Coordinating supply chains : a bilevel programming approach

Coordinating Supply Chains: a Bilevel Programming Approach

Ton G. de Kok, Gabriella Muratore

IEIS, Technische Universiteit, 5600 MB Eindhoven, The Netherlands

April 2009

Abstract

In this paper we formulate the coordination and optimization problem of an extended supply chain as a bilevel program. As of today no such approach exists in relevant literature. In our opinion this model better represents the dynamics of today supply chains, comprised of separately owned entities each striving to maximize their own profit and still in need of collaborating and trading to supply the market. Bilevel programs are NP hard even in the linear case. We provide an efficient heuristic to find good solutions.

Keywords:

1 Introduction

In this paper we address the problem of coordinating and optimizing an extend supply chain, i.e., a system made up of suppliers, manufacturers, distributors, carriers and retailers who collaborate and trade in order to reach the common objective of supplying products and services to the final customers. We would like to find a supply chain plan which satisfies a multi item stochastic demand over a time horizon T. The plan will specify the stream of orders to each element in the supply chain, i.e., the induced demand over the appropriate horizon for each element in the supply chain. The plan will also specify the inventory and backorders levels, over the time horizon, for each entity in the supply chain. Finally, for each entity, the plan specifies a detailed schedule of releases, which take into account resources capacities, processing lead times and local constraints.

Before proceeding further let us analyze which are the benefits of such a plan. First of all, depending on the final demand pattern, induced demand can be zero for some entities so that we can dynamically adapt the structure of the supply chain to take advantage of competition, selecting the best supply chain structure for a given stream of final demand. Hence, we are merging together a strategic decision, i.e., the optimal supply chain structure, with a tactical decision, i.e., the actual induced demand at each supply chain entity for a given demand pattern. Traditionally, these decisions are hierarchical and significant savings can be reached by joining these two decisions problems. We are also merging other two hierarchical decision: inventory and resource capacity planning, a tactical decision, with scheduling, an execution level type of decision. Basic optimization theory tells us that potential savings can be significant.

The next question to be asked is what type of model can be used to solve this optimization problem. First of all, in order to separate the stochastic issues from the deterministic one, we consider, as in [de Kok, Fransoo 2003] and [Spitter 2005], a rolling schedule approach in which demand is forecasted over a time horizon and in which only current decisions are implemented at every entity. We will be assuming that safety stocks are computed separately, before the optimal plans are computed, so that these are input to the problem. Hence, we have a deterministic optimization problem that theoretically can be solved by a mathematical program: the objective function optimizes system wide supply chain costs, as if the entire supply chain were owned by the same entity, subject to system wide constraints and local constraints. There are two issues with this approach: a computational issue and a modeling issue. The computational issue is that this mathematical program, modeling a realistic supply chain, can not be solved in reasonable time. The modeling issue is that a mathematical program does not take into account hierarchy nor competing objectives. The mathematical programming approach was certainly suitable when many companies were highly vertically integrated, basically from raw materials to retailing, so that a system wide optimal solution was the right target and a system wide solution could be enforced at every level of the chain. Now, though, supply chains are comprised of many independent firms, each with a own objective function that can be in contrast with a system wide optimal solution. A system wide optimal solution can be applied only if there are side payments that appropriately divide the savings among competing players, [Chacon 2003]. We believe that an alternative approach that solves both the computational and the modeling issues is to use a bilevel optimization program in which the objective function still consists of optimizing system wide performances, e.g., minimizing costs, subject to each player optimizing its own objective function, e.g., profits,. The program will also have upper level constraints which coordinate the supply chain and lower level constraints which optimize each player resources. For a nice review of bilevel and multilevel programs the reader is referred to [Vicente,Calamai 1994] and [Colson et. al. 2005]. The concept of bilevel program can be seen as a generalization of a Stackelberg game and there have been applications in the military field, production and marketing since the seventies. So far, though, there are not bilevel or multilevel program approach to optimizing supply chains. Our program will minimize an objective function consisting of purchasing costs, production and transportation costs, inventory and backlog costs. The program is constrained in order to satisfy final demand, i.e., having the right amount of products at the right time and at the right location. Also, the program is constrained in order to maximize each player profit subject to local constraints, representing usually production and transportation constraints. We will assume that each player will implement a just in time policy, consistent with the rolling schedule approach, and when ordering it will select the least costly purchase pattern. A similar rational attitude is assumed for the consumers that will select the cheapest retailers. Each entity in the supply chain has a price function and a cost function. Both functions depend on the demand the entity faces over the time horizon T as well by a given current state. The cost function depends also on the chosen plan. In the next section we will formulate the problem while in the second we will give a heuristic to find a good solution. The heuristic repeatedly solves a mixed integer program with a non linear objective function. In the last section we give a simple method to represent this non linear mixed integer program as a linear mixed integer program. The reason for this modification relies on the fact that methods to solve linear mixed integer programs are more mature than methods to solve non linear mixed integer programs so that the corresponding software have comparatively better performances. Computational work is ongoing.

2 Problem Formulation

We have a general, capacitated supply chain represented, as usual, as a network in which each node represents an element in the supply chain and in which a direct link between two nodes represents a possible flow of goods or services between the two adjacent nodes. The supply chain is providing q items to the final market and there are a total of m items in the supply chain. A forecast of the q items over a time horizon T is given as well as exogenous forecast for each intermediate item. Each resource will define its own safety stock s_i for each item for any time period. Each resource in the network is characterized by a current state, that gives information about some relevant key indicators. When entity i is faced with a stream of demand $d_i = (O_{1i111}, \dots O_{jilst}, \dots, O_{nimrT})$ (where O_{jilst} is the order of item l placed at i from customer j at time t to be transhipped via transportation mode s) covering the entire time horizon T, it will select the plan p that optimize its profit function. Hence, the local problem faced by any entity i is:

$$Max \ \Pi_i = Rev_i(Q_i, state) - C_i^T(Q_i, p_i, state)$$

$$s.t.$$

$$Q_i \leq d_i$$

$$p_i(Q_i) \ \textit{feasible towards prodution and/or transportation constraints}$$

Here $Rev_i(Q_i, state)$ is the revenue collected by producing and/or transhipping Q_i and $C_i^T(Q_i, p_i, state)$ is the relative total cost. Beside the usual costs we also consider the cost of modifying plan, i.e., we cost the distance between the optimal plan and the current plan codified in *state*, $c(d_{dist}(p_i, p^{old}))$. This is to properly trade off the additional revenue of modified forecast and the replanning cost that can cause delays in other scheduled jobs. Practical application would suggest how this cost is relevant. The distance function can be any relevant distance, from the euclidean to a makespan distance. Note that the same optimal plan p_i^* can correspond to many possible combination of customers orders. We assume though that if $Q_i^* < d_i$ the plan p_i^* is modified in such a way that $p_i^*(Q_i^* + \epsilon)$ would be unfeasible for any vector ϵ. Similarly, one could modify the revenue function in such a way that $Q_i^* = d_i$ always. We distinguish between a production plan and a transportation plan. The production plan $p_i^p = (..., K_{it}, G_{it}, m_{it}, M_{it}, L_{it}^p, ...)$ is characterized, for each time t, by an output vector K_{it}, an input vector G_{it}, a minimum load m_{it}, a maximum capacity M_{it} and a production lead time L_{it}^p. The output vector represents all the items produced by resource i during the production lead time. It will be represented by $[k_{i1t}, \ldots, k_{imt}]$. The input vector represents all the items needed to produce the output. It will be represented by $[g_{i1t}, \ldots, g_{ikt}]$. W.l.g. let us assume that $k_{i1t} = 1$. Then a unit order release at time t will need g_{ijt} for all j to produce k_{ijt} for all j. Hence, if we release at time t an order R_{it} it will produce $R_{it}k_{ijt}$ for any output item j and it will need $R_{it}g_{ijt}$ for any input item. We assume a finite capacity modeled by the constraints $R_{it} \leq M_{it}$ i.e., we can not release more than $M_{it}k_{ijt}$ of output. Similarly the transportation plan $p_{is}^{tr} = (..., Q_{ijlt}^{min}, Q_{ijlt}^{max}, L_{ijlst}^{tr}...)$ is characterized by Q_{ijlst}^{min}, a minimum load for transhipping item l at time t from i to j via transportation mode s, Q_{ijlst}^{max}, the maximum capacity and L_{ijlst} the transportation lead time. The production cost function $C_i^p(R_i K_i, p_i^p, state)$ depends on the actual releases R_i over the time horizon, by the chosen production plan and by the current state. Similarly, the transportation cost function $C_{is}^t(O_{is}, p_{is}^t, state)$ depends by the actual orders, by the chosen plan and by the current state. The price charged to client j, $p_{ij}(O_{jis}, state_i, state_s) = p^p(O_{ji}, state_i) + \sum_{s \in S_{ijl}} p_s^t(O_{jis}, state_s)$ is divided in a purchasing price $p^p(O_{ji}, state_i)$, which depends on the total purchasing pattern of client j over the time horizon and by the current state of entity i, and in a transportation price $p_s^t(O_{jis}, state_s)$ which depends on the transportation mode s, on the purchasing pattern along that transportation mode and by the current state of transhipper s. In this way we indifferently allow in house transportation capability as well the use of third party logistic. Note that the state indicators used in the cost function and those used in the price function are usually different. For example, the state parameter in the price function could signal an excess amount of inventory of some item and hence some discounting scheme. We will distinguish furthermore two type of resources in the network: those resources that have input and output inventories I_{ijt}^{inp}, I_{ijt}, e.g., production centers, and those resources that have only one type of inventories, I_{ijt}, e.g., whareouses/distribution centers, retailers. The problem that a benevolent central planner would solve is that of meeting forecasted demand at the minimum cost while satisfying supply chain constraints. Costs taken into consideration are purchasing costs, production and transportation costs, inventory and backlogging costs. The possible contrast between the social allocation and each player self interest is best represented via a bilevel program in which the leader is the social planner while the followers are the separated entities. Before we describe the program let us summarize all variables and parameters needed.

n :	number of resources (nodes) in the network
m :	total number of items in the system
q :	total number of end items
D_{lt} :	exogenous demand of end item l at time t
d_{ilt} :	amount of end demand of item l faced by i at time t
D_{ilt} :	exogenous demand of item l faced by i at time t
s_i :	safety stock at entity i
O_i :	demand (customer orders) faced by entity i over the time horizon
p_{ilt} :	unit price for item l charged by entity i at time t for exogenous demand
O_{ijlst} :	amount of item l ordered by customer i from supplier j via transportation mode s at time t
Q_{ijlst}^{min} :	minimum order quantity for item l transhipped via transportation mode s from entity i to entity j at time t
Q_{ijlst}^{max} :	maximum order quantity for item l transhipped via transportation mode s from entity i to entity j at time t
L_{ijlst}^{tr} :	ATP transportation lead time of item l from supplier i to customer j at time t

R_{it} :	release at resource i at time t
R_i :	release at resource i over the entire horizon
m_{it} :	available to promise, minimum load at resource i at time t
M_{it} :	finite, available to promise, capacity at resource i at time t
$K_{it} = (k_{i1t}, \ldots, k_{imt})$:	output vector at resource i at time t
$G_{it} = (g_{i1t}, \ldots, g_{imt})$:	input vector at resource i at time t
L^p_{it} :	ATP production lead time at resource i at time t
$\overrightarrow{O}_{ij}$:	purchasing pattern from customer i to entity j over the time horizon
$\overrightarrow{O}_{ijs}$:	purchasing pattern from customer i to entity j
	over the time horizon via transportation mode s
$state_i =:$	current, given state at i
$p_{ij}(\overrightarrow{O}_{ji}, state_i, state_{\overrightarrow{s}}) =:$	$(p^p_{ij}(\overrightarrow{O}_{ji}, state_i) + \sum_{s \in S_{ij}} p^s_{ij}(\overrightarrow{O}_{jis}, state_s)$
	price charged to customer j from entity i for the purchasing pattern of client j.
	It is made of two parts: a purchasing price and a transportation price
$C^p_i(R_i, p^p_i, state_i)$:	production cost at entity i
$C^t_{is}(O_{is}, p^t_{is}, state_{is})$:	transportation cost cost at entity is
I_{ilt} :	inventory level of item l at resource i at time t
B_{ilt} :	backlog level of item l at resource i at time t
$h_{ilt}(I_{ilt})$:	holding cost of item l at resource i at time t
$\beta_{ilt}(B_{ilt})$:	backlog cost of item l at resource i at time t
J_{il} :	suppliers index set, providing item l to entity i
J_i :	index set of suppliers to i
Cl_{il} :	client index set, ordering item l from i
S_{ijl} :	transportation mode index set, carrying item l from i to j
S_{ij} :	transportation mode index set from i to j
S_i :	index of transportation modes available at i

$Rev_i(O_i; state)$: revenue incurred at i by selling O_i when in *state*

$C_i^T(O_i; p_i^p; state)$: total cost incurred at i by producing O_i according to production plan p_i^p when in *state*

$Rev_{is}(O_{is}, state_{is})$: revenue incurred at *(is)* by selling O_{is} when in *state*

$C_{is}^T(O_{is}, p_i^{tr}, state_{is})$: total cost incurred at *(is)* by transhipping O_{is} according to plan p_i^{tr} when in *state*

The bilevel mathematical model is as follow:

$$Min \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ilt}(d_{ilt}+D_{ilt}) + \sum_{i=1}^{n}\sum_{j\in J_i}(p_{ji}^p(\overrightarrow{O}_{ij}, state_j) + \sum_{s\in S_{ji}} p_{ji}^s(\overrightarrow{O}_{ijs}, state_s))+$$

$$+\sum_i (C_i^p(R_iK_i, p_i^p, state) + \sum_i\sum_{s\in S_i} C_{is}^t(\overrightarrow{O}_{is}, p_{is}^t, state^s) + \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{l=1}^{m}(h_{ilt}(I_{ilt}) + h_{ilt}(I_{ilt}^i) + \beta_{ilt}(B_{ilt}))$$

$s.t.$

$$\sum_{i:i\ retailer} d_{ilt} = D_{lt} \qquad \forall\ l=1,\dots q,\ \ \forall\ t=1,\dots T$$

$$d_{ilt} = 0 \qquad \textit{if retailer } i \textit{ does not sell item } l$$

$$d_{ilt} \geq 0 \qquad \textit{otherwise}$$

$$I_{ilt} - B_{ilt} = I_{ilt-1} - B_{ilt-1} + \sum_{j\in J_{il}}\sum_{s:S_{jil}}\sum_{\tau:\tau+L^{tr}_{jils\tau}=t} O_{ijls\tau} - d_{ilt} - s_{ilt} \qquad \forall\ \textit{end item } l \textit{ and } \forall\ \textit{retailer } i \textit{ selling item } l$$

$$I_{ilt} - B_{ilt} = I_{ilt-1} - B_{ilt-1} + \sum_{j\in J_{il}}\sum_{s:S_{jil}}\sum_{\tau:\tau+L^{tr}_{jils\tau}=t} O_{ijls\tau} - \sum_{j:j\in Cl_{il}}\sum_{s:S_{ijl}} O_{jilst} - D_{ilt} - s_{ilt} \qquad \forall\ \textit{intermediate items } l \textit{ and resources } i \textit{ with only one type of inventory}$$

$$I_{ilt} - B_{ilt} = I_{ilt-1} - B_{ilt-1} + \sum_{\tau:\tau+L^{pr}_{i\tau}=t} R_{i\tau}k_{il\tau} - \sum_{j\in Cl_{il}} O_{jilt} - D_{ilt} - s_{ilt} \qquad \forall\ \textit{items } l \textit{ and all resources } i \textit{ with two types of inventories}$$

$$I^i_{ilt} - B^i_{ilt} = I^i_{ilt-1} - B^i_{ilt-1} + \sum_{j:J_{il}}\sum_{s:S_{jil}}\sum_{\tau:\tau+L^{tr}_{jils\tau}=t} O_{ijls\tau} - R_{it}g_{ilt} - s_{ilt} \qquad \forall\ \textit{items } l \textit{ and all resources } i \textit{ with input inventories}$$

$$I_{il0} = \overline{I}_{il0} \qquad \forall\ \textit{items } l \textit{ and all relevant resources } i$$

$$I^i_{il0} = \overline{I}^i_{il0} \qquad \forall\ \textit{items } l \textit{ and all relevant resources } i$$

$$m_{it} \leq R_{it} \leq M_{it} \qquad \forall\ i=\{1,\dots,n\},\ \forall\ t=\{1,\dots,T\}$$

$$R_{it} \leq \frac{I^{inp}_{ilt}}{g_{ilt}} \qquad \textit{for all input item } l,\ \forall\ i,\ \forall\ t$$

$$Q_{ijlst}^{min} \leq O_{ijlst} \leq Q_{ijlst}^{max} \qquad \forall\ i, j, s, t$$

$$p_i^p = (..., m_{it}, M_{it}, G_{it}, K_{it}, L_{it}, ...) \qquad \forall\ i$$

$$p_{is}^{tr} = (..., Q_{ijlst}^{min}, Q_{ijlst}^{max}, L_{ijlst}^{tr}, ...) \qquad \forall\ (is)$$

$$Max\ \ Rev_i(x_i^p, state_i) - C_i^T(x, p_i^p, state_i) \qquad \forall\ i$$

$$s.t.$$

$$x_i^p \leq R_i K_i$$

plan p_i^p can produce x_i^p and satisfies all production constraints at entity i

$$Max\ \ Rev_{is}(x_{is}^{tr}), state_{is}) - C_{is}^T(x_{is}, p_{is}^{tr}, state_{is}) \qquad \forall\ i$$

$$s.t.$$

$$x_{is}^{tr} \leq O_{is}$$

plan p_{is}^{tr} can tranship x_{is}^{tr} and satisfies all transportation constraints at entity *(is)*

3 Problem Solution

This is a difficult problem to solve to optimality in its general form. We should at least have some assumptions on the cost functions and on the feasible sets. If we though would like to keep its generality (and hence its wide applicability in practical situation) we need to use heuristic. We would like our heuristic to perform well and being possible optimal or close to it.

W.l.g. we can assume that each entity production and transportation costs can be written as $C_i^T(x, p_i, state_i) = C_i(x, state_i) + C_i(x, p, state_i)$ i.e., in a part that does not depend on the specific plan and another part that does instead depend on the specific plan. Just to give an example, costs that do not depend on the specific plan are indirect material, labor payments, overhead, etc, while set up costs are obviously dependent on the specific plan. In order to solve the previous problem we adopt the following strategy: for each entity i we compute the set $Sol_i = \{x_1, x_2, ..., x_p)$ such that $Rev_i(x_j, state_i) - C_(x_j, state_i) \geq Rev_i(y, state_i) - C_(y, state_i)\ \forall\ \ i \in Sol_i$ and $\forall y \notin Sol_i$. Sol_i is hence the set of the p most profitable production (transportation) patterns for entity i without considering costs associated to the plans. Then for each of this pattern we compute the least cost feasible production (transportation) plan. If a feasible plan does not exists for some of the x_i we drop it from Sol_i. Note that if the costs associated with these plan are more or less the same then Sol_i would be the set of the p optimal solutions also for the entire objective function. In any case Sol_i is a good approximation. Let us call P_i the set of the p best plan for entity i. We associate to each of these plans a $0-1$ variable z_{iw} and we index all the relevant parameters by the chosen plan. We extend also the production and transportation function to be zero when the decision variable is 0 and the same as before otherwise. $C^p_{iw}(R_iK_{iw}, p^p_{iw}, state, z_{iw}) = 0$ if $z_{iw} = 0$ and $C^p_{iw}(R_iK_{iw}, p^p_{iw}, state, z_{iw}) = C^p_{iw}(R_iK_{iw}, p^p_{iw}, state)$ if $z_{iw} = 1$. Similarly for the transportation function.

Hence we can consider the following non linear, integer program:

$$Min \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{l=1}^{q} p_{ilt}(d_{ilt} + D_{ilt}) + \sum_{i=1}^{n}\sum_{w=1}^{p}(\sum_{j\in J_i} p^p_{ji}(\overrightarrow{O}_{ijw}, state_j) + \sum_{s\in S_{ji}} p^s_{ji}(\overrightarrow{O}_{ijws}, state_s)) +$$

$$+\sum_{i}\sum_{w=1}^{p} C^p_{iw}(R_iK_{iw}, p^p_{iw}, state, z_{iw}) + \sum_{i}\sum_{s\in S_i}\sum_{w=1}^{p} C^t_{isw}(\overrightarrow{O}_{is}, p^t_{isw}, state^s, z_{iw}) + \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{l=1}^{q}(h_{ilt}(I_{ilt}) + h_{ilt}(I^i_{ilt}) + \beta_{ilt}(B_{ilt}))$$

$$s.t.$$

$$\begin{array}{rcll}
\sum_{i:i\ retailer} d_{ilt} & = & D_{lt} & \forall\ l = 1, \dots q,\ \ \forall\ t = 1, \dots T \\
d_{ilt} & = & 0 & \textit{if retailer } i \textit{ does not sell item } l \\
d_{ilt} & \geq & 0 & \textit{otherwise} \\
I_{ilt} - B_{ilt} & = & I_{ilt-1} - B_{ilt-1} + \sum_{j\in J_{il}}\sum_{s:S_{jil}}\sum_{w=1}^{p}\sum_{\tau:\tau+L^{tr}_{jilsw\tau}=t} O_{ijlsw\tau} - d_{ilt} & \forall\ \textit{end item } l \textit{ and } \forall\ \textit{retailer } i \textit{ selling item} \\
I_{ilt} - B_{ilt} & = & I_{ilt-1} - B_{ilt-1} + \sum_{j\in J_{il}}\sum_{s:S_{jil}}\sum_{w=1}^{p}\sum_{\tau:\tau+L^{tr}_{jilsw\tau}=t} O_{ijlsw\tau} - \sum_{j:j\in Cl_{il}}\sum_{s:S_{ijl}} O_{jilst} - D_{ilt} & \forall\ \textit{intermediate items } l \textit{ an} \\
 & & & \textit{resources } i \textit{ with only one type of inventory} \\
I_{ilt} - B_{ilt} & = & I_{ilt-1} - B_{ilt-1} + \sum_{w=1}^{p}\sum_{\tau:\tau+L^{pr}_{iw\tau}=t} R_{iw\tau}k_{ilw\tau} - \sum_{j\in Cl_{il}} O_{jilt} - D_{ilt} & \forall\ \textit{items } l \textit{ and all resources } i \\
 & & & \textit{with two types of inventories} \\
I^i_{ilt} - B^i_{ilt} & = & I^i_{ilt-1} - B^i_{ilt-1} + \sum_{j:J_{il}}\sum_{s:S_{jil}}\sum_{w=1}^{p}\sum_{\tau:\tau+L^{tr}_{jilsw\tau}=t} O_{ijlsw\tau} - \sum_{w=1}^{p} R_{iwt}h_{ilwt} & \forall\ \textit{items } l \textit{ and all resources } i \\
 & & O_{ijlst} = \sum_{w=1}^{p} O_{ijlswt} & \\
 & & & \textit{with input inventories} \\
 & & I_{il0} = \overline{I}_{il0} & \forall\ \textit{items } l \textit{ and all relevant resources } i \\
 & & I^i_{il0} = \overline{I}^i_{il0} & \forall\ \textit{items } l \textit{ and all relevant resources } i
\end{array}$$

$$z_{iw} m_{iwt} \le R_{iwt} \le M_{iwt} z_{iw} \qquad \forall\ i = \{1,\dots,n\},\ \forall\ t = \{1,\dots,T\}$$

$$R_{iwt} \le \frac{I^{inp}_{ilt}}{G_{iwlt}} \qquad \textit{for all input item } l,\ \forall\ i,\ \forall\ t$$

$$z_{jw} Q^{min}_{jilswt} \le O_{ijlswt} \le Q^{max}_{jilswt} z_{jw} \qquad \forall\ i,j,s,t$$

$$\sum_{w=1}^{p} z_{iw} \le 1$$

$$p^{p}_{iw} = (..., m_{iwt}, M_{iwt}, G_{iwt}, K_{iwt}, L_{iwt}, ...) \qquad \forall\ i$$

$$p^{tr}_{iws} = (..., Q^{min}_{ijlswt}, Q^{max}_{ijlswt}, L^{tr}_{ijlswt}, ...) \qquad \forall\ (is)$$

$$z_i \in \{0,1\}$$

What we are doing here is splitting each resource i into p resources and requiring that exactly one of these images being active. By solving the previous program to optimality two things can happen: the problem is infeasible or it is feasible. If it is infeasible we need to add more plans i.e., increasing p, otherwise let d_i^* and p_i^* the optimal allocation for each player. We solve each player profit maximization problem with d_i^* as upper bound. We find an optimal plan $\tilde{p}_i$ and an optimal quantity $\tilde{Q}_i$. We add the new plan $\tilde{p}_i$. If $\tilde{Q}_i \neq d_i^*$ we remove p_i^* and recompute the optimal allocation. We stop when $\tilde{Q}_i = d_i^*$ for all i. In order to prove convergence result we need to add some hypothesis. In particular the procedure converge very quickly if the profit function is increasing up to capacity, i.e. selling more is better than selling less. This is because $\tilde{Q}_i = d_i^*$ and once feasibility is reached we need to add only one round of plans. As to feasibility, in a well functioning market the most profitable plans for each company should be able to cover the market, otherwise there would be space for competition.

4 Price and cost modeling

We will assume the following price and cost structures. For each player i we have a discount unit production price function, i.e.,

$$\begin{array}{ll} p^{p}_{ijl1} & \text{if } O_{jil} \in [q_{il1}, q_{il2}[\\ p^{p}_{ijl2} & \text{if } O_{jil} \in [q_{il2}, q_{il3}[\\ .... & \\ p^{p}_{ijlv} & \text{if } O_{jil} \in [q_{ilv}, q_{ilv+1}] \end{array}$$

$$p^{p}_{ijl1} > p^{p}_{ijl2} > ... > p^{p}_{ijlv}$$

Here O_{jil} is the total amount of item l ordered by customer j to supplier i over the entire horizon T along any transportation mode. Hence $O_{jil} = \sum_{t=1}^{T} \sum_{s \in S_{ijl}} O_{ijlst}$.

We also define other variables OP_{ijlk} for $k = 1, \dots v$ and we consider the following additional constraints:

$$p^p_{ijl} = \sum_{k=1}^{v} I^p_{ilk} p^p_{ijlk}$$

$$\sum_{k=1}^{v} I^p_{ilk} = 1 w_{jil}$$

$$O_{jil} = \sum_{k=1}^{v} OP_{jilk}$$

$$I^p_{ilk} q_{ilk} \leq OP_{jilk} \leq I^p_{ilk} q_{ilk+1} \ \ \forall \ k = 1, ...v$$

$$w_{jil}, I^p_{ilk} \in \{0, 1\}$$

Here I^p_{ilk} is an indicator function that select the appropriate price and $w_{jil} \in \{0, 1\}$ indicates if an order for item l is placed from j to i. The relative total production price charged from i to j for item l is then:

$$p^p_{ijl} O_{ijl}$$

We will assume the following transportation price function. A fixed cost c^F_{ijlst} if transportation mode s is selected at time t and a unit transportation cost per each mode

$$\begin{array}{rl} p^t_{ijlst1} & \text{if } O_{jilts} \in [q_{ilst1}, q_{ilst2}[\\ p^t_{ijlst2} & \text{if } O_{jilst} \in [q_{ilst2}, q_{ilst3}[\\ .... & \\ p^t_{ijlstu} & \text{if } O_{jilst} \in [q_{ilstu}, q_{ilstu+1}] \end{array}$$

We do not require here prices to be in order so that by increasing v, i.e., by reducing the interval length we can represent any function.

Here O_{jilst} is the total amount of item l shipped from i to j with transportation mode s at time t.

Other price structure can also be defined.

We have then

$$p^t_{ijlst} = \sum_{k=1}^{u} I^t_{ijlstk} p^t_{ijlstk}$$

$$\sum_{k=1}^{u} I^t_{ijlstk} = S_{jilst}$$

$$O_{jilst} = \sum_{k=1}^{u} OP^t_{jilstk}$$

$$I^t_{ijlstk} q_{ijlstk} \leq OP^t_{jilstk} \leq I^t_{ijlstk} q_{ijlstk+1} \;\; \forall\; k = 1, ...u$$

$$S_{jilst}, I^t_{ijlstk} \in \{0, 1\}$$

Here S_{jilst} indicates if an order for item l placed from j to i at time t will be carried along transportation mode s and I^t_{ijlstk} selects the price.

The total transportation price is hence

$$\sum_{t=1}^{T} \sum_{s=1}^{num_{ijlt}} c^F_{ijlst} S_{jilst} + p^t_{ijlst} O_{jilst}$$

where num_{ijlt} is the number of transportation mode from i to j for item l at time t.

As to the cost function we will have a production and a transportation cost for each plan. We will have a fixed cost if a plan (production or transportation cost) is selected and a unit cost function that will be approximated as before by a step function. We have a fixed production cost for plan w CP^F_w and a unit cost function per item per plan

$$c^p_{ilw} = \sum_{k=1}^{u} Ind_{ilwk} c^p_{ilwk}$$

$$\sum_{k=1}^{u} Ind_{ilwk} = z_{iw}$$

$$Ord_{ilw} = \sum_{t=1}^{T} K_{iwtl} R_{iwt}$$

$$Ord_{ilw} = \sum_{k=1}^{u} Ind_{ilwk} Orselect_{ilwk}$$

$$Ind_{ilwk} q_{ilwk} \leq Orselect_{ilwk} \leq Ind_{ilwk} q_{ilwk+1} \;\; \forall\; k = 1, ...u$$

$$Ind_{ilwk} \in \{0, 1\}$$

Similarly for the transportation cost. In the end we can write the following MIP:

$$Min \ \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ilt}(d_{ilt}+D_{ilt}) + \sum_{i=1}^{n}\sum_{j\in J_i}\sum_{l=1}^{m}\sum_{k=1}^{numbreak} p^p_{jilk}OP^p_{ijlk} + \sum_{t=1}^{T}\sum_{s=1}^{num_{ijl}} c^F_{ijlst}z_{jilst} + \sum_{k=1}^{numbreak} p^{tr}_{ijlst}OP^{tr}_{jilstk} +$$

$$\sum_{i=1}^{n}\sum_{w=1}^{v}(c^F_{iw}z_{iw} + \sum_{k=1}^{numbreak} c^p_{iwk}OR_{iwk}) + \sum_{i}\sum_{j}\sum_{s}\sum_{w}(c^{tF}_{ijsw}z_{ijsw} + \sum_{k=1}^{numbreak} c^t_{ijswk}OS_{ijswk}) +$$

$$+ + \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{l=1}^{m}(h_{ilt}I_{ilt} + h_{ilt}I^i_{ilt} + \beta_{ilt}B_{ilt})$$

s.t.

$$O_{ijl} = \sum_{t=1}^{T}\sum_{s\in S_{ijl}} O_{ijlst}$$

$$O_{ijl} = \sum_{k=1}^{numbreak} OP^p_{ijlk}$$

$$\sum_{k=1}^{numbreak} I^p_{jlk} = z_{ijl}$$

$$I^p_{jlk}q_{jlk} \le OP^p_{ijlk} \le I^p_{jlk}q_{jlk+1} \ \ \forall\ k = 1, ...numbreak$$

$$z_{jil}, I^p_{jlk} \in \{0,1\}$$

$$\sum_{k=1}^{numbreak} I^t_{ijlstk} = z_{jilst}$$

$$O_{jilst} = \sum_{k=1}^{numbreak} OP^{tr}_{jilstk}$$

$$I^t_{ijlstk}q_{ijlstk} \le OP^{tr}_{jilstk} \le I^t_{ijlstk}q_{ijlstk+1} \ \ \forall\ k = 1, ...numbreak$$

$$z_{jilst}, I^t_{ijlstk} \in \{0,1\}$$

$$OR_{iw} = \sum_{k=1}^{numbreak} OR_{iwk}$$

$$OR_{iw} = \sum_{t=1}^{T} R_{iwt}$$

$$IO^p_{iwk}q_{iwk} \le OR_{iwk} \le IO^p_{iwk}q_{iwk+1} \ \ \forall\ k = 1, ...numbreak$$

$$\sum_{k=1}^{numbreak} IO^{p}_{iwk} = z_{iw}$$

$$z_{iw}, IO^{p}_{iwk} \in \{0,1\}$$

$$O_{ijsw} = \sum_{t=1}^{T}\sum_{l=1}^{m} O_{ijlswt}$$

$$O_{ijsw} = \sum_{k=1}^{numbreak} O_{ijswk}$$

$$IO^{t}_{ijswk} q_{ijswwk} \leq O_{ijswk} \leq IO^{t}_{ijswk} q_{ijswk+1} \ \ \forall\ k = 1, ...numbreak$$

$$\sum_{k=1}^{numbreak} IO^{t}_{ijswk} = z_{ijsw}$$

$$z_{ijsw}, IO^{t}_{ijswk} \in \{0,1\}$$

$$
\begin{aligned}
\sum_{i:i\ retailer} d_{ilt} &= D_{lt} && \forall\ l = 1,\dots q,\ \ \forall\ t = 1,\dots T\\
d_{ilt} &= 0 && \textit{if retailer } i \textit{ does not sell item } l\\
d_{ilt} &\geq 0 && \textit{otherwise}\\
I_{ilt} - B_{ilt} &= I_{ilt-1} - B_{ilt-1} + \sum_{j\in J_{il}}\sum_{s:S_{jil}}\sum_{w=1}^{p}\sum_{\tau:\tau+L^{tr}_{jilsw\tau}=t} O_{ijlsw\tau} - d_{ilt} && \forall\ \textit{end item } l \textit{ and } \forall\ \textit{retailer } i \textit{ selling item } l\\
I_{ilt} - B_{ilt} &= I_{ilt-1} - B_{ilt-1} + \sum_{j\in J_{il}}\sum_{s:S_{jil}}\sum_{w=1}^{p}\sum_{\tau:\tau+L^{tr}_{jilsw\tau}=t} O_{ijlsw\tau} - \sum_{j:j\in Cl_{il}}\sum_{s:S_{ijl}} O_{jilst} - D_{ilt} && \forall\ \textit{intermediate items } l \textit{ and}\\
& && \textit{resources } i \textit{ with only one type of inventory}\\
I_{ilt} - B_{ilt} &= I_{ilt-1} - B_{ilt-1} + \sum_{w=1}^{p}\sum_{\tau:\tau+L^{pr}_{iw\tau}=t} R_{iw\tau}k_{ilw\tau} - \sum_{j\in Cl_{il}} O_{jilt} - D_{ilt} && \forall\ \textit{items } l \textit{ and all resources } i\\
& && \textit{with two types of inventories}\\
I^{i}_{ilt} - B^{i}_{ilt} &= I^{i}_{ilt-1} - B^{i}_{ilt-1} + \sum_{j:J_{il}}\sum_{s:S_{jil}}\sum_{w=1}^{p}\sum_{\tau:\tau+L^{tr}_{jilsw\tau}=t} O_{ijlsw\tau} - \sum_{w=1}^{p} R_{iwt}h_{ilwt} && \forall\ \textit{items } l \textit{ and all resources } i\\
& O_{ijlst} = \sum_{w=1}^{p} O_{ijlswt}\\
& && \textit{with input inventories}\\
& I_{il0} = \overline{I}_{il0} && \forall\ \textit{items } l \textit{ and all relevant resources } i\\
& I^{i}_{il0} = \overline{I}^{i}_{il0} && \forall\ \textit{items } l \textit{ and all relevant resources } i\\
& z_{iw}m_{iwt} \leq R_{iwt} \leq M_{iwt}z_{iw} && \forall\ i = \{1,\dots,n\},\ \forall\ t = \{1,\dots,T\}
\end{aligned}
$$

$$R_{iwt} \leq \frac{I^{inp}_{ilt}}{G_{iwlt}} \qquad \textit{for all input item } l, \ \forall\ i, \ \forall\ t$$

$$z_{jw}Q^{min}_{jilswt} \leq O_{ijlswt} \leq Q^{max}_{jilswt}z_{jw} \qquad \forall\ i, j, s, t$$

$$\sum_{w=1}^{p} z_{iw} \leq 1$$

$$p^{p}_{iw} = (..., m_{iwt}, M_{iwt}, G_{iwt}, K_{iwt}, L_{iwt}, ...) \qquad \forall\ i$$

$$p^{tr}_{iws} = (..., Q^{min}_{ijlswt}, Q^{max}_{ijlswt}, L^{tr}_{ijlswt}, ...) \qquad \forall\ (is)$$

$$z_i \in \{0, 1\}$$

References

[Chacon 2003] G. Chacon, Supply chain coordination with contracts, *Handbooks in Operations Research and Management Science: Supply Chain Management*, North Holland, 2003.

[Colson et. al. 2005] B. Colson, P. Marcotte, G. Savard, Bilevel programming: A survey, *4OR* 3:87-107, 2005.

[de Kok, Fransoo 2003] Ton G. de Kok, Jan C. Fransoo, Planning Supply Chain Operations: Definition and Comparison of Planning Concepts, *Handbooks of Operations Research: Supply Chain Management: Design, Coordination and Operation*, 2003.

[Spitter 2005] J. M. Spitter Rolling Schedule Approaches for Supply Chain Operations Planning, *Phd thesis Technology Department Univeristeit Eindhoven*, 2005.

[Vicente,Calamai 1994] L.N. Vicente, P.H. Calamai Bilevel and Multilevel Programming: A Bibliography Review, *Journal of Global Optimization* 5: 291-306, 1994.