Instabilities in fluid mixtures

Instabilities in Fluid Mixtures

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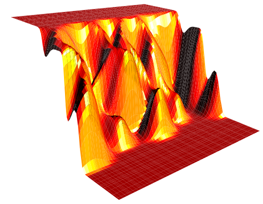
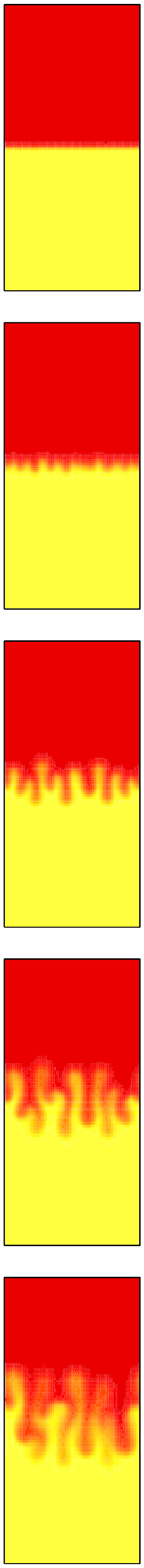


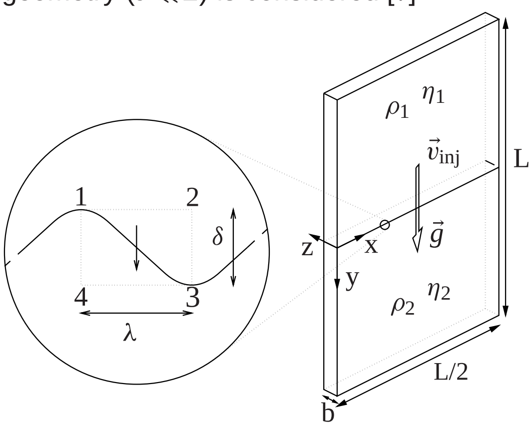


1. Introduction

To investigate macrovoid formation in porous membranes [?], the mechanical and thermodynamical stability of inhomogeneous, multi-component fluid systems in a flow field is considered.

2. Displacement Flows

First the stability of an isothermal, miscible, binary displacement flow in a Hele-Shaw geometry ($b \ll L$) is considered [?].



Stability depends on the pressure difference

$$p_4 - p_3 = (\rho_2 - \rho_1)g\delta - (\tilde{\eta}_2 - \tilde{\eta}_1)v_{inj}\delta$$

where $\tilde{\eta} = 12\eta/b^2$. The perturbation is unstable if $p_3 > p_4$.

The governing equations are given by

$$\nabla\cdot\vec{v} = -\zeta\nabla\cdot(D\nabla c), \quad \zeta = (\rho_1 - \rho_2)/\rho_2$$
$$dc/dt + c\nabla\cdot\vec{v} = \nabla\cdot(D\nabla c)$$
$$\rho d\vec{v}/dt = -\nabla p - \tilde{\eta}(\vec{v} + \vec{v}_{inj}) + \rho\vec{g}$$
$$\rho = \rho_1 c + \rho_2(1-c)$$
$$\eta = \eta(c)$$
$$D = D(c)$$

A typical time evolution of the density field in case $\rho_1 > \rho_2$ (Rayleigh-Taylor instability), for constant viscosity and diffusion is shown on the left.

3. Thermodynamics

To include surface tension and phase separation a Ginzburg-Landau type free energy functional [?] is used

$$F[c] = \int \{f(c) + \frac{1}{2}\kappa(\nabla c)^2\} dV$$

To obtain the governing equation, the Fickian diffusion terms must be replaced by Cahn-Hilliard terms [?]

$$M\left(\frac{\partial^2 f}{\partial c^2} - \kappa\nabla^2\right)\nabla^2 c$$

and a force, induced by composition gradients, must be added to the momentum equation

$$-\kappa\nabla\cdot(\nabla c \otimes \nabla c)$$

4. Conclusions

- The miscible model does not predict all the qualitative aspects of macrovoid formation in porous membranes.
- The Ginzburg-Landau model describes interfaces with a finite thickness, which justifies the use of a continuum surface tension method [?].

5. References

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