

## Fifteen years of experience with modelling courses in the Eindhoven program of applied mathematics

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## **Fifteen Years of Experience with Modelling Courses in the Eindhoven Program of Applied Mathematics**

**Jacob Perrenet**

Eindhoven School of Education, Eindhoven University of Technology  
[j.c.perrenet@tue.nl](mailto:j.c.perrenet@tue.nl)

**Ivo Adan**

Department of Mechanical Engineering, Eindhoven University of Technology  
[i.j.b.f.adan@tue.nl](mailto:i.j.b.f.adan@tue.nl)

### **Abstract**

The curriculum of Applied Mathematics at the Eindhoven University of Technology (TU/e) in the Netherlands includes a series of modelling projects: the so-called Modelling Track. This track was introduced in the curriculum fifteen years ago and has a specific educational approach. Mathematics that may be useful in the projects is not necessarily taught in courses preceding the modelling projects. Moreover, during the projects, students have to use their current skills and knowledge or even have to learn (or discover) new techniques by themselves. Overall, this teaching method has been quite successful in terms of students' results and satisfaction. Project coaches have always been recruited from the entire department and gradually, the majority of the staff has become pleased with this method of modeling education. Throughout fifteen years, the structure and content of the series of projects have evolved. This article is based on reflections concerning these changes expressed by the coordinator of the mathematical modelling education (the second author) and the educational advisor (the first author), both of whom have been closely involved in the track's development over the years. These changes will be described and their external as well as internal causes will be identified. Examples of external causes are developments of technical phenomena in society, university-wide educational innovations, and a change in the overall structure of the university's academic calendar. An example of an internal cause is the variety in background of the project coaches. Finally, strengths and weaknesses of the track will be analyzed. The purpose of the article is to share the experiences with this way of teaching mathematical modelling in higher education and give advice to others who want to implement it.

**Keywords:** mathematical modelling, higher education, program changes.

### **1. Introduction**

Mathematical modelling involves much more than applying mathematics. To be successful, more is needed than mathematical knowledge and problem solving skills alone. The ability to translate a problem into a manageable mathematical one, a broad overview of mathematics, communication skills, common sense, and intuition for practical problems are essential as well. Mathematical modelling has been done for centuries, but developed rapidly in the last 60 years. Its importance strongly increased in industry, government and scientific research. In higher education, mathematical modelling is characteristic to an applied mathematics curriculum at an engineering university compared to pure mathematics at a general university. One could even claim that not one engineer can do without! In the Netherlands, mathematical modelling and the teaching of it in higher education started during and shortly after World War II (Alberts, 1998) with the work of Van Danzig, who tried to give a mathematical basis to the statistical work of the just established Mathematical Centre (nowadays: Centre for Mathematics and Informatics) and with the work of Timman, who is one of the most important founding fathers of the education of mathematical engineering. At Eindhoven, mathematical modelling has been part of the Applied Mathematics curriculum of the Eindhoven University of Technology for several decennia. However, only during the last fifteen years, in the so-called Modelling Track, the modelling education found its present form. We will describe its origin, its

characteristics and its development. Concrete examples will be presented as well as educational reflections. The start will be an historical overview.

Even before the implementation of the Modelling Track, some mathematical modelling education was part of the Applied Mathematics curriculum. In the next section, we will describe how mathematical modelling was taught within the so-called Modelling Practical within the old curriculum, which in those days had a length of four years.

In 1995, the Department of Mathematics and Computing Science introduced the Modelling Track into the curriculum, after the government had given permission to technical universities to extend their programs from four to five years. The introduction was a response to the employers' complaint about graduates' poor ability to apply their theoretical knowledge in practice. We will describe the design, the structure, the supervision, and the assessment of the projects.

We will give some typical examples of problems, representative for the three domains of application within the curriculum. Throughout the years, problems disappeared and new ones appeared, and also the involvement of industry has grown by providing an increasing number of projects. We will discuss this development.

In 1998, the university decided to introduce Design Based Learning (DBL) in all its curricula, in this way complying with the needs of employers for graduates' applicative and collaborative abilities. DBL means professionalization, activation, co-operation, creativity, integration and multidisciplinary. For the Modelling Track this innovation might signify the incorporation of the DBL features, e.g. working with larger groups or integration with other courses. However, compared to other TU/e programs no major changes were implemented in the curriculum. We will describe the minor changes, such as structuring student interaction and teacher supervision – and we will explain why major changes failed to appear.

The extension of the Modelling Track with a reflection course in the third year, as the conclusion of portfolio activities at the end of every project, could indeed be called a major change. We will describe and evaluate the reflection assignments, which were implemented for the first time in 2003.

In 2006, the university changed its calendar year from a trimester to a semester structure. This caused the extension of second year projects. After one year of experience, a midterm project review had to be implemented in order to help students planning their project activities.

For many years, only little modelling methodology had been taught explicitly during the Modelling Track. As it became more and more clear how much variation occurred in the more implicit coaching on methodology by the staff, it was decided in 2007 to introduce methodology lectures. This change and the one described before, although of a very different nature, gave more structure to the Modelling Track. We will describe both changes in more detail and discuss their necessity.

Throughout the years, some changes occurred, in the student population as well as in the teacher population. However, these changes did not influence the Modelling Track significantly.

We will conclude the article with a reflection on the various changes in the Modelling Track and the factors which caused them to occur. We will also analyze the strengths and weaknesses of the track.

## **2. Prehistory: Mathematical Modelling Education Before the Modelling Track**

In this section we will briefly describe how mathematical modelling was taught within the so-called Modelling Practical in the second year of the old curriculum of four years length.

The set-up of the practical was a weekly meeting in class, during which the teacher presented a new problem. The problems were formulated in non-mathematical terms, but the essence was rather academic. Some examples will be presented below.

### *Road through a swamp*

In a municipality a city and a village are located, 11 km apart from each other. The village is in a swamp, and the city is at the border of the swamp. The border is assumed to be straight. The distance of the village to the border of the swamp is 8 km. The municipality has decided to construct a road from this city to the village. Road construction is more expensive in the swamp than on solid ground. So it may be possible to save costs by constructing part of the road along the border of the

swamp. The costs per km on solid ground are known, namely 660 K Dutch guilders per km. The costs per km in the swamp are only exactly known afterwards, but a lower and upper bound can be provided: these costs will in between 1000 and 2000 K Dutch guilders per km. The municipality now considers performing an additional investigation to estimate more accurately the costs per km in the swamp. The costs for this investigation may of course not exceed the possible savings due to a more accurate estimate. The assignment is to determine an upper limit for the investigation costs, so that these costs will never exceed the possible savings for the road construction.

#### *Traffic lights*

On a bridge, one of the lanes is blocked over a distance of 500 meters due to road maintenance. Traffic from both directions is sent alternating over the free lane, which is controlled by traffic lights. The speed limit on the free lane is 20 km per hour. The traffic intensity in one direction is 800 cars per hours, while 300 cars per hour in the other direction. The assignment is to develop a good traffic light control.

#### *Breeding bacteria*

In a research project, bacteria are cultured in a nutrient solution, where they can multiply by division. For a certain type of bacteria one observes that the division process stops once the concentration of bacteria reaches a certain value. Then, diluting the solution by addition of a certain amount of nutrients, the division process starts again, and stops when the concentration of bacteria has reached the same value as before the dilution. Repeated dilution always gives the same result. One attempts to explain this phenomenon, by assuming that the bacteria secrete a substance which stops the division process when a sufficiently high concentration has been reached. The assignment is to investigate whether this hypothesis (and if necessary further assumptions) explains the final fixed concentration of the bacterial culture.

During the weekly meeting, typically in the afternoon, the new problem would be discussed, analyzed and, step by step, solved under strict guidance of the teacher. At the end of the afternoon, most students would know how the problem “had to be solved” and they were supposed to write, together with another student, a clear report on the problem, which had to be delivered at the end of the week. Throughout the years some changes occurred in the Modelling Practical. For example, the report on a modelling problem would be written by one of the students in the group, while the other was supposed to read the report critically and provide (written) comments. Also, in the last week of the Modelling Practical, each group had to give a presentation of one of the modelling problems. In this way, each student solved about ten problems (under strict guidance).

Looking back at the way of teaching and learning of mathematical modelling in this practical, we observe that the difference with the way of teaching and learning nowadays in the Mathematical Modelling Track is not so much in the problem content: some of the problems of the Modelling Practical could be used at the start of the Modelling Track, although further on in the track, current problems are bigger. Neither is the difference located in the learning of communicative skills. It is mainly located in learning how to *independently* perform the activities of the mathematical modelling cycle: from a real-life problem to a mathematical model, to a mathematical solution, and to an interpretation of that solution in terms of the original problem (see further section 3.2). In the practical, problems were solved in class in a single afternoon session under supervision of the teacher, whereas in the current setting, students spend a whole semester struggling independently with a modelling problem. In the practical, one approach and solution of a problem would be presented as the correct one, while in the Modelling Track several approaches and solutions are acceptable. In the rest of this paper we will look at the Modelling Track and its development in more detail.

### **3. Start of the Modelling Track**

#### **3.1 The curriculum**

Applied Mathematics, or Mathematical Engineering as it was called for some years, started as a five year’ program in 1995. Before that, it was a four year’ program. After the implementation of the Bachelor-Master structure in 2002, there has been a separated Bachelor program and several Master

programs. The number of students starting in the first (Bachelor) year is about 30. The main courses in the Bachelor program concern basic knowledge and skills in mathematics and applications in operational management, digital communication and technology. The regular didactical forms are lectures, practicals and projects. The Modelling Track mainly consists of a series of compulsory projects in the first and second year. The students perform these projects in pairs. The Modelling Track originally consisted of two courses in the first year, taking up 10 percent of that year's study load, and two courses in the second year for about 20 percent of that year's load. Later, it was completed with a reflection course (see section 6) in the third year.

### 3.2 The Modelling Track

The aim of introducing the modelling projects into the curriculum was to improve the students' ability to solve practical problems, as in general, professional mathematical engineering problems are hardly ever presented as mathematical problems, and once identified, they are often not purely mathematical problems, but of multidisciplinary nature. The overall learning goal of the Modelling Track was, and still is, learning how to solve practical problems, posed in non-mathematical language, by using mathematical methods.

Firstly, this means the ability to complete the so-called modelling cycle that essentially consists of the following steps:

- Problem analysis: mapping out the problem using common sense;
- Problem translation: formulating a mathematical model for the problem;
- Mathematical analysis: mathematical elaboration of (sub-) problems;
- Implementation: implementation of the solution into a computer program;
- Retranslation: translation back to the original problem.

Secondly, it means the ability to work along project lines, with the following characteristics:

- Working according to a plan;
- Collaborating in small teams;
- Communicating verbally with colleagues and interested lay-men;
- Giving a verbal presentation to colleagues and interested lay-men;
- Reporting on paper to colleagues and interested lay-men;
- Conducting interviews;
- Critically following fellow students' modelling activities.

In total each student solves about seven problems, of which most are much bigger than the ten problems of the earlier practical described in section 3.1. (Apart from that, as will become clear further on, the students also reflect and comment on many problems solved by their peers). In the beginning of the program, the students do small projects with intensive coaching. Gradually the projects become larger and more complex; on the other hand, the students work more independently. Each project is related to a different application area (i.e. operational management, digital communication or technology); every student pair gets its own specific project. During a period of three months the students work approximately one day per week on their project. Each pair has its own website where it presents its plans, progress and products. Integrated with the project work in the first and second year are courses on verbal presentation, report writing, the document preparation system LaTeX, project planning, library usage, and interviewing. For a more detailed description, see Perrenet & Adan (2002).

Gray (1998) distinguishes three ways of presenting mathematical modelling: 1) various mathematical systems applied in various areas, 2) several mathematical systems, each applied in a single area, and 3) a single area of application. Our approach is nearest to the first option, but it is important to remember, that sometimes even the mathematics to be applied has to be discovered and studied by the students themselves as part of the project. Students may have to develop, explore and apply 'new' mathematics, taking various levels of problem details into account. For example, it can happen that, by including more details, a (too) complicated differential equation is obtained, which cannot be solved analytically, but for which the students 'discover' a method to produce a numerical solution. Or, if students realize that 'waiting' is an important issue for their problem, they may decide to explore queuing theory, a subject that is taught at the Master level, and apply queuing models that fit their needs. Like in practice, the students are brought into a situation where it is not clear

beforehand what (and even if) mathematics is useful to solve (certain aspects of) the problem. So the approach enhances learning how to model as well as learning mathematics through modelling, as well as learning to apply mathematics. Within the variety of perspectives on modelling in mathematics education as discussed by Kaiser and Sriraman (2006) our approach is closest to the so-called pragmatic perspective of Pollak: focussing on the goal of learning to apply mathematics, but in the broad sense as described above.

Assessment is based on a written report and a presentation at the end of the project. Generally both members of a pair get the same grade.

## 4. Problems and Their Development

### 4.1 Characteristics

A problem represents a certain domain of application: Digital communication, Operational management and Technology. The problem is posed in natural (non-mathematical) language. The translation from natural language to mathematical language – an important part of the mathematical modelling process – is a task the student should perform. A problem is open, which means that several interpretations and several approaches are possible. A problem has a certain degree of complexity (throughout the years in the Bachelor program, this complexity will increase). Problems at the start only ask for secondary school knowledge. We will give three examples of ‘simple’ problems meant for freshmen.

#### *Switch-over costs at a machine*

Consider a machine at which a number of different tasks have to be performed. The switch-over costs are  $C_{ij}$  if product  $j$  is manufactured just after product  $i$ . If manufacturing starts with task  $i$ , then starting costs are  $C_i$ . Develop a method to determine the optimal manufacturing order. Write a computer program for this purpose. Vary the number of products from 4 to 10 and make a choice for the switch-over costs. Examine how computing time grows.

#### *Traffic casualties*

At a particular intersection, everyday a small probability  $p$  exists that an accident occurs. Most of the times it concerns only damage to the bodywork. The number of casualties is  $k$  with probability  $q(k)$ , where  $q(0)=0.8$ ,  $q(1)=0.16$ ,  $q(2)=0.03$ ,  $q(3)=0.008$  and  $q(4)=0.002$ . We are interested in the yearly number of casualties. How to determine the probability of  $k$  casualties for a year for  $k=0, 1, 2, \dots$ ? Compute these probabilities.

#### *Telephone market*

In recent years a lot of competition has been created in the telephone market. Many companies bid for the caller’s favor. To get a better view on the customer’s behavior, a company collects data on the new customer’s bills. The bill of the first month is studied. This bill is retrieved from 200 customers, see the Excel file attached [an Excel file is given with a realistic series of 200 amounts]. The assignment is to investigate these data and to make a report for the general manager. Can you give any advice to the general manager given these data?

Further in the Modelling Track, problems may ask for more and more skills and knowledge from the curriculum, sometimes even for knowledge and skills not yet taught but within reach.

Problems are delivered by staff members from the three domains of application. The staff member who delivers a problem becomes the problem owner. Sometimes (but nowadays more often) a problem comes from outside the university. Each student pair is supervised by a staff member. The supervisor and problem owner are different persons, playing different roles in the modelling process; the supervisor monitors and steers (if necessary) the modelling process, whereas the problem owner provides problem specific information and gives feed-back on proposed solutions. Only in the first year of the curriculum, both roles are performed by the same staff member.

Generally, the problem owner does not know all the details of possible solutions beforehand; sometimes even the problem owner does not know a solution at all. In practice, the problem delivering process is not trivial. At one hand, problems do not come in equal numbers from every domain: for example, sometimes not enough Digital Communication or Technical problems are delivered. At the other hand, not all staff members deliver problems in natural language: sometimes a problem is delivered in mathematical language already, even with the accompanying instruction to apply a certain method. The latter format is not accepted as a suitable problem description.

We will give some examples of problems meant for more advanced students, starting with problems from the first years that the Modelling Track was implemented, and concluding with some examples of recent years.

## 4.2 Early examples

### *Operational management: Buffer design*

On a machine in a candy factory marshmallows are sealed in fixed batches. The marshmallows are transported to this machine on a conveyor belt. The arrival process is irregular, i.e., the inter-arrival times of the marshmallows vary. However, the machine works continuously at constant speed. A vertical buffer is positioned between the conveyor belt and the sealing machine, to prevent the machine from sealing too many “empty positions”. Develop a model to determine the optimal buffer size.

### *Digital communication: Teletext*

Teletext is an on-line information medium available on television. It usually takes some time before a selected information page appears on the screen. Develop a model to explain the delay, and if possible, propose a method to reduce the delay.

### *Technology: Noise reduction by wings*

For the design of a new aeroplane one has to take into account international regulations for the production of noise. In these regulations it is stated that the amount of noise produced by an aeroplane landing or taking off should not exceed certain thresholds. Most of the noise produced by aeroplanes comes from their engines; their location within the aircraft is possibly a significant factor in the noise production: above the wings at the tail, or below them. The claim is that in the first case the wings yield a considerable reduction of noise. Develop a model to estimate the reduction of noise.

## 4.3 New examples

### *Operational management: Roundabouts*

Nowadays everywhere in the Netherlands, junctions are being replaced by roundabouts. The claim is that traffic flows faster through roundabouts. Is that true?

### *Digital communication: Blogs*

Blogs have become a common way to present (any kind) of information on the web. Looking at the various characteristics of any recently accessible blog, would there be a systematic way to predict its future popularity (for instance in terms of number of visits) and thus to classify a new blog as potentially popular?

### *Technology: Tsunami*

Tsunamis are extremely high waves (caused by earthquakes) with sometimes disastrous consequences. Mathematical models play an important role in modern warning systems for tsunamis. Investigate the causes and damaging consequences of tsunamis and develop a simple model to describe the propagation of tsunamis. By making use of available geophysical data, try to use this model to predict whether tsunamis are a potential risk for the Netherlands.

Reasons to remove and replace certain problems are twofold. Firstly, problem owners can lose interest, because after some years, the chance that students come up with something new becomes small. Secondly, the context can get outdated. Teletext is a disappearing medium; blogs are from the

world of today and tsunamis have recently tormented Asia. Of course, this is a relative argument, as shown by the water clock example below. Very out-of-date problems may become interesting again.

*Technology: Water clock*

Three thousand years ago the Egyptians used a simple bin of clay to keep track of time; see Figure 1 below. The clock consists of a bin, which is filled with water every day. The water in the bin pours out very slowly from a hole in the bottom, and time is measured by means of small dashes inside the bin, where each dash indicates an hour. The striking feature is that the dashes are distributed *evenly* inside the bin. This is only possible if the side of the bin has exactly the adequate angle, and if the ancient Egyptians were precisely aware of this angle. Develop a model for the water clock and use this model to determine the adequate angle of the bin.



**Figure 1:** Water clock used by the ancient Egyptians

The last example originates from outside the university. The client is a research and consultancy office.

*Operational management: Air quality*

Vehicle-actuated traffic signal control in the Netherlands is being optimized for traffic-scientific reasons (e.g., minimizing delay of vehicles). Generally, it is assumed that this optimization also favors the air quality. However, there are grounds to assume that optimization for air quality demands other requirements than traffic-scientific requirements alone. A model is asked for that does justice to the air-quality requirements without making extreme concessions to traffic-scientific preconditions.

## **5. Mathematical Modelling and Design Based Learning**

### **5.1 Design Based Learning, a University-wide Innovation**

In its Institutional Plan for 1998-2001, the university announced that it would develop a single university wide educational philosophy for university-based education of engineers: Design Based Learning, DBL for short (Wijnen, 2000). Conforming to the needs of employers, the thorough field knowledge of TU/e engineers has to go hand in hand with the ability to critically apply that knowledge in an industrial setting and in multidisciplinary teams of designers. The activity of designing is a central activity of professional engineers which occurs in many variations, such as designing products, processes, models, systems, structures, etc. It depends on the specific engineering discipline whether one should view designing more as creating, collaborating and integrating, making procedures or problem solving. The DBL curriculum should have the following characteristics of the designing process in the engineering profession: Professionalization, Activation, Co-operation, Creativity, Integration, and Multidisciplinarity. The main motives for introducing DBL summed up by Wijnen (2000) were the following:

- To improve the quality of education;
- To increase the level of competence orientation;
- To reinforce the coherence between education and research;



- To strengthen cohesion and coherence within the TU/e;
- To achieve innovation of technical systems.

For each of the ten Master programs the eight departments outlined one typical curriculum innovation project to be carried out in two years. In some cases, it meant the intensification of an existing activity, in other cases, projects involved new activities. Finally, the goal was set to work towards multidisciplinary projects with students from two departments working together on a multidisciplinary problem. The project for Applied Mathematics will be described in the next section.

## 5.2 DBL in the Modelling Track

For mathematical engineering the core activity is solving problems from multidisciplinary origin by means of mathematical methods. Designing could be translated as mathematical modelling. The natural choice for the DBL activities in the program of Applied Mathematics was the project "New Forms of Modelling", aiming at the Modelling Track. Further elaboration and formulation of new forms for the Modelling Track were considered to be important challenges for this project as well as a further standardization of assessment (Wijnen, 2000). The development of multidisciplinary projects and the improvement of mathematics education as part of programs of other departments were also part of the project.

A small task force analyzed the Modelling Track to answer the following question: To what extent are the characteristics of Design Based Learning already present in the Modelling Track? Professionalization is present: through the years problems become more realistic. Also, students have to take more initiative themselves as the responsibility is transferred more and more from the supervisor to the group, thus activation is an aspect of the Modelling Track. Co-operation is present, but only on a small scale as the groups only have two members. Some of the projects really ask for creativity, others have more the character of an assignment. Throughout the consecutive years of the program, more and more subject matter of the courses is applicable. However, real integration with parallel mathematics courses is not present. Some multidisciplinary is present as some projects come from an external client (e.g., the air quality problem, described before).

Observations of students in action were carried out. Observing students' project presentations showed that students are present at presentations of other couples, but not really involved. The supervisors and problem owners are usually the ones asking questions about the work. As a new element to stimulate discussion by students, so-called 'discussion groups' were brought in: every student pair is assigned to another pair, with another problem, and required to open the discussion with some questions, after the presentation. Collaborative preparation between pairs is allowed. Although at first, students were not really happy with this innovation, the teachers viewed it as a useful training for students in constructively criticizing the methods and results of peers. This approach resulted in a little strengthening of characteristics of activation, co-operation and professionalization. Two plans were suggested for more structural innovations. The first plan was to integrate the track with one or more parallel courses, like described by Kjersdam and Enemark (1994) for the curriculum at Aalborg University in Denmark and by Perrenet (1998) for a part of the curriculum at Maastricht University. The second plan consisted of the introduction of groups of at least four students in order to strengthen the DBL-characteristic of co-operation. However, both plans were turned down by the mathematics department. The integration plan was turned down because the support for the Modelling Track among faculty was supposedly not strong enough yet for such a structural innovation. For a detailed discussion, see Perrenet and Adan (2002) and see section 11.5. The groups-of-four plan was turned down because of the majority's opinion that mathematics students do not need the experience of working in such monodisciplinary groups. For a detailed discussion, see also Perrenet and Adan (2002) and see section 7. While many TU/e departments really innovated their curriculum by introducing project work, Applied Mathematics only implemented small changes. Besides the introduction of discussion groups, a handbook was constructed to support staff and more detailed assessment criteria were developed for reports and presentations. For some years interdepartmental projects existed, often solving problems with some mathematical modelling aspects. However, this innovation never developed university wide, because collaboration between departments turned out to

be hard (Perrenet and Mulders, 2002). Currently these interdepartmental projects have disappeared from the Bachelor program (see also section 7).

## 6. Academic Reflection

Recently, in 2002, the Bachelor-Master system has been introduced in Dutch higher education. Therefore the distinction between university education and other forms of higher education has become more important. The TU/e decided to strengthen the aspect of academic education in its study programs. In 2002 several changes were carried out in the mathematics curriculum in this direction, such as the implementation of a course on mathematics and its usage in historical perspective. Another change was the introduction of a reflective portfolio in the Modelling Track and the completion of the track with a series of reflective activities in the course called Modelling 5.

### 6.1 The Portfolio

As a preparation for the reflective activities in Modelling 5, students have to build up a portfolio during the Modelling 1 to 4 courses. They have to store their project reports and products together with notes and remarks about the discussions with staff or peers. Also they have to reflect on their work after completing each modelling project by answering questions like “When looking back, would it have been possible to handle the assignment in a better way, especially from the mathematical point of view?” and “What is the social importance of the assignment?”

### 6.2 Overview of reflective activities

Important aspects of academic education are the competences to criticise and improve ones own methods and products, to be aware of the social impact of ones own work and to be able to reflect on ones own educational progress. For Modelling 5, a number of assignments were constructed to make these learning goals operational. These assignments generally consist of writing a short essay according to certain guidelines. Drafts are read and generally commented by the teacher<sup>1</sup> as well as read and specifically commented by two peers. After feedback a final version has to be delivered. At every assignment the students choose suited modelling projects from their portfolio.

In this section we only elaborate the assignments directly related to modelling. Besides those there was (a) an assignment asking for reflection on the students’ own learning attitude and study methods as a freshman compared to as a near graduate, and (b) an assignment about making choices in ones own educational career. We will describe the various types of modelling related reflection in more detail by giving the assignments as presented to the students.

#### *Reflection on the activity of modelling*

Comment on the following statement: “Modelling is essentially a three step process, consisting of a language step (from problem to model), a calculation step (from model to solution) and an interpretation step (from solution back to the problem).” Construct a more detailed scheme and confront this scheme with your own former modelling work from your portfolio.

#### *Mathematical reflection*

If you had to do the modelling project concerned again, would you do it in a mathematically different way? Would you use other methods or the same method more thoroughly? Did you learn anything since that is useful for this project? What new aspects you would take into account now, which you did not think of earlier?

#### *Social reflection*<sup>2</sup>

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<sup>1</sup> The first author of this article.

<sup>2</sup> The questions are inspired on the interactive study package STUDIO, in use at the TU/e and the Delft University of Technology. See <http://www.studio.tbm.tudelft.nl/studio/>.

Analyze your modelling project on the two following social aspects:

Firstly, the *actors* aspect: Which actors (groups, organisations, institutions, persons) have an interest in the issue of the problem and its solution? What is each actor aiming for? What interests are at stake for the various actors? Are the various actors satisfied or not with the present situation? In what way can the various actors influence the situation? Answer these questions at least for the following four types of actors (but do not leave out other important actors):

- Technology producers, such as universities or production companies
- Technology users, such as consumers or companies
- Technology regulators, such as governments or standardization organisations
- Technology advisors, such as engineering or designing offices, commissions and policy staff members.

Secondly, the aspect of *history and future*: Does the problem possess a history? Are former solutions tried out? Is it possible to predict certain short-term or long-term consequences of the solution, intended or unintended?

Conclude your analysis by arguing which social aspects are the most important and whether it would be possible to take into account these aspects in the modelling process.

Find at least one scientific article or policy memorandum on your subject, summarize it and incorporate it in your essay.

## 7. Integration of Communication Skills

Throughout the years, even in the ‘prehistoric’ Modelling Practical, communication skills were trained in the context of the problem at hand. However, substantive changes occurred concerning who was the trainer and what was trained. (A partly trial-and-error search over the years for the optimal structure and contents of skills training connected to project work can be observed more often, c.f., Perrenet, Aerts & van der Woude (2003)).

In the beginning, all training was done by staff from the own department. Later on, it was realized, that for skills such as writing and giving a presentation, a communication expert from *outside* the mathematics department would be more suitable. It was important however, that this trainer was already connected to the TU/e and familiar with the technology context. Also the content of the training was developed in interaction with a mathematics staff member.

*Report writing* and *giving a presentation* can be considered as the communication core skills. Modern technical aspects like *using LaTeX* and *reporting on the web* were added as internal training when these aspects became relevant. *Use of library systems* was incorporated as a follow-up of a visitation committee’s advice (see section 10.1). *Interviewing* and *planning* were added, firstly, because these skills were relevant, but secondly, probably because suited training had been centrally developed, as a consequence of the university wide DBL innovation (see section 5).

*Giving feedback to peers* was added during presentations and also practiced in the concluding modelling course (see section 6) without explicit training. Similarly, *various reflective skills* were incorporated into the portfolio activities (see also section 6).

*Writing for lay man* is considered an important skill. It is practiced in the curriculum, but not integrated into the Modelling Track. It is done as part of the individual final Bachelor project (more a research project than a design project).<sup>3</sup>

While in almost all other programs *collaboration in groups* of six or more was trained and practiced in DBL, the majority of the staff turned down the idea of groups larger than two in the first and second year. The development of modelling skills should get priority over the development of group skills. It was believed that participation and the development of individual creativity would be stronger in a pair (Perrenet & Adan, 2002).

*Collaboration in multidisciplinary groups* was propagated university wide, as part of DBL, but the implementation of interdepartmental project work was not successful because of organizational problems (Perrenet & Mulders, 2002). Therefore, although the idea of larger, even multidisciplinary, groups was accepted for the third year, this specific implementation failed.

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<sup>3</sup> In fact, it is coordinated by the first author.

Finally, after many years, a mathematical modelling project, with use of larger groups, will be implemented in the Master program of Industrial and Applied Mathematics. This program is the first choice for students who wish to continue their studies after the Bachelor program of Applied Mathematics. See further under ‘opportunities’ in section 11.

## 8. The Need for Structure

### 8.1 Midterm Review

In 2006 the university went from a trimester system to a semester system. This meant that the longer projects of year two were more spread out than before. The first experience was that planning had become a problem for the students: because of the longer period students were more tended to postpone their work to the end and to give a superficial mid-term presentation (since hardly anything could be presented). It was important to stimulate the students to work seriously at their project already in the first half of the semester. The solution was to implement a strict evaluation halfway through period. Every group got the choice to take two assignments, both spread over half a semester, or one assignment spread over the whole semester. Midterm, every group gives a presentation and delivers a report. For a half-semester assignment this is the final presentation; for the whole-semester assignment this is a half way presentation. In both cases a report has to be delivered too. For the whole-semester assignment this report is supposed to contain a detailed problem description and description of the first modelling steps, as well as an outline of the project plan in the second half of the semester. In both cases the work in the first half of the semester will be graded. In this way, a big assignment can be forced to end halfway, because of too little work done by the students or because that it was unlikely that the assignment would be completed successfully in the second half.

The first experience with this new system is that in exceptional cases only, whole-semester projects really have to be put to an end at midterm.

### 8.2 Modelling Methodology

Throughout the years it became more and more clear, that there are many modelling perspectives and methodologies. Better than leaving this implicit and dependent on the accidental lessons of a supervisor in one or more modelling projects, it was decided in 2008 to implement four specific lectures about mathematical modelling, given by experts from various fields. We will give a short description of each lecture and the reason why it was chosen.

#### *Methodology of mathematical modelling*

This lecture presents a general overview of the methodology of mathematical modelling as it is applied in the different areas of application. In particular attention is paid to the mathematical modelling cycle, and different types of models (e.g. first principle, stochastic, statistical and data models). This lecture is concluded with an assignment for the students: for their project at hand, they have to argue why a mathematical model is needed and they have to elaborate the first step of the modelling cycle, i.e. the problem specification.

#### *The practice of consultancy from the operational management perspective*

The director of a consultancy company in quantitative modelling (with a high “density of mathematics”) presents his views on problem solving and mathematical modelling, based on his rich experience of many years of “working in the field”. For example, he explains that at the start of a project it is crucial to find the (real) question behind the question and he points out pitfalls in problem solving (such as “do not always use a the same tool for the job”).

#### *Methodology of mathematical modelling from an industrial applications perspective*

In this lecture the director of the post-doctoral Mathematics for Industry program presents his personal views on the role and aim of mathematical modelling in general, and more specifically in the context of industrial (and technical) applications.

### *Statistical consultancy from an engineering problem solving perspective*

In this lecture mathematical modelling is viewed through the eyes of a statistician.

It is still too early to evaluate the effect of these lectures. The question remains whether their impact will be significant. Because there is no uniform study path – every student has a unique series of projects – it will be hard to relate the methodology explicitly to the individual projects at hand. Without this relation, the methodology lectures bear the risk of lacking practical significance to the students.

## **8.3 Representation of the Modelling Cycle**

In the years 2009, 2010 a study was done on the variation of the representation of the modelling cycle's steps (see section 3.2) by students and professionals. The impression was that, over the years, students showed a great diversity in representation of the modelling cycle at the end of the track. This diversity has been investigated in detail and compared to the representations of the professionals. The schemes of 80 students and 20 teachers, in which they give their vision of the modelling cycle in more detail, have been analyzed with respect to variables such as: validity, verification, iteration, communication, and complexity. Indeed, there existed in both groups, students and teachers, much diversity on a lot of the variables. Only on iteration (one is passing the modelling cycle more than once) the groups were systematically different. This aspect was significantly more present in the teachers' representations. The presence of the aspect of communication was quite low (in both groups). In a discussion with the teachers about the results of this investigation the value of the variables under investigation was recognized. The ascertained diversity was no problem to most of the teachers, as in their opinion, the variation is inherent to the modelling process. However, the aspect of communication, especially with the client, should be stressed more. For further details, see Perrenet & Zwaneveld (submitted).

## **9. Students and Teachers**

### **9.1 Non-mathematics Students from the University**

Nowadays, the Bachelor program in Eindhoven has a major-minor structure with a minor of half a year. The minor Applied Mathematics contains the course Modelling B and is accessible for students from respectively Chemical Technology, Industrial Engineering, Computer Science, Applied Physics and Mechanical Engineering. It turns out that these students are quite successful; in some of the projects these students are even able to integrate the contents of other courses in the Applied Mathematics minor (e.g. the course Linear Programming modelling) with the project at hand.

### **9.2 Mathematics Students from Another University**

The Eindhoven University of Technology cooperates with the Hasselt University in Flanders. Flemish mathematics students can choose to follow part of their studies in Eindhoven. In their program part of the Modelling Track is incorporated, namely Modelling B. It turns out that, although for some Flemish students the idea of working with significant self-responsibility at an open problem is quite new, most of them are quite successful, due to their solid mathematical knowledge base.

### **9.3 Changes in Secondary (Mathematics) Education**

Throughout the years major changes occurred in secondary mathematics education. Decrease of time available, increase of the role of the graphic calculator and formula card, less attention for memorization of formulas, more attention for communication skills, less attention for content knowledge, shift in the role of the teacher from lecturer to facilitator, etc. Many complaints came from the universities about the dramatic decrease of the mathematics knowledge and skills of the freshmen, probably caused by the combination of these factors. Surprisingly (or not), the students' functioning in the Modelling Track was not hampered at all. Increase in 'soft' skills probably compensated for decrease in hard knowledge.

### 9.3 Student Satisfaction

Regularly students fill in questionnaires about the quality of courses and group discussions are organized. Results point out that students are quite satisfied with the track. By example, the mean score at the item ‘The project work increased my understanding of the work of a mathematical engineer’ always scores about 4 at an agreement scale from 1 to 5. In group discussions many students express their satisfaction with the freedom to develop their own solutions. A small minority prefers to be told before how to find solutions. Concerning the reflection assignments, described in section 6.2, we can conclude, that most students more or less appreciate the reflective activities and the interaction with other students about the modelling projects. However, every year, there is a substantial minority of the students, who only do the assignments, because they have to; they prefer to do mathematics without such reflections. See also Perrenet and Ter Morsche (2004).

### 9.4 The Staff

From the start of the Modelling Track, the decision had been to involve many staff members. Better let many staff members coach a student team each than let a few staff member coach many teams. In the first way, enthusiasm and involvement would spread throughout the department. However, to be able to successfully participate in this modelling course, the staff involved should have some experience in mathematical modelling in industry or at least, they should enjoy the challenges posed by solving real-life problems. Part of the staff is indeed really involved and interested in industrial projects, e.g. through the Mathematics for Industry program or the applied mathematics ‘laboratory’ LIME (Laboratory for Industrial Mathematics Eindhoven) of the Mathematics department; they are well equipped to participate in the Modelling Track. On the other hand, part of the staff is more theoretically oriented and not so much interested in applications; hence, being unconvinced of the necessity of teaching modelling skills (“the students can learn that later in practice”), they might still pose a threat for the existence of the mathematical modelling courses in the future. The staff’s opinion has not been systematically evaluated, but from many informal discussions with the coordinator (the second author) the impression is that only about 20% of the staff is unsatisfied with the modelling courses.

## 10. Comments from Outside the Department

### 10.1 Visitation reports

Higher-education programs are evaluated every five years by a committee of the Association of Dutch Universities VSNU. In 2002, the committee reported (VSNU, 2002, pp. 58): “One of the most important developments in recent years is the implementation of the Modelling Track. Students as well as graduates have a very positive opinion about this track. It prepares for the often very vague problem statements as they occur in practice. In this track also activities such as ‘communicating and reporting’ are treated in a natural way.”

In 2007, the next committee, Quality Assurance Netherlands Universities (QANU), reported (QANU, 2007, pp. 29): “The committee was impressed by the Modelling Track at the Eindhoven University of Technology; it was broad, well designed and elaborated.”

### 10.2 Academic Profile

At the TU/e, all programs are evaluated to measure their academic profile by use of interviewing all teaching staff about their ambitions for each course. An academic engineer should:

- Be competent in one or more scientific disciplines;
- Be competent in doing research;
- Be competent in designing;
- Have a scientific approach;

- Possess basic intellectual skills;
- Be competent in co-operating and communicating;
- Take account of the temporal and the social context.

The profiles show to what extent students should spend their time at developing competences in these areas. Moreover, the profiles show to what extent students should perform typical academic thinking and activities such as analyzing, synthesizing, abstracting and concretizing. See also Meijers, van Overveld, & Perrenet (2005). A special study took place to assess the role of the Modelling Track within the Bachelor program. The results show that the Modelling Track is crucial for the curriculum profile of the Bachelor program. It strengthens the competence area of designing. At the level of individual competencies, some competencies are already at the Master's level, such as reformulating ill-structured complex design problems, creativity and synthetic skills for complex design problems, independently producing and executing a design plan. Also, in the modelling projects, students have to create and apply mathematics while taking much detail of the problem situation into account (high level of concretising). Finally, within a modelling project, they have to use their analytic and synthetic skills at several levels of complexity, zooming in and zooming out of the problem situation. On the other hand, the level of abstraction aimed for in the Modelling Track is somewhat lower, compared to many other courses in the curriculum. For a more detailed description and a discussion of the results, see Perrenet & Adan (in print).

## 11. Conclusions and Reflection

Looking back at the creation and the development of the Modelling Track within the program of Applied Mathematics of the TU/e, we conclude that this method of teaching mathematical modelling has been mainly successful. As time went by, some improvements have been implemented. We will discuss those and sum up the reasons or causes. We will conclude with a SWOT-analysis of the track.

### 11.1 Changes in the Modelling Track and Their Causes

We have showed and discussed changes in problem contents, integration with communication skills and course structure. Most changes came from outside the department, from the level of the university or even higher education in general. The change of the TU/e program structure from trimester to semester was necessary to match the program structure of other universities of technology in the Netherlands. The changes in the use of communication skills did not occur in isolation at this university but were part of national or even international general educational trends of student centred education. In many higher education programs all over the world, project work as well as the use of a reflection portfolio was implemented. A final factor is, of course, the development of technology in general and in the discipline of applied mathematics specifically. In short, external factors like general educational trends, university politics and technology development, were much more important than the internal factor of experience: as stated before, the majority of students and staff are quite satisfied with the track. The involvement of regional companies, for providing problems as well as lectures, is an example of self organized improvement.

### 11.2 Strengths

One of the strengths of our organization of the modelling courses is that many staff members are involved. There is a great diversity in problems and supervisors' background, so the Modelling Track offers a representational view on many areas of mathematics and their applications. Working in the modelling projects as described, stimulates independence, taking initiative and creative and critical thinking.

Recently, the involvement of regional companies as clients has increased. This means more authentic problems and knowledge from the field, as well as career orientation. Participating companies are, e.g., CQM (Consultants in Quantitative Methods), Quintiq (Advanced planning and scheduling software) and DTV (Sustainable mobility).

### 11.3 Weaknesses

As we are quite satisfied with the track, it is hard to list its weaknesses. About the aspect of larger groups, the authors have a different opinion (see further Perrenet & Adan, 2002). We mentioned the involvement of many staff members as a strength, because of the broad support. However, it also bears the risk of having not one shared view about what modelling ‘really is’. For that reason, the lectures on the methodology of mathematical modelling have been introduced. Finally, in section 10.2 it was mentioned that the level of abstraction was generally lower than in the other courses of the program. However, this is not a weakness for the program as a whole.

### 11.4 Opportunities

During the fifteen years of its existence, the Modelling Track stopped with the end of the Bachelor program. However, this fall (2010) a modelling week will be implemented in the Master program. The main goals are that Master students will learn about aspects of modelling within an industrial context, and to stimulate collaboration between Master students from the local Applied Mathematics Bachelor programs and Master students from another, often non-Dutch, Bachelor program. Experience from the post-Master program Mathematics for Industry (in which international modelling weeks have been organized already for many years) will be used and there will be cooperation with LIME. It will be a full time experience, prepared by a series of communication skills trainings, including attention for intercultural differences.

### 11.5 Threats

Although throughout the years a majority of the department’s staff have accepted the Modelling Track as a useful part of the curriculum, still now and then voices ask for restructuring modelling education directed at application only and after ‘enough’ mathematics knowledge and skills have been taught. Moreover, the trend of student centred education is challenged, especially within the context of mathematics education, by a back to basics trend (read: back to pure algebra). See also Perrenet & Adan (in press).

Quite another type of threat is the fact that the track during these fifteen years has been coordinated by the same enthusiastic stimulator of this kind of mathematical modelling education. What would happen if he would choose for another direction in his career? Ideally, the track should be the responsibility of a chair in mathematical modelling (which is not yet present at the TU/e).

### 11.6 Advice

*Would we advise this kind of mathematical modelling education in other Bachelor programs of Applied Mathematics?* Yes. However, in some parts of the world cultural factors could hamper its success. If the teacher is seen as the expert who knows everything and the student as the one who has to do what the teacher tells him or her to do, students as well as teachers would have problems with their role. See also Hofstede (2005).

*Would we advise its implementation in Master programs of Applied Mathematics?* Yes. Not only would this give the opportunity to collaborate in larger (inter)national teams, it would also provide the track with a more extended and solid base in Bachelor, Master and post-Master programs.

*What about secondary education?* In the Netherlands, mathematical modelling is mentioned in the national examination program. Only part of the cycle is explicitly assessed in the central examination, as in general in problems, the model is provided along with the problem statement. This lacks the essentials of mathematical modelling and strongly influences educational practice. Of course, problems should be less complex and smaller in secondary education. However, at least for the central mathematics examination concluding secondary education, preparing for university studies in mathematics, science and technology, the whole cycle should be assessed. But even this kind of ‘directed’ modelling is better than replacing it by more practice in solving algebra problems, as is happening in the Netherlands nowadays. See also Perrenet & Adan (in press). Mathematical modelling



might get a much larger group of secondary education students interested in mathematics and its applications or at least make them see the use and importance of it, than exercises in equation solving, which only kindle enthusiasm in the limited group of lovers of pure algebra.

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