

# Nonlinear observer design for parametric roll resonance

**Citation for published version (APA):**

Nijmeijer, H., Belleter, D. J. W., Breu, D. A., & Fossen, T. I. (2012). Nonlinear observer design for parametric roll resonance. In N. Themelis, A. D. Paprikolaou, & K. J. Spyrou (Eds.), *Proceedings on Stability of Ships and Ocean Vehicles, 23-28 August 2012, Athens, Greece* (pp. 699-705). National Technical University.

**Document status and date:**

Published: 01/01/2012

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

**General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.tue.nl/taverne](http://www.tue.nl/taverne)

**Take down policy**

If you believe that this document breaches copyright please contact us at:

[openaccess@tue.nl](mailto:openaccess@tue.nl)

providing details and we will investigate your claim.



# Nonlinear Observer Design for Parametric Roll Resonance

Dennis J.W. Belleter, *Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands, [d.j.w.belleter@student.tue.nl](mailto:d.j.w.belleter@student.tue.nl)*

Dominik A. Breu, *Centre for Ships and Ocean Structures, Norwegian University of Science and Technology, Norway, [breu@itk.ntnu.no](mailto:breu@itk.ntnu.no)*

Thor I. Fossen, *Department of Engineering Cybernetics, Norwegian University of Science and Technology, Norway, [fossen@ieee.org](mailto:fossen@ieee.org)*

Henk Nijmeijer, *Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands, [h.nijmeijer@tue.nl](mailto:h.nijmeijer@tue.nl)*

## ABSTRACT

In this work a nonlinear two degrees-of-freedom (DOF) ship model is considered. For this model a state observer is designed in this paper. The type of state observer that is chosen is an Extended Kalman filter (EKF). First the theory for the design of an EKF is presented. Using this theory an observer in the form of an EKF is then designed to estimate the wave frequency and wave encounter frequency for the ship experiencing parametric roll resonance. The performance of the proposed EKF is verified by a simulation study. In the simulation study the estimates of the observer are compared against fictive measurements generated by a measurement generation model. The model determines the necessary control input to obtain a desired velocity and contains a wave spectrum model to excite the ship model.

**Keywords:** *Parametric roll, Nonlinear system, Observer design, Extended Kalman filter*

## 1. INTRODUCTION

Parametric roll resonance is a dangerous nonlinear resonance phenomenon for ships that can cause roll oscillations up to  $\pm 40^\circ$  (France et al., 2003). Certain ship types such as fishing vessels and container ships have been shown to be especially prone to parametric roll resonance in moderate to heavy head or stern seas, with long crested waves. Parametric roll is induced by variations in the ship's transverse stability during a wave passage. (Carmel, 2006; France et al., 2003)

A lot of research has been done to model parametric roll and the ship's dynamics during parametric roll, see for instance Fossen and

Nijmeijer (2012) and Neves and Rodríguez (2005) and the references therein. Recent active control approaches are based on changing the ship's forward velocity (Holden et al., 2012), resulting in a Doppler shift of the frequency at which the ship is meeting the waves, the wave encounter frequency. Frequency detuning control can be used to alter the wave frequency out of a (ship specific) frequency range where the ship is susceptible to parametric roll (Holden et al., 2012).

Assuming constant heading, the wave encounter frequency is dependent on the wave frequency and the ship's speed. Frequency detuning control generally assumes the knowledge of both. However, while the ship's



speed is generally known the wave frequency is not and is nontrivial to obtain by measurements in a practical situation at sea. Consequently, a state observer to estimate the wave frequency and wave encounter frequency is crucial for the applicability of the frequency detuning control.

The 2-DOF ship model under investigation is a state-space model with five states. These states are the roll angle  $\varphi$ , the roll rate  $\dot{\varphi}$ , the velocity in surge direction  $u$ , the wave frequency  $\omega_0$  and the wave encounter frequency  $\omega_e$ . This results in the state vector:

$$x = [\varphi, \dot{\varphi}, u, \omega_0, \omega_e]^T \quad (1)$$

The roll angle, roll rate, and the velocity in surge direction are assumed to be measured. The wave frequency and wave encounter frequency need to be estimated by the state observer.

The paper is organized as follows. In Section 2 the model of the ship under investigation is introduced. The theory for the design of an EKF is introduced in Section 3. Followed by a simulation study in Section 4 where the EKF is used to estimate the wave frequency and wave encounter frequency.

## 2. THE SHIP MODEL

In this section the ship model under investigation with state vector (1) is introduced.

Table 1. Model parameters

Sym.	Quantity
$m_1$	mass and added mass in surge
$d_1$	linear damping in surge
$\mu$	control input in surge
$m_2$	added mass and inertia in roll
$d_2$	linear damping in roll
$B_i$	disturbance amplitude
$w_i$	velocity disturbance frequency

$\overline{GM}_a$	amplitude of change in meta-centric height
$\overline{GM}_m$	mean value of meta-centric height
$\rho$	density of water
$\nabla$	water displacement of the ship
$g$	gravitational acceleration

The roll dynamics for the ship in Table 1 is given by:

$$m_2 \ddot{\varphi} + d_2 \dot{\varphi} + k(1 + A \cos(\omega_e t)) \varphi = 0 \quad (2)$$

with  $A = \overline{GM}_a / \overline{GM}_m$  and  $k = \rho g \nabla \overline{GM}_m$ . This roll dynamics is a simplified version of the models in Fossen (2011).

The surge dynamics is given by:

$$m_1 \dot{u}(t) + d_1 u(t) = \mu + \sum_i B_i \cos(\omega_i t) \quad (3)$$

where  $\mu$  is the control input in surge and  $\sum_i B_i \cos(\omega_i t)$  is a disturbance due to a sinusoidal wave spectrum. The wave frequency is assumed to be constant and the wave encounter frequency can be obtained by (Fossen, 2011):

$$\omega_e(u(t), \omega_0, \beta_w) = \left| \omega_0 - \frac{\omega_0^2}{g} U(t) \cos(\beta_w) \right| \quad (4)$$

The assumption is made that the velocity in sway is much smaller than the velocity in surge such that the speed of the ship is given by  $U(t) = \sqrt{u^2(t) + v^2(t)} \approx u(t)$ . Moreover, it is assumed that the ship is sailing in head seas ( $\beta_w = 180^\circ$ ). Hence, (4) is reduced to:

$$\omega_e(u(t), \omega_0) = \left| \omega_0 + \frac{\omega_0^2}{g} u(t) \right| \quad (5)$$

Using (2)–(5) a state-space model for the system can be formulated as:



$$\dot{x} = f(x, \mu, t) = \begin{bmatrix} x_2 \\ \frac{-(d_2 x_2 + k(1 + A \cos(x_5 t))x_1)}{m_2} \\ \frac{\mu}{m_1} - \frac{d_1 x_3}{m_1} + \sum_i B_i \cos(\omega_i t) \\ 0 \\ x_4^2 \left( \frac{\mu}{m_1} - \frac{d_1 x_3}{m_1} + \sum_i B_i \cos(\omega_i t) \right) \\ g \end{bmatrix} \quad (6)$$

$$y = h(x) = [x_1, x_2, x_3]^T$$

### 3. DESIGN OF THE EKF

In this section an algorithm for a discrete-time EKF obtained from Simon (2006) will be presented. However, to implement the algorithm the model (6) needs to be discretized first. The discretization takes the form:

$$\begin{aligned} x_k &= F_{k-1} x_{k-1} + G_{k-1} \mu_{k-1} + w_{k-1} \\ y_k &= H_k x_k + v_k \end{aligned} \quad (7)$$

where

$$\begin{aligned} F_{k-1} &= \exp\left(\frac{\partial f}{\partial x} \Big|_{\hat{x}_{k-1}^+} T\right) \\ G_{k-1} &= \int_0^T \exp\left(\frac{\partial f}{\partial x} \Big|_{\hat{x}_{k-1}^+} \tau\right) d\tau \frac{\partial f}{\partial \mu} \\ H_k &= \frac{\partial y_k}{\partial x} \Big|_{\hat{x}_k^-} \end{aligned} \quad (8)$$

Here  $T$  is the sampling time chosen as 0,1 seconds and  $w_{k-1}$  and  $v_k$  are Gaussian white noise processes with zero mean and covariance matrices  $Q$  and  $R$ , respectively. The covariance matrix of the measurement noise  $R$  is chosen as a constant diagonal matrix with entries based on the variances of the sensors, that is,

$$R = \text{diag}(\sigma_\varphi^2, \sigma_\phi^2, \sigma_u^2)$$

The estimation of the process noise covariance matrix  $Q$  is more complicated. For the measured states it should represent the uncertainty in the model equations. Note that for equations that are modeled exactly, like the first equation of (6), process noise can be omitted or given a small positive value for numerical purposes. For the states to be estimated it should represent the covariance of the noise driving the state estimates. These driving noise terms for the wave frequency and wave encounter frequency should not be chosen too large or the filter might become too aggressive in its adjustments. Moreover the state estimates might converge to the wrong value if  $Q$  is too large. Especially the entry for the wave frequency should be chosen very small since estimation of the wave frequency is a parameter estimation problem. Making  $Q$  a constant diagonal matrix yields:

$$Q = \text{diag}(Q_\varphi, Q_\phi, Q_u, Q_{\omega_0}, Q_{\omega_s})$$

Now the EKF can be initialized by:

$$\begin{aligned} \hat{x}_0^+ &= E(x_0) \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned} \quad (9)$$

Here  $\hat{x}_0^+$  denotes the original estimate for the states and  $P_0^+$  is the initial state error covariance.

Then, at each discrete-time step of the Kalman filter the following has to be calculated. First the time update is done:

$$\begin{aligned} P_k^- &= F_{k-1} P_{k-1}^+ F_{k-1}^T + Q \\ \hat{x}_k^- &= f_{k-1}(\hat{x}_{k-1}^+, \mu_{k-1}, 0) \end{aligned} \quad (10)$$

with  $P_k^-$  the error covariance matrix of the error  $\hat{x}_{k-1}^+ - \hat{x}_k^-$ . In the time update the model is applied to the system to update the state estimate. The state error covariance is updated using the linearized system matrix form (8), the a posteriori state error covariance from the



previous time step and the covariance of the process noise.

The measurement update can then be done as in Simon (2006):

$$\begin{aligned} K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k (y_k - h_k(\hat{x}_k^-, 0)) \\ P_k^+ &= (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k R K_k^T \end{aligned} \quad (11)$$

where  $K_k$  is the Kalman gain,  $\hat{x}_k^+$  is the a posteriori state estimate after the  $k$ -th time step and  $P_k^+$  is the error covariance matrix of the error  $\hat{x}_k^+ - \hat{x}_k^-$ . The Kalman gain is calculated using the linearized measurements of (8) the a priori state error covariance and the covariance of the measurement noise. The estimate of the states is adapted by multiplying the error in the measured variables with the Kalman gain. Finally the a posteriori state error covariance is calculated using the Kalman gain and the covariance of the measurement noise.

As mentioned in Simon (2006), alternate formulations for  $K_k$  and  $P_k^+$  can be chosen. However the formulations chosen here are to guarantee that  $P_k^+$  is symmetric and positive definite as long as  $P_k^-$  is positive definite.

#### 4. FREQUENCY ESTIMATION

The EKF of Section 3 will now be applied to the ship model (6) to create an EKF for the ship model. The EKF will be analyzed in a simulation study. However, to perform the measurement update, the Kalman filter needs a measurement of the roll angle, roll rate, and the surge velocity. Hence, a model to generate a fictive measurement is created first. This model includes a second-order wave model and a velocity controller. The wave model is based on the second-order wave model found in Fossen (2011) and takes the form:

$$h(s) = \frac{2\lambda\omega_0\sigma s}{s^2 + 2\lambda\omega_0 s + \omega_0^2} \quad (12)$$

where  $\lambda$  is a damping coefficient and  $\sigma$  is a constant describing the wave intensity. Both of these parameters are determined by a linearization of the wave spectrum as in Fossen (2011). The velocity controller is designed such that it keeps the velocity at a pre-described reference value  $r$ . The desired velocity  $u_d$  is generated by the second-order reference model:

$$\ddot{u}_d + 2\xi\omega_p\dot{u}_d + \omega_p^2 u_d = \omega_p^2 r \quad (13)$$

with  $\omega_p = 0,1 \text{ rad/s}$  and relative damping  $\xi = 1$ .

Combining this with the surge model of (3) yields:

$$\begin{aligned} \dot{u} &= -\frac{d_1}{m_1} u + \frac{1}{m_1} \mu \\ \mu &= m_1 \left( \dot{u}_d + \frac{d_1}{m_1} u_d \right) \end{aligned} \quad (14)$$

The following acceleration profile is obtained:

$$\dot{u} = \dot{u}_d + \frac{d_1}{m_1} (u - u_d) \quad (15)$$

The wave disturbance and the measurement noise are added to the velocity after numerical integration to create a disturbed velocity profile. In addition to the wave model and the velocity controller the measurement model also has dynamical equations for the roll angle and roll rate. The EKF tracks these profiles.

For the unmeasured variables a confidence interval for the estimates is also given. This confidence interval is calculated using the diagonal elements of the state error covariance matrix resulting in:

$$\hat{x}_{99\%} = \hat{x} \pm 3\sqrt{P_{ii}}$$



A simulation study is now performed in which the EKF needs to estimate a wave frequency of 0,4684rad /s. In the simulation study the ship accelerates to a constant velocity of 5,66m /s. This velocity is chosen such that the wave encounter frequency is about twice the ship's natural roll frequency given the specified wave frequency. As a consequence, the ship is experiencing parametric rolling which is apparent from the roll angle and roll rate plots in Figure 1. The initial conditions for the simulation study are chosen as:

$$[\varphi \quad \dot{\varphi} \quad u \quad \omega_0 \quad \omega_e]^T = \left[ \frac{2\pi}{90} \quad \frac{\pi}{90} \quad 5 \quad \frac{1}{2} \quad 0.62 \right]^T$$

The simulation results for the measured variables can be seen in Figure 1. It can clearly be seen from Figure 1 that the estimation of the measured variables roll angle, roll rate, and surge velocity is trivial and they are estimated nearly perfect. For the velocity the EKF is tuned such that the filter does not follow all the disturbances on the velocity. This improves the estimate of the wave encounter frequency, which strongly depends on the velocity. Note that the maximum roll angle that is achieved in the simulation is approximately 0,5 rad which corresponds to about 29° and that the ship rolls with an angular speed of 0,15 rad/s which corresponds to about 9°/s. This is quite a heavy resonance. The resonance is lost after about 2400 seconds. The effect of this on the estimation of the unknown frequencies will be demonstrated later.

The results of the estimation of the wave frequency can be seen in Figure 2. Figure 2 has a plot of the wave frequency estimate including the confidence interval and a plot with a detail of the mean of the wave frequency estimate.

From Figure 2 it can be seen that the mean is estimated quite well until the resonance is lost; after this the estimate retains a bias. This is due to the fact that the estimation of the wave frequency is a parameter estimation process.

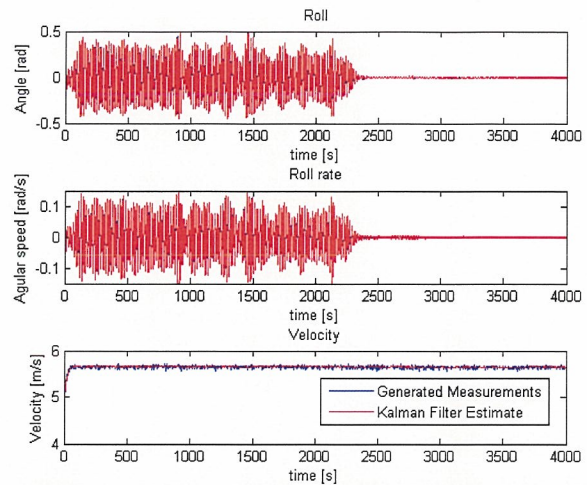


Figure 1: EKF frequency estimation of the measured states for the ship in resonance

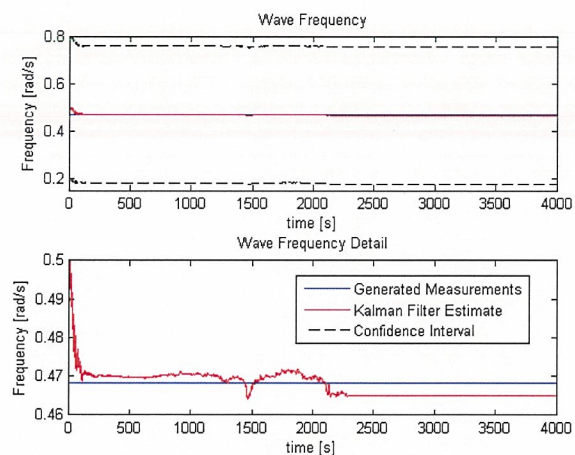


Figure 2: EKF frequency estimation of the wave frequency with detail of the mean estimate

This means that the estimate for the wave frequency is not adjusted in the time update (10) since  $\dot{\omega}_0 = 0$ . Hence, the state estimate is only adjusted by the measurement update (11). This makes the adjustment of the state estimate dependent on the size of the Kalman gain and the error in the measured variables. When the resonance is lost at about 2400 seconds, the error in the state estimate of the roll angle also reduces significantly and the wave frequency state estimate is hardly adjusted.

From Figure 2 it can also be seen that the confidence interval for the wave frequency estimate only converges very slowly. This can



also be attributed to the fact that the estimation of the wave frequency is a parameter estimation process. The adaptation of the state error covariance of the wave frequency in (10) is governed by the small process noise term driving the estimate of the wave frequency.

Figure 3 shows the results of the estimation of the wave encounter frequency. The top plot in Figure 3 shows the wave encounter frequency estimate and its confidence interval. The bottom plot in Figure 3 shows a detail of the estimate of the wave encounter frequency and its confidence interval.

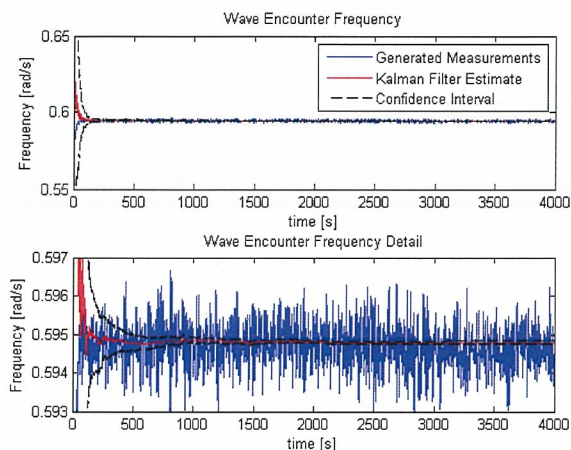


Figure 3: EKF frequency estimation of the wave encounter frequency with a detail of the mean estimate

From Figure 3 it can be seen that the estimate for the wave encounter frequency converges to the correct value quite fast. Moreover the confidence interval also converges to within acceptable bounds quite fast. Note here that just like with the estimation of the velocity the filter is tuned such that the estimate tracks the mean of the wave encounter frequency and does not try to track all the disturbances. This would cause a filter that is too aggressive and is more likely to diverge. A close inspection of the wave encounter frequency estimate shows that when the resonance is lost, the estimate of the wave encounter frequency is adjusted less and the confidence interval increases a little. This is because the noise terms become more dominant for small values of the states. When the noise is larger with respect to the states the

uncertainty in the state estimates becomes larger. Hence, the state error covariance increases.

Good initial performance of the filter is obtained by a suitable choice of the initial state error covariance. For the measured states this is not an issue, however for the unmeasured states it is. If the initial error covariance is chosen too small the filter will be sluggish and it will take longer for the estimate to converge or it might not converge at all. If the initial state error covariance is chosen too large the filter will react very aggressive in the beginning which can cause the state estimate to overshoot the true value and oscillate around it for a while. Both under tuning and over tuning of the initial state error covariance can cause the wave frequency to diverge. Hence, it is important to have a good initial estimate of the wave frequency when initializing the filter. Figure 4 shows the wave frequency and wave encounter frequency for a filter with large initial error covariance to illustrate the mentioned problems.

For convergence the choice of the process noise covariance is also important. Large values for the process noise covariance will allow the filter to track the noisy signals of the velocity in Figure 1 and the wave encounter frequency in Figure 3 more closely. However, large values for the process noise covariance will also cause the filter to be more sensitive to e.g. a loss of the resonance which can cause the state estimate of the wave frequency and wave encounter frequency to diverge and their confidence interval to increase. However the filter will always need some process noise to adjust the estimates.

## 5. CONCLUSIONS

An EKF used as a state observer has been designed to estimate the wave frequency and wave encounter frequency of a ship experiencing parametric roll. This observer is designed on the basis of a ship model.

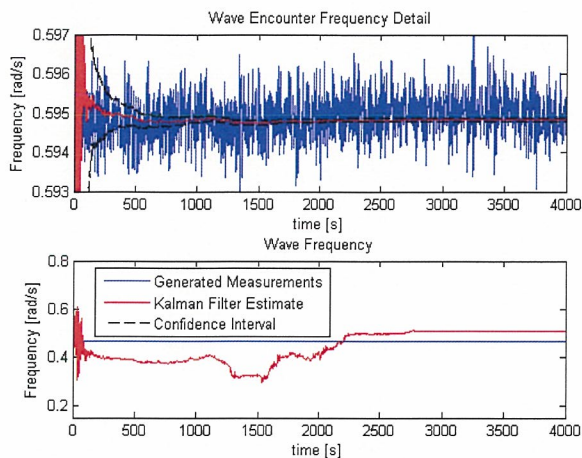


Figure 4: Wave encounter frequency and wave frequency estimate with large initial error covariance

The estimation of the wave frequency proved to be quite difficult due to the fact that the estimation of the wave frequency is a parameter estimation process. It has been shown that this parameter estimation process needs the parametric resonance to readily adjust the state estimate. Moreover the confidence interval only converges very slowly since it is only adapted by the small process noise term driving the filter. The estimate for the mean of the wave encounter frequency is good and converges fast. The confidence interval for the wave encounter frequency also converges to an acceptable value quite fast. It has to be noted though that estimation of the wave encounter frequency also becomes more difficult when the resonance is lost. The filter is sensitive to tuning of the initial state error covariance matrix and to the tuning of the process noise covariance matrix. This can cause a long time for convergence of especially the wave frequency. Hence, for good performance of the filter and convergence it is important to make a suitable choice for the process noise error covariance and the initial state error covariance.

## 6. REFERENCES

- Carmel, S. M., 2006. "Study of parametric rolling event on a panamax container vessel." Journal of the Transportation Research Board, vol. 1963, pp. 56–63.
- Fossen, T. I., 2011. Handbook of Marine Craft Hydrodynamics and Motion Control. Wiley.
- Fossen, T. I. and Nijmeijer, H. (eds.), 2012. Parametric Resonance in Dynamical Systems. Springer.
- France, W., Levadou, M., Treacle, T. W., Paulling, J. R., Michel, R. K., and Moore, C., 2003. "An investigation of head-sea parametric rolling and its influence on container lashing systems." Marine Technology, vol. 40, No.1, pp. 1–19.
- Holden, C., Breu, D. A., and Fossen, T. I., 2012. "Frequency Detuning Control by Doppler Shift." Fossen, T. I. and Nijmeijer, H. (eds.), Parametric Resonance in Dynamical Systems, Springer.
- Neves, M. A. S. and Rodríguez, C. A., 2005. "On unstable ship motions resulting from strong non-linear coupling." Ocean Engineering, vol. 33, pp. 1853–1883.
- Simon, D., 2006. Optimal state estimation. Wiley.