

On the application of electron cyclotron emission imaging to the validation of theoretical models of magnetohydrodynamic activity

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On the application of electron cyclotron emission imaging to the validation of theoretical models of magnetohydrodynamic activity^{a)}

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Two-dimensional (2D) imaging of electron temperature perturbations provides a powerful constraint for validating theoretical models describing magnetohydrodynamic plasma behavior. In observation of Alfvén wave induced temperature fluctuations, electron cyclotron emission imaging provides unambiguous determination of the 2D eigenmode structure. This has provided support for nonperturbative eigenmode solvers which predict symmetry breaking due to poloidal flows in the fast ion population. It is shown that for Alfvén eigenmodes, and in cases where convective flows or saturated perturbations lead to nonaxisymmetric equilibria, electron plasma displacements oriented parallel to a gradient in mean temperature are well defined. Furthermore, during highly dynamic behavior, such as the sawtooth crash, highly resolved 2D temperature behaviors yield valuable insight. In particular, addressing the role of adiabatic heating on time scales much shorter than the resistive diffusion time through the additional diagnosis of local electron density allows progress to be made toward a comprehensive understanding of fast reconnection in tokamak plasmas.

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I. INTRODUCTION

Modern electron cyclotron emission imaging (ECEI) systems with advanced capabilities and versatile highly flexible coupling optics¹ are now in operation on the ASDEX-U,² DIII-D,³ and KSTAR (Refs. 4 and 5) tokamaks. A new system is to be installed on the EAST tokamak in March 2011. In each case, ECEI provides powerful data that serve as a unique validation tool for theoretical models. In cases where effects such as adiabatic heating and flows along isothermal surfaces are not important, ECEI provides an unambiguous description of the plasma phenomenon and an opportunity to observe predicted behavior in an experimental setting. This is demonstrated in the example of imaging Alfvén eigenmodes on DIII-D.⁶ Many plasma phenomena, however, are not readily differentiated by a single component of the plasma displacement. This is true in the case of convective instabilities which arise during the ramp phase of the sawtooth oscillation. Though two distinct magnetohydrodynamic (MHD) modes, oscillating at different frequencies, arise during very different plasma conditions, their temperature perturbations

cannot be differentiated by ECEI. Only the plasma displacement perpendicular to a magnetic flux surface produces a temperature perturbation, and so ECEI does not provide direct observation of the poloidal flow. The result is that both modes satisfy a necessary, but not sufficient, condition for being produced by quasi-interchange instability in that they exhibit smoothly varying radial displacements.

This paper concludes with a closer look at the sawtooth crash and the identification of an important ambiguity which remains central to a discussion of magnetic reconnection in tokamak plasmas. Though collective heat flow corresponding to a localized penetration of the inversion radius has now been observed on TEXTOR,⁷⁻⁹ ASDEX-U, DIII-D, and KSTAR,¹⁰ the topology of magnetic reconnection during the sawtooth crash has not been conclusively resolved. Due to the structure of the evolving precursor mode and the dynamic nature of the activity leading up to the crash, uncertainties remain in relating observed temperature behavior to particle and energy transport. The role of possible localized increases in kinetic pressure is yet to be adequately explored, motivating the continued development of diagnostic tools for localized density measurement, with capabilities analogous to those demonstrated by ECEI.

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II. VALIDATION OF NONPERTURBATIVE ALFVÉN EIGENMODE MODELING

Both toroidicity induced (TAEs) and reversed-shear induced Alfvén eigenmodes (RSAEs) are observed in the current ramp phase of neutral beam heated plasmas on DIII-D,¹¹ thus ECEI is well suited to imaging the temperature perturbation resulting from these eigenmodes as it provides a localized electron temperature measurement, unambiguously identifying the 2D structure of the temperature perturbation. Furthermore, ECEI readily images core plasma fluctuations without the inherent detractors of chord averaging or inference from measurement made at the edge. This is particularly important for the imaging of RSAEs which are constituted by a single dominant m/n mode and therefore confined to a region of low shear near a minimum in the safety factor, or q , profile.

In general, the equation of state which determines the electron temperature fluctuation, δT_e , in terms of a plasma displacement, ξ , may be written as¹²

$$\frac{\delta T_e}{\langle T_e \rangle} = -\xi \cdot \frac{\nabla \langle T_e \rangle}{\langle T_e \rangle} - (\gamma - 1) \nabla \cdot \xi, \quad (1)$$

where γ is the adiabatic constant and bracketed quantities are mean values obtained by ensemble averaging. The mode frequency of TAEs and RSAEs on DIII-D is small compared to the electron transit frequency, and so kinetic theory dictates that the electron temperature is isothermal in the direction of the magnetic field.¹³ Furthermore, because the modes of interest are transverse wave eigenmodes, there is no net displacement along any closed field line. Therefore, the right hand term of Eq. (1), which represents the contribution of adiabatic heating, may be neglected. Reduction of Eq. (1) to a purely convective temperature perturbation allows for a unique solution of the radial plasma displacement, ξ_ψ , defined in terms of the toroidal flux function, γ , by $\xi_\psi \equiv \xi \cdot \nabla \psi / |\nabla \psi|$.

Providing a definitive description of the radial displacement, ECEI data may be directly compared with theoretical models which are elucidated by codes such as NOVA (Refs. 14 and 15) and TAEFL.^{16,17} In the ideal MHD code NOVA, the eigenmode structure is determined by the thermal plasma equilibrium. The perturbed toroidal flux function is defined as a set of even harmonic components such that $\psi = \sum_{m,n} \times \psi_{mn} \cos(m\theta - n\zeta)$. The poloidal and toroidal mode numbers are identified by m and n , respectively, while the poloidal and toroidal angles are defined by θ and ζ . This description adequately reproduces important features, such as the mode frequency and radial amplitude envelope. However, deviations from the symmetric mode predicted by NOVA are observed by ECEI. The nonperturbative code TAEFL allows symmetry breaking due to the influence of fast ion kinetic effects. TAEFL is a hybrid gyrofluid-MHD code which includes a description of the fast ion population through a closed set of gyrofluid moment equations. These equations couple to both even and odd terms of the perturbed flux function $\psi = \sum_{m,n} [\psi_{mn,c} \cos(m\theta - n\zeta) + \psi_{mn,s} \sin(m\theta - n\zeta)]$.

As shown in Fig. 1, ECEI confirms the predictions of the TAEFL model and serves as a tool for further validation.⁶ The inclusion of acoustic modes by way of coupling to the electron density continuity equation provides excellent

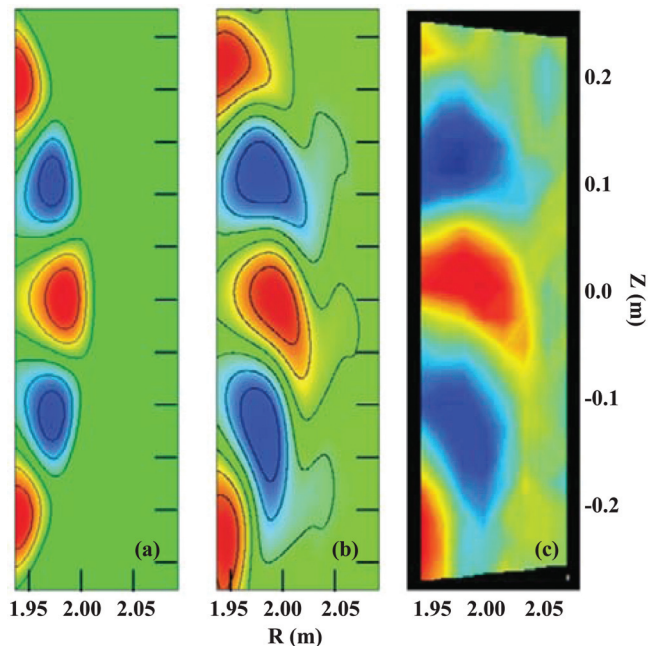


FIG. 1. (Color) ECEI measures subtle features of Alfvén eigenmode structure predicted by nonperturbative modeling. In (a), the symmetric RSAE structure predicted by ideal MHD is shown. TAEFL demonstrates that fast ion contributions may alter the eigenmode structure, inducing symmetry breaking as shown in (b). Experimental measurement by ECEI (c) confirms these predictions providing an opportunity for model validation.

agreement in mode frequency while having a minimal effect on the eigenmode structure. An elimination of $E \times B$ flows in the simulation is also found to be of little consequence. However, distortions of the 2D eigenmode are sensitive to the fast ion kinetic pressure profile. In addition to implications for ion transport which arise from a more refined description of the eigenmode structure—which includes radial phase variation—the observed sensitivity to the fast ion kinetic pressure profile suggests that further investigation may lead to a capability for diagnosing some aspects of the fast ion distribution.

III. CONVECTIVE INSTABILITIES OBSERVED DURING THE SAWTOOTH RAMP

Elongated neutral beam heated plasmas on DIII-D exhibit long-lived precursors and other oscillating modes which are evident in the MHD spectrum obtained by ECEI. A dominant mode of interest is the $m/n = 1/1$ precursor mode which is observed as an oscillation at the plasma rotation frequency. This mode exhibits a saturated temperature perturbation of approximately 5% and persists up to the crash time. Increasing plasma triangularity has been shown to suppress the precursor $m/n = 1/1$ mode for “bean” shaped plasmas,¹⁸ evidence that this precursor is of a quasi-interchange nature.

The $m/n = 1/1$ precursor mode is analyzed by means of reconstructing the complete poloidal mode structure under an assumption of rigid-body rotation. A subset of ECEI data is collected near the midplane at evenly spaced time intervals during a single period of mode rotation. Each consecutive frame is then mapped to a consecutive angular position in the poloidal plasma cross-section in order to visualize in a

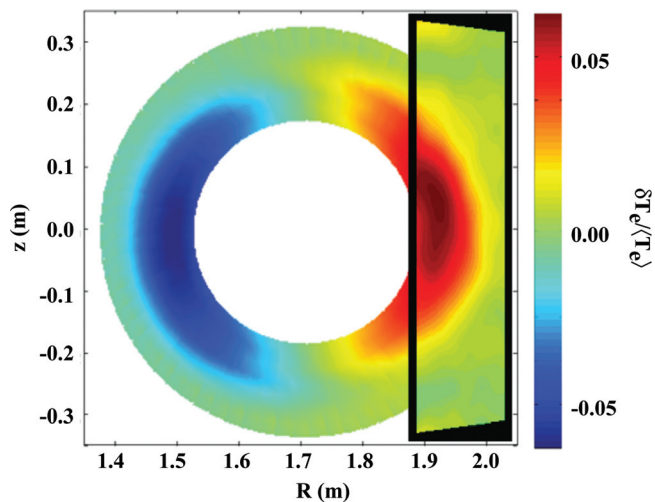


FIG. 2. (Color) The $m/n = 1/1$ temperature perturbation produced by the saturated displacement of the sawtooth precursor is shown. The complete instantaneous view of the ECEI diagnostic is shown at right overlaid on a reconstruction of the poloidal mode structure. Reconstruction is performed by mapping data collected near the midplane at consecutive time intervals to increasing poloidal angle under an assumption of rigid-body rotation.

single image the behavior of the mode beyond the instantaneous field of view. The fundamental oscillation identified in the autocorrelated ECEI power spectrum is isolated by means of digital frequency filtering. Several harmonic frequencies are included so as not to arbitrarily constrain the shaping of the mode. The perturbed quantity $\delta T_e / \langle T_e \rangle$ is described by symmetric hot and cold temperature islands, peaking off axis and diametrically opposed in the poloidal plane. This mode structure is shown in Fig. 2.

The MHD oscillations analyzed here are the result of saturated displacements. It is therefore reasonable to presume that they result in a nonaxisymmetric equilibrium for which kinetic pressure and temperature are conserved on flux surfaces. This provides a justification, different from that invoked for Alfvén eigenmodes, for neglecting adiabatic heating. Evaluation of Eq. (1) then results in a smoothly varying radial displacement which is a necessary, but not sufficient, condition of the perturbation resulting from a quasi-interchange instability.¹⁹

The quasi-interchange model predicts the formation of a Rayleigh–Taylor type convection cell.²⁰ Poloidal electric fields in a region of low magnetic shear produce an outward flow which displaces the hot core plasma toward a stationary rational flux surface with magnetic winding number $q = 1$. Colder plasma is drawn into the core, and the flows are connected by a broad region of return flow centered on the $q = 1$ surface. It may be said with certainty that the radial displacement producing the observed temperature perturbation is approximately 8 cm on axis, decreasing monotonically with radius. It may further be inferred that the lines of convection in the core region are parallel, as implied by the $\cos \theta$ dependence of ξ_ψ . However, poloidal flow is directed along isothermals and therefore produces no temperature perturbation. Only radial flow is imaged, and therefore ambiguity remains.

During early phases of the sawtooth ramp, particularly when the temperature profile remains flattened throughout

the core for an extended portion of the sawtooth period, a down-shifted rotating mode is also observed. The frequency shift of this mode relative to the plasma rotation frequency is comparable to the electron drift-diamagnetic frequency and of the appropriate sign. It is observed in both “oval” and bean shaped plasmas¹⁸ and its nature remains unresolved. However, the associated temperature fluctuation imaged by ECEI is indistinguishable from that of the precursor $1/1$ oscillation. This encourages caution and illustrates that a quasi-interchange instability does not produce a unique temperature perturbation eigenmode.

IV. LOCALIZED RECONNECTION AND THE SAWTOOTH CRASH

Collective heat flow through a narrow region of the inversion radius, indicating the formation of an X-point and localized magnetic reconnection,⁹ has now been observed in circular and oval shaped elongated plasmas on TEXTOR, ASDEX-U, DIII-D, and KSTAR. This observation has defied the assertion that global stochasticization of the magnetic field acts as a trigger for the sawtooth crash. However, the topology of the reconnecting field lines and the mechanisms, which govern the characteristic time scale of the process, remain unresolved.

Excellent images of the sawtooth crash are obtained by ECEI when reconnection occurs within the view of the diagnostic. Figure 3 shows a sequence of images from a sawtooth crash on DIII-D. They capture the formation of an opening at the inversion radius, through which the heat from the core plasma is expelled. If one may assume that the observed temperature perturbations are due to divergence free convective flows, then determination of the reconnected topology is relatively straightforward.²¹ However, the possibility of adiabatic processes contributing to the observed temperature

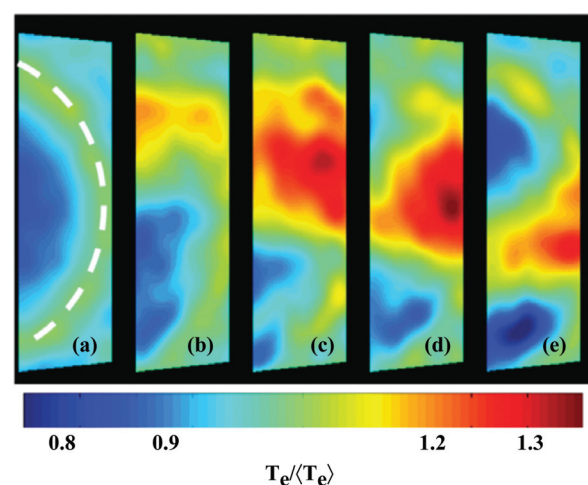


FIG. 3. (Color) A localized reconnection resulting in a sawtooth crash is imaged by ECEI. In (a), an accumulation of heat in the mixing zone has resulted in an annulus of elevated temperature before the crash begins. The estimated position of the $q = 1$ surface is indicated by the dashed white line. In (b), penetration of the inversion radius begins with the formation of an apparent “pressure finger.” In (c) and (d), the opening widens, with heat being expelled. As the opening closes in (e), complete ejection of heat from the core results in a flattening of the temperature profile.

perturbation during this dynamic behavior makes such a simple interpretation unlikely to be accurate.

During the final milliseconds of precursor activity preceding the sawtooth crash, there is a fast evolution of the precursor mode structure. The peak of the mode migrates outward toward the inversion radius. Simultaneously, the mode becomes more concentrated, appearing to contract both poloidally and toroidally until reconnection begins with a characteristic scale at least as small as the spatial resolution of the diagnostic. The implied confluence of plasma trajectories would suggest a finite convergence in the displacement during this time. Though the isothermal property of the magnetic field line allowed adiabatic compression to be neglected for Alfvén eigenmodes described in Sec. II, the sawtooth precursor remains a predominantly 1/1 mode occurring in the vicinity of the $q = 1$ surface. Therefore, compression perpendicular to a magnetic field line represents the compression of a complete flux tube, and the relevant component of the anisotropic adiabatic tensor is the effective perpendicular element, γ_{\perp} . Determining the contribution of adiabatic heating has enormous relevance to imaging the sawtooth crash as it allows one to determine whether the observed heat flow is representative of a plasmoid ejection¹⁰ or rather a heat pulse which propagates independent of flow in the background plasma medium.

Determining the contribution of adiabatic compression to temperature perturbation may require simultaneous measurement of the local electron density perturbation, described by the continuity equation

$$\frac{\delta n_e}{\langle n_e \rangle} = -\xi \cdot \frac{\nabla \langle n_e \rangle}{\langle n_e \rangle} - \nabla \cdot \xi. \quad (2)$$

For typical plasma parameters and time averaged profiles observed before the sawtooth crash, a hypothetical plasma displacement similar to that described in Sec. III and consistent with the formation of a convective cell produces a temperature perturbation of 5% and a corresponding density perturbation of 1%. The proportionality of these perturbations is determined by the difference in local profile gradient. If poloidal flow is constrained, however, a gradient in the radial displacement produces a region of enhanced local kinetic pressure. When the adiabatic constant is chosen such that the contributions of compression and adiabatic heating dominate the observed density and temperature perturbations, the local ratio of perturbations is determined by the value of the adiabatic constant chosen. Therefore, a dominance of either convection or adiabatic heating is easily distinguished by the local ratio of perturbed quantities.

It is important to note that the line integrated density fluctuation produced by a significant compression of the plasma would not differ from a convective perturbation in cases where the region of compression is small, as is suggested by ECEI measurements. Therefore, diagnostics which rely on chord averaged measurements would be unable to provide any distinction. The ideal complementary density diagnostic for resolving the dynamics of the sawtooth crash would have capabilities comparable to ECEI and provide a 2D localized measurement. One such diagnostic technique

which has been proposed is microwave imaging reflectometry (MIR).^{22,23} The development of MIR systems for DIII-D and KSTAR (Ref. 24) is currently underway. Multiple probing frequencies are planned, allowing multiple cut-off layers at evenly spaced radial positions to be imaged. Advancements in quasi-optical imaging developed for ECEI systems find direct application in MIR, where a similar arrangement of detector arrays will require designs with comparable versatility.

V. CONCLUSION

ECEI is a versatile and uniquely capable diagnostic technique which has provided immediate contributions to plasma physics. In the case of imaging Alfvén eigenmodes, ECEI is particularly well suited as a tool for the validation of theoretical modeling. Temperature fluctuations associated with these modes provide an unambiguous determination of the eigenmode structure. A comparison to the predictions of ideal MHD models provides a method of diagnosing both plasma shape and the evolving q profile. Furthermore, subtle features such as radial phase variation have implications for ion transport and are essential to the exploration of connections between the eigenmode structure and the phase space distribution of fast ions.

Wherever adiabatic compression may be neglected, periodic temperature perturbations are uniquely related to plasma displacements. However, only displacements parallel to a gradient in the mean temperature produce a perturbation, while poloidal displacements generally produce no temperature perturbation and must be inferred by other means. In the case of saturated convective displacements which give rise to long-lived oscillations during sawtooth activity, the observation of a smoothly varying radial displacement is a necessary, but not sufficient, condition for describing the displacement as the result of a quasi-interchange instability. The observation of multiple modes which exhibit indistinguishable radial displacements, however, underscores the need for other data which may conclusively identify the formation of a convective cell.

In the final moments leading to the sawtooth crash and inversion of the temperature profile, precursor temperature perturbations become increasingly localized. One interpretation of this evolution is a peaking of the poloidal distribution of ξ_{ψ} , from a $\cos \theta$ dependence to one of $\cos^2 \theta$ and higher order terms. This suggests an inhibition, or even reversal, of poloidal return flow. Mode structure precludes neglecting adiabatic heating without knowledge of the effective adiabatic constant observed perpendicular to a field line, motivating further exploration of local kinetic pressure dynamics with the goal of determining absolutely the underlying plasma displacement. The development of imaging diagnostics for simultaneous, localized density measurement with capabilities analogous to ECEI promises to bring the experimentalist closer than ever before to an unambiguous diagnosis of the magnetic topology during fast reconnection in tokamak plasmas.

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