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by

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Abstract

The paper reports the development and application of a SPH (smoothed particle hydrodynamics) based simulation of rapid filling of pipelines, for which the rigid-column model is commonly used. In this paper the water-hammer equations with a moving boundary are used to model the pipe filling process, and a mesh-less Lagrangian particle approach is employed to solve the governing equations. To assign boundary conditions with time-dependent (upstream) and constant (downstream) pressure, the SPH pressure boundary concept proposed recently in literature is used and extended. Except for imposing boundary conditions, this concept also ensures completeness of the kernels associated with particles close to the boundaries. As a consequence, the boundary deficiency problem encountered in conventional SPH is remedied. The employed particle method with the SPH pressure boundary concept aims to predict the transients occurring during rapid pipe filling. It is validated against laboratory tests, rigid-column solutions and numerical results from literature. Results obtained with the present approach show better agreement with the test data than those from rigid-column theory and the elastic model solved by the box scheme. It is concluded that SPH is a promising tool for the simulation of rapid filling of pipelines with undulating elevation profiles.

Keywords: Rapid filling of pipelines; Undulating elevation profile; SPH

1. Introduction

Fluid transients in liquid-conveying pipelines involve large pressure variations, which may cause considerable damage. Water hammer is probably the best known and extensively studied phenomenon in this respect [1]. Rapid filling of an empty pipeline with undulating elevation profile may occur under gravity and by pumping. While the water column is driven by a high head, air is expelled by the advancing water column. If the generated air flow is not seriously blocked by valves, the water column

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grows with little adverse pressure and may attain a high velocity. When the advancing column impacts a sharp bend or a partially closed valve, severe water-hammer pressures occur [2, 3, 4]. Also, water column separation may occur at high elevation points of the pipeline. It changes the hydraulics significantly and may cause pressure surges more harmful than the initial water hammer when the separated columns rejoin [5, 6]. Therefore, better understanding of the rapid filling process is of high importance. A reliable model that can predict the magnitude of the water column velocities, the possible occurrence of column separation and the induced overpressure in the system is highly desirable.

For the 1D modelling of the rapid filling of pipelines, the rigid-column model [5] based on a set of ODEs is commonly used. It gives reasonable results as long as the flow remains axially uniform. When the water column is disturbed somewhere in the system, pressure oscillations along its length or even column separation may occur and the rigid-column model will fail. The elastic model based on a set of PDEs for unsteady flow in conduits [1] is capable of dealing with potential fast transients in rapid pipe filling. However, the elastic model with a moving boundary is difficult to solve using traditional mesh-based methods. A recent attempt is the fully implicit box or Preismann finite-difference scheme, employed by Malekpour and Karney [7]. This method uses a fixed spatial grid and a flexible temporal grid, where the Courant number is time dependent. The obtained results gave acceptable agreement with the laboratory tests by Liou and Hunt [5]. However, a serious and unsolved numerical convergence problem occurred due to an uncontrollable large Courant number.

In this paper, the SPH particle method is employed to solve the full elastic model with a moving boundary. The SPH computations are compared with laboratory measurements, rigid-column theory and numerical results of the box scheme. Good agreements are obtained, especially in the deceleration phase of the filling process, where the SPH results completely coincide with the laboratory tests. The present Lagrangian particle model, which takes the moving boundary into account in a natural way, is a promising tool for slow, intermediate and fast transients in the pipe filling process.

2. Governing equations

Consider a pipeline equipped with a valve, with upstream a reservoir and downstream open to air as sketched in Fig. 1. Two pipe segments with different slopes represent a simple undulating elevation profile. The valve is located at a distance L_0 from the inlet. After the valve is opened, the water will advance into the pipe. At the early phase of the filling, the driving reservoir pressure dominates and induces a high acceleration up to a maximum velocity. With its length and velocity increasing, inertia and skin friction decelerate the water column. Sometime after the water column arrived at the end of the pipeline, a steady flow will develop.

The following assumptions are made:

- The pipe segment that has been filled remains full and a well-defined front exists. This assumption allows for a one-dimensional model to be used.
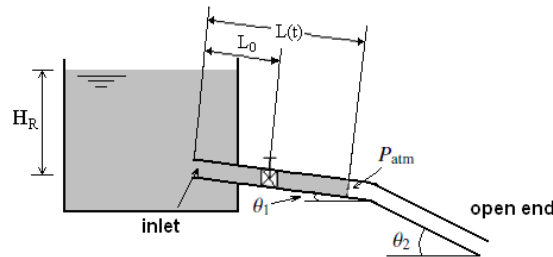

Fig. 1. Definition sketch of filling of a pipeline with undulating elevation profile.

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- The air in the empty pipe can flow out with negligible resistance, and consequently it has no effect on the motion of the water column.
- The Darcy-Weisbach friction law developed for steady pipe flow can be used. This is a reasonable assumption for turbulent pipe flows.
- The resistance of the open valve is negligible.
- The compressibility is taken into account through the wave speed, while the density remains constant.

The transient flow in a pipe is governed by the following 1D continuity and momentum equations [1]:

$$\frac{dP}{dt} = -\rho c^2 \frac{\partial V}{\partial x}, \tag{1}$$

$$\frac{dV}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g\sin\theta - \frac{\lambda V |V|}{2D}, \tag{2}$$

in Lagrangian form with

$$\frac{dx}{dt} = V, \tag{3}$$

where P = pressure, V = velocity, ρ = water density, c = speed of sound, g = gravitational acceleration, θ = pipe inclination angle, λ = friction factor, D = pipe diameter, x and t denote spatial coordinate and time, respectively, and d/dt is the material derivative. The initial conditions are

$$V(x,0) = 0 \text{ and } P(x,0) = \rho g(H_R + x\sin\theta) \ \ (0 < x \le L_0), \tag{4}$$

where L_0 is initial water column length. The upstream and downstream head is taken respectively

$$P(0,t) = \rho g(H_R - \frac{V^2}{2g} - K\frac{V^2}{2g}) \text{ and } P(L(t),t) = 0. \tag{5}$$

in which K = the entrance loss coefficient and $L(t)$ = the water column length. The velocity head and entrance head losses have been included in the upstream boundary condition.

3. SPH method

In SPH the spatial derivative of a function f is approximated by

$$\frac{\partial f_a}{\partial x} = \frac{\partial f}{\partial x}(x_a) = \sum_b \frac{m_b}{\rho_b}(f_a - f_b)\frac{dW_{ab}}{dx_a} \tag{6}$$

with

$$\frac{dW_{ab}}{dx_a} = \frac{\text{sign}(x_a - x_b)}{h^2}\begin{cases} -2q + 1.5q^2, & 0 \le q < 1, \\ -0.5(2-q)^2, & 1 \le q < 2, \\ 0, & q > 2, \end{cases} \tag{7}$$

where the subscripts a and b are the indices of the particles, m_b and ρ_b are the mass and density of particle b, $W(x$-$x_b, h)$ is the kernel function with h the smoothing length, and $q=r_{ab}/h$ with $r_{ab}=|x_a$-$x_b|$ the distance between the particles. The kernel used herein is the cubic spline function; see [8] for details about SPH.

Replacing the spatial derivatives in Eqs. (1) and (2) with the approximation (6), one obtains the discrete SPH formulation given by the ODEs

$$\frac{dP_a}{dt} = -c^2 \sum_b m_b(V_a - V_b)\frac{dW_{ab}}{dx_a}, \tag{8}$$

$$\frac{dV_a}{dt} = -\frac{1}{\rho^2}\sum_b m_b(P_a - P_b + \Pi_{ab})\frac{dW_{ab}}{dx_a} + g\sin\theta_a - \frac{\lambda V_a |V_a|}{2D}, \tag{9}$$

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The fact that the density is nearly constant has been used in the derivation of Eqs. (8) and (9). To alleviate possible oscillations at sharp wave fronts, an artificial viscosity term Π_{ab} has been added to the momentum equation. It has the following form

$$\Pi_{ab} = \frac{-c\,h}{\rho}\,\min\!\left(\frac{(V_a - V_b)(x_a - x_b)}{r_{ab}^2 + 0.01h^2};\ 0\right). \tag{10}$$

The rate of change of particle position is

$$\frac{dx_a}{dt} = V_a. \tag{11}$$

4. Boundary conditions

To impose the boundary conditions given by Eq. (5), the novel SPH pressure boundary concept proposed by Kruisbrink *et al.* [9] is employed and extended. Assume that at time t fluid particle r (reservoir) is the one closest to the reservoir and that its velocity is V_r (previous time step or initial value) (see Fig. 2a). To apply the upstream boundary pressure, a set of particles with spacing Δx (Δx is the initial fluid particle spacing) is placed in the reservoir. Their velocity is V_r and their pressure is $P_{in} = \rho g[H_R - (1+K)V_r^2/(2g)]$. The number of pressure inlet particles, N_{pip}, depends on the smoothing length h, as the kernel associated with particle r needs to be fully supported. Since the radius of the kernel is $2h$, to meet the above requirement an integer $N_{pip} > 2h/\Delta x$ must be taken. When a pressure inlet particle enters the pipe, it becomes a fluid particle and a new pressure particle is generated in the reservoir. The pressure boundary condition at the moving water front can be imposed in the same way. Suppose that at time t particle f (front) is located at the water front and its velocity is V_f (see Fig. 2b). A set of pressure particles is placed downstream of particle f. The pressure of these particles is zero, and their velocity is set equal to V_f. The number of pressure outlet particles, N_{pop}, should be an integer larger than $2h/\Delta x$ too. With the defined pressure inlet and outlet particles, all fluid particles are fully and properly supported.

5. Numerical results and conclusion

The SPH method is applied to Liou and Hunt's [5] experiments. One is referred to [5] for the details of the test rig, which comprises a 6.66 m long pipe of 22.9 mm inner diameter. The calibrated steady friction factor is 0.0245 and the entrance-loss coefficient is 0.8. A realistic speed of sound $c = 1000$ m/s is used in SPH, and there are about 650 particles when the pipe is full. Figure 3 compares the predicted velocities against water column length (measured from inlet) with the measurement of Liou and Hunt [5], their rigid-column results and the solution of Malekpour and Karney [7]. Among the results from the different models and methods, the SPH solution agrees the best with the measurement, although the maximum velocity is not fully reached. The solution of the box scheme [7] matches the late phase of the filling process well, but under-predicts the velocity in the early phase. The rigid-column results are presented in three different curves labelled as 0, 10D and 20D, where 10D and 20D represent the length of a virtual
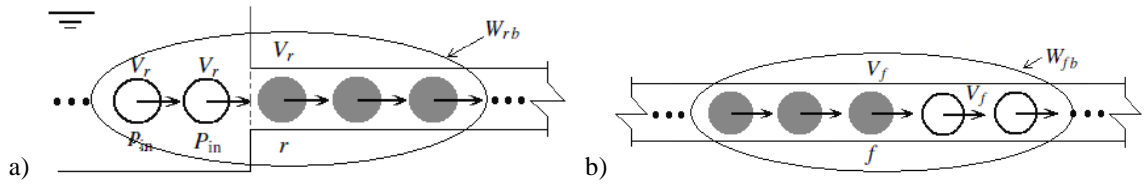

Fig. 2. Illustration of pressure particles for (a) upstream inlet condition and (b) downstream moving water front.
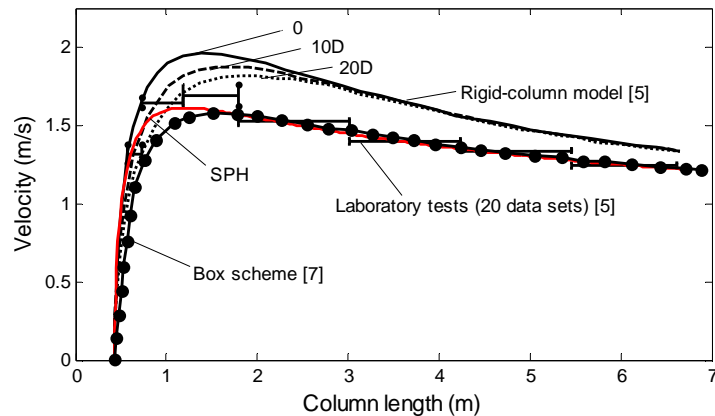
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Fig. 3. Velocity vs. column length for laboratory tests [5], rigid-column model [5], and elastic model solved by the box scheme [7] and the present SPH method.

pipe segment ahead of the inlet [5]. In fact, a better solution can be obtained without adding any virtual pipe if the velocity head were included in the upstream boundary condition used by Liou and Hunt [5]. This has also been demonstrated in [7, 10, 11].

SPH seems to be a viable method for simulating pipe filling processes. Although it has been applied herein to a relatively slow filling process, waterhammer – due to possible column impact – has been taken into account in the formulation.

Acknowledgements

References

[1] Wylie EB, Streeter VL, Suo LS. Fluid Transients in Systems. Prentice-Hall, Englewood Cliffs, 1993.

[2] Guo Q, Song CSS. Surging in urban storm drainage systems. J. Hydraul. Engrg., 116, 12 (1990): 1523–37.

[3] Zhou F, Hicks FE, Steffler PM. Transient flow in a rapidly filling horizontal pipe containing trapped air. J. Hydraul. Engrg., 128, 6 (2002): 625–34.

[4] De Martino G, Fontana N, Giugni M. Transient flow caused by air expulsion through an orifice. J. Hydraul. Engrg., 134, 9 (2008): 1395–99.

[5] Liou CP, Hunt WA. Filling of pipelines with undulating elevation profiles. J. Hydraul. Engrg., 122, 10 (1996): 534–39.

[6] Bergant A, Simpson AR, Tijsseling AS. Water hammer with column separation: A historical review. J. Fluid. Struct., 22 (2006), 135–71.

[7] Malekpour A, Karney BW. Rapidly filling analysis of pipelines using an elastic model. 10th Int. Conf. on Pressure Surges, Edinburgh, UK, 2008, 539–52.

[8] Liu GR, Liu MB. Smoothed Particle Hydrodynamics: A Meshfree Particle Method. World Scientific, Singapore, 2003.

[9] Kruisbrink ACH, Pearce F, Yue T, Cliffe A, Morvan H. SPH concepts for continuous wall and pressure boundaries. The 6th Int. SPHERIC Workshop, Hamburg, Germany, 2011.

[10] Malekpour A, Karney BW. Rapid filling analysis of pipelines with undulating profiles by the Method of Characteristics. ISRN Applied Mathematics (2011), Article ID 930460.

[11] Razak T, Karney BW. Filling of branched pipelines with undulating elevation profiles. 10th Int. Conf. on Pressure Surges, Edinburgh, UK, 2008, 473–87.

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