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Introduction

For studying mixing phenomena it's useful to have the possibility of tracking deforming fluid volumes. This can be done efficiently using an adaptive front tracking. Fig. 1 shows the deformation of a blob in a 2D time-periodic cavity flow [1], produced by successive motion of top and bottom wall.

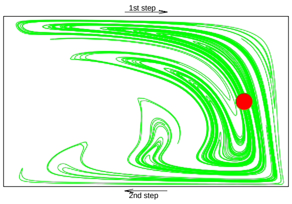


Fig. 1 *Result of the blob advection after 5 periods (initial configuration is shown in red)*

However, when studying long term mixing behaviour, the blob advection method is useless because of an exponential increasing number of needed points. Therefore, a complementary technique is required, that provides prediction of the mixed state for long term mixing.

Mapping approach for periodic flows

For periodic flows a possible solution involves the usage of discretized representation (*coarse grain density* [2]) of the flow domain, which is subdivided into a large number of cells and the average concentration c_i^0 of the marked fluid in each cell is determined (for an example see fig. 2).

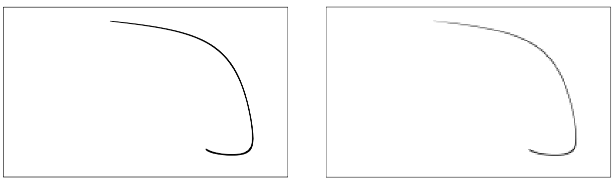


Fig. 2 *Blob, tracked for 1 period (left) and its coarse grain density representation using the mesh* 200×120 *(right)*

For each cell the evolution of the fluid volume during one period of the flow is computed. As a result an essentially sparse matrix A is built with A_{ij} being the relative amount of the fluid, occupying initially cell i that ends up in a cell j after 1 period. The concentration after 1 period is then computed as $c_i^1 = \sum_j A_{ji} c_j^0$. Applying the same mapping n times, the concentration distribution after n periods can be computed.
In fig. 3 the discretized shape of the blob, explicitly tracked for 5 periods is compared with the result of mapping from 1 period to 5 periods.

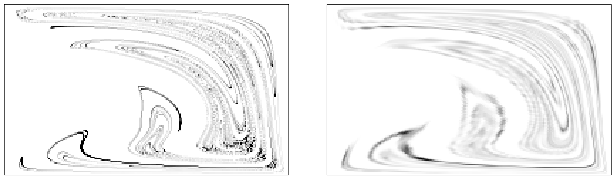


Fig. 3 *Blob after 5 periods: explicitly tracked and discretized (left) and determined via mapping (right)*

The mapping correctly predicts the overall structure, position and direction of the dye striations. The drawback: artificial "diffusion" is introduced. This results in a rather poor reversibility of mapping even if the flow itself is reversible (compare the images in fig. 4).

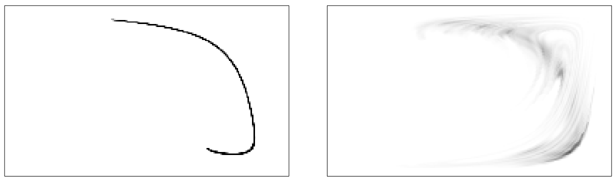
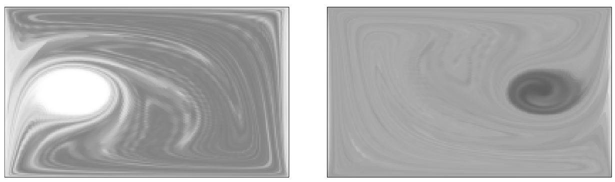


Fig. 4 *Initial distribution (left) and the result of successive mapping 2 periods forward and next 2 periods backward (reversed protocol of motion)*

The advantage of mapping is the tremendous speed-up of the computations: tedious computations to build the matrix A are performed only once for a given flow. This gives a possibility to predict mixing patterns for much longer times than accessible with direct tracking. The most important is that it allows to quickly find the zones of poor mixing (fig 5a) or even to evaluate the way to eliminate them (fig 5b).



(a) **(b)**

Fig. 5 ***a)*** *Prediction of mixing pattern after 20 periods;* ***b)*** *the same but with the wall motion sequence altered after 10 periods*

Future work:

The mapping technique with a coarse grain density description will be applied to:

- transient time-periodic flows
- space-periodic flows (static mixers)

References:

[1] MELESHKO, V.V. AND PETERS, G.W.M.: *Phys. Let. A* **216**.
[2] TUCKER, III, C.L.: in Mixing in Polymer Processing, ed. Rauwendaal, C., Marcel Dekker Inc., N.Y., 1991.