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# Effect of Commonality on Spare Parts Provisioning Costs for Capital Goods

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## Abstract

Machines at customers have to be provided with spare parts upon failure. Consider a number of groups of machines, for each of which a target aggregate fill rate or target average response time (waiting time) should be met. Between groups, commonality exists, i.e., some parts occur in the material breakdown structure of machines in multiple groups. Instead of using separate stocks per group of machines, we study the potential benefits of exploiting commonality by using a shared stock for all groups together. For this purpose, we formulate a multi-item single-site spare parts inventory model, with the objective to minimize the spare parts provisioning costs, i.e., inventory holding and transportation costs, under the condition that all service level constraints are met. We develop a heuristic solution procedure using a decomposition approach as in Dantzig-Wolfe decomposition, in order to obtain both a heuristic solution and a lower bound for the optimal costs. In a case study and a numerical experiment, we show that significant reductions in spare parts provisioning costs can be obtained by using shared stocks. Furthermore, we show how the size of the potential benefits behaves as a function of the number of groups, the percentage of commonality and the occurrence of commonality in cheap or expensive items.

**Keywords:** Inventory control, spare parts, system approach, commonality, service level constraints.

## 1 Introduction

This paper aims to provide managerial insights into the effect of commonality on spare parts provisioning costs for capital goods. Although the developed insights may apply to other industries

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as well, our focus is on the capital goods industry because especially in that industry, spare parts inventory management plays an important role. Usually, spare parts are expensive, and therefore managerial attention is justified. Spare parts models are desired to be multi-item models, i.e., rather than focussing on the performance for an individual item, they focus on the performance for all items together because that is what the customer really bothers. These models have been studied since the late sixties, starting with Feeney and Sherbrooke [2] and Sherbrooke [4]. Commonality, i.e., the fact that parts are used in more than one machine type, has received attention in several papers (see Van Mieghem [5] and the references therein). As far as we know, however, little research exists on commonality in multi-item spare parts models. This paper is devoted to this topic.

In Section 2, we present a multi-item single-site spare parts inventory model with commonality, followed by an analysis of this model in Section 3. The core part of the paper regarding insights into the effect of commonality is in Section 4, where we present a case study and a numerical experiment. The paper is concluded in Section 5.

## 2 Model

Consider a number of machines at one or more customers. Machines consist of multiple parts, also referred to as stock-keeping units (SKU-s). Each SKU is assumed to be either a consumable part or a repairable part (repairable parts with condemnation constitute a mixture; they could be captured as well but would require a slightly more complicated notation than used below). The machines are divided into *groups* (a group may also be seen as a customer class). Usually, such a group contains all machines at one customer or all machines of a specific machine family or machine type at one customer, but other compositions of groups are possible as well. Between groups, commonality exists, i.e., some parts occur in the material breakdown structure of machines in multiple groups. Let  $I$  denote the set of SKU-s, with  $|I| \geq 1$ , and let  $J$  denote the set of groups, with  $|J| \geq 1$  ( $|J| = 1$  can be used for the situation with a separate stock per group). For each SKU  $i \in I$  and group  $j \in J$ , failures (demands) are assumed to occur according to a Poisson process with constant rate  $m_{i,j}$  ( $\geq 0$ ). If SKU  $i$  does not occur in the material breakdown structure of machines in group  $j$ , then  $m_{i,j} = 0$  by definition. Let  $\mu_i := \sum_{j \in J} m_{i,j}$ ,  $i \in I$ . Let  $M_j := \sum_{i \in I} m_{i,j}$ ,  $j \in J$ , and assume that  $M_j > 0$ .

If one of the parts of a machine fails, the machine is down and the defective part has to be replaced by a spare part. A failure of a machine is always caused by one defective part, and can be remedied by replacing that part only. All requests for spare parts are sent to one warehouse. This warehouse coordinates spare parts provisioning to the customers. If a requested part  $i \in I$  is available at the warehouse, it is delivered immediately. Otherwise, an emergency shipment takes place to fulfill the demand. For the stock at the warehouse, such a demand can be considered a

lost sale. The average time for an emergency shipment from the supplier to the warehouse is  $t_i^{em}$  and the corresponding costs are  $c_i^{em}$ .

For each SKU  $i \in I$ , the stock in the warehouse is controlled by a base stock policy with base stock level  $S_i$ . The holding costs per time unit for one unit of SKU  $i$  are  $c_i^h$ . When a part  $i$  in the warehouse stock is used to fulfill customer demand, a ready-for-use part  $i$  arrives in the warehouse to refill the stock after a regular replenishment lead time with mean  $t_i^r (\geq t_i^{em})$ . Per SKU, lead times are independent and identically distributed, and lead times for different SKU-s are independent. Let  $c_i^r (\leq c_i^{em})$  denote the costs related to a regular replenishment shipment.

In case of a consumable, an emergency shipment may come either from another source as for regular replenishments, or from the same source. In the latter case, an emergency shipment may mean that a faster transportation channel is used than for regular replenishments. For a repairable, an emergency shipment may mean that for one of the parts in the repair shop the repair is done (or finished) against the highest possible speed (in case of a zero base stock level, this has to be the part that just failed). Alternatively, it may mean that a part is obtained from another source. In that case, we assume that the part that just failed is sent back to this other source (either immediately or after repair), so that the inventory position remains constant.

Customers strive for minimal down time of their machines, and therefore, for each group a service level constraint is defined with respect to spare parts provisioning by the warehouse. Constraints are defined in terms of a *maximum average waiting time* (response time)  $\hat{W}_{j,obj}$  for the aggregate stream of requests from group  $j \in J$ . For SKU  $i$ , let  $W_i(S_i)$  and  $\beta_i(S_i)$ ,  $i \in I$ , denote the average waiting time and *item fill rate*, respectively. For each  $j \in J$ , let  $\hat{W}_j(S)$  denote the average waiting time per request for the aggregate demand stream of for that group, where  $S := \{S_i\}_{i \in I}$  denotes an overall policy for all SKU-s. The behavior of the physical stock of SKU  $i$  is as in an Erlang loss system with arrival rate  $\mu_i$  and mean service time  $t_i^r$ , and hence  $\beta_i(S_i)$  is equal to one minus the Erlang loss probability:

$$\beta_i(S_i) = 1 - \frac{(\mu_i t_i^r)^{S_i} / S_i!}{\sum_{k=0}^{S_i} (\mu_i t_i^r)^k / k!}. \quad (1)$$

Further,

$$W_i(S_i) = (1 - \beta_i(S_i)) t_i^{em}, \quad i \in I, \quad (2)$$

and the average waiting times  $\hat{W}_j(S)$  are weighted sums of the average waiting times  $W_i(S_i)$  for individual SKU-s, with the fractions  $m_{i,j}/M_j$  as weights:

$$\hat{W}_j(S) = \sum_{i \in I} \frac{m_{i,j}}{M_j} W_i(S_i), \quad j \in J. \quad (3)$$

Let  $r_i \in \{0, 1\}$ ,  $i \in I$ , denote whether for SKU  $i$  the parts in the replenishment pipeline are counted as inventory ( $r_i = 1$ ), or not ( $r_i = 0$ ). Naturally,  $r_i = 1$  for repairables and  $r_i = 0$  for consumables that are supplied from an external source. For consumables that are supplied by a

source within the same company, both  $r_i = 0$  and  $r_i = 1$  may occur. For each SKU  $i$ , the expected number of parts in the pipeline is  $\mu_i \beta_i(S_i) t_i^r$  (according to Little's law), and hence the inventory holding costs per time unit are given by

$$c_i^h [S_i - (1 - r_i) \mu_i \beta_i(S_i) t_i^r].$$

Transportation costs per time unit for SKU  $i$  are

$$\mu_i [\beta_i(S_i) c_i^r + (1 - \beta_i(S_i)) c_i^{em}] = \mu_i c_i^r + \mu_i (1 - \beta_i(S_i)) (c_i^{em} - c_i^r).$$

Note that the first term  $\mu_i c_i^r$  is independent of  $S_i$ . We define  $c_i(S_i)$  as the spare parts provisioning costs that depend on  $S_i$  (also called relevant costs):

$$c_i(S_i) := c_i^h [S_i - (1 - r_i) \mu_i \beta_i(S_i) t_i^r] + \mu_i (1 - \beta_i(S_i)) (c_i^{em} - c_i^r). \quad (4)$$

The objective is to minimize the total spare parts provisioning costs subject to the aggregate waiting time constraints for the groups. Our optimization problem is as follows:

$$\begin{aligned} (P) \quad & \min \quad \sum_{i \in I} c_i(S_i) \\ & \text{subject to} \quad \sum_{i \in I} \frac{m_{i,j}}{M_j} W_i(S_i) \leq \hat{W}_{j,obj}, \quad j \in J, \\ & \quad \quad \quad S_i \in \mathbb{N}_0, \quad i \in I, \end{aligned}$$

with  $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ . The optimal costs of Problem (P) are denoted by  $C_P$ .

Notice that straightforward application of the described model constitutes a situation with shared stock for all groups. The situation with a separate stock per group can be obtained either by a slight modification of the input data (removing the commonality property by replacing common SKU-s by group-specific SKU-s for each group it occurs in) or by solving Problem (P) for each group individually. Summarizing, the model provides a framework to compare the use of separate stocks per group to the use of a shared stock for all groups together.

**Remark.** In case the average emergency shipment times  $t_i^{em}$  are the same for all SKU-s, i.e.,  $t_i^{em} = t^{em}$  for all  $i \in I$ , the average waiting time constraints may be rewritten as

$$\sum_{i \in I} \frac{m_{i,j}}{M_j} \beta_i(S_i) \geq \hat{\beta}_{j,obj}, \quad j \in J,$$

with

$$\hat{\beta}_{j,obj} = 1 - \frac{\hat{W}_{j,obj}}{t^{em}}.$$

Thus, in that case, the average waiting time constraints are equivalent to aggregate fill rate constraints.

### 3 Analysis

A lower bound for the optimal costs  $C_P$  of Problem ( $P$ ) may be obtained by a decomposition and column generation method which reveals close similarity to Dantzig-Wolfe decomposition for linear programming problems (see Dantzig and Wolfe [1] for a general description of that method). In Subsection 3.1, we describe the decomposition and column generation method for our problem. Next, in Subsection 3.2, we present a way to obtain a feasible solution for Problem ( $P$ ), i.e., an upper bound on  $C_P$ .

#### 3.1 Lower bound

Like in Dantzig-Wolfe decomposition, a *Master Problem* is introduced in which the variables of our original problem are expressed as convex combination of columns that contain all possible values for the decision variables in the original problem. Let  $K := \mathbb{N}_0$  denote the set of base stock policies for each of the SKU-s  $i \in I$ . Let  $S_i^k$ ,  $i \in I, k \in K$ , denote the (fixed) base stock level of policy  $k$  for SKU  $i$ , and let  $x_i^k \in \{0, 1\}$ ,  $i \in I, k \in K$ , be a variable indicating whether policy  $k$  for SKU  $i$  is chosen ( $x_i^k = 1$ ) or not ( $x_i^k = 0$ ). Relaxing the integrality constraint on  $x_i^k$ ,  $i \in I, k \in K$ , a suitable Master Problem related to Problem ( $P$ ) is defined as follows:

$$\begin{aligned}
 (MP) \quad & \min && \sum_{i \in I} \sum_{k \in K} c_i(S_i^k) x_i^k \\
 & \text{subject to} && \sum_{i \in I} \sum_{k \in K} \frac{m_{i,j}}{M_j} W_i(S_i^k) x_i^k \leq \hat{W}_{j,obj}, \quad j \in J, && (MP.1) \\
 & && \sum_{k \in K} x_i^k = 1, \quad i \in I, && (MP.2) \\
 & && x_i^k \geq 0, \quad i \in I, k \in K.
 \end{aligned}$$

The optimal costs of Problem ( $MP$ ) are denoted by  $C_{MP}$ . Notice that the relaxation of the integrality condition on  $x_i^k$ ,  $i \in I, k \in K$ , in Problem ( $MP$ ) allows for fractional values of  $x_i^k$ ,  $i \in I, k \in K$  and thus corresponds to allowing *randomized policies*. Therefore,  $C_{MP}$  constitutes a lower bound on  $C_P$ .

Besides Problem ( $MP$ ), a Restricted Master Problem, Problem ( $RMP$ ), is defined that for each SKU  $i \in I$  only considers a small subset  $K_i \subseteq K$  of columns (policies). The optimal costs of Problem ( $RMP$ ) are denoted by  $C_{RMP}$ . For each SKU  $i \in I$ , let  $K_i$  initially consist of one policy  $k$ , with  $S_i^k := \min\{S_i | W_i(S_i) \leq \hat{W}_{j,obj}, j \in J, S_i \in \mathbb{N}_0\}$ . For given  $K_i$ ,  $i \in I$ , Problem ( $RMP$ ) can be solved using the simplex method, if the number of variables is at least  $|I| + |J|$ . Notice that we satisfy that condition, since our choice of  $|I|$  initial policies implies  $|I|$  variables  $x_i^k$  and the  $|J|$  service level constraints ( $MP.1$ ) lead to  $|J|$  slack variables. Furthermore, notice that our choice of initial policies constitutes a feasible solution for Problem ( $RMP$ ).

After solving Problem (*RMP*) with the  $|I|$  initial policies, we are interested in policies that were not yet considered, but that would improve the solution of Problem (*RMP*) if they were added. To check if such policies exist, we solve, for each SKU  $i \in I$ , a so-called *column generation subproblem* that for that SKU generates a policy with the lowest reduced cost coefficient. Given an optimal solution for Problem (*RMP*), let  $u_j \leq 0$ ,  $j \in J$ , denote the dual variables (shadow prices) related to the  $|J|$  service level constraints (*MP.1*), and let  $v_i$ ,  $i \in I$ , denote the dual variables related to the  $|I|$  convexity constraints (*MP.2*). Then, for our Problem (*RMP*), the column generation subproblem for an SKU  $i \in I$ , is as follows.

$$\begin{aligned}
(SUB(i)) \quad \min \quad & c_i(S_i) - \sum_{j \in J} u_j \frac{m_{i,j}}{M_j} W_i(S_i) - v_i \\
\text{subject to} \quad & S_i \in \mathbb{N}_0.
\end{aligned}$$

Let  $S_{SUB(i)}$  denote an optimal policy (base stock level) for Problem (*SUB(i)*), and let  $C_{SUB(i)}$  denote the cost of an optimal solution of Problem (*SUB(i)*), i.e., the lowest reduced cost coefficient. It can be shown that  $\beta_i(S_i)$  is increasing and concave on its entire domain  $\mathbb{N}_0$  (see Lemma 2 in Kranenburg and Van Houtum [3], p.13). Since  $c_i(S_i)$  consists of a term that is linear in  $S_i$ , and terms having  $\beta_i(S_i)$  with a non-positive coefficient (see (4)), and the objective function of Problem (*SUB(i)*) further consists of another term containing  $\beta_i(S_i)$  (via  $W_i(S_i)$ ) with a non-positive coefficient and a constant term  $v_i$ , the objective function of Problem (*SUB(i)*) is convex in  $S_i$ . This implies that Problem (*SUB(i)*) can be solved in a straightforward way. If there exists an optimal policy  $S_{SUB(i)}$  with a negative reduced cost coefficient  $C_{SUB(i)}$  for SKU  $i$ , this policy is added to  $K_i$ .

As long as a policy with a negative reduced cost coefficient exists for one or more SKU-s, adding columns to Problem (*RMP*) and solving Problem (*RMP*) is done iteratively. If for none of the SKU-s  $i \in I$  a policy with negative reduced cost can be found, the obtained solution for Problem (*RMP*) is optimal for Problem (*MP*) as well. This iterative procedure is finite, as can be seen as follows. We start the procedure with  $|I|$  policies that constitute a feasible solution with costs  $C_{RMP}$ . In each iteration, we add for each item a policy if there exists one with negative reduced cost, i.e., a policy that would decrease  $C_{RMP}$  if it would be added. The number of policies that satisfies this condition is finite, and therefore the number of iterations is finite.

In total,  $|I| + |J|$  variables are in the basis. Notice that at most  $|J|$  SKU-s will have fractional  $x_i^k$ -values because the convexity constraints (*MP.2*) require that for each  $i$  at least one  $x_i^k$  is a basic variable. Furthermore, notice that the number of service level constraints (*MP.2*) that is satisfied with equality is at least equal to the number of SKU-s that has fractional  $x_i^k$ -values.

### 3.2 Upper bound

After a lower bound has been found for Problem (P), as described above, an upper bound, i.e., a feasible solution for Problem (P), can be determined as follows.

If none of the  $x_i^k$ -values in the solution of Problem (MP) is fractional, the obtained solution of Problem (MP) is feasible for Problem (P) as well.

If fractional  $x_i^k$ -values do occur, however, we need to apply some further steps. Since the number of fractional  $x_i^k$ -values is at most  $|J|$ , and, usually,  $|J|$  is very small compared to  $|I|$ , it would be reasonable to select for each of these few SKU-s, the policy with a non-zero  $x_i^k$ -value and the highest base stock level. Obviously, this results in a feasible solution for Problem (P), but it may lead to a somewhat loose upper bound if one of these SKU-s happens to be expensive. Therefore, another procedure is proposed to obtain an upper bound.

Select for each SKU  $i \in I$  the policy with a non-zero  $x_i^k$ -value and the lowest base stock level  $S_i^k$ , and refer to this policy as policy  $k'$ . Notice that  $c_i(D) \geq c_i(S_i^{k'})$  for  $D > S_i^{k'}, D \in \mathbb{N}$ . This holds because  $\beta_i(D) \geq \beta_i(S_i^{k'})$ , and thus if  $c_i(D) < c_i(S_i^{k'})$ , a feasible solution would exist with  $x_i^{k'} = 0$  that has lower costs than the current solution of Problem (MP). This would contradict to the choice of  $k'$ . Let  $S_i := S_i^{k'}$  and  $S := \{S_i\}_{i \in I}$ . Notice that  $\hat{W}_j(S) > \hat{W}_{j,obj}$  for at least one  $j$ . This holds because  $x_i^{k'}$  is fractional for at least one  $i$ . It can be verified that for this  $i$  it holds that  $\beta_i(S_i^{k'}) < \beta_i(S_i^l)$ , and thus that  $W_i(S_i^{k'}) > W_i(S_i^l)$ , and furthermore that  $c_i(S_i^{k'}) < c_i(S_i^l)$ , with  $l$  denoting any other policy with fractional  $x_i^l$ -value, and that  $x_i^{k'} = 1$  would lead to an infeasible solution, i.e., a solution with  $\hat{W}_j(S) > \hat{W}_{j,obj}$  for at least one  $j$ .

We evaluate a number of neighbors  $S^i$ ,  $i \in I$ , of  $S$  and select the best one. This step is repeated until we have obtained a feasible solution for Problem (P). Neighbor  $S^i$  is the set of policies with for all SKU-s identical base stock levels as for  $S$ , except for SKU  $i$ , for which the base stock level is increased with one unit. Neighboring policies are evaluated with respect to the decrease in distance to the target average waiting times per unit cost increase,

$$\frac{\sum_{j \in J} [\hat{W}_j(S) - \hat{W}_{j,obj}]^+ - \sum_{j \in J} [\hat{W}_j(S^i) - \hat{W}_{j,obj}]^+}{c_i(S_i^i) - c_i(S_i)},$$

with  $[a]^+ := \max\{0, a\}$ , and the neighbor for which this value is largest is selected.

## 4 Numerical Results

In this section, we numerically study the potential benefits of exploiting commonality by using a shared stock for all machine families together instead of using a separate stock per machine family. We show results for a case study that we have done with data of ASML, in Subsection 4.1, and results of a numerical experiment with a smaller data set, in Subsection 4.2. The model has been implemented in AIMMS 3.4, and XA is used as solver for the linear programming problems.



Data set	$M_1/M_2$	$CP_1$	$CP_2$	$(\hat{W}_{1,obj}, \hat{W}_{2,obj})$ = (0.10, 0.10)	$(\hat{W}_{1,obj}, \hat{W}_{2,obj})$ = (0.10, 0.05)	$(\hat{W}_{1,obj}, \hat{W}_{2,obj})$ = (0.05, 0.10)	$(\hat{W}_{1,obj}, \hat{W}_{2,obj})$ = (0.05, 0.05)
1	8.95	0.12	0.23	0.96	0.93	0.96	0.94
2	0.40	0.10	0.12	0.96	0.97	0.96	0.97
3	33.22	0.12	0.26	0.93	0.94	0.93	0.94
4	2.37	0.12	0.25	0.93	0.93	0.93	0.92
5	0.75	0.10	0.19	0.97	0.97	0.96	0.96
6	0.57	0.19	0.32	0.93	0.94	0.91	0.91
7	1.25	0.18	0.30	0.94	0.95	0.93	0.93
8	10.00	0.12	0.26	0.92	0.93	0.92	0.93

Table 1: Results case study ASML

For each instance, we define the commonality percentage  $CP_j$  for group  $j$  as

$$CP_j := \frac{|(i|m_{i,j} < \mu_i, m_{i,j} > 0)|}{|(i|m_{i,j} > 0)|}, \quad j \in J.$$

It can be interpreted as the percentage of SKU-s in group  $j$  that is common (i.e., these parts also receive demand from at least one other group).

#### 4.1 Case study at ASML

In the case study, we consider 8 data sets of ASML, corresponding to 8 different local warehouses. For each data set, we have  $|J| = 2$  groups. The average number of SKU-s per group is about 700, and varies between 400 and 1000. Failure rates are low, on average 0.25 per year, and vary between 0.0005 and 40 per year. The relative size of the groups, expressed in terms of  $M_1/M_2$ , and  $CP_1$  and  $CP_2$  are given in Table 1. On average,  $CP_j$  is 0.19. For the 10% most expensive SKU-s per group, the commonality percentages are about the same as the depicted values, which gives us an indication that for all groups, the occurrence of commonality is equally distributed over cheap and expensive SKU-s. Emergency replenishment costs  $c_i^{em}$  and times  $t_i^{em}$  and regular replenishment costs  $c_i^r$  and times  $t_i^r$  are SKU-independent. At ASML,  $t_i^r = 14$  days (= 2 weeks), and  $t_i^{em} = 1$  day. The value for  $(c_i^{em} - c_i^r)$  is set equal to 0.1 times the average holding costs  $c_i^h$ . We set  $r_i = 1$  for all  $i$ , so pipeline stock is included in holding costs. For each data set, we compare using shared and separate stock, and we do that for four situations:  $(\hat{W}_{1,obj}, \hat{W}_{2,obj}) \in \{(0.10, 0.10), (0.10, 0.05), (0.05, 0.10), (0.05, 0.05)\}$  (the targets  $\hat{W}_{j,obj}$  are expressed in days). In Table 1, the spare parts provisioning costs for the shared stock situations are depicted as a fraction of the spare parts provisioning costs in the separate stock cases.

As can be seen from Table 1, on average 6% can be saved in spare parts provisioning costs in the ASML data sets. Target fill rate levels and the relative sizes of the groups seem to have little

influence on this.

Besides in comparisons between shared and separate stock, we are interested in the performance of the method. On average, it took 13 seconds to run this model on a Pentium 4 computer, where the maximum duration was 26 seconds. The method provides an upper bound ( $UB$ ) for the optimal costs  $C_P$  of Problem ( $P$ ), i.e., a feasible solution, and a lower bound ( $LB$ ). We define the relative distance  $G$  between both bounds as

$$G := \frac{UB - LB}{LB},$$

and observed that the method generates quite good solutions:  $G$  is on average 0.06% and at most 0.3% in the considered cases.

## 4.2 Numerical experiment

In addition to the case study, we performed a numerical experiment as well, in which we varied characteristics that were given in the case study. In the experiment, we chose arbitrarily 100 SKU-s from one group in one data set of ASML and checked their representativeness by plotting failure rates versus prices. This showed a pattern similar to the complete set of SKU-s for that group. For these 100 SKU-s, we studied several scenarios for situations with 2 and 5 groups. We let all groups have these 100 SKU-s (with their failure rates and prices). In these scenarios, we varied the  $CP_j$ -value from 0% to 100% in steps of 20% (within a scenario we assumed  $CP_j$  equal for all  $j$ ). Furthermore, we varied the commonality setting, indicating if commonality occurs in cheap or expensive SKU-s or equally distributed over all SKU-s. According to these settings, we declared an SKU in a scenario either as completely common (i.e., occurring in all groups) or as group-specific. Regular replenishment costs and times, emergency replenishment costs and times, and the  $r_i$  were chosen identically to the values in the case study, and  $\hat{W}_{j,obj} = 0.05$  for all  $j$ . For these settings, we show in Figures 1 and 2 graphically the savings of using shared stocks compared to using separate stocks (i.e., 0% commonality). Of course, at 100% commonality, the commonality setting is indifferent. From Figures 1 and 2, it can be seen that enormous savings can be obtained if the commonality percentage  $CP_j$  is high. Furthermore, if the number of groups increases, the benefits of using shared instead of separate stocks increases as well. Thirdly, commonality in expensive SKU-s is from a managerial point of view much more interesting than having commonality mainly in cheap SKU-s. Even if  $CP_j$  is about 40%, only a small saving can be obtained if the commonality occurs in cheap SKU-s only. Lastly, in Figure 1 with commonality setting 'equal' and  $CP_j = 20\%$  ASML's situation can be positioned.

Again, the performance of the method was quite well. The gap  $G$  was reasonably larger than in the case study, most likely because of the smaller number of SKU-s considered. Its average value was 2.7%, and in one case,  $G$  was 12%. The computation time was on average less than 2 seconds (at most 3).

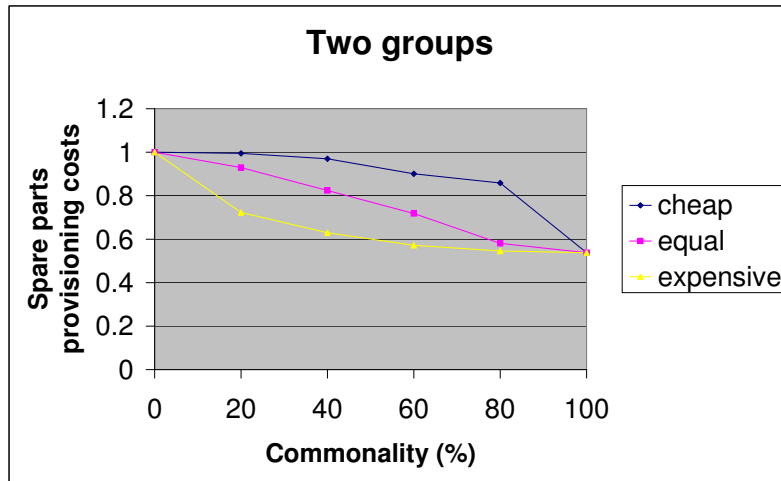


Figure 1: Spare parts provisioning costs for two groups

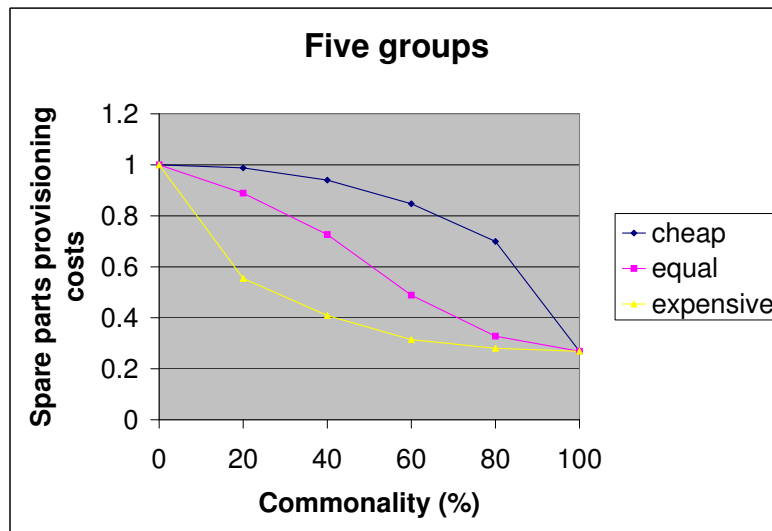


Figure 2: Spare parts provisioning costs for five groups

## 5 Conclusions

We have developed a multi-item, single-stage spare parts inventory model with multiple groups to study the effect of commonality on spare parts provisioning costs for capital goods. A case study in which we studied several data sets of ASML, an original equipment manufacturer in the semiconductor industry, showed that on average 6% reduction can be obtained in spare parts provisioning costs if stocks for different groups are shared. In a numerical experiment with a smaller data set, we showed that a larger number of groups increases potential benefits considerably. Also, we have seen that the savings obtained by shared stocks are significantly affected by the commonality percentage and the degree to which the commonality occurs in the expensive SKU-s.

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