

Decay of swirl in turbulent pipe flows

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Decay of swirl in turbulent pipe flows

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de Rector Magnificus, prof. dr. J.H. van Lint, voor een commissie aangewezen door het College van Dekanen in het openbaar te verdedigen op dinsdag 19 januari 1993 om 16.00 uur

door

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geboren te Eindhoven

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Dit proefschrift is goedgekeurd door de promotoren prof. dr. ir. G.J.F. van Heijst en prof. dr. ir. H.W.M. Hoeijmakers en co-promotor: dr. K. Krishna Prasad

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1. Introduction

1.1 Flow measurement in large-scale transport systems for natural gas

In the past decades the use of gaseous fuels has grown sharply. Consequently, the importance of accurate measurement of large amounts of gas has been a major interest of all gas transport companies. This has resulted in sophisticated calibration techniques and in the development of accepted flow standards. However, recent studies reveal that, the uncertainty in the measurement of large volumes of gas must be estimated to be, at best, 0.7% of the actual flow (van der Kam (1990)). According to this study the origin of the uncertainty lies primarily in the metering device, and is caused by uncertainties in the flow standards and by installation effects of the device in the metering station. The present study is devoted to the characterization of the flow upstream of the flow metering device and thus deals with the second cause of uncertainty described above.

For accurate flow measurements most flow metering devices, e.g. orifice plates and turbine meters, need a certain length of straight pipe upstream of the device. This allows the decay of the disturbances introduced by bends, valves and other components normally encountered in transport systems for natural gas. If the length of the straight pipe is sufficiently large the flow will obtain a sufficiently developed state to allow accurate flow measurements. The length required depends on the type of metering device and on several parameters such as, the Reynolds number of the flow, the wall roughness and of course the type of the disturbances.

Traditionally, the length of the straight piece of pipe upstream of the flow meter needed for the flow to settle down, has been determined from previous experience. However, often this design requirement is obtained from results for flows at different Reynolds numbers or with different disturbances. Recent research has shown that the straight pipe, for example prescribed by the ISO standard on orifice plate metering and used in existing installations, might have an inadequate length for certain types of upstream disturbances. The result is that this uncertainty on installation requirements leads to an increased uncertainty in the results of the flow metering.

An obvious possibility to reduce the uncertainty in flow metering is the in-situ calibration of the metering devices. However, for economical and practical reasons this option is not feasible. A second possibility is to condition the flow by means of flow straighteners. However, even though flow straighteners may remove certain types of disturbances very efficiently, they do not deliver ideal flow conditions in all situations. Furthermore, the disturbances introduced by the straighteners themselves, again need a straight length of pipe to settle down. Finally, straighteners will introduce undesirable extra pressure losses. The present study is devoted to a third alternative: Prediction of the influence of specific pipe arrangements on the flow to determine the correct installation of the flow meter. Obviously, an important part of the prediction of the installation effects is the prediction of the decay of disturbances present in the flow. As mentioned above recent research has shown that the upstream pipe length requirements as posed by the metering standards (e.g. ISO-5167 or ANSI/API-2530) might be inadequate for certain types of disturbances. A plausible cause for the inadequacy is an under-prediction of the relaxation lengths, i.e. the length needed for a disturbance to decay fully. As illustrated by, for example Klein (1981) the distance required for a full development of turbulent pipe flows may exceed 140 pipe diameters. In contrast, in most standards for flow metering and in most standard textbooks (e.g. Schlichting (1967)) the flow is considered to be fully developed after 25 to 100 pipe diameters.

A disturbance which is notorious for its slow decay, is the disturbance generated by a combination of two out-of-plane bends of the pipe. When the gas passes through such a



Figure 1.1: A schematic view of swirling pipe flow

configuration the flow acquires an axial vorticity component, which results in the so-called "swirling" flow. The flow downstream of the bends is characterized by streamlines with a helical shape (Fig. 1.1). In addition to the slow decay of the swirl, swirl also affects the performance of the metering devices considerably. In a turbine meter, as Salami (1984) concludes, even a swirl angle of 2° means a likely error of 2% in the measured mass flow.

Clearly, near a surface metering station, in underground transport systems for natural gas, a combination of two out-of-plane bends is very likely to be encountered, see Fig. (1.2).



Figure 1.2: A schematic view of the pipe manifold in the 3×12^n orifice-plate meterrun "Oude Statenzijl Noord" of Dutch Gasunie

Hence, for the prediction of installation effects in large-scale transport systems for natural gas, the study of swirling pipe flows is very relevant. The work described in this thesis is devoted to the prediction of the decay of disturbances with swirl in turbulent pipe flows.

1.1.1 Experiments on the decay of swirl

Although the effect of swirl on flow meter performance was already observed in the early 1900's, detailed experiments were only possible with the advent of advanced experimental techniques, such as hot-wire, hot-film probes and laser-doppler velocimetry. Consequently, the amount of detailed experimental work available in the literature is limited. Work found in the literature aimed at the prediction of the decay of swirl in turbulent flows is equally limited. The first study in this connection has been performed by Kreith & Sonju (1965). Other studies have been performed by, Nystom & Padmanabhan (1985), McManus et al. (1985), Mottram & Rawat (1986), Algifri et al. (1987), Halsey (1987), Mattingly & Yeh, (1988) and Kitoh (1991). Perusal of these studies reveals that apparently the decay of swirl is not fully understood, although all studies agree in the sense that they predict a slow decay of the swirl. It is reported that typical "half distances" for swirl decay are in the order of 50 pipe diameters. However, the reported decay rates exhibit a considerable scatter. For example, Mottram & Rawat (1986) predict that the swirl decays with a decay coefficient λ , operating through the factor $exp(-\lambda x/D)$, equal to $\lambda = 0.5f$, where f is the friction factor for fully developed flow as defined by Blevins (1984). On the other hand Nystom & Padmanabhan (1985) and Halsey (1987) suggest that a better estimate would be a λ of 0.75 f.

Furthermore, most experiments mentioned here consider only one type of swirl, are performed for only one Reynolds number and are performed in pipes with fixed degree of roughness. As a consequence, the relevance of these studies to the metering problem is limited. Most experimental work is performed at a much lower Reynolds number (typical values are $Re = \mathcal{O}(10^8)$) than the Reynolds number occurring in large-scale transport systems for gaseous fuels (typical values are $Re = \mathcal{O}(10^7) \dots \mathcal{O}(10^8)$). For a successful extrapolation of the experimental results to operational conditions, detailed knowledge on the decay rate of the disturbance is needed of the effect of parameters such as the Reynolds number, the wall roughness, and the swirl type. As illustrated by the scatter in the reported decay coefficients, apparently the knowledge for a reliable extrapolation is incomplete.

Though, for the purpose of obtaining more information on the effect of parameters such as the Reynolds number, the wall roughness, and the swirl type, in this study some exploratory measurements at a low Reynolds number will be reported, the emphasis is put on the numerical modelling of the decay of swirl in turbulent pipe flows, rather than on performing extensive parameter studies through measurements

1.1.2 Numerical modelling of the decay of swirl in turbulent pipe flows

Since the early eighties, numerical prediction of turbulent flows has become common practice for engineering purposes. Since then a massive amount of work concerning the prediction of turbulent flows is reported in the open literature. However, very little work is directly applicable to the problem of the relaxation of disturbances in pipe flows. Contrary to the bulk of the work for engineering purposes, the geometry of the problem at hand is very simple. However, the requirement on the accuracy of the prediction of the development of the pipe flow is stronger than the commonly required accuracy for engineering purposes. Since the numerical prediction of turbulent flows involves modelling of certain aspects of the flow, this modelling has to be considered very carefully for its effect on the accuracy of the prediction of the decay of the swirl.

In the present study the turbulence modelling is based on the so-called one-pointclosure schemes for turbulence. At present and in the foreseeable future, in an engineering environment, only these schemes seem to be among the few feasible for the prediction of turbulent flows at high Reynolds numbers. Unfortunately the one-point closures are not, and will not be, the ultimate answer to the prediction of turbulent flows. Especially the modelling of the class of "complex" flows, to which the swirling flows belong, provides serious difficulties. Since the one-point-closure schemes heavily rely on empirical input, for every new class of flow much work is required to tune the coefficients in the scheme.

Based on predictions of flows in complex geometries like industrial burners and vortex tubes, it appears that for flows with swirl most researchers agree that only second-order closure schemes are capable of capturing the physics of the flow. However, for relatively simple swirling flows, like the swirling flow in a straight pipe, not much information is available in literature and it is felt that additional research is necessary.

1.2 Thesis overview

In the second chapter of this thesis an overview is given of the most widely used models of turbulence. A description is given of the assumptions on which the models are based, starting from the most general closure scheme, the Reynolds Stress Model of turbulence. To reveal some of the conceptual differences the lower-order schemes are derived from the more general schemes. Focus is on the modelling of flows with swirl.

Chapter 3 deals with the numerical techniques used in this study. Most of these techniques are existing techniques, though not often used for pipe flows with swirl.

In chapter 4 the prediction of axi-symmetric swirling pipe flows will be discussed in detail. Employing existing turbulence closure schemes we will derive simplified expressions for the turbulent stresses. These simplified expressions allow us to assess the predicted effects of swirl on the turbulence as predicted by the various methods. A detailed description is given of the consequences of the various assumptions on the prediction of the decay of swirl in turbulent pipe flows.

The question whether the assumption of axisymmetry in swirling pipe flows is valid will be addressed in chapter 5. Numerical experiments are used to investigate the stability of the flow against non-axisymmetric disturbances.

In chapter 6 experimental results are presented. The results presented in this chapter are first results of an extensive systematic study of turbulent pipe flows with swirl.

Finally, in chapter 7 the results of the preceding chapters are summarized and the numerically obtained predictions are compared with the results of the experiment. The thesis is concluded by a discussion on the feasibility of numerical predictions of installation effects.

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2. Turbulence modelling and swirl

2.1 Introduction

Though it is generally accepted that the basic physics of turbulence is captured by the combination of the continuity equation, the time-dependent Navier-Stokes equations and the energy equation, 1^{-2} limitations in computer capacity make it impossible to solve the equations directly for most flows in practical situations of technological interest. Many of the flows encountered in practice involve complex geometries and are of a three-dimensional and non-homogeneous nature. The result is that virtually all scientific and engineering calculations of non-trivial turbulent flows at high Reynolds-number involve some kind of modelling. The only class of models capable of handling the wide variety of flows encountered in an engineering environment is the class of one-point closures. In this chapter an overview of the most commonly used one-point-closure schemes is given.

A "difficult" class of flows from the modelling point of view is the class of the "complex" flows. They distinguish themselves from "simple" flows in the sense that instead of to one strain the flow is subjected to additional rates of strain. These additional strains can be caused by velocity gradients in other directions than the main strain, curvature of streamlines, buoyancy and coriolis forces. As discussed by Bradshaw (1973) these secondary strains can have surprisingly strong effects on the turbulence structure.

A relatively simple "complex" flow is a swirling flow. A swirling flow is a flow with a mean vorticity component in the direction of the main flow. The structure of this flow indeed strongly differs from the structure of the simple shear flow. Hence not all turbulence closures are capable to correctly describe this flow. To appreciate some of the causes of the deficiencies of some turbulence closures we will start by describing the most general one-point-closure schemes, the schemes based on transport equations for the second-order moments of the turbulent motions. Emanating from these closures we will derive the lowerorder closure schemes, in order to address the conceptual differences between the models. At the end of this chapter a few typical examples of calculations of swirling flows using the different turbulence closures are given.

^{1.} Here we will often denote the combination of the continuity, Navier-Stokes and energy equations, simply by the term "Navier-Stokes equations".

^{2.} For the flows under consideration in this study, the energy equation is superfluous.

2.2 Second-order-closure schemes

2.2.1 The Reynolds-averaged Navier-Stokes equations

The motion of fluids is governed by the continuity equation and the Navier-Stokes equations. In an incompressible Newtonian flow these equations are written as,

$$\nabla \cdot \vec{U} = 0$$
 (2.1)

and

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \nabla\right) \vec{U} + \frac{1}{\rho} \nabla P - \nu \nabla^2 \vec{U} = \vec{F}$$
(2.2)

where

 \vec{U} denotes the velocity,

P the pressure,

 ν the kinematic viscosity,

 ρ the density of the fluid

and

\vec{F} the external forces acting on the fluid.

The Navier-Stokes equations are a set of non-linear equations. A measure for the nonlinearity is the Reynolds number,

$$Re = \frac{UL}{v}$$

with

U denoting a typical velocity scale, in this work the mean velocity U_{mean} is used,

and

L denoting a typical length scale, in this work often the radius R, or the diameter D, of the pipe is used.

For small values of the Reynolds number the non-linear character is not significant. Disturbances in the flow will be damped by viscous effects (represented by $\nu \nabla^2 \vec{U}$), and the flow will be stable. Hence the equation can be solved by numerical means. When the Reynolds number becomes higher, the relative importance of the viscous terms will diminish. The possibility arises that above a critical magnitude of Re disturbances in the flow will grow. Time-dependent vortex-like structures, in turbulence terminology often denoted as "eddies", appear in the flow. The largest eddies can remain in existence by extracting energy from the mean flow. Owing to processes like vortex stretching the energy contained in these eddies is transferred to smaller scale eddies. The Reynolds number based on this small scale is of order unity and the energy contained in the small-scale eddies is eventually dissipated by molecular effects. The cascade of eddies will have a strong effect on the large-scale structure of the flow. Apart from extracting energy from the mean flow, the eddies also serve as a carrier of momentum. Hence, in turbulent flows, the exchange of momentum will be

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strongly enhanced as compared to the stable laminar flow. Thus if one wants to describe such unstable flows, one has to take all these eddy-like structures into account. However, in most flows relevant to engineering purposes the resolution of the small scale structures is beyond the capabilities of the current generation of computers. An illustration is given by Speziale (1991): "To gain appreciation of the task, consider the fact that economically feasible direct simulations of a turbulent pipe flow at a Reynolds number of 500,000---a turbulent flow that, although nontrivial, is far from the most difficult encountered—would require a computer 10 million times faster than the CRAY YMP!" So in general obtaining a solution by numerical means is not feasible.

To obtain a "solvable" problem the mathematical artefact of turbulent stresses is introduced. These apparent stresses are obtained by splitting the flow-field quantities into a mean (time averaged for "steady" flows and ensemble averaged for time-dependent flows) part and a fluctuating part, i.e.:

$$ec{U}_{ ext{total}} = ec{U} + ec{u}, \ P_{ ext{total}} = P + p \ ext{and} \ ec{F}_{ ext{total}} = ec{F} + ec{f}$$

Here the uppercase symbols denote the mean components, the lower case symbols the fluctuating components.

These expressions are then substituted into the original Navier-Stokes equations. The next step is to apply the averaging procedure again on the equations. The result resembles the original Navier-Stokes equations but contains an extra term, the divergence of the second-order correlations of the fluctuating parts.

$$\nabla \cdot \vec{U} = 0$$
 (2.3)

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \nabla\right) \vec{U} + \frac{1}{\rho} \nabla P - \nu \nabla^2 \vec{U} + \nabla \cdot \left(\begin{array}{ccc} uu & uv & uw \\ uv & vv & vw \\ uw & vw & ww \end{array}\right) = \vec{F}$$
(2.4)

The second-order velocity correlations are usually denoted as "Reynolds stresses". They serve, like the viscous terms, as a distribution mechanism for momentum. Hence the unstable, essentially time-dependent, character of the original equations is weakened. Since the effect of all small-scale structures on the mean-flow quantities is now contained in the Reynolds-stress tensor, Eq. (2.4) is solvable by conventional numerical methods, provided that the Reynolds stresses can be expressed in terms of mean-flow quantities. In order to obtain a closed set of equations at least six additional equations have to be provided. This is known as the closure problem for turbulent flows.

2.2.2 The Reynolds-stress transport equations

To solve the closure problem of turbulence a relation has to be established between the Reynolds stresses and the mean flow. It seems natural to investigate the interaction of Reynolds stresses and the mean flow by formulation of transport equations for the Reynolds stresses. The successive steps in the derivation of a transport equation for any component of the Reynolds-stress tensor are: • Subtract the mean-momentum equations from the full momentum equations, the fluctuating momentum equations are obtained. In index notation this yields;

$$\frac{\partial u_i}{\partial t} + U_j \frac{\partial u_i}{\partial x_j} = -u_j \frac{\partial U_i}{\partial x_j} - \frac{\partial \left(u_i u_j - \overline{u_i u_j}\right)}{\partial x_j} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} \,. \tag{2.5}$$

- Multiply the equation for u_i by u_j , multiply the equation u_j by u_i and add the results.
- Apply the average operation to the resulting expression, and rearrange the different terms.

The result is the following set of transport equations for the Reynolds stresses,

Rate of change
$$\frac{\partial \overline{u_{i}u_{j}}}{\partial t} + U_{k}\frac{\partial \overline{u_{i}u_{j}}}{\partial x_{k}} =$$
Production: P_{ij}

$$-\left(\frac{\overline{u_{i}u_{k}}\frac{\partial U_{j}}{\partial x_{k}} + \overline{u_{j}u_{k}}\frac{\partial U_{i}}{\partial x_{k}}\right)$$
Pressure-strain interaction: Φ_{ij}

$$+\frac{\overline{p}\left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}}\right)}{p\left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{k}}\right)}$$
(2.6)
Dissipation: $-\epsilon_{ij}$

$$-\frac{\partial}{\partial x_{k}}\left(\overline{u_{i}u_{j}u_{k}} + \frac{\overline{pu_{i}}}{\rho}\delta_{jk} + \frac{\overline{pu_{j}}}{\rho}\delta_{ik} - \nu\frac{\partial \overline{u_{i}u_{j}}}{\partial x_{k}}\right)$$

The procedure outlined above does not automatically yield this representation of the transport equations. They are obtained after some rearranging of terms in the original equations. The rearrangement presented above results in terms with a clear physical interpretation (which is given at the side of each term).

2.2.3 Interpretation of terms in the Reynolds-stress transport equations

The terms in Eq. (2.6) can be interpreted as follows:

- Rate of change of the turbulent stresses This term represents the usual transport term for a quantity, here $\overline{u_i u_j}$, convected by the flow.
- Production of the turbulent stresses P_{ij} The production term represents the transfer of energy from the mean components of the flow to the fluctuating components.
- Pressure-strain interaction, Φ_{ij} The pressure-strain term represents the redistribution of energy between the components of the Reynolds-stress tensor. In order to represent only redistribution of

energy it is necessary that this term is traceless. This can be achieved by a special decomposition of the original pressure terms, that results from the last step in the procedure outlined above

$$\overline{u_j \frac{\partial p}{\partial x_i}} = \frac{\partial \overline{u_j p}}{\partial x_i} - \overline{p \frac{\partial u_j}{\partial x_i}} = \frac{\partial \overline{u_j p}}{\partial x_k} \delta_{ik} - \overline{p \frac{\partial u_j}{\partial x_i}}.$$

The last term on the right-hand side is traceless, which can be shown using the continuity equation. The traceless terms are combined in the pressure-strain term. The non-traceless remainders are included in the diffusion term.

- Viscous dissipation of the turbulent stresses ε_{ij}
 This term arises from a decomposition of the viscous terms.
- Diffusive transport of the turbulent stresses The diffusion consists of three contributions.
 - A turbulent-diffusion term
 This term involves the third-order correlations of the velocity.
 - A pressure-diffusion term
 This is the non-traceless part of the pressure-velocity correlation.

A viscous-diffusion term
 Again this term is the result of a rearrangement of terms.

Apart from terms depending on the mean-flow quantities only and the Reynolds stresses themselves, the Reynolds-stress transport equations also contain triple-velocity and velocity-pressure correlations. Thus the formulation of the transport equations for the second-order correlations has led to new unknown correlations of still higher order. To obtain a closed set of equations these higher-order correlations have to be expressed in terms of known quantities, such as mean velocity, mean pressure and the Reynolds stresses themselves.

2.2.4 The modelling of the Reynolds-stress transport equations

For a highly turbulent flow in parts of the flow field far away from walls or other interfering objects it is possible to find approximations for the higher-order terms in the Reynolds-stress transport equations in terms of known quantities. These approximations are individually fitted to a set of simple "test" flows in which the different effects they represent can be considered in isolation, i.e. flow problems dominated by one of the effects. To obtain some degree of generality for the case where several effects are simultaneously to be accounted for, the minimum requirements the approximations should obey are:

- invariance: each of the approximations must be independent of the coordinate system used;
- realizability: the approximations must yield physically realistic results, e.g., the total kinetic energy contained in the Reynolds-stress tensor must remain posi-

tive and certain symmetry conditions have to be preserved (Schwarz inequality) (Schumann 1976); and

• accuracy: for each of the individual "test" flows the predictions have to be accurate. Even the set minimum requirements leads to depressingly complicated approximate expressions for the terms that require modelling. In the past they were viewed too complicated for practical use Lumley (1978). However, recently some proposals have been put forward for models satisfying complete realizability Shih *et al.* (1991) for a limited class of flows.

Most approximations are derived under the assumption of the "high-Reynoldsnumber hypothesis" which implies that:

- the large-scale motions are not affected by viscosity (the coefficients are independent of Re); and
- the small-scale motions are isotropic (the viscous dissipation does not depend on geometrical effects).

The terms in Eq. (2.6) which do contain higher-order correlations and thus need modelling are the diffusion, pressure-strain and dissipation terms. Especially the accuracy of the model for the pressure-strain interaction term proves to be important for realistic predictions of flows employing turbulence models based on the Reynolds-stress equations.

2.2.5 The model for the pressure-strain interaction term, Φ_{ij}

The basic form of the pressure-strain interaction model is due to Rotta (1951), Naot *et al.* (1970) and Launder *et al.* (1975). It is based on the notion that:

- the pressure-strain interaction only redistributes energy;
- a turbulent flow not subjected to any strain tends to become isotropic; and
- in turbulent flow subject to a single mean strain the anisotropy of the turbulence tends to grow.

Hence the pressure-strain interaction term consists of two redistribution mechanisms:

- the "return to isotropy" term, $\Phi_{ij,1}$; and
- the "rapid part", $\Phi_{ij,2}$.

The return-to-isotropy term arises due to the mutual interaction of the fluctuating velocity components only. The simplest model to mimic this effect is linear in the anisotropy tensor (Rotta 1951), i.e.

$$\Phi_{ij,1} = -C_1 \epsilon a_{ij} , \qquad (2.7)$$

with

 C_1 a dimensionless constant,

 the rate-of-dissipation of the total energy contained in the Reynolds-stress tensor,

and

 a_{ij} the anisotropy tensor given by,

$$a_{ij} = rac{\left(\overline{u_i u_j} - rac{2}{3} \delta_{ij} k
ight)}{k}$$

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with

k the contraction of $\overline{u_i u_j}$ given by:

$$k=\frac{\overline{u_iu_i}}{2},$$

the turbulent kinetic energy.

The value of C_1 reported in literature varies between 1.5 and 5.0.

Some extensions to this linear approximation have been proposed by for example Lumley (1978) and Reynolds (1984). At this moment none of these extensions has obtained a wide acceptance.

The "rapid part" is caused by the interaction of the mean flow with the fluctuating velocity components. The corresponding model is based on the symmetry properties of an exact expression for the pressure strain interaction where the pressure term itself does not appear (derived using a Poisson equation for the pressure). For the "rapid part" Launder, Reece and Rodi (1975) suggested,

$$\Phi_{ij,2} = -\frac{c_2 + 8}{11} \left(P_{ij} - \frac{2}{3} \delta_{ij} P \right)$$
$$-\frac{30c_2 - 2}{55} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) k$$
$$-\frac{8c_2 + 2}{11} \left(D_{ij} - \frac{2}{3} \delta_{ij} P \right)$$
(2.8)

with

 c_2 a dimensionless constant, P_{ij} the production of $\overline{u_i u_j}$, see Eq. (2.6), P the contraction of P_{ij} ,

and

 D_{ij} given by,

$$D_{ij} = -\left(\overline{u_i u_i} \frac{\partial U_l}{\partial x_j} + \overline{u_j u_l} \frac{\partial U_l}{\partial x_i}\right)$$

In literature the model due to Launder *et al.* is sometimes referred to as the "Quasi-Isotropic" model (QI model). Launder *et al.* found the first group of terms on the righthand side of Eq. (2.8) to be the dominant one, so a somewhat simpler model proposed by Naot *et al.* (1970) should provide a good approximation:

$$\Phi_{ij,2} = -C_2 \left(P_{ij} - \frac{2}{3} \delta_{ij} P \right) \tag{2.9}$$

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The constant C_2 in Eq. (2.9) will differ somewhat in magnitude from the coefficient of the first term involving c_2 in Eq. (2.8), this in order to compensate for the omitted terms in Eq. (2.8). This model is sometimes referred to as the "Isotropization of Production" model (IP model).

Experimentally isolating the effects of the redistribution of energy, or "pressure scrambling", between the components of the Reynolds-Stress tensor by the action of shear, is not a realistic option. Instead the model is tuned to an analytical model of turbulence for the case of isotropic turbulence subjected to a fast distortion. This theory (Rapid Distortion Theory, Batchelor & Proudman (1954)) is based on the observation that, on a time scale which is small enough, the interaction of the turbulence with itself is negligible. As a consequence the fluctuating momentum equations, Eq. (2.5) may be linearized and solved for the fluctuating velocity components. The "rapid-part" of the pressure-strain is required to conform to this RDT limit. Expression (2.8) conforms automatically to this limit, irrespective of the magnitude of c_2 . The second form, Eq. (2.9) satisfies the RDT limit only when C_2 takes a value of 0.6.

According to Fu *et al.* (1987) the shorter form, Eq. (2.9), contains a deficiency. Depending on the rotation of the reference frame it predicts a different behaviour. The reason is that individually the production term and the convection term in the Reynolds-stress transport equations, Eq. (2.6), depend on the kinematic state of the frame of reference. However, it can be shown that the difference between the production term and the convection term does not depend on the rotation of the frame of reference. As a remedy Fu *et al.* (1987) propose the following form for the rapid part

$$\Phi_{ij,2} = -C_2 \left[\left(P_{ij} - \frac{2}{3} \delta_{ij} P \right) - \left(C_{ij} - \frac{2}{3} \delta_{ij} C \right) \right]$$
(2.10)

with

$$C_{ij}$$
 – the convection of Reynolds stresses, given by

$$C_{ij} = U_l \frac{\partial \overline{u_i u_j}}{\partial x_l}$$

and

C the contraction of C_{ij} ,

$$C = \frac{C_{ii}}{2}$$

In general, the flow configurations for which the models are used hardly ever resemble the test flows that are used to tune the coefficients of the model. Hence, the magnitude of these coefficients cannot be considered rigidly fixed. On the contrary, when one wants to conform to simple flows, like a developed boundary-layer flow, extra conditions are posed on the value of the coefficients of the pressure-strain interaction. If all transport terms of turbulence are neglected, which indeed is a valid assumption in the outer region of a developed boundary layer, the model for the Reynolds-stress transport equations simplify drastically. Comparison of these approximate transport equations with the measured stress levels indicates that for the IP pressure-strain model,

$$\frac{1-C_2}{C_1} \approx 0.27.$$
 (2.11)

It is clear that in general it will be impossible to simultaneously satisfy all conditions with a single set of coefficients. Especially when the IP model is used, the consequence is that different sets of coefficients have to be used for different flows. The "standard" coefficient set for the pressure-strain interaction is obtained by relaxing the constraint for "Return to isotropy" and giving priority to the "Rapid distortion" result, $C_2 = 0.6$. In combination with the boundary-layer condition, Eq. (2.11), this results in,

$$C_1 = 1.5$$
 and $C_2 = 0.6$.

For flows with swirl or streamline curvature Gibson & Younis (1986) argued that the coefficients should obey a second relation, also based on a comparison of measured stress levels in a curved flow and the simplified transport equations for the Reynolds stresses in a curved flow,

$${2-C_2\over C_1}pprox 0.57$$
 .

Combining the condition set by the flat boundary layer, Eq. (2.11), and one for the curved boundary layer results in,

$$C_1 = 3.0$$
 and $C_2 = 0.3$.

Finally, for buoyant flows the flat boundary-layer condition, Eq. (2.11), imposes

$$\frac{1-C_2}{C_1}\approx 0.22$$

Hence, Gibson & Launder (1978) proposed

$$C_1 = 1.8$$
 and $C_2 = 0.6$

for flows with buoyancy effects. For clarity the different sets that are in use for the different flows are summarized in Table (2.1).

2.2.6 The rate of dissipation of turbulence, ϵ_{ij}

The basis for the dissipation model is the concept of local isotropy. In the high Reynoldsnumber limit it is assumed that the small-scale motions are isotropic. Since the viscous dissipation takes place at the smallest length scales it is assumed that,

$$\epsilon_{ij} = \frac{2}{3} \delta_{ij} \epsilon \tag{2.12}$$

where ϵ denotes the rate of dissipation of the turbulent kinetic energy, k

Flow	C_1	C_2	Reference
standard	1.5	0.6	Launder et al. (1975)
curvature	3.0	0.3	Gibson & Younis (1986)
buoyancy	1.8	0.6	Gibson & Launder (1978)

Table 2.1: A few examples of the coefficients used for the IP pressure-strain model for different type of flows

However, also the rate of dissipation, ϵ , of the turbulent kinetic energy is unknown. Again a transport equation provides an expression for ϵ . In the high-Reynolds-number limit two terms in the transport equation for ϵ are dominant:

- generation of small-scale vorticity due to "self stretching" of turbulence; and
- viscous destruction of small-scale vorticity.

Most commonly used is the following model, which is compatible with decaying grid turbulence and also provides a source term necessary for shear layers,

$$\operatorname{Convection}(\epsilon) + \operatorname{Diffusion}(\epsilon) = \left(C_{\epsilon 1} \frac{P}{\epsilon} - C_{\epsilon 2}\right) \frac{\epsilon^2}{k} \ .$$

Since the diffusion of fluctuating vorticity is of minor importance it is approximated with a relatively crude gradient diffusion model. The resulting transport equation reads as,

$$\frac{\partial \epsilon}{\partial t} + U_l \frac{\partial \epsilon}{\partial x_l} = \left(C_{\epsilon 1} \frac{P}{\epsilon} - C_{\epsilon 2} \right) \frac{\epsilon^2}{k} + C_{\epsilon} \frac{\partial}{\partial x_l} \left(\frac{\epsilon}{k} \frac{\overline{u_k u_l}}{\partial x_k} \frac{\partial \epsilon}{\partial x_k} \right)$$
(2.13)

where

 $C_{\epsilon 1}, C_{\epsilon 1}, C_{\epsilon 2}$ denote dimensionless constants.

Eq. (2.13) is subject to much criticism and numerous amendments have been proposed, for example by Pope (1978) to correct for the otherwise anomalous predictions for the roundjet flow, and by Bardina *et al.* (1985) to account for effects of system rotation. What is considered to be a major weakness is that Eq. (2.13) neglects any rotational strain. Hence for isotropic decaying grid turbulence it yields the *same decay rate* independent of the rotation of the frame of reference. In strong contrast to this result, experiments and numerical simulations indicate that the decay rate of turbulent kinetic energy can be strongly reduced by a system rotation (Bardina*et al.* (1985), Traugott (1958) and Wigeland & Nagib (1978)).

In fact the rate of dissipation, ϵ , fixes a length scale, \mathcal{L} , of the large-scale motions,

 $k^{rac{3}{2}}/\epsilon \propto \mathcal{L}$

Obviously, for conceptual reasons it is very easy to criticize the construction of a turbulence macro scale based on small-scale information. Moreover, this definition of length scale contains no directional information. Especially for flows with shear in multiple directions this appears inadequate. Some researchers have attempted to develop an equation for the length scale based on a two-point velocity correlation tensor (Wolfshtein (1980)). For homogeneous turbulence it can be shown that the resulting models are equivalent to the standard form of the model for the dissipation-rate equation. It is also possible, without too much loss of generality, to derive the dissipation equation from the two-point correlation tensor, see Speziale (1989). Hence it is concluded that at least some of the criticism mentioned above is not justified.

None of the proposed modifications to the approximate dissipation-rate equation did result in a generally well-behaved model applicable to wide class of flows. Therefore, they did not find a wide acceptance in the CFD community. Eq. (2.13) is still the most widely used and successful form for the dissipation equation, even though it contains a number of serious deficiencies. In the light of this Speziale (1989) stated that "the kind of ad hoc adjustments in the modelled dissipation rate equation that have been considered during the past decade appear to be counter-productive".

2.2.7 The diffusive transport of Reynolds stresses

Owing to the relative minor importance of the diffusion of the Reynolds stresses, the models for the diffusion term received not as much attention as the pressure-strain and dissipation terms. Mostly only one of the three diffusion terms is taken into account, namely the turbulent diffusion of the turbulent stresses given by the third-order correlation. The model for this term is again inspired by a transport equation for this third-order correlation. Upon drastic simplification of this transport equation one arrives at the following expression for $\overline{u_i u_j u_k}$.

$$\overline{u_i u_j u_k} = -C_s \frac{k}{\epsilon} \left(\overline{u_i u_l} \frac{\partial \overline{u_j u_k}}{\partial x_l} + \overline{u_j u_l} \frac{\partial \overline{u_i u_k}}{\partial x_l} + \overline{u_k u_l} \frac{\partial \overline{u_i u_j}}{\partial x_l} \right)$$
(2.14)

Even this expression generates a massive amount of terms in anything but a simple shear flow in a cartesian coordinate system. For this reason expression (2.14) is often simplified into:

$$\overline{u_i u_j u_k} = -C_s^* \frac{k}{\epsilon} \left(\frac{\partial u_i u_j}{\partial x_l} \right)$$
(2.15)

which is simply expression (2.14) with the first two terms on the right-hand side discarded. As a result of this simplification expression (2.15) does not satisfy the invariance condition. Despite this fundamental shortcoming expression (2.15) has been used with about equal success as expression (2.14), indicating the minor role the diffusion of the stresses plays in the transport equations for the Reynolds-stresses.

2.2.8 The basic Reynolds-stress turbulence model

If we collect all modelled terms, substitute them into the Reynolds-stress transport equations, combine the equations for the Reynolds stresses with the equation for the dissipation rate and the equations for the mean flow variables one ends up with a closed set of equations. The combination of Reynolds-stress transport equations and the equation for the dissipation rate is usually denoted as "the Reynolds-Stress Model" (RSM) of turbulence. It is emphasized that this basic version of the model is capable of handling flows at high Reynolds number, far away from walls, without buoyancy effects and chemical reactions and only in a stationary frame of reference. It has the following form:

Rate of change
$$\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} =$$

Production

Pressure-strain interaction $-C_1 \frac{\epsilon}{k} \left(\overline{u_i u_j} - \frac{2}{3} \delta_{ij} k \right) - C_2 \left(P_{ij} - \frac{2}{3} \delta_{ij} P \right)$ Dissipation $-\frac{2}{3} \delta_{ij} \epsilon$ (2.16)

 P_{ii}

Diffusive transport
$$-\frac{\partial}{\partial x_k} \left[C_{*}^* \frac{k}{\epsilon} \left(\frac{\partial \overline{u_i u_j}}{\partial x_l} \right) \right]$$

and

$$\frac{\partial \epsilon}{\partial t} + U_l \frac{\partial \epsilon}{\partial x_l} = \left(C_{\epsilon 1} \frac{P}{\epsilon} - C_{\epsilon 2} \right) \frac{\epsilon^2}{k} + \frac{\partial}{\partial x_l} \left(C_{\epsilon} \frac{k}{\epsilon} \overline{u_k u_l} \frac{\partial \epsilon}{\partial x_k} \right)$$
(2.13)

For most flow configurations, defining appropriate initial and boundary conditions needed for these equations may be far from trivial. Since the model is not suited for regions close to solid walls, the "boundary" conditions have to be prescribed in a region far away from solid walls. Often these "boundary" conditions are based on empirical relations. Defining, a proper initial condition is especially difficult for the rate of dissipation ϵ . For most flows of practical interest, direct measurement of ϵ is not feasible. Hence, yet again the initial condition for ϵ must be derived using empirical relations.

Though the model was developed during the seventies and the potential of it was appreciated from the beginning, it took almost two decades until the practical use of the model reached some degree of acceptance. In 1981 at the land-mark Stanford Conference, Kline *et al.* (1982), the Evaluation Committee declined to acknowledge a demonstrated superiority of the second-order closures over traditional closures. According to Launder (1989) one of the reasons was the limited computational capability, resulting in, for example, solutions with unacceptably high levels of numerically induced diffusion. However, since computer power has undergone a continual growth, it is only now that the real capabilities of the second-order closure schemes are gaining full appreciation.

2.3 Derivation of lower-order models

2.3.1 The Algebraic Stress Model

Now that a model for the second-order turbulence closure scheme is established, it is possible to derive lower-order schemes from it. The first "closure schemes" were developed much carlier than the RSM, the first one being the mixing-length model of Prandtl (1925). Hence, by deriving the low-order schemes from the second-order closures we reverse the chronological order of events. However, by using this ordering of the presentation of the various turbulence models the connections between the closure schemes and more importantly the differences in approximations used, can be elucidated more clearly.

As indicated above, at the time the Rcynolds-stress model was developed the computational capabilities were inadequate to make use of an "expensive" closure scheme like the RSM for a wide class of flows. In an attempt to reduce the costs of 7 additional partial differential equations the Algebraic Stress Model was developed. The ASM is based upon the observation that the only terms containing derivatives in the equations of the RSM of the Reynolds stresses are the convection and diffusion terms. Remove these terms and the equations become algebraic in the stresses. Instead of completely removing the differential forms of the Reynolds stresses, Rodi (1976) proposed to approximate the convection and diffusion terms by terms which are algebraic in the Reynolds stresses, i.e.

$$\operatorname{Convection}(\overline{u_i u_j}) \approx \frac{\overline{u_i u_j}}{k} \operatorname{Convection}(k)$$

and

$$\operatorname{Diffusion}(\overline{u_iu_j}) pprox rac{\overline{u_iu_j}}{k} \operatorname{Diffusion}(k)$$
 .

In a stationary flow the total transport, i.e. the convection minus the diffusion, of the turbulent kinetic energy equals the difference of production and dissipation of turbulence

$$\operatorname{Convection}(k) - \operatorname{Diffusion}(k) = P - \epsilon$$

Hence,

$$\text{Convection}(\overline{u_i u_j}) - \text{Diffusion}(\overline{u_i u_j}) \approx \frac{\overline{u_i u_j}}{k} \left(P - \epsilon\right) \; .$$

Substituting this expression into the basic RSM equations (2.16) yields the ASM equations.

$$\frac{\overline{u_i u_j}}{k} = \frac{(1 - C_2) \left(P_{ij}/\epsilon - \frac{2}{3} \delta_{ij} P/\epsilon \right)}{C_1 + P/\epsilon - 1} + \frac{2}{3} \delta_{ij}$$
(2.17)

One unknown remains in these expressions, the turbulent kinetic energy k. An equation for k may be found by contracting the approximate Reynolds-stress transport equations (2.16). This yields a transport equation for k.

$$\frac{\partial k}{\partial t} + U_l \frac{\partial k}{\partial x_l} = P - \epsilon + \frac{\partial}{\partial x_l} \left(C_k \frac{k}{\epsilon} \overline{u_m u_l} \frac{\partial k}{\partial x_m} \right)$$
(2.18)

with C_k a constant of order unity.

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Instead of seven non-linear partial differential equations the closure now consists of two non-linear partial differential equations, one for ϵ and one for k, and a system of six algebraic equations. One of the consequences of the algebraic approximation of the transport terms is that memory effects in the turbulence are neglected. The Reynolds-stress tensor becomes an instantaneous function of the velocity field.

2.3.2 Two-equation eddy-viscosity models

In most calculations for engineering purposes the $k - \epsilon$ turbulence model is still used. Its merits are simplicity, robustness and economy. However, it is based on further drastic assumptions on the structure of the turbulence. Compared to the RSM it has a more limited applicability, especially in flows with secondary strains. The connection with the RSM can be appreciated by considering a simple shear flow, outside the immediate vicinity of walls, with $\partial U_1/\partial x_2$ as only non-zero strain (Fig. (2.1)). According to Eq. (2.17), the



Figure 2.1: A flow subject to a single shear

dominant stress in the 1-component of the Navier-Stokes equation, $\overline{u_1u_2}$ can be expressed as,

$$\overline{u_1u_2} = -\frac{1-C_2}{C_1+P/\epsilon-1} \frac{k}{\epsilon} \overline{u_2}^2 \frac{\partial U_1}{\partial x_2}$$

In the same manner also $\overline{u_2^2}$ can be determined from Eq. (2.17).

$$\overline{u_2}^2 = -\frac{2}{3} \left(\frac{1 - C_2}{C_1 + P/\epsilon - 1} \frac{P}{\epsilon} - 1 \right) k$$

Substituting the last expression into the former results in:

$$-\overline{u_1u_2} = \frac{2}{3} \frac{1-C_2}{C_1} \frac{C_1 + C_2 P/\epsilon - 1}{\left(C_1 + P/\epsilon - 1\right)^2} \frac{k^2}{\epsilon} \frac{\partial U_1}{\partial x_2}$$

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In a simple shear flow, like the outer region of a turbulent boundary-layer flow, the total production, P and the rate of dissipation, ϵ , will be equal (local equilibrium). By assuming local equilibrium and isotropy of the turbulent stresses everywhere this result is generalized and the Boussinesq-approximation for the turbulent stresses is obtained.

$$-\overline{u_i u_j} = \nu_\ell \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k$$
(2.19)

with u_t given by $u_t = C_{\mu}k^2/\epsilon$ and C_{μ} a constant, for most flows $C_{\mu} = 0.09$

Together with the transport equations for k and ϵ again a closed set of equations is obtained. However, a gradient approximation of the diffusion terms in the k and ϵ transport equation is more consistent with an effective viscosity model for the turbulent stresses. Thus these equations simplify into,

$$\frac{\partial k}{\partial t} + U_l \frac{\partial k}{\partial x_l} = P - \epsilon + \frac{\partial}{\partial x_l} \left(\frac{\nu_l}{\sigma_k} \frac{\partial k}{\partial x_l} \right)$$
(2.20)

and

$$\frac{\partial \epsilon}{\partial t} + U_l \frac{\partial \epsilon}{\partial x_l} = \left(C_{\epsilon 1} \frac{P}{\epsilon} - C_{\epsilon 2} \right) \frac{\epsilon^2}{k} + \frac{\partial}{\partial x_l} \left(\frac{\nu_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_l} \right)$$
(2.21)

Together, Eq. (2.19), (2.20) and (2.21) form the $k - \epsilon$ turbulence model. The closure scheme now consists of two coupled non-linear partial differential equations and an explicit expression for $\overline{u_i u_j}$. The $k - \epsilon$ model is still a transport model, i.e. it allows transport of turbulence. However, coefficients like C_{μ} are evaluated assuming local equilibrium. Hence models like the $k - \epsilon$ model must be considered as first-order corrections for "local equilibrium" models like the Prandtl mixing-length model.

The $k-\epsilon$ model is just one of the many possible two-equation eddy-viscosity models. In general any combination of quantities can be used that yield the correct dimension for the eddy viscosity, i.e.

$$\left[\nu_{t}\right]=\left[U\right]\left[L\right],$$

where [U] denotes a typical velocity scale and [L] a typical length scale of the flow. In the $k-\epsilon$ model the velocity scale is provided by the square root of the turbulent kinetic energy, $k^{1/2}$, while the length scale is provided by a combination of k and ϵ , $k^{3/2}/\epsilon$. Examples of alternative two-equation turbulence models are, the $k-\omega$ model of Wilcox (1988), the $k-\omega^2$ model of Wilcox & Rubesin (1980), the q-f model of Smith (1984) and most recently the k-r model of Speziale *et al.* (1992).

2.3.3 One-equation and mixing-length models

Instead of formulating an equation to fix the length scale one can also explicitly specify a length scale. The turbulence model then reduces to,

$$u_i = C^*_\mu \sqrt{kL}$$

 $\mathbf{21}$

the Kolmogorov-Prandtl expression, and

$$\frac{\partial k}{\partial t} + U_l \frac{\partial k}{\partial x_l} = P - C_D k^{3/2} / L + \frac{\partial}{\partial x_l} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial x_l} \right)$$

where L denotes a typical length-scale for momentum exchange, and C_D a constant.

The problem of course is to find an expression for L. In general it is only possible in very simple boundary-layer-like or jet-like flows. In these simple cases the length scale is determined by arguments based on a dimensional analysis. An important assumption in this analysis is again that of "local equilibrium".

The last simplification is to assume that "local equilibrium" is obeyed everywhere. Hence, all transport terms of turbulence disappear and the transport equation for the turbulent kinetic energy reduces to:

$$P = \epsilon \text{ or } 1 = P/\epsilon = C_{\mu} \frac{k^2}{\epsilon^2} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

Substituting $\epsilon = k^{3/2}/L$ and using $\nu_t = C_{\mu}k^{1/2}L$ for the eddy viscosity yields,

$$\nu_{i} = C_{\mu}^{-3/2} L^{2} \left[\frac{\partial U_{i}}{\partial x_{j}} \left(\frac{\partial U_{i}}{\partial x_{j}} + \frac{\partial U_{j}}{\partial x_{i}} \right) \right]^{1/2}$$
(2.22)

Thus the closure scheme has been reduced to finding an expression for L.

2.4 Modifications of turbulence models

2.4.1 Wall effects and low-Reynolds-number adaptations

All models described in the preceding sections are based on the "high-Reynolds-number hypothesis". Of course this hypothesis is often not applicable. Examples are flows with strong favourable pressure gradients, where relaminarization can occur, or flows in the immediate vicinity of a solid wall, where turbulent fluctuations in the direction normal to the wall are suppressed. In these situations the standard turbulence closures fail. Either one has to abandon the turbulence closure schemes, for example by using empirical relations to characterize the turbulence, or one has to modify the turbulence model. The first approach is often used in wall-bounded flows where the wall region is approximated using a logarithmic law of the wall. Since it is an empirical relation its applicability is limited. In flows with strongly curved streamlines, flows close to separation, or flows close to reattachment the velocity profile and the turbulence structure may deviate from the logarithmic law of the wall. For accurate predictions of, for example the wall shear stress, the second approach, of modifying the turbulence models may be more appropriate.

The basic Reynolds Stress Model, Eqs (2.16) is not fully satisfactory even before the immediate vicinity of a solid wall is reached. The model for the pressure-strain interaction

is derived assuming a non-bounded domain. In order to account for the wall effects in the region "not-too-near" to a solid wall, wall-reflection terms are added to the pressure-strain model, e.g. Launder *et al.* (1975). These extra terms are relatively small in magnitude, but have a long range ($\sim 1/x_2$, were x_2 is the distance to the wall, see Fig. (2.2)).



Figure 2.2: The flow close to a solid wall

In the immediate vicinity of the wall, apart from wall reflection effects, also the effect of viscosity must be taken into account. The most important effects are:

- In the viscous sublayer the molecular transport will be larger than the turbulent transport.
- the scales of the energy-containing eddies and the dissipative motions overlap. The dissipative motions do not obey local isotropy.
- the large scale motions are likely to be influenced by viscosity. The coefficients will be functions of at least Re_t (= $k^2/\nu\epsilon$).

An indication for the required form of the modifications can be extracted from the limiting behaviour of the Reynolds stresses in the viscous sublayer. This behaviour can be understood by employing Taylor series expansions for the fluctuating velocity components, e.g. $u_1 = a_1x_2 + b_1x_2^2 + \ldots$, with x_2 taken normal to the wall and the coefficients a_i, b_i, \ldots to be random functions of t, x_1 and x_3 , but not of x_2 . In a developed wall layer, the continuity equation imposes,

$$\begin{array}{rcl} \overline{u_1}^2 &=& \overline{a_1}^2 \, x_2^2 + \dots \, , \\ \overline{u_2}^2 &=& \overline{b_2}^2 \, x_2^4 + \dots \, , \\ \overline{u_3}^2 &=& \overline{c_1}^2 \, x_2^2 + \dots \, , \\ \overline{u_1} \overline{u_2} &=& \overline{a_1} \overline{b_2} \, x_3^3 + \dots \, , \\ \text{and} & k &=& \frac{1}{2} \left(\overline{a_1}^2 + \overline{c_1}^2 \right) \, x_2^2 + \dots \end{array}$$

Thus as x_2 approaches zero,

- $\overline{u_2^2}$ tends to zero faster than $\overline{u_1^2}$ and $\overline{u_3^2}$;
- $\overline{u_1u_2}/k$ tends to zero; and
- $\overline{u_1 u_2} / \sqrt{\overline{u_1}^2} \sqrt{\overline{u_2}^2}$ may be uniform across the sublayer.

Using the same Taylor-series expansion it is also easy to verify that at the wall the rate of dissipation can be written as

$$\frac{\epsilon}{k} = \epsilon_{11}/\overline{u_1}^2 = \epsilon_{33}/\overline{u_3}^2 = \frac{1}{2}\epsilon_{12}/\overline{u_1}\overline{u_2} = \frac{1}{4}\epsilon_{22}/\overline{u_2}^2$$

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and that the complete pressure-strain interaction, $\frac{1}{p}\left(u_i\frac{\partial p}{\partial x_i}+u_j\frac{\partial p}{\partial x_i}\right)$, will vanish at the wall. Thus the structure of the turbulence in the sublayer clearly differs from the structure in the high-Reynolds-number limit, illustrating the need for low-Reynolds-number modifications.

The first efforts towards low-Reynolds-number extensions for second-order closures were made by Dalcy *et al.* (1970), and Hanjalic & Launder (1976). Though not asymptotically correct the extended models performed quite well, e.g. Lai & So (1990). An overview of more recent efforts in this direction is given by Hanjalic (1990), Launder & Shima (1989) and Lai & So (1990).

For the class of eddy-viscosity models much more work was done on low-Reynoldsnumber extensions. A review is given by Patel *et al.*(1985), who analysed eight different extensions. In general the modifications of the $k - \epsilon$ model consist of:

- viscous diffusion is explicitly represented in the momentum, the k and the ϵ transport equations;
- a wall damping function is introduced in the Boussinesq-approximation to account for the suppression of fluctuations normal to the wall;
- the $C_{t,2}$ coefficient in the dissipation rate equation is made dependent on Re_t in order to accommodate to the final stage of decaying grid turbulence;
- extra source terms are included in the equation for the rate of dissipation; and
- for numerical convenience often $\bar{\epsilon} = \epsilon \epsilon_{wall}$ is used as dependent variable.

According to Patel *et al.* (1985) most of the proposed modifications lack a sound physical basis and are tailored to one specific type of flow.

2.4.2 Modifications for curved flows

The standard $k - \epsilon$ model, being an effective viscosity model, will have difficulties in accounting for the effects of multiple strains. An effective viscosity model assumes the stresses and strains to be aligned, which is not necessarily the case for flows subject to more than one strain. However, in the case of "mildly" complex flows, where one strain dominates, the effect of a secondary strain on the structure of the turbulence may be predictable.

A simple example of a "mildly" complex flow is a flow with a density gradient perpendicular to the direction of the mean flow. Depending on the sign of the density gradient the typical length scale on which momentum transfer occurs is either increased or decreased. Analogous to flows with a density gradient Bradshaw (1973) proposed a similar behaviour for flows with streamline curvature. Depending on the sign of the gradient of the angular momentum the flow is stabilized or destabilized, i.e. the typical length scale of turbulence is changed.

$$\frac{l_m}{l_{m0}} = 1 - \beta \frac{c}{\partial U_1 / \partial x_2} = 1 + \beta R i$$

with

 $\begin{array}{ll} \beta & \text{ an empirical coefficient, often this constant is large, $\mathcal{O}(10)$,} \\ e & \text{ the extra strain acting on the fluid, for example, U_1/R,} \\ & & & & \\ & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & &$

and Ri a Richardson number for curved flows, in this case $Ri = \frac{U_1/R}{\partial U_1/\partial x_2}$.

For flows with longitudinal curvature of the streamlines the extra strain ϵ , will be of the order $\epsilon \sim U_1/R$ where R denotes the radius of the curvature (see Fig. 2.3). For flows



Figure 2.3: A flow with longitudinal curvature

with swirl, e will be of the order $e \sim \Omega$, where Ω denotes the angular velocity.

For the class of the two-equation eddy-viscosity models more general adaptations are possible. Again the basic assumption is that primarily the length scales are affected by streamline curvature. In the Boussinesq-approximation an effective viscosity is proposed, constructed from the product of a typical velocity scale and a typical length scale,

$$u_t = C_\mu \left(k^{1/2}
ight) \left(k^{3/2}/\epsilon
ight) \; .$$

The curvature effects can be accounted for in two ways, directly through a modification of the coefficient C_{μ} , or indirectly by a modification of the equation determining the length scale (in this case the ϵ equation).

Examples of the latter approach are given by Launder *et al.* (1977), Rodi (1979) and Shreenivasan & Padmanabhan (1980). In the approach of Launder *et al.* (1977) the sink term in the ϵ -equation is modified according to

Sink =
$$C_{\epsilon,2} \left(1 - C_c R i_t\right) \frac{\epsilon^2}{k}$$
,

where for longitudinal curvature

 C_c denotes a coefficient in the range $0 \dots 0.5$

and

Rit denotes a turbulent Richardson-number defined by,

$$Ri_t = \frac{k^2}{\epsilon^2} \frac{U_1}{R} \frac{\partial U_1 R}{\partial x_2}$$

For flows with swirl the same correction is used, however with a different value of C_c . It is easily appreciated that for high rotation rates this modification is not well-behaved because the energy dissipation rate can become negative. Rodi *et al.* (1979) used a similar approach but partly to overcome the problems with Launder's modification they modified the source term in the ϵ -equation instead. Leschziner & Rodi (1981), Pourahmadi & Humphrey (1985) and Pougare & Lakshminirayana (1983) are examples of the first approach mentioned above. Instead of modifying the ϵ -equation, they adapted the coefficient C_{μ} in order to reflect effects of streamline curvature. For a 2-D channel flow, Pourahmadi & Humphrey (1985) proposed a single expression for C_{μ} as a function of the flow field.

$$C_{\mu}^{1/2} = 2\sqrt{Q}\cos\left[\frac{1}{3}\cos^{-1}\left(RQ^{-2/3}\right)\right] - \frac{S}{3}$$

Q, R and S represent here complicated functions of P, ϵ and the velocity gradients.

Leschziner & Rodi (1981) used a similar approach, also making the coefficient C_{μ} a function of flow-field quantities. Pougare & Lakshminirayana (1983) used two different expressions for C_{μ} for the momentum exchange in streamwise and spanwise direction to account for a non-alignment of stress and strain. An overview of several attempts to sepsitize the eddy-viscosity models to additional strains is given by Lakshminirayana (1986).

The above mentioned modifications have one thing in common, they are all meant to replace the RSM or ASM closure scheme. However, as already indicated in section (2.2.6) also the RSM or ASM model contain deficiencies with respect to some aspects of rotating flows. An expected effect of system rotation on the dissipation rate of turbulence is neither correctly reflected by the basic RSM and ASM nor by the modifications mentioned above.

Intuitively, it is clear that an effect of rotation must be present. The transport of energy from the large-scale eddies towards the small-scale eddies is caused by vortexvortex interaction. Clearly, a large-scale background rotation will alter the dynamics of this interaction. In some situations the presence of strong rotational effects even reverses the direction of the energy cascade. Hence, at least one expects rotation to slow down the energy cascade. Indeed, this trend has been observed experimentally, e.g. Traugott (1958) and Wigeland & Nagib (1978), in decaying grid turbulence. To account for this effect Bardina *et al.* (1985) suggested an extra sink term in the ϵ -equation,

$$\operatorname{Sink}_{\operatorname{extra}} = -C_{\Omega} \in \Omega$$

linear in the rotation rate, Ω and linear in the dissipation itself. To account for rotation effects in non-homogeneous flows Bardina *et al.* (1985) proposed,

$${
m Sink}_{\,{
m extra}} = - C_\Omega \; \epsilon \; \sqrt{\Omega_{ij}\Omega_{ij}}/2$$

where

$$\Omega_{ij}$$
 is given by,

$$\Omega_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right)$$

However, this generalized modification again leads to a model that becomes ill-behaved for situations in which a large rotation is combined with a strong deformation.

2.5 Calculations of swirling flows

2.5.1 Calculations based on eddy-viscosity models

As indicated in section 2.4.2 one can expect eddy-viscosity based models to be inadequate for swirling flows. According to the literature the inability of the $k - \epsilon$ model to reflect the salient features of swirling flow is the assumed proportionality of the stress tensor and the strain tensor. A classical example of the performance of the $k - \epsilon$ model is given by Boysan & Swithenbank (1982), see Fig. 2.4. In contrast to this poor performance, the ASM



Figure 2.4: Predicted and experimental cross-sectional velocity profile in a vortex tube, Boysan & Swithenbank (1982), — ASM model, $--k \in model$, $\circ \circ \circ experiment$, W_0 : inlet velocity, τ_0 : radius of exit tube.

predictions appear to be much better. Nevertheless, in other geometries the deficiencies of the $k - \epsilon$ model seem less pathological. Especially when some of the above mentioned modifications of the $k - \epsilon$ model are used, results have been claimed to be quite satisfactory.

For boundary-layer-like situations the modifications of the ϵ -equation due to Launder (1977) and Rodi (1979) have proven to be workable. For example, the mean velocity components in a flow over a spinning cylinder could be predicted quite satisfactorily. However, application of such modifications in a more complex geometry like two confined co-axial swirling jets were much less successful (Shreenivasan & Padmanabhan (1980)).

Calculations of swirling pipe flows using some simple modifications for the $k - \epsilon$ model were performed by different authors. Examples can be found in Frith & Duggins (1985) and in Kobayashi & Yoda (1987). By varying the coefficient C_{μ} and by using different values of C_{μ} for the different components of the Reynolds-stress tensor it was possible to find an acceptable agreement of predictions and measurements.

2.5.2 Calculations with Reynolds-stress and Algebraic-stress models

With the continuous growth of computer capacity and the advent of efficient numerical algorithms the accent on the applications have shifted from eddy-viscosity models towards Reynolds-stress and Algebraic-stress models. While for flows with swirl in relatively sim-

ple geometries modified eddy-viscosity models seem feasible, for more complex geometries second-order closures seem indispensable. Relatively much work with second-order schemes was done in combustor-like geometries, where apart from swirl also strong recirculation zones are present. Indeed calculations by Hogg & Leschziner (1989) for such geometries indicate that the $k - \epsilon$ model structurally overpredicts the level of the shear-stress components. Hence a transition from a supercritical state to a subcritical state of the flow, akin to a hydraulic jump as can occur in free-surface flow, is not captured by the $k - \epsilon$ model. In the results of a calculation employing a RSM this phenomenon was represented.

A similar study, performed by Visser *et al.* (1987), compared predictions employing a $k - \epsilon$ model, a RSM and an ASM. Much attention was paid to the effect of the rate of confinement of the swirling jets. Visser *et al.* concluded that for a low confinement rate the use of a second-order closure model was essential for an adequate prediction. For a high confinement rate the performance of the $k - \epsilon$ model, the ASM and the RSM was comparable. Although the prediction of the length of the recirculation zone proved to be accurate and the prediction of the mean velocity components to be satisfactory, the predictions were not perfect. In all cases differences up to an order of 200% were observed between the predicted and measured components of the Reynolds-stress tensor.

Finally, for a burner-like assembly, Nikooy & Mongia (1991) studied the effect of the specific form of the model used for the pressure-strain interaction. Their results indicated that, when using the standard values for the coefficients, the use of the QI or the IP model for the "rapid part" of the pressure-strain interaction did not yield large differences in the predictions. However, varying the values of the coefficients in the pressure-strain model did produce significant differences. It appeared that the coefficients defined by Gibson & Younis (1986) yielded superior results.

Gibson & Younis (1986) performed an extensive analysis of the physics behind the pressure-strain term (see section 2.4.2) in jet-like geometries. With their set of coefficients they managed to obtain a better correlation of measurement and prediction for both the mean-flow quantities and the components of the Reynolds-stress tensor. However, as shown in Fig. (2.5), still large differences between measurement and prediction occurred in the far-field region of the jet.

Fu et al. (1987) performed calculations for the same flow problem but found that it was necessary to modify the model for the pressure-strain interaction (section 2.2.5). With this modification the agreement of measurement and prediction also improved. In addition they concluded that models based on an algebraic approximation of the turbulent stresses are not appropriate for an adequate prediction of the properties of a non-confined swirling flow.

2.5.3 Concluding remarks

In this chapter we presented an overview of the well-known one-point turbulence models. For virtually all non-trivial flows likely to be encountered in an engineering environment only calculations employing members of the class of one-point turbulence models may be expected to be feasible. However, depending on the characteristics of the flow not all members of this class are capable of accurately predicting the physics of the flow.



Figure 2.5: Predicted and measured distributions of the circumferential velocity in a single swirling jet, Gibson & Younis (1986). — Gibson & Younis pressure-strain model, – – – Launder et al. pressure-strain model, * * * experiment.

For "simple" flows like boundary-layer flows or pipe flows, models based on an effective viscosity perform very well. For flows involving more than one length scale or subject to multiple strains the eddy-viscosity models cannot represent a number of features of the turbulence which may be vital for a correct description of the flow. For these "complex" flows the second-order turbulence closure schemes, like the RSM and the ASM, appear to be indispensable. However, experience with these models is still limited, and unlike the eddy-viscosity closures, where the $k - \epsilon$ model became the "de facto" standard, no definitive variant has emerged.

The flow to be studied in this thesis, a turbulent pipe flow with decaying swirl, is an example of a flow subject to more than one strain. According to Bradshaw's (1973) nomenclature it must be described as being of a "complex" nature. However, in contrast with the "complexity" of the physics, the geometry of the problem at hand is very simple. It is this simplicity of the geometry that distinguishes this study from the majority of studies reported in open literature. Due to the simplicity of the geometry, the development of a turbulent swirling pipe flow is largely governed by the turbulent stresses. Most studies reported in the literature deal with flows in more complex geometries. Examples found in the literature include the prediction of the flow through bends, in industrial ovens, and through turbines. Flows in these geometries are likely to be influenced by more than the turbulent stresses alone. Extra stresses, for example, pressure forces, coriolis forces or centrifugal forces, may be important as well. Hence, an inaccurate description of the turbulent stresses is expected to be not as critical as it will be for the description of the relaxation of swirl in turbulent pipe flow. A second special feature of the study of the development of a turbulent swirling flow is the aspect ratio of the geometry. Experimental studies (see chapter 1) indicate that the distance required for the decay of the swirl may be large, L/D = O(100). Especially when second-order closure schemes are used and when the elliptic character of the governing equations is retained the demands on computer resources may become large as well.

Third and last special feature is that due to the simplicity of the geometry the characteristics of the mean flow are almost fixed. Since the swirl occurring in large-scale gas transport systems is relatively low and the decay of the swirl is slow, for example regions with recirculation are not likely to occur. Hence, the streamlines will have a helical shape with a slowly varying pitch.

The three above mentioned features characteristic for a turbulent pipe flow with a decaying swirl:

• the large influence of the turbulent stresses;

• the extent of the domain of interest;

and

• the almost "fixed" character of the mean flow,

distinguish this flow from the flows generally encountered in an engineering environment. These features also affect choice of the numerical treatment as well as the selection of the turbulence model.

Since the swirl is known to decay slowly, the elliptical character of the flow may expected to be weak. When the elliptical character of the flow is neglected, the economy of the numerical treatment may be enhanced dramatically by using a "marching" procedure. As a result a very fine discretization in radial direction becomes feasible which will enhance the accuracy of the prediction. However, second-order turbulence closure schemes are seldom used in combination with parabolized equations. Moreover, when the full closure schemes are retained, potentially the "well-posedness" of the system of equations may be destroyed. To be consistent with the "parabolization" of the system of equations, these turbulence closures must be simplified as well. In the present study this parabolization will be followed.

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3. Numerical aspects of predicting turbulent pipe flows

3.1 Introduction

The study of turbulent pipe flow differs in several aspects from the study of turbulent flows normally encountered in the application of computational fluid dynamics for engineering purposes where the geometry considered is often of a complex nature. As a result of the geometrical complexity the flow is governed by different dynamical mechanisms. For example, pressure forces, centrifugal forces, effects due to density gradients and turbulent stresses may be equally important.

In this study the geometry is of a simple nature, namely a straight section of a pipe. However, the flow in this pipe can be distinguished from the flows mentioned above because the aspect ratio of the geometry is large. Typically the length scale in longitudinal direction exceeds the diameter of the pipe by a factor of about $\mathcal{O}(10^2)$. Owing to this large aspect ratio the physics of the problem is simplified considerably.

A non-swirling developing pipe flow is largely governed by the equilibrium of pressure forces and turbulent stresses. The numerical treatment of the turbulent pipe flows may benefit from the corresponding simplification of the physics. The Navier-Stokes equations reduce from elliptic to a form that is of a parabolic nature. For problems of a parabolic nature no boundary conditions have to be prescribed at the downstream boundary of the computational domain. As a result a marching procedure can be used for the numerical integration of the equations. The fully 3-D problem can be reduced to a sequence of 2-D problems in successive cross-flow planes while the axisymmetric quasi-2-D problem can be reduced to a sequence of quasi-1-D problems. Thus the formulation of the numerical scheme may be less complex, the stability of the calculation may be enhanced and the computational effort is reduced considerably.

Historically, simple viscous flows like pipe flows were amongst the first ones to be tackled numerically. However, with the advent of large computers and sophisticated numerical techniques the algorithms taking advantage of the parabolic nature of some flows fell into disuse. Extensive, commercially available, software packages have been developed, aiming at the numerical solution of the Navier-Stokes equations for a wide range of applications. While these codes offer a large flexibility and efficiency for most complex flows, the flows that are of a parabolic nature are not treated efficiently by these codes.

For the prediction of the full development of a swirling pipe flow, algorithms that do not take the parabolic character of the flow into account, are less suited. Due to the long distance it takes before the swirl decays the demands on computer resources will be extreme. In chapter 4 it will be shown that the decay of swirl is largely driven by the wallshear stress. Therefore, in this work special emphasis is put on the solution in the region close to the wall. This necessitates a detailed treatment of this region. For turbulent flows it implies that low-Reynolds-number models have to be used instead of the conventional "law of the wall". Again the application of a marching procedure simplifies the use of the low-Reynolds-number models. Finally, most commercially available codes are available as a binary only. Adaptations of the turbulence models implemented in such codes is either difficult or impossible.

One of the aims of the work presented here is to study the capabilities of different turbulence models in predicting swirling pipe flows. For this purpose a code specially tailored to the prediction of pipe flows has been developed. For reasons of economy this code is based on a parabolic description of the flow. However, more emphasis was put on the flexibility of the code than on its efficiency. The code was designed to optimize the ease of the development and test process of turbulence models rather than the pure computational efficiency.

The code developed for this work is based on the finite-element software package SEPRAN. The SEPRAN package consists of a "tool box" of subroutines. With these subroutines it is possible to construct finite-element methods for solving a wide variety of problems. Flows that can be handled range from simple potential flows to viscous threedimensional and time-dependent flows. The standard software package does not contain the possibility of using two-equation or ASM and RSM turbulence models. However, the SEPRAN package is fully open and well-documented. Hence, the SEPRAN package can be adapted easily.

In the present approach only the proprocessing part, the procedures for solving the system of linear equations, and the postprocessing part of SEPRAN were utilized. Of course, for the case of axisymmetric parabolic problems, the preprocessing and postprocessing needs are only limited. The most important part of the SEPRAN package used in this work is the set of subroutines for solving a system of algebraic equations. Since SEPRAN is a finite-element package the system of equations is solved directly. While a direct solver is inefficient for large systems of equations (number of equations $\propto \mathcal{O}(10^3 \dots 10^4)$), a direct solution procedure is feasible for smaller systems of equations. Due to the parabolization, the problem reduces—for axisymmetric flows—to a series of quasi-1-D problems and small systems of equations. Hence, even though it is based on a direct solution procedure the resulting algorithm will be sufficiently fast.

In this chapter a description is given of the numerical treatment of the developing turbulent pipe flow. First the question of the parabolization of the Navier-Stokes equation will be addressed, then aspects such as stability and accuracy of simple parabolic "model" equations will be treated and finally a description is given of the implementation of the reduced Navier-Stokes equations in a finite element environment.

The work described in this chapter is inspired to a large extent by material in standard textbooks. The most important sources of information used here are Anderson *et al.* (1984), Baker (1985), Fletcher (1988a), (1988b) and Hirsch (1990).

3.1.1 The reduced Navier-Stokes equation and boundary-layer equations

A large class of flow problems is characterized by the appearance of a dominant flow direction. Non-swirling pipe flow belongs to this class of flows. Due to the large aspect ratio $(R/L \ll 1)$ of the geometry, the length scale relevant for changes in the axial flow component is large compared to the length scale connected with the variations of the cross-flow-plane velocity component. For axisymmetric flow an order-of-magnitude analysis of the continuity equation, written in cylindrical coordinates,

$$\frac{\partial}{\partial x}(rU) + \frac{\partial}{\partial r}(rV) = 0$$

assuming that the gradients in axial direction scale with 1/x and the gradients in radial direction scale with 1/R, gives the expected smaller order of magnitude of the radial velocity component

$$rac{V}{U} = \mathcal{O}(rac{R}{x}) \ll 1.$$

In principle the derivation of the boundary-layer equations or of the reduced Navier-Stokes equations must be performed for laminar and turbulent flows separately. However, the method of obtaining the reduced Navier-Stokes equations or boundary-layer equations is similar. Moreover, often the effects of turbulence are modelled with an effective viscosity. Arguments applying to laminar flows thus will also apply to turbulent flows modelled with an effective viscosity. According to Rubin (1984), in straight pipes the flow can be categorized in four types. The type of flow depends on the distance from the entrance. The four categories are (Fig. 3.1):



Figure 3.1: Flow regimes in a developing pipe flow according to Rubin (1984)

- Immediate entrance flow, x is order O(R/Re).
- Entry region flow, x is order $\mathcal{O}(R)$.
- Fully viscous flow, x is order O(RRe).
- Fully developed flow, $x \gg RRe$.

(Note that for turbulent flows Re must be replaced by a Reynolds number based on an "eddy" viscosity, $Re^* = UR/\nu_{eddy}$)

In the first region the flow experiences severe gradients close to the wall of the pipe where the inflow velocity is suddenly reduced to zero at the wall. For an accurate description of the flow field in this region it is required to apply the full Navier-Stokes equations. In the other three regions the Navier-Stokes equations can be simplified.

In the second region, O(1) < x/R < O(Re) a boundary layer starts to develop at the wall. Its development can be described by a simplified set of equations. Most salient features of these simplified equations are:

- the assumption that the diffusion of axially directed momentum may be neglected in streamwise direction;
- the pressure is constant across the boundary layer; and
- the velocity component normal to the wall may be approximated using the continuity equation.

For a boundary layer developing along the wall of a pipe, up to order $\mathcal{O}((R/x)^2)$ and in non-dimensional form, the axial momentum, radial momentum and continuity equation are approximated by,

(using
$$x = x'/R$$
, $\tau = \tau'/R$, $U = U'/U_{mean}$, $V = V'/U_{mean}$
and $P = P'/\rho U_{mean}^2$)
axis momentum

$$\frac{\partial}{\partial x}\left(U^{2}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(rUV\right)=-\frac{dP}{dx}+\frac{1}{Re}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)\right],$$

radial momentum

$$\frac{\partial P}{\partial r} = 0$$

and

continuity

$$rac{\partial}{\partial x}(rU)+rac{\partial}{\partial r}(rV)=0\;,$$

respectively.

The flow in the core region is considered inviscid. Here the flow is dominated by the equilibrium of the axial pressure gradient and the acceleration of the flow in axial direction. Actually, the elliptic character of the equations is reflected in the pressure gradient. However, also in the core region the axial pressure gradient may be considered uniform to first order. Thus, in a numerical scheme, the axial pressure gradient may be determined by the enforcement of conservation of the total mass flow, i.e.

$$\frac{\partial P}{\partial x}^{n+1} = \frac{\partial P}{\partial x}^n + \lambda \varepsilon_{\text{mass flow}}^n$$

with λ representing an appropriate Lagrangian multiplier, and $\varepsilon_{\text{mass flow a}}$ "mass defect" caused by truncation errors. As a result the above set of equations may also be used for the core region and a marching procedure is feasible.

Sufficiently far downstream, x/R = O(Re), the "boundary layer" fills the entire cross-sectional area and the flow must be considered viscous throughout the cross-section of the pipe. Eventually, $x/R \gg O(Re)$, the variation of the velocity distributions in the axial direction will have disappeared and the flow can be considered fully developed. In these last two regimes one may expect that the application of a marching scheme based on the equations mentioned above is most appropriate.

For an axisymmetric swirling flow the situation is slightly more complicated. The system of equations is extended with the equation for the conservation of angular momentum. Furthermore, the approximation of the equation for the conservation of radial momentum changes. The simplest adaptation of this equation is to assume an equilibrium between the radial pressure gradient and the centrifugal forces. However, even in this simple formulation the feasibility of a marching scheme can be endangered. As, the magnitude of the swirl changes or even as the distribution of angular momentum changes in downstream direction, also the radial distribution of the pressure will change. As a result, at a given axial position, the axial pressure gradient may be non-uniform. Near the pipe centre-line adverse pressure gradients can occur, while in the near-wall region the pressure gradient may still be favourable. For too strong adverse pressure gradients a parabolic description is invalid.

The radial redistribution of angular momentum and axial momentum are closely coupled. Strong redistribution of angular momentum often implies stagnant or reversed flow near the pipe axis. This effect is closely related to the vortex-breakdown phenomenon often encountered in vortex tubes, see Benjamin (1962).

A flow in solid-body rotation in a pipe of constant diameter, , at sufficiently high rotation rates, can sustain inertial wave motions and allow upstream wave propagation. Consequently, in general a parabolic description of the flow is invalid. For low rates of rotation inertial wave motions cannot be sustained any more. For a solid-body rotation the critical swirl intensity can be calculated. Expressed in terms of a "swirl number",

$$S = \frac{\int\limits_{0}^{R} UWr^2 dr}{R\int\limits_{0}^{R} U^2 r dr}.$$

according to Benjamin (1962) for a solid-body rotation, the critical value appears to be S = 0.96. For lower swirl intensities information of a downstream disturbance can no longer propagate upstream and a parabolic description will be valid. The critical swirl number depends on the distribution of angular momentum and the shape of the axial velocity distribution. For a general case, a universal critical swirl number cannot be defined.

Generally, swirl induced in piping systems is of a low intensity, S < 0.5. Moreover, to first approximation, the distribution of angular momentum as generated by for example

a combination of out of plane bends, may be considered as the distribution of angular momentum due to solid-body rotation. Thus we do not expect that the vortex-breakdown phenomenon is of large relevance for the metering problem.

Up to order $\mathcal{O}((R/x))^2$, an axisymmetric pipe flow up to moderate swirl intensities can be approximated by,

axial momentum

$$\frac{\partial}{\partial x}\left(U^{2}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rUV\right) = -\frac{\partial P}{\partial x} + \frac{1}{Re}\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial U}{\partial r}\right]\right),\tag{3.1}$$

radial momentum

$$\frac{W^2}{r} = \frac{\partial P}{\partial r},\tag{3.2}$$

angular momentum

$$\frac{\partial}{\partial x}(UW) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2VW) = \frac{1}{Re}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left[\frac{\partial W}{\partial r} - \frac{W}{r}\right]\right)$$
(3.3)

and

continuity

$$\frac{\partial}{\partial x}\left(rU\right) + \frac{\partial}{\partial r}\left(rV\right) = 0.$$
(3.4)

In general the swirling flow as encountered in piping systems will not be axisymmetric. For the description of non-axisymmetric pipe flows, even without swirl, the boundarylayer equations are clearly inadequate. In this case both the cross-flow-plane momentum equations have to be taken into account. Similar to the boundary-layer approach in these additional equations the axial diffusion terms may be neglected. However, as demonstrated, for example by Fletcher (1988b), additional measures have to be taken to obtain a wellposed initial value problem. In the system of momentum equations and the continuity equation the elliptic character is not only reflected in the diffusion terms but also in the pressure terms. Due to this character in downstream direction exponentially growing modes are possible. In order to prevent the occurrence of these modes additional simplifications of the system of equations have to be made. For a non-swirling flow a sufficient measure to obtain a well-posed system of equations is to neglect the pressure variation in the cross-flow-plane direction in the axial momentum equation. For example, the axial pressure gradient in the cross-flow plane may be taken equal to the axial pressure gradient at the centre line. Since in this study a finite-element-method is employed, a curvi-linear mesh is not needed for an adequate representation of the cross-sectional area. Hence, without loss of accuracy the system of equations may be represented in a cartesian coordinate system. Up to order $\mathcal{O}((R/x)^2)$ the equations are,

axial momentum

$$\frac{\partial}{\partial x}\left(U^{2}\right) + \frac{\partial}{\partial y}\left(UV\right) + \frac{\partial}{\partial z}\left(UW\right) = -\frac{d}{dx}P_{\text{centre-line}} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial U}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial U}{\partial z}\right]\right) , \quad (3.5)$$

cross-flow-plane momentum, y component

$$\frac{\partial}{\partial x}(UV) + \frac{\partial}{\partial y}\left(V^{2}\right) + \frac{\partial}{\partial z}(VW) = -\frac{\partial P}{\partial y} + \frac{1}{Rc}\left(\frac{\partial}{\partial y}\left[\frac{\partial V}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial V}{\partial z}\right]\right) , \qquad (3.6)$$

cross-flow-plane momentum, z component

$$\frac{\partial}{\partial x}(UW) + \frac{\partial}{\partial y}(VW) + \frac{\partial}{\partial z}\left(W^2\right) = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial W}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial W}{\partial z}\right]\right)$$
(3.7)

and

continuity

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0.$$
(3.8)

Similar to the axisymmetric flow the centre-line pressure gradient can be obtained by imposing conservation of total mass flow.

For a strongly swirling flow, a uniform axial pressure gradient is clearly inaccurate. To balance the centrifugal forces, the swirl induces a cross-flow-plane pressure gradient. Similar to the situation of an axisymmetric flow with swirl, the decay of the swirl induces a non-uniform distribution of the axial pressure gradient. However, this situation is more complex. Since non-axisymmetric flows regimes are allowed, also the cross-flow-plane pressure variation may be non-axisymmetric. Moreover, since the centre of swirl may vary in downstream direction even without an appreciable decay, still a substantial variation of the axial pressure gradient within the pipe cross-section may occur. Finding an accurate description for the axial pressure gradient without destroying the well-posedness of the scheme is not trivial in this situation.

3.2 Initial value problems

3.2.1 A finite-element formulation for parabolic problems

The system of equations for the prediction of developing pipe flow can be viewed upon as a non-linear initial-value problem. Combined with, for example, a two-equation turbulence model it yields a set of coupled partial differential equations (pde's). Apart from the continuity equation and the equation for the cross-flow-plane pressure variation, the equations have a structure comparable to the structure of a time-dependent diffusion problem. In this analogy the axial coordinate serves as the time-like coordinate. For time-dependent diffusion problems a wealth of integration schemes is available, both explicit and implicit. However, the choice of a "time-integration" method for a parabolized set of flow equations may be limited by the characteristics of the problem. Explicit schemes have a maximum admissible integration step that is given by a Courant-Friedrichs-Lewy -like condition for temporal stability, e.g.

$$rac{lpha\Delta au}{\Delta\xi^2} < \mathcal{O}(1) \; ,$$

For the parabolized Navier-Stokes equations, this CFL condition often leads to servere restrictions on the integration step size in the time-like direction. In most situations the flow will conform to a no-slip condition at the pipe wall. As a result the effective "time" step

$$\Delta \tau_{\rm eff} = \Delta x / U\left(\xi\right)$$

will exhibit a large variation in the pipe cross-section. Owing to the no-slip condition at the pipe wall the magnitude of the axial velocity component $U(\xi)$, becomes small as the wall is approached. Thus the effective "time" step is much larger in the near-wall region than in the region close to the axis of the pipe. For a parabolized convection-diffusion equation the condition for the integration step must be rewritten as,

$$\frac{\alpha \Delta \tau}{U(\xi) \Delta \xi^2} < \mathcal{O}(1), \tag{3.9}$$

with $U(\xi)$ a typical measure for the velocity in the time-like direction within the interval $\Delta \xi$

and $\Delta \tau$ the integration step in the time-like direction

Expression (3.9) shows that the maximum admissible integration step may be limited drastically. Close to the pipe wall not only a fine cross-flow-plane grid must be used for turbulent flows, also the magnitude of the velocity will be small in the near-wall region. Especially for predictions employing a low-Reynolds-number turbulence model, condition (3.9) poses an unacceptably severe restriction on the integration step. In this particular case the first grid point must lie at a distance from the wall of typically one viscous unit. For a turbulent pipe flow this results in a maximum integration step of typically,

$$\Delta x/D < \mathcal{O}\left(Re_D u^*/U_{msan}
ight)^{-1}$$
.

where u^{\bullet} represents the friction velocity based on the wall-friction.

For a pipe flow at $Re_D = 10^4$ this yields,

 $\Delta x/D < \mathcal{O}\left(10^{-3}\right)$.

Thus for predictions of developing turbulent pipe flows, where the length of the domain is of $\mathcal{O}(100D)$, with a low-Reynolds-number turbulence model explicit integration schemes do not appear to be efficient. In the present work only implicit schemes are used. As will be shown later on, implicit schemes do not pose such strong restrictions on the integration step.

The remainder of this section is devoted to the description of a second-order accurate integration scheme for non-linear initial value problems.

Consider a time-dependent quasi-linear diffusion problem,

$$\frac{\partial}{\partial t}\phi = \nabla \cdot \left[\alpha\left(\phi\right)\nabla\phi\right] \,. \tag{3.10}$$

where ϕ denotes an arbitrary quantity and α a diffusion coefficient that may depend on ϕ .

Using any finite-element discretization scheme Eq. (3.10) can be discretized in space. The result is a set of ordinary differential equations,

$$\frac{\partial}{\partial t} \left(\mathbf{M} \cdot \vec{\phi} \right) = \mathbf{S}(\alpha) \cdot \vec{\phi} . \tag{3.11}$$

Here

 $\tilde{\phi}$ denotes the solution vector. (It is the approximation of discrete ϕ in the solution space.)

M denotes a linear "mass" operator,

and

denotes a "stiffness" operator.

This equation is integrated using the following integration scheme,

$$\left(\mathbf{M} \cdot \tilde{\boldsymbol{\phi}} \right)_{t+\Delta t} - \left(\mathbf{M} \cdot \tilde{\boldsymbol{\phi}} \right)_{t} = \Delta t \left[\Theta \left(\mathbf{S}(\boldsymbol{\alpha}) \cdot \tilde{\boldsymbol{\phi}} \right)_{t+\Delta t} + (1 - \Theta) \left(\mathbf{S}(\boldsymbol{\alpha}) \cdot \tilde{\boldsymbol{\phi}} \right)_{t} \right] +$$
 (3.12)

$$\delta(\Theta - \frac{1}{2}) \mathcal{O}(\Delta t) + \mathcal{O}(\Delta t^{2}) ,$$

(where δ denotes the Dirac-delta function).

5

For $\Theta = 0$ it is the first-order explicit Euler scheme, for $\Theta = 1$ the first-order implicit Euler scheme and for $\Theta = 1/2$ it is the second-order Crank-Nicolson integration scheme. Rearrangement of terms gives

$$\left(\left(\mathbf{M} - \Theta \Delta t \mathbf{S} \right) \cdot \tilde{\phi} \right)_{t+\Delta t} = \left(\left(\mathbf{M} + (1 - \Theta) \Delta t \mathbf{S} \right) \cdot \tilde{\phi} \right)_{t} .$$
(3.13)

If **M** and **S** do not depend on ϕ , Eq. (3.13) can be solved directly. However, in general either **M** and/or **S** will be a function of ϕ . In that case the Jacobian of the left-hand side of Eq. (3.13) has to be evaluated and the solution at time $t + \Delta t$ must be found by, for example, a Newton iteration. If Eq. (3.13) is rewritten into,

$$\mathbf{F}(\phi) = \left(\left(\mathbf{M}(\tilde{\phi}_{t+\Delta t}) - \Theta \Delta t \, \mathbf{S}(\tilde{\phi}_{t+\Delta t}) \right) \cdot \bar{\phi}_{t+\Delta t} \right) -$$

$$\left(\left(\mathbf{M}(\tilde{\phi}_{t}) + (1 - \Theta) \, \Delta t \, \mathbf{S}(\tilde{\phi}_{t}) \right) \cdot \tilde{\phi}_{t} \right) = 0 ,$$
(3.14)

and the Jacobian of $\mathbf{F}(\tilde{\phi}_{t+\Delta t})$ is denoted by,

$$\mathbf{J}(\tilde{\phi}_{t+\Delta t}) = \frac{\partial \mathbf{F}(\phi_{t+\Delta t})}{\partial \tilde{\phi}_{t+\Delta t}}$$
(3.15)

the Newton iteration scheme to get $\phi_{t+\Delta t}$ can be expressed as,

$$\mathbf{J}(\mathbf{F})^{it} \cdot (\tilde{\phi}_{t+\Delta t}^{it+1} - \tilde{\phi}_{t+\Delta t}^{it}) = -\mathbf{F}(\tilde{\phi})_{t+\Delta t}^{it} \quad , it = 0, 1, \dots$$
 (3.16)

The advantage of using a parabolic scheme will be clear. Instead of a solution vector containing the values of the unknowns in the complete solution domain, now the solution vector $\phi_{t+\Delta t}$, contains only the values of the unknowns at one axial position in the cross-flow plane. For an axisymmetric pipe flow the parabolization reduces a two-dimensional problem to a sequence of quasi one-dimensional problems, while a non-axisymmetric problem reduces from a three-dimensional problem to a sequence of quasi two-dimensional problems.

3.2.2 Finite element discretization

Thus far no attention has been given to the details of the discretization in space. As stated in the introductory section of this chapter, the finite-element type of discretization is employed by the software package SEPRAN. In essence, in a finite-element formulation a measure for the error of the discrete solution is minimized. This error measure is often denoted as the energy norm I, see Baker (1985).

Assuming that the differential equation may be written as,

$$\mathcal{L}(w) = 0 \tag{3.17}$$

with L(w) denoting a differential operator L working on a function w,

then for a Galerkin method of weighted residuals the energy norm is defined as,

$$I=\int_{\Omega}wL\left(w\right)d\Omega$$

Here Ω denotes the solution domain.

In general the unknown function w is approximated by a sum of K "basis functions".

$$w pprox ilde{w} = \sum_{i=1}^K a_i arphi_i$$

The discrete approximation of w, \tilde{w} will not obey Eq. (3.17) exactly, so

$$L(\tilde{w}) = R.$$

The Galerkin method of weighted residuals minimizes the residual in the sense that it makes the residual orthogonal to the space spanned by the "basis functions".

$$\int_{\Omega} \varphi_j L\left(\tilde{w}\right) d\Omega = 0, \quad \text{for} \quad j = 1, \dots, K.$$

Introducing the approximation for \tilde{w} yields the desired algebraic set of equations,

$$\int_{\Omega} \varphi_j L\left(\sum_{i=1}^K a_i \varphi_i\right) d\Omega = 0, \quad \text{for} \quad j = 1, \dots, K.$$

For a linear operator the order of summation and the integration can be exchanged, yielding a set of linear equations for the amplitudes of the basis functions a_i , i = 1(1)K

$$0 = \sum_{i=1}^{K} a_i \int_{\Omega_e} \varphi_j L(\varphi_i) \, d\Omega_e, \quad \text{for} \quad j = 1, \dots, K.$$
(3.18)

If the coefficients a_i are ordered in a column vector **a**, and the integrals $\int_{\Omega_e} \varphi_j \mathcal{L}(\varphi_i) d\Omega_e$ in a matrix **L**, Eq. (3.18) may be rewritten as,

 $\mathbf{L}\cdot\mathbf{a}=0.$

The structure of the matrix L depends on the form of the operator L. For the classes of problems described in this study, three basic structures can be distinguished:

- a "mass" type operator, L(u) = f(x, y, z)u;
- a "convection" type operator, $L(u) = (\vec{g}(x, y, z) \cdot \nabla) u$; and
- a "diffusion" type operator, $L(u) = \nabla \cdot (h(x, y, z) \nabla u)$.

Here f(x, y, z), g(x, y, z) and h(x, y, z) denote arbitrary functions of the spatial coordinate or the solution at a previous iteration or at a previous axial station.

Discretization of these operators yields "mass", "convection" and "diffusion" matrices, respectively. Partial differential equations like the Navier-Stokes equations or the timedependent diffusion equation mentioned at the beginning of this section can be discretized "term by term" using these matrices. For non-linear terms an iteration scheme can be built using combinations of these operators.

Apart from the nature of the operator to be discretized, the resulting matrices also depend on the choice of the "basis function". In the finite-element method the basis functions are chosen such that they are only non-zero in a small part—i.e. element—of the total solution domain. As a result the matrices will have a banded structure. The matrices can be assembled from simple "element matrices",

$$\mathbf{L} = [\mathbf{L}_{s}]_{s}$$

Here $[\ldots]_e$ represents the finite-element assembly algorithm, see Baker (1985).

For the construction of the element matrices L_e integrations over the element subdomains have to be performed. Thus,

$$\mathbf{L} = \left[\int_{\Omega_s} \varphi_j L\left(\varphi_i\right) d\Omega_s \right]_s$$

with i, j now ranging from $1, \ldots, N$ where N is the number of nodes in the element.

Hence the process of discretization is reduced to the integration of the differential equations over the element domains using the appropriate weight and basis functions. The algorithm for the assembly of the large matrix from the element matrices is a standard component of finite-element codes. However, especially in two and three dimensions this assembly process is not a trivial task. In order to reduce the bandwidth of the large matrix the ordering of the elements is critical. The SEPRAN package offers several strategies for optimising the bandwidth and/or the profile of the matrix, see Segal (1984). In the present study the assembly algorithm of the SEPRAN package is used. However, for the element-wise integration a dedicated "tool box" was written to obtain a flexible yet simple means of discretizing arbitrary second-order partial differential equations. In the next section a description is given of the structure of this "tool box".

3.2.3 Implementation of a finite-element discretization using linear basis functions

The construction of discretization schemes for arbitrary partial differential equations can be reduced to the formulation of integration schemes for the separate element domains. Often the subroutines in which these integration rules are implemented are denoted as "the element". For each new differential equation a new "element" must be implemented. However, by regarding pde's as combinations of the basic terms described in the previous section, new "elements" can be assembled from combinations of a small number of basic "elements". Provided that the basic terms are discretized in their most general appearance, elements for complex sets of pde's, like combinations of turbulence models and the Navier-Stokes equations, can be assembled from these basic terms with relative ease.

The building blocks for the discretization of the equations considered in the present study are,

"mass" terms;

- "convection" terms; and
- "diffusion" terms.

The simplest term to discretize is the "mass" term. The nomenclature of this term is due to its occurrence in the time-dependent part of arbitrary momentum equations. In structural mechanics this term is often proportional to the mass of the system. The characteristic feature of mass terms is that they are proportional to the magnitude of the unknown function rather than to its derivatives. In general a term of this type can be written as

$$f(t, x, y, z, \phi^{it})\phi,$$

with $f(t, x, y, z, \phi^{it})$ representing an arbitrary function.

Clearly, the discretized form of this term depends strongly on the choice of the weight functions and that of the basis functions. In fluid dynamics a frequent choice for these functions is to use Lagrangian interpolation polynomials. In one dimension, using linear polynomials, the "mass" element matrix consist of a 2×2 matrix. For an element spanning the interval $x_i \leq x \leq x_{i+1}$ the "mass" element matrix yields,

$$M_{mn} = \int_{x_i}^{x_{i+1}} f(t, x, \phi^{it}) \lambda_m \lambda_n dx.$$
 (3.19)

The Lagrangian polynomials, λ_m and λ_n , for this interval are given by,

$$\lambda_k(\xi) = \begin{cases} \frac{\xi_2 - \xi}{\xi_2 - \xi_1} & \text{for } k = 1\\ \frac{\xi - \xi_1}{\xi_2 - \xi_1} & \text{for } k = 2 \end{cases}$$

For the interval $x \in [x_i, x_{i+1}]$, $\xi = x$, $\xi_1 = x_i$ and $\xi_2 = x_{i+1}$.

Since the unknown function ϕ is approximated with combinations of linear interpolation polynomials, an obvious approximation for $f(t, x, \phi^{ii})$ is also a combination of Lagrangian polynomials. For the interval $x_i \leq x \leq x_{i+1}$

$$f(t, x, \phi^{it}) \approx f(t, \xi_1, \phi_1^{it}) \lambda_1(\xi) + f(t, \xi_2, \phi_2^{it}) \lambda_2(\xi),$$

with $\xi = x$, $\xi_1 = x_i$, $\xi_2 = x_{i+1}$, $\phi_1 = \phi(x_i)$ and $\phi_2 = \phi(x_{i+1})$.

Using this approximation and the integration rule for Lagrangian polynomials,

$$\int_{\Delta x} \lambda_1^p \lambda_2^q \, dx = \Delta x \frac{p! \, q!}{(p+q+1)!}$$

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-	

Ŷ,

the element integral (3.19) can be expressed in terms of product of a vector and a "hypermatrix",

$$M_{mn} = \frac{\Delta x}{12} \{f_m, f_n\} \cdot \begin{pmatrix} \left\{ \begin{array}{c} 3\\1 \end{array}\right\} & \left\{ \begin{array}{c} 1\\1 \end{array}\right\} \\ \left\{ \begin{array}{c} 1\\1 \end{array}\right\} & \left\{ \begin{array}{c} 1\\1 \end{array}\right\} \\ \left\{ \begin{array}{c} 1\\3 \end{array}\right\} \end{pmatrix}, \qquad (3.20)$$

where the "product" refers to the terms within the curly brackets. The only terms that need evaluation at the moment of discretization are $f(t, \xi_1, \phi_1^{it})$ and $f(t, \xi_2, \phi_2^{it})$. When the values of these terms are known, the discretization of an arbitrary term with the structure of a "mass" term merely reduces to a matrix-vector multiplication.

After some manipulation also arbitrary convection and diffusion terms can be discretized using a matrix-vector multiplication concept. Since the different "hypermatrices" for the different families of terms are geometrically determined, they can be calculated in advance. With these pre-calculated "hypermatrices" the development of a discretization scheme for more complex pde's becomes simple. A differential equation like,

$$f(x)\phi + \frac{\partial}{\partial x}(g(x)\phi) + \frac{\partial}{\partial x}\left(h(x)\frac{\partial\phi}{\partial x}\right) = 0$$

can be discretized so that,

 $\mathbf{L}\cdot\tilde{\phi}=0\,,$

with the use of the element matrices,

$$L_{e} = \{f_{1}, f_{2}\} \cdot \left\{ \begin{array}{c} M_{e,1} \\ M_{e,2} \end{array} \right\} + \\ \{g_{1}, g_{2}\} \cdot \left\{ \begin{array}{c} C_{e,1} \\ C_{e,2} \end{array} \right\} + \\ \{h_{1}, h_{2}\} \cdot \left\{ \begin{array}{c} D_{e,1} \\ D_{e,2} \end{array} \right\}.$$

Here f_1, f_2, \ldots represent the values of f(x), g(x) and h(x) at the first and second node of the element under consideration.

Thus, with few subroutines where the matrix-vector multiplication is carried out, the definition of the discretization scheme can be reduced to a series of subroutine calls.

Instead of using the integration rule for Lagrangian polynomials, other integration schemes can be used. In general most turbulence closures are characterized by the strongly non-linear character of the equations. Consequently, integrating terms with strongly nonlinear factors f(x), g(x) and h(x) may not be very accurate. In these situations it is often more accurate to use a Gauss quadrature. For linear elements in one dimension, a twopoint rule will generally be sufficient. When a Gauss quadrature is used, the discretization can still be described in terms of a matrix vector product. However, the entries in the "hypermatrices" will have a different value and instead of the values of $f(x, \phi)$, $g(x, \phi)$ and $h(x, \phi)$ at the nodes of the element now the values of these functions at the "Gauss" points have to be evaluated.

For two-dimensional problems, a similar procedure as demonstrated above for the one-dimensional case can be used for the discretization scheme. Instead of one-dimensional line elements, now triangles are used for the discretization in space. Again linear basis functions are used together with a four-point Gauss quadrature. Also in the two-dimensional case the use of the Gauss-quadrature improves the accuracy in some situations. An example of the effect of the integration method on the accuracy of the solution is given in Fig. (3.2).



Figure 3.2: Convection of a concentration packet in a solid-body velocity field after one full revolution for $C = \Delta_t / \omega R \Delta_c = 0.1$ and $Pe_w = 10^9$. (a) Initial condition; (b) Lagrangian integration rule; (c) Gauss quadrature

It presents the result after one revolution of a concentration distribution in velocity field due to solid-body rotation. The concentration distribution is not centered at the axis of rotation. Since the problem is described in a cartesian formulation the velocity components in the convection terms are non-linear functions of the coordinates. Due to diffusion the concentration distribution will decay. However, since the diffusion coefficient is chosen extremely small, after one revolution the concentration profile should have undergone little change compared to the initial distribution given in Fig. (3.2a). Comparison of Figs (3.2b) and (3.2c) shows that the result of the method based on the four-point Gauss quadrature is in much closer agreement with the initial distribution than the result of the method based on the Lagrangian integration rule. Apparently, the approximation of the velocity components with linear polynomials causes appreciably larger dispersion errors than the approximation of the element integrals by the four-point Gauss quadrature.

3.2.4 Some properties of the parabolic finite-element scheme

In the preceding sections the integration method and discretization scheme have been defined. In this section some of the basic properties of the algorithm are studied. First an assessment of the stability of the scheme will be made, then the accuracy of the scheme will be discussed briefly.

All turbulence closure schemes used in the present study consist of additional pde's solved simultaneously with the flow equations. Like the flow equations, these additional pde's have the structure of non-linear transport equations. However, not only the turbulence model pde's are non-linear themselves, also the coupling with the flow equations is generally non-linear. In order to obtain a robust scheme, an "as stable as possible" integration method has to be chosen. For parabolized transport equations this leads to an implicit integration scheme.

For systems of strongly non-linear equations it is difficult to derive general rules for the stability of a numerical scheme. However, when the scheme is stable for equations which are locally linearized by "freezing" the coefficients, and when the stability is independent of the magnitude of the coefficients, it is most likely that the scheme is also stable when applied to the non-linear equations. Here we will study the stability properties of the finite-element/Crank-Nicolson scheme by considering the linear test problem as described by

$$U\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial y}\left(\alpha\frac{\partial\phi}{\partial y}\right) + \beta\phi , \qquad (3.21)$$

with boundary conditions

$$\phi(x, -1) = 0$$
 and $\phi(x, 1) = 0$,

and initial condition

$$\phi(0,y)=\phi_0(y) \ .$$

In this equation

 α represents the diffusion coefficient, ($\alpha > 0$),

 β represents a "source" coefficient with an arbitrary magnitude.

For constant U the solution to this equation is given by

$$\phi(x,y) = \sum_{l=0}^{\infty} \phi_l c^{x(\beta - \lambda_l^2 \alpha)/U} \cos(\lambda_l y)$$
(3.22)

with

 $\lambda_l = (2l+1)\pi$

and

$$\phi_l = \int_{-1}^1 \phi_0(y) \cos(\lambda_l y) dy$$

For $\beta > \pi^2 \alpha$ at least one eigenfunction will be growing. In this situation the solution will grow indefinitely at least if the initial condition contains the appropriate mode. For $\beta < \pi^2 \alpha$

all eigenfunctions and thus the total solution will decay to zero for large values of x. Using the methods described in the preceding sections Eq. (3.22) can be discretized which results in a system of algebraic equations. The stability of the integration scheme can be studied using a Neumann-like stability analysis, see Anderson *et al.*(1984) or Hirsch (1990). In this analysis an arbitrary error in terms of a discrete Fourier series is introduced in the solution. The condition for stability is that: for $\beta < \pi^2 \alpha$, the amplitude of the separate modes will not grow; or for $\beta > \pi^2 \alpha$, the amplitude of the modes will not grow faster than that of the solution itself. For an integration scheme obtained by discretization with linear elements of width h in y direction and by using the Θ -integration scheme (Eq. 3.12) the condition for stability is

$$\frac{\frac{Uh}{12}\left[4\cos(\kappa_n h)+8\right]+\left(1-\Theta\right)\left[\frac{\beta h\Delta x}{12}\left(4\cos(\kappa_n h)+8\right)+\frac{\alpha\Delta x}{2h}\left(2\cos(\kappa_n h)-2\right)\right]}{\frac{Uh}{12}\left[4\cos(\kappa_n h)+8\right]-\Theta\left[\frac{\beta h\Delta x}{12}\left(4\cos(\kappa_n h)+8\right)+\frac{\alpha\Delta x}{2h}\left(2\cos(\kappa_n h)-2\right)\right]}\right| < 1,$$
(3.23)

for all permitted values of κ_n ,

where κ_n represents the wave number in discrete Fourier space.

The strongest growing mode will be the mode with $\kappa_n h \ll 1$. For small $\kappa_n h$ Eq. (3.23) may be simplified using the approximation,

$$\cos(\kappa_n h) \approx 1 - \kappa_n^2 h^2$$

For a Crank-Nicolson integration scheme ($\Theta = 1/2$) the stability condition (3.23) will be satisfied for all Δx if

$$\kappa_n^2 \alpha - \beta > 0 . \tag{3.24}$$

The permitted values of κ_n are,

$$\kappa_{\max} = [2(N-1)+1]\pi \text{ and } \kappa_{\min} = \pi$$

where N represents the number of nodes.

Substituting the smallest value for κ_n in Eq. (3.24) yields,

$$\pi^2\alpha-\beta>0,$$

which exactly coincides with the condition for a decaying solution of the exact problem. Thus the finite-element/Crank-Nicolson scheme is unconditionally stable for equations with "not too strong" source terms.

For equations with stronger source terms the situation is more complex. As stated above the scheme can be considered "stable" when the growth rate of the errors is smaller than the growth rate of the solution itself. However, the exact solution will only contain these growing modes in case the initial solution contains the corresponding eigenfunctions. Due to the finite accuracy, the numerical approximation of the solution will contain all possible eigenfunctions, including the growing eigenfunctions. In this sense the scheme is unstable. However, in general a uniform initial condition will be chosen, $\phi(0, y) = \phi_0$. Thus, all eigenfunctions are present in the initial conditions. The stability of the scheme is determined by the growth rate of corresponding modes. Following Eq. 3.23, for small integration steps, $\beta \Delta x/h \ll 1$ and $\alpha \Delta x/Uh^2 \ll 1$, the error growth rate can be approximated by

$$\frac{\frac{1+\frac{\Delta x}{2}(\beta-\alpha\kappa_n^2)}{1-\frac{\Delta x}{2}(\beta-\alpha\kappa_n^2)}}{1-\frac{\Delta x}{2}(\beta-\alpha\kappa_n^2)}$$

if again only the long wavelength errors are considered, $(\kappa_n h \ll 1)$. The growth rate of the n-th eigenfunction can be approximated by,

$$1 + \Delta x (\beta - \alpha \lambda_n^2)$$
.

It can be verified that for corresponding λ and κ and integration steps with

$$\Delta x (eta - lpha \kappa_n^2) \ll 1$$

the growth rate of the error is smaller than the growth rate of the solution itself and the scheme is stable. Clearly, the integration process becomes unstable when the integration step becomes too large,

$$\Delta x(\beta - \alpha \kappa_n^2) = \mathcal{O}(1) \; .$$

As the source term becomes stronger the maximum admissible integration step becomes smaller, the equation exhibits a "stiff" character.

Even though this test equation is linear, and much simpler than the equations of for example a $k - \epsilon$ turbulence model, the study of the stability of the linear test problem clarifies some of the properties of the finite element/Crank-Nicolson scheme when it is applied to the equations of the $k - \epsilon$ model. A salient feature of turbulence models like the $k - \epsilon$ model is that in large parts of the physical domain the source and dissipation terms are much larger than diffusion or convection terms. Recall that the "logarithmic law of the wall" is even based on the assumption that the transport terms of turbulence are negligible compared to the source and dissipation terms. However, rather than considering the equations separately, clearly the system of equations must be analyzed. For a flow close to "local equilibrium" the eigenvalues of the "frozen" system of equations are small. Therefore the source terms are effectively small in this situation. However, during the development of a wall layer, a strong imbalance of production and dissipation may occur. Here the eigenvalues of the "frozen" system of equations are small, even though an implicit integration in marching direction is used.

The second important property of the scheme that must be investigated is the accuracy of the numerical approximation. Potentially, the numerical error of the finite-element/Crank-Nicolson scheme will originate from two sources;

- the truncation error due to the Crank/Nicolson integration; and
- the truncation error due to the finite-element discretization.

The convection/diffusion equation with a source term, Eq. (3.21) will also be used to study the accuracy of the scheme. If one assumes the spatial discretization to introduce no errors the accuracy of the scheme is simply given by the familiar expression for the accuracy of the second-order Crank/Nicolson integration method,

$$\epsilon \approx C \frac{\partial^2 \phi}{\partial x^2} (\Delta x)^2,$$

where C represents a dimensionless coefficient.

By using an adaptive procedure to determine the step size Δx , the error caused by the integration scheme can be kept small without seriously reducing the efficiency of the procedure.

The error in the solution of Eq. (3.21) caused by the finite accuracy of the finite element scheme can be investigated by assuming that the integration in x-direction is exact. In this case the accuracy of the scheme can be studied by analyzing an eigenvalue problem. Solutions of Eq. (3.21) of the form $\phi(x, y) = \phi_i(y) \exp(\frac{\lambda_i}{U}x)$ will obey,

$$(-\lambda_i+\beta)\phi_i+\frac{\partial}{\partial y}\left(\alpha\frac{\partial\phi_i}{\partial y}\right)=0$$

or,

$$-\lambda_i'\phi_i + \frac{\partial^2\phi_i}{\partial y^2} = 0. \tag{3.25}$$

with $\lambda'_i = \frac{\lambda_i}{H} - \beta$

The solutions of Eq. (3.25) can be looked upon as the result of the minimization of the functional

$$\int_{\Omega} \left(\frac{d\phi}{dy} \frac{d\phi}{dy} \right) \, dy \; ,$$

under the constraint

$$\int_{\Omega}\phi^2 dy = 1 ,$$

with λ'_i acting as an appropriate Lagrangian multiplier. Owing to the finite accuracy of the finite-element discretization, Eq. (3.25) will not be satisfied identically by the numerical approximation of ϕ_i . Instead the finite-element solution procedure minimizes the residual in the sense that it attempts to obtain a residual that is orthogonal to the space spanned by the eigenfunctions, $\bar{\phi}_i$:

$$(\tilde{\phi}_i, L\tilde{\phi}_i) = \min(\tilde{\phi}_e, L\tilde{\phi}_e)$$

with

$$(\phi,\psi)=\int_{\Omega}\phi\psi d\Omega$$

 $L\phi = \frac{\partial^2 \phi}{\partial t} - \lambda' \phi,$

and

 $\tilde{\phi}_{e}$ a function that ranges over the total finite-element solution space.

The eigenvalues λ' produced by the finite-element algorithm can be examined using the Wronskian of Eq. (3.25),

$$\mathcal{W}(\phi) = \frac{(\phi, L\phi)}{(\phi, \phi)} \tag{3.26}$$

If the first N eigenvalues of Eq. (3.25) are ordered like, $\lambda_1 < \lambda_2 < \ldots < \lambda_N$ then the approximation of the first eigenvalue will be equal to the minimum of $\mathcal{W}(\phi_e)$ provided ϕ_e ranges over all possible functions in the finite-element subspace. However, since solving Eq. (3.25) is equivalent with solving the Laplace equation under the constraint

$$(\phi,\phi)=1$$

finding the lowest eigenvalue is completely equivalent with finding the finite-element solution. This insight gives the opportunity to study the accuracy of the eigenvalues as calculated by the finite-element algorithm.

Suppose the solution of the finite-element algorithm can be expressed as

$$\phi_i = \phi_i + \epsilon_i$$

where

 ϕ_i denotes the exact solution of Eq. (3.25)

and

 ϵ_i the difference between the exact solution and its numerical approximation, $\bar{\phi_i}$

then the lowest eigenvalue can be expressed as

$$\lambda_1 = (\bar{\phi}_1, \bar{L}\bar{\phi}_1) + (\epsilon_1, L\epsilon_1) \tag{3.27}$$

since for all i, $(\phi_i, \phi_i) = 1$ and $(\epsilon_i, \tilde{\phi}_i) = 0$. However, the exact eigenvalue of Eq. (3.25) equals $(\phi_1, L\phi_1)$ and hence

$$\lambda_1 = \ddot{\lambda}_1 + (\epsilon_1, L\epsilon_1) . \tag{3.28}$$

The second term on the right-hand side of Eq. (3.28) is a quadratic form. This implies that the finite-element approximation of the first eigenvalue overpredicts the exact value by an amount of $(\epsilon_1, L\epsilon_1)$. An expression for $(\epsilon_1, L\epsilon_1)$ is given by Baker (1985) on the basis of the energy norm of the finite-element solution for a Laplace equation. For linear basis and test functions this norm satisfies,

$$(\epsilon_1, L\epsilon_1) \le C\Delta_{\epsilon}^2 \|\bar{\phi}''\|^2, \tag{3.29}$$

with

 Δ_c representing a typical measure for the mesh size, $\|\tilde{\phi}''\|$ the norm of the second derivative of the exact solution with respect to y

and

C a constant depending on the mesh distribution.

N	$\ln(A(x)/A(0))$	$\int_{-1}^{1} [\phi(0,y) - (A(0)/A(x)) * \phi(x,y)] dy$
2	5.172	$2.30 * 10^{-11}$
4	1.275	$1.77 * 10^{-16}$
8	0.3177	$3.61 * 10^{-16}$
16	0.0793	$1.55 * 10^{-15}$
32	0.0198	$2.59 * 10^{-15}$

Table 3.1: The amplitude at x = 100 of the first eigenfunction as a function of the number of elements, calculated with the finite-element/Crank-Nicolson integration scheme. ($\alpha = 1/10$, $\beta = x^2 \alpha$ and U = 1)

The consequence of this result is that the evolution of the modes in the solution are structurally overpredicted even for the case of an exact integration procedure in axial direction. An illustration of the finite-element solution of Eq. (3.21) is given in Table (3.1). In this calculation the parameters are chosen such that the first eigenvalue equals zero, i. e.

$$\lambda_1 = (\beta - \alpha \pi^2)/U = 0 \; .$$

As initial condition the first eigenfunction is chosen. Hence, the initial distribution, $\phi(0, y)$, should remain undisturbed in axial direction. An error in the "decaying" constant will surface immediately in the amplitude of the profile, $\phi(x, y)$. However, according to the analysis above the shape of the profile must not be affected by the integration process. Examining the second column of Table (3.1) indeed shows that the amplitude of the eigenfunction grows, though slower with increasing number of elements. Furthermore it shows that the growth rate—or the error in the predicted eigenvalue—scales with the reciprocal of the number of elements squared. Finally, the third column shows that, except for the twoelement result, the shape of the profile remains virtually unchanged. Hence, the calculated profiles are indeed eigenfunctions of the discretized system.

3.3 A parabolic algorithm for predicting turbulent pipe flow

3.3.1 Axisymmetric pipe flow

In this section the algorithm for axisymmetric turbulent pipe flow is presented. The algorithm is based on the finite-element/Crank-Nicolson scheme presented above. This results in 2-D problem with the axial coordinate forming the marching direction and the radial coordinate the spatial direction for which the finite-element discretization is used. To take the effect of turbulence into account a low-Reynolds-number version of the $k - \epsilon$ model (see sections 2.3.2 & 2.4) is used. Since the flow is assumed to exhibit a parabolic character not all terms of the turbulence model are retained. Terms of order $\mathcal{O}((x/D)^2)$, for example $\frac{\partial}{\partial x}(\overline{uw})$ in the equation for the conservation of angular momentum, are neglected. The resulting equations for U, V, W, P, k and ϵ are respectively: Axial momentum: U

$$\frac{\partial}{\partial x}(U^2) + \frac{1}{r}\frac{\partial}{\partial r}(rUV) = -\frac{\partial P}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{Re}\frac{\partial U}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}(r\overline{u}\overline{v})$$
(3.30)

Continuity: V

$$\frac{\partial}{\partial x}(rU) + \frac{\partial}{\partial r}(rV) = 0 \tag{3.31}$$

Angular momentum: W

$$\frac{\partial}{\partial x}(UW) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2VW) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{Re}\left[\frac{\partial W}{\partial r} - \frac{W}{r}\right]\right) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\overline{v}w)$$
(3.32)

Radial momentum: P

$$\frac{\partial P}{\partial r} = \frac{W^2}{r} \tag{3.33}$$

Turbulent kinetic energy: k

$$\frac{\partial}{\partial x}(Uk) + \frac{1}{r}\frac{\partial}{\partial r}(rVk) = P_k - \epsilon + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{(1/Re + \nu_i)}{\sigma_k}\frac{\partial k}{\partial r}\right) + F_{\text{low-Reynolds}}$$
(3.34)

Rate of dissipation: ϵ

$$\frac{\partial}{\partial x}(U\epsilon) + \frac{1}{r}\frac{\partial}{\partial r}(rV\epsilon) = (C_{\epsilon_1}P_k - C_{\epsilon_2}\epsilon)\frac{\epsilon}{k} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{(1/R\epsilon + \nu_t)}{\sigma_{\epsilon}}\frac{\partial\epsilon}{\partial r}\right) + G_{\text{low-Reynolds}}$$
(3.35)

Production of turbulent kinetic energy

$$P_{k} = -w \frac{\partial U}{\partial r} - v w \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right)$$
(3.36)

Reynolds-shear stress

$$\overline{uv} = -\nu_t \frac{\partial U}{\partial r} \tag{3.37}$$

Reynolds-shear stress

$$\overline{vw} = -\nu_t \left(\frac{\partial W}{\partial r} - \frac{W}{r}\right) \tag{3.38}$$

and

Eddy viscosity

$$\nu_t = C_\mu k^2 / \epsilon \tag{3.39}$$

(Note that in this formulation ν_t is automatically non dimensional)

In these equations C_{μ} , C_{ϵ_1} , C_{ϵ_2} , σ_k and σ_{ϵ} represent dimensionless coefficients. The low-Reynolds extension used in this work is the extension due to Chien (1982), which consists of extra terms in the k and ϵ transport equation. Furthermore some of the coefficients are made dependent on a local Reynolds number and the distance to the pipe wall, y (= 1 - r):

$$F_{\text{low-Reynolds}} = -\frac{1}{Re} \frac{2k}{y^2}$$

$$G_{\text{low-Reynolds}} = -\frac{1}{Re} \frac{2\epsilon}{y^2} e^{-y^4}$$

$$C_{\mu} = C_{\mu,\infty} \left(1 - e^{-0.0115y^4}\right)$$

$$C_{\epsilon_2} = C_{\epsilon_2,\infty} \left(1 - 0.22e^{(Re_t/\beta)^2}\right)$$

$$Re_t = Re \frac{k^2}{\epsilon}$$

$$y^4 = Re y U_{\text{total}}$$

The system of equations, Eqs (3.30) to (3.35) consists of four parabolized convection/diffusion equations and two first-order differential equations. The four convection/diffusion equations, Eqs (3.30), (3.32), E(3.34) and (3.35) are solved simultaneously using the algorithm described above. Thus at every integration step in x-direction a system of four one-dimensional elliptical boundary-value problems has to be solved, and consequently eight boundary conditions have to be provided. These are

$$\begin{array}{ll} \frac{\partial U}{\partial r} = 0 \ {\rm at} \ r = 0 & {\rm and} & U = 0 \ {\rm at} \ r = R \\ W = 0 \ {\rm at} \ r = 0 & {\rm and} & W = 0 \ {\rm at} \ r = R \\ \frac{\partial k}{\partial r} = 0 \ {\rm at} \ r = 0 & {\rm and} & k = 0 \ {\rm at} \ r = R \\ \frac{\partial \epsilon}{\partial r} = 0 \ {\rm at} \ r = 0 & {\rm and} & \epsilon = 0 \ {\rm at} \ r = R \\ \end{array}$$

The two other equations are first-order differential equations and can be solved by integration in the radial direction. The boundary conditions for these equations are,

$$V = 0$$
 at $r = R$
 $P = P_{\text{wall}}$ at $r = R$

However, as stated in the preceding sections, a strong interaction may exist between the distribution of angular momentum and the axial velocity field. This interaction is maintained through the pressure gradient. The stability of the solution procedure may benefit from a strongly implicit coupling. Hence, the pressure is calculated simultaneously with these four convection/diffusion equations. To simplify the coding of the algorithm, the equation for the radial momentum equation Eq. (3.33) is differentiated with respect to r. As a result an elliptic boundary-value problem is created.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(W^2\right)$$
(3.40)

Eq. (3.40) is solved using the same algorithm as used for the four transport equations, but with the convection velocity U set equal to zero. Since the order of the equation has been raised by one, one extra boundary condition must be supplied,

$$\frac{\partial P}{\partial r} = 0 \text{ at } r = 0$$

The continuity equation, Eq. (3.31), is solved separately. It can be solved by straightforward integration in radial direction. However for reasons of simplicity, also this equation is differentiated with respect to r and thus transformed to an elliptic boundary-value problem. As an extra boundary condition

$$V = 0$$
 at $r = 0$

is supplied. In the parabolic formulation, the radial velocity, V, is assumed to be small. Thus, an explicit coupling with the other equations is not expected to seriously affect the stability of the scheme.

To close the system of equations the pressure boundary condition at the wall has to be prescribed. The wall pressure can be calculated by enforcing global mass conservation. At each iteration a "mass-defect" is used to update the magnitude of the wall pressure, by

$$P_{\text{wall}}^{n+1}(x) = P_{\text{wall}}^n(x) + \lambda_{\text{mean}}(x)(\Phi^n(x) - \Phi_0)/\Phi_0 ,$$

$$\Phi^n(x) = \int_0^R U^n(x, r) r dr,$$

$$\Phi_0 \text{ representing a measure for the inlet mass flow$$

and

with,

 Φ_0 representing a measure for the inlet mass flow, λ an appropriate Lagrangian multiplier.

With the specification of the pressure at the wall the system of equations is complete. When the system of equations is discretized using linear one-dimensional basis functions it turns out that for approximately 60 elements the scheme yields grid-independent results for a fully developed turbulent pipe flow at $Re_D = 10^5$. As illustrated in Figs (3.3) & (3.4), the experimentally determined axial velocity distribution and the pressure drop are reproduced satisfactorily by the algorithm. As described in the preceding sections the scheme is not unconditionally stable for source-dominated equations. Hence, for a turbulent flow one may expect that in regions far from "local equilibrium" the size of the integration step in axial direction must be limited. For example, when a uniform flow is used as inlet condition,

locally strong source terms will be present in the equations. It appears that to maintain stability, the step size must be limited to 10^{-5} times the diameter of the pipe. However,



Figure 3.3: The axial velocity distribution near the wall in a developed pipe flow at $Re_D = 10^5$ (60 elements).

downstream of the immediate vicinity of the inlet the step size may be increased, e.g. 1 diameter behind the inlet the allowable step size is already of the order 0.1 * D. In the final stages of the development of the flow, no limitation of the step size is necessary. Owing to the initially "stiff" character of the equations a large part of the computational effort must be spent in finding the solution in the region of the first few diameters, even though for the present example the results are physically meaningless.

3.3.2 Non-axisymmetric pipe flow

The description of non-axisymmetric pipe flows is more complex than the description of symmetric pipe flows. Instead of a quasi two-dimensional problem, for the nonaxisymmetric flow a quasi three-dimensional problem must be solved. In general the boundary-layer approximation of the Navier-Stokes equations is not appropriate for this class of flows. Instead another reduced form of the Navier-Stokes equation must be used to describe the development of the flow. The evolution of the cross-flow-plane velocity components is described by momentum equations rather than by the continuity equation. Thus in the numerical representation, continuity is not automatically enforced and special measures have to be taken.

In this section an algorithm for the solution of the reduced Navier-Stokes equations is presented. The algorithm follows the work of Baker (1985). Continuity is enforced by using the concept of a penalty differential constraint within the finite-element solution algorithm. The variation of the pressure in circumferential direction is evaluated using a "pressure-Poisson" equation.

Since a finite-element method is used, there is no strong need to make use of a curvilinear coordinate system. As illustrated in Fig. (3.5), the cross-section of the pipe is discretized using triangular elements. For such elements it is most convenient to write the equations in a cartesian coordinate system. In this coordinate system the conservative form of the parabolized equations describing laminar flow is,



Figure 3.4: Comparison of the predicted pressure coefficient for a smooth wall in a developed pipe flow and the experimentally determined pressure coefficient

x-momentum

$$\frac{\partial}{\partial x}\left(U^{2}\right) + \frac{\partial}{\partial y}\left(UV\right) + \frac{\partial}{\partial z}\left(UW\right) = -\frac{d}{dx}\tilde{P} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial U}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial U}{\partial z}\right]\right) ,\qquad(3.5)$$

with \vec{P} representing the pressure at an arbitrary but fixed point in the cross-sectional plane,

y-momentum

$$\frac{\partial}{\partial x}(UV) + \frac{\partial}{\partial y}\left(V^2\right) + \frac{\partial}{\partial z}(VW) = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial V}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial V}{\partial z}\right]\right) , \qquad (3.6)$$

z-momentum

$$\frac{\partial}{\partial x}(UW) + \frac{\partial}{\partial y}(VW) + \frac{\partial}{\partial z}\left(W^2\right) = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial W}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial W}{\partial z}\right]\right) , \quad (3.7)$$

and

continuity

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0.$$
(3.8)

In general Eqs (3.5) - (3.8) does not accurately describe swirling flows (see section 5.3). The approximation in Eq. (3.5) of a uniform axial pressure gradient, needed to obtain a "well posed" system of equations, may not be accurate for appreciable levels of swirl. However, the cross-flow-plane variation of the axial pressure gradient scales with the square of the swirl intensity. Thus for low swirl intensities the assumption that the axial pressuregradient is uniform, is not grossly in error. Similar to the algorithm for axisymmetric



Figure 3.5: An example of the discretization of the pipe cross-section using triangular elements (laminar flow)

flow the variation of the pressure in axial direction is obtained by enforcing "global mass conservation".

Contrary to the axial momentum equation, for the two cross-flow-plane momentum equations the variation of the pressure in the cross-flow plane cannot be neglected. An equation for the cross-flow-plane pressure distribution is constructed by taking the "divergence" of the y-momentum and z-momentum equations. This leads to,

$$\frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = -\frac{\partial}{\partial y} \left(\frac{\partial}{\partial x} UV + \frac{\partial}{\partial y} V^2 + \frac{\partial}{\partial z} VW - \frac{1}{Re} \left[\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right] \right) -$$
(3.41)
$$\frac{\partial}{\partial z} \left(\frac{\partial}{\partial x} UW + \frac{\partial}{\partial y} VW + \frac{\partial}{\partial z} W^2 - \frac{1}{Re} \left[\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} \right] \right) .$$

For a given velocity distribution Eq. (3.41) yields an elliptic boundary value problem for the pressure distribution in the cross-flow plane. The pressure distribution that satisfies this quasi-linear boundary-value problem can be expressed as the sum of the solution of the homogeneous equation P^h , and the particular solution P^p , i.e.

$$P = P^h + P^p ,$$

where P^{h} , satisfies

$$\frac{\partial^2 P^h}{\partial y^2} + \frac{\partial^2 P^h}{\partial z^2} = 0. \qquad (3.42)$$

When viscous effects are neglected, the no-slip condition at the pipe wall implies that

$$\frac{\partial P}{\partial n} = 0$$
 at the wall of the pipe. (3.43)

By assuming that both P^p and P^h satisfy this boundary condition, the pressure distribution can be obtained. Equation (3.42) together with boundary condition (3.43) leads to the solution of the homogeneous problem, i.e.

$$P^{h} = \text{constant}$$
.

 P^{h} is completely determined by imposing $P^{h} = \tilde{P}$ at, for example, the axis of the pipe as an additional boundary condition. The particular pressure field belonging to this choice for P^{h} , must satisfy $P^{p} = 0$ at the axis of the pipe. However, as a result of the simplicity of the solution of the homogeneous problem, the "total" pressure may be calculated directly by imposing $P = \tilde{P}$ at the axis of the pipe.

Having established an expression for the pressure distribution, now only the enforcement of the continuity equation remains. To enforce continuity, instead of the continuity equation (3.8), an extra variable Φ is introduced. At the p-th iteration level Φ satisfies,

$$\frac{\partial^2 \Phi^p}{\partial y^2} + \frac{\partial^2 \Phi^p}{\partial z^2} = \frac{\partial U^p}{\partial x} + \frac{\partial V^p}{\partial y} + \frac{\partial W^p}{\partial z} + \beta \left(\frac{\partial^2 \Phi^{p-1}}{\partial y^2} + \frac{\partial^2 \Phi^{p-1}}{\partial z^2}\right)$$
(3.44)
for $p = 1$ (1) N

The boundary conditions for Eq. (3.44) are,

$$\frac{\partial \Phi}{\partial n} = 0$$
 at the wall of the pipe (3.45)

and

$$\Phi = 0$$
 at, at least, one point on the wall of the pipe (3.46)

The cross-flow-plane momentum equations, Eqs (3.6) & (3.7), are "enriched" with a term proportional to the gradient of Φ in y-direction and z-direction, respectively. The purpose of this extra term is to penalize deviations from continuity. Including this "penalty" term, the y-momentum and z-momentum equation become:

$$\frac{\partial}{\partial x}(UV) + \frac{\partial}{\partial y}\left(V^{2}\right) + \frac{\partial}{\partial z}(VW) = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial V}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial V}{\partial z}\right]\right) - \gamma \frac{\partial \Phi}{\partial y} \quad (3.47)$$

and

$$\frac{\partial}{\partial x}\left(UW\right) + \frac{\partial}{\partial y}\left(VW\right) + \frac{\partial}{\partial z}\left(W^{2}\right) = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left(\frac{\partial}{\partial y}\left[\frac{\partial W}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{\partial W}{\partial z}\right]\right) - \gamma \frac{\partial \Phi}{\partial z} \quad (3.48)$$

The value for the coefficients β and γ must be chosen in such a manner that at every station in x-direction the magnitude of Φ_p decreases to zero during the iteration process. An appropriate value for γ is found to be,

$$\gamma = U(y,z;x)/\Delta x$$
 .

Instead of the obvious choice for β , $\beta = 1$, in the present calculations choosing a value of

 $\beta = 0.5$

yielded a slightly faster convergence.

With the description of the method to enforce continuity the algorithm for the calculation of the 3-dimensional parabolized method is complete. The equations used by the algorithm are the momentum equations (3.5), (3.47) and (3.48), the pressure Poisson equation (3.41) and the penalty constraint equation (3.44). The equations are solved simultaneously using the non-linear Crank-Nicolson integration scheme used for the case of axisymmetric flow.

The above algorithm is based on the 3DPNS algorithm developed by Baker (1985). To improve convergence for the swirling flow case some minor changes were made. For example, for swirling flow the non-axisymmetric character of the boundary conditions for Φ , Eqs (3.45) and (3.46) causes a strong sensitivity of the algorithm to oscillations, resulting in a small allowable step in marching direction. Employing an axisymmetric boundary condition instead,

$$\Phi = 0$$
 everywhere on $r = R$

improves the damping of scheme dramatically and results in a much larger allowable step in marching direction. Also the value of the coefficients β and γ given above is different from the original value in the 3DPNS of Baker (1985) algorithm.

To check the algorithm two test cases are considered:

- the case of an axisymmetric pipe flow developing from a uniform inlet condition; and
- the case of the decay of a axisymmetric swirl in a flow field with a uniform axial velocity.

For the first test case, Fig. (3.6) shows the comparison between the predicted axial velocity distribution obtained with the present algorithm and the distribution obtained with an axisymmetric two-dimensional elliptic penalty-finite-element algorithm. It follows from Fig. (3.6) that the agreement of both results is excellent.

The second test case concerns the decay of an axisymmetric swirl in a uniform axial velocity field. In a cylindrical coordinate system the equation describing the decay of the swirl reduces to a linear equation when the axial velocity distribution is uniform. Hence the decay of the swirl may be expressed in terms of a series of decaying eigenfunctions. Fig. (3.7) compares the computed decay of some initial swirl distribution with the analytical decay of the lowest eigenfunction. Since the initial swirl distribution does not coincide with the lowest eigenfunction, initially the decay of the swirl must be stronger than the decay of this single eigenfunction. However, further downstream all but the eigenfunction with the lowest eigenvalue will have decayed fully and the calculated decay rate must coincide with the analytical decay rate of the lowest eigenfunction. Fig. (3.7) demonstrates that the analytical decay rate is indeed reproduced accurately by the present algorithm.



Figure 3.6: Comparison of a developing pipe flow at $Re_D = 10^3$ predicted with the (a) 3-dimensional parabolic algorithm (linear triangular elements), and (b) an axisymmetric elliptic penalty finite-element algorithm (quadratic triangular elements)

3.4 Concluding remarks

In this chapter the algorithms were presented that will be used for the study of the decay of swirl in a turbulent pipe flow. The algorithms are based on a parabolization of the Navier-Stokes equations, allowing for a marching solution procedure. For axisymmetric swirling pipe flows the parabolization results in a system of equations akin to the boundary-layer equations. For non-axisymmetric swirling pipe flow the parabolization of the equations leads to the "reduced Navier-Stokes equations".

In the parabolized formulation of the Navier-Stokes equations the coordinate in the direction of the pipe axis serves as a "time-like" coordinate. In this direction the equations are integrated using a Crank-Nicolson integration scheme. In the successive cross-flow-planes a finite-element discretization method is used.

For turbulent flows, the equations of the turbulence model are solved simultaneously with the momentum equations. Both the momentum equations and the equations of the turbulence model have the structure of a transport equation. Hence, they are solved with the same Crank-Nicolson/finite element algorithm. A salient feature of the equations of the turbulence model is that they contain strong source terms. In large regions of the physical domain the source terms may be much larger than the convection and the diffusion terms. This may lead to "stiffness" of the equations. Even though the Crank-Nicolson/finite element algorithm does not suffer from restrictions on the permissable magnitude of the step in axial direction due to a Courant-Friedrichs-Lewy condition, the "stiffness" limits



Figure 3.7: The predicted decay of swirl in a uniform axial flow

the maximum allowable integration step in axial direction. If uniform inlet conditions for all flow quantities are set and low-Reynolds-number turbulence models are used, the axial integration step is restricted strongly. It turns out that the initial integration step in axial direction must be limited to $\mathcal{O}(10^{-5}D)$. However, the allowable integration step may rapidly be increased. One diameter downstream of the inlet plane the allowable stepsize is already of $\mathcal{O}(10^{-1}D)$. Further downstream, no limitation is necessary anymore.

In this study much emphasis is put on an accurate prediction of decay coefficients. In the present algorithm the accuracy of the decay coefficient is determined by two sources of error:

- the truncation error of the Crank-Nicolson procedure; and
- the truncation error of the finite-element discretization.

The first error may be reduced by limiting the stepsize in regions of large axial variations. The second error is independent of the axial variation of the solution and can be limited by an accurate discretization of the cross-flow-plane only. For a linear parabolized convection/diffusion problem, the error in the decay coefficient due to the finite accuracy of the cross-flow-plane discretization scales with the energy norm $(\epsilon, L\epsilon)$, see Eq. (3.29), of the discretization error.

For a non-axisymmetric flow, to obtain a "well-posed" problem it is necessary to approximate the pressure gradient in axial direction by a uniform value. Consequently, for high levels of swirl the present method may not be accurate.

The primary motivation for the parabolic approach is that in many cases considered in the study of installation effects of metering devices the elliptic character of the system of equations that describe the flow is weak. Furthermore, in general the swirl generated in pipings systems is weak as well. The secondary, though important, motivation is that the memory and CPU requirements remain modest even though the domain of interest may be large. For the class of problems considered in this study, computing times for the parabolic algorithm are at least an order of magnitude less than the computing times needed for algorithms for the fully three-dimensional formulation of the problem.

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4. Turbulent swirling pipe flow

4.1 Introduction

Swirling flows often occur in complex geometries. Examples are flows in cyclones or in combustor-like geometries. Swirl is used for the separation of compounds or for the stabilization of flames. Important aspect that are considered in numerical studies of these flows are for example, the question whether or not recirculation occurs, the length of such a recirculation region and the intensity of turbulence within the recirculation region. In order to provoke recirculation, the swirl in these devices is relatively high. The aspect ratios of the devices considered are generally low, the length of the device seldomly exceeds the width by more than a factor of 10. As a result of these geometrical properties, the elliptic character of the equations that describe the flows in these devices is essential.

The flows considered in this study are of a different nature. The intensity of the swirl is quite low compared to that in cyclones or combustors, recirculation regions will not be present and the length of the domain of interest is generally long. Hence, at least in parts of the domain, the equations that describe the flow have a weak elliptic character only.

As far as the metering practice concerns, details of the turbulence structure are also of secondary importance. One may expect that the performance of a metering device depends on the global characteristics of the flow field, rather than on local details.

Therefore, our first goal is to study the behaviour of one or two global measures that characterize the flow. Suitable integral measures to characterize the swirl may be, for example, the integral amount of angular momentum or a swirl angle averaged over the pipe cross-section. One of the objectives of this study is to find accurate predictions of the axial development of such measures. Accurate predictions of detailed velocity distributions or of all second-order moments of the fluctuations are only secondary goals.

In a pipe flow the decay of the swirl is solely caused by viscous forces, i.e. angular momentum is dissipated by the action of the wall-shear stress. Most of the radial transport of angular momentum, from the core of the flow to the wall, is achieved by the turbulent stresses. Hence, the exact structure of the turbulence will govern the decay of the swirl, so that the choice of the turbulence model is of a vital importance.

According to the literature (see for example Kline *et al.* (1982), or chapter 2) the calculation of swirling flows with the classical $k-\epsilon$ model has its shortcomings. It is believed that a number of aspects essential for swirling flow cannot be captured by this model. One of these aspects is that in swirling flows the stress and the strain tensor are not aligned. The $k - \epsilon$ model, being an effective viscosity model, will not reflect this. As has been demonstrated before (Fig. 2.4, Boysan & Swithenbank (1982)) especially the prediction of

the distribution of the circumferential velocity depends stongly on the correct description of the stress tensor. As a consequence, for swirling flows it has become common practice to use either the more complete Algebraic Stress Model or even the full Reynolds Stress Model. (Gibson & Younis (1986), Hogg & Leschziner (1989)). Only these models have the potential to capture the relevant parts of the physics of the flow. However, for a pipe flow, application of the full RSM is considered beyond the scope of the present investigation.

For simple geometries, like flows over curved or spinning surfaces, attempts have been made to extend the standard $k - \epsilon$ model to flows with curved streamlines (see section 2.4.2). For the class of flows considered in these studies this approach proved to be successful (section 2.5.1). However, Shreenivasan's (1980) results using these adaptations for the case of two counter-rotating confined swirling jets proved to be unsatisfactory.

Without doubt the swirling pipe flow is closer related to the confined swirling jet than to a curved or swirling boundary layer. Hence, simple modifications of a $k - \epsilon$ model may not be sufficient to describe the flow correctly. On the other hand since the swirl decays slowly, one may expect that the turbulent stresses can follow the changes in the distributions of the mean velocity components. This implies that memory effects in the Reynolds-stress tensor might be of minor importance, and that an algebraic approximation of the transport of turbulence may assumed to be valid.

In this chapter we will assess some results of calculations of turbulent pipe flows with a decying swirl, employing a number of turbulence models. Our goal is to identify the most important mechanisms, rather than obtaining a complete description. Hence, only the simpler variants of the models described in chapter 2 will be considered. Furthermore, in the present chapter the flow is considered to be axisymmetric. As a starting point we will use an algebraic representation of the transport terms in the Reynolds-stress equations.

4.2 Modelling turbulent swirling pipe flow

4.2.1 On the applicability of an ASM model in a swirling pipe flow

Although in general the stresses and strains will not be aligned, any algebraic stress model assumes that their orientation is an instantaneous function of the flow field. This assumption is valid provided that the transport terms in the Reynolds-stress transport equations can be neglected or can be approximated by terms algebraic in the stresses themselves. The classical ASM approximation is due to Rodi (1976). Noting that,

$$\frac{\partial \overline{u_i u_j}}{\partial x} = \frac{\overline{u_i u_j}}{k} \frac{\partial k}{\partial x} + k \frac{\partial}{\partial x} \left(\frac{\overline{u_i u_j}}{k} \right)$$

Rodi proposed,

$$Transport\left(\overline{u_i u_j}\right) \approx \frac{\overline{u_i u_j}}{k} Transport\left(k\right) . \tag{4.1}$$

A minimum requirement for this approximation to hold is that all gradients of the quantity $\frac{u_i u_j}{L}$ are small,

$$k\frac{\partial}{\partial x_l}\left(\frac{\overline{u_i}\overline{u_j}}{k}\right) \ll \frac{\overline{u_i}\overline{u_j}}{k}\frac{\partial k}{\partial x_l}, \quad \text{for} \quad l = 1, 2, 3.$$
(4.2)
Since the transport of the turbulent kinetic energy equals the difference between the its production P, and its dissipation ϵ , the transport of the Reynolds stresses can be approximated by

$$Transport\left(\overline{u_i u_j}\right) = \frac{u_i u_j}{k} \left(P - \epsilon\right) , \qquad (4.3)$$

provided that condition (4.2) is met. However, for flows with curved streamlines this approximation is not necessarily very accurate. The reason can be appreciated best by considering the convection of the turbulent stresses in a curvilinear coordinate system.

For flows with curved streamlines it is often advantageous to use a curvilinear coordinate system. Instead of condition (4.2), now a covariant equivalent of Eq. (4.2) is a necessary condition for the ASM approximation to be valid. In an arbitrary curvi-linear coordinate system the left-hand side of Eq. (4.2) will transform according to,

$$k\nabla_l\left(\frac{\overline{u^iu^j}}{k}\right) = k\frac{\partial}{\partial x_l}\left(\frac{\overline{u^iu^j}}{k}\right) + \Gamma^i_{lm}\overline{u^ju^m} + \Gamma^j_{lm}\overline{u^iu^m}.$$
(4.4)

In general the Christoffel symbols, Γ_{bc}^{α} , will not be small and the covariant equivalent of condition (4.2) will not be valid. Instead of the transport approximation Eq. (4.3), now an equivalent for curvilinear coordinate systems has to be used that explicitly contains the curvature-induced transport terms:

Transport
$$(\overline{u_i u_j}) = \frac{\overline{u_i u_j}}{k} (P - \epsilon) + C_{ij}$$
 (4.5)

where C_{ij} represents all the curvature-induced terms and consists of a convection contribution, $C_{c,ij}$ and a diffusion contribution, $C_{d,ij}$.

In a cylindrical (x, r, φ) -coordinate system, with corresponding velocity components, U, V and W, the curvature-induced convection terms can be shown to have the following form.

$$C_{c,xx} = 0$$

$$C_{c,xx} = +2 \frac{W}{r} \overline{v}\overline{w}$$

$$C_{c,\varphi\varphi} = -2 \frac{W}{r} \overline{v}\overline{w}$$

$$C_{c,\varphi\varphi} = -\frac{W}{r} \overline{u}\overline{w}$$

$$C_{c,\varphi\varphi} = +\frac{W}{r} \overline{u}\overline{v}$$

$$C_{c,r\varphi} = -\frac{W}{r} (\overline{v^2} - \overline{w^2})$$

Upon transformation of these expressions back into a cartesian representation it appears that the swirl continuously exchanges energy between the various components of the Reynolds-stress tensor. The energy exchange, or "convection scrambling" is caused by the curvature of the streamlines. Of course, when analysing the same flow in a cartesian coordinate system, without the detour in the cylindrical coordinate system, the ASM approximation yields no energy-exchange terms!

The cause of this paradox is that by combining the ASM approximation with some choice for the coordinate system, one assumes that along the streamlines of the flow the "convection scrambling" of the Reynolds-stresses is fully represented by the terms induced by the curvature of the coordinate system. Necessary conditions for the validity of this assumption are:

- the symmetry of the turbulence field must coincide with the symmetry of the coordinate system; and
- the curvature of the streamlines must be reflected by the coordinate system.

As such the validity or invalidity of the ASM approximation is determined by whether or not the chosen coordinate system "matches" the flow. Expression (4.4) shows that the "convection scrambling" terms are algebraic in the stresses. Thus, the "convection scrambling" effect is potentially large and accurate predictions with an ASM are only possible when this effect is properly accounted for.

For a swirling pipe flow the natural choice for a coordinate system is a cylindrical coordinate system. In such a flow it is likely that to first approximation the turbulence field will have a cylindrical symmetry as well. Thus the first condition is satisfied. The second condition poses problems in swirling flows. Firstly, in the case of a recirculation region the curvature of the streamlines will be quite different from the curvature of the coordinate curves. Secondly, the streamlines in a swirling flow will have a helical shape, which does not coincide with the shape of one of the coordinate curves. However, it can easily be appreciated that compared to the case of circular streamlines, the helical shape of the streamlines causes no additional redistribution of energy, provided that the pitch of the streamlines is constant. Hence a necessary condition for an accurate representation of the convection of the Reynolds stresses by the ASM approximation is that, at least locally, the pitch of the streamlines may be considered constant. Hence, accurate predictions of swirling pipe flow with an ASM are restricted to flows,

- without recirculation
- \mathbf{and}
- with a slowly decaying swirl.

4.2.2 The ASM in a cylindrical coordinate system

In the preceding section we showed that in a swirling pipe flow the ASM-approximation of the convection of Reynolds-stresses is possibly inaccurate. However, the simplification of the transport terms by the ASM not only applies to the convection of turbulent stresses but also to the diffusion of these stresses. The curvilinear character of the coordinate system again induces curvature contributions in the diffusion term. Like the curvature-induced convection terms, the curvature-induced diffusion terms may be large in magnitude, and thus have to be accounted for in an ASM representation. Appropriate curvature-induced diffusion terms are not as easily obtained as the curvature-induced convection terms. Using the curvature terms of the full Reynolds-stress equations is not an option, since they are essentially non-linear in the stresses and still contain derivatives of the Reynolds stresses. Moreover, there is no need to approximate the curvature-induced terms more accuratly than the conventional ASM approximation of the diffusion terms. Hence, for example in codes like FLUENT they are neglected altogether. However, since the geometry of interest is so simple, an intermediate approach may be possible. Rather than simply ignoring these terms we search for a formulation that is at least consistent with Eq. (4.1) but preserves the typical "tensor-like" diffusion properties. A formulation that meets this condition can be found by defining,

$$lpha_{ij} = rac{\overline{u_i u_j}}{k} \quad ext{with} \quad rac{\partial lpha_{ij}}{\partial x_l} = 0$$

and rewriting the diffusion term in covariant form,

$$Diff(\overline{u_i u_j}) = \nabla_l \left(g^{lm} \nu_{stress} \nabla_m \left(k \alpha^{ij} \right) \right), \tag{4.6}$$

with

 ν_{stress} denoting a turbulent diffusivity

and

 g^{lm} denoting the metric tensor.

Using this expression for the diffusion term one arrives at the following approximation for the curvature-induced diffusion terms,

$$Diff(\overline{u_i u_j}) \simeq \overline{u_i u_j} * \frac{Diff(k)}{k} + C_{d,ij}$$
(4.7)

,

where for a cylindrical coordinate system $C_{d,ij}$ is given by,

$$\begin{split} \mathcal{C}_{d,11} &= 0 , \\ \mathcal{C}_{d,22} &= -\frac{1}{r^2} \, \nu_{stress} \, \left(\overline{v^2} - \overline{w^2} \right) \\ \mathcal{C}_{d,33} &= \frac{1}{r^2} \, \nu_{stress} \, \left(\overline{v^2} - \overline{w^2} \right) , \\ \mathcal{C}_{d,12} &= -\frac{1}{r^2} \, \nu_{stress} \, \overline{uv} , \\ \mathcal{C}_{d,13} &= -\frac{1}{r^2} \, \nu_{stress} \, \overline{uw} , \\ \mathcal{C}_{d,23} &= -\frac{4}{r^2} \, \nu_{stress} \, \overline{vw} . \end{split}$$

By introducing these expressions in the ASM equations (4.5), all curvature induced transport processes are accounted for. However, the introduction of these terms is only meaningful if the magnitude of these terms is comparable to the curvature-induced convection terms. Comparing the expressions for $C_{c,ij}$ with those for $C_{d,ij}$ indeed shows that neglecting all curvature-induced diffusion terms will probably not be justified in all cases. Near the axis of the pipe the diffusion terms may become large and of the same order of magnitude as the convection terms. However, it has to be kept in mind that the curvature-induced diffusion terms are the result of an approximation. This approximation is similar to the approximation of the diffusion term in the $k - \epsilon$ turbulence model. Just, one of the identified weaknesses of the $k - \epsilon$ model is the description of the diffusion in the neighbourhood of a symmetry plane or an axis of symmetry.

Now that the transport terms of the Reynolds stresses have been simplified, the ASM can be completed by choosing a model for the pressure-strain interaction. In chapter 2 three basically different models for the "rapid part" of the pressure-strain term were given, the QI model, the IP model and the rotation-rate independent FIP model of Fu *et al.* (1987). Of these three, the QI pressure-strain model is the most complex model. Considering the form of the QI model (Eq. (2.8)), it is seen that the structure of the expressions for the stresses becomes rather complex and too bulky to analyse conveniently. Moreover, for swirling flows no decisive advantages of the QI-model over the IP-model were found (section 2.5.2).

Generally, also the pressure-strain model due to Fu *et al.* (1987) (FIP model) provides problems in an algebraic representation. Apart from production terms it also contains a convection term. Since we are using an algebraic representation only an approximation for the convection term is available. However, in the foregoing section we showed that the ASM yields accurate results for a limited class of flows only. Naturally, if one only considers this class of flows, the ASM approximation can be used in the pressure-strain model as well. According to Fu *et al.* (1987) the rapid part of the pressure strain has to be corrected with a term equal to,

$$C_2\left(C_{ij}-\frac{1}{3}\delta_{ij}C_{kk}\right).$$

Substitution of the generalized ASM approximation for the convection terms yields,

$$C_2\left[\left(\frac{\overline{u_i u_j}}{k} - \frac{1}{3}\delta_{ij}\right)C(k) + \mathcal{C}_c\right],\tag{4.8}$$

where C(k) denotes the convection of k.

However, a prerequisite for an accurate description of the convection by terms algebraic in the stresses, is that the swirl should decay slowly. This implies that both the gradients in axial direction and the radial velocity component are small. In this case the convection of the turbulent kinetic energy k, will be small as well. To first order we can neglect the convection of k, C(k), in Eq. (4.8).

Summarizing, for a swirling pipe flow, the ASM system of equations can be written as,

$$\frac{u_i u_j}{k} = \varphi_1 P_{ij} / \epsilon + \varphi_2 \delta_{ij} + \varphi_3 \left(\mathcal{C}_{c,ij} + \mathcal{C}_{d,ij} \right) / \epsilon, \tag{4.9}$$

for the IP pressure-strain model and

$$\frac{\overline{u_i u_j}}{k} = \varphi_1 \left(P_{ij} + C_{c,ij} \right) / \epsilon + \varphi_2 \, \delta_{ij} + \varphi_3 \, C_{d,ij} / \epsilon, \tag{4.10}$$

for the FIP pressure-strain model,

with,

$$\varphi_1 = \frac{1-C_2}{C_1+P/\epsilon-1}, \quad \varphi_2 = \frac{C_1+C_2P/\epsilon-1}{C_1+P/\epsilon-1} \quad \text{and} \quad \varphi_3 = \frac{1}{C_1+P/\epsilon-1}$$

 \mathbf{and}

 C_1 and C_2 the "return-to-isotropy" coefficient and the "rapid-part" coefficient, respectively.

4.2.3 Solving the Algebraic-Stress equations

For a given mean flow field, a given magnitude of k and a given magnitude of ϵ , both systems of equations, Eqs (4.9) and Eqs (4.10) are quasi-linear in the unknown components of the Reynolds-stress tensor. The non-linear character of these equations is weak and stems from the quantity P/ϵ in the coefficients φ_1, φ_2 and φ_3 . In most flows, the quotient of production and dissipation varies slowly. Therefore the magnitude of P/ϵ may be assumed to be known also. With all these assumptions the system of ASM equations reduces to a system of 6 linear equations for 6 unknowns and a closed-form solution can be obtained. In general this solution will be rather bulky and unattractive for further analysis.

However, consistent with the ASM-approximation for the convection of the Reynolds-stresses, it can be assumed that locally the flow is "frozen". Instead of allowing for a very slow development, locally the development in axial direction is neglected altogether. For the "frozen" flow, the subsystem consisting of the equations for $\overline{v^2}$, $\overline{w^2}$ and \overline{vw} is independent from the subsystem consisting of the equation for $\overline{u^2}$, \overline{uv} and \overline{uw} . In the assumption of a "frozen" pipe-flow simple closed-form solutions of the ASM equations emerge.

In the next sections we will consider the solution of the ASM-equations. Since the \overline{vv} component of the Reynolds-stress tensor is the dominant term in the equation for the circumferential momentum we will first concentrate on this component.

4.2.4 The radial transport of circumferentially directed momentum

The solution of the ASM-subsystem for $\overline{v^2}$, $\overline{w^2}$ and \overline{vw} depends strongly on the specific form of the curvature-induced convection and diffusion terms in the ASM equations. Here we will consider just three possibilities:

omission of all curvature-induced terms,

omission of the curvature-induced diffusion terms only,

and

including all curvature-induced terms.

Apart from the form of the curvature terms the solution also depends on the choice of the pressure-strain model. Here we will start with the IP model. In the second part of this

section we will also analyse the results obtained with the FIP model for the "rapid part". For the three situations mentioned above the expression for \overline{vw} becomes:

$$\overline{vw}_0 = -\frac{2}{3}\varphi_1\varphi_2\frac{k^2}{\epsilon}\left(\frac{\partial W}{\partial r} - \frac{W}{r}\right)$$
(4.11)

$$\overline{v}\overline{w}_1 = \frac{\overline{v}\overline{w}_0}{1 + Ri_{\overline{v}\overline{w}}} \tag{4.12}$$

with

$$Ri_{\overline{vw}} = 4\frac{k^2}{\epsilon^2} \left(\varphi_1 + \varphi_3\right) \frac{W}{r} \left(\varphi_1 \frac{\partial W}{\partial r} + \varphi_3 \frac{W}{r}\right)$$

and

$$\overline{v}\overline{w}_{2} = \frac{\overline{v}\overline{w}_{0}\left(1 + 4\varphi_{3}\frac{k}{\epsilon}\nu_{sircss}/r^{2}\right)}{\left[1 + 4\varphi_{3}\frac{k}{\epsilon}\nu_{sircss}/r^{2}\right]^{2} - 4\varphi_{1}^{2}\varphi_{3}\frac{k^{3}}{\epsilon^{3}}\nu_{sircss}\left[\frac{\partial W}{\partial r} - \frac{W}{r}\right]^{2}/r^{2} + Ri_{\overline{v}\overline{w}}}$$
(4.13)

When all curvature-induced terms are neglected, i.e. Eq. (4.11), the solution of the ASMsystem reduces to the classical Boussinesq-approximation for the Reynolds stresses. The expression for \overline{vw} consists of a dimensionless group of constants, a group of turbulent quantities with a dimension of a viscosity and a group representing the deformation of the mean flow field. Expressions (4.12) and (4.13) reveal that introduction of curvature terms results in a correction factor on the original Boussinesq-approximation. For more complex curvature terms the correction factor also becomes more complex.

It should be noted here that in the layer close to the wall the structure of the expression for \overline{vw} with the curvature induced convection terms included (Eq. (4.12)), is comparable to the expression for \overline{vw} for curved streamlines as proposed by Bradshaw (1973). His expression for \overline{vw} is based on a Richardson number for curved flows (section 2.4.2). For the circular pipe the equivalent of Bradshaw's expression is of the form,

$$\overline{v}\overline{w} = \overline{v}\overline{w}_0 \ (1 - \beta Ri) \tag{4.14}$$

where Ri denotes the Richardson number and β a case-dependent constant.

For small Ri Eq. (4.14) is approximately equal to,

$$\overline{vw} = \overline{vw}_0 \, \frac{1}{1 + \beta R_i} \; .$$

According to Bradshaw (1973) the Richardson number equals the ratio of the extra strain acting on the fluid due to the curvature of the streamlines and the original strain, i.e.

$$Ri \propto \Omega/rac{\partial U}{\partial r}$$

with Ω representing the angular velocity.

It follows from Eq. (4.12) that apparently the Richardson number for swirling flows is of the form,

$$Ri \propto \frac{k^2}{\epsilon^2} (\varphi_1 + \varphi_3) \frac{W}{r} \left(\varphi_1 \frac{\partial W}{\partial r} + \varphi_3 \frac{W}{r} \right)$$
(4.15)

which differs from the Ri-number used by Bradshaw. Expression (4.15) suggests that the Richardson number is formed by the quotient of a *production*-like term and the dissipation of turbulence.

Due to the no-slip condition at a solid wall, this Richardson number will be small close to the wall. However, further away from the wall the circumferential velocity will be much larger and we may expect a larger Richardson number. When the Richardson number is large and positive the resulting shear stress will be very small and consequently the radial transport of circumferentially directed momentum will be reduced.

In turbulence folklore (Bradshaw (1973), Rodi (1979), Launder et al. (1977)) often a similarity is pointed out between rotating flows and flows with a vertical density gradient. The presence of a density gradient has a stabilizing or destabilizing influence on the flow, depending on the direction of the gradient; In a gravity field a flow with a density gradient opposite to the direction of the gravity field will be stabilized, while in the case that the gravity field and the density gradient are in the same direction the flow will be destabilized. As a consequence the momentum exchange can either increase or decrease. Equivalently, in a rotating flow the curvature can be either stabilizing or de-stabilizing. By analogy with the Rayleigh circulation theorem for axisymmetric three-dimensional disturbances, which states that the flow will be stable for flow with a positive gradient of angular momentum and unstable in the opposite case, one may expect that in a turbulent swirling pipe flow the intensity of the radial velocity fluctuations increases when the gradient of angular momentum is positive and decreases in the opposite case. Consequently, one may expect that the radial transport of momentum may increase or decreases. Thus, the "neutral" distribution of cicumferential velocity which is neither stabilized nor destabilized is the distribution which satisfies.

$$\frac{\partial}{\partial r}(rW) = 0$$
 or $W \sim r^{-1}$.

According to Eq. (4.12) the neutral profile is given by

$$\varphi_1 \frac{\partial W}{\partial r} + \varphi_3 \frac{W}{r} = 0$$

ог,

$$W \sim r^{-\varphi_3/\varphi_1} = r^{-1/(1-C_2)}$$
 (4.16)

Expression (4.16) shows that the neutral profile for momentum exchange is directly coupled to the coefficient of the "rapid part" of the pressure-strain model (see section 2.2.5).

A similar analysis can be performed when the FIP model is used for the "rapid part". Of course, changing the pressure-strain model does not affect the "curvatureless" result. However, the result obtained with the curvature-induced convection terms included is affected:

$$\overline{vw_1} = \frac{\overline{vw_0}}{1 + Ri_{\overline{vw}}^*} \tag{4.17}$$

with

$$Ri_{\overline{vvv}}^* = 8\frac{k^2}{\epsilon^2} \varphi_1^{-2} \frac{W}{r} \left(\frac{\partial W}{\partial r} + \frac{W}{r}\right) \; , \label{eq:River}$$

Contrary to the result obtained with the IP model, Eq. (4.12), this result predicts a "neutral stability profile" which is independent of the values of the coefficients of the pressure-strain model, i.e.

$$W \sim r^{-1}$$
. (4.18)

By comparing expression (4.12) and expression (4.17) it appears that the reduction of the radial exchange of circumferential momentum in a stabilizing velocity profile may be quite different for the two models. For a flow subjected to a solid-body rotation, $W = \Omega r$, the denominator of both expressions (4.12) and (4.17) can be expressed as,

$$1 + \alpha \frac{k^2}{\epsilon^2} \Omega^2,$$

where Ω denotes the angular velocity.

If local equilibrium is assumed $(P/\epsilon = 1)$, α is given by,

$$\alpha = \begin{cases} 4\left(\frac{2-C_2}{C_1}\right)^2 & \text{for the IP model,} \\ 16\left(\frac{1-C_2}{C_1}\right)^2 & \text{for the FIP model.} \end{cases}$$

Using the standard coefficients for the pressure-strain model (see section 2.2.5),

$$C_1 = 1.5$$
 and $C_2 = 0.6$

the value of α for these models is

$$\alpha \approx \begin{cases}
3.5 & \text{for the IP model,} \\
1.1 & \text{for the FIP model.}
\end{cases}$$

However, when applying the Gibson & Younis (1986) coefficients for the IP model,

$$C_1 = 3.0$$
 and $C_2 = 0.3$

the magnitude of α equals:

lpha pprox 1.3 .

Clearly, the predicted stabilization effect using the FIP model with the standard coefficients is in much better agreement with the magnitude predicted by the conventional IP model with the Gibson/Younis coefficients. This also explains that the performance of both models in a swirling-jet geometry is equally successful (section 2.5.2).

Thus far we analysed the consequences of including the curvature-induced convection terms. Now we will consider the effects on \overline{vw} of the curvature-induced diffusion terms (Eq. (4.13)). Although it appears to be much more complicated, expression (4.13) has a structure that is similar to the result according to Eq. (4.12). However, in the limit for $r \to 0$ both results differ considerably. While expression (4.12) predicts that the effective viscosity $\propto \overline{vw} / \left(\frac{\vartheta W}{\vartheta r} - \frac{W}{r}\right)$, stays finite in this limit, expression (4.13) predicts a zero effective viscosity irrespective of the magnitude of the swirl.

4.2.5 The radial transport of axially directed momentum

Since the \overline{uv} -component of the Reynolds-stress tensor is one of the dominant terms in the x-momentum equation, in this section this component will be analysed in some detail. To find an expression for \overline{uv} for the case of a pipe flow with swirl the second sub-system of the set of ASM-equations will be solved. This sub-system consists of the equations for $\overline{u^2}$, \overline{uv} and \overline{uw} .

First the analysis will be restricted to two possible treatments of the curvatureinduced terms: to neglect them totally; or to include the curvature-induced convection terms only. For the IP model the expressions for \overline{uv} are

$$\overline{uv}_0 = -\frac{2}{3}\varphi_1\varphi_2\frac{k^2}{\epsilon}\frac{\partial U}{\partial r}$$
(4.19)

and

$$\overline{uv_1} = \overline{uv_0} \frac{1 + Ri_{\overline{u}\overline{v}}}{(1 + Ri_{\overline{u}\overline{w}}/4)(1 + Ri_{\overline{v}\overline{w}})}$$
(4.20)

with

$$Ri_{\overline{\psi\psi}} = \frac{k^2}{\epsilon^2} \left(\varphi_1 + \varphi_3\right) \frac{W}{r} \left(\varphi_1 \frac{\partial W}{\partial r} + \left(3\varphi_1 + \varphi_3\right) \frac{W}{r}\right)$$

The corresponding expressions obtained with the FIP model can be found by simply replacing φ_3 by φ_1 .

As expected, the result of the first approximation is again the classical Boussinesq expression for \overline{uv} . The result of the second approximation is slightly more complex than the corresponding result for \overline{vw} . However, when the swirl-induced curvature of the streamlines is small enough, both correction factors will be similar.

When the pipe wall is approached, the curvature can indeed be regarded as being small, in the sense that the regular deformation terms are much larger than the curvature terms, $\frac{W}{r} \ll \frac{\partial W}{\partial r}$. In this situation the \overline{vw} -component can be approximated by,

$$\overline{vw_1} \simeq \frac{\overline{vw_0}}{1 + 4\frac{k^2}{\epsilon^2}(\varphi_1 + \varphi_3)\frac{W}{r}\varphi_1\frac{\partial W}{\partial r}}$$
(4.21)

while the *uv*-component can be approximated by

$$\frac{uv_1}{1+4\frac{k^2}{\epsilon^2}(\varphi_1+\varphi_3)}\frac{W}{r}\varphi_1\frac{\partial W}{\partial r}$$
(4.22)

It appears that for a relatively small, swirl-induced, curvature of the streamlines, at least as far as the \overline{uv} and \overline{vw} are concerned, the commonly quoted "non-alignment" of the stress tensor and the strain is not present. Instead the momentum exchange in the wall layer is increased compared to the momentum exchange in a non-swirling wall layer. Thus the effect can be described as a change of the characteristic length scale of the turbulence. In a qualitative sense this result agrees with the arguments put forward by Bradshaw (1973), Rodi (1979) and by Launder *et al.* (1977). They hypothesised that the change in length scale is the most important effect in a swirling boundary layer and their adaptations of the $k - \epsilon$ model also predict an increase of this length scale.

At a larger distance from the wall a misalignment of the stresses (\overline{uv} and \overline{vw}) and the strains ($\frac{\partial U}{\partial r}$ and $\frac{\partial W}{\partial r} - \frac{W}{r}$) occurs. In a solid-body rotation the misalignment effect is relatively small compared to the stabilization effect. However, when the circumferential velocity profile conforms to the "neutral" profile (section 4.2.4) both effects are of the same magnitude. Expression (4.20) shows that, contrary to the exchange of circumferentially directed momentum, the exchange of axially directed momentum is increased. At this point the predictions from the modifications due to Bradshaw, Rodi and Launder may be expected to differ considerably from the second-order closure-scheme predictions.

When the curvature induced diffusion terms are added to the equations, near the axis of the pipe, \overline{uv}_2 exhibits the same anomalous behaviour as \overline{vw}_2 ;

$$\overline{uv} = \frac{k}{\epsilon} \left(\varphi_1 \overline{v^2} \frac{\partial U}{\partial r} - \overline{uv} \frac{\varphi_3}{r^2} \, \nu_{stress} \right)$$

or,

$$\overline{uv} = \frac{\varphi_1 \overline{v^2} \frac{\partial U}{\partial r}}{1 + \frac{k}{\epsilon} \frac{\varphi_3}{r^2} v_{stress}}$$

For the limiting situation of $(r \to 0)$ again the quotient of the stress \overline{uv} and the strain $\frac{\partial U}{\partial r}$, decreases to zero.

Although it is a straightforward consequence of Rodi's ASM approximation (Eq. (4.1)) these results show that the gradient diffusion model (Eq. (4.6)) that underlies these results is clearly inadequate. Close to the axis of the pipe the application of the gradient-diffusion model leads to "unphysical" results. Hence we are forced to relinquish the inclusion of curvature-induced diffusion terms, even though they are potentially important.

4.2.6 Closure

In the foregoing sections we have presented an analysis of the behaviour of an Algebraic Stress Model in an axisymmetric "frozen" swirling pipe flow. It was shown that for a flow with strongly curved streamlines the ASM is possibly inaccurate. Only when the coordinate system closely matches the flow considered, accurate results may be expected. In general the character of the flow is not known beforehand and the choice of the coordinate system cannot be based on the flow at hand. Thus the applicability of a model based on an algebraic representation of the transport of turbulence is limited. However, for a "frozen" swirling pipe flow, the simplicity of the geometry and the simplicity of the flow allows the use of an ASM. As a consequence, the results of an ASM will only be accurate to first order in the case of a decaying swirling pipe flow. In addition to an analysis of the applicability of ASM-type models, an assessment was made of the results predicted by the ASM. The main results are:

- The transformation of an Algebraic Stress Model into a cylindrical coordinate system is not trivial. The approximation of the diffusion of the Reynolds stresses poses problems. The straightforward application of the gradient-diffusion model, needed to obtain algebraic expressions, produces unacceptable results. Therefore the curvature-induced diffusion terms are neglected, though close to the axis of the pipe the neglected terms are potentially important.
- If one assumes that the ratio P/ϵ is known, it is possible to formulate closed-form expressions for the Reynolds stresses. If one furthermore assumes that the variations in axial direction may be neglected and if one uses a simple model for the pressure-strain correlation, these expressions are relatively simple and suitable for further analysis.
- Different models for the pressure-strain interaction yield different expressions for the Reynolds stresses. However, by choosing appropriate coefficients the results of the models may be similar.
- The closed-form expressions show that the swirl may have large effects on the structure of the turbulence. Momentum exchange as caused by the turbulent motions can either be increased or decreased by swirl. Near the pipe axis the model predicts a strong decrease of the exchange of momentum.
- In contrast to the flow near the axis the pipe, the momentum exchange in the wall region is increased by the swirl. In the wall region the predicted amplification of the exchange of axially directed momentum and circumferentially directed momentum appear to be same. As such the ASM predictions can be viewed upon as correction factors on the mixing length and are in agreement with the intuitive models of Bradshaw (1973), Launder (1977) and Rodi (1979).
- In a flow without a radial gradient of angular momentum, intuitively one expects the turbulent exchange in radial direction to be neither increased nor decreased. Only the pressure-strain model due to Fu *et al.* (1987), Eq. (2.10), yields a "neutral" profile for the exchange of circumferentially directed momentum that coincides with this profile. The conventional IP model for the "rapid part" profile exists.

An analysis as performed here contains the risk that some of the conclusions on the structure of the turbulence are based on mere artefacts of the model. However, although they are far from perfect, the building blocks of the model can be regarded as being "sound". Effects like the individual production of each component of the turbulent stresses and the "pressure scrambling" of these components are accounted for.

Having a detailed understanding of the behaviour of the model in a simple flow will certainly be of use for the interpretation of predictions in more complex flow situations. Moreover, also the interpretation of experimental results may benefit from the insight gained by this analysis. Hence, in the next sections we will proceed with a more detailed description of the structure of the axisymmetric swirling pipe flow, as predicted by an ASM-type model.

4.3 The structure of a turbulent pipe flow with a decaying swirl

4.3.1 An adapted Boussinesq approximation for swirling pipe flow

In this section we will describe the general features of a decaying swirl in a turbulent pipe flow as predicted by ASM-type models. As indicated in section 4.2.1 the ASM approximation is valid only for a limited class of flows. As long as recirculation does not occur and as long as the swirl decays slowly, the swirling pipe flow belongs to the class of flows where an ASM is applicable. The strong constraints owing to these conditions, causes the application of a full ASM closure to be somewhat controversial. Since the approximation of the convection terms neglects the extra "curvature scrambling" caused by the axial gradients, taking these gradients into account in the ASM equations appears to be inconsistent. Therefore, without loss of accuracy the axial gradients may be neglected when the ASM system of equations is solved. The ASM model then yields closed form expressions for the stresses (section 4.2.3). According to section 2.3.2 the expression for uv in a "single-strain" flow can be connected to the $k - \epsilon$ model by replacing the group of dimensionless quantities $\frac{2}{3} \varphi_1 \varphi_2$ by the coefficient C_{μ} . The expressions for \overline{uv} and \overline{vw} obtained by neglecting all curvature terms, i.e. Eq. (4.19) and Eq. (4.11) respectively, have the same structure as the expression for \overline{uv} in a "single strain" flow. In this sense, when the curvature effects are neglected, the ASM is equivalent to the conventional $k - \epsilon$ model.

When the curvature terms are added, according to Eq. (4.20) and Eq.(4.12) the curvature effect can be accounted for by a set of simple correction terms on the "curvatureless" expressions for the stresses, i.e.

 $\overline{uv} = \overline{uv}_{k-\epsilon} \times \text{correction factor}_{\overline{uv}}$

and

$$\overline{vw} = \overline{vw}_{k-\epsilon} \times \text{correction factor}_{\overline{vw}}$$
.

These expressions open the possibility to modify a $k - \epsilon$ type of method for developing pipe flow, into a method for swirling pipe flow. Hence, it is possible to use the algorithm



Figure 4.1: Radial distribution of the circumferential velocity component obtained with the full ASM (FLUENT) and the modified Boussinesq approximation (present algorithm) for the turbulent stresses. $(U_{inlef} = 1.0, W_{inlet} = r/R, Re_D = 10^6, x/D = 35, N=100)$

as described in chapter 3 for the prediction of the development of pipe flow with swirl. Indeed, the modified Boussinesq approximation and the full ASM yield very similar velocity distributions. The largest difference between the results of these two methods occur in the distribution of the circumferential velocity component, see Fig. (4.1). The difference in the magnitude of both distributions stems from a difference in the treatment of the boundary condition at the wall. In the calculation employing the full ASM, a logarithmic law of the wall is used, while in the calculation employing the modified $k - \epsilon$ model a "low-Reynoldsnumber" model approach is followed. Moreover, also application of a full Reynolds-stress closure scheme results in very similar distributions (Chen (1992)), indicating that neglecting the memory effects in the Reynolds-stress tensor is justified for the class of flows considered. Hence, in this study the modified $k - \epsilon$ model is employed only.

4.3.2 Regimes of swirl decay

As it turns out, the decay of swirl in a developing pipe flow is a complex process in which several mechanisms participate and interact mutually. However, the process can be divided into several regimes in which different mechanisms dominate. The three main parts of the decaying process are:

- a redistribution part;
- a stable "mode" part; and
- an eddy-viscosity part.



Figure 4.2: The circumferential velocity distribution in the eddy-viscosity part. $(U_{inlet} = 1.0, W_{inlet} = r/R, x/D = 100, Re_D = 10^{4}, N=100)$

The reason for this specific division is based on the form of the ASM corrections for the shear stresses. These corrections show that the momentum exchange is influenced by the circumferential velocity distribution. Hence the last part of the three is clear: at the end of the decay the swirl will have become so small that its influence on the exchange of momentum has become negligible. As Fig. (4.2) shows, the character of the circumferential velocity distribution is comparable to the one of the axial velocity distribution in an almost developed pipe flow, showing the largest deformation close to the wall of the pipe and the smallest deformation near the axis of the pipe. Furthermore, comparison of Figs (4.2) and (4.3) shows that in this region the shape of the radial distribution as predicted by the conventional $k - \epsilon$ model, indicating that all effects of streamline curvature on the structure of the turbulence have disappeared indeed.

Upstream of the eddy-viscosity part the phenomena are radically different. Here the swirl is strong enough to have a profound effect. Expressions (4.12) and (4.20) show that momentum exchange is strongly affected by the swirl. The consequences are dramatic. It appears that the effect is so strong that the flow behaves as if it were non-viscous. In the core region of the pipe the flow virtually "freezes", at least as far as the distribution of the circumferential velocity component concerns. It preserves its shape along a large part of the total length considered. Moreover, it appears that there are fixed types of distributions, or "modes". The shape of the distributions scems to lock into a shape that strongly depends on the initial profile. At least three types of "modes" can be distinguished;



Figure 4.3: The circumferential velocity distribution computed using the conventional $k - \epsilon$ model at different axial stations. (U_{inlet} = 1.0, W_{inlet} = τ/R , $Re_D = 10^6$, N=100)

- Solid-body rotation
- Centered-vortex rotation
- Wall-jet

Examples of these three "modes" are shown in Figs (4.4) to (4.6). The first "mode" is characterized by a large region with solid-body rotation near the axis of the pipe, the second "mode" by a concentrated central region of high axial vorticity and the third "mode" by a region of low vorticity near the pipe axis.

Having established that the flow apparently "locks" into certain states, the distinction between the first and the second part is also obvious. The upstream condition will in general not coincide exactly to one of the "modes" the flow tends to "lock" in. Hence there must be a region of large changes in the distribution of angular momentum. This redistribution can have a large effect on the axial velocity profile. In a parabolized formulation of the flow problem the pressure distribution is dominated by an equilibrium of the pressure gradient and centrifugal forces. Provided the swirl is strong enough and the redistribution is fast enough, one may expect that locally the axial pressure gradient becomes positive. At the same time the radial momentum transport by the turbulent stresses is strongly reduced. As a result of both effects the axial velocity distribution may show the well-known "dip" near the pipe axis which is so characteristic for swirling flows. The development of this "dip" in the axial velocity profile and the strong redistribution of angular momentum is closely related to the "vortex breakdown" phenomenon (see also section 3.1.1). Indeed vortex breakdown is only observed at high enough swirl intensities (see for example (Visser *et al.* 1987)). Clearly, a parabolized formulation of the flow problem will not be



Figure 4.4: An example of the solid-body-rotation "mode", using the Gibson & Younis pressure-strain coefficients. ($Re_D = 10^5$, N=100)



Figure 4.5: An example of the centered-vortex-rotation "mode", using the Gibson & Younis pressure-strain coefficients. ($Re_D = 10^8$, N=100)



Figure 4.6: An example of the wall-jet "mode", using the Gibson & Younis pressure-strain coefficients. $(Re_D = 10^5, N=100)$

valid when the flow approaches the regime where "vortex breakdown" occurs. Moreover, approaching the "vortex-breakdown" regime will also be detrimental for the accuracy of any algebraic approximation of the transport of the turbulent stresses. As discussed in section 4.2.1 the ASM approximation is only accurate when the streamlines "match" with the coordinate curves. Close to "vortex breakdown" this condition is not fulfilled. Furthermore the "vortex-breakdown" phenomenon is connected with rapid changes in the flow. Memory effects, that are not accounted for in the ASM, may then become important. Thus the applicability of the present approach is not warranted in the redistribution region. As a consequence, for strong swirl the predictions obtained with the modified $k - \epsilon$ model, may not be reliable for the initial stages of the development of the swirl. However, for the initial velocity distributions and swirl intensities considered here, the dip in the distribution of the second and third decay region, starting with a detailed description of the stable "mode" region.

4.3.3 A detailed description of the stable "mode" region

The most characteristic part in the decaying process is the part we denoted with the term "stable mode decay". Owing to the action of the curvature terms in a large part of the pipe cross-section the flow will behave as if it were inviscid. It is obvious that changes of the time-averaged quantities can only take place over a long distance in axial direction. Hence it seems that in the central region especially the circumferential velocity profile appears to be "frozen". As far as the axial velocity profile is concerned it can be noted that, similar to



Figure 4.7: The acceleration of the flow in axial direction. ($W_{inlet} = r/R$, $U_{inlet} = 1$, $Re_D = 10^5$, N=100)

the flow in the redistribution part, the axial momentum flux will be dominated by the axial pressure gradient. However, since in this region the distribution of angular momentum is almost constant—apart from a slow decay of the integral amount of angular momentum—it is unlikely that close to the core of the swirl the axial pressure gradient remains positive. Hence, eventually a plateau of high axial velocity may develop. This is demonstrated in Fig. (4.7) for the case of solid-body-rotation inlet condition.

Clearly the above mentioned acceleration effect must be reflected in the circumferential velocity profile. Owing to the acceleration in axial direction the axial directed vortex tubes will be stretched. Indeed Figs (4.7) & (4.8) show that in downstream direction the circumferential velocity profiles are steepened near the axis of the pipe and that the change of the slope is proportional to the magnitude of the axial velocity.

Since the flow in the core region is apparently dominated by inviscid-flow mechanisms, it appears that the different "modes" mentioned in the preceding section are nothing more than the remnants of the upstream profiles. In other words when a solid-body-rotation velocity distribution is offered as initial condition, the flow will lock into the "solid-body mode", when a swirling wall jet is offered as initial condition the flow will lock into the "wall jet mode".

A more interesting region is the region in between the immediate vicinity of the wall, where viscous forces are clearly dominant, and the region near the axis of the pipe where the flow is governed mainly by inviscid mechanisms. Because in the largest part of this region the magnitude of the circumferential velocity is still high, according to Eqs (4.12)and (4.20) viscous effects will be weak. The shape of the distribution that develops in this region can be appreciated by assuming that the flow may be considered inviscid. In



Figure 4.8: The steepening of the circumferential velocity distribution as result of the acceleration of the flow in axial direction. ($W_{inlet} = r/R$, $U_{inlet} = 1$, $Re_D = 10^8$, N=100)

inviscid flow with distributed vorticity, the vorticity tends to concentrate in regions with a high level of vorticity, the so-called vortex cores. For the case of a swirling pipe flow this would result in a concentration of axially directed vorticity near the axis of the pipe and an acceleration of the flow in axial direction. Clearly, the remaining velocity distribution in the region between the, more and more in itself concentrating, core and the wall contains little or no vorticity anymore. Of course, the distribution that belongs to a rotating flow without vorticity is the potential-flow distribution (free-vortex distribution), i.e. $W \propto 1/r$.

Since the flow is only inviscid to first approximation one may expect that the shape of the radial distribution of the circumferential velocity resembles the free-vortex distribution. Indeed, the computations do not show a prefect "free-vortex" shape of the velocity distribution. Instead it appears that the shape is determined by the specific form of the pressure-strain approximation. Comparison of results of computations employing different coefficients and different models suggests that the profile in this region is determined by the pressure-strain model (Fig. (4.9)). The expression for \overline{vw} predicts a "neutral" velocity distribution for momentum exchange $W \propto 1/r^p$, that depends on the specific form and the coefficients of the pressure-strain approximation (see section 4.2.4)). Fig. (4.9) suggests that a correlation exists between the actual profile and this "neutral" profile.

In any case, close to the wall the "free-vortex-like" behaviour must break down. Since the integral amount of axial vorticity contained in the pipe cross section must be zero, the large "core vorticity" must be balanced by an equal but opposite directed amount of vorticity. As argued in the foregoing paragraphs, due to the strong reduction of momentum exchange in the intermediate region the vorticity contents of the region between wall and



Figure 4.9: The effect of the choice of the pressure-strain model on the distribution of the circumferential velocity component in the "free-vortex" region. ("Centered-vortex-rotation mode", $Re_D = 10^5$, N=100)

centre is low. Hence, the "core vorticity" is largely balanced by the "wall vorticity". As a result the circumferential velocity distribution close to the wall will show a boundary-layer like structure, see Fig. (4.10).

4.3.4 A description of a turbulent swirling boundary layer

For understanding the structure of the wall layer in a swirling pipe flow it is necessary to realize that the swirling wall layer may distinguish itself in two ways from a conventional parallel wall layer. Firstly, the length scale of the turbulence can be affected by the swirl as a result of "curvature-scrambling" effects. Secondly, contrary to the ordinary wall layer which is governed by one length scale only, in the swirling wall layer one can think of a second length scale, the radius of the curved fluid motion. We will analyse both possibilities.

A problem in analysing the first possibility is whether expressions like Eqs (4.12) and (4.20) for \overline{vw} and \overline{uv} are valid close to a wall. These expressions are based on simple models for the pressure-strain interaction. A deficiency of these simple models is that they predict equal $\overline{u_2}^2$ and $\overline{u_3}^2$ in a simple shear flow with $\partial U_1/\partial x_2$ as the only strain rate, see Fig. (4.11). In experiments the two quantities are different, in a single shear flow $\overline{u_2}^2$ will be smaller than $\overline{u_1}^2$ and $\overline{u_3}^2$. Owing to wall-reflection effects in the pressurestrain interaction the imbalance of the normal stresses will become even stronger when approaching a wall. Very close to a wall low-Reynolds-number effects become important as well. These effects are neither accounted for in expression (4.20) and expression (4.12) nor in any other prediction method based on simple pressure-strain models like the IP model. In contrast, the QI model (section 2.2.5) for the rapid part does predict a different magnitude



Figure 4.10: The circumferential velocity near the pipe wall. ("Solid-body-rotation mode", $Re_D = 10^3$, N=100)



Figure 4.11: The coordinate system for a flow subjected to a single shear

for the stress components and, when extended with the proper wall-reflection terms and low-Reynolds-number terms, satisfactory predictions of the wall region of turbulent pipe flows can be obtained, see Lai & So (1990). However, the QI model is not as successful in predicting swirling flows as the IP model, see Nikooy & Mongia (1991). Taking into account these shortcomings it is unlikely that expressiona (4.20) and (4.12) will be successful in predicting the swirling wall layer.

As stated in section 4.3.1, expressions (4.20) and (4.12) are merely used to incorporate effects of swirl in the Boussinesq approximation. Turbulence models based on the Boussinesq-approximation, like the $k - \epsilon$ model, are tailored to "single-strain" flows. Especially when low-Reynolds-number modifications are included, flows like turbulent pipe flows can be predicted quite well. Apparently the anisotropic character of "single-strain" flows or flows near walls is accounted for by a proper choice of the coefficients in the model. Martinuzzi et al. (1989) concluded that for turbulent pipe flows low-Reynolds-number $k - \epsilon$ models are to be preferred above algebraic-stress models.

If one assumes that the effect of swirl on the structure of the wall layer is considerably smaller than the wall effects, clearly the approach of incorporating effects of swirl into the Boussinesq-approximation can be used close to the wall also. Therefore in our calculations the $k - \epsilon$ model with the low-Reynolds-number modification due to Chien (1982) was used.

Eqs (4.21) and (4.22) in section 4.2.5 show that the dominant stress and dominant strain components are aligned provided that $\frac{W}{r} \ll \frac{\partial W}{\partial r}$. Close to the pipe wall this condition will be satisfied. Equations (4.22) and (4.21) also show that for small swirl angles these expressions can be considered as a swirl-corrected version of the classical mixing-length expression and that the correction factor will have a "Bradshaw-like" structure, i.e.

$$l_m = l_0 \left(1 - 4 \frac{k^2}{\epsilon^2} \left(\varphi_1 + \varphi_3 \right) \varphi_1 \frac{W}{r} \frac{\partial W}{\partial r} \right) \quad \text{for the IP pressure-strain model}$$
(4.23)

with,

$$\frac{k^2}{\epsilon^2} \frac{W}{r} \frac{\partial W}{\partial r} \ll 1$$

In contrast to the core region of the flow, according to Eq. (4.23), momentum transfer is enlarged in the wall region $(\frac{\partial W}{\partial r} < 0)$. Since the increase of the momentum transfer is small, a "law of the wall"-like behaviour for the total velocity $\sqrt{U^2 + W^2}$ may be expected. When the swirl is strong enough we may expect a deviation from the "law of the wall"-like behaviour.

The second possibility for the swirling wall layer to distinguish itself from a conventional wall layer, is the possibility that more than one length scale play a role. An obvious candidate for a second scale is the curvature of the streamlines. However, since this second length scale is always larger than the pipe radius, this effect will be notable only when the Reynolds number is small enough. Only then will the ratio of the internal length scale of the wall layer and the curvature of the streamlines be close enough to unity to bave an effect.

How strong both effects are is difficult to judge in advance. Hence we will use the computed results to invstigate this matter. First we consider the results of a calculation of swirling pipe flow with a conventional low-Reynolds-number $k - \epsilon$ turbulence model (Fig. (4.12)). In this figure the swirl angle $(tan^{-1}(\frac{W}{U}))$, normalized with the limiting swirl angle at the wall, is plotted as a function of wall units. Since a conventional $k - \epsilon$ model is used, the second effect—the "Bradshaw" effect—is excluded. When the curvature of the streamlines is important one may expect, for example, a dependence of the swirl angle on the distance to the wall. Such a dependence would clearly show that the flow in the wall region is governed by more than one length scale. The results in Fig. (4.12) show that for low Reynolds number ($Re_D = 10^4$) an effect is clearly visible. Starting at the inner edge of the logarithmic region the effect grows as the distance to the wall grows. As expected, at higher Reynolds numbers ($Re_D = 10^6$) the effect is much weaker. Though the variation of the swirl-angle is qualitatively the same, the y^+ at which the effect is noticable has now moved to the outer edge of the logarithmic region. However, the fact that at this Reynolds



Figure 4.12: The swirl angle in the wall region at different Reynolds numbers calculated with the Chien low-Reynolds-number $k - \epsilon$ turbulence model.

number there is still a notable effect is somewhat surprising. If the behaviour of the flow at the edge of the logarithmic region is influenced by a process with an intrinsic length scale considerably larger than the pipe radius, why do we observe a logarithmic region at all?

The next step is to include also the curvature-correction terms. Results of calculations with these terms included are shown in Fig. (4.13). The main extra effect of the swirl, the change in length scale, should be reflected in the distribution of the velocity magnitude. Fig. (4.13) shows that at the moderate swirl intensities we are concerned with, almost no dependence is present.

In conclusion, the predictions of the models suggest that at moderate swirl intensities, the structure of the swirling wall layer is comparable to the structure of the nonswirling wall layer provided the Reynolds-number is high enough. Up to—and beyond—the outer edge of the conventional logarithmic region the swirl does not have a notable effect on the mean flow. As a consequence the use of conventional wall-laws for numerical predictions is a realistic option. When the Reynolds-number is too low ($Re \leq 10^5$), curvature effects are present in the logarithmic layer and conventional wall laws cannot be used any more.

4.3.5 The rate of decay of the swirl

As announced in section 4.1 a subject of even greater concern in a swirling pipe flow than a detailed prediction of the velocity field is the prediction of the full development in axial direction of a swirling flow. To study this development it is advantageous to define an integral measure to characterize the swirl. An appropriate measure is the "swirl number"



Figure 4.13: The velocity magnitude in the wall region in a swirling pipe flow at a $Re_D = 10^5$ calculated with an ASM-corrected $k - \epsilon$ turbulence model.

S. In the literature the swirl number S is often used to characterize the flow. However, the definition of S varies. Here we will use a definition directly inspired by the moment of the W-transport equation,

$$S = 2\pi \int_{0}^{R} U(rW) r dr / \left(\pi R^{3} U_{mean}^{2}\right)$$
(4.24)

As defined here, S denotes the total axial flux of angular moment. The reason for this choice becomes apparent as follows. Multipying the φ -component of the Reynolds-averaged momentum equation by r and integrating the resulting expression over the pipe cross section defines,

$$\int_{0}^{1} \frac{\partial}{\partial x} \left[UW \right] r^{2} dr + \int_{0}^{1} \left[\frac{1}{r^{2}} \frac{\partial}{\partial r} \left(r^{2} VW \right) + \right] r^{2} dr =$$
$$- \int_{0}^{1} \left[\frac{1}{r^{2}} \frac{\partial}{\partial r} \left(r^{2} vw \right) \right] r^{2} dr + \int_{0}^{1} \left[\frac{1}{r^{2}} \frac{\partial}{\partial r} \left(\frac{r^{2}}{Re} \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right) \right) \right] r^{2} dr.$$

Introducing Eq. (4.24) and evaluating the integrals yields,

$$\frac{\partial S}{\partial x'} = -2 \tau_{\varphi, wall}, \qquad (4.25)$$
$$\tau_{\varphi, wall} = \frac{1}{R\epsilon} \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right)_{wall}.$$

with

Expression (4.25) shows that the development of S is determined by the wall shear stress only.

In the literature (see chapter 1) the decay of swirl in turbulent pipe flow is often described as being exponential with the downstream distance. As was shown in section 4.3.2, for small swirl intensities the flow regime can be referred to as the "eddy-viscosity regime". In this decay region the swirl does not have a notable influence on the turbulence structure any more. Here an exponential decay may take place. The wall shear stress in a swirling pipe flow can be related to the wall shear stress in a non-swirling pipe flow as follows,

$$\frac{\tau_{\text{wall}}}{\tau_{\text{wall,non-swirl}}} = \frac{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} \left[r \sqrt{U^2 + W^2} \right] \right)}{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} \left[r U_{\text{non-swirl}} \right] \right)}$$

The circumferential component of the wall shear stress can be expressed as,

$$\frac{\tau_{\varphi, \text{wall}}}{\tau_{\text{wall}}} = \frac{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rW]\right)}{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} \left[r\sqrt{U^2 + W^2}\right]\right)}$$

and combining both expressions gives,

$$\frac{\tau_{\varphi,\text{wall}}}{\tau_{\text{wall,non-swirl}}} = \frac{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rW] \right)}{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rU_{\text{non-swirl}}] \right)}.$$

Since it is assumed that the structure of the turbulence is not affected by the swirl $U_{\nu,non-nwirl}$ may be approximated by U_{ν} and,

$$\frac{\tau_{\varphi,\text{wall}}}{\tau_{\text{wall,non-swirl}}} = \frac{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rW]\right)}{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rU]\right)}.$$

Furthermore, according to section 4.3.4, the ratio W/U is constant across the viscous sublayer. Hence,

$$\frac{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rW]\right)}{\lim_{r \to 1} \left(\frac{1}{r} \frac{\partial}{\partial r} [rU]\right)} = \lim_{r \to 1} \left(\frac{W}{U}\right)$$

which in turn represents the pitch of the limiting streamline at the pipe wall. Since the shape of the radial distribution of W is assumed to be constant inx', one may assume that this pitch is proportional to S,

$$\lim_{r\to 1} \left(\frac{W}{U}\right) = \beta S.$$

 β denotes a dimensionless coefficient or "shape factor"

Finally, $\tau_{non-swirl}$ can be approximated using the friction factor for fully developed flow,

$$\tau_{non-switl} = \frac{f}{8}$$

where f denotes the friction factor, Blevins (1984).

Combining the above expressions and substituting in Eq. (4.25) gives

$$\frac{\partial S}{\partial x'} = -\frac{\beta f}{4}S,\tag{4.26}$$

yielding the expected exponential decay.

At higher intensities, the swirl does not exhibit an exponential decay any more. The wall shear stress will become a function of the swirl number, and a function of the "swirl mode" (Fig. (4.14)). The way the wall shear stress depends on the mode is predictable, the more the swirl is concentrated near the pipe wall, the higher the wall shear stress will be. For low swirl intensities the different curves representing the different "modes" collapse onto one curve. In this region the coefficient β in equation (4.26) can be evaluated. Since the exponential decay law is based on the assumption that the swirl has no effect on the turbulence structure, its magnitude may depend only on the Reynolds number. In contrast to the assumption of Mottram & Rawat (1986), according to the present computational results, both the friction factor f, and the shape factor β in Eq. (4.26) depend on the Reynolds number. The magnitude of β for a range of Reynolds numbers is given in Table (4.1).

Re	1 • 104	$3 \cdot 10^{4}$	$5 \cdot 10^4$	$1 \cdot 10^{3}$	$3 \cdot 10^{5}$	$5 \cdot 10^{5}$	1 - 106
β	1.97	2.05	2.08	2.10	2.10	2.10	2.10

Table 4.1: The shape factor β as a function of Reynolds number

In contrast to the strong sensitivity of the circumferential component of the wall shear stress to the "decay mode", the sensitivity to the pressure-strain model that is employed is small, Fig. (4.15). This in spite of the fact that the shape of the circumferential velocity distribution in the "free-vortex" region is determined by the model for the pressurestrain interaction, see Fig. (4.9). However, the effect of the choice of the pressure-strain



Figure 4.14: The circumferential component of the wall shear stress as a function of swirl number calculated for three "modes". (Rep = 10^5 , N=100, Gibson & Younis pressure-strain coefficients)



Swirl number

Figure 4.15: The circumferential component of the wall shear stress as a function of swirl number calculated with different pressure-strain models and coefficients. ("Solid-body-rotation mode", $Re_D = 10^5$, N=100)

model is most notable far from the pipe wall. The velocities close to the wall layer do not seem too different. Hence, the resulting shear stress and consequently the decay of the swirl will be comparable for different models.

4.3.6 Closure

In the second part of this chapter a detailed study was carried out into the structure of a decaying swirl in a turbulent axisymmetric pipe flow as predicted by a modified $k-\epsilon$ model. The most important aspect of the flow is that according to this model the momentum exchange in radial direction is strongly reduced in large parts of the pipe cross-section. As a direct consequence, the rate of development of a swirling pipe flow is much smaller than the rate of development of a non-swirling pipe flow.

For the description of the decay of swirl, the integral amount of angular momentum flux offers a suitable measure. The decrease of the integral amount of angular momentum flux is governed by the wall shear stress only. When the ASM is used as a first-order correction to account for the effects of swirl in a conventional low-Reynolds-number turbulence model, there are no curvature effects in the region adjacent to the pipe wall. If the Reynolds number is high enough the region that is not affected by the streamline curvature extends throughout the logarithmic layer. Thus the choice for the pressure-strain model does not influence the prediction of the wall shear stress much. As a result the decay of swirl is not critically dependent on the paricular choice for the pressure-strain model.

In contrast, the decay of the swirl depends strongly on the initial distribution of swirl. Owing to the suppression of momentum exchange the initial character of the core region of the flow is preserved. The magnitude of the wall shear stress is determined by the velocity distribution in the region between the wall region and the core region. If the vorticity is concentrated mainly in the core region the magnitude of the wall shear stress is low even though the swirl number may be high. Vice versa, if the vorticity is concentrated close to the wall the magnitude of the shear stress will be high.

In experimental studies, the decay of swirl is often fitted to an exponential decay curve, see chapter 1. However, the reported coefficients for the decay of swirl exhibit a large scatter. The results presented in this chapter indicate that part of the explanation for this scatter may be found in the strong effect on the wall shear stress of the "swirl mode" and the region of decay.

On either side of the "stable mode" region the flow phenomena observed are different. Downstream of this region the swirl is too weak to affect the turbulence structure, and the decay of the swirl can be described by an exponential decay law. Upstream of the "stable mode" region the flow is characterized by rapid changes in the distribution of momentum as well as the structure of the turbulence. An approach based on an algebraic approximation of the turbulent stresses may not be accurate here.

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5. Aspects of non-axisymmetry in a decaying swirling flow

5.1 Introduction

Disturbances commonly encountered in flows through pipes may exhibit a nonaxisymmetric character. The reason is that most disturbances are caused by components like elbows and valves that do not possess any symmetry with respect to the pipe axis. Thus the velocity distribution downstream of these components will lack symmetry with respect to the axis.

For non-swirling pipe flows the development of non-axisymmetric disturbances does not seem to yield new, essentially different, regimes of flow development. In contrast, for swirling flows the development of a slightly non-axisymmetric flow may be quite different from the development of an axisymmetric flow. In this chapter a preliminary survey of the subject will be made.

For non-axisymmetric swirling flows several scenarios are possible for the flow to develop. For example, the non-axisymmetric character of the flow may decay faster than the swirl itself, leaving non-axisymmetric aspects of the decay process limited to the initial parts of the pipe. A second possibility is that the the non-axisymmetric character of the flow is amplified by the swirl. In the latter scenario the non-axisymmetric character is an important feature of the decaying swirling flow. Also it is likely that the non-axisymmetric character of the swirl distribution will be reflected in the axial velocity profile. Since the swirl decays, one may expect that eventually the axial velocity profile becomes symmetric again. If this scenario is followed still different possibilities exist for the development of the swirl distribution. Either the swirl becomes symmetric as well, or the non-axisymmetric character of the swirl distribution is retained throughout the decay process. For this latter possibility, in the limiting situation where the strength of the swirl approaches zero the centre of the swirl will maintain its off-axis position.

It is difficult to predict which of these decay scenarios is actually followed. Clearly, the different phenomena will be governed by a competition between viscous effects, that may restore symmetry of the flow, and the essentially inviscid interaction of axially directed vorticity and the cross-flow-plane components of the vorticity, that may enhance nonaxisymmetric modes in the flow. The non-viscous interaction of the axial and azimuthal vorticity components is rather complex. Even in a non-viscous axisymmetric pipe flow the interaction between the axial and azimuthal motions is described as being "rather puzzling" Batchelor (1957). Analytical solutions that describe this interaction are only possible for a limited class of initial conditions. Hence, even for the non-viscous flows the prediction of non-axisymmetric swirling flows seems only possible using numerical techniques.

However, as already indicated in chapter 3, the accurate numerical treatment of nonaxisymmetric swirling flows is not trivial. Since the cross-flow-plane velocity components may be quite strong for this class of flow, a fine mesh in the cross-flow plane is necessary. Combined with the large decay length of the swirl, the accurate simulation of the flow poses very large demands on computer resources. Thus a fully three-dimensional solution procedure does not seem feasible. An option to reduce the excessive needs for computer power is again to parabolize the flow equations. As was mentioned in chapter 3, the parabolization of the flow equations in a situation with strong cross-flow-plane velocity components is not a straightforward extension of the parabolization of the flow equations for the symmetric case. In order to obtain a "well-posed" initial-value problem special measures have to be taken for the calculation of the pressure.

An extra step in complexity is the prediction of a decaying turbulent flow. As demonstrated in the preceding chapters, turbulence models based on an eddy-viscosity concept are not always suited for predicting swirling flows. In some situations, for example for flows with strong swirls, second-order closure schemes appear to be necessary. However, for the metering problem the swirl levels are only moderate. For the swirl intensities relevant to the metering problem, the differences between the predictions of second-order turbulence models and those of eddy-viscosity models are only small. Hence, application of higherorder closure schemes for this class of flows does not seem appropriate. Furthermore, the application of second-order closure schemes also poses extra demands. As was argued in the preceding chapters, second-order closure schemes based on algebraic approximations of the turbulent stresses are not suitable. For accurate algebraic approximations of the transport of the turbulent stresses it is necessary that the curvature of the streamlines is properly accounted for by the coordinate system used. In general this condition will not be met for a flow that appreciably deviates from axisymmetry. Only full Reynolds-Stress models may be expected to handle these flows accurately. Reynolds-Stress models again pose large demands on computer resources. Due to the limited relevance for the metering problem, the difficulties in the numerical treatment and the necessity of elaborate and computationally expensive turbulence closures, a simulation of non-axisymmetric swirling flow at moderate or high swirl intensity does not appear to be opportune within the framework of this study.

Nevertheless, some information about the extent to which decaying swirling pipe flows tend to become non-axisymmetric is important. In this chapter we will attempt to gain some insight into the problem of a non-axisymmetric swirling flow. First some attention will be given to the literature, calculations and experiments concerning non-axisymmetric swirling pipe flows and finally some computed results for steady non-axisymmetric flows at a low swirl intensity are presented.

5.2 Experimental, theoretical and numerical work on non-axisymmetric swirling pipe flows

Much theoretical work on non-axisymmetric aspects of swirling flows is devoted to the stability of Poiseuille flow in axially rotating pipes (see for example Landman (1990), Pedley (1969) and Topolosky & Akylas (1988)). Pedley (1969) showed that in the "rapidrotation" limit $(U/\omega R \ll 1)$ the solid-body rotation may have a strong destabilizing effect on the otherwise stable Poiseuille flow, causing a linear instability at Reynolds numbers as low as 83. The linear spiral instability waves found by Pedley (1969) are essentially time dependent and rotate in the direction opposite to the direction of the basic swirl. Topolosky & Akylas (1988) performed numerical simulations of a rotating Poiseuille flow. For the "slow-rotation" limit they found periodic perturbations with a helical symmetry. As the axial pressure gradient was kept fixed in these simulations, the axial mean flow induced by the helical waves caused a flux defect. In certain cases the defect appeared to be as large as 40-50% of the undisturbed mass flux. Landman (1990) solved the Navier-Stokes equations subject to imposed helical symmetry with a fixed pitch. A limited study was carried out in the parameter space of the the axial and azimuthal Reynolds numbers and the pitch of the imposed helical symmetry. The steadily rotating waves found by Topolosky & Akylas (1988) were observed to undergo a series of bifurcations, from periodic to a-periodic variations with time.

Clearly, these studies show that rotation is capable of destabilizing pipe flows and that the instabilities may have a non-axisymmetric structure. However, a complex time dependence as outlined in these studies may prevent practically feasible turbulent flow calculations based on one-point closure schemes. On the other hand, the flows considered in the studies mentioned above, are quite different from the flows we are concerned with. For the present study they are only relevant in the sense that they indicate that symmetry of swirling pipe flows is not always possible. To our knowledge, for turbulent swirling pipe flows, no extensive theoretical or numerical studies have been performed. For this class of flows the only sources of information are experimental results.

Indeed, in some experimental studies including the experiments described in chapter 6, asymmetry of the velocity distribution has been reported. Even though the initial velocity profiles were carefully kept symmetric, in the experiments performed in our laboratory (see chapter 6) at some distance downstream of the "swirl generator" the flow exhibited a non-axisymmetric character. Further downstream, as the intensity of the swirl became much weaker, not only the axial velocity distribution but also the circumferential velocity distribution became more symmetric again.

In a study of swirl produced by two out-of-plane bends performed at the NBS by Mattingly & Yeh (1988) the experimental results suggest that the centre of the swirl follows a helical path. The same result was found by Kito (1984). Also in this experiment, which was specifically devoted to the study of the asymmetry, the initially symmetric swirling pipe flow became non-axisymmetric at some distance downstream of the entrance region. Similar to the experiments of Mattingly & Yeh (1988) the centre of the swirl followed a helical trajectory and the centre of the swirl was observed to "rotate" in the same direction as the swirl itself. Contrary to the theoretical results of laminar flow in rotating pipes, no periodic variation with time was observed in these experiments. However, since these experimental studies were performed in turbulent flows, a periodicity may be difficult to detect. Any periodicity in the measured signals is likely to be swamped by the fluctuations induced by the turbulence. Moreover, the energy contained in the non-axisymmetric modes is likely to enter the turbulence cascade immediately. In this respect periodic or quasi-periodic modes can be considered as the large-scale eddies of the turbulence.

Indeed in none of the experiments mentioned above, nor in the experiment described in chapter 6, a clear periodicity was found. In this experiment, for the purpose of investigating a possible periodicity, also an exploratory visualization study was performed. Small air bubbles were injected close to the symmetry axis of the swirl generator. The centrifugal acceleration caused by the swirling motion causes the bubbles to remain in the centre of the swirl. Thus, non-axisymmetry and periodicity are easily detected. However, while a slight non-axisymmetry was visible in the bubble path, periodicity was not found. Therefore it appears that the decaying turbulent swirling pipe flow can be considered as a "steady" flow. As a consequence the computation of turbulent swirling flow employing a steady-flow method is considered to be feasible.

5.3 Numerical simulation

In this section we will computationally study the effects of non-axisymmetry on the decay of the swirl and the effects of swirl on the non-axisymmetry of the flow.

A problem encountered in the study of non-axisymmetric flows is how to quantify the deviations from axisymmetry. In experiments this deviation is often quantified by measuring the position of the centre of the swirl. Disadvantage of a measure like this is its local nature. For example, a growth in the displacement of the centre of the swirl not necessarily implicates a growth in the non-axisymmetry of the velocity distribution. The nonaxisymmetry as indicated by the swirl centre may be different from the non-axisymmetry of the axial flow field, while also the angular distribution of angular momentum is not necessarily correctly characterized by the position of the centre of the swirl. A more appropriate method to quantify non-axisymmetry is to use a combination of integral quantities. For the axial velocity distribution an appropriate measure is a "centre of mass flow" defined as,

$$\vec{r}_m = \int_{\Omega} \vec{r} U_x d\Omega / \int_{\Omega} U_x d\Omega , \qquad (5.1)$$

with Ω representing the cross-sectional area of the pipe.

An equivalent measure for the asymmetry of the swirl is for example a "centre of angular momentum flow",

$$\vec{r}_{\theta} = \int_{\Omega} \vec{r} U_x U_{\theta} |\vec{r}|^2 d\Omega / \int_{\Omega} U_x U_{\theta} |\vec{r}|^2 d\Omega .$$
(5.2)

Using the measures defined above, the evolution of the non-axisymmetry can be followed easily.

As was shown already in section 3.3.2, for following the evolution of the asymmetry in a swirling pipe flow the application of the reduced Navier-Stokes equations is a necessity. In contrast to symmetric swirling flow, in a non-axisymmetric swirling flow potentially strong convection effects may occur in the cross-flow plane. Furthermore, the circumferential velocity components cannot be considered small any more. Contrary to a non-swirling pipe flow, or a symmetric swirling flow, in which the radial velocity components scale with,

$$U_{\text{cross-flow-plane}} = \mathcal{O}\left(rac{R}{L}U_{ ext{axial}}
ight) \,,$$

in a non-axisymmetric swirling pipe flow the cross-flow-plane velocity components are partly independent of the axial velocity field. Instead, they must be considered as the sum of three contributions:

• a "scalar-potential" part (V_{Φ}, W_{Φ}) given by

$$V_{\Phi} = rac{\partial \Phi}{\partial y}, \quad W_{\Phi} = rac{\partial \Phi}{\partial z} \quad ext{and} \quad
abla^2 \Phi = rac{\partial U}{\partial x};$$

• a "vector-potential" part (V_{Ψ}, W_{Ψ}) given by

$$V_{\Psi}=rac{\partial\Psi}{\partial z}, \hspace{0.3cm} ext{and} \hspace{0.3cm} W_{\Psi}=-rac{\partial\Psi}{\partial y} \hspace{0.3cm} ; \hspace{0.3cm} ext{and} \hspace{0.3cm}$$

• an inviscid part (V_i, W_i) caused by the interaction of the axial and the cross-flowplane vorticity components. (For example the flow induced by the extra axial vorticity produced by "vortex stretching" in an axially accelerating flow.)

The magnitude of the "scalar-potential" part and the inviscid part of the velocity may be assumed to be small, see Briley & McDonald (1984). The magnitude of the "vector-potential" part of the cross-flow-plane velocity field is determined by the initial swirl intensity and is independent from the development of the axial velocity distribution. Thus the reduced Navier-Stokes equations are needed for an accurate description of the flow development.

However, as shown in chapter 3, the approximation of the pressure field might pose problems. Due to the pressure-continuity coupling the "well posedness" of the system of equations may be destroyed. To guarantee that the system of equations remains "well posed" it is necessary that the coupling between the axial pressure gradient and the continuity equation is removed. In a symmetric swirling flow this may be achieved by restricting the radial variation of the axial pressure gradient such that it is a function of the φ component of the cross-flow-plane velocity only. In a non-axisymmetric flow an equivalent procedure to obtain the cross-flow-plane variation of the axial pressure gradient is to use a Poisson equation for the pressure (section 3.3.2). To obtain an "uncoupled" pressure field all "scalar-potential" contributions in the cross-flow-plane velocity field must be removed from the pressure-Poisson equation. Since in this algorithm the separate components of the cross-flow-plane velocity distribution are not known, this procedure is not feasible. Thus we are forced to neglect the radial variation of the axial pressure gradient. Clearly, for non-swirling flows the approximation of a uniform axial pressure gradient is consistent with the parabolization concept. For swirling flow the approximation of a uniform axial pressure gradient may be invalid. To assess the validity of this approximation we will perform an order-of-magnitude analysis of the pressure term in the axial momentum equation.

Assume that the flow can be categorized as a "fully viscous flow" Rubin (1984). In this situation the axial pressure gradient can be approximated with,

$$rac{\partial P}{\partial x} = \mathcal{O}(rac{U_{\mathrm{mean}}}{R^2})$$

Owing to the evolution of the azimuthal velocity distribution the axial pressure gradient is "modulated" with a term proportional to the square of the circumferential velocity and proportional to the reciprocal of a typical length scale \mathcal{L} , for the downstream evolution of the swirl,

$$rac{\partial \Delta P}{\partial x} = \mathcal{O}(rac{U_{
m swjrl}^2}{\mathcal{L}}) \; .$$

The evolution length scale is proportional to $\alpha Re R$, were α is of order 10^{-1} and hence,

$$rac{\partial \Delta P}{\partial x} = \mathcal{O}(rac{U_{\mathrm{swirl}}^2}{lpha Re R}) \; .$$

A necessary condition for the validity of the uniform axial pressure gradient in the crossflow plane is

$$\frac{\partial \Delta P}{\partial x} \ll \frac{\partial P}{\partial x}$$
,
 $\alpha U_{\text{awirl}}^2 \ll U_{\text{mean}}^2$.

от

The result is that the calculations based on a uniform axial pressure gradient in the crossflow plane must be restricted to low swirl intensities. In the calculations presented here $U_{\rm swirl}/U_{\rm mean} = \mathcal{O}(10^{-2})$ to $\mathcal{O}(10^{-1})$.

For turbulent flows, as a consequence of the low admissible swirl intensity, the turbulent stresses may be modelled with relatively simple turbulence closures. It follows from the computational results reported in chapter 4 that the predictions using eddy-viscosity turbulence models and second-order closures coincide for low swirl intensities. Hence, for the present turbulent-flow calculations a simple mixing-length model will be used.

In the next sections we will present results for non-axisymmetric swirling flow. First results of a simulation of a laminar flow will be presented, next results for a turbulent flow are presented. The calculations are based on the parabolic penalty type of algorithm that has been presented in section 3.3.2.

5.3.1 Laminar non-axisymmetric swirling pipe flow

The first results presented here concern the evolution of an arbitrary but symmetric inlet condition. In this calculation the distribution of the axial velocity component is that of



Figure 5.1: The path of the COM (a) and COAM (b) in a swirling pipe flow. (Symmetric inlet condition, initial swirl number, $S_{initial} = 0.08$, laminar flow, $Re_D = 2 * 10^3$)

a fully developed flow, the inlet condition for the circumferential velocity distribution is given by

$$V \sim z(1-y^2-z^2)$$
 and $W \sim -y(1-y^2-z^2)$.

Fig. (5.1) shows the evolution of the position of the "centre of mass flow" (COM), see Eq. (5.1), and the "centre of angular momentum flow" (COAM), see Eq. (5.2). Though, even for the initial profile, the finite resolution of the discretization causes the COM and the COAM to be located off the centre of the pipe, no appreciable asymmetry develops. However, the evolution of the position of the COM and the COAM does not seem to be completely arbitrary. Possibly, the kinematics of the flow affects the development of this discretization-error-induced asymmetry. Since the error in the position of the COM and COAM induced by the discretization error, is not negligible, conclusions concerning the stability of the flow against non-axisymmetric perturbations cannot be drawn from this result.

Next a non-axisymmetric perturbation in the initial profile is introduced. The amplitude of this perturbation is small compared to the "basic" flow, but large compared to the discretization-induced errors. Naturally there is a large freedom in the choice the type of perturbation that can be chosen. Here two extreme types are chosen. The first type (I)consists of a strictly two-dimensional dipole-like structure in the cross-flow plane. It can be viewed upon as induced by a vector potential,
$$V_p\sim rac{\partial \Psi_p}{\partial x} \quad ext{and} \quad W_p\sim -rac{\partial \Psi_p}{\partial y} \; ,$$

with

 $\Psi_p \sim y (1-y^2-z^2)^2$ as x-component of a vector potential

The second type (II) perturbation consists of a disturbance of the axial velocity distribution only,

$$U_p \sim y(1-y^2-z^2) \; .$$

The type I disturbance shifts the centre of the swirl to an off-axis position, the type II disturbance shifts the position where the axial velocity is maximum to an off-axis position (Fig. (5.2)). In pipe flows with swirl generated by out-of-plane bends, the non-axisymmetric character of the flow appears to be very similar to the character of the type II disturbance, see Mattingly (1988). The evolution in downstream direction of the type



Figure 5.2: An example of the initial velocity distribution for (a), the type I disturbance, contours of the axial component of the axial velocity and (b) the type II disturbance, contours of the axial velocity component

II disturbance is shown in Fig. (5.3) for $0 \le x/D \le 50$ and for a different magnitude of the initial disturbance. According to Fig. (5.3) the initial asymmetry of the profile indeed grows. Furthermore, Fig. (5.3) shows that the magnitude of the asymmetry scales with the magnitude of the initial disturbance.

Apart from the development of an increasingly more asymmetric velocity distribution, the swirl contained in the flow also decays. Since the driving force of the asymmetry decays, one may expect that eventually the axial velocity profile should become symmetric again. Figs (5.4) and (5.5) show the evolution of the position of the COM and COAM



Figure 5.3: The evolution of the position of the COM and COAM as a function of the initial disturbance. The relative magnitude of the disturbance (Amplitude perturbation/Amplitude base flow) is a) 1%, b) 2% and c) 5%, Type II disturbance, laminar flow,, initial swirl number, $S_{initial} = 0.08$, laminar flow, $Re_D = 2 * 10^3$



Figure 5.4: The path of the (a) COM and the (b) COAM for $0 \le x/D \le 200$, type I disturbance, laminar flow, initial swirf number, $S_{initial} = 0.08$, laminar flow, $\mathcal{R}e_D = 2 * 10^3$



Figure 5.5: The path of the (a) COM and the (b) COAM for $0 \le x/D \le 200$, type II disturbance, laminar flow, initial swirl number, $S_{initial} = 0.08$, laminar flow, $Re_D = 2 * 10^3$

with increasing distance along the pipe. They show that when the swirl intensity is low, the COM as well as the COAM returns to the axis of the pipe. Apparently non-axisymmetric modes are triggered only if the swirl is strong enough, as is illustrated by Fig. (5.6). This figure shows the evolution of the asymmetry for an identical initial condition but for different values of the Reynolds number. According to Fig. (5.6), for $Re_D = 2 \times 10^3$ asymmetry is growing, for $Re_D = 4 \times 10^2$ the asymmetry initially neither grows or decays, while for $Re_D = 2 \times 10^2$ the asymmetry decays.

The results described above show that for swirls that are strong enough, the symmetric swirling mode is a meta-stable state. Any initially non-axisymmetric disturbance tends to grow in the downstream direction. The extent to which the flow becomes non-axisymmetric scales with the magnitude of the initial disturbance. Thus the flow exhibits a linear spatial instability. The growth rate of the instability is determined by the ratio of the swirl intensity and the viscous forces, i.e. a circumferential Reynolds number based on the swirl velocity. When this ratio is smaller than a critical value the non-axisymmetric modes decay; when it exceeds the critical value, the non-axisymmetric modes grow. Owing to the decay of the swirl, the driving force, eventually all asymmetry must disappear.

It must be kept in mind that for laminar flows there is no guarantee that the flow remains stationary. As indicated in section 5.2 instationary modes are possible. When nonstationary modes do occur, the possibility exists that within a short distance downstream of the initial disturbance the instationary modes will dominate the flow. Clearly, in this situation none of the steady-state results shown above will occur in practice.





Figure 5.6: The path of the COM and the COAM for $0 \le x/R \le 0.05 * Re_D$ and different Reynolds numbers a) $Re_D = 2 * 10^2$, b) $Re_D = 4 * 10^2$, c) $Re_D = 2 * 10^3$, laminar flow, type II disturbance

5.3.2 Turbulent non-axisymmetric swirling pipe flow

For the turbulent pipe flow a mixing-length turbulence model has been applied. In this model the turbulent stresses are given by,

$$\overline{u_i u_j} = \nu_t \frac{\partial U_i}{\partial x_j}$$

The eddy viscosity is prescribed by a mixing length formula,

$$\nu_{t} = (\kappa y_{n})^{2} \sqrt{\frac{\partial U_{i}}{\partial x_{j}} \left[\frac{\partial U_{i}}{\partial x_{j}} + \frac{\partial U_{j}}{\partial x_{i}} \right]},$$

with y_n representing the "normal" distance from the wall and κ the von Karman coefficient.

For regions close to the pipe wall, the van Driest damping of the mixing length l_m , is used,

$$l_m = \kappa y_n * \left(1 - e^{Re_\tau/A^+} \right)$$

with $Re_{\tau} = \sqrt{\tau/\rho} y_n/\nu$ and $A^+ = 26$.

Since the largest velocity gradients occur close to the wall, in the discretization the elements become rapidly smaller close to the wall, see Fig. (5.7). Similar to the calculations



Figure 5.7: The element distribution for the turbulent flow calculations.

for laminar flow, the initial velocity distribution is taken as a fully developed axial velocity distribution. Added to this developed axial velocity distribution is a symmetric swirl component. Contrary to the laminar case, the position of the COM and the COAM remain stable throughout the length of the pipe considered. Even when a type I disturbance is used as perturbation to the initial velocity distribution, both the position of the COM and of the COAM return quickly to the axis of the pipe, see Fig. (5.8). When a type II disturbance is used as perturbation, the asymmetry is more pronounced, but still the non-axisymmetry is much weaker than is observed in laminar flow, see Fig. (5.8).

There are two possible explanations for the apparent higher stability of turbulent flows against non-axisymmetric perturbations. The first reason is that in the turbulent flow at $Re_D = 2 * 10^4$ the momentum exchange is stronger than in a laminar flow at $Re_D = 2 * 10^3$. Thus the effective circumferential Reynolds number is lower and the flow will be more stable. Moreover, even though the Reynolds number in the laminar flow case is a factor ten lower, the decay of swirl in the turbulent flow case is faster than in the laminar flow case. Thus the driving force of the asymmetry disappears more quickly than in the laminar case.

A second explanation for the enhanced stability for the turbulent flow is found in the shape of the axial velocity distribution. Away from the wall the axial velocity distribution is much flatter than the developed axial velocity distribution in a laminar flow. Therefore, convection effects in the axial momentum equation, caused by a non-axisymmetric crossflow-plane velocity distribution, are much smaller than in a laminar flow. On the other hand, in the region close to the wall, where large velocity gradients do occur, the wall forces the flow to be symmetric. Thus a part of the interaction between the axially directed



Figure 5.8: The evolution of a non-axisymmetric swirl in a turbulent flow, $Re_D = 10^4$, type I disturbance



Figure 5.9: The evolution of a non-axisymmetric swirl in a turbulent flow, $Re_D = 10^4$, type II disturbance



Figure 5.10: The decay of the swirl for a symmetric and a non-axisymmetric flow

vorticity and the cross-flow-plane vorticity components is suppressed and again stability is enhanced. Indeed, it turns out that also laminar swirling flows are much more stable when the initial axial velocity distribution is more uniform.

Finally, in Fig. (5.10) the effect of the asymmetry of the swirl distribution on the decay rate is shown. When the non-axisymmetric circumferential velocity distribution is considered to be a superposition of a symmetric swirl and a dipole-like perturbation, one would expect that, to first order, asymmetry is not relevant for the rate of decay of the swirl. Due to the mirror-symmetric character of the perturbation, the contribution of the perturbation to the wall-shear stress cancels. However, for a turbulent flow, the turbulent momentum exchange is affected by asymmetry. Even a simple mixing-length approximation for the eddy viscosity is capable of accounting for this effect. Such a mixing-length model predicts that the non-axisymmetric part of the velocity distribution causes a non-axisymmetric eddy-viscosity distribution. As a result the symmetry of the wall-shear stress distribution is destroyed. Consequently, a non-axisymmetric velocity distribution may be reflected in the decay rate of the swirl. The results presented in Fig. (5.10) show that the effect is very small. For the flows considered in this chapter the decay rate is not affected by the asymmetry.

5.4 Closure

In this chapter an exploratory study was carried out into non-axisymmetric aspects of laminar and turbulent swirling pipe flows. Our own experiments, experiments described in the literature and theoretical studies have indicated that symmetry with respect to the pipe axis is not self-evident. Even a simple flow like a Poiseuille flow in an axially rotating pipe, may exhibit strong and still not fully understood instabilities.

In this study some simulations were performed to investigate the spatial stability of swirling flow in a stationary pipe. For laminar flows, above a critical swirl intensity, the flow appears to be linearly unstable for non-axisymmetric disturbances. The driving force that triggers these disturbances is the swirl itself. Since the swirl decays with distance along the pipe, the driving force for the non-axisymmetric modes disappears gradually. Eventually, the swirl intensity will become too small to sustain the non-axisymmetric modes and the flow becomes axisymmetric again. This implies that the asymmetry of the flow is of importance during the first stages of the decay only.

A turbulent flow with a mild swirl appears to be more stable against non-axisymmetric disturbances than a laminar flow. Two effects may be responsible for the stronger resistance to asymmetry. Compared to a laminar flow, the exchange of momentum is relatively stronger in a turbulent flow. Thus the effective Reynolds number based on swirl may be lower for a turbulent flow. The second reason is that distribution of the axial velocity is much more uniform than the distribution for a Poiseuille flow. This prevents a strong interaction of the axial and the cross-flow-plane components of the vorticity vector.

On the basis of a mixing-length hypothesis for the turbulent stresses one may expect that the decay of the swirl is influenced by asymmetry. However, the computational results indicate that this effect is negligible. Hence, for low swirl intensities, asymmetry needs not to be considered for an accurate description of the decay of the swirl.

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6. Experiments

6.1 Introduction

In this chapter experimental results concerning swirl in a turbulent pipe flow are presented. The results presented here are first results of a large and exploratory experimental study of pipe flows with swirl, see Steenbergen (1992). The objective of the measurements presented here has been to study the decay of swirl in flows relevant to the problem of flow metering. As noted in chapter 1, swirl in transport systems for natural gas may be produced by a combination of two out-of-plane bends. Swirl produced by this mechanism, is seldomly very intense, is concentrated near the pipe wall and is never axisymmetric.

In the present work swirl is introduced into the flow by means of an assembly of guide vanes, rather than by a combination of out-of-plane bends. By using adjustable guide vanes for generating swirl, a large number of different velocity distributions can be generated. Moreover, contrary to the swirling flow produced by out-of-plane bends, the velocity distribution produced by guide vanes can easily be made axisymmetric. Clearly, axisymmetric velocity distributions will ease the interpretation of the measurements considerably. However, although the swirl is introduced differently, in this study care is taken to produce velocity distributions not too different from the profiles typical for the flow produced by a combination of bends.

In earlier experimental studies of swirling pipe flows (for example Kitoh (1991)) the initial swirl intensity was high (S > 1.0), the initial velocity distribution exhibited reversal of the axial component and a strong concentration of axial vorticity near the axis of the pipe. In contrast, in the present experiment the intensity of the initial swirl is moderate $(S \le 0.5)$, no flow reversal occurs and the concentration of the axial vorticity near the axis of the pipe is not very pronounced.

6.2 Experimental method and flow configuration

6.2.1 Flow configuration

A schematic representation of the experimental setup using water as the flow medium is given in Fig. (6.1). Its main parts are the "swirl generator", the test section with a circular cross-section and an inner diameter of 0.07 m and a total length of 20 m and the measuring section. A constant flow of water is maintained by controlling the water level in a supply vessel. The level is regulated using a combination of pumps, each equipped with a simple on-off water-level sensor. The maximum realizable volume flow in the water circuit is about



Figure 6.1: A schematic view of the experimental set-up

80 m^3/hr . The maximum variation in a specified flow rate is less than 2%. The maximum attainable Reynolds number based on the mean velocity and the pipe diameter is 300,000.

Swirl is introduced in the flow by means of a "swirl generator", see Fig. (6.2). The generator consists of an assembly of guide vanes mounted onto a central body within a wider section of the tube. Flow reversal near the axis of the pipe is prevented by allowing a part of the water to flow through a channel passing centrally through the central body. By changing the diameter of the central channel and adjusting the angle of the guide vanes, a large variety of initial velocity distributions may be realized. In the series of measurements described here, one setting of the vane angle, central channel diameter (20 mm) and flow rate was chosen.



Figure 6.2: The swirl generator

The test section consists of a set of straight brass pipes of different lengths, all with an inner diameter of 0.07 m. The pipes were manufactured according to DIN 17660 and DIN 1755 standards. However, the pipes used in the test section were specially selected to meet additional criteria. These extra criteria were defined to guarantee that the pipe system itself does not introduce disturbances to the flow. These criteria were on:

- straightness, only pipes without a visible curvature were used;
- internal roundness, the maximum variation allowed was 2×10^{-4} m; and
- wall thickness, the maximum allowed eccentricity of the inner and outer diameter was $5 \times 10^{-4} m$.

Special pipe-to-pipe connections were designed to give a minimum discontinuity between two consecutive pipes. Measurements of the static pressure drop have indicated that the pipes may be considered as almost "hydraulically" smooth. The mean roughness parameter obtained from the Moody diagram was $\epsilon/D = 10^{-5}$.

The LDV measurements (section 6.2.2) have been carried out in the specially built measuring section, see Fig. (6.3). The measuring section consists of an ordinary pipe section. To allow optical access to the pipe, in three sub-sections of approximately 1 inner diameter long, the brass pipe wall was replaced by a thin (100 μ m) polyester film. To minimize undesired optical effects and to stabilize the polyester film, the polyester sections are contained in water-filled boxes with flat windows. The pressure in the boxes is kept at a slightly lower level than the pressure in the pipe. This is achieved by externally connecting the boxes to a point downstream. Thus, effectively the polyester sections are very rigid and it may be assumed that they preserve their cylindrical shape. The complete measuring section can be rotated around the axis of the pipe to allow for measurements at different inclination angles. It may be placed anywhere in the test section. Apart from the first eight diameters, measurements are possible at every downstream position.



Figure 6.3: One sub-section of the measuring section

6.2.2 Laser doppler velocimeter

The velocities are measured using a two-component Laser Doppler Velocimeter (LDV) (DANTEC). The system is operated in the reference beam mode. For each measurement within the pipe, a combination of three independent measurements using three different inclination angles (-45° , 0° and 45°), yields the three velocity components and all components of the Reynolds-stress tensor.

The measurement volume has a length of 1000 μm and a diameter of 100 μm . As a reference point for the orientation of the measurement volume, the axis of the pipe is

used. The position of the axis is determined using a signal produced by the polyester pipe wall. When the measurement volume is placed on the polyester section of the pipe wall, a strong continuous signal with a frequency equal to the shift frequency is produced. A traverse of the measurement volume through the pipe wall in the direction normal to the wall, yields an approximately symmetric intensity profile as a function of the distance along the traverse. By definition, the position of the pipe wall was chosen to be at the symmetry plane of the intensity profile. By combining three measurements of the wall position at three inclination angles the position of the axis of the pipe is determined.

The measurement volume is positioned by positioning the LDV system as a whole. For this purpose the LDV system is mounted onto a rotatable two-degree-of-freedom traversing system operated by stepping motors. A more extensive description of the experimental set-up and measuring system can be found in Steenbergen (1992).

6.3 Results

Measured results for the turbulent pipe flow with swirl are shown in Figs (6.4) to (6.11). In total, three "traverses" ¹ at three different inclination angles were made. However, the non-axisymmetric character of the measured velocity distributions appeared to be weak (see also chapter 5). Since the measured distributions appeared to be almost axisymmetric, here only the results of the traverse in the y = 0-plane are presented. The complete results of the experiments can be found in Steenbergen (1992). The Reynolds number based on the mean axial velocity and the pipe diameter was 50,000. The initial swirl number S (Eq. (4.24)) is estimated to be $S_{\rm initial} \approx 0.42$. Figs (6.4) and (6.5) show the mean axial velocity component. At the first station, (x/D = 8.7), an elevated plateau of higher axial velocity is visible. Possibly, this plateau is related to the plateau that is predicted by the calculations obtained with the modified $k - \epsilon$ model, see section 4.3.3. However it may also be a remnant of a "jct-like" axial flow generated by the hole in the central body of the "swirl generator". Further downstream, this plateau disappears and the axial velocity distribution develops rapidly towards a remarkably flat distribution (x/D = 36, 50, 70). At x/D = 96 the character of the distribution of U starts to resemble the character of a fully developed pipe flow. However, the difference between this distribution and the fully developed distribution is still appreciable, illustrating the long distance needed for the decay of swirling pipe flow.

The distribution of the mean circumferential velocity components is shown in Figs (6.6) and (6.7). Even though the flow passing through the central channel of the "swirl generator" is not subjected to any circumferential excitation, and a low level of vorticity near the axis of the pipe might be expected, already at x/D = 8.7 the circumferential velocity distribution exhibits a concentrated central region with a high level of vorticity. The character of the circumferential velocity profile at the first station resembles the characteristic distribution of the circumferential velocity found in other experiments, such as

^{1.} Note that in fact one "traverse" consists of three different traverses performed with three different orientations of the measurement volume with respect to the traversing direction.



Figure 6.4: The mean axial velocity component as function of the distance from the axis of the pipe at different axial positions in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$, along the horizontal traverse through the centre of the pipe, $0 < \pi/D < 40$



Figure 6.5: The mean axial velocity component as function of the distance from the axis of the pipe at different axial positions in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$, along the horizontal traverse through the centre of the pipe, 40 < x/D < 100



Figure 6.6: The mean circumferential velocity component as function of the distance from the axis of the pipe at different axial positions in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$, along the horizontal traverse through the centre of the pipe, 0 < x/D < 40



Figure 6.7: The mean circumferential velocity component as function of the distance from the axis of the pipe at different axial positions in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$, along the horizontal traverse through the centre of the pipe, 40 < x/D < 100

Kitoh (1991) and Algifri et al. (1987).

According to Kitch (1991) and Algifri et al. (1987) the circumferential velocity distribution can be divided into three regions, a core region which rotates as a solid body, an intermediate region or "free vortex" region containing only a small amount of vorticity and a wall region where the circumferential velocity sharply decreases to zero as a function of the distance to the wall. At x/D = 8.7 the present circumferential velocity distribution clearly exhibits a "solid-body-rotation" core region. However, the concentration of the axial vorticity component near the axis of the pipe is not as pronounced as in the experiment of Kitoh (1991), see Fig. (6.12). Furthermore, the "free-vortex" character of the outer region is only weak. The sharp decrease of circumferential velocity as a function of the distance from the axis of the pipe in the wall region is again clearly reproduced by the present measurements. At the second station x/D = 22, the three-region character has disappeared. In the region close to the axis, the magnitude of the axial vorticity component has decreased considerably and no "free-vortex" region can be distinguished any more. At this station the deviations from axisymmetry appear to be the most pronounced. Further downstream, see Fig. (6.7), the "solid-body-rotation" region expands until it fills the crosssectional area of the pipe almost completely and the flow returns to an axisymmetric state. At larger distances from the "swirl generator", where the intensity of the swirl has become low, the shape of the velocity distribution remains almost constant.

Regrettably, due to an unforeseen technical problem, reliable measurements of the turbulent shear stresses could not be obtained. A strong external signal with a frequency close to the shift frequency jammed the diode signals and effectively prevented the reliable evaluation of the turbulent shear stresses. The measurements of the normal stresses however, did not suffer from this problem.

The development of these components of the Reynolds-stress tensor is shown in Figs (6.8) to (6.11). Near the axis of the pipe, the distributions of the normal-stresses exhibit a strong peak at the first station downstream of the "swirl generator", see Fig. (6.8). Furthermore, at this axial station, the turbulence appears to be approximately isotropic. Apart from the region close to the pipe wall the three normal components are of the same order of magnitude. At the next station, as Fig. (6.9) shows, the level of the $\overline{u^2}$ -component has decreased considerably near the axis. Here the radial distribution of the $\overline{u^2}$ -component of the Reynolds-stress tensor is almost uniform. In strong contrast with this is the behaviour of the v^2 -component and the w^2 -component of the Reynolds-stress tensor. The value of these stress components remains approximately of the same order of magnitude and the radial distributions of the two components remain peaky. In downstream direction the magnitude of $\overline{v^2}$ and $\overline{w^2}$ decreases gradually. However the peaky radial distributions may be distinguished until x/D = 50. At larger distances downstream of the "swirl generator" this striking difference in the distribution of energy between the three normal components of the Reynolds-stress tensor gradually develops towards a more isotropic distribution, see Fig. (6.11). Here the distributions of the normal stresses start to resemble the radial distributions of a fully developed non-swirling pipe flow, showing that the $\overline{u^2}$ component has the largest magnitude and that the normal stresses are smallest at the axis of the pipe.



Figure 6.8: The normal components of the Reynolds-stress tensor as function of the distance from the axis of the pipe at x/D = 8.7 along the horizontal traverse through the centre of the pipe in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$,



Figure 6.9: The normal components of the Reynolds-stress tensor as function of the distance from the axis of the pipe at x/D = 22 along the horizontal traverse through the centre of the pipe in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$,



Figure 6.10: The normal components of the Reynolds-stress tensor as function of the distance from the axis of the pipe at $\pi/D = 36$ along the horizontal traverse through the centre of the pipe in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$,



Figure 6.11: The normal components of the Reynolds-stress tensor as function of the distance from the axis of the pipe at x/D = 70 along the horizontal traverse through the centre of the pipe in a turbulent pipe flow with swirl at $Re_D = 5 \times 10^4$,

6.4 Comparison with results of other experiments

In this chapter results have been presented of an experimental investigation concerning a turbulent pipe flow with swirl. In this experiment emphasis was put on a type of swirl comparable to the swirl that is likely to be encountered in large-scale transport systems for natural gas. However, unlike the velocity distributions encountered in transport systems for natural gas, the velocity distributions in the present experiment are almost axisymmetric. In this sense the present experiment can be distinguished from earlier experiments described in literature. In the experiments described in recent literature (e.g. Kitoh (1991), Algifri et al. (1987) and Mattingly & Yeh (1988)), either the swirl is introduced by means of a device similar to the device used in the present experiment, or the swirl is generated by a combination of out-of-plane bends. The flow in the former type of experiments is characterized by a central zone of recirculation attached to the swirl generator, see Fig. (6.12). The flow produced by a combination of out-of-plane bends is characterized.



Figure 6.12: Some typical radial distributions of the circumferential and axial velocity components as measured by Kitob (1991) in a turbulent pipe flow at $Re_D = 10^8$

by a much smoother distribution of swirl and by a low swirl intensity, see Figs (6.13) & (6.14). In the distribution of the axial flow no recirculation zones or even "dips" are present. However, immediately downstream of the bends the axial and the circumferential velocity profiles are strongly non-axisymmetric.

The results presented here reflect features characteristic for both types of experiments. Close to the swirl generator, the flow exhibits the three-region character—a "solidbody-rotation" core, a "free-vortex" annular region and a "wall" region—typical for the



x/0

Figure 6.13: Some typical radial distributions of the circumferential velocity component as measured by Mattingly et al. (1988) in a turbulent pipe flow at $Re_D = 10^5$ (Note that z denotes the downstream direction here)



x/D

Figure 6.14: Some typical radial distributions of the axial velocity component as measured by Mattingly et al. (1988) in a turbulent pipe flow at $Re_D = 10^5$

first group of experiments. However, contrary to the radial distributions of the circumferential velocity component measured by Kitoh (1991), which keep a high radial gradient along a large part of the pipe, in the present experiment the "solid-body-rotation" core starts to expand immediately in the cross-sectional plane. From a relatively short distance behind the swirl generator on, the circumferential velocity profile is in closer resemblance with the results obtained by Mattingly & Yeh (1988) than that measured by Kitoh (1991).

The radial distributions of the axial velocity component measured in the present experiment differ from the results in both types of experiments described above. Apart from the region close to the "swirl generator" the radial distributions of the axial velocity component at the different axial positions are rather uniform. Neither the results of Kitoh (1991) and Algifri *et al.* (1987) nor the results of Mattingly & Yeh (1988) exhibit comparable distributions. Clearly, the central recirculation region distinguishes the axial velocity distributions of Kitoh (1991) and Algifri *et al.* (1987) from the present results. In the results obtained by Mattingly & Yeh (1988) the radial distribution of the axial velocity component is strongly affected by the non-axisymmetric character of the flow. Finally, one of the most striking features of the present results is the behaviour of the normal Reynolds-stresses in the "solid-body-rotation" core of the flow. In the literature, no comparable effect has been reported. Since the flow is not subjected to a strong deformation in this region, it appears that the development in downstream direction of the normal components is not caused by extra swirl-induced production of turbulence. It may be assumed that the mutual interaction of the small-scale turbulent eddies is affected by the large-scale background vorticity caused by the swirl, or even that a strong direct interaction exists between the large-scale background vorticity distribution and the small-scale turbulent eddies.

The comparison of the results presented in this chapter and some typical results reported in the literature clearly indicates the diversity of the problem of a decaying swirl in a turbulent pipe flow. Although in some aspects the different results show a degree of resemblance, in general each of the results also appear to have strongly different characteristics. The differences may be partly caused by differences in the method of swirl generation or in the intensity of the swirl. However, the comparison of the results of the experiments indicates that the differences in the initial velocity distribution for these experiments also cause the swirl to develop differently in downstream direction.

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7. Comparison and discussion

7.1 Comparison

In this section we will compare the measurements presented in chapter 6 with predictions for axisymmetric flow based on the turbulence models presented in chapter 4. The comparison is carried out for a Reynolds number of 50,000 and a moderate value of initial swirl. The profiles of the mean flow quantities measured at the first axial station (x/D = 8.7) are used as an initial condition for the computation. For the region between the pipe wall and the measuring point closest to the wall, the initial velocity distribution is obtained by interpolation according to the logarithmic law of the wall.

The k and ϵ inlet conditions are obtained by a relaxation method: to the axial and angular momentum equations an additional "force" term F is added. For example for the axial momentum equation this term is

$$F = C_{\text{force}}(U_{\text{initial}}(r; 0) - U(r; x)),$$

with

 C_{force} representing a constant of magnitude $\mathcal{O}(10^1)$, U_{initial} the velocity distribution obtained from experiment and U the present velocity distribution.

is added. With these force terms added, the solution of the continuity equation, the three momentum equations and the k and ϵ equations, is allowed to evolve until it reaches a fully developed state. In this state the radial distribution of, for example, U(r; x) will be very similar to the distribution of $U_{\text{initial}}(r; 0)$ provided that C_{force} is large enough. The k, ϵ and velocity distributions belonging to this solution are used as the initial condition for the actual calculation, without F added.

In this comparison we consider the standard $k - \epsilon$ model and the swirl-corrected $k - \epsilon$ model, using the Gibson/Younis pressure-strain coefficients, described in chapter 4. Neither the full ASM nor the RSM model are considered in this comparison. As indicated in chapter 4 the results of the ASM are almost identical to the results of the swirl-corrected $k - \epsilon$ model. According to Chen (1992) the results of calculations employing an RSM based on the Gibson & Younis (1986) pressure-strain approximation are, at least in a qualitative sense, in agreement with the results of the computations using the swirl-corrected $k - \epsilon$ model

Fig. (7.1) shows a comparison of the predicted and measured radial distributions of the circumferential velocity W at four axial positions. Neither the swirl-corrected k = 1



Figure 7.1: A comparison of the computed radial distributions of the circumferential velocity component W_i and the measured distributions at different axial stations. $Re_D = 5 \times 10^4$

 ϵ model (M-k-eps) nor the standard $k - \epsilon$ model (k-eps) appears to be capable of representing the true behaviour of the flow with swirl. The swirl-corrected $k - \epsilon$ model predicts an efficient reduction of the radial exchange of angular momentum by the swirl, that results in a "three-region" character of the radial distribution of W with a distinct "free-vortex" region along a large part of the pipe. In all experiments reported by other authors, performed at a higher swirl intensity (Algifri (1987) and Kitoh (1991)), a similar "three-region" behaviour is found.

The standard $k - \epsilon$ model predicts a radial exchange of angular momentum that is not affected by the swirl. It results in a rapid development of the radial distribution of the circumferential velocity component towards a "self-similar" distribution that is preserved up to the last station considered. This behaviour is in better agreement with the results of experiments using out-of-plane bends as swirl generating mechanism.

The measured distributions of W show that neither of these two scenarios is followed. While in the first part of the pipe (x/D < 50) the radial distributions exhibit a "threeregion" character, in the second part of the pipe (x/D > 50) the measured distribution of W develops towards an almost perfect "solid-body-rotation" velocity distribution. Though not as pronounced as predicted by the swirl-corrected $k - \epsilon$ model, the measured radial distributions of W suggest that in the initial stages of the decay the radial exchange of momentum is reduced. In contrast, the experimental results suggest that during the final stages of the decay an effective radial exchange of momentum occurs. The almost perfect "solid-body-rotation" velocity distribution even suggests that in the second part of the pipe (x/D > 50) the radial exchange of momentum in the experiment is more effective



Figure 7.2: A comparison of the computed radial distributions of the axial velocity component U, and the measured distributions at different axial stations. $Re_D = 5 \times 10^4$

than the radial exchange predicted by the standard $k - \epsilon$ model.

The measured and predicted axial velocity distributions, Fig. (7.2), partly confirm this suggestion. While the results of the calculation based on the standard $k - \epsilon$ model resemble the "developed pipe-flow distribution" already at x/D = 36, the measured distribution develops to an almost uniform distribution. Only beyond x/D = 100 the experimentally determined distributions of U reaches a state comparable to the fully developed state, see Steenbergen (1992).

The predictions of the swirl-corrected $k - \epsilon$ model exhibit a central jet-like plateau in the distribution of U that is not present in the measurements, see Fig. (7.2). Thus, also the strong reduction of the radial exchange of axially directed momentum, that is predicted by the corrected $k - \epsilon$ model, is not reflected by the measurements.

Keeping in mind the large discrepancies between the calculated velocity distributions and the measured distributions, it is no surprise that the experimentally determined rate of decay of the swirl differs from the calculated rate of decay. A comparison of the measured and the calculated decay rates is shown in Fig. (7.3). However, it appears that the differences between the measured and calculated decay rate are far less dramatic than the differences occurring in the distributions of the mean velocity profiles. Partly this can be explained by the observation that the decrease of the total amount of swirl is determined by the wall-shear stress only, see Eq. (4.25). The magnitude of the wall-shear stress is determined by the magnitude of the mean velocity close to the wall. Differences in the velocity distribution close to the axis of the pipe will not affect the decay rate as long as the magnitude of the predicted mean velocity close to the wall is the same as the measured



Figure 7.3: A comparison of the measured rate of decay of swirl and the calculated rate of decay. $(Re_D = 5 \times 10^4)$

velocity.

It must be noted here that since the expression for the integral amount of axial angular momentum flux, Eq. (4.24) contains a weight factor r^2 , the determination of S from experiment is very sensitive to the method of interpolation in the region between the point closest to the wall where data is taken and the pipe wall. For the determination of S from the experimental results we assume that the wall-layer behaves according to the "law of the wall", i.e. the flow is in perfect "local equilibrium" and the velocity vector is not skewed. Assuming that the friction velocity varies between $0.1 \ge u^*/U_{\text{mean}} \ge 0.05$ along the pipe, the point closest to the wall where data is measured lies at the outermost edge of the logarithmic region $(r/R \approx 0.9, 250 \ge y^+ \ge 125)$. Since the Reynolds number used in the present experiment is not very high $(Re_D = 5 \times 10^4)$ the assumption of a "law of the wall" behaviour may be inappropriate, especially in the first part of the pipe. Consequently, an uncertainty in the magnitude of S may be expected. A first estimate for the maximum magnitude of the uncertainty in the value of S, is found by comparing the value of S obtained using a logarithmic interpolation and the value of S obtained using a linear interpolation between r/R = 0.9 and r/R = 1.0. It turns out that the difference in the magnitude of S resulting from the different treatment of the wall region is $\Delta S/S = 10 - 15\%$.

Keeping in mind the uncertainty in the magnitude of S obtained from the experimental data, we conclude that the main observation made in the comparison of the radial distributions of circumferential and axial velocity component is confirmed by Fig. (7.3). Since the experimentally found decay of S is faster than the decay according to the "swirlcorrected" $k - \epsilon$ model, it seems that the radial exchange of momentum is underpredicted by this model.

The decay of the integral amount of angular momentum predicted by the standard $k - \epsilon$ model seems in much better agreement with the measurements. However, at the end of the decay region the measured decay seems even faster than the decay predicted by the standard $k - \epsilon$ model, indicating a larger radial exchange of momentum than predicted by any of the models considered in this study.

7.2 Discussion

with

An important aspect of the flows considered in this study is the assumption of a cylindrical symmetry in the distribution of the flow quantities. The circumferential velocity component drives a continuous redistribution of energy between the various components of the Reynolds-stress tensor only when this symmetry is present. Without this "convectionscrambling" effect the predicted radial exchange of momentum is much stronger (see section 4.2.4). The measured distributions of the normal components of the Reynolds-stress tensor show that, apart from the region close to the pipe wall, the levels of $\overline{v^2}$ and $\overline{w^2}$ are approximately the same, while the magnitude of $\overline{u^2}$ level is much lower (Fig. (6.8) to Fig. (6.10)). Furthermore, the radial distribution of $\overline{u^2}$ is almost uniform. Hence the "curvature-scrambling" effect is not as pronounced as it is assumed to be within the ASM. It may be expected that the typical distribution of energy between the normal components of the Reynolds-stress tensor will have its effect on the magnitude of the shear stresses. Indeed, when the ASM equations (e.g. Eq. (4.9)) are solved, using a given distribution of the normal-stress components similar to the measured distribution, i.e.

$$\overline{u^2} = 2lpha(r)k$$
 and $\overline{v^2} = \overline{w^2} = (1 - lpha(r))k$
 $0 < \alpha(r) < \frac{1}{3}$

the resulting expression for \overline{vw} predicts a radial exchange of momentum comparable to the corresponding standard $k - \epsilon$ expressions, namely

$$\overline{vw} = -(1-\alpha) \varphi_1 \frac{k^2}{\epsilon} \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right).$$
(7.1)

Thus, apart from the comparison of the computed and measured distributions of the mean circumferential and axial velocity components, also the measured radial distributions of the normal-stresses suggest that the reduction of radial momentum transport predicted by the swirl-corrected $k - \epsilon$ model is too strong.

A final indication that the swirl-corrected $k - \epsilon$ model exaggerates the reduction of the shear stresses is obtained when the predicted reduction of the radial momentum transport is artificially weakened. This can be effectuated by replacing, the correction factors $Ri_{\overline{uv}}$ and $Ri_{\overline{vw}}$, in Eqs (4.20) and (4.12), by $\gamma Ri_{\overline{uv}}$ and $\gamma Ri_{\overline{vw}}$, with γ a reduction



Figure 7.4: A comparison of the computed radial distributions of the axial velocity and the measured distributions at different axial stations. $Re_D = 5 \times 10^4$, modified $k - \epsilon$ model, $\gamma = 0.5$ & $\gamma = 1.0$



Figure 7.5: A comparison of the computed radial distributions of the circumferential velocity and the measured distributions at different axial stations. $Re_D = 5 \times 10^4$, modified $k - \epsilon$ model, $\gamma = 0.5 \& \gamma = 1.0$

factor, i.e.

$$uv_1 = \overline{uv_0} * \frac{1 + \gamma Ri_{\overline{uv}}}{(1 + \gamma Ri_{\overline{vw}})(1 + \gamma Ri_{\overline{vw}}/4)}$$

and

$$\overline{vw_1} = \frac{\overline{vw_0}}{1 + \gamma Ri_{\overline{vw}}} \,.$$

Figs (7.4) and (7.5) show that even for $\gamma = 0.5$ the dramatic differences between the measurements and the computations are reduced considerably.

Clearly, the above observations merely suggest that some of the discrepancies between measurement and computation may partly be explained by the characteristic distribution of energy between the normal components of the Reynolds-stress tensor. Of course, the question that remains is, where this specific distribution of energy originates from. The conventional pressure-strain models do not offer an explanation. Near the axis of the pipe where the anisotropy between the cross-flow-plane velocity fluctuations and the axial velocity fluctuations is the most explicit, the deformation of the mean velocity distributions is minimal. Hence, the production terms of turbulence are small in this region and zstrong anisotropy can only be maintained by the pressure-strain interaction terms. However, it appears that the popular models for the pressure-strain interaction are not capable of reflecting the experimentally observed anisotropy. Since the production terms are small, the "rapid-part" of the Gibson-Younis (1986) pressure-strain model is small as well. The "rapid-part" of the pressure-strain model due to Fu et al. (1987) only contains the contribution due to convection. Also this contribution does not redistribute energy between the cross-flow-plane components of the velocity fluctuations and the axial component of the velocity fluctuations. Thus, also the pressure-strain model due to Fu et al. (1987) cannot explain the distribution observed in the experiment.

For a perfect solid-body rotation without a deformation of the axial velocity distribution, even the Launder, Reece and Rodi model (1975) (LRR-model) for the pressure-strain interaction, which for a simple shear flow allows the strongest anisotropy between the normal components, does not predict a redistribution between the cross-flow-plane and the axial components of the normal stresses. For the idealized swirl flow the LRR-pressurestrain term predicts that the sum of $\overline{v^2}$ and $\overline{w^2}$ remains independent of the rate of rotation:

$$\overline{v^2} = k \, \left(rac{2\,arphi_2}{3} - rac{2\,\Omega\,(arphi_C - arphi_D + arphi_P)\,arvee \overline{w}}{\epsilon}
ight) \qquad ext{and}$$

$$\overline{w^2} = k \left(\frac{2 \varphi_2}{3} + \frac{2 \Omega \left(\varphi_C - \varphi_D + \varphi_P \right) \overline{vw}}{\epsilon} \right)$$

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Figure 7.6: A local Rossby number determined from experiment as a function of the distance to the axis of the pipe for a number of axial stations.

so that,

$$\overline{v^2} + \overline{w^2} = \frac{4}{3} \varphi_P \, k \, ,$$

with
$$\varphi_2 = \frac{11(C_1 + P/\epsilon - 1) + (9c_2 - 1)P/\epsilon}{11(C_1 + P/\epsilon - 1)},$$

 $\varphi_P = \frac{3 - c_2}{11(C_1 + P/\epsilon - 1)},$
 $\varphi_D = \frac{-8c_2 - 2}{11(C_1 + P/\epsilon - 1)},$
 $\varphi_C = \frac{1}{C_1 + P/\epsilon - 1},$

where C_1 and c_2 represent the pressure-strain coefficients belonging to the pressure-strain interaction model (Eqs. (2.8) and (2.7)), and Ω represents the rotation rate.

Without a continuous supply of energy to the cross-flow-plane components of the velocity fluctuations one may expect that the magnitude of these components will decrease rapidly. For this particular distribution of energy between the normal components of the Reynolds-stress tensor, both the conventional "return-to-isotropy" term and the conventional dissipation term in the transport equations for the Reynolds-stresses will act to diminish the magnitude of both $\overline{v^2}$ and $\overline{w^2}$.

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One of the identified weaknesses of all present one-point closure schemes is their inability to take effects of system rotation into account. For example studies by Bardina *et al.* (1985) and Traugott (1958) clearly show that the dissipation of turbulence is reduced by system rotation. Furthermore, the simulations of Bardina *et al.* (1985) show that the primary effect of rotation is a redistribution of energy in wave-number space, leading to an increase of the length scales in the flow, especially those along the axis of rotation.

For the present experiment, an appropriate indication for the importance of rotation effects as described by Bardina *et al.* (1985) and Traugott (1958), is given by a local Rossby number, for example defined as,

$$\mathcal{R}\sigma = \sqrt{\frac{D_{ij}D_{ij}}{\Omega_{ij}\Omega_{ij}}} \tag{7.2}$$

with

$$\Omega_{ij} = rac{\partial U_i}{\partial x_j} - rac{\partial U_j}{\partial x_i}$$
 and $D_{ij} = rac{\partial U_i}{\partial x_j} + rac{\partial U_j}{\partial x_i}$

For $Ro \ll 1$ rotation effects are expected to be important, while for $Ro \geq 1$ rotation effects will be small. Fig. (7.6) shows that in the region close to the axis of the pipe, the total rotation is large compared to the total deformation of the mean velocity distribution. Hence one may expect that in the present flow, close to the axis of the pipe, rotation effects are important.

Weinstock (1981), (1982) and Weinstock and Burk (1985) showed that for a simple shear flow the LRR model provides a good approximation for the "rapid-part" of the pressure-strain interaction and that discrepancies between predicted stress levels and measured stress levels must be attributed to Rotta's (1951) "return-to-isotropy" approximation. Rather than provoking a "return-to-isotropy" the pressure-strain term should resist to a large anisotropy. The formal difference between the Rotta (1951) "return-to-isotropy" term and the term derived by Weinstock (1981), (1982) and Weinstock and Burk (1985) is that the "return-to-isotropy" coefficient C_1 , is different for all components of the Reynolds-stress tensor and varies with the anisotropy of the flow. Furthermore, Weinstock (1981), (1982) shows that the "return-to-isotropy" coefficients are sensitive to the shape of the spectrum of the turbulence.

Taking into consideration the relative magnitude of the rate of rotation compared to that of the rate of deformation of the flow, clearly at least part of the explanation for the incorrectly predicted distribution of energy among the normal-components of the Reynolds-stress tensor may be found in the effect of rotation on the distribution of energy in wavenumber space, demonstrated by Bardina *et al.* (1985), combined with sensitivity of "return-to-isotropy" term to the spectrum of the turbulence predicted by Weinstock (1981), (1982) and Weinstock and Burk (1985).

Alas, the studies of Bardina *et al.* (1985), and Weinstock (1981), (1982) and Weinstock and Burk (1985) apply only to flows very different from the swirling pipe flow. In literature, no expressions are given for the "return-to-isotropy" coefficients $C_{1,ij}$, for rotating flows. Furthermore studies on the effect of rotation on for example the spectra or the

dissipation in non-homogeneous and non-isotropic turbulence have not yet been carried out. In the flow subject of the present study the distribution of turbulence is non-homogeneous and non-isotropic. Hence, if the flow near the axis of the pipe is indeed dominated by rotation, it seems that none of the presently used one-point-closure schemes is capable of capturing the experimentally observed distribution of energy between the normal components of the Reynolds-stress tensor. Even the simplest expressions for the pressure-strain interaction show that the poor prediction of the normal-stresses may lead to an underprediction of the turbulent shear stresses and consequently to an underprediction of the decay of swirl.

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8. Concluding remarks

Since the occurrence of swirl is a principle source of error in flow metering, knowledge of the rate of decay of swirl is of utmost importance for the prediction of the installation effects of flow meters. In piping systems, swirl is usually generated by two consecutive out-of-plane bends. Current standards (c.g. ISO-5167 or ANSI/API-2530) for flow metering do not fully recognize swirl as a source of error and hence do not seem fully adequate for defining the installation requirements in the presence of swirl. As a result, flow-measuring devices are often employed in non-ideal installations, leading to an enhanced uncertainty in the flow metering results.

Experimental work aimed at the decay of swirl has been performed in the past but does not seem to be conclusive. Furthermore, most experimental work has been performed under conditions very different from the conditions relevant to the practice of large-scale transport systems for natural gas.

In this thesis we investigated the possibility of numerically predicting the decay of swirl in a turbulent pipe flow. The prime motivation of this study is the desire to obtain a fundamental understanding of the decay process and the desire to obtain a tool to translate experimental results for the decay of swirl obtained in the laboratory at sub-scale conditions to conditions relevant to full-scale transport systems for natural gas. Accurate predictions of the decay length of swirl would open the possibility to assess installation effects on flow metering devices, thereby reducing the uncertainty in the flow-metering results.

It appears that numerical predictions of turbulent swirling pipe flow depend critically on the quality of the model for the Reynolds stresses. This suggests that accurate predictions are only possible if "second-order" turbulence closure schemes are used. Only these models have the potential to capture the most important mechanisms governing the decay of swirl in a turbulent pipe flow. For the case of a straight section of a pipe, and for slowly decaying swirls, the simpler versions of the "second-order" turbulence closure schemes can be shown to reduce to a simple correction to the well-known $k - \epsilon$ model of turbulence.

The main effect of the correction consists of a change of the length-scale on which the momentum exchange occurs. For the case of a swirling pipe flow, the "second-order" models predict that, apart form the region close to the wall of the pipe, the length scales are reduced, leading to a reduction of the radial exchange of momentum. The reduction of the radial mixing of momentum results in strong memory effects and a strong dependence of the rate of decay of the swirl on the initial velocity distributions.

In the region close to the wall of the pipe the momentum exchange may be enhanced. However, this effect appears to be weak. For flows at a high Reynolds number $(Re_D > O(10^6))$, the computational results suggest that in this region the velocity distribution obeys the "logarithmic law of the wall". Also the angle between the velocity vector and the axis of the pipe remains constant in this region.

In general the flows encountered in pipe systems will be non-axisymmetric. From a computational point of view non-axisymmetric swirling flows are difficult to handle. A fully three-dimensional representation of the governing equations easily leads to unacceptable demands on computer resources. On the other hand, a parabolization of the system of equations, without destroying the "well-posedness" of the system requires careful analysis. For high Reynolds number, a preliminary study of non-axisymmetric swirling pipe flows at low swirl intensities shows that the non-axisymmetric character of the flow is pathological. In this situation even an initially axisymmetric swirling flow develops to a non-axisymmetric flow. The flow becomes axisymmetric again, only upon almost complete decay of the swirl. For low levels of swirl, the effect of non-axisymmetric velocity distributions on the rate of decay of the swirl can be shown to be negligible.

The results of the computations are only partly in agreement with the experimental results. As predicted by the computations, initially the velocity distribution develops towards a non-axisymmetric distribution, while further downstream the distribution returns to an axisymmetric distribution again. The predicted pronounced reduction of the radial exchange of momentum is not obscrved in the measurements. The three-region character of the radial distribution of the circumferential velocity is recognizable very close to the "swirl generator" only. This suggests that the reduction of the radial exchange of momentum exchange is overpredicted by the models. The downstream development of the swirl number S, determined from the experiment, supports this observation. The experimentally found decay is faster than the computed one.

An explanation for the discrepancies between computation and measurement may be found in possible effects of rotation on the dissipation of turbulence and on the "returnto-isotropy" interaction. In the models used in the present calculations these effects are not present. Theoretical studies and direct simulations of, and experiments in, much simpler flows, i.e. homogeneous, isotropic turbulence, suggest that an effect of rotation on the rate of dissipation of the turbulence and on the "return-to-isotropy" interaction maybe important.

As a final note we conclude that, even though the decay of the swirl is not reproduced to great accuracy by any of the models used, the models used in the calculations do capture some of the important mechanisms present in swirling pipe flows. Hence, the computational methods may serve as a tool to obtain a first order assessment of the effects of for example Reynolds number, inlet conditions or wall roughness on the decay of swirl. In this fashion the computational study of swirling pipe flows does contribute to the interpretation of experimental results, and the extrapolation of experimental results from sub-scale conditions to full-scale operational conditions.

Summary

For the international transport of large amounts of gaseous fuels through piping systems, accurate measurement of the total volume flow is of utmost economic importance. Recent research has shown that the accuracy of the two standard measuring devices, i.e. orifice meters and turbine meters, for large-scale flow measurement in pipes, is not adequate in all situations. The inaccuracy is assumed to be caused by "installation effects", i.e. caused by non-ideal flow conditions at the metering device. A notorious cause of error is the appearance of an axially directed vorticity component in the flow. A flow with an axially directed vorticity component, or "swirling" flow, is known to develop very slowly and to have a large effect on flow meters performance.

This thesis describes a study initiated by Dutch Gasunie. The study is aimed at the description of the development of a turbulent flow with swirl in a straight pipe. Since extensive experimental studies are not feasible under conditions relevant to large-scale transport systems for natural gas, in this study emphasis was put on the numerical modelling of turbulent pipe flows with swirl.

For the class of flows with "secondary strains", to which swirling flows belong, most researchers agree that only "second-order" turbulence models, like the Reynolds Stress and Algebraic Stress Model, are capable of capturing the important physical phenomena. However, for the simple geometry of a straight pipe, for low swirl intensities and for axisymmetric flows, the some variants of these "second-order" turbulence models can be shown to reduce to a much simpler form. This simpler form may considered as a modification to the well-known $k - \epsilon$ model of turbulence. In this thesis this modified version of the $k - \epsilon$ model is used to study the development of a turbulent pipe flow with swirl.

The most important effect that is predicted by the modified $k - \epsilon$ model is, compared to the exchange that is predicted by the conventional $k - \epsilon$ model, a strong reduction of the radial exchange of momentum. This reduction results in appreciable memory effects in the flow. Decaying lengths for swirl depend strongly on the initial velocity distribution. For very low swirl intensities the modifications to the $k - \epsilon$ model can be shown to have no effect.

In general, flows encountered in pipe system will be non-axisymmetric. Preliminary computations show that even for axisymmetric initial conditions, swirling pipe flows tend to become non-axisymmetric. The velocity distribution becomes axisymmetric again, only upon the swirl has decayed almost completely. However, the effect of the non-axisymmetric velocity distribution on the decay rate of the swirl appears to be minimal.

The numerical results are only partly confirmed by the first results of an experimental study. The experimental results indicate that the suppression of radial momentum exchange is not as strong as predicted by the modified $k - \epsilon$ model. It appears that, apart from the first stages of the decay, the predictions of the conventional $k - \epsilon$ model are even better than the predictions of the modified $k - \epsilon$ model and the "second-order" turbulence models.

The experimental results suggest that, contrary to the assumptions of the one-point turbulence closures, the non-linear interactions between the small-scale turbulent eddies are affected by the large-scale background rotation caused by the swirl. Furthermore, the experimental results show, instead of an expected strong anisotropy between the circumferential and radial velocity fluctuations, a strong anisotropy between the axial and the cross-flow-plane velocity fluctuations. This effect is completely missed by the modified $k - \epsilon$ model and the popular variants of the RSM and ASM! Yet, the measured anisotropy of the turbulent fluctuations can be shown to have a "restoring" effect on the radial exchange of momentum.

The predictions obtained with the corrected $k - \epsilon$ model, the RSM and the ASM provide a conservative estimate of the swirl decay rate and consequently also for the length required for the swirl to decay. Compared to the measured rate of decay the conventional $k - \epsilon$ model predicts a higher rate of decay in the initial stages of the decay, while in the final stages of the decay a slightly lower rate of decay is predicted.

Samenvatting

Voor het grootschalige internationale transport van gassen en vloeistoffen door pijpleidingen is een nauwkeurige debietmeting van het allergrootste economische belang. Recent onderzoek heeft aangetoond dat de nauwkeurigheid van de twee standaardtechnieken gebruikt voor pijpstromingen, de meetflens en de turbinemeter, in een aantal situaties nict voldoende is. Aangenomen wordt dat de onnauwkeurigheid veroorzaakt wordt door installatie effecten, dat wil zeggen door niet ideale aanstroomcondities van de meter. Een beruchte oorzaak van meetfouten is de aanwezigheid van een axiaal gerichte rotatie in de stroming. Bekend van een pijpstroming met een axiale rotatiecomponent, ofwel een pijpstroming met "swirl", is dat deze bijzonder langzaam ontwikkelt en dat de debietmeting sterk wordt beïnvloed.

Het onderzoek beschreven in dit proefschrift is gestart op initiatief van de N.V. Nederlandse Gasunie. In dit onderzoek wordt gepoogd een beschrijving te geven van de ontwikkeling van pijpstromingen met swirl. Omdat turbulentiemetingen moeilijk uitvoerbaar zijn bij de hoge Reynoldsgetallen waarvan in praktijk sprake is, wordt in de hier beschreven studie de nadruk gelegd op de numerieke modellering van pijpstromingen met swirl.

Voor de klasse van stromingen met "secondary strains", waartoe de pijpstroming met swirl behoort, wordt in het algemeen aangenomen dat alleen de "second-order" turbulentie modellen, zoals het Reynolds Stress Model (RSM) en het Algebraic Stress Model (ASM), in staat zijn de essentiële verschijnselen van deze klasse van stromingen te beschrijven. Voor een eenvoudige stroming als de axi-symmetrische pijpstroming met swirl, kunnen een aantal versies van de "second-order" sterk vereenvoudigd worden. Ze kunnen in deze vereenvoudigde vorm beschouwd worden als een correctie op het bekende $k - \epsilon$ turbulentie model. In het hier beschreven onderzoek wordt een gecorrigeerde versie van het $k - \epsilon$ turbulentie model gebruikt.

Het belangrijkste effect van de correctie is dat, vergeleken met de impuls-uitwisseling voorspeld door het standaard $k - \epsilon$ model, de impuls-uitwisseling sterk onderdrukt wordt. Deze onderdrukking van de impuls-uitwisseling heeft tot gevolg dat de stroming een sterk "geheugen" heeft. De lengte nodig voor het verdwijnen van de swirl hangt hierdoor sterk af van de intrecconditie van de stroming. Voor hele lage swirlintensiteiten kan worden aangetoond dat de correcties geen effect hebben.

Pijpstromingen zoals ze in praktijk voorkomen zullen in het algemeen niet axisymmetrisch zijn. Verkennende berekeningen tonen aan, dat zelfs voor symmetrische begincondities een pijpstroming met swirl de neiging heeft asymmetrisch te worden. Pas wanneer de swirl bijna helemaal is uitgedempt wordt de stroming weer axi-symmetrisch. Het effect van de asymmetrie op demping van swirl blijkt minimaal te zijn. De numerieke resultaten worden slechts in beperkte mate bevestigd door de experimenten. De experimentele resultaten geven aan dat de onderdrukking van de impulsuitwisseling veel minder sterk is dan wordt voorspeld door het gecorrigeerde $k - \epsilon$ model. Behalve gedurende de eerste paar diameters, levert het standaard $k - \epsilon$ model betere resultaten op dan het gecorrigeerde $k - \epsilon$ model en de meer algemene "second-order" turbulentiemodellen. De experimentele resultaten suggeren dat, in tegenstelling tot de basisveronderstellingen van de genoemde turbulentiemodellen, de niet-lineaire interacties tussen de turbulente eddies op de kleinste schaal worden beïnvloed door de grootschalige achtergrondrotatie. Verder geven de experimentele resultaten, in plaats van een algemeen verwachte sterke anisotropie tussen de radiaal en azimutaal gerichte turbulente snelheidsfluctuaties, juist een sterke anistropie tussen de axiale fluctuaties enerzijds en de radiale en azimutale fluctuaties anderzijds. Dit effect wordt volledig gemist door zowel bet gecorrigeerde $k - \epsilon$ model als door de ASM en RSM modellen. Van deze gemeten anisotropie van de normaalspanningen kan worden aangetoond dat deze de impuls-uitwisseling weer kan bevorderen.

De voorspellingen van bovenstaande modellen geven een behoudende schatting voor de snelheid waarmee een swirl in een pijpstroming uitdempt. Het standaard $k - \epsilon$ model geeft, vergeleken met de gemeten demping, aanvankelijk een te sterke demping, terwijl het voor lage swirlintensiteiten een te zwakke demping geeft.
Curriculum vitae

9 november 1961:	Geboren in Eindhoven
juni 1980:	Diploma Atheneum b
februari 1987:	Diploma Technische Natuurkunde, Technische Universiteit Eindhoven
juni 1987- januari 1989:	Vervulling dienstplicht aan de Technische Universiteit Eindhoven
februari 1989 juli 1992:	In dienst van de Technische Universiteit Eindhoven als toegevoegd onderzoeker
oktober 1992:	In dienst van TNO TU-Delft (TPD)

Stellingen behorende bij het proefschrift "Decay of swirl in turbulent pipe flows" door R.R. Parchen

1. De veronderstelling dat er loslating optreedt bij de instroming van de glottis zoals voorgesteld door Ishizaka (1972) en Miller (1988) is het gevolg van het toepassen van een onaanvaardbare vereenvoudiging van de geometrie van de glottis, waarbij de glottis wordt voorgesteld als een contractie met scherpe randen. De veronderstelling dat het drukverlies gekoppeld is aan een vena-contracta factor is daarom onrealistisch en dient achterwege gelaten te worden.

Ishizaka, K. & Matsudara, M., (1972), Speech Communication Research Laboratory, Monograph no. 8 Miller, J.A., et al., (1988), J. Sound and Vibration, **121**, 277

2. De storing in het gasdistributiesysteem in Goirle (1988), waarbij de eindverbruikers een druk van ongeveer 3 bar aangeboden kregen in plaats van de gewenste 30 mbar was het gevolg van de incompressibele benadering van de wet van Bernoulli bij het ontwerp van het regelsysteem. Het over het hoofd zien van de beperkte geldigheid van deze wet benadrukt het belang van het opnemen van het vak gasdynamica in het verplichte curriculum voor de studie van werktuigbouwkundig ingenieur.

Kolkman, M., (1992), Afstudgerverslag, Hogeschool Enschede

 Bij de beschrijving van demping van akoestische golven in turbulente pijpstroming is het essentieel dat de eindige relaxatietijd van de turbulentie in rekening gebracht wordt.

Ronneberger, D., & Ahrens, C.D., (1977), J. Fluid Mech., 83, 433 Peters, M.C.A.M, ct al., (1992), Submitted to J. Fluid Mech.

4. Het verdient aanbeveling om wandschuifspanningen in pulserende turbulente pijpstromingen, in plaats van rechtstreeks met behulp van hete draad- of hete filmopnemers, indirect met een multi-microfoon methode te bepalen.

Reijnen, A.J., (1992), Afstudeerverslag R-1183-A, vakgroep Transportfysica, Technische Universiteit Eindhoven Mankbadi, R.R., Liu, J.T.C., (1992), J. Fluid Mech., 238, 55 5. De veronderstelling van Fletcher & Rossing dat de amplitude van het akoestische veld in een fluit alleen begrensd wordt door de, door de eindige breedte van de jet bij het labium, verzadigende fluctuerende volumestroom leidt tot een voorspelling van de akoestische amplitude welke tenminste één orde groter is dan de experimenteel waargenomen amplitudes. De kwalificatie van deze overeenkomst door Fletcher & Rossing als zijnde "redelijk goed" is misplaatst. Een model waarbij het al in 1969 door Coltman gesuggereerde niet-lineaire effect van wervelafschudding in rekening gebracht wordt, levert namelijk wêl de correcte orde van grootte van de amplitude op (Fabre, 1992).

Fletcher, N.H. & Rossing, T.D., (1991), The Physics of Musical Instruments, Springer Verlag
Coltman, J.W., (1969), J. Acoust. Soc. Am., 46, 477
Fabre, B., (1992), Proefschrift, Le Mans

6. Kitoh (1991) typeert het buitengebied van een pijpstroming met swirl als zijnde van het "vrije wervel" type. Berekeningen met Reynolds Stress en Algebraïsche Stress modellen geven aan dat de distributie van azimutale snelheid sterk afhangt van de details van het gebruikte model. De term "vrije wervel" suggereert echter dat visceuze effecten verwaarloosbaar zouden zijn en is daarom misleidend.

Kitob, O., (1991), J. Fluid Mech., 225, 445

7. De veronderstelling dat een turbulente stroming onderworpen aan een solid-body rotatie gestabiliseerd zou worden, gaat voorbij aan de mogelijk destabiliserende interactie tussen de axiaal en azimutaal gerichte vorticiteit. De door Bradshaw (1969) geponeerde analogie met een stroming onderworpen aan een temperatuurgradiënt wordt daarom in het algemeen ten onrechte geeiteerd.

Bradshaw, P., (1969), J. Fluid Mech., 36, 177

8. Om het gedrag van een oscillerende stroming bij hoge frequentie in een porcus materiaal te onderzoeken, kan men volstaan met het berekenen van een stationaire stroming.

Smeulders, D.M.J., (1992), Proefschrift, Technische Universiteit Eindhoven

9. De door Girschick en Chiu (1990) voorgestelde uitdrukking voor de homogene nucleatiesnelheid in een oververzadigde damp berust op een niet gerechtvaardigde toekenning van macroscopische eigenschappen aan "clusters" bestaande uit één molecuul. De verbeterde overeenkomst tussen de nieuwe theorie en sommige experimentele resultaten berust dan ook op toeval.

Girschik, S.L., Chiu, C.P., (1990), J. Chem. Phys., 93, 1273

 Een verbod van voorruiten en andere vormen van rijwindbescherming in het gemotoriseerde verkeer zal de verkeersveiligheid in belangrijke mate verhogen.