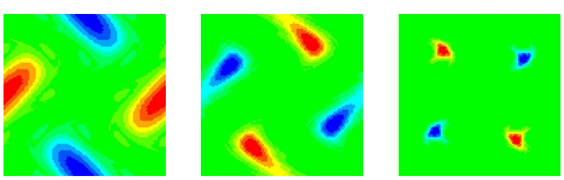
The present study proposes an efficient and accurate way for the eigenmode representation of arbitrary concentration distributions based on orthogonalization of the truncated eigenvector basis of the mapping matrix Φ. Mapping matrix concerns purely advective transport and for any initial distribution of concentration $\boldsymbol{C}_0$, concentration $\boldsymbol{C}_n$ after n periods can be found [1],

$$\boldsymbol{C}_n = (\underbrace{\Phi(\Phi(...(\Phi}_{\text{n times}}\boldsymbol{C}_0)...).$$

The mapping admits eigenmode decomposition for the whole spectrum of eigenmodes [1]. The rapid decay of eigenmodes of Φ implies that eigenmode decomposition can be limited by M dominant modes for the concentration distribution $\boldsymbol{C}_m$,

$$\boldsymbol{C}_m = \sum_{k=1}^{M} C_k^0 \lambda_k^m \boldsymbol{v}_k + \mathcal{O}(\lambda_{M+1}^m),$$

where $\{\lambda_k, \boldsymbol{v}_k\}$ are eigenvalue-eigenvector pairs. By dominant we understand eigenmodes with $|\lambda_k|$ close to 1. Second dominant eigenvector represents the structure of the flow domain, see fig. 1.



T=0.56 T=0.8 T=1.12

Fig. 1 *Structure of the second dominant eigenvector in the case of TPSF.*

Dominant eigenmodes can be divided into groups according to the region where they are non-zeros, see fig. 2.

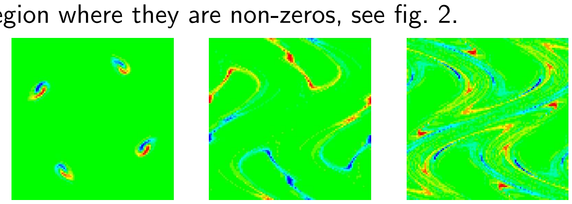
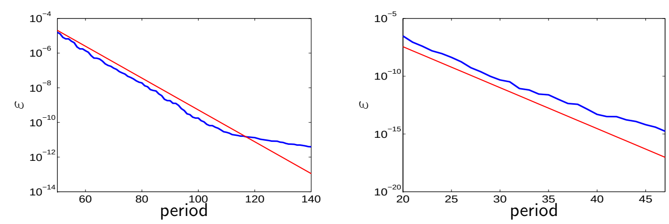


Fig. 2 *Structures of dominant eigenvectors in the case of TPSF: T=0.8.*

Results

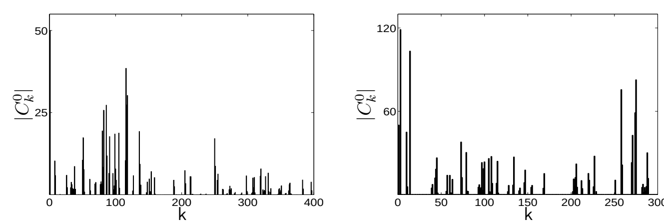
Coefficients C_k^0 entirely depends on the initial distribution of $\boldsymbol{C}_0$ and can be found by constructing a system of equation based on orthogonalized dominant eigenvectors. This system is build only for M dominant eigenvectors and does not require computation of the whole spectrum of eigenmodes. For the analysis of the truncated eigenmode decomposition were considered two cases of TPSF: $T = 0.56$ and $T = 1.6$ and number of dominant eigenmodes $M = 400$ and $M = 300$, respectively. Fig.3 illustrates the error ε between results obtained by truncated eigenmode decomposition and mapping method.



T=0.56 T=1.6

Fig. 3 *Error between results obtained by using truncated eigenmode decomposition and mapping method.*

The accuracy of the truncated decomposition can be regulated by the number of dominant eigenvectors M. Fig.4 illustrates magnitudes of the coefficients in the two cases of TPSF for initial concentration when top half of the flow domain has concentration $\boldsymbol{C} = 1$ and bottom $\boldsymbol{C} = 0$.



T=0.56 T=1.6

Fig. 4 *Magnitudes of coefficients* $|C_k^0|$.

Coefficients C_k^0 define which eigenmodes are excited for such $\boldsymbol{C}_0$ and only these modes are needed to describe mixing.

Conclusion

Eigenmode decomposition allows to perform physical analysis of the mixing. Controlling contribution of eigenmodes, quality of the mixture can be improved. Investigation of eigenmode decomposition allows to improve the control on the mixing.

References:

[1] M. K. Singh, M.F.M. Speetjens, P.D. Anderson 2009 Eigenmode analysis of scalar transport in distributive mixing. *Phys. Fluids* **21**, 093601