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# A New Approximate Evaluation Method for Two-Echelon Inventory Systems with Emergency Shipments

**Erhun Özkan · Geert-Jan van Houtum · Yasemin Serin**

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**Abstract** We consider the control of repairable spare parts in a network consisting of a central warehouse, a central repair facility, and multiple local warehouses. Demands for spare parts occur at the local warehouses. If a local warehouse is out of stock, then an arriving demand is satisfied by an emergency delivery from the central warehouse or the central repair facility. Such emergency shipments are common practice for networks that support technical systems with high downtime costs, and it is important to take them into account when the inventory is optimized. Our main contribution consists of the development of a new approximate evaluation method. This method gives accurate approximations for the key performance measures, as we show via numerical analysis. The method is also fast and thus can easily be incorporated in existing (greedy) heuristic optimization methods. Our method outperforms the approximate evaluation method of Muckstadt and Thomas (1980), as we also show via the numerical analysis. Finally, we show that the performance of the system is rather insensitive to the leadtime distribution of the repairs at the central repair facility, which implies that our method works well for generally distributed repair leadtimes.

**Keywords** Spare parts · two-echelon system · emergency deliveries · approximate evaluation

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## 1 Introduction

The management of spare parts becomes more and more important in the capital goods industry. For many technical systems, downtime costs are high and thus broken parts have to be replaced by spare parts as quickly as possible. Spare parts may be kept on stock in networks by the user itself, or by Original Equipment Manufacturers or third parties. Spare parts networks typically consist of local warehouses at close distance of installed systems and one or more layers of central and regional warehouses. In such networks different types of flexibilities have been built in to react as quickly as possible on failures of technical systems. If a local warehouse is out of stock at the moment that a demand arrives, then it is made possible to sent a part from a neighboring local warehouse and/or directly from a higher level warehouses. These options are denoted as lateral and emergency shipments, respectively. Which options are used/possible depends on geographical factors and on the arrangements that have been made with, e.g. with logistics service providers and external repair centers.

We consider repairable spare parts in a two-echelon system, consisting of a central warehouse, a central repair facility, and multiple local warehouses. The repair facility, which is assumed to have an infinite repair capacity, supplies the central warehouse, and the central warehouse supplies the local warehouses. We assume a continuous review, one-for-one replenishment policy in the network (i.e., base stock control), which is a common policy in the spare parts literature. As an illustration, we describe the supply chain of spare parts at Nedtrain, a train maintenance company in the Netherlands. Nedtrain has thousands of different repairables in its supply chain. The repairables have a wide price range; their price can reach up to tens of thousands of euros. A failure of a critical repairable causes downtime of the train until the broken part is replaced by a ready-for-use part, and downtime costs per hour are very high. Thus the availability of a critical repairable is very important to Nedtrain. When a demand for a repairable arrives at a local warehouse, it is supplied by the local warehouse if there is on-hand stock in the warehouse. If the demanded repairable is not available in the stock of the local warehouse, then the central warehouse is checked and if there is on-hand stock in there, an emergency shipment is made from the central warehouse. If even the central warehouse does not have on-hand stock, an emergency shipment is done from the repair facility to the local warehouse. An emergency shipment time is shorter than a normal replenishment time, and as a result, this supply option is costly. Managing this kind of inventory systems needs quantitative models which take the use of emergency deliveries into account.

There are many studies on inventory control in spare parts networks. Sherbrooke (1968) developed the METRIC (Multi-Echelon Technique for Repairable Item Control) model for two-echelon systems, without lateral and emergency deliveries. Via the METRIC approach, expected backorder levels at all local warehouses can be computed, under base stock control and given base stock levels. In his approach, Sherbrooke (1968) approximates the realized replenishment leadtimes for the local warehouses by independent and deterministically distributed leadtimes. Graves (1985) developed exact and approximate evaluation procedures for multi-echelon systems. In the approximate method, Graves (1985) fits a negative binomial distribution at the first two moments of pipeline stocks and this approximation gives much more accurate results than the METRIC approximation.

Rustenburg et al. (2003) generalized Graves' exact and approximate evaluation methods to multi-echelon, multi-indenture systems. Sherbrooke (1968) also developed a heuristic optimization method for the minimization of the total stock of multiple items under a constraint for the total number of backorders in the whole system. Wong et al. (2007) developed multiple heuristics for the same optimization problem but then with a constraint per local warehouse. Basten (2010) and Basten et al. (2009) looked at the integrated optimization of spare parts stocks and the places where parts are repaired ('Level Of Repair Analysis').

Muckstadt and Thomas (1980) extended the work of Sherbrooke (1968) to systems with emergency deliveries from the central warehouse and central repair facility. Their focus is on the (heuristic) optimization of the base stock levels, which builds on the approximate evaluation method introduced in their paper. They also compare centralized and decentralized decision making. Hausman and Erkip (1994) improved the decentralized case of Muckstadt and Thomas (1980) and showed that the performance of the improved single-echelon model is within 3% and 5% of the multi-echelon model of Muckstadt and Thomas (1980).

Axsäter et al. (2004) considered a two-echelon inventory system in which emergency deliveries are done only from the central repair facility to the local warehouses. They assumed that the emergency delivery time exceeds regular replenishment leadtimes from the central warehouse to the local warehouses. Different from the other studies, Axsäter et al. (2004) assumed that the central warehouse also receives direct customer demands and this stream of demands has priority over the replenishment orders of the local warehouses. They use critical inventory levels at the central warehouse to differentiate between the demand streams. Axsäter et al. (2004) also derived an approximate upper and lower bound for the total system cost and developed a heuristic approach to determine the optimal policy parameters.

Axsäter (1990) developed an approximate evaluation for two-echelon systems with lateral shipments. Alfredsson and Verrijdt (1999) considered two-echelon systems with both lateral and emergency shipments. In case of stock-out at a local warehouse when a demand occurs, they first check other local warehouses for a lateral shipment, then they check the central warehouse for an emergency shipment, and lastly they make an emergency shipment from the repair facility if needed. Because of the lateral shipments, which are possible between all pairs of local warehouses (full pooling), they can aggregate all stocks in the local warehouses to calculate the fractions of demands satisfied by emergency shipments from the central warehouse and repair facility. For the latter step, they make use of a two-dimensional Markov process for the central and local stock, and its numerically calculated steady-state distribution. As a result, their approximate method becomes very time-consuming for already medium high base stock levels. Alfredsson and Verrijdt (1999) also executed a sensitivity analysis with respect to the distribution of the repair leadtimes of the central repair facility and the distribution of the transportation times between the central warehouse and the local warehouses. They found that the performance parameters are almost 100% insensitive for these distributions.

Grahovac and Chakravarty (2001) considered the same system as Alfredsson and Verrijdt (1999), but then without the possibility of an emergency shipment for the central repair facility (and thus with the possibility of backordering at the local warehouses). A second difference is that, in case of stock-out at a local warehouse when a demand occurs, they first check the central warehouse for an emergency shipment, and next they check other local warehouses for a lateral shipment. Lastly, they considered emergency trigger inventory levels at the local warehouses, i.e, they allow lateral shipments not only when there is a stock-out, but at arbitrarily chosen levels of net stock. They used a similar iterative solution methodology as Axsäter (1990). They also showed that sharing of stock (via the emergency and lateral shipments) often, but not always, reduces overall system costs. Moreover, the optimal emergency trigger inventory levels were $-1$ in most of the cases, which implies that anticipation of future demand is often not beneficial.

Wong et al. (2005) developed a heuristic optimization method for a single-echelon, multi-location, multi-item system with lateral shipments and under the assumption of emergency shipments. The emergency shipments may e.g. come from a central warehouse with practically unlimited stock. The heuristic optimization builds on exact evaluations via Markov processes. Kranenburg and van Houtum (2009) considered the same system as Wong but then with a form of partial pooling instead of full pooling. In their system, only a limited number of main local warehouses is allowed to provide lateral shipments. They developed an approximate evaluation method in which the demand processes for lateral shipments are modeled as Poisson overflow processes (which fits in the line started by Axsäter (1990)). In addition, they developed an effective and efficient greedy heuristic for the minimization of total inventory and lateral and emergency shipment costs subject to mean waiting time constraint at the local warehouses. They showed that using only some of the local warehouses as lateral transshipment sources is enough to get the most of the benefits of full pooling. Their work has been implemented at ASML, a manufacturer of lithography machines for the production of semiconductors. There are many more studies available with respect to systems with lateral shipments; for an overview, see Paterson et al. (2011).

In some networks, the use of emergency shipments is strongly preferred above lateral shipments Lateral shipments may be more expensive than emergency shipments, e.g. because the local warehouses are geographically dispersed and/or procedures for lateral shipments are not well organized. Or, lateral shipments are even excluded by ensuring that the repair facility can always provide an emergency delivery. In the situation of Nedtrain, as discussed above, lateral shipments are not completely excluded, but they are seen as undesired exceptions, and thus they are excluded for the inventory planning at the tactical level. Surprisingly, such networks with emergency shipments but without lateral shipments, have hardly been studied in the literature. Until now, only the work of Muckstadt and Thomas (1980) is available for such networks.

Our main contribution is as follows. We introduce a new approximate evaluation method for two-echelon systems with emergency shipments but without lateral shipments. We will show that our method performs significantly better than the approximate evaluation of Muckstadt and Thomas (1980). Our method is accurate and fast, and thus can well be used in greedy heuristic optimization methods for the multi-item version of our model. Such greedy optimization methods are e.g. used for the minimization of inventory holding and emergency and lateral shipment costs subject to aggregate waiting time constraints per local warehouse, and they have been shown to work very well for closely related systems; see Wong et al. (2005), Wong et al. (2007), and Kranenburg and van Houtum (2009). Without doubt, we can assume that they also work well for our system. In our numerical analysis, we also show that the performance of our system is almost 100% insensitive to the distribution of the repair leadtimes at the central repair facility, which implies that our method works well for generally distributed repair leadtimes.

Notice that exact evaluation for the system studied in this paper would be possible via a Markov analysis, under the assumption of exponential distributions for the repair leadtimes and the transportation times from the central warehouse to the local warehouses. But that would require a numerical solution of multi-dimensional Markov processes and then we would obtain long computation times for already medium high base stock levels.

Notice also that, although we use the terminology of repairable spare parts in this paper, our model applies more generally. Consumable spare parts fit equally well into the same framework. The central repair facility is then replaced by an external supplier.

The organization of this paper is as follows. In Section 2, we described our model. In Section 3, we describe our approximate evaluation method and we summarize the method of Muckstadt and Thomas (1980). Next, in section 4, we test our method via a numerical analysis and we execute the sensitivity analysis with respect to the repair lead time distribution. We conclude in Section 5.

## 2 Model Description

We consider a single-item, two-echelon inventory model with one central warehouse, denoted by index 0, and $N$ $(\geq 0)$ local warehouses. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be the set of local warehouses. In addition, there is a central, external repair facility, where all failed parts are being returned and repaired.

Demands for spare parts occur at the local warehouses. We assume that demands at local warehouse $n$ arrive according to a Poisson process with a constant rate $m_n$ $(> 0)$. Each demand at a local warehouse $n$ stems from a failure of a part in a technical system. For each demand, one of the following procedures is applied (see also Figure 1):

1. If local warehouse $n$ has a part on stock, then it satisfies the demand itself. In this case, there is no delay in satisfying the demand. The failed part is sent to the repair facility (it may flow via the local and central warehouse). Further, the local warehouse places a replenishment order for one ready-for-use part at the central warehouse, and the central warehouse places an order for one unit at the repair facility.
2. If local warehouse $n$ is out of stock and there is at least one part on stock at the central warehouse, then the demand is satisfied from the central warehouse. In this case, the part is delivered via a fast emergency shipment, which leads to a delay in satisfying the demand of on average $t_n^{CW}$ time units. The failed part is sent to the repair facility, and at the same time the central warehouse places an order for one ready-for-use unit at the repair facility.
3. If both local warehouse $n$ and the central warehouse is out of stock, then a part is delivered from the external repair facility. We assume that the repair facility has always a possibility to provide a spare part. E.g., it may finish the repair of one of the parts in the repair shop via an emergency procedure. This leads to a delay of on average $t_n^{RF}$ time units. The failed part is sent to the repair facility.

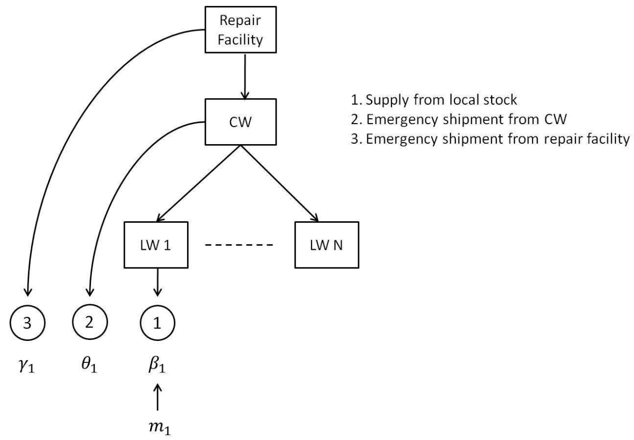



Fig. 1: Demand fulfillment process

Under these procedures, the inventory position remains at a constant level at each of the warehouses. Let $S_n$ be the constant level for warehouse $n$, $n \in \mathcal{N} \cup \{0\}$. Equivalently, we may say that the inventory is controlled by a base stock policy, and $S_n$ is the base stock level at warehouse $n$.

The replenishment leadtime for local warehouse $n$ is deterministic and denoted by $t_n$. Obviously, replenishments get delayed when the central warehouse is out of stock. The external repair facility is assumed to follow a given planned leadtime, denoted by $t_0$. This implies that every order for a ready-for-use part placed by the central warehouse will be delivered after exactly $t_0$ time units. This is equivalent to modeling the repair facility as an ample server with deterministic service times $t_0$.

The main performance measures that need to be determined are directly related to the demand streams at the local warehouse. For the demand stream at local warehouse $n \in \mathcal{N}$, we want to determine:

- $\beta_n$: the steady-state fraction of demands that is satisfied by local warehouse $n$ itself. This measure is also denoted as the *fill rate* of local warehouse $n$.
- $\theta_n$: the steady-state fraction of demands that is directly satisfied by the central warehouse.
- $\gamma_n$: the steady-state fraction of demands that is directly satisfied by the central repair facility.

Notice that, by definition,

$$\beta_n + \theta_n + \gamma_n = 1, \qquad \forall n \in \mathcal{N}. \tag{1}$$

The fractions $\beta_n$, $\theta_n$, and $\gamma_n$ are visualized in Figure 1.

In the rest of this paper, we will focus on the (approximate) evaluation of the fractions $\beta_n$, $\theta_n$, and $\gamma_n$. In optimization problems, one often minimizes relevant costs subject to constraints that are related to downtime or availability of the supported technical systems. E.g., one may have a constraint in terms of the mean expected waiting/delay time until demands at local warehouse $n$ are fulfilled, for which it holds that

$$W_n = \theta_n t_n^{CW} + \gamma_n t_n^{RF}.$$

A typical total cost function would consist of inventory holding costs (for all parts in stock and in repair or in transport from the central warehouse to a local warehouse) and extra costs for demands fulfilled from the central warehouse and the repair facility:

$$C^{Total} = h \sum_{n=0}^{N} S_n + \sum_{n=1}^{N} m_n(\theta_n C_n^{CW} + \gamma_n C_n^{RF}),$$

where $h$ represents the inventory holding cost parameter, $C_n^{CW}$ represents the costs that are made for an emergency shipment from the central warehouse to local warehouse $n$, and $C_n^{RF}$ represents the costs made for an emergency delivery from the repair facility to local warehouse $n$. For both the $W_n$ and $C^{Total}$, extended expressions are obtained when one wants to optimize over multiple items. As we see, quantities such as the waiting times $W_n$ and total costs $C^{Total}$ are easily obtained from the $\beta_n$, $\theta_n$, and $\gamma_n$.

## 3 Solution Procedures

In this part, we describe our new approximate evaluation method and we summarize the approximate evaluation method of Muckstadt and Thomas (1980).

### 3.1 Approximate Evaluation Method

Our approximate evaluation procedure starts with an iterative solution procedure which iteratively calculates the fill rates $\beta_n$ at the local warehouses and the expected delay at the central warehouse. In each iteration, first the fill rates $\beta_n$ are calculated under a given delay at the central warehouse, and next the expected delay at the central warehouse is calculated under given fill rates $\beta_n$. Below, we first describe these two steps per iteration. Then we summarize the iterative procedure. Finally, we give the approximations for the fractions $\theta_n$ and $\gamma_n$.

*Calculating the fill rates*

The replenishment leadtime of local warehouse $n$ is given by $t_n$. This time may be seen as the planned leadtime. When a replenishment order is placed at the central warehouse, its fulfilment will be delayed, and thus the realized leadtime is longer. Let $W_0$ be the mean delay for an arbitrary replenishment order at the central warehouse. Notice, that replenishment orders from different local warehouse experience statistically the same delays. Let $LT_n$ denote the mean of realized replenishment leadtimes for local warehouse $n$. Then,

$$LT_n = t_n + W_0. \tag{2}$$

These realized leadtimes depend on the on-hand stock distribution at the central warehouse. The higher the basestock level at the central warehouse, the shorter the mean delay $W_0$. And, higher basestock levels at the local warehouses have a decreasing effect on the stream of requests for emergency shipments at the central warehouse and thus also a decreasing effect on the stream of emergency deliveries from the repair facility (i.e., more demand has to be satisfied by the central warehouse), which then may lead to a slightly longer mean delay. In our analysis, all basestock levels are given, but the basestock levels at the local warehouses are correlated with the fill rates $\beta_n$ and thus $W_0$ depends on the $\beta_n$, and vice versa. For the initial computation of the $\beta_n$, we assume a zero delay, i.e., $W_0 = 0$.

The fill rates $\beta_n$ are computed per local warehouse $n \in \mathcal{N}$. We pretend that the realized leadtimes for replenishment orders at local warehouse $n$ are independent and identically distributed (so, this is an approximate step). Demands arrive according to a Poisson process with rate $m_n$. Because of the emergency deliveries from the central warehouse and the repair facility, there is no backordering of demand. From the perspective of the local warehouse $n$, demand that is not satisfied from stock, can be seen as lost demand. This implies that the local warehouse $n$ behaves the same as an Erlang loss system (i.e., an $M|G|c|c$ queue). Each unit of stock may be seen as a server that is occupied for on average $LT_n$ time units when it servers a demand. In fact, the steady-state behavior of the number of outstanding replenishment orders ($= S_n$ minus the on-hand stock) is identical to the steady-state behavior of the number of occupied servers in an Erlang loss system with $S_n$ servers, arrival rate $m_n$, and mean service time $LT_n$. As a result, the fill rate $\beta_n$ may be obtained as the percentage of accepted customers in the equivalent Erlang loss system.

For a general Erlang loss system with $c$ servers and offered load $\rho$ (the product of the arrival rate and the mean service time), let $L(c, \rho)$ denote the Erlang loss probability (i.e., the percentage of customers that is not served). It is known that

$$L(c, \rho) = \frac{\frac{\rho^c}{c!}}{\sum_{x=0}^{c} \frac{\rho^x}{x!}}$$

The fill rate at local warehouse $n$ is then obtained by the following formula:

$$\beta_n = 1 - L(S_n, (m_n \cdot LT_n)) \tag{3}$$

*Calculating the Expected Delay in the Central Warehouse*

Suppose now that values for the fill rates $\beta_n$ are known. We now want to estimate the mean delay $W_0$ at the central warehouse.

We model the process for the inventory level at the central warehouse as a birth-death process, i.e., a continuous-time Markov process with states $x \leq S_0$. Notice that the inventory level is equal to the on-hand stock minus the backordered replenishment orders from the local warehouses. Per local warehouse $n$, there is a demand stream of replenishment orders and a demand stream for emergency shipments. The first demand stream has rate $m_n\beta_n$ and is assumed to be a Poisson process (that this process is a Poisson process is an approximation). Demands from this stream are immediately satisfied if the central warehouse has at least one part on stock (i.e., a strictly positive inventory level) and otherwise they are backordered. The second stream has rate $m_n(1-\beta_n)$ and is also assumed to be a Poisson process (this is also an approximation). Demands from this stream are immediately satisfied if the central warehouse has at least one part on stock and otherwise they are lost (i.e., they will be satisfied by the repair facility via an emergency delivery). All above demand streams are assumed to be independent of each other and independent of the actual inventory level at the central warehouse. Obviously, this is also an approximate step. As a result of the above assumptions, the total demand stream at the central warehouse is a Poisson process with rate

$$\sum_{n\in\mathcal{N}}(m_n\beta_n + m_n(1-\beta_n)) = \sum_{n\in\mathcal{N}} m_n = m_0$$

as long as the inventory level at the central warehouse is strictly positive, and it is a Poisson process with rate

$$m_0' = \sum_{n\in\mathcal{N}} m_n\beta_n \tag{4}$$

when the inventory level is zero or strictly negative.

A second approximate step that we make is that the deterministic leadtimes $t_0$ at the central warehouse are replaced by exponential leadtimes with the same mean, i.e., by exponential times with rate $\mu_0 = \frac{1}{t_0}$. It will be shown that the steady-state behavior of the whole system is rather insensitive for the probability distribution of these repair leadtimes; see Subsection 4.2. As stated in the introduction, a similar insensitivity was also observed by Alfredsson and Verrijdt (1999) for their two-echelon system with lateral and emergency shipments. Hence, this approximation will not lead to strong deviations, and it facilitates that the inventory level process can be modeled as a birth-death process.

A last step that we make is that we truncate the state space of the birth-death process. Because backorders can only occur when replenishment orders are placed, and the number of outstanding replenishment orders at local warehouse $n$ can never be more than $S_n$, the number of backorders at the central warehouse can never be more than $\bar{S} = \sum_{n\in\mathcal{N}} S_n$. Hence, we truncate the states $x$ with $x < -\bar{S}$. This completes the construction of the birth-death process; the resulting process is depicted in Figure 2.

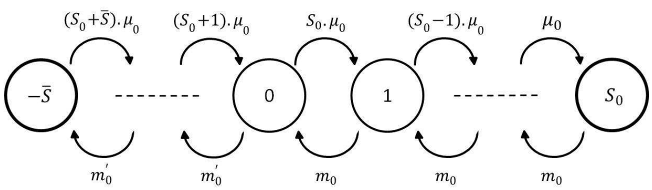



Fig. 2: Birth-death process for the inventory level at the central warehouse

The mean delay $W_0$ now follows from the steady-state distribution of the birth-death process. Let the steady-state distribution be denoted by $\{\pi_x\}$. The steady-state probabilities satisfy the following (steady-state) equations:

$$\pi_x = \begin{cases} \frac{m_0'}{(S_0-x)\mu_0}.\pi_{x+1}, & -\bar{S} \leq x < 0 \\ \frac{m_0}{(S_0-x)\mu_0}.\pi_{x+1}, & 0 \leq x < S_0 \end{cases} \tag{5}$$

By these equations, they can all be expressed as a function of $\pi_{S_0}$ and $\pi_{S_0}$ itself follows from the normalization. Next, the mean number of backordered demands, $B_0$, follows from

$$B_0 = \sum_{x=-\bar{S}}^{-1} (-x)\pi_x, \tag{6}$$

and, by the Little's law, we find (notice that the rate for the total stream of replenishment orders is $m_0'$)

$$W_0 = \frac{B_0}{m_0'} \tag{7}$$

*Iterative algorithm for the approximation method*

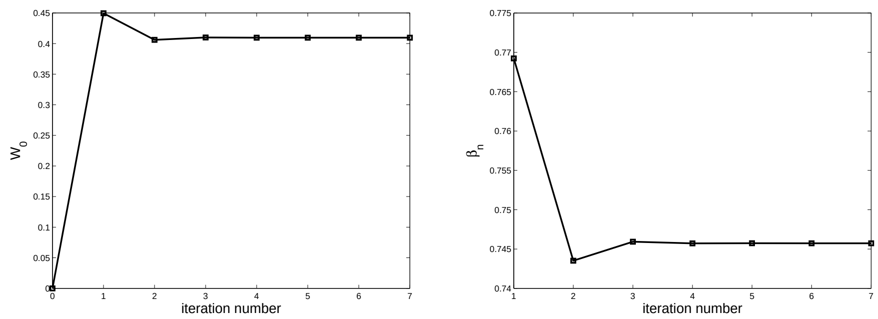
We obtain the following iterative algorithm for the computation of the fill rates $\beta_n$, $n \in \mathcal{N}$, and the mean delay $W_0$:

**Step 1**: $W_0 := 0$.
**Step 2**: Compute $\beta_n$ via (2) and (3), $\forall n \in N$.
**Step 3**: Compute $W_0$ via (4), (5), (6), and (7).
**Step 4**: Repeat *Step 2* and *Step 3* until $W_0$ does not change more than $\epsilon$.

With respect to the convergence of this algorithm, we have no theoretical results, but, as often with this type of algorithms, we obtained convergence for all instances used in our numerical study. The setup of the numerical study and the outcomes are reported in Section 4. Here, we pay attention to the convergence. Figure 3a and 3b show convergence of $W_0$ and $\beta_n$ for instance 62 of the symmetric instances, i.e., the instance with $N = 20$, $m_n = 0.1$ demands per day for all $n \in \mathcal{N}$, $t_0 = 20$ days, $t_n = 3$ days for all $n \in \mathcal{N}$, $S_0 = 40$, $S_n = 1$ for all $n \in \mathcal{N}$.

As we see in Figure 3a, the initial value of $W_0$ is 0. In this case the corresponding $\beta_n$ becomes the largest because the lower the expected delay in the central warehouse, the lower the lead times and the higher the fill rate at each local warehouse. In the following iteration, $W_0$ becomes the largest, because the fill rates $\beta_n$ are largest and the higher the fill rates, the higher the number of replenishment orders from the central warehouse and the higher the delay for these orders. Afterwards, $\beta_n$ becomes the lowest as seen in iteration number 2 in Figure 3b. Then, $W_0$ becomes the second lowest, and so on. At each even numbered iteration, the $\beta_n$ and $W_0$ decrease, and at each odd numbered iteration, the $\beta_n$ and $W_0$ increase. At each iteration, the differences of the values for the $\beta_n$ and $W_0$ with the values of the previous iteration decrease, and we obtain convergence.

The algorithm is robust with respect to the initial value of $W_0$. We experimented with different starting values, and for all possible initial values of $W_0$, we obtained convergence.




(a) $W_0$ at each iteration in instance 62

(b) $\beta_n$ at each iteration in instance 62

Fig. 3: Convergence of $W_0$ and $\beta_n$ in the instance 62 of symmetric instances

*Calculation of the $\theta_n$ and $\gamma_n$*

We finally approximate the fractions of demands satisfied by an emergency delivery from the central warehouse and the repair facility, respectively. Let $IL_n$ and $IL_0$ be random variables which denote the inventory level in local warehouse $n$ and the central warehouse, respectively. Then, it holds that

$$\theta_n = \mathbf{P}(IL_0 > 0, IL_n = 0)$$

By conditioning to "$IL_0 > 0$", we obtain

$$\theta_n = \mathbf{P}(IL_n = 0 | IL_0 > 0) \cdot \mathbf{P}(IL_0 > 0).$$

The probability $\mathbf{P}(IL_0 > 0)$ may be estimated from the birth-death process to compute $W_0$ in the last iteration of the iterative algorithm; we estimate $\mathbf{P}(IL_0 > 0)$ by $\sum_{x=1}^{S_0} \pi_x =: \beta_0$. For the conditional probability $\mathbf{P}(IL_n = 0 | IL_0 > 0)$, we pretend that the central warehouse has a strictly positive inventory level for a very long time. Then the behavior of the inventory level at local warehouse $n$ will be conform an Erlang loss system with mean service times $t_n$ instead of $LT_n$ (see the step to compute the fill rates $\beta_n$ in the iterative algorithm). This leads to the following approximation:

$$\mathbf{P}(IL_n = 0 | IL_0 > 0) \approx L(S_n, m_n t_n).$$

For $\theta_n$, we thus obtain

$$\theta_n \approx \beta_0 L(S_n, m_n t_n). \tag{8}$$

Finally, $\gamma_n$ can be calculated by substituting $\beta_n$ and $\theta_n$ into (1).

## 3.2 The method of Muckstadt and Thomas (1980)

We briefly summarize the approximate evaluation method of Muckstadt and Thomas (1980), which is a sequential solution procedure without iterations. It first approximates the mean delay at the central warehouse, and after that the $\beta_n$, $\theta_n$, and $\gamma_n$ at the local warehouses are computed.

First consider the central warehouse in isolation. They ignore the effect of demands that are fulfilled by the repair facility via an emergency delivery. They assume that the total demand stream is a Poisson process with rate $m_0$ and the number of backordered demands can grow to infinity. The steady-state behavior is then equal to that of an $M|G|\infty$ queue with arrival rate $m_0$ and mean service time $t_0$. By Palm's theorem, the steady-state probability for $x$ occupied servers within this queueing system equals

$$\pi_x = \frac{1}{x!}.(m_0.t_0)^x.e^{-(m_0.t_0)}, \qquad x \geq 0.$$

The probability distribution for the inventory level $IL_0$ at the central warehouse is then approximated by:

$$\mathbf{P}(IL_0 = y) = \pi_{S_0-y}, \qquad y \leq S_0.$$

Next, the mean on-hand stock $I_0$, the mean number of backorders $B_0$, and the mean delay $W_0$ are obtained by:

$$I_0 = \sum_{y=1}^{S_0} y\pi_{S_0-y}, \;\; B_0 = \sum_{y=-\infty}^{-1} -x\pi_{S_0-y} = I_0 - E(IL_0) = I_0 - (S_0 - m_0 t_0), \;\; W_0 = \frac{B_0}{m_0}.$$

The second step for the computation of the $\beta_n$, $\theta_n$, and $\gamma_n$ is executed as follows. Per local warehouse $n \in \mathcal{N}$, first the realized replenishment leadtime is approximated by $LT_n = t_n + W_0$ (as in (2) in our method). Then, the fill rate $\beta_n$ is approximated by $\beta_n = 1 - L(S_n, (m_n \cdot LT_n))$ (as in (3)). The fractions $\theta_n$ and $\gamma_n$ are approximated by

$$\theta_n = \beta_0(1-\beta_n), \tag{9}$$

$$\gamma_n = (1-\beta_0)(1-\beta_n) = 1 - \beta_n - \theta_n, \tag{10}$$

where $\beta_0 = \mathbf{P}(IL_0 > 0) = \sum_{y=1}^{S_0} \pi_{S_0-y}$.

When comparing our new approximate evaluation method to the method of Muckstad and Thomas, we see differences at two points:

- The approximation of $W_0$: There we use a more refined approximation, where we take into account that unfilled requests for emergency deliveries at the central warehouse are satisfied by the repair facility and thus they are lost for the central warehouse itself. To incorporate this effect, we make use of the fill rates $\beta_n$, and thus we need to iterate.
- The approximation for $\theta_n$: In our pproximation, we use equation (8). use equation (9), which is equivalent to approximating $\theta_n = \mathbf{P}(IL_0 > 0, IL_n = 0)$ by $\mathbf{P}(IL_0 > 0)\mathbf{P}(IL_n = 0)$, i.e., by assuming that the inventory levels at the central warehouse and local warehouse $n$ behave independently. In our approximation, we use equation (8), where we take the dependency into account in a certain way.

## 4 Numerical Analysis

This section consists of two parts. We first test the accuracy of our approximation method in Subsection 4.1. After that, we investigate the sensitivity of the performance for the repair leadtime distribution in Subsection 4.2.

## 4.1 Accuracy of the Approximate Evaluation Method

In this subsection, we compare our approximation method with exact results obtained by simulation and with the method of Muckstadt and Thomas (1980).

We consider 96 different instances for our numerical experiment. The input parameters are the number of local warehouses ($N$), the demand rates $m_n$, the repair lead time $t_0$, the planned replenishment leadtimes $t_n$ of the local warehouses, and the base stock levels $S_n$ at the central warehouse and all local warehouses. Among all instances, 64 of them are symmetric, where the $m_n$, $S_n$, and $t_n$ are the same for all local warehouses, and the remaining 32 instances are asymmetric.

In both the symmetric and asymmetric instances, we take the following numbers for $N$: 2, 4, 10, 20. We choose a wide range for $N$, because there are companies keeping spare parts on stock in only a couple of local warehouses as well as companies with many local warehouses. In the symmetric instances, 3 different values for $m_n$ are used: 0.01, 0.04, and 0.1 demands per day. The $t_n$ is equal to 3 days in each instance, and two values are assumed for $t_0$: 5 and 20 days. The base stock levels $S_n$ are chosen such that the performance measures of the system are within different ranges.

In the asymmetric cases, we determined the $m_n$ and $t_n$ for all instances by the following formulas:

$$m_n = m_{n-1} + \Delta_m, \quad n \geq 2,$$
$$t_n = t_{n-1} + \Delta_t, \quad n \geq 2,$$

where $\Delta_m$ and $\Delta_t$ are chosen constants per instance. The parameters $m_1$, $t_1$, $\Delta_m$, and $\Delta_t$ of each instance can be seen in Table 2. We choose the base stock levels $S_n$ from the set $\{1, 2, 3\}$, such that they are nondecreasing in $n$ (because of the increasing $m_n$). The column with "$S_n = 1$", "$S_n = 2$" and "$S_n = 3$" of Table 2 shows the local warehouses with base stock level 1, 2, and 3, respectively. For instance, the value of instance 21 at the column "$S_n = 1$" is "$1 - 3$", which means that the base stock level is 1 for the local warehouses 1, 2, and 3.

We implemented the simulation in the Arena Simulation Software. At each instance, we determined the warm-up period and total run time such that each local warehouse will get at least 10.000 demands in the warm-up period and 50.000 demands in total. We made 100 replication runs at each instance.

Table 1 and Table 2 show the results of the symmetric and asymmetric instances respectively. **M1**, **M2**, and **M3** represent the exact results (via simulation), the results of our approximation method, and the results of the approximate method of Muckstadt and Thomas (1980), respectively. In Table 1, the exact results can be seen with their 95% confidence intervals. As one can see, the simulated results have been determined with a high absolute precision. In Table 2, the results show the average values of the performance measures with respect to all local warehouses. We calculated the average values for each instance by the following formulas:

$$\beta_n^{aver} = \frac{1}{N}\sum_{n=1}^{N}\beta_n \ , \qquad \theta_n^{aver} = \frac{1}{N}\sum_{n=1}^{N}\theta_n \ , \qquad \gamma_n^{aver} = \frac{1}{N}\sum_{n=1}^{N}\gamma_n \ ,$$

The computation time for both method **M2** and method **M3** is quite short. The average computation time per instance for the methods **M2** and **M3** is less than 4 and 2 milliseconds, respectively, at an Intel Core2 Duo 3GHz computer.

The results in Tables 1 and 2, show that our method **M2** clearly ourperforms method **M3** with respect to the approximation of the $\beta_n$. When **M3** is accurate, **M2** is also accurate. Method **M3** has large deviations from the exact values in several cases, and **M2** is still quite close to the exact values in those cases. With respect to the approximation of the $\theta_n$ and $\gamma_n$, the picture is less clear, but it is clearer when we compute the differences for groups of instances.

In Tables 3 and 4, we see differences **M2** and **M1**, and **M3** and **M1**, for the symmetric and asymmetric instances, respectively. For groups of instances with 2, 4, 10, and 20 local warehouses, we have computed the average of the difference, the average of the absolute difference and the maximum absolute difference respectively. In the asymmetric instances, we computed the average of the absolute differences of each performance measure in the following way. We first computed the absolute differences for each performance measure at each warehouse and instance. Then we computed the average values of the absolute differences for all warehouses at each instance, and then took the average over all instances with 2, 4, 10, 20 warehouses.

According to the results, our approximation method **M2** is accurate at each of the performance measures. The absolute differences over all instances for $\beta_n$, $\theta_n$, and $\gamma_n$ are less than 0.0067, 0.0129, and 0.0114, respectively. For $\beta_n$, the absolute differences are low for all values of $N$. For $\theta_n$ and $\gamma_n$, very low absolute differences are obtained for high values of $N$ and larger absolute differences are obtained for low values of $N$. The latter is most likely due to the stronger dependence between inventory levels at the central and local warehouse(s) when $N$ is low.

With respect to the average difference, our method **M2** gives better results than method **M3** for $\beta_n$ and $\theta_n$. However, for $\gamma_n$, method **M3** gives slightly better results. With respect to average absolute and maximum absolute differences, **M2** gives much better results than **M3** for all performance measures. This means that our method **M2** dominates method **M3**.

An interesting result is that both **M2** and **M3** has a tendency to overestimate $\theta_n$. For the symmetric instances, **M2** overestimated $\theta_n$ at each instance and **M3** overestimated $\theta_n$ in 56 of the 64 instances. And similar results can be observed for the asymmetric instances (this follows from the results in Table 4 for $\theta_n^{aver}$, and we see this also when looking at the underlying values for the $\theta_n$). We can analyze this result by considering (8) and (9). Table 5 depicts the values of $\beta_0$ (i.e., the fraction of time that the central warehouse has a strictly positive inventory level) as obtained by **M1**, **M2**, and **M3**, respectively. The exact results (**M1**) are given with their 95% confidence intervals. In Table 6, the average difference between **M2** and **M1** with respect to $\beta_0$, the average of the absolute difference, and the maximum absolute difference are given, and similarly for the differences between **M3** and **M1**.

Table 1: Results of the symmetric instances

| | | | | | | $\beta_n$ | | | $\theta_n$ | | | $\gamma_n$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | $N$ | $m_n$ | $(t_0, t_n)$ | $S_0$ | $S_n$ | M1 | M2 | M3 | M1 | M2 | M3 | M1 | M2 | M3 |
| 1 | 2 | 0.01 | (5,3) | 1 | 1 | 0.9696 ± 0.0002 | 0.9686 | 0.9686 | 0.0004 ± 0.0000 | 0.0264 | 0.0284 | 0.0300 ± 0.0002 | 0.0050 | 0.0030 |
| 2 | | | (20,3) | 1 | 1 | 0.9454 ± 0.0002 | 0.9401 | 0.9388 | 0.0003 ± 0.0000 | 0.0196 | 0.0410 | 0.0542 ± 0.0002 | 0.0403 | 0.0202 |
| 3 | | 0.04 | (5,3) | 1 | 1 | 0.8775 ± 0.0003 | 0.8671 | 0.8657 | 0.0046 ± 0.0001 | 0.0725 | 0.0900 | 0.1179 ± 0.0003 | 0.0604 | 0.0443 |
| 4 | | | | 2 | 1 | 0.8915 ± 0.0003 | 0.8895 | 0.8894 | 0.0813 ± 0.0002 | 0.1006 | 0.1038 | 0.0272 ± 0.0002 | 0.0098 | 0.0068 |
| 5 | | | | 1 | 2 | 0.9893 ± 0.0001 | 0.9897 | 0.9897 | 0.0002 ± 0.0000 | 0.0043 | 0.0069 | 0.0105 ± 0.0001 | 0.0060 | 0.0034 |
| 6 | | | (20,3) | 1 | 1 | 0.7164 ± 0.0004 | 0.7083 | 0.6575 | 0.0022 ± 0.0000 | 0.0278 | 0.0692 | 0.2814 ± 0.0004 | 0.2639 | 0.2734 |
| 7 | | | | 3 | 1 | 0.8704 ± 0.0003 | 0.8595 | 0.8510 | 0.0640 ± 0.0002 | 0.0855 | 0.1167 | 0.0656 ± 0.0002 | 0.0550 | 0.0323 |
| 8 | | | | 2 | 2 | 0.9645 ± 0.0002 | 0.9707 | 0.9697 | 0.0001 ± 0.0000 | 0.0034 | 0.0159 | 0.0354 ± 0.0002 | 0.0259 | 0.0144 |
| 9 | | 0.1 | (5,3) | 1 | 1 | 0.7141 ± 0.0004 | 0.6894 | 0.6739 | 0.0152 ± 0.0001 | 0.0952 | 0.1200 | 0.2707 ± 0.0004 | 0.2154 | 0.2061 |
| 10 | | | | 2 | 1 | 0.7581 ± 0.0004 | 0.7445 | 0.7397 | 0.1211 ± 0.0003 | 0.1744 | 0.1915 | 0.1209 ± 0.0003 | 0.0812 | 0.0688 |
| 11 | | | | 1 | 2 | 0.9293 ± 0.0003 | 0.9282 | 0.9269 | 0.0015 ± 0.0000 | 0.0126 | 0.0269 | 0.0692 ± 0.0002 | 0.0592 | 0.0462 |
| 12 | | | | 4 | 2 | 0.9663 ± 0.0002 | 0.9661 | 0.9661 | 0.0284 ± 0.0002 | 0.0328 | 0.0332 | 0.0053 ± 0.0001 | 0.0010 | 0.0006 |
| 13 | | | (20,3) | 1 | 1 | 0.4428 ± 0.0004 | 0.4741 | 0.3560 | 0.0030 ± 0.0001 | 0.0206 | 0.0118 | 0.5542 ± 0.0004 | 0.5053 | 0.6322 |
| 14 | | | | 2 | 1 | 0.5592 ± 0.0004 | 0.5535 | 0.4246 | 0.0174 ± 0.0001 | 0.0520 | 0.0527 | 0.4234 ± 0.0004 | 0.3945 | 0.5227 |
| 15 | | | | 4 | 2 | 0.8964 ± 0.0003 | 0.8957 | 0.8764 | 0.0054 ± 0.0001 | 0.0159 | 0.0536 | 0.0982 ± 0.0003 | 0.0884 | 0.0701 |
| 16 | | | | 4 | 3 | 0.9629 ± 0.0002 | 0.9741 | 0.9723 | 0.0002 ± 0.0000 | 0.0015 | 0.0120 | 0.0369 ± 0.0002 | 0.0244 | 0.0157 |
| 17 | 4 | 0.01 | (5,3) | 1 | 1 | 0.9675 ± 0.0002 | 0.9665 | 0.9665 | 0.0011 ± 0.0000 | 0.0239 | 0.0274 | 0.0314 ± 0.0002 | 0.0096 | 0.0061 |
| 18 | | | (20,3) | 1 | 1 | 0.9229 ± 0.0003 | 0.9170 | 0.9155 | 0.0007 ± 0.0000 | 0.0134 | 0.0380 | 0.0765 ± 0.0003 | 0.0697 | 0.0465 |
| 19 | | 0.04 | (5,3) | 1 | 1 | 0.8567 ± 0.0003 | 0.8480 | 0.8458 | 0.0087 ± 0.0001 | 0.0500 | 0.0693 | 0.1346 ± 0.0003 | 0.1020 | 0.0849 |
| 20 | | | | 3 | 1 | 0.8918 ± 0.0003 | 0.8908 | 0.8907 | 0.0919 ± 0.0003 | 0.1022 | 0.1041 | 0.0163 ± 0.0001 | 0.0070 | 0.0052 |
| 21 | | | | 2 | 2 | 0.9920 ± 0.0001 | 0.9921 | 0.9921 | 0.0003 ± 0.0000 | 0.0052 | 0.0064 | 0.0077 ± 0.0001 | 0.0027 | 0.0015 |
| 22 | | | (20,3) | 1 | 1 | 0.6409 ± 0.0004 | 0.6369 | 0.5952 | 0.0016 ± 0.0000 | 0.0095 | 0.0165 | 0.3575 ± 0.0004 | 0.3536 | 0.3883 |
| 23 | | | | 3 | 1 | 0.8082 ± 0.0004 | 0.7853 | 0.7587 | 0.0277 ± 0.0002 | 0.0477 | 0.0917 | 0.1641 ± 0.0004 | 0.1670 | 0.1496 |
| 24 | | | | 3 | 2 | 0.9591 ± 0.0002 | 0.9644 | 0.9631 | 0.0005 ± 0.0000 | 0.0025 | 0.0140 | 0.0404 ± 0.0002 | 0.0332 | 0.0229 |
| 25 | | 0.1 | (5,3) | 1 | 1 | 0.6635 ± 0.0004 | 0.6498 | 0.6314 | 0.0167 ± 0.0001 | 0.0453 | 0.0499 | 0.3198 ± 0.0004 | 0.3049 | 0.3187 |
| 26 | | | | 2 | 1 | 0.7272 ± 0.0004 | 0.7094 | 0.6967 | 0.0634 ± 0.0002 | 0.1082 | 0.1231 | 0.2094 ± 0.0004 | 0.1825 | 0.1802 |
| 27 | | | | 4 | 2 | 0.9636 ± 0.0002 | 0.9630 | 0.9629 | 0.0179 ± 0.0001 | 0.0288 | 0.0318 | 0.0184 ± 0.0001 | 0.0082 | 0.0053 |
| 28 | | | | 4 | 3 | 0.9959 ± 0.0001 | 0.9961 | 0.9961 | 0.0008 ± 0.0000 | 0.0029 | 0.0034 | 0.0033 ± 0.0001 | 0.0011 | 0.0006 |
| 29 | | | (20,3) | 1 | 1 | 0.3778 ± 0.0003 | 0.4033 | 0.3279 | 0.0008 ± 0.0000 | 0.0043 | 0.0002 | 0.6214 ± 0.0003 | 0.5925 | 0.6719 |
| 30 | | | | 2 | 1 | 0.4492 ± 0.0003 | 0.4597 | 0.3570 | 0.0039 ± 0.0001 | 0.0131 | 0.0019 | 0.5468 ± 0.0004 | 0.5272 | 0.6410 |
| 31 | | | | 4 | 2 | 0.7790 ± 0.0004 | 0.7798 | 0.7281 | 0.0008 ± 0.0000 | 0.0032 | 0.0115 | 0.2201 ± 0.0004 | 0.2170 | 0.2604 |
| 32 | | | | 6 | 3 | 0.9468 ± 0.0003 | 0.9557 | 0.9514 | 0.0002 ± 0.0000 | 0.0007 | 0.0093 | 0.0530 ± 0.0003 | 0.0436 | 0.0393 |
| 33 | 10 | 0.01 | (5,3) | 2 | 1 | 0.9696 ± 0.0002 | 0.9693 | 0.9693 | 0.0189 ± 0.0001 | 0.0265 | 0.0279 | 0.0115 ± 0.0001 | 0.0041 | 0.0028 |
| 34 | | | (20,3) | 3 | 1 | 0.9540 ± 0.0002 | 0.9515 | 0.9507 | 0.0131 ± 0.0001 | 0.0199 | 0.0333 | 0.0329 ± 0.0002 | 0.0286 | 0.0159 |
| 35 | | 0.04 | (5,3) | 1 | 1 | 0.8200 ± 0.0004 | 0.8162 | 0.8107 | 0.0071 ± 0.0001 | 0.0177 | 0.0256 | 0.1729 ± 0.0004 | 0.1661 | 0.1637 |
| 36 | | | | 3 | 1 | 0.8807 ± 0.0003 | 0.8773 | 0.8758 | 0.0545 ± 0.0002 | 0.0744 | 0.0840 | 0.0648 ± 0.0002 | 0.0483 | 0.0402 |
| 37 | | | | 4 | 2 | 0.9928 ± 0.0001 | 0.9928 | 0.9928 | 0.0031 ± 0.0001 | 0.0055 | 0.0061 | 0.0041 ± 0.0001 | 0.0017 | 0.0010 |
| 38 | | | (20,3) | 4 | 1 | 0.7191 ± 0.0004 | 0.7062 | 0.6553 | 0.0063 ± 0.0001 | 0.0128 | 0.0146 | 0.2746 ± 0.0004 | 0.2810 | 0.3301 |
| 39 | | | | 10 | 1 | 0.8751 ± 0.0003 | 0.8671 | 0.8602 | 0.0733 ± 0.0003 | 0.0807 | 0.1002 | 0.0515 ± 0.0002 | 0.0521 | 0.0396 |
| 40 | | | | 6 | 2 | 0.9545 ± 0.0002 | 0.9584 | 0.9556 | 0.0005 ± 0.0000 | 0.0013 | 0.0085 | 0.0450 ± 0.0002 | 0.0403 | 0.0359 |
| 41 | | 0.1 | (5,3) | 2 | 1 | 0.6563 ± 0.0004 | 0.6496 | 0.6232 | 0.0150 ± 0.0001 | 0.0243 | 0.0152 | 0.3287 ± 0.0004 | 0.3261 | 0.3616 |
| 42 | | | | 5 | 1 | 0.7440 ± 0.0004 | 0.7345 | 0.7206 | 0.1019 ± 0.0003 | 0.1237 | 0.1231 | 0.1541 ± 0.0004 | 0.1418 | 0.1563 |
| 43 | | | | 8 | 2 | 0.9646 ± 0.0002 | 0.9643 | 0.9642 | 0.0237 ± 0.0002 | 0.0291 | 0.0310 | 0.0117 ± 0.0001 | 0.0066 | 0.0048 |
| 44 | | | | 10 | 3 | 0.9966 ± 0.0001 | 0.9966 | 0.9966 | 0.0027 ± 0.0001 | 0.0032 | 0.0033 | 0.0007 ± 0.0000 | 0.0002 | 0.0001 |
| 45 | | | (20,3) | 10 | 1 | 0.5911 ± 0.0004 | 0.5705 | 0.4346 | 0.0169 ± 0.0001 | 0.0285 | 0.0028 | 0.3921 ± 0.0004 | 0.4010 | 0.5625 |
| 46 | | | | 16 | 1 | 0.7047 ± 0.0004 | 0.6774 | 0.5745 | 0.0828 ± 0.0003 | 0.0936 | 0.0666 | 0.2125 ± 0.0004 | 0.2290 | 0.3589 |
| 47 | | | | 17 | 2 | 0.9079 ± 0.0003 | 0.9043 | 0.8833 | 0.0068 ± 0.0001 | 0.0100 | 0.0258 | 0.0853 ± 0.0003 | 0.0856 | 0.0909 |
| 48 | | | | 25 | 2 | 0.9613 ± 0.0002 | 0.9608 | 0.9601 | 0.0263 ± 0.0002 | 0.0286 | 0.0337 | 0.0124 ± 0.0001 | 0.0106 | 0.0063 |
| 49 | 20 | 0.01 | (5,3) | 3 | 1 | 0.9699 ± 0.0002 | 0.9698 | 0.9698 | 0.0224 ± 0.0001 | 0.0268 | 0.0278 | 0.0077 ± 0.0001 | 0.0034 | 0.0024 |
| 50 | | | (20,3) | 3 | 1 | 0.9186 ± 0.0003 | 0.9155 | 0.9112 | 0.0040 ± 0.0001 | 0.0077 | 0.0211 | 0.0774 ± 0.0003 | 0.0767 | 0.0676 |
| 51 | | 0.04 | (5,3) | 2 | 1 | 0.8263 ± 0.0003 | 0.8240 | 0.8160 | 0.0085 ± 0.0001 | 0.0139 | 0.0169 | 0.1653 ± 0.0003 | 0.1621 | 0.1672 |
| 52 | | | | 5 | 1 | 0.8813 ± 0.0003 | 0.8789 | 0.8768 | 0.0578 ± 0.0002 | 0.0706 | 0.0775 | 0.0609 ± 0.0002 | 0.0505 | 0.0457 |
| 53 | | | | 7 | 2 | 0.9931 ± 0.0001 | 0.9932 | 0.9932 | 0.0044 ± 0.0001 | 0.0057 | 0.0061 | 0.0025 ± 0.0001 | 0.0011 | 0.0008 |
| 54 | | | (20,3) | 10 | 1 | 0.7708 ± 0.0004 | 0.7598 | 0.7022 | 0.0101 ± 0.0001 | 0.0154 | 0.0129 | 0.2191 ± 0.0004 | 0.2248 | 0.2849 |
| 55 | | | | 20 | 1 | 0.8854 ± 0.0003 | 0.8823 | 0.8784 | 0.0871 ± 0.0003 | 0.0903 | 0.0987 | 0.0275 ± 0.0002 | 0.0274 | 0.0228 |
| 56 | | | | 15 | 2 | 0.9783 ± 0.0002 | 0.9804 | 0.9796 | 0.0016 ± 0.0000 | 0.0024 | 0.0075 | 0.0201 ± 0.0001 | 0.0171 | 0.0129 |
| 57 | | 0.1 | (5,3) | 5 | 1 | 0.6847 ± 0.0004 | 0.6796 | 0.6443 | 0.0195 ± 0.0001 | 0.0264 | 0.0104 | 0.2958 ± 0.0004 | 0.2940 | 0.3453 |
| 58 | | | | 10 | 1 | 0.7534 ± 0.0004 | 0.7480 | 0.7339 | 0.1227 ± 0.0003 | 0.1350 | 0.1218 | 0.1239 ± 0.0003 | 0.1170 | 0.1442 |
| 59 | | | | 12 | 2 | 0.9620 ± 0.0002 | 0.9618 | 0.9614 | 0.0191 ± 0.0001 | 0.0238 | 0.0269 | 0.0189 ± 0.0001 | 0.0144 | 0.0117 |
| 60 | | | | 15 | 3 | 0.9965 ± 0.0001 | 0.9965 | 0.9965 | 0.0025 ± 0.0001 | 0.0031 | 0.0032 | 0.0010 ± 0.0000 | 0.0004 | 0.0003 |
| 61 | | | (20,3) | 30 | 1 | 0.7053 ± 0.0004 | 0.6850 | 0.5537 | 0.0641 ± 0.0002 | 0.0718 | 0.0193 | 0.2305 ± 0.0004 | 0.2433 | 0.4270 |
| 62 | | | | 40 | 1 | 0.7544 ± 0.0004 | 0.7457 | 0.7013 | 0.1596 ± 0.0003 | 0.1613 | 0.1431 | 0.0860 ± 0.0003 | 0.0930 | 0.1556 |
| 63 | | | | 30 | 2 | 0.8869 ± 0.0003 | 0.8855 | 0.8475 | 0.0024 ± 0.0001 | 0.0039 | 0.0066 | 0.1107 ± 0.0003 | 0.1107 | 0.1459 |
| 64 | | | | 40 | 2 | 0.9481 ± 0.0002 | 0.9464 | 0.9402 | 0.0160 ± 0.0001 | 0.0181 | 0.0286 | 0.0360 ± 0.0002 | 0.0355 | 0.0312 |

Table 2: Results of the asymmetric instances

| Instance | $N$ | $m_1$ | $\Delta_m$ | $(t_0, t_1)$ | $\Delta_t$ | $S_0$ | $S_n = 1$ | $S_n = 2$ | $S_n = 3$ | $\beta_n^{aver}$ | | | $\theta_n^{aver}$ | | | $\gamma_n^{aver}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | M1 | M2 | M3 | M1 | M2 | M3 | M1 | M2 | M3 |
| 1 | 2 | 0.01 | 0.01 | (5,3) | 0 | 1 | 1-1 | 2-2 | - | 0.9831 | 0.9827 | 0.9827 | 0.0004 | 0.0133 | 0.0149 | 0.0165 | 0.0040 | 0.0024 |
| 2 | | 0.04 | 0.04 | (5,3) | 0 | 2 | 1-1 | 2-2 | - | 0.9323 | 0.9301 | 0.9300 | 0.0342 | 0.0571 | 0.0615 | 0.0335 | 0.0128 | 0.0085 |
| 3 | | | 0.04 | (20,3) | 0 | 2 | 1-1 | 2-2 | - | 0.8223 | 0.8181 | 0.7886 | 0.0081 | 0.0230 | 0.0652 | 0.1696 | 0.1589 | 0.1462 |
| 4 | | | 0 | (5,2) | 1 | 3 | 1-1 | 2-2 | - | 0.9575 | 0.9573 | 0.9573 | 0.0396 | 0.0422 | 0.0424 | 0.0029 | 0.0005 | 0.0003 |
| 5 | | 0.1 | 0.1 | (5,3) | 0 | 3 | 1-1 | 2-2 | - | 0.8277 | 0.8198 | 0.8180 | 0.1009 | 0.1361 | 0.1472 | 0.0714 | 0.0441 | 0.0348 |
| 6 | | | 0.1 | (5,3) | 0 | 4 | 1-1 | 2-2 | - | 0.8326 | 0.8301 | 0.8297 | 0.1410 | 0.1556 | 0.1591 | 0.0264 | 0.0143 | 0.0112 |
| 7 | | | 0.1 | (20,3) | 0 | 4 | 1-1 | 2-2 | - | 0.6722 | 0.6602 | 0.5376 | 0.0236 | 0.0490 | 0.0699 | 0.3042 | 0.2908 | 0.3925 |
| 8 | | | 0 | (20,2) | 1 | 4 | 1-1 | 2-2 | - | 0.8094 | 0.7864 | 0.7400 | 0.0339 | 0.0566 | 0.1127 | 0.1567 | 0.1570 | 0.1473 |
| 9 | 4 | 0.01 | 0.01 | (5,3) | 0 | 3 | 1-4 | - | - | 0.9310 | 0.9307 | 0.9307 | 0.0644 | 0.0679 | 0.0683 | 0.0046 | 0.0014 | 0.0010 |
| 10 | | 0.04 | 0.01 | (5,3) | 0 | 3 | 1-2 | 3-4 | - | 0.9306 | 0.9296 | 0.9295 | 0.0490 | 0.0607 | 0.0635 | 0.0205 | 0.0097 | 0.0070 |
| 11 | | | 0.01 | (20,3) | 0 | 3 | 1-2 | 3-4 | - | 0.8131 | 0.8035 | 0.7780 | 0.0079 | 0.0165 | 0.0411 | 0.1790 | 0.1800 | 0.1809 |
| 12 | | | 0 | (5,2) | 0.5 | 2 | 1-2 | 3-4 | - | 0.9500 | 0.9483 | 0.9482 | 0.0220 | 0.0365 | 0.0419 | 0.0280 | 0.0152 | 0.0099 |
| 13 | | 0.1 | 0.02 | (5,3) | 0 | 2 | 1-1 | 2-4 | - | 0.8423 | 0.8363 | 0.8292 | 0.0140 | 0.0312 | 0.0457 | 0.1436 | 0.1325 | 0.1251 |
| 14 | | | 0.02 | (5,3) | 0 | 4 | 1-1 | 2-4 | - | 0.8891 | 0.8851 | 0.8838 | 0.0556 | 0.0764 | 0.0855 | 0.0553 | 0.0385 | 0.0307 |
| 15 | | | 0.02 | (20,3) | 0 | 10 | 1-1 | 2-3 | 4-4 | 0.8596 | 0.8478 | 0.8238 | 0.0335 | 0.0423 | 0.0720 | 0.1069 | 0.1099 | 0.1041 |
| 16 | | | 0 | (20,2) | 0.5 | 5 | 1-1 | 2-3 | 4-4 | 0.8009 | 0.7924 | 0.7476 | 0.0051 | 0.0103 | 0.0251 | 0.1940 | 0.1973 | 0.2272 |
| 17 | 10 | 0.01 | 0.01 | (5,3) | 0 | 6 | 1-8 | 9-10 | - | 0.9002 | 0.8997 | 0.8995 | 0.0875 | 0.0930 | 0.0944 | 0.0123 | 0.0073 | 0.0061 |
| 18 | | 0.04 | 0.01 | (5,3) | 0 | 10 | 1-6 | 7-10 | - | 0.8857 | 0.8857 | 0.8857 | 0.1114 | 0.1127 | 0.1129 | 0.0028 | 0.0016 | 0.0014 |
| 19 | | | 0.01 | (20,3) | 0 | 20 | 1-4 | 5-10 | - | 0.9094 | 0.9058 | 0.9016 | 0.0565 | 0.0603 | 0.0724 | 0.0341 | 0.0339 | 0.0259 |
| 20 | | | 0 | (5,2) | 0.2 | 4 | 1-4 | 5-10 | - | 0.9593 | 0.9588 | 0.9587 | 0.0269 | 0.0329 | 0.0354 | 0.0138 | 0.0083 | 0.0059 |
| 21 | | 0.1 | 0.01 | (5,3) | 0 | 8 | 1-3 | 4-10 | - | 0.8604 | 0.8567 | 0.8520 | 0.0635 | 0.0767 | 0.0831 | 0.0761 | 0.0666 | 0.0649 |
| 22 | | | 0.01 | (5,3) | 0 | 10 | 1-1 | 2-8 | 9-10 | 0.9286 | 0.9278 | 0.9272 | 0.0478 | 0.0552 | 0.0585 | 0.0236 | 0.0171 | 0.0142 |
| 23 | | | 0.01 | (20,3) | 0 | 25 | 1-1 | 2-6 | 7-10 | 0.8893 | 0.8839 | 0.8553 | 0.0136 | 0.0169 | 0.0295 | 0.0971 | 0.0992 | 0.1151 |
| 24 | | | 0 | (20,2) | 0.2 | 20 | 1-1 | 2-4 | 5-10 | 0.9522 | 0.9513 | 0.9472 | 0.0113 | 0.0133 | 0.0248 | 0.0365 | 0.0354 | 0.0279 |
| 25 | 20 | 0.01 | 0.005 | (5,3) | 0 | 8 | 1-16 | 17-20 | - | 0.8918 | 0.8906 | 0.8895 | 0.0747 | 0.0828 | 0.0859 | 0.0334 | 0.0267 | 0.0246 |
| 26 | | 0.04 | 0.005 | (5,3) | 0 | 12 | 1-12 | 13-20 | - | 0.8791 | 0.8782 | 0.8772 | 0.0933 | 0.0996 | 0.1015 | 0.0276 | 0.0222 | 0.0213 |
| 27 | | | 0.005 | (20,3) | 0 | 45 | 1-10 | 11-20 | - | 0.8996 | 0.8992 | 0.8985 | 0.0932 | 0.0941 | 0.0956 | 0.0072 | 0.0067 | 0.0059 |
| 28 | | | 0 | (5,2) | 0.1 | 5 | 1-15 | 16-20 | - | 0.9146 | 0.9130 | 0.9117 | 0.0394 | 0.0490 | 0.0555 | 0.0459 | 0.0380 | 0.0328 |
| 29 | | 0.1 | 0.005 | (5,3) | 0 | 20 | 1-6 | 7-20 | - | 0.8708 | 0.8703 | 0.8696 | 0.1116 | 0.1155 | 0.1159 | 0.0176 | 0.0142 | 0.0145 |
| 30 | | | 0.005 | (5,3) | 0 | 15 | 1-6 | 7-16 | 17-20 | 0.8754 | 0.8735 | 0.8688 | 0.0563 | 0.0633 | 0.0645 | 0.0683 | 0.0632 | 0.0667 |
| 31 | | | 0.005 | (20,3) | 0 | 50 | 1-6 | 7-13 | 14-20 | 0.8477 | 0.8404 | 0.7885 | 0.0253 | 0.0284 | 0.0223 | 0.1270 | 0.1311 | 0.1892 |
| 32 | | | 0 | (20,2) | 0.1 | 37 | 1-6 | 7-15 | 16-20 | 0.8969 | 0.8906 | 0.8679 | 0.0283 | 0.0307 | 0.0392 | 0.0748 | 0.0787 | 0.0930 |

Table 3: Results of the symmetric instances for groups of instances

| | Average Difference | | | | | |
|---|---|---|---|---|---|---|
| | $\beta_n$ | | $\theta_n$ | | $\gamma_n$ | |
| Instances | M2-M1 | M3-M1 | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| $N = 2$ | -0.0022 | -0.0242 | 0.0250 | 0.0393 | -0.0228 | -0.0151 |
| $N = 4$ | -0.0015 | -0.0227 | 0.0140 | 0.0226 | -0.0124 | 0.0001 |
| $N = 10$ | -0.0060 | -0.0291 | 0.0079 | 0.0093 | -0.0020 | 0.0197 |
| $N = 20$ | -0.0039 | -0.0256 | 0.0046 | 0.0017 | -0.0007 | 0.0239 |
| All $N$ | -0.0034 | -0.0254 | 0.0129 | 0.0182 | -0.0095 | 0.0072 |
| | Average Absolute Difference | | | | | |
| | $\beta_n$ | | $\theta_n$ | | $\gamma_n$ | |
| Instances | M2-M1 | M3-M1 | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| $N = 2$ | 0.0083 | 0.0261 | 0.0250 | 0.0393 | 0.0228 | 0.0372 |
| $N = 4$ | 0.0079 | 0.0238 | 0.0140 | 0.0229 | 0.0128 | 0.0269 |
| $N = 10$ | 0.0065 | 0.0292 | 0.0079 | 0.0131 | 0.0061 | 0.0319 |
| $N = 20$ | 0.0042 | 0.0257 | 0.0046 | 0.0106 | 0.0039 | 0.0310 |
| All $N$ | 0.0067 | 0.0262 | 0.0129 | 0.0215 | 0.0114 | 0.0317 |
| | Maximum Absolute Difference | | | | | |
| | $\beta_n$ | | $\theta_n$ | | $\gamma_n$ | |
| Instances | M2-M1 | M3-M1 | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| $N = 2$ | 0.0313 | 0.1345 | 0.0800 | 0.1048 | 0.0575 | 0.0993 |
| $N = 4$ | 0.0255 | 0.0922 | 0.0448 | 0.0640 | 0.0326 | 0.0942 |
| $N = 10$ | 0.0274 | 0.1565 | 0.0218 | 0.0295 | 0.0166 | 0.1705 |
| $N = 20$ | 0.0204 | 0.1517 | 0.0128 | 0.0448 | 0.0127 | 0.1965 |
| All $N$ | 0.0313 | 0.1565 | 0.0800 | 0.1048 | 0.0575 | 0.1965 |

Table 4: Results of the asymmetric instances for groups of instances

| | Average Difference | | | | | |
|---|---|---|---|---|---|---|
| | $\beta_n$ | | $\theta_n$ | | $\gamma_n$ | |
| Instances | M2-M1 | M3-M1 | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| $N = 2$ | -0.0065 | -0.0316 | 0.0189 | 0.0364 | -0.0123 | -0.0047 |
| $N = 4$ | -0.0054 | -0.0182 | 0.0113 | 0.0239 | -0.0059 | -0.0057 |
| $N = 10$ | -0.0019 | -0.0072 | 0.0053 | 0.0116 | -0.0034 | -0.0044 |
| $N = 20$ | -0.0025 | -0.0131 | 0.0052 | 0.0073 | -0.0026 | 0.0058 |
| All $N$ | -0.0041 | -0.0175 | 0.0102 | 0.0198 | -0.0061 | -0.0023 |
| | Average Absolute Difference | | | | | |
| | $\beta_n$ | | $\theta_n$ | | $\gamma_n$ | |
| Instances | M2-M1 | M3-M1 | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| $N = 2$ | 0.0075 | 0.0317 | 0.0189 | 0.0364 | 0.0141 | 0.0268 |
| $N = 4$ | 0.0065 | 0.0182 | 0.0113 | 0.0239 | 0.0097 | 0.0157 |
| $N = 10$ | 0.0024 | 0.0073 | 0.0053 | 0.0116 | 0.0048 | 0.0089 |
| $N = 20$ | 0.0026 | 0.0131 | 0.0052 | 0.0089 | 0.0049 | 0.0147 |
| All $N$ | 0.0048 | 0.0176 | 0.0102 | 0.0202 | 0.0084 | 0.0165 |
| | Maximum Absolute Difference | | | | | |
| | $\beta_n$ | | $\theta_n$ | | $\gamma_n$ | |
| Instances | M2-M1 | M3-M1 | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| $N = 2$ | 0.0425 | 0.1431 | 0.0381 | 0.1034 | 0.0308 | 0.0884 |
| $N = 4$ | 0.0380 | 0.0941 | 0.0292 | 0.0524 | 0.0252 | 0.0615 |
| $N = 10$ | 0.0233 | 0.0625 | 0.0167 | 0.0510 | 0.0163 | 0.0483 |
| $N = 20$ | 0.0191 | 0.0870 | 0.0142 | 0.0238 | 0.0152 | 0.1108 |
| All $N$ | 0.0425 | 0.1431 | 0.0381 | 0.1034 | 0.0308 | 0.1108 |

Firstly, we see from Table 5 that **M2** underestimates $\beta_0$ at all instances except the symmetric instance 63, and **M3** underestimates $\beta_0$ at all 96 instances. The explanation of this result is that the approximate method **M2** is based on the implicit assumption that the inventory levels at the local warehouses are independent from the inventory level at the central warehouse. More precisely, when analyzing the central warehouse behavior, we assume that, independent of the ac-

Table 5: Results for $\beta_0 = P(IL_0 > 0)$

| Symmetric Instances | | | | | | | | Asymmetric Instances | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | M1 | M2 | M3 | Ins. | M1 | M2 | M3 | Ins. | M1 | M2 | M3 |
| 1 | 0.9337 ± 0.0002 | 0.9050 | 0.9048 | 33 | 0.9222 ± 0.0001 | 0.9102 | 0.9098 | 1 | 0.8713 ± 0.0002 | 0.8608 | 0.8607 |
| 2 | 0.7133 ± 0.0003 | 0.6736 | 0.6703 | 34 | 0.7135 ± 0.0002 | 0.6837 | 0.6767 | 2 | 0.9101 ± 0.0002 | 0.8794 | 0.8781 |
| 3 | 0.7734 ± 0.0003 | 0.6769 | 0.6703 | 35 | 0.1963 ± 0.0001 | 0.1655 | 0.1353 | 3 | 0.4148 ± 0.0003 | 0.3547 | 0.3084 |
| 4 | 0.9682 ± 0.0001 | 0.9394 | 0.9384 | 36 | 0.7535 ± 0.0002 | 0.6942 | 0.6767 | 4 | 0.9953 ± 0.0001 | 0.9921 | 0.9921 |
| 5 | 0.6785 ± 0.0003 | 0.6708 | 0.6703 | 37 | 0.8624 ± 0.0002 | 0.8576 | 0.8571 | 5 | 0.8950 ± 0.0002 | 0.8203 | 0.8088 |
| 6 | 0.3422 ± 0.0003 | 0.2598 | 0.2019 | 38 | 0.1417 ± 0.0002 | 0.1196 | 0.0424 | 6 | 0.9686 ± 0.0001 | 0.9377 | 0.9344 |
| 7 | 0.8668 ± 0.0003 | 0.7980 | 0.7834 | 39 | 0.8164 ± 0.0003 | 0.7534 | 0.7166 | 7 | 0.3974 ± 0.0004 | 0.2951 | 0.1512 |
| 8 | 0.5526 ± 0.0005 | 0.5305 | 0.5249 | 40 | 0.2184 ± 0.0003 | 0.2103 | 0.1912 | 8 | 0.6234 ± 0.0005 | 0.5129 | 0.4335 |
| 9 | 0.5684 ± 0.0003 | 0.4126 | 0.3679 | 41 | 0.1436 ± 0.0001 | 0.1054 | 0.0404 | 9 | 0.9916 ± 0.0000 | 0.9858 | 0.9856 |
| 10 | 0.8744 ± 0.0003 | 0.7555 | 0.7358 | 42 | 0.6564 ± 0.0002 | 0.5360 | 0.4405 | 10 | 0.9228 ± 0.0001 | 0.9022 | 0.9004 |
| 11 | 0.4059 ± 0.0003 | 0.3778 | 0.3679 | 43 | 0.8863 ± 0.0002 | 0.8703 | 0.8666 | 11 | 0.2878 ± 0.0003 | 0.2452 | 0.1851 |
| 12 | 0.9876 ± 0.0001 | 0.9812 | 0.9810 | 44 | 0.9694 ± 0.0001 | 0.9682 | 0.9682 | 12 | 0.8382 ± 0.0003 | 0.8112 | 0.8088 |
| 13 | 0.1485 ± 0.0003 | 0.0894 | 0.0183 | 45 | 0.1620 ± 0.0003 | 0.1237 | 0.0050 | 13 | 0.3555 ± 0.0002 | 0.3068 | 0.2674 |
| 14 | 0.3518 ± 0.0005 | 0.2255 | 0.0916 | 46 | 0.5430 ± 0.0004 | 0.4055 | 0.1565 | 14 | 0.8057 ± 0.0003 | 0.7507 | 0.7360 |
| 15 | 0.5335 ± 0.0006 | 0.4741 | 0.4335 | 47 | 0.3315 ± 0.0005 | 0.3000 | 0.2211 | 15 | 0.5632 ± 0.0005 | 0.4883 | 0.4090 |
| 16 | 0.4648 ± 0.0007 | 0.4433 | 0.4335 | 48 | 0.8763 ± 0.0004 | 0.8536 | 0.8432 | 16 | 0.2061 ± 0.0004 | 0.1789 | 0.0996 |
| 17 | 0.8472 ± 0.0002 | 0.8192 | 0.8187 | 49 | 0.9289 ± 0.0001 | 0.9203 | 0.9197 | 17 | 0.9588 ± 0.0000 | 0.9423 | 0.9392 |
| 18 | 0.4973 ± 0.0002 | 0.4587 | 0.4493 | 50 | 0.2867 ± 0.0002 | 0.2657 | 0.2381 | 18 | 0.9923 ± 0.0000 | 0.9886 | 0.9880 |
| 19 | 0.5437 ± 0.0002 | 0.4663 | 0.4493 | 51 | 0.1505 ± 0.0001 | 0.1301 | 0.0916 | 19 | 0.8137 ± 0.0003 | 0.7685 | 0.7363 |
| 20 | 0.9720 ± 0.0001 | 0.9537 | 0.9526 | 52 | 0.7137 ± 0.0002 | 0.6586 | 0.6288 | 20 | 0.8753 ± 0.0002 | 0.8597 | 0.8571 |
| 21 | 0.8163 ± 0.0003 | 0.8092 | 0.8088 | 53 | 0.8931 ± 0.0001 | 0.8898 | 0.8893 | 21 | 0.6787 ± 0.0003 | 0.6131 | 0.5615 |
| 22 | 0.1158 ± 0.0002 | 0.0888 | 0.0408 | 54 | 0.1637 ± 0.0002 | 0.1439 | 0.0433 | 22 | 0.8454 ± 0.0002 | 0.8167 | 0.8043 |
| 23 | 0.5449 ± 0.0004 | 0.4456 | 0.3799 | 55 | 0.8825 ± 0.0002 | 0.8428 | 0.8122 | 23 | 0.3438 ± 0.0004 | 0.3080 | 0.2042 |
| 24 | 0.4097 ± 0.0004 | 0.3906 | 0.3799 | 56 | 0.3947 ± 0.0003 | 0.3827 | 0.3675 | 24 | 0.5431 ± 0.0006 | 0.5095 | 0.4703 |
| 25 | 0.2806 ± 0.0002 | 0.1964 | 0.1353 | 57 | 0.1496 ± 0.0002 | 0.1145 | 0.0293 | 25 | 0.8391 ± 0.0001 | 0.7987 | 0.7776 |
| 26 | 0.6079 ± 0.0003 | 0.4688 | 0.4060 | 58 | 0.6929 ± 0.0002 | 0.5851 | 0.4579 | 26 | 0.8785 ± 0.0001 | 0.8460 | 0.8266 |
| 27 | 0.8811 ± 0.0003 | 0.8595 | 0.8571 | 59 | 0.7326 ± 0.0003 | 0.7105 | 0.6968 | 27 | 0.9626 ± 0.0001 | 0.9511 | 0.9415 |
| 28 | 0.8613 ± 0.0003 | 0.8574 | 0.8571 | 60 | 0.9186 ± 0.0002 | 0.9169 | 0.9165 | 28 | 0.6912 ± 0.0002 | 0.6505 | 0.6288 |
| 29 | 0.0238 ± 0.0001 | 0.0184 | 0.0003 | 61 | 0.4149 ± 0.0003 | 0.3109 | 0.0432 | 29 | 0.9297 ± 0.0001 | 0.9055 | 0.8887 |
| 30 | 0.0813 ± 0.0002 | 0.0566 | 0.0030 | 62 | 0.8025 ± 0.0003 | 0.6989 | 0.4790 | 30 | 0.6135 ± 0.0002 | 0.5621 | 0.4915 |
| 31 | 0.1004 ± 0.0003 | 0.0962 | 0.0424 | 63 | 0.1115 ± 0.0003 | 0.1156 | 0.0432 | 31 | 0.3141 ± 0.0004 | 0.2770 | 0.1056 |
| 32 | 0.2218 ± 0.0005 | 0.2116 | 0.1912 | 64 | 0.5811 ± 0.0005 | 0.5395 | 0.4790 | 32 | 0.4820 ± 0.0004 | 0.4292 | 0.2963 |

Table 6: Average, average of the absolute, and maximum absolute differences for $\beta_0$

| | Symmetric Ins. | | Asymmetric Ins. | |
|---|---|---|---|---|
| | M2-M1 | M3-M1 | M2-M1 | M3-M1 |
| Average Diff. | -0.0435 | -0.0864 | -0.0394 | -0.0791 |
| Absolute Diff. | 0.0437 | 0.0864 | 0.0394 | 0.0791 |
| Maximum Abs. Diff. | 0.1559 | 0.3865 | 0.1105 | 0.2462 |

tual inventory level, there is always a Poisson demand stream with rate $m_n\beta_n$ for replenishment orders placed by local warehouse $n$ and a Poisson demand stream with rate $m_n(1-\beta_n)$ for emergency shipment requests placed by local warehouse $n$ ($n \in \mathcal{N}$). This leads to the approximate birth-death process for the behavior of the inventory level at the central warehouse as depicted in Figure 2. However, in the true system, we have a positive correlation between the inventory level $IL_0$ at the central warehouse and the inventory levels $IL_n$ at the local warehouses $n \in \mathcal{N}$. Hence, in the true system, the total stream of emergency shipment requests will have a higher rate than $\sum_{n\in\mathcal{N}} m_n(1-\beta_n)$ when $IL_0 \leq 0$, and the stream of replenishment orders will have a lower rate than $\sum_{n\in\mathcal{N}} m_n\beta_n$. Hence, in Figure 2, the rate $m_0'$ for transitions to the left when $IL_0 \leq 0$ is an overestimation, and this leads to an underestimation of $\beta_0$. Notice that the bounding of the state space (at state $-\bar{S}$) reduces the effect of the overestimation of the transitions to the left when $IL_0 \leq 0$.

Method **M3** underestimates $\beta_0$ more than our method **M2**, as seen in Table 5 and 6. This is explained by the fact that **M3** assumes that the demand rate coming to the central warehouse is always equal to $m_0$. Hence, the transitions to the left when $IL_0 \leq 0$ are even further overestimated. Further, no bounding of the state space is assumed.

Although method **M2** generally underestimates $\beta_0$, it overestimates $\theta_n$, which is determined via equation (8). The reason is that $L(S_n, m_nt_n)$ generally overestimates $P(IL_n = 0|IL_0 > 0)$. In the true system, there is a positive correlation between $IL_0$ and $IL_n$. If there is positive stock at the central warehouse, then it is less likely to have zero stock at a local warehouse. Apparently, the relative overestimation of $P(IL_n = 0|IL_0 > 0)$ is larger than the relative underestimation of $\beta_0$. Lastly, because method **M2** generally overestimates $\theta_n$, it has a tendency to underestimate $\gamma_n$ because of (1). Similarly, although **M3** underestimates $\beta_0$ in all instances, it generally overestimates $\theta_n$, which is determined by (9). The reason is that $1 - \beta_n$ severely overestimates $P(IL_n = 0|IL_0 > 0)$. Again, this is because of the ignored positive correlation between $IL_0$ and $IL_n$.

Notice that all over- and underestimations become smaller when the correlations between $IL_0$ and the $IL_n$ become less strong. This is typically so when we have higher numbers of local warehouses.

### 4.2 Sensitivity Analysis

In our model, we assumed that the repair leadtime at the repair facility is deterministic. This leadtime has been denoted by $t_0$. However, this assumption may not always hold in real-life situations. One may have some variability in this leadtime. Therefore, in this section, we analyze the sensitivity of the system performance with respect to the distribution of the repair leadtime. We consider four different distributions. The first distribution is the *deterministic distribution*, cf. the assumption in our model. As we mentioned in Section 3, we assumed an exponential distribution for the repair leadtime in our approximate evaluation method (in the step to determine the mean delay $W_0$). So, the second distribution that we consider is the *exponential distribution*, with mean time $t_0$. The other two distributions are with a coefficient of variation of 0.5 and 2, respectively. We choose an *Erlang-4 distribution* as the third distribution. This distribution has a coefficient of variation of 0.5; its scale parameter is chosen such that the mean is equal to $t_0$. For the fourth distribution, we choose a *lognormal distribution*, with parameters such that the coefficient of variation is 2 and the mean equals $t_0$. We used simulation to get the results under each distribution, and we generated these results for the symmetric instances with 4 and 10 local warehouse (32 instances in total). We created 95% confidence intervals for the differences in the $\beta_n$, $\theta_n$, and $\gamma_n$ between the deterministic case and each of the remaining cases. The results are listed in Table 7.

The differences in Table 7 are all quite small. Additional results are generated in Tables 8 and 9. The percentages of the intervals containing 0 are depicted in Table 8. The average difference, average absolute difference, and maximum absolute difference of the deterministic case with the other distributions are depicted in Table 9.

In Table 8, we see that the most sensitive performance measure to the repair leadtime distribution is $\theta_n$, which is also the one that our approximation method estimates worst among the three performance measures $\beta_n$, $\theta_n$, and $\gamma_n$. In Table 8, we see that all values are very low. The average differences and average absolute differences are all even below 0.003. Hence, we may conclude that the performance is rather insensitive for the repair leadtime distribution. We also made some simulation runs for the asymmetric instances to check the sensitivity, and we got similar results.

Table 7: Results of the sensitivity analysis

| | $\beta_n$ | | | $\theta_n$ | | | $\gamma_n$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Ins. | Erlang-Det. | Expo.-Det. | Log.-Det. | Erlang-Det. | Expo.-Det. | Log.-Det. | Erlang-Det. | Expo.-Det. | Log.-Det. |
| 17 | -0.0002 ± 0.0002 | -0.0004 ± 0.0002 | -0.0006 ± 0.0002 | 0.0015 ± 0.0001 | 0.0057 ± 0.0001 | 0.0078 ± 0.0001 | -0.0013 ± 0.0002 | -0.0053 ± 0.0002 | -0.0072 ± 0.0002 |
| 18 | -0.0004 ± 0.0004 | -0.0006 ± 0.0004 | -0.0001 ± 0.0004 | 0.0001 ± 0.0000 | 0.0010 ± 0.0000 | 0.0011 ± 0.0000 | 0.0003 ± 0.0004 | -0.0004 ± 0.0004 | -0.0001 ± 0.0004 |
| 19 | -0.0009 ± 0.0005 | -0.0022 ± 0.0005 | -0.0003 ± 0.0005 | 0.0030 ± 0.0001 | 0.0103 ± 0.0002 | 0.0139 ± 0.0002 | -0.0020 ± 0.0004 | -0.0081 ± 0.0004 | -0.0110 ± 0.0005 |
| 20 | -0.0001 ± 0.0004 | -0.0003 ± 0.0004 | -0.0004 ± 0.0004 | 0.0002 ± 0.0004 | 0.0018 ± 0.0004 | 0.0026 ± 0.0004 | 0.0000 ± 0.0002 | -0.0015 ± 0.0002 | -0.0022 ± 0.0002 |
| 21 | -0.0001 ± 0.0001 | -0.0001 ± 0.0001 | -0.0002 ± 0.0001 | 0.0004 ± 0.0000 | 0.0014 ± 0.0000 | 0.0019 ± 0.0000 | -0.0003 ± 0.0001 | -0.0013 ± 0.0001 | -0.0017 ± 0.0001 |
| 22 | -0.0004 ± 0.0006 | -0.0006 ± 0.0006 | -0.0011 ± 0.0007 | 0.0001 ± 0.0001 | 0.0005 ± 0.0001 | 0.0005 ± 0.0001 | 0.0004 ± 0.0006 | 0.0001 ± 0.0006 | 0.0005 ± 0.0007 |
| 23 | -0.0013 ± 0.0005 | -0.0021 ± 0.0005 | -0.0024 ± 0.0006 | 0.0006 ± 0.0002 | 0.0019 ± 0.0002 | 0.0021 ± 0.0002 | 0.0007 ± 0.0005 | 0.0002 ± 0.0005 | 0.0003 ± 0.0006 |
| 24 | -0.0001 ± 0.0003 | -0.0002 ± 0.0003 | -0.0002 ± 0.0003 | 0.0000 ± 0.0000 | 0.0001 ± 0.0000 | 0.0002 ± 0.0000 | 0.0001 ± 0.0003 | 0.0001 ± 0.0003 | 0.0000 ± 0.0003 |
| 25 | -0.0013 ± 0.0006 | -0.0028 ± 0.0006 | -0.0035 ± 0.0006 | 0.0022 ± 0.0002 | 0.0065 ± 0.0002 | 0.0088 ± 0.0002 | -0.0009 ± 0.0005 | -0.0037 ± 0.0006 | -0.0053 ± 0.0006 |
| 26 | -0.0024 ± 0.0006 | -0.0048 ± 0.0005 | -0.0006 ± 0.0006 | 0.0044 ± 0.0003 | 0.0125 ± 0.0003 | 0.0161 ± 0.0003 | -0.002 ± 0.0005 | -0.0078 ± 0.0005 | -0.0100 ± 0.0006 |
| 27 | -0.0002 ± 0.0003 | -0.0004 ± 0.0003 | -0.0005 ± 0.0003 | 0.0006 ± 0.0002 | 0.0029 ± 0.0002 | 0.0039 ± 0.0002 | -0.0004 ± 0.0002 | -0.0025 ± 0.0002 | -0.0035 ± 0.0002 |
| 28 | -0.0001 ± 0.0001 | -0.0001 ± 0.0001 | -0.0001 ± 0.0001 | 0.0002 ± 0.0000 | 0.0006 ± 0.0000 | 0.0009 ± 0.0001 | -0.0001 ± 0.0001 | -0.0006 ± 0.0001 | -0.0008 ± 0.0001 |
| 29 | -0.0002 ± 0.0005 | -0.0004 ± 0.0005 | -0.0002 ± 0.0006 | 0.0000 ± 0.0000 | 0.0001 ± 0.0000 | 0.0001 ± 0.0000 | 0.0001 ± 0.0005 | 0.0003 ± 0.0005 | 0.0000 ± 0.0006 |
| 30 | -0.0003 ± 0.0005 | -0.0004 ± 0.0005 | -0.0006 ± 0.0006 | 0.0002 ± 0.0001 | 0.0005 ± 0.0001 | 0.0006 ± 0.0001 | 0.0001 ± 0.0005 | -0.0001 ± 0.0005 | 0.0000 ± 0.0006 |
| 31 | 0.0000 ± 0.0006 | -0.0005 ± 0.0006 | -0.0008 ± 0.0006 | 0.0000 ± 0.0000 | 0.0001 ± 0.0000 | 0.0001 ± 0.0000 | -0.0001 ± 0.0006 | 0.0004 ± 0.0006 | 0.0007 ± 0.0006 |
| 32 | 0.0000 ± 0.0004 | -0.0002 ± 0.0004 | -0.0001 ± 0.0004 | 0.0000 ± 0.0000 | 0.0000 ± 0.0000 | 0.0000 ± 0.0000 | 0.0000 ± 0.0004 | 0.0002 ± 0.0004 | 0.0000 ± 0.0004 |
| 33 | -0.0001 ± 0.0002 | -0.0001 ± 0.0002 | -0.0001 ± 0.0002 | 0.0005 ± 0.0002 | 0.0018 ± 0.0002 | 0.0025 ± 0.0002 | -0.0004 ± 0.0002 | -0.0017 ± 0.0001 | -0.0023 ± 0.0001 |
| 34 | -0.0002 ± 0.0003 | -0.0003 ± 0.0003 | -0.0004 ± 0.0003 | 0.0001 ± 0.0002 | 0.0005 ± 0.0002 | 0.0006 ± 0.0002 | 0.0001 ± 0.0003 | -0.0002 ± 0.0003 | -0.0002 ± 0.0003 |
| 35 | -0.0002 ± 0.0005 | -0.0004 ± 0.0005 | -0.0007 ± 0.0005 | 0.0005 ± 0.0001 | 0.0019 ± 0.0001 | 0.0026 ± 0.0001 | -0.0003 ± 0.0005 | -0.0014 ± 0.0005 | -0.0020 ± 0.0005 |
| 36 | -0.0005 ± 0.0004 | -0.0008 ± 0.0004 | -0.0011 ± 0.0004 | 0.0013 ± 0.0003 | 0.0047 ± 0.0003 | 0.0066 ± 0.0003 | -0.0008 ± 0.0003 | -0.0039 ± 0.0003 | -0.0054 ± 0.0004 |
| 37 | 0.0000 ± 0.0001 | -0.0001 ± 0.0001 | 0.0000 ± 0.0001 | 0.0002 ± 0.0001 | 0.0007 ± 0.0001 | 0.0009 ± 0.0001 | -0.0001 ± 0.0001 | -0.0006 ± 0.0001 | -0.0008 ± 0.0001 |
| 38 | -0.0002 ± 0.0006 | -0.0005 ± 0.0006 | -0.0006 ± 0.0006 | 0.0001 ± 0.0001 | 0.0004 ± 0.0001 | 0.0004 ± 0.0001 | 0.0001 ± 0.0006 | 0.0001 ± 0.0006 | 0.0001 ± 0.0006 |
| 39 | -0.0003 ± 0.0004 | -0.0007 ± 0.0004 | -0.0008 ± 0.0004 | 0.0001 ± 0.0004 | 0.0006 ± 0.0004 | 0.0007 ± 0.0004 | 0.0003 ± 0.0003 | 0.0001 ± 0.0003 | 0.0001 ± 0.0004 |
| 40 | 0.0000 ± 0.0003 | -0.0001 ± 0.0003 | -0.0001 ± 0.0003 | 0.0000 ± 0.0000 | 0.0001 ± 0.0000 | 0.0001 ± 0.0000 | 0.0000 ± 0.0003 | 0.0000 ± 0.0003 | 0.0001 ± 0.0003 |
| 41 | -0.0002 ± 0.0005 | -0.0006 ± 0.0005 | -0.0007 ± 0.0006 | 0.0005 ± 0.0002 | 0.0014 ± 0.0002 | 0.0019 ± 0.0002 | -0.0003 ± 0.0005 | -0.0008 ± 0.0006 | -0.0011 ± 0.0006 |
| 42 | -0.0011 ± 0.0005 | -0.0022 ± 0.0005 | -0.0028 ± 0.0005 | 0.0020 ± 0.0004 | 0.0062 ± 0.0004 | 0.0081 ± 0.0004 | -0.0009 ± 0.0005 | -0.0040 ± 0.0005 | -0.0053 ± 0.0005 |
| 43 | -0.0001 ± 0.0003 | -0.0002 ± 0.0003 | -0.0003 ± 0.0003 | 0.0003 ± 0.0002 | 0.0015 ± 0.0002 | 0.0019 ± 0.0002 | -0.0002 ± 0.0002 | -0.0012 ± 0.0002 | -0.0017 ± 0.0002 |
| 44 | 0.0000 ± 0.0001 | 0.0000 ± 0.0001 | 0.0000 ± 0.0001 | 0.0001 ± 0.0001 | 0.0002 ± 0.0001 | 0.0002 ± 0.0001 | 0.0000 ± 0.0000 | -0.0002 ± 0.0000 | -0.0002 ± 0.0000 |
| 45 | -0.0006 ± 0.0006 | -0.0008 ± 0.0006 | -0.0001 ± 0.0006 | 0.0005 ± 0.0002 | 0.0009 ± 0.0002 | 0.0010 ± 0.0002 | 0.0001 ± 0.0006 | -0.0002 ± 0.0006 | 0.0000 ± 0.0006 |
| 46 | -0.0012 ± 0.0006 | -0.0021 ± 0.0006 | -0.0022 ± 0.0006 | 0.0009 ± 0.0004 | 0.0018 ± 0.0004 | 0.0021 ± 0.0004 | 0.0003 ± 0.0006 | 0.0003 ± 0.0006 | 0.0002 ± 0.0007 |
| 47 | -0.0003 ± 0.0004 | -0.0004 ± 0.0005 | -0.0005 ± 0.0005 | 0.0001 ± 0.0001 | 0.0003 ± 0.0001 | 0.0003 ± 0.0001 | 0.0002 ± 0.0004 | 0.0001 ± 0.0005 | 0.0002 ± 0.0005 |
| 48 | -0.0001 ± 0.0003 | -0.0002 ± 0.0003 | -0.0002 ± 0.0003 | 0.0000 ± 0.0002 | 0.0002 ± 0.0002 | 0.0002 ± 0.0002 | 0.0001 ± 0.0002 | 0.0000 ± 0.0002 | 0.0000 ± 0.0002 |

Table 8: Percentage of intervals containing 0 for each performance measure

| Performance Measure | Erlang-Det. | Expo.-Det. | Log.-Det. |
|---|---|---|---|
| $\beta_n$ | 72% | 56% | 38% |
| $\theta_n$ | 34% | 3% | 3% |
| $\gamma_n$ | 59% | 50% | 47% |

Table 9: Average, average absolute and maximum absolute differences between the deterministic and the remaining ditribution cases

| | Average Difference | | | Average Absolute Diff. | | | Maximum Absolute Diff. | | |
|---|---|---|---|---|---|---|---|---|---|
| Case | $\beta_n$ | $\theta_n$ | $\gamma_n$ | $\beta_n$ | $\theta_n$ | $\gamma_n$ | $\beta_n$ | $\theta_n$ | $\gamma_n$ |
| Erlang-Det. | -0.0004 | 0.0006 | -0.0002 | 0.0004 | 0.0006 | 0.0004 | 0.0024 | 0.0044 | 0.0020 |
| Expo.-Det. | -0.0008 | 0.0022 | -0.0014 | 0.0008 | 0.0022 | 0.0015 | 0.0048 | 0.0125 | 0.0081 |
| Log.- Det. | -0.0010 | 0.0028 | -0.0018 | 0.0010 | 0.0028 | 0.0020 | 0.0060 | 0.0161 | 0.0110 |

The above results imply that our approximate evaluation method works also well for systems with a generally distributed repair leadtime.

## 5 Conclusion

In this study, we derived an accurate and fast approximate evaluation method for two-echelon spare parts systems with emergency shipments. We also showed that our method outperforms the method of Muckstadt and Thomas (1980). Further, we showed that the performance measures of our system are roughly insensitive to the repair leadtime distribution, which increases the applicability of our approximation method. In future research, we can consider extensions of our method to networks with both emergency and lateral shipments and to networks with more than two echelon levels.

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