Synchronous digital transmission over multiple channel systems

SYNCHRONOUS DIGITAL TRANSMISSION OVER MULTIPLE CHANNEL SYSTEMS

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OVER

MULTIPLE CHANNEL SYSTEMS

PROEFSCHRIFT

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CONTENTS

SUMMARY

This thesis deals with the problem of detecting synchronous data sequences, which are transmitted over multiple channel systems and disturbed by noise, intersymbol and interchannel interference.

Chapter 1 starts with definitions of intersymbol and interchannel interference. Multidimensional interference is the term used to describe the combined effect of these two disturbances. The multiple channel communication model, to be considered in this thesis, is described after a short historical introduction.

Chapter 2 is devoted entirely to linear receivers. First of all the structure of the optimal linear receiving filter is derived. This filter consists of two parts, called the multiple matched filter and the multiple tapped delay line. It is found that this structure, which is valid for the criterion of minimum symbol error probability and the criterion of minimum symbol error probability under the zero-forcing constraint, is the equivalent of the structure found by Kaye and George applying the mean square error criterion. Furthermore, the multidimensional Nyquist criterion is defined, which fits Shnidman's generalized Nyquist criterion. A simple expression is derived for the error probability of systems satisfying this multi-dimensional Nyquist criterion. Then optimum realizable (i.e. finite length) multiple tapped delay lines are considered and algorithms are given to calculate the tap coefficients in several practical situations. At the end of the chapter, two experiments are described, to which the theory developed for linear receivers is applied. These examples concern the transmission of four binary data sequences over a cable,

consisting of four identical wires, which are symmetrically situated inside a cylindrical, conducting shield. The experiments were conducted at both 5 Mbit/s per channel and 50 Mbit/s per channel.

In Chapter 3 maximum likelihood receivers are investigated. To apply the concepts of maximum likelihood sequence estimation, the statistical sufficiency of the multiple matched filter output samples is proved first of all. Then two maximum likelihood sequence estimation algorithms are generalized for maximum likelihood vector sequence estimation. To apply the vector Viterbi algorithm a multiple whitened matched filter is defined. The vector Ungerboeck algorithm uses the sampled output of the multiple matched filter directly. The latter algorithm avoids the multiple tapped delay line and is essentially no more complicated than the first one. An analysis of the error performance of this kind of receivers shows that, under a certain constraint, for moderate and large signal-to-noise ratios the symbol error probability is as good as if multidimensional interference were absent. Finally, some attention is paid to maximum a posteriori receivers.

The main conclusion of these investigations is that multidimensional interference is a generalization of intersymbol interference. Several important concepts from the intersymbol interference literature can be generalized for multidimensional interference.

ABBREVIATIONS

AGN	additive Gaussian noise
CCGN	colored, correlated, Gaussian noise
ICI	interchannel interference
ISI	intersymbol interference
MAP	maximum a posteriori
MDI	multidimensional interference
ML	maximum likelihood
MMF	multiple matched filter
MTDL	multiple tapped delay line
MWMF	multiple whitening matched filter
SNR	signal-to-noise ratio
WUGN	white, uncorrelated, Gaussian noise

LIST OF SYMBOLS

a_{jl}	input symbol at input j at instant lT
a_{j0_k}	$j^{\underline{th}}$ element of $\underline{x}_{0_k}$
$a_{in}(t)$	element of $A(t)$
A	$\sum_{l=-\infty}^{\infty}{}' \, \|\| \sum_{j=-N}^{N} C_j V_{l-j} \|\| + \|\|Z\|\|$
A^*	minimum value of A
$\bar{A}$	value of A at $(C_{-N}^*,\ldots,C_k^* + E_k,\ldots,C_n^*)$
$A(t)$	arbitrary matrix, the elements of which consist of time functions
$b_{nj}(t)$	element of $B(t)$
B	auxiliary matrix
$B(t)$	arbitrary matrix, the elements of which consist of time functions
$\|\|B\|\|_2$	$\max_{\underline{x} \neq \underline{0}} \dfrac{\|\|B\underline{x}\|\|_2}{\|\|\underline{x}\|\|_2}$
c_{njl}	tap coefficients of MTDL; $n,j^{\underline{th}}$ element of the matrix C_l
C	composite matrix consisting of the C_l matrices
C_l	matrix of tap coefficients of MTDL after l delays
C_T	composite matrix consisting of the C_l^T matrices
C^T	the transposed matrix of C
C^{-1}	the inverse matrix of C
C_k^*	correction matrices that lead to a minimum value of A
$C(D)$	matrix D-transform of the C_l matrix sequence
d	smallest difference between two output levels
D	delay operator
$D^2(s_l,s_{l+1})$	distance of an observation to the transition from state s_l to state s_{l+1}

$\underline{e}_l$	error vector at instant lT
e_{il}	$i^{\underline{th}}$ component of $\underline{e}_l$
$\underline{e}(D)$	$\hat{\underline{x}}(D) - \underline{x}(D)$; D-transform of the error sequence of an error event
$\|\|\underline{e}_l\|\|_2$	$\sqrt{\sum_i e_{il}^2}$; Euclidean norm of the error vector
E	the set of all possible error events
$E[.]$	expectation of the stochastic variable between the brackets
E_δ	subset of error events with $\delta(\varepsilon) = \delta$
E_k	small deviation of the matrix C_k^*
$f_{nj}(t)$	impulse response from input j to output n of the system consisting of the cascade connection of the multiple channel, (the MMF) and the MTDL
F	quantity to be used for the up-dating of a metric
F_l	the matrix of impulse responses of the multiple channel in cascade with (the MMF) and the MTDL, evaluated at instant lT
F_k^*	$\sum_{j=-N}^{N} C_j^* V_{k-j}$
$F(D)$	matrix D-transform of the F_l matrix sequences
G	$[\langle R^T(t), R(t)\rangle]^{-1}$
$h_{ni}(t)$	impulse response from input i to output n of the linear receiving filter
H	the length of an error event is $H+N$
$H(D)\ H^T(D^{-1})N_0$	spectral factorization of $\Phi(D)$
i	integer index
I	$M \times M$ identity matrix
I_0	worst case MDI

I_n	MDI at output n
j	integer index
J	metric
J_n	auxiliary functional
$J_l(\ldots,\underline{\xi}_{l-1},\underline{\xi}_l)$	$-2 \sum_{n=-\infty}^{l} \underline{\xi}_n^T \underline{v}_n + \sum_{n=-\infty}^{l} \sum_{k=-\infty}^{l} \underline{\xi}_n^T V_{n-k} \underline{\xi}_k$
J_l	survivor metric at instant lT
$J\{\underline{\xi}(D)\}$	metric of the input sequence $\underline{\xi}(D)$
k	integer index
k_{ij}	element of K
K	$<A(t),B(t)>$; inner product of $A(t)$ and $B(t)$
l	integer index
L	number of elements of the input alphabet
m	integer index
M	number of inputs/outputs
M_0	$\sum_{l=-\infty}^{\infty}{}' \lVert V_l \rVert$
n	integer index
$\underline{n}$	$<R^T(t),\underline{n}(t)>$; sampled noise signal at the output of the MMF
$\underline{n}_l{}'$	sample values at instant lT of the noise at the outputs of the MMF
$\underline{n}(t)$	vector noise at the output of the multiple channel system
$n_i(t)$	additive noise waveform at output i of the multiple channel system
$\underline{n}_r(t)$	relevant vector noise

$\underline{n}(D)$	vector D-tranform of noise samples at the output of the multiple channel system
$\underline{n}'(D)$	vector D-tranform of the noise samples at the output of the MMF
N	for linear correction the length of the MTDL is $2N$; at the Viterbi algorithm this length is N
N_i	double-sided density of the noise spectrum of $n_i(t)$
N_0	double-sided density of the noise spectra if WUGN disturbance of the channel output is assumed
O	$M\times M$ all zero matrix
$p(.)$	probability density function of the stochastic variable in the parenthesis
P	matrix of transition probabilities
$P(s)$	auxiliary matrix
$Pr(e)$	symbol error probability
$Pr(\varepsilon)$	probability of the event ε
$Q(x)$	$\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{\alpha^2}{2}}\,d\alpha$
$Q(-s)\ Q^T(s)$	spectral factorization of $\Phi_{\underline{nn}}(s)$
$r_{ij}(t)$	impulse response from input j to output i of the multiple channel system
R	composite matrix of the R_l matrices
R_T	composite matrix of the R_l^T matrices
R_l	the matrix of impulse responses of the multiple channel system, evaluated at instant lT
$R(t)$	matrix of impulse responses of the multiple channel system

$R(D)$	matrix D-transform of the R_l matrix sequence
$R(D,t)$	matrix consisting of the chip D-transforms of the elements of $R(t)$
s	bilateral Laplace variable
$\underline{s}$	$\langle R^T(t),\ \underline{s}(t)\rangle$; sampled output of the MMF if the multiple channel system is excited by a single input vector and if noise is absent
s_l	state of a finite state machine at instant lT
$\underline{s}(t)$	vector signal at the output of the multiple channel system, if this system is excited by a single input vector and in the absence of noise
$s_i(t)$	signal at output i of the multiple channel system, if this system is excited by a single input vector and in the absence of noise
$s_{nk}(t)$	response at output n of the receiving filter if the channel is excited by the single input vector $\underline{x}_{0_k}$
t	time
t_s	sampling instant
T	time between successive transmissions
U	composite matrix consisting of the I and O matrices
$\underline{u}(t)$	received vector signal at transmission of the vector sequence $\underline{x}(D)$
v_{ijl}	$i,j^{\underline{\underline{th}}}$ element of V_l
$\underline{v}$	equivalent received signal vector
$\underline{v}_l$	sampled output of the MMF at instant lT
$\underline{v}(t)$	equivalent received vector signal
$\underline{v}(D)$	vector D-transform of the sequence $\underline{v}_l$

$v_{mj}(t)$	impulse response from input j to output m of the cascade connection of the multiple channel system and the MMF
V	composite matrix of the V_l matrices
V_l	the matrix of impulse responses of the multiple channel system in cascade with the MMF, evaluated at $t=lT$
$V(D)$	D-transform of the V_l matrix sequence
$\|\|V_l\|\|_\infty$	$\max_i \{\sum_j \|v_{ijl}\|\}$
$w_{mn}(t)$	impulse response from input n to output m of the multiple whitened matched filter
$W(D,t)$	matrix consisting of the chip D-transforms of the $w_{mn}(t)$
$\underline{x}$	arbitrary vector
$\underline{x}_l$	input vector that is transmitted at instant lT
$\underline{x}_{0_k}$	one of the L^M possible input vectors at $t=0$
$\underline{x}(D)$	D-transform of the input vector sequence $\underline{x}_l$
$\hat{\underline{x}}(D)$	estimate of $\underline{x}(D)$
$y_i(s_l,s_{l+1})$	$i^{\underline{th}}$ element of $\underline{y}(s_l,s_{l+1})$
$\underline{y}$	sampled output of the multiple whitening filter
$\underline{y}(s_l,s_{l+1})$	output vector associated with the transition from state s_l to state s_{l+1}
$\underline{y}(D)$	D-transform of the sampled output of the multiple whitened matched filter in the absence of noise
$\underline{z}_l$	sampled output of the multiple whitened matched filter at instant lT
z_{il}	$i^{\underline{th}}$ component of $\underline{z}_l$
$\underline{z}(D)$	D-transform of the $\underline{z}_l$ sequence
Z	auxiliary matrix with diagonal elements equal to zero

Z^*	$-I + \sum_{j=-N}^{N} C_j^* V_{-j}$
α	$2\|\|V_0^{-1}\|\|_2 \sum_{l=0}^{H} \underline{e}_l^T \underline{n}'_l$
$\underline{\alpha}$	auxiliary vector
α_{nkj}	component of $\underline{\alpha}$
$\underline{\beta}$	auxiliary vector
β_{nkj}	component of $\underline{\beta}$
δ	auxiliary variable
δ_0	minimum non-zero value of the Euclidian norm of the error vector $\underline{e}_i$
δ_m, δ_{nj}	Kronecker delta; $\delta_m = \begin{cases} 1, & m=0 \\ 0, & m\neq 0 \end{cases}$; $\delta_{nj} = \begin{cases} 1, & n=j \\ 0, & n\neq j \end{cases}$
δ_{min}	minimum value out of the set Δ
$\delta(\varepsilon)$	magnitude of the error event ε
$\delta(t-lT)$	unit impulse at instant lT
Δ	the set of all possible values of $\delta(\varepsilon)$
ε	error event
ε_1	sub-event
ε_2	sub-event
ε_2'	sub-event
θ_{kijl}	$k,i^{\underline{th}}$ element of the matrix V_{l-j}
λ	auxiliary variable
λ_{nkl}	Lagrange multiplier
$\lambda(\underline{\xi}_l)$	contribution of a certain transition in the trellis to the probability of a certain path
$\lambda_{min}(V_0)$	smallest eigenvalue of V_0
$\underline{\xi}_l$	possible value of $\underline{x}_l$

$\underline{\xi}(D)$	possible transmitted vector sequence
$\prod_{i=0}^{H}$	repeated multiplication over the index starting with $i=0$ and up to and including $i=H$
ρ_{nkjl}	$n,k^{\underline{th}}$ element of the matrix R_{l-j}
σ_n^2	noise variance at output n of the linear receiving filter
Σ_l'	summation over l excluding the term with $l=0$
$\phi_{nm}(D)$	D-transform of the sequence $\phi_{nm}(lT)$
$\phi_{nm}(\rho)$	cross-correlation of the noise waveforms at the MMF outputs n and m
$\Phi_{\underline{nn}}(s)$	Laplace transform of the correlation matrix of the noise processes $n_i(t)$
$\Phi_{\underline{yy}}(s)$	Laplace transform of the correlation matrix of the noise processes at the outputs of the multiple filter $Q^{-1}(s)$
$\Phi(D)$	spectral matrix of the output noise
$\Phi_{ww}(D)$	$\langle W(D^{-1},t),\ W^T(D,t)\rangle$

CHAPTER 1

INTRODUCTION

In this thesis we shall investigate the transmission of digital signals over multiple channel systems, where each channel is used to transmit a data sequence.

Apart from intersymbol interference (ISI), interchannel interference (ICI) can be one of the major problems in such a multiple channel digital transmission system. ISI is a disturbance of an output signal by symbols that originate from the corresponding input but that are shifted in time with respect to the symbol under consideration. ICI is a disturbance of an output signal by symbols that do not originate from the corresponding input but from input symbols that belong to neighbouring channels. Because the equalization of the ISI also changes the ICI at the output and the other way round, only a simultaneous treatment of these two phenomena can be succesful in combating the overall degradation.

It was first pointed out by Shnidman [1] that ISI and crosstalk between multiplexed signals are essentially identical phenomena. Kaye and George worked out this idea by investigating the transmission of multiplexed signals over multiple channel and diversity systems [2]. The author of this thesis has given a unified theory for treating ISI and ICI as one type of disturbance [3, 4]. He introduced the name multidimensional interference (MDI) for the combined effect of ISI and ICI.

In this thesis a number of techniques known from the ISI literature are generalized to MDI. Examples of systems to which these methods can be applied, are multiwire cables and multichannel radio systems that make use of perpendicular polarized waves in a common frequency band.

The transmission systems to be considered in this thesis have M inputs and M outputs. To each input j a data sequence $\Sigma_l a_{jl} \delta(t-lT)$ with $l = \ldots,-1,0,1,\ldots$ is applied, which it is desired to detect at the receiving end of the communication system. The symbols a_{jl} are elements of the alphabet $\{0,1,\ldots,L-1\}$. Except in those sections where it is otherwise stated, these symbols are chosen equiprobable and independent of each other.

In the present investigations a linear, dispersive and time invariant multiple channel model is assumed (Fig. 1.1). This means that there is

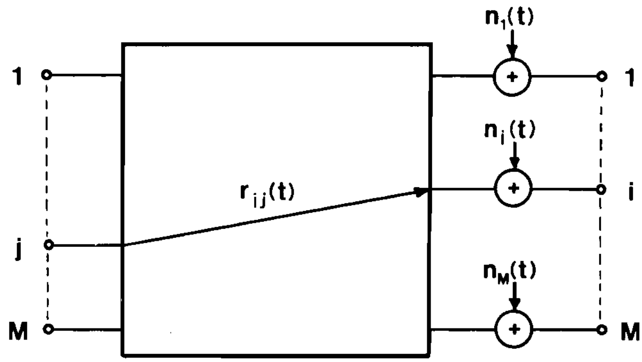

Fig. 1.1 Multiple channel communication model.

a linear relation between each input and each output signal and that the output signal due to the excitation of more than one input is the sum of the individual responses to the inputs in question. The relation between input j and output i is denoted by the impulse response $r_{ij}(t)$. All these responses are assumed to be square-integrable and of finite duration. Furthermore we assume that the output signals are disturbed by MDI and additive, zero-mean, Gaussian noise (AGN). Each output i is corrupted by a different noise waveform $n_i(t)$.

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CHAPTER 2

OPTIMUM LINEAR RECEIVERS

By means of an optimum linear receiver and symbol-by-symbol detection on each channel output an estimate is made of the several input sequences. The receiving filter is assumed to be linear in the sense described in Chapter 1. This configuration is included in the more general structure considered by Kaye and George [1]. In this thesis a technique is used that leads to an optimum structure for both the zero-forcing and minimum error probability criterion, instead of the minimum mean square error criterion used by Kaye and George. The linear relation between input i and output n of the receiving filter is characterized by the impulse response $h_{ni}(t)$ (see Fig. 2.1).

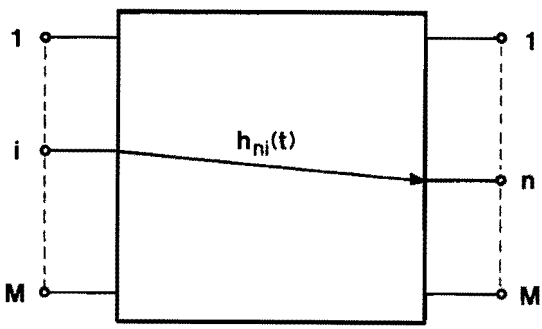

Fig. 2.1 Multiple linear receiving filter.

In this chapter we develop optimum solutions for the linear multiple channel receiving filter in several more or less theoretical and practical conditions.

2.1 The structure of the optimum linear receiving filter

Assuming that the noise processes $n_i(t)$ are white and uncorrelated, the noise variance at output n of the causal receiving filter can be written as

$$\sigma_n^{\,2} = \sum_{i=1}^{M} N_i \int_0^{\infty} h_{ni}^{\;2}(\tau)\, d\tau \tag{2.1}$$

where N_i is the double-sided density of the noise spectrum of $n_i(t)$. Investigating the optimum structure of the linear receiving filter a technique presented in [2] and [3] is used. This implies that all signal values contributing to the possible sample values of the signal at output n are fixed. Then the noise variance $\sigma_n^{\,2}$ is minimized, subject to these constraints. Defining the input vector

$$\underline{x}_l \triangleq \begin{bmatrix} a_{1l} \\ a_{2l} \\ \cdot \\ \cdot \\ \cdot \\ a_{Ml} \end{bmatrix} \tag{2.2}$$

the constraints are found by considering the sample values of the signals at output n due to the L^M possible input vectors $\underline{x}_l$. The latter sample values are found in the following way. Assume that at time $t=0$ the single vector $\underline{x}_{0_k}$, being one of the L^M possible input vectors, is applied to the input of the channel. Then the response at output n of the receiving filter evaluated at the instant t_s+lT, is given by

$$s_{nk}(t_s+lT) = \sum_{j=1}^{M} a_{j0_k} \sum_{i=1}^{M} \int_0^{\infty} h_{ni}(\tau) r_{ij}(t_s+lT-\tau)\, d\tau \, . \tag{2.3}$$

In the minimization process these values for all k and l must be kept constant, therefore we have to minimize the functional

$$J_n = \sum_{i=1}^{M} N_i \int_0^{\infty} h_{ni}^2(\tau)\, d\tau \; +$$

$$- \; 2 \sum_{k=1}^{L^M} \sum_{l} \lambda_{nkl} \sum_{j=1}^{M} a_{j0_k} \cdot \sum_{i=1}^{M} \int_0^{\infty} h_{ni}(\tau) r_{ij}(t_s+lT-\tau)\, d\tau$$

$$l = \ldots,-1,0,1,\ldots \tag{2.4}$$

where λ_{nkl} are Lagrange multipliers.

Applying the calculus of variations to (2.4) yields

$$h_{ni}(t) = \frac{1}{N_i} \sum_{k=1}^{L^M} \sum_{l} \lambda_{nkl} \sum_{j=1}^{M} a_{j0_k} \; r_{ij}(t_s+lT-t). \tag{2.5}$$

For the sake of simplicity we take

$$N_i \triangleq N_0 \qquad\qquad i = 1,\ldots,M. \tag{2.6}$$

This assumption and the assumptions that the noise processes $n_i(t)$ are white and uncorrelated are not a restriction of the generality, as is shown in Appendix 2.6.1. with

$$c_{njl} = \frac{1}{N_0} \sum_{k=1}^{L^M} a_{j0_k} \lambda_{nkl} \tag{2.7}$$

Equation (2.5) reduces to

$$h_{ni}(t) = \sum_{j=1}^{M} \sum_{l} c_{njl} r_{ij}(t_s+lT-t). \tag{2.8}$$

The structure of the receiving filter follows from this equation. Each component $h_{ni}(t)$ consists of a bank of matched filters, the outputs of which are added together. The output signals of the components $h_{ni}(t)$ with $i = 1,\dots,M$ are added, such forming the $n^{\underline{th}}$ output of the receiving filter. Assuming that t_s is greater than the longest duration of all $s_{nk}(t)$, then a simplification of the receiving filter is possible. Fig. 2.2 depicts the result for $M=3$. For ease of notation the time axis is shifted such that $t_s=0$. At each filter input i we see an array of filters matched to the particular responses at channel output i due to the individual excitation of the several channel inputs. Then all the outputs of the filters matched to the responses due to the same channel input are added to form the primed outputs $1'-2'-3'$. This part of the filter we call the multiple matched filter (MMF) (inputs $1-2-3$ and outputs $1'-2'-3'$). Each primed output is followed by a delay line with elements D giving a delay T. Each element of these delay lines is, with a weighting coefficient c_{njl}, connected with each of the M output adding circuits. This part of the filter we call the multiple tapped delay line (MTDL) (inputs $1'-2'-3'$ and outputs $1''-2''-3''$). The weighting coefficients c_{njl} have to be chosen so as to satisfy the optimization criterion. In the case of the minimum symbol error probability criterion it is impossible to find an analytical solution for the set $\{c_{njl}\}$. By means of a numerical optimization method an approximation can be found. A system satisfying the zero MDI criterion offers two

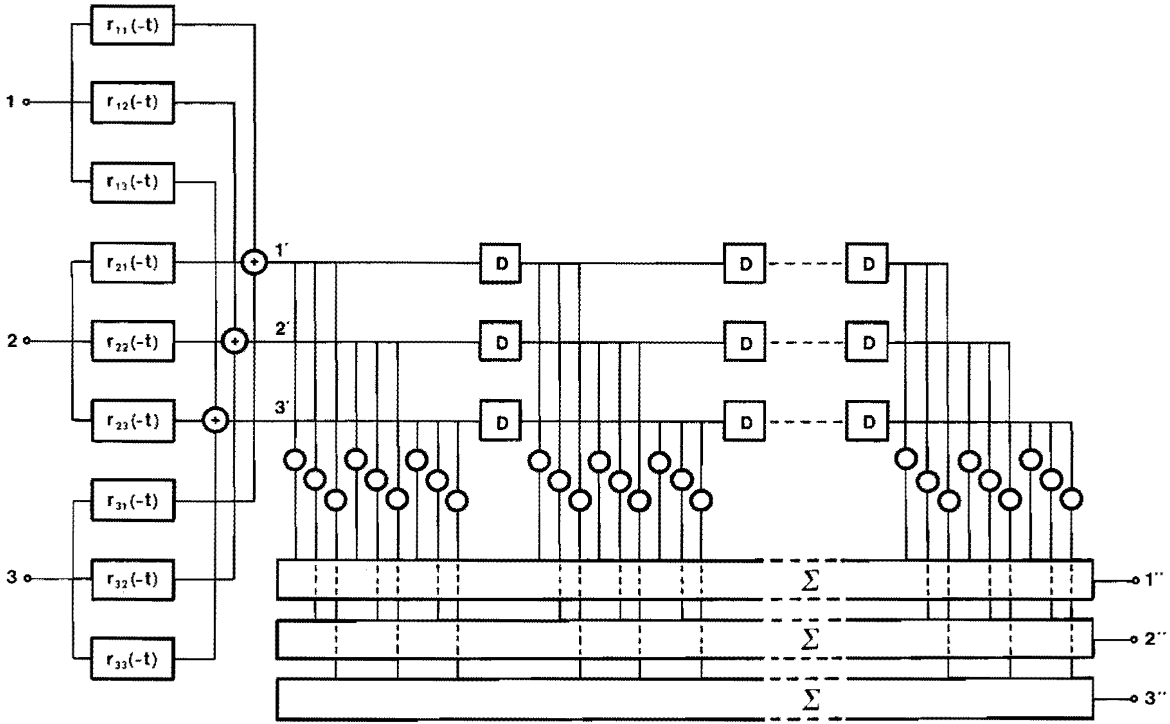

Fig. 2.2 Structure of the multiple linear receiving filter.

advantages. Firstly, the tap coefficients can be calculated rather easily, as will be shown in Section 2.4. Secondly, the practical realization is easily checked by means of the eye pattern.

2.2 The multidimensional Nyquist criterion

Denote the impulse responses of the cascade connection of the channel, the MMF and the MTDL, evaluated at the discrete instants lT by

$$F_l \triangleq \begin{bmatrix} f_{11}(lT) & f_{12}(lT) & . & . & . & f_{1M}(lT) \\ f_{21}(lT) & f_{22}(lT) & . & . & . & f_{2M}(lT) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ f_{M1}(lT) & f_{M2}(lT) & . & . & . & f_{MM}(lT) \end{bmatrix} \tag{2.9}$$

with $f_{nj}(t)$ the response at output n of this system as the result of a delta excitation at $t=0$ at input j.

Further we define the D-transform

$$F(D) \triangleq \sum_l F_l D^l \tag{2.10}$$

where D is the delay operator.

A measure for MDI is now defined as

$$I_n \triangleq \frac{\sum_l \sum_{j=1}^{M} \left| f_{nj}(lT) \right| - \left| f_{nn}(0) \right|}{\left| f_{nn}(0) \right|} \tag{2.11}$$

which is called the worst-case distortion at output n due to MDI. The overall worst-case MDI distortion is given by

$$I_0 \triangleq \max_n (I_n). \tag{2.12}$$

The terms "zero MDI" and "zero-forcing" are used here if $I_0 = 0$. By means of (2.10) we formulate a multidimensional Nyquist criterion. This criterion turns out to be similar to Shnidman's generalized Nyquist criterion [4].

THEOREM 2.1

The multiple channel transmission system described by (2.10) satisfies the multidimensional Nyquist criterion if

$$F(D) = I \tag{2.13}$$

where I is the $M \times M$ identity matrix.

It will be clear from the foregoing that for a system satisfying the multidimensional Nyquist criterion the MDI will be zero.

Now let us consider the channel in cascade with the MMF as a multiple channel system with M inputs and M outputs. The impulse response from input j to output m of this system is called $v_{mj}(t)$ and can be written as

$$v_{mj}(t) = \sum_{i=1}^{M} r_{ij}(t) * r_{im}(-t) \tag{2.14}$$

where * means convolution. Define

$$V_l \triangleq \begin{bmatrix} v_{11}(lT) & v_{12}(lT) & . & . & . & v_{1M}(lT) \\ v_{21}(lT) & v_{22}(lT) & . & . & . & v_{2M}(lT) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ v_{M1}(lT) & v_{M2}(lT) & . & . & . & v_{MM}(lT) \end{bmatrix} \tag{2.15}$$

and

$$V(D) \triangleq \sum_l V_l D^l. \tag{2.16}$$

The MTDL is also a multiple linear filter. For this system we define

$$C_l \triangleq \begin{bmatrix} c_{11l} & c_{12l} & . & . & . & c_{1Ml} \\ c_{21l} & c_{22l} & . & . & . & c_{2Ml} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ c_{M1l} & c_{M2l} & . & . & . & c_{MMl} \end{bmatrix} \tag{2.17}$$

and

$$C(D) \triangleq \sum_l C_l D^l. \tag{2.18}$$

With (2.8), (2.10), (2.16) and (2.18) it follows that

$$F(D) = C(D)\ V(D). \tag{2.19}$$

In Section 2.4 we shall give a procedure to calculate the tap coefficients described by $C(D)$.

2.3 The error probability of systems satisfying the multidimensional Nyquist criterion

If in a multiple channel transmission system it is possible to satisfy the multidimensional Nyquist criterion and the system has an optimum constraint receiver as described in the foregoing, the mean symbol error probability of channel n of such a system is denoted by

$$Pr(e_n) = 2\,\frac{L-1}{L}\,Q\left(\frac{d}{2\sigma_n}\right) \tag{2.20}$$

where the $Q(.)$-function as defined in [5, p. 82] is given by

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{\beta^2}{2}}\, d\beta \tag{2.21}$$

and d is the smallest difference between two output levels. As the smallest difference between two elements of the input alphabet is taken unity and because of (2.13), d equals one. The noise variance at output n is calculated from (2.1), (2.6) and (2.8) and by dropping the causality

$$\sigma_n^2 = \sum_m \sum_l \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{M} N_0 c_{njm}\, c_{nkl} \int_{-\infty}^{\infty} r_{ik}(lT-\tau) r_{ij}(mT-\tau)\, d\tau\,. \tag{2.22}$$

The impulse response from input j to output n, evaluated at the instant mT, can be written as

$$f_{nj}(mT) = \sum_{l} \sum_{i=1}^{M} \sum_{k=1}^{M} c_{nkl} \int_{-\infty}^{\infty} r_{ik}(lT-\tau) r_{ij}(mT-\tau)\, d\tau . \tag{2.23}$$

From (2.13) and [4] it follows that for systems satisfying the multi-dimensional Nyquist criterion

$$f_{nj}(mT) = \delta_m \, \delta_{nj} \tag{2.24}$$

where

$$\delta_m \triangleq \begin{cases} 0 & m \neq 0 \\ 1 & m = 0 \end{cases}$$
$$\delta_{nj} \triangleq \begin{cases} 0 & n \neq j \\ 1 & n = j . \end{cases} \tag{2.25}$$

Substituting (2.23), (2.24) and (2.25) reduces Equation (2.22) to the simple form

$$\sigma_n^2 = N_0 c_{nno} \tag{2.26}$$

which, if substituted in (2.20) gives for the symbol error probability of channel n

$$Pr(e_n) = 2 \frac{L-1}{L} \; Q\left(\frac{1}{2\sqrt{N_0 c_{nno}}}\right). \tag{2.27}$$

2.4 The optimum finite length multiple tapped delay line

The index l of the $C(D)$ sequence runs from minus infinity to plus infinity and in consequence the MTDL becomes infinitely long. In practice it has to be of finite length and in this case (2.13) cannot be satisfied exactly. If the MTDL is of length $2N+1$ the question arises how the tap settings, given by the matrices $C_{-N},\ldots,C_N$ have to be chosen to minimize the worst-case MDI distortion as given in (2.11). The following method is closely related to that in [6, Section 6.1.1]. From (2.19) it follows that

$$F_l = \sum_{j=-N}^{N} C_j V_{l-j}\,. \tag{2.28}$$

It is assumed that

$$V_0 = I \tag{2.29}$$

and

$$f_{nn}(0) = 1 \qquad\qquad n=1,\ldots,M. \tag{2.30}$$

If $V_0 \neq I$ it can be made equal to the identity matrix by following the MMF by a multiple channel system with transfer V_0^{-1}. This presupposes the existence of V_0^{-1}. However, for most practical systems the matrix V_0^{-1} exists or can be made to exist. From (2.28) it follows that

$$f_{ni}(lT) = \sum_{j=-N}^{N} \sum_{k=1}^{M} c_{nkj}\, \Theta_{kijl} \tag{2.31}$$

where Θ_{kijl} is the $k,i^{\underline{th}}$ component of V_{l-j}. The assumptions (2.29) and (2.30) lead to

$$c_{nn0} = 1 - \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j \ \delta_{nk})\ c_{nkj}\ \Theta_{knj0}\ . \tag{2.32}$$

By means of this Equation (2.31) is rewritten as

$$f_{ni}(lT) = \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j \ \delta_{nk})\ c_{nkj}\ (\Theta_{kijl}-\Theta_{knj0}\Theta_{ni0l})+\Theta_{ni0l}\ . \tag{2.33}$$

According to (2.11) and (2.33) the worst-case MDI distortion at output n becomes

$$I_n = \sum_{l} \sum_{i=1}^{M} (1-\delta_l \ \delta_{ni})\ \left| f_{nk}(lT)\right| =$$

$$= \sum_{l} \sum_{i=1}^{M} (1-\delta_l \ \delta_{ni})\ \left| \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})c_{nkj}(\Theta_{kijl}-\Theta_{knj0}\Theta_{ni0l})+\right.$$

$$\left. + \Theta_{ni0l}\ \right| =$$

$$= \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})c_{nkj}\ \ [\sum_{l} \sum_{i=1}^{M} (1-\delta_l\delta_{ni})(\Theta_{kijl}-\Theta_{knj0}\Theta_{ni0l})\ .$$

$$.\ sgn\ \{\ f_{ni}(lT)\}] + [\sum_{l} \sum_{i=1}^{M} (1-\delta_l\delta_{ni})\Theta_{ni0l}\ sgn\ \{f_{ni}(lT)\}] \tag{2.34}$$

where

$$sgn\ \{f_{ni}(lT)\} \triangleq \begin{cases} +1 & f_{ni}(lT) \geq 0 \\ -1 & f_{ni}(lT) < 0 \end{cases} \tag{2.35}$$

The function given by (2.34) is well defined, because $r_{ij}(t)$ is square integrable and of finite duration. Observe from (2.34) that I_n is a continuous, piecewise-linear function of the tap settings $\{c_{nkj}\}$. In this equation the coefficients of the c_{nkj} are constant over certain regions of the $\{(2N+1)M-1\}$-dimensional space of definition for $\{c_{nkj}\}$. At the breakpoints the coefficients get new values because at least one of the output sample values $f_{ni}(lT)$ changes its sign. I_n cannot achieve its minimum between breakpoints where the function is linear; thus at least one value $f_{ni}(lT)$ must be zero at the minimum. This requirement can be used to eliminate one of the variables c_{nkj}. The reduced equation is of the same piecewise-linear form, requiring at least one more output sample value $f_{ni}(lT) = 0$. Continuing this line of reasoning we arrive at the conclusion that at least $(2N+1)M-1$ output samples $f_{ni}(lT)$ must be zero at the minimum. Those $(2N+1)M-1$ equations together with (2.32) are sufficient to determine the tap settings $\{c_{nkj}\}$. The question remains which set of $(2N+1)M-1$ output samples has to be taken to achieve minimum worst-case MDI distortion at output n. Linear programming techniques can be used for solving this problem. Discussion of these techniques is outside the scope of this thesis.

In situations where all V_l are circulant matrices [7], all worst-case MDI distortions at the outputs of the MMF will be equal to each other. From symmetry considerations it follows that the C_l and thus all V_l matrices must also be circulant matrices in those cases. Thus all worst-case MDI distortions at the outputs of the receiving filter have the same value. Now the worst-case MDI distortion at the outputs of the MMF is represented by $\Sigma_l' ||V_l||_\infty$, where (see [8, Chapter 1])

$$||V_l||_\infty \triangleq \max_i \{\sum_j |v_{ijl}|\} \tag{2.36}$$

and v_{ijl} is the $i,j^{\underline{th}}$ component of V_l. The worst-case MDI distortion at the outputs of the receiving filter is represented by

$$I_n = \sum_l{}' ||F_l||_\infty + ||F_0 - I||_\infty \quad n = 1,\ldots,M. \tag{2.37}$$

In the situations described above linear programming can often be avoided, thanks to the following theorem.

<u>THEOREM 2.2</u>

Assume that:

1/ $V_0 = I$

2/ $\Sigma_l{}'||V_l||_\infty$ represents the worst-case MDI distortion at the output of the MMF

3/ $\Sigma_l{}'||V_l||_\infty < 1$

4/ $\Sigma_l{}'||F_l||_\infty + ||F_0 - I||_\infty$ represents the worst-case MDI distortion at the output of the receiving filter.

Then the worst-case MDI distortion at the output of the receiving filter is minimal for those tap settings which cause $F_0 = I$ and $F_l = 0$, $|l| \leq N$, $l \neq 0$.

This theorem, the proof of which is given in Appendix 2.6.2, is a generalization of a theorem derived by Lucky for ISI [6, p.138]. The condition $\Sigma_l{}'||V_l||_\infty < 1$ means that in the binary case $(a_{jl} \epsilon \{0,1\})$ the eye at the MMF outputs is not closed.

The tap settings as follow from Theorem 2.2 are calculated in the following manner. Define the composite matrices

$$C \triangleq \begin{bmatrix} C_{-N} \\ C_{-N+1} \\ \cdot \\ \cdot \\ \cdot \\ C_N \end{bmatrix} , \tag{2.38}$$

$$C_T \triangleq \begin{bmatrix} C_{-N}^T \\ C_{-N+1}^T \\ \cdot \\ \cdot \\ \cdot \\ C_N^T \end{bmatrix} , \tag{2.39}$$

$$V \triangleq \begin{bmatrix} V_0 & V_1 & \cdot & \cdot & \cdot & V_{2N} \\ V_{-1} & V_0 & \cdot & \cdot & \cdot & V_{2N-1} \\ V_{-2} & V_{-1} & V_0 & \cdot & \cdot & V_{2N-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ V_{-2N} & V_{-2N+1} & \cdot & \cdot & \cdot & V_0 \end{bmatrix} \tag{2.40}$$

and

$$U \triangleq \begin{bmatrix} O \\ \cdot \\ \cdot \\ \cdot \\ O \\ I \\ O \\ \cdot \\ \cdot \\ \cdot \\ O \end{bmatrix} \tag{2.41}$$

where O is the $M{\times}M$ all-zero matrix. To satisfy Theorem 2.2 we have the equation

$$C_T{}^T V = U^T \quad . \tag{2.42}$$

This equation is further simplified by means of (2.14), (2.15) and (2.40); it is obvious that

$$V^T = V \tag{2.43}$$

so that

$$VC_T = U. \tag{2.44}$$

An important property of the worst-case MDI at output n as a function of the tap settings $\{c_{nkj}\}$, is given by

THEOREM 2.3

The worst-case MDI distortion I_n given by Equation (2.34), is a convex function of the $(2N+1)M-1$ variables c_{nkj}, $k=1,\ldots,M$, $|j|\leq N$, $k\neq n\wedge j=0$.

For the proof of this theorem two arbitrary tap settings of the MTDL are denoted by the $\{(2N+1)M-1\}$ component vectors $\underline{\alpha}$ and $\underline{\beta}$. The convexity of I_n is proved if for any two settings $\underline{\alpha}$ and $\underline{\beta}$ and for all allowable λ

$$I_n\ [\lambda\underline{\alpha}+(1-\lambda)\underline{\beta}]\ \leq \lambda I_n\ [\underline{\alpha}] + (1-\lambda)I_n\ [\underline{\beta}] \qquad 0\leq\lambda\leq 1. \tag{2.45}$$

From (2.34) it follows

$$I_n\ [\lambda\underline{\alpha}+(1-\lambda)\underline{\beta}] = \sum_l \sum_{i=1}^{M} (1-\delta_l\delta_{ni})\Big| \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})\lambda\alpha_{nkj} \cdot$$

$$\cdot(\theta_{kijl}-\theta_{knj0}\theta_{ni0l}) + \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})(1-\lambda)\beta_{nkj} \cdot$$

$$\cdot(\theta_{kijl}-\theta_{knj0}\theta_{ni0l}) + \theta_{ni0l}\ \Big| =$$

$$= \sum_l \sum_{i=1}^{M} (1-\delta_l\delta_{ni})\Big|\lambda\{ \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})\alpha_{nkj}(\theta_{kijl}-\theta_{knj0}\theta_{ni0l}) +$$

$$+\theta_{ni0l}\} +$$

$$+ (1-\lambda)\{ \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})\beta_{nkj}(\theta_{kijl}-\theta_{knj0}\theta_{ni0l}) + \theta_{ni0l}\}\Big|$$

$$\leq \lambda \sum_l \sum_{i=1}^{M} (1-\delta_l\delta_{ni})\Big| \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j\delta_{nk})\alpha_{nkj}(\theta_{kijl}-\theta_{knj0}\theta_{ni0l}) + \theta_{ni0l}\Big|+$$

$$+ (1-\lambda) \sum_{l} \sum_{i=1}^{M} (1-\delta_l \delta_{ni}) \left| \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_j \delta_{nk}) \beta_{nkj} (\theta_{kijl} - \theta_{knj0}\theta_{ni0l}) + \right.$$

$$\left. + \theta_{ni0l} \right| \tag{2.46}$$

and

$$I_n[\lambda \underline{\alpha} + (1-\lambda)\underline{\beta}] \leq \lambda I_n[\underline{\alpha}] + (1-\lambda) I_n[\underline{\beta}]. \tag{2.47}$$

The most important property of convex functions is in our case the fact that they possess no local minima other than their absolute minimum. Thus any minimum of I_n found by whatsoever method must be the absolute minimum of the worst-case MDI distortion at output n.

In systems where the noise does not play an important role the MMF can be omitted and the correction of the MDI distortion can be applied directly to the channel response. For this situation we define

$$R_l \triangleq \begin{bmatrix} r_{11}(lT) & r_{12}(lT) & . & . & . & r_{1M}(lT) \\ r_{21}(lT) & r_{22}(lT) & . & . & . & r_{2M}(lT) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ r_{M1}(lT) & r_{M2}(lT) & . & . & . & r_{MM}(lT) \end{bmatrix} \tag{2.48}$$

and

$$R(D) \triangleq \sum_{l} R_l D^l. \tag{2.49}$$

The overall transmission is then given by

$$F(D) = C(D)\, R(D). \tag{2.50}$$

It is obvious that Theorem 2.2 is also valid with V_l replaced by R_l. And with

$$R \triangleq \begin{bmatrix} R_0 & R_1 & . & . & . & R_{2N} \\ R_{-1} & R_0 & . & . & . & R_{2N-1} \\ R_{-2} & R_{-1} & R_0 & . & . & R_{2N-2} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ R_{-2N} & R_{-2N+1} & . & . & . & R_0 \end{bmatrix} \tag{2.51}$$

the correction in accordance to Theorem 2.2 can be calculated from the equation

$$C_T^{\,T} R = U^T \tag{2.52}$$

or equivalent

$$R^T\, C_T = U. \tag{2.53}$$

Sometimes it is possible to choose the sampling instant such that $R_l=0$ for $l<0$. The matrix sequence $C(D)$ starts now with $l=0$ too, giving a simplification of the algorithm for calculating the C_l matrices. Applying Theorem 2.2 the tap coefficients are determined by the recurrence relation

$$C_0 = R_0^{-1}$$

$$C_l = -R_0^{-1} \sum_{i=0}^{l-1} R_{l-i} C_i \qquad l \geq 1. \tag{2.54}$$

If the noise is negligible and the MMF is omitted, the MDI correction circuit can also be inserted at the transmitting end, allowing a realization of the MTDL in the form of M shift registers with resistors. As a result the overall transmission now becomes

$$F(D) = R(D)C(D). \tag{2.55}$$

Consider again a finite length MTDL with $C_{-N},\ldots,C_N$. Then

$$F_l = \sum_{j=-N}^{N} R_{l-j} C_j. \tag{2.56}$$

From this equation it follows that

$$f_{ni}(lT) = \sum_{j=-N}^{N} \sum_{k=1}^{M} \rho_{nkjl} c_{kij} \tag{2.57}$$

with ρ_{nkjl} the $n,k^{\underline{th}}$ component of R_{l-j}. At the minimization of one of the I_n of (2.34) only $(2N+1)M-1$ of the weighting coefficients were determined. Minimizing (2.11) by substituting (2.57), however, determines $(2N+1)M^2-1$ elements of the set $\{c_{nkj}\}$.

For this reason (2.30) is not valid now. There is only one degree of freedom and we take

$$f_{11}(0) = 1. \tag{2.58}$$

Assumption (2.29) is still valid, so that

$$c_{110} = 1 - \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_{k1}\delta_j)\rho_{1kj0}c_{k1j}. \tag{2.59}$$

Substituting (2.57) and (2.59) in (2.11) yields

$$\begin{aligned} I_n &= \frac{1}{|f_{nn}(o)|}\Big[\sum_{l} \sum_{i=1}^{M} \Big| \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_{k1}\delta_{i1}\delta_j)c_{kij}\rho_{nkjl}+c_{110}\rho_{n10l}\Big|\Big]-1= \\ &= \frac{1}{|f_{nn}(0)|}\Big[\sum_{l} \sum_{i=1}^{M} \Big| \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_{k1}\delta_{i1}\delta_j)c_{kij}\rho_{nkjl}+\rho_{n10l}+ \\ &\quad - \rho_{n10l} \sum_{j=-N}^{N} \sum_{k=1}^{M} (1-\delta_{k1}\delta_j)\rho_{1kj0}c_{k1j}\Big|\Big]-1. \end{aligned} \tag{2.60}$$

If the I_n of (2.60) is minimized for one value of n all c_{nkj} are determined, thus leaving no control over the remaining I_n. It makes sense in this situation to minimize I_0 (see (2.12)). However, this minimization problem cannot easily be solved by means of a linear programming technique. Other computer minimization methods must be looked for. It is easy to show that Theorem 2.2 is now also valid with V_l replaced by R_l and F_l given by (2.56).

With

$$R_T \triangleq \begin{bmatrix} R_0^{\ T} & R_1^{\ T} & \cdot & \cdot & \cdot & R_{2N}^{\ T} \\ R_{-1}^{\ T} & R_0^{\ T} & \cdot & \cdot & \cdot & R_{2N-1}^{\ T} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{-2N}^{\ T} & R_{-2N+1}^{\ T} & \cdot & \cdot & \cdot & R_0^{\ T} \end{bmatrix} \tag{2.61}$$

the solution for $C(D)$ is given by

$$R_T^{\ T} C = U. \tag{2.62}$$

If it is possible to choose the sampling instant such that $R_l=0$ for $l<0$, the solution for $C(D)$ is as given in (2.54).

2.5 Examples

Example 2.5.1

As a first example we implemented the transmission of binary data over a multiwire cable, consisting of four identical wires which are symmetrically situated inside a cylindrical shield (see Fig. 2.3). Each wire was used as a transmission channel with the cylindrical shield

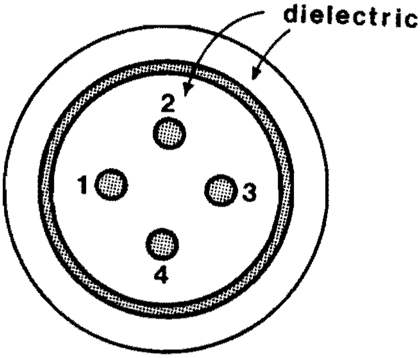

Fig. 2.3 Cross section of the 4-wire cable

as common return. The cable has a length of 1 km and the bit rate is taken 5 Mbit/s for each channel. In this example the length of the cable, the bit rate and the transmitted signals are such that the noise can be neglected. We have measured the following matrices

$$R_0 = \begin{bmatrix} 1 & 0.24 & 0.13 & 0.24 \\ 0.24 & 1 & 0.24 & 0.13 \\ 0.13 & 0.24 & 1 & 0.24 \\ 0.24 & 0.13 & 0.24 & 1 \end{bmatrix}$$

$$R_1 = 0.26\ I$$

$$R_2 = 0.11\ I$$

$$R_3 = 0.07\ I$$

$$R_4 = 0.04\ I. \qquad (2.63)$$

It can be verified that $\Sigma_l{}'||R_l R_0^{-1}||<1$ and since the R_l are circulant matrices, Theorem 2.2 can be applied. The calculated C_l matrices according to (2.54) are

$$C_0 = \begin{bmatrix} 1 & -0.21 & -0.03 & -0.21 \\ -0.21 & 1 & -0.21 & -0.03 \\ -0.03 & -0.21 & 1 & -0.21 \\ -0.21 & -0.03 & -0.21 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.31 & 0.12 & -0.01 & 0.12 \\ 0.12 & -0.31 & 0.12 & -0.01 \\ -0.01 & 0.12 & -0.31 & 0.12 \\ 0.12 & -0.01 & 0.12 & -0.31 \end{bmatrix} . \qquad (2.64)$$

Because all R_l and C_l are circulant matrices, $\Sigma_l{}'||R_l R_0^{-1}||$ represents the worst-case MDI at the channel output. Moreover, the filter output matrices F_l are also circulant matrices [7] and thus $\Sigma_l{}'||F_l||$ represents the worst-case MDI at the filter output, which shows that the use of Theorem 2.2 was justified. In the realization of the MTDL tap coefficients equal to or smaller than 0.03 are omitted because these values do not give a substantial improvement of the eye opening. All elements of C_2, C_3, etc. are smaller than 0.03, hence, they are not given in (2.64).The MTDL is implemented with four shift registers at the transmitting end which are connected to the cable by means of resistors. Fig. 2.4 shows the eye pattern at the receiving end when all wires are excited. The fact that this eye is closed can be verified from (2.63). Fig. 2.5 shows the eye pattern of the system characterized by $R(D)R_0^{-1}$ which means that a multiple channel system with transfer R_0^{-1} is placed between the transmitter and the transmissing end of the cable. The eye pattern of this system is not closed, hence, $\Sigma_l'||R_lR_o^{-1}||<1$, which is also verified from (2.63) and (2.64). Finally, Fig. 2.6 shows the eye pattern of the equalized system that is quite satisfactory.

Example 2.5.2

In this example the cable of the previous example is excited in its modes [9] at a bit rate of 50 Mbit/s for each mode. Owing to imperfections in the structure of the cable, the ICI is rather severe at the given bit rate. So MDI correction will be necessary. For the several modes the ratios of the wire voltages are as given in Table 2.1.

	mode nr.			
	1	*2*	*3*	*4*
wire nr.				
1	*1*	*1*	*0*	*1*
2	*1*	*0*	*1*	*-1*
3	*1*	*-1*	*0*	*1*
4	*1*	*0*	*-1*	*-1*

Table 2.1

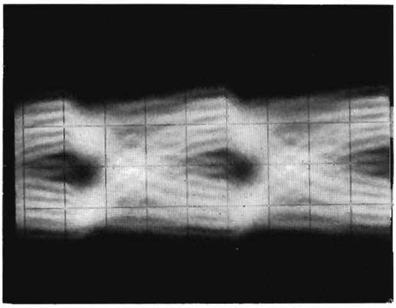

Fig. 2.4 The eye pattern of the unequalized system of Example 2.5.1.

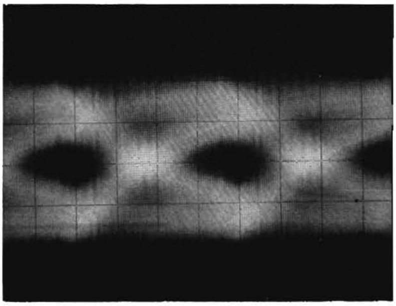

Fig. 2.5 The eye pattern of the system $R(D)R_0^{-1}$ of Example 2.5.1.

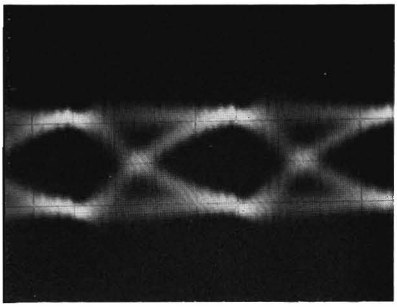

Fig. 2.6 The eye pattern of the equalized system of Example 2.5.1.

At an appropriate value of the sampling instant the following matrices were measured:

$$R_{-1} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 \\ 7.000 & 0.500 & -0.150 & 0.400 \\ 0.500 & -0.350 & 0.400 & -0.650 \\ 0.000 & 0.450 & -0.250 & 0.550 \end{bmatrix}$$

$$R_{0} = \begin{bmatrix} 29.125 & 1.550 & -0.100 & 1.800 \\ -5.625 & 15.250 & -0.200 & 1.250 \\ -5.000 & 0.200 & 16.000 & 1.300 \\ -2.000 & 0.200 & 0.350 & 7.400 \end{bmatrix}$$

$$R_{1} = \begin{bmatrix} 7.875 & -1.500 & -0.700 & -1.750 \\ -3.000 & 6.850 & 0.250 & -1.200 \\ -1.875 & 0.200 & 6.000 & 2.150 \\ -0.750 & -0.550 & 0.950 & 4.050 \end{bmatrix}$$

$$R_{2} = \begin{bmatrix} -5.875 & -0.800 & -1.000 & -1.100 \\ -0.375 & -0.900 & 0.000 & -0.350 \\ 0.375 & 0.050 & -1.000 & 0.200 \\ 0.000 & -0.150 & 0.150 & 0.500 \end{bmatrix}$$

$$R_{3} = \begin{bmatrix} -4.750 & 0.100 & 0.000 & -0.100 \\ 0.000 & -1.450 & 0.000 & -0.050 \\ 0.500 & 0.000 & -1.500 & -0.150 \\ 0.000 & -0.050 & 0.000 & -0.200 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} -2.750 & 0.100 & 0.100 & 0.050 \\ 0.000 & -1.150 & 0.000 & 0.000 \\ 0.250 & 0.000 & -1.100 & -0.150 \\ 0.000 & 0.000 & 0.000 & -0.300 \end{bmatrix}$$

$$R_5 = \begin{bmatrix} -1.625 & 0.000 & 0.100 & 0.000 \\ 0.000 & -0.800 & 0.000 & 0.000 \\ 0.000 & 0.000 & -0.750 & -0.100 \\ 0.000 & 0.000 & 0.000 & -0.200 \end{bmatrix}$$

$$R_6 = \begin{bmatrix} -0.875 & 0.000 & 0.000 & 0.000 \\ 0.000 & -0.450 & 0.000 & 0.000 \\ 0.000 & 0.000 & -0.350 & 0.000 \\ 0.000 & 0.000 & 0.000 & -0.100 \end{bmatrix}. \tag{2.65}$$

Because these matrices do not satisfy the constraints of Theorem 2.2, the latter cannot be applied to achieve an optimum MTDL. For correction at the receiving end a linear programming procedure was used to calculate the optimum tap settings. The result is

$$C_{-1} = \begin{bmatrix} 0.00092 & 0.00027 & -0.00018 & 0.00062 \\ -0.01479 & 0.00009 & 0.00009 & 0.00187 \\ 0.00007 & 0.00200 & -0.00202 & 0.00707 \\ -0.00117 & -0.00411 & 0.00252 & -0.01051 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 0.03167 & -0.00362 & 0.00060 & -0.00829 \\ 0.02117 & 0.06215 & 0.00087 & -0.02141 \\ 0.00943 & -0.00268 & 0.06378 & -0.01792 \\ 0.00765 & 0.00069 & -0.00441 & 0.14299 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.00635 & 0.00519 & 0.00139 & 0.01231 \\ -0.00989 & -0.02664 & -0.00134 & 0.03219 \\ -0.00282 & 0.00162 & -0.02278 & -0.00206 \\ -0.00272 & 0.00579 & -0.00420 & -0.07633 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.00832 & -0.00203 & 0.00063 & -0.00470 \\ 0.00597 & 0.01625 & 0.00104 & -0.02166 \\ 0.00276 & -0.00104 & 0.01290 & 0.00575 \\ 0.00197 & -0.00463 & 0.00410 & 0.03472 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0.00181 & 0.00089 & 0.00000 & 0.00214 \\ -0.00174 & -0.00341 & -0.00030 & 0.01235 \\ 0.00006 & 0.00065 & -0.00073 & -0.00465 \\ 0.00051 & 0.00307 & -0.00297 & -0.01245 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0.00325 & -0.00012 & 0.00053 & -0.00030 \\ 0.00183 & 0.00483 & 0.00017 & -0.00655 \\ 0.00110 & -0.00046 & 0.00346 & 0.00203 \\ 0.00050 & -0.00162 & 0.00102 & 0.00924 \end{bmatrix}$$

$$C_5 = \begin{bmatrix} 0.00212 & 0.00029 & 0.00007 & 0.00070 \\ 0.00015 & 0.00028 & -0.00003 & 0.00390 \\ 0.00082 & 0.00025 & 0.00119 & -0.00152 \\ 0.00055 & 0.00116 & -0.00089 & -0.00316 \end{bmatrix}$$

$$C_6 = \begin{bmatrix} 0.00189 & 0.00009 & 0.00037 & 0.00017 \\ 0.00108 & 0.00160 & 0.00005 & -0.00170 \\ 0.00047 & -0.00017 & 0.00108 & 0.00008 \\ 0.00038 & -0.00041 & 0.00019 & 0.00252 \end{bmatrix}, \qquad (2.66)$$

giving rise to the following values of the worst-case MDI distortions

$$I_1 = 0.13072$$
$$I_2 = 0.07953$$
$$I_3 = 0.08422$$
$$I_4 = 0.10051. \qquad (2.67)$$

The linear programming procedure is rather complicated as compared to the calculation of the tap coefficients that yield $F_0 = I$ and $F_l = 0,\ l = -1,1,\dots,6$. This latter method gives the following tap settings

$$C_{-1} = \begin{bmatrix} 0.00092 & 0.00027 & -0.00018 & 0.00062 \\ -0.01479 & 0.00009 & 0.00009 & 0.00187 \\ 0.00007 & 0.00200 & -0.00202 & 0.00707 \\ -0.00117 & -0.00411 & 0.00252 & -0.01051 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 0.03167 & -0.00362 & 0.00060 & -0.00829 \\ 0.02117 & 0.06215 & 0.00086 & -0.02140 \\ 0.00943 & -0.00268 & 0.06378 & -0.01792 \\ 0.00765 & 0.00069 & -0.00441 & 0.14299 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.00635 & 0.00519 & 0.00139 & 0.01231 \\ -0.00989 & -0.02664 & -0.00134 & 0.03219 \\ -0.00282 & 0.00162 & -0.02278 & -0.00206 \\ -0.00272 & 0.00579 & -0.00420 & -0.07633 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.00832 & -0.00203 & 0.00063 & -0.00470 \\ 0.00597 & 0.01625 & 0.00104 & -0.02166 \\ 0.00276 & -0.00104 & 0.01290 & 0.00575 \\ 0.00197 & -0.00463 & 0.00410 & 0.03472 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0.00181 & 0.00089 & 0.00000 & 0.00214 \\ -0.00174 & -0.00341 & -0.00030 & 0.01235 \\ 0.00006 & 0.00065 & -0.00073 & -0.00465 \\ 0.00051 & 0.00307 & -0.00297 & -0.01245 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0.00325 & -0.00012 & 0.00053 & -0.00030 \\ 0.00183 & 0.00483 & 0.00017 & -0.00655 \\ 0.00110 & -0.00046 & 0.00346 & 0.00203 \\ 0.00050 & -0.00162 & 0.00102 & 0.00924 \end{bmatrix}$$

$$C_5 = \begin{bmatrix} 0.00212 & 0.00029 & 0.00007 & 0.00070 \\ 0.00015 & 0.00028 & -0.00003 & 0.00390 \\ 0.00082 & 0.00025 & 0.00119 & -0.00152 \\ 0.00055 & 0.00116 & -0.00089 & -0.00316 \end{bmatrix}$$

$$C_6 = \begin{bmatrix} 0.00189 & 0.00009 & 0.00037 & 0.00017 \\ 0.00108 & 0.00160 & 0.00005 & -0.00170 \\ 0.00047 & -0.00017 & 0.00108 & 0.00008 \\ 0.00038 & -0.00041 & 0.00019 & 0.00252 \end{bmatrix}, \qquad (2.68)$$

and MDI distortions

$$\begin{aligned} I_1 &= 0.13072 \\ I_2 &= 0.08008 \\ I_3 &= 0.08422 \\ I_4 &= 0.10051. \end{aligned} \qquad (2.69)$$

Note that only I_2 differs from that of (2.67). In correspondence with this fact only the second row of the C_l matrices differs at a few places with that of (2.66). The conditions of Theorem 2.2 are sufficient but not necessary. In many practical cases where these conditions are not satisfied, the tap settings that yield $F_0 = I$ and $F_l = 0$, $l = -N,\ldots,-1,1,\ldots,N$ will nevertheless give an optimum or satisfactoring solution, as is demonstrated in this example.

For correction at the transmitting end a procedure was used to minimize I_0 (see 2.12). The results are

$$C_{-1} = \begin{bmatrix} 0.00085 & 0.00024 & -0.00016 & 0.00056 \\ -0.01496 & 0.00001 & 0.00014 & 0.00169 \\ 0.00008 & 0.00196 & -0.00202 & 0.00701 \\ -0.00092 & -0.00394 & 0.00247 & -0.01014 \end{bmatrix}$$

$$C_{0} = \begin{bmatrix} 0.03170 & -0.00361 & 0.00059 & -0.00826 \\ 0.02129 & 0.06220 & 0.00082 & -0.02127 \\ 0.00933 & -0.00268 & 0.06375 & -0.01786 \\ 0.00772 & 0.00064 & -0.00434 & 0.14280 \end{bmatrix}$$

$$C_{1} = \begin{bmatrix} -0.00637 & 0.00518 & 0.00140 & 0.01230 \\ -0.00997 & -0.02667 & -0.00132 & 0.03210 \\ -0.00281 & 0.00162 & -0.02277 & -0.00206 \\ -0.00261 & 0.00584 & -0.00424 & -0.07621 \end{bmatrix}$$

$$C_{2} = \begin{bmatrix} 0.00832 & -0.00203 & 0.00063 & -0.00469 \\ 0.00601 & 0.01626 & 0.00103 & -0.02162 \\ 0.00272 & -0.00105 & 0.01289 & 0.00576 \\ 0.00202 & -0.00463 & 0.00412 & 0.03469 \end{bmatrix}$$

$$C_{3} = \begin{bmatrix} 0.00180 & 0.00088 & 0.00000 & 0.00214 \\ -0.00177 & -0.00342 & -0.00029 & 0.01232 \\ 0.00005 & 0.00065 & -0.00073 & -0.00465 \\ 0.00059 & 0.00309 & -0.00298 & -0.01241 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0.00325 & -0.00012 & 0.00053 & -0.00030 \\ 0.00184 & 0.00483 & 0.00017 & -0.00654 \\ 0.00107 & -0.00046 & 0.00346 & 0.00203 \\ 0.00055 & -0.00162 & 0.00104 & 0.00924 \end{bmatrix}$$

$$C_5 = \begin{bmatrix} 0.00212 & 0.00029 & 0.00007 & 0.00066 \\ 0.00015 & 0.00028 & -0.00003 & 0.00390 \\ 0.00061 & 0.00020 & 0.00119 & -0.00174 \\ 0.00055 & 0.00116 & -0.00088 & -0.00288 \end{bmatrix}$$

$$C_6 = \begin{bmatrix} 0.00201 & 0.00016 & 0.00042 & 0.00038 \\ 0.00000 & 0.00117 & 0.00095 & -0.00152 \\ -0.00007 & -0.00026 & -0.00057 & 0.00014 \\ -0.00203 & -0.00181 & -0.00126 & -0.00133 \end{bmatrix} , \qquad (2.70)$$

and

$$I_1 = 0.13177$$
$$I_2 = 0.13177$$
$$I_3 = 0.13177$$
$$I_4 = 0.13177 . \qquad (2.71)$$

As a starting point for the above procedure we used the solution found by taking $F_0 = I$ and $F_l = 0,\ l = -1,1,\ldots,6$

$$C_{-1} = \begin{bmatrix} 0.00085 & 0.00024 & -0.00016 & 0.00056 \\ -0.01496 & 0.00001 & 0.00014 & 0.00169 \\ 0.00008 & 0.00196 & -0.00202 & 0.00701 \\ -0.00092 & -0.00394 & 0.00247 & -0.01015 \end{bmatrix}$$

$$C_{0} = \begin{bmatrix} 0.03170 & -0.00361 & 0.00059 & -0.00826 \\ 0.02129 & 0.06220 & 0.00082 & -0.02127 \\ 0.00933 & -0.00268 & 0.06375 & -0.01786 \\ 0.00772 & 0.00064 & -0.00434 & 0.14280 \end{bmatrix}$$

$$C_{1} = \begin{bmatrix} -0.00637 & 0.00518 & 0.00140 & 0.01230 \\ -0.00997 & -0.02667 & -0.00132 & 0.03210 \\ -0.00281 & 0.00162 & -0.02277 & -0.00206 \\ -0.00261 & 0.00584 & -0.00424 & -0.07621 \end{bmatrix}$$

$$C_{2} = \begin{bmatrix} 0.00832 & -0.00203 & 0.00063 & -0.00469 \\ 0.00601 & 0.01626 & 0.00103 & -0.02162 \\ 0.00272 & -0.00105 & 0.01289 & 0.00576 \\ 0.00202 & -0.00463 & 0.00412 & 0.03469 \end{bmatrix}$$

$$C_{3} = \begin{bmatrix} 0.00180 & 0.00088 & 0.00000 & 0.00214 \\ -0.00177 & -0.00342 & -0.00029 & 0.01232 \\ 0.00005 & 0.00065 & -0.00073 & -0.00465 \\ 0.00059 & 0.00309 & -0.00298 & -0.01241 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0.00325 & -0.00012 & 0.00053 & -0.00030 \\ 0.00184 & 0.00483 & 0.00017 & -0.00654 \\ 0.00107 & -0.00046 & 0.00346 & 0.00203 \\ 0.00055 & -0.00162 & 0.00104 & 0.00924 \end{bmatrix}$$

$$C_5 = \begin{bmatrix} 0.00212 & 0.00029 & 0.00007 & 0.00070 \\ 0.00015 & 0.00028 & -0.00003 & 0.00390 \\ 0.00082 & 0.00025 & 0.00119 & -0.00152 \\ 0.00055 & 0.00116 & -0.00089 & -0.00316 \end{bmatrix}$$

$$C_6 = \begin{bmatrix} 0.00180 & 0.00008 & 0.00038 & 0.00014 \\ 0.00161 & 0.00166 & 0.00012 & -0.00161 \\ 0.00044 & -0.00017 & 0.00109 & 0.00013 \\ 0.00033 & -0.00037 & 0.00016 & 0.00259 \end{bmatrix} \qquad (2.72)$$

with

$$I_1 = 0.15809$$
$$I_2 = 0.07889$$
$$I_3 = 0.05600$$
$$I_4 = 0.04133 \ . \qquad (2.73)$$

This last named solution was implemented using four shift registers with resistor matrices. The eye patterns at the outputs of this implementation are given in Figs. 2.7, 2.8, 2.9 and 2.10. Althouth these eye patterns are not as good as those of Example 2.5.1 Fig. 2.6, they were found to be good enough for perfect reconstruction of the four

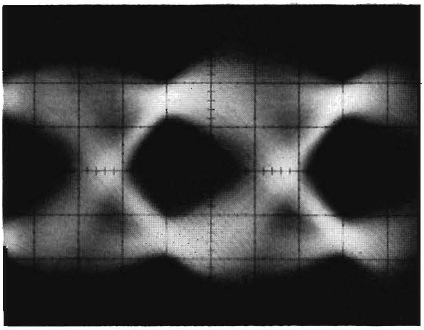

Fig. 2.7 Eye pattern of the equalized mode 1 of Example 2.5.2

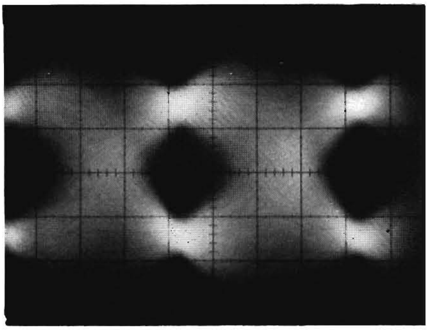

Fig. 2.8 Eye pattern of the equalized mode 2 of Example 2.5.2.

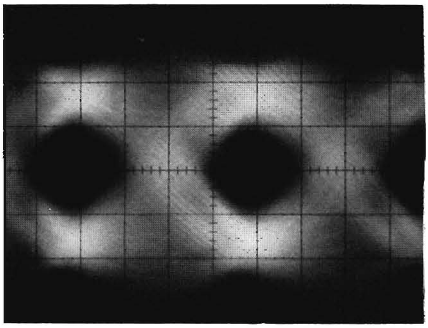

Fig. 2.9 Eye pattern of the equalized mode 3 of Example 2.5.2.

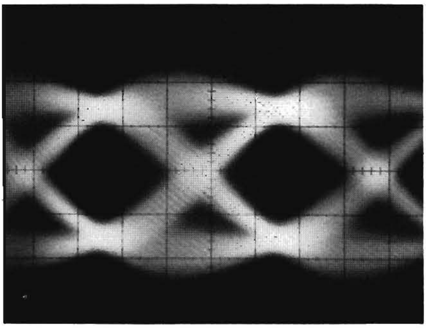

Fig. 2.10 Eye pattern of the equalized mode 4 of Example 2.5.2.

input sequences.

2.6. Appendices

Appendix 2.6.1

In this appendix we show that the assumptions that the noise processes $n_i(t)$ are white and uncorrelated do not constitute a restriction of the generality, i.e. a system not satisfying these assumptions can be transformed into a system that does meet the requirements. The proof starts with the remark that the spectral matrix (which is the Laplace transform of the correlation matrix) of the input noise can be factored, according to [10], as

$$\Phi_{\underline{nn}}(s) = Q(-s)\, Q^T(s) \tag{2.74}$$

where s is the bilateral Laplace variable. Assuming that we have a system with transfer matrix $P(s)$ such that the spectral matrix of the output noise is the identity matrix if the input spectral matrix is given by

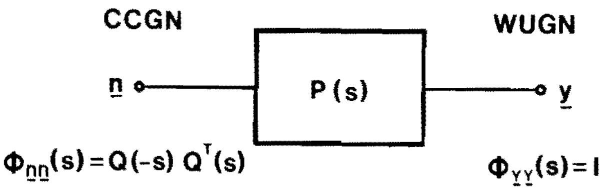

Fig. 2.11 Multiple noise whitening filter

(2.74), then the spectral matrix of the output $\underline{y}$ of $P(s)$ is written as follows [10]

$$\Phi_{\underline{\underline{yy}}}(s) = P(-s)\ Q(-s)\ Q^T(s)\ P^T(s) \tag{2.75}$$

(see Fig. 2.11). From this it follows that

$$P(s) = Q^{-1}(s) \tag{2.76}$$

satisfies the requirement of white, uncorrelated output noise. A procedure for finding a $Q(s)$ such that both $Q(s)$ and $Q^{-1}(s)$ are stable is also given in [10]. Now we shall further investigate the multiple matched filter (MMF) for colored, correlated Gaussian noise (CCGN). The several impulse responses $r_{ij}(t)$ of the multiple channel system are written in a matrix $R(t)$ as given below

$$R(t) \triangleq \begin{bmatrix} r_{11}(t) & r_{12}(t) & . & . & . & r_{1M}(t) \\ r_{21}(t) & r_{22}(t) & . & . & . & r_{2M}(t) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ r_{M1}(t) & r_{M2}(t) & . & . & . & r_{MM}(t) \end{bmatrix}. \tag{2.77}$$

From (2.76) it follows that the multiple channel transmission system with transfer matrix $R(s)$ disturbed by colored, correlated Gaussian noise with spectral matrix $\Phi_{\underline{\underline{nn}}}(s)$ can be replaced by a multiple channel transmission system with transfer matrix $Q^{-1}(s)\ R(s)$ disturbed by white, uncorrelated, Gaussian noise (WUGN) (see Fig. 2.12). As the inverse of $Q^{-1}(s)$ exists it follows from the theorem of reversibility [5, p. 222] that the insertion if this filter does not affect the optimality of the receiver to be found for the given channel. The MMF for the system depicted in Fig. 2.12 is given by

$$[Q^{-1}(-s)\ R(-s)]^T = R^T(-s)[Q^T(-s)]^{-1}. \tag{2.78}$$

Note that the MMF for the system with impulse response matrix $R(t)$ disturbed by WUGN is given by $R^T(-t)$. So the MMF for the original system can be written as

$$R^T(-s)[Q^T(-s)]^{-1}Q^{-1}(s) = R^T(-s)\ [\Phi^T_{\underline{nn}}(s)]^{-1} \tag{2.79}$$

(see Fig. 2.13). This MMF we call multiple whitening matched filter (MWMF).

Appendix 2.6.2

Proof of Theorem 2.2.

The proof of this theorem consists of two parts. First of all we prove that $F_0 = I$ and then this result is used to show that $F_l = 0$, $l = -N,\ldots,-1,1,\ldots,N$. Let $\{V_l\}_{l=-\infty}^{\infty}$ be given with $V_0 = I$ and let

$$M_0 = \sum_{l=-\infty}^{\infty}{}' \,||V_l|| < 1. \tag{2.80}$$

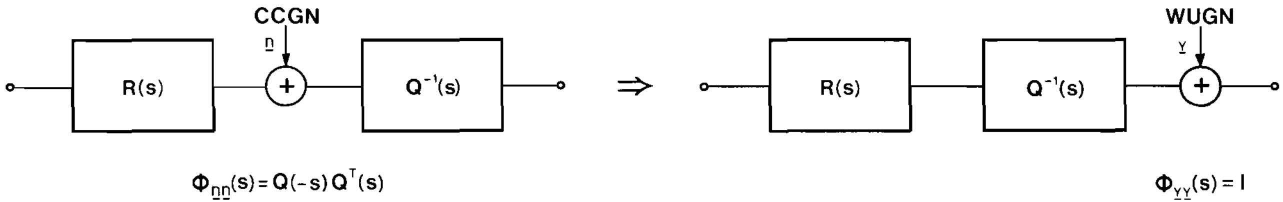

Fig. 2.12 The system $R(s)$ disturbed by CCGN is replaced by the system $Q^{-1}(s)\ R(s)$ disturbed by WUGN.

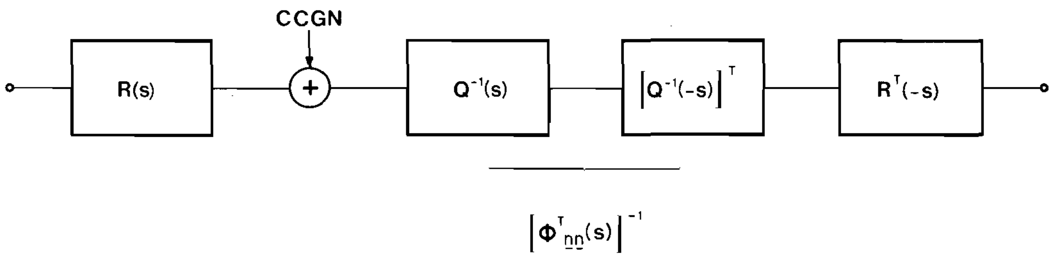

Fig. 2.13 The multiple channel system disturbed by CCGN in cascade with the multiple whitening matched filter.

Let

$$F_l = \sum_{j=-N}^{N} C_j V_{l-j} \qquad l = \ldots,-1,0,1,\ldots \tag{2.81}$$

Assume that the diagonal elements of F_0 are all unity, so that

$$F_0 = I + Z = \sum_{j=-N}^{N} C_j V_{-j} \tag{2.82}$$

where Z is a matrix with diagonal elements equal to zero. From this equation it follows that

$$Z = -I + \sum_{j=-N}^{N} C_j V_{-j}. \tag{2.83}$$

Let

$$A = \sum_l{}' ||F_l|| + ||Z|| = \sum_l{}' || \sum_{j=-N}^{N} C_j V_{l-j} || + ||Z|| =$$

$$= \sum_l{}' || \sum_{j=-N}^{N} C_j V_{l-j} || + || -I + \sum_{j=-N}^{N} C_j V_{-j} ||. \tag{2.84}$$

Let A be minimal at $(C_{-N}^{*},\ldots,C_N^{*})$ and let its value there be A^*. Consider A at $(C_{-N}^{*},\ldots,C_0^{*}+E_0,\ldots,C_N^{*})$ and let its value there be $\bar{A}$. Now we must have

$$A^* \leqslant \bar{A}. \tag{2.85}$$

From (2.84) it follows

$$\bar{A} = \sum_{l}' \Big|\Big| \sum_{j=-N}^{N} C_j^* V_{l-j} + E_0 V_l \Big|\Big| + \Big|\Big| -I + \sum_{j=-N}^{N} C_j^* V_{-j} + E_0 V_0 \Big|\Big|$$

$$\leq \sum_{l}' \Big|\Big| \sum_{j=-N}^{N} C_j^* V_{l-j} \Big|\Big| + \sum_{l}' ||E_0||.||V_l|| + \Big|\Big| -I + \sum_{j=-N}^{N} C_j^* V_{-j} + E_0 \Big|\Big|$$

$$= A^* - ||Z^*|| + ||E_0|| \sum_{l}' ||V_l|| + ||Z^* + E_0|| \qquad (2.86)$$

where

$$Z^* \triangleq -I + \sum_{j=-N}^{N} C_j^* V_{-j} . \qquad (2.87)$$

Choose

$$E_0 = -\delta Z^* \qquad 0<\delta\leq 1 . \qquad (2.88)$$

By means of (2.88), Equation (2.86) becomes

$$\bar{A} - A^* \leq \delta ||Z^*|| (M_0 - 1) . \qquad (2.89)$$

From (2.89) it follows that

$$||Z^*|| = 0 \qquad (2.90)$$

because otherwise there is a contradiction with (2.85). Now Let

$$A = \sum_{l=-\infty}^{\infty}{}' \Big|\Big| \sum_{j=-N}^{N} C_j V_{l-j} \Big|\Big| \qquad (2.91)$$

under the constraint

$$\sum_{j=-N}^{N} C_j V_{-j} = I . \tag{2.92}$$

The matrix C_0 will be used to satisfy this constraint

$$C_0 = I - \sum_{j=-N}^{N}{}' C_j V_{-j} . \tag{2.93}$$

We shall show that a minimum for A occurs if

$$F_l = \sum_{j=-N}^{N} C_j V_{l-j} = 0 \qquad l = -N,\ldots,-1,1,\ldots,N. \tag{2.94}$$

Proof:

By means of (2.93) Equation (2.91) can be written as

$$A = \sum_{l=-\infty}^{\infty}{}' \left|\left| \sum_{j=-N}^{N}{}' C_j (V_{l-j} - V_{-j} V_l) + V_l \right|\right| . \tag{2.95}$$

Let A be minimal at $(C^*_{-N},\ldots,C^*_N)$ and let its value there be A^*. Consider A at $(C^*_{-N},\ldots,C^*_k+E_k,\ldots,C^*_N)$ and let its value there be $\bar{A}$. Now we must have again $\quad k\neq 0$

$$A^* \leqslant \bar{A} . \tag{2.96}$$

From (2.95) it follows that

$$\bar{A} = \sum_{l=-\infty}^{\infty}{}' \left|\left| \sum_{j=-N}^{N}{}' C_j (V_{l-j} - V_{-j} V_l) + V_l + E_k (V_{l-k} - V_{-k} V_l) \right|\right|$$

$$\leqslant \quad A^* - ||F_k^*|| + \sum_{\substack{l=-\infty \\ l\neq k}}^{\infty}{}' ||V_{l-k}||.||E_k|| + \sum_{\substack{l=-\infty \\ l\neq k}}^{\infty}{}' ||V_l||.||E_k||.||V_{-k}|| +$$

$$+ ||F_k^* + E_k(I - V_{-k}V_k)|| \tag{2.97}$$

where

$$F_k^* \triangleq \sum_{j=-N}^{N} C_j^* V_{k-j} . \tag{2.98}$$

Choose

$$E_k = -\delta F_k^*(I - V_{-k}V_k)^{-1} \qquad 0<\delta\leqslant 1 \tag{2.99}$$

which is possible if the inverse of $(I - V_{-k}V_k)$ exists. Since $M_0<1$ we can say that $||V_k||<1$ and $||V_{-k}||<1$ for $k\neq 0$, so that

$$||V_{-k}V_k|| \leqslant ||V_{-k}||.||V_k|| < 1. \tag{2.100}$$

In general a matrix $(I - B)$ is regular if $||B||<1$, as will be shown below. Suppose $(I - B)$ to be singular, then there must be a vector $\underline{x} \neq \underline{0}$ such that

$$(I - B)\underline{x} = \underline{0} , \tag{2.101}$$

so that $\underline{x} = B\underline{x}$ and

$$||\underline{x}|| = ||B\underline{x}|| \leqslant ||B||.||\underline{x}|| < ||\underline{x}||. \tag{2.102}$$

This latter inequality implies a contradiction. Thus $(I - B)$ must be regular. Hence, the inverse of $(I - V_{-k}V_k)$ exists. Moreover, we have

$$||(I - V_{-k}V_k)^{-1}|| \leq \frac{1}{1-||V_{-k}||.||V_k||} \quad . \tag{2.103}$$

By means of (2.99) and (2.103) equation (2.97) becomes

$$\bar{A}-A^* \leq ||F_k^*||[-1+ \frac{\delta}{1-||V_{-k}||.||V_k||} \{M_0-||V_{-k}||+(M_0-||V_k||)||V_{-k}||\}+1-\delta]$$

$$\leq \frac{\delta||F_k^*||}{1-||V_{-k}||.||V_k||}[M_0-||V_{-k}||+M_0||V_{-k}||-||V_k||.||V_{-k}||-1+||V_k||.||V_{-k}||]$$

$$= \frac{\delta||F_k^*||}{1-||V_{-k}||.||V_k||}[(M_0-1)(1+||V_{-k}||)]. \tag{2.104}$$

From (2.104) it follows that

$$||F_k^*|| = 0 \tag{2.105}$$

because otherwise there is a contradiction with (2.96).

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CHAPTER 3

MAXIMUM LIKELIHOOD AND MAXIMUM A POSTERIORI RECEIVERS

In Chapter 2 it was found that several concepts known from the ISI literature can be generalized to MDI. Recently, maximum likelihood sequence estimation of data disturbed by noise and ISI received considerable attention [1], [2] and [3]. Now the question arises as to whether these concepts can also be generalized to sequences transmitted over multiple channel systems where the output data are disturbed by noise and MDI. In this chapter this question is answered to the affirmative.

3.1 The statistical sufficiency of the multiple matched filter output

With the input vector sequence we associate the vector D-transform

$$\underline{x}(D) \triangleq \sum_{l} \underline{x}_l \, D^l \qquad l = \ldots,-1,0,1,\ldots \tag{3.1}$$

where D is the delay operator.

In this section we shall show that if the multiple matched filter (MMF), as defined in Chapter 2 and [4], is used as multiple linear receiving filter, then the sampled outputs of this MMF form a set of sufficient statistics for estimating the vector input sequence $\underline{x}(D)$.

The impulse responses $r_{ij}(t)$ are considered as elements of a matrix as in Chapter 2

$$R(t) \triangleq \begin{bmatrix} r_{11}(t) & r_{12}(t) & . & . & r_{1M}(t) \\ r_{21}(t) & r_{22}(t) & . & . & r_{2M}(t) \\ . & . & . & . & . \\ . & . & . & . & . \\ r_{M1}(t) & . & . & . & r_{MM}(t) \end{bmatrix} \tag{3.2}$$

which defines the behaviour of the multiple channel system. If the MMF is described analogously it will be clear that its response is denoted by $R^T(-t)$. Assume that the multiple channel system is excited by a single input vector $\underline{x}$. Denoting the signal at output i of the multiple channel system by $s_i(t)$, we can write the total system output as a vector

$$\underline{s}(t) \triangleq \begin{bmatrix} s_1(t) \\ s_2(t) \\ . \\ . \\ s_M(t) \end{bmatrix} \tag{3.3}$$

called the vector output signal. The noise is also given as a vector

$$\underline{n}(t) \triangleq \begin{bmatrix} n_1(t) \\ n_2(t) \\ . \\ . \\ n_M(t) \end{bmatrix} \tag{3.4}$$

called the vector noise.

In the following we shall several times use the inner-product of matrices, the elements of which consist of time functions.

Such a product is denoted as

$$K = \langle A(t), B(t) \rangle \tag{3.5}$$

and defined by

$$k_{ij} \triangleq \sum_n \int_{-\infty}^{\infty} a_{in}(t) b_{nj}(t) dt. \tag{3.6}$$

The sampled output of the MMF, in the absence of noise, is given by the signal vector

$$\underline{s} = \langle R^T(t), \underline{s}(t) \rangle. \tag{3.7}$$

The inverse transformation from signal vector to output vector signal is

$$\underline{s}(t) = R(t) G \underline{s} \tag{3.8}$$

where G is a matrix to be determined. Substituting (3.8) in (3.7) gives

$$G = [\langle R^T(t), R(t) \rangle]^{-1}. \tag{3.9}$$

This matrix equals the inverse of the sampled transfer of the multiple channel system $R(t)$ in cascade with the MMF given by $R^T(-t)$. This latter transfer was called V_0 in Chapter 2 and we have seen there that we must require our systems to satisfy the existence of the matrix G according to (3.9).

In absence of the signal the sampled output, due to noise only, can be written as

$$\underline{n} = \langle R^T(t), \underline{n}(t) \rangle. \tag{3.10}$$

According to (3.10) the relevant vector noise [5, Chapter 4], being that part of the input vector noise represented by the projection of $\underline{n}(t)$ onto the signal space, is denoted by

$$\underline{n}_r(t) = R(t)G\underline{n}\ . \tag{3.11}$$

By means of the definition

$$\underline{v} \triangleq \underline{s} + \underline{n} \tag{3.12}$$

the equivalent received vector signal is written as

$$\underline{v}(t) = R(t)G\underline{v} \tag{3.13}$$

which means that for the sampled output it makes no difference whether the true received vector signal $\underline{s}(t) + \underline{n}(t)$ or the vector signal $\underline{v}(t)$ is presented to the input of the MMF. Writing out (3.13) yields

$$\underline{v}(t) = R(t)G\underline{v} = R(t)G\underline{s} + R(t)G\underline{n} = \underline{s}(t) + \underline{n}_r(t)\,. \tag{3.14}$$

Thus $R(t)$ is a basis for the signal space spanned by both $\underline{s}(t)$ and $\underline{n}_r(t)$ [5, Chapter 4], which proves that the sampled MMF output is a sufficient statistic for estimating a single input vector $\underline{x}$. This sufficiency for single input vectors $\underline{x}$ is also valid for sequences of input vectors with finite support (see [1] and [5]). Hence, we have the following

THEOREM 3.1

If a vector $\underline{x}_l$ is transmitted at each instant lT, then the vector output sequence

$$\underline{v}(D) \triangleq \sum_{l} \underline{v}_l D^l \qquad l = \ldots,-1,0,1,\ldots \tag{3.15}$$

forms a set of sufficient statistics for estimating the vector input sequence $\underline{x}(D)$.

3.2 The multiple whitened matched filter

Now consider the system consisting of the channel in cascade with the MMF as a multiple channel system with M inputs and M outputs. As in Chapter 2 the impulse response from input j to output n of this system is called $v_{nj}(t)$ and can be written as

$$v_{nj}(t) = \sum_{i=1}^{M} r_{in}(-t) * r_{ij}(t) = \sum_{i=1}^{M} \int_{-\infty}^{\infty} r_{in}(\tau-t) r_{ij}(\tau) d\tau \tag{3.16}$$

where $*$ means convolution. Again define

$$V_l \triangleq \begin{bmatrix} v_{11}(lT) & v_{12}(lT) & . & . & v_{1M}(lT) \\ v_{21}(lT) & v_{22}(lT) & . & . & v_{2M}(lT) \\ . & . & . & . & . \\ . & . & . & . & . \\ v_{M1}(lT) & v_{M2}(lT) & . & . & v_{MM}(lT) \end{bmatrix} \tag{3.17}$$

and

$$V(D) \triangleq \sum_{l} V_l \, D^l \qquad l = \ldots,-1,0,1,\ldots \tag{3.18}$$

From (3.16) it is evident that (3.18) is equivalent to

$$V(D) = \langle R^T(D^{-1},t), R(D,t) \rangle \tag{3.19}$$

where $R(D,t)$ is a matrix with elements consisting of the chip D-transforms [1] of the elements of $R(t)$.

As the matrix of impulse responses of the MMF is $R^T(-t)$ the cross-correlation of the output noise signals at outputs n and m is given by

$$\phi_{nm}(\rho) = \sum_{i=1}^{M} N_0 \int_{-\infty}^{\infty} r_{in}(-t) r_{im}(-t-\rho) dt$$

$$= \sum_{i=1}^{M} N_0 \int_{-\infty}^{\infty} r_{in}(t) r_{im}(t-\rho) dt \tag{3.20}$$

Sampling this function, we define its D-transform as

$$\phi_{nm}(D) \triangleq \sum_{l} \phi_{nm}(lT)\, D^l \qquad l = \ldots,-1,0,1,\ldots \tag{3.21}$$

If all $\phi_{nm}(D)$ are collected in a matrix we obtain the spectral matrix

$$\Phi(D) = N_0 \langle R^T(D,t), R(D^{-1},t)\rangle . \tag{3.22}$$

Relation (3.22) can readily be verified by means of (3.20). In [6] and [7] it is shown that a matrix $H(D^{-1})$ can be found such that

$$\Phi(D) = N_0 H(D) H^T(D^{-1}) \tag{3.23}$$

with both $H(D^{-1})$ and $H^{-1}(D^{-1})$ stable and nonanticipatory. Comparing (3.19), (3.22) and (3.23) it is obvious that

$$V(D) = H(D^{-1}) H^T(D) . \tag{3.24}$$

Now we conclude that the sampled output of the MMF can be written as

$$\underline{v}(D) = H(D^{-1}) H^T(D) \underline{x}(D) + H(D^{-1}) \underline{n}(D) \tag{3.25}$$

where $\underline{n}(D)$ is the sampled input noise vector sequence.

The output noise

$$\underline{n}'(D) = H(D^{-1})\underline{n}(D) \tag{3.26}$$

is colored Gaussian with spectral matrix $\Phi(D)$. This follows from

$$E[H(D)\underline{n}(D^{-1})\{H(D^{-1})\underline{n}(D)\}^T] =$$

$$E[H(D)\underline{n}(D^{-1})\underline{n}^T(D)H^T(D^{-1})] = N_0H(D)H^T(D^{-1}). \tag{3.27}$$

From (3.25) it is seen that the output noise is whitened by the operation

$$\underline{z}(D) \triangleq H^{-1}(D^{-1})\underline{v}(D) = H^T(D)\underline{x}(D) + \underline{n}(D) = \underline{y}(D) + \underline{n}(D) \tag{3.28}$$

which means physically that the MMF is followed by a multiple tapped delay line (MTDL) (see Chapter 2 and [4]) with transfer $H^{-1}(D^{-1})$. It has been mentioned in the foregoing that $H^{-1}(D^{-1})$ is stable and nonanticipatory and thus realizable. The MMF followed by the MTDL is called multiple whitened matched filter and is characterized by its chip D-transform

$$W(D,t) \triangleq H^{-1}(D^{-1})R^T(D^{-1},t). \tag{3.29}$$

If the impulse response from input n to output m is denoted by $w_{mn}(t)$, the set of functions $w_{mn}(t-kT)$ is orthonormal in both time and space as is seen from

$$\begin{aligned}\Phi_{ww}(D) &= \langle W(D^{-1},t),W^T(D,t)\rangle = \\ &= H^{-1}(D)\langle R^T(D,t),R(D^{-1},t)\rangle\{H^{-1}(D^{-1})\}^T = \\ &= H^{-1}(D)V(D^{-1})\{H^T(D^{-1})\}^{-1} = \\ &= H^{-1}(D)H(D)H^T(D^{-1})\{H^T(D^{-1})\}^{-1} = I.\end{aligned} \tag{3.30}$$

In this previous section we concluded that $\underline{v}(D)$ forms a set of sufficient statistics for estimating $\underline{x}(D)$, but $\underline{z}(D)$ is found by the reversible linear transformation $H^{-1}(D^{-1})$ on $\underline{v}(D)$. Thus $\underline{z}(D)$ also forms a set of sufficient statistics for estimating $\underline{x}(D)$. These results are summarized in the following

THEOREM 3.2

Let $R(t)$ be the matrix of impulse responses of the multiple channel transmission system and $H(D^{-1})H^T(D)$ a factorization of

$$V(D) = \langle R^T(D^{-1},t),R(D,t)\rangle \tag{3.31}$$

such that both $H(D^{-1})$ and $H^{-1}(D^{-1})$ are stable and nonanticipatory. Then the multiple filter whose chip D-transform is

$$W(D,t) = H^{-1}(D^{-1})R^T(D^{-1},t) \tag{3.32}$$

is realizable and is called a multiple whitened matched filter. Its sampled outputs give a vector sequence

$$\underline{z}(D) = H^T(D)\underline{x}(D) + \underline{n}(D) \tag{3.33}$$

which is a sufficient statistic for estimating the vector input sequence $\underline{x}(D)$. The noise vector sequence is white in both time and space.

The multiple whitened matched filter found in this section is a generalized version of the whitened matched filter derived in [1].

3.3 The vector Viterbi algorithm

In the preceding sections we have derived a structure giving a set of sufficient statistics for estimating the input vector sequence of a multiple channel transmission system from the observations of the output. This output is disturbed by MDI and noise. As the noisy parts of the multiple whitened matched filter output samples are Gaussian and uncorrelated, hence, independent the Viterbi algorithm can be used to perform ML estimation of the vector input sequence $\underline{x}(D)$. The vector Viterbi algorithm is a vector version of the algorithm used to make ML estimations on digital sequences and which is extensively described in [1] and [2]. The vector sequence $\underline{y}(D)$ may be considered to be generated by a multiple finite state machine, driven by an input vector sequence $\underline{x}(D)$ (see Fig. 3.1.). We define the state s_l at time lT of this finite state machine by

$$s_l \triangleq \{\underline{x}_{l-1},\ldots,\underline{x}_{l-N}\} \qquad l = \ldots,-1,0,1,\ldots \tag{3.34}$$

where N is the degree of the matrix polynomial $H^T(D)$ (see (3.28)). There are L^{NM} distinct states. We can depict the successive states of the multiple finite state machine, together with all allowable transitions, in a trellis diagram [1], [8] and [9]. Given the observations $\underline{z}_l$, the log likelihood of a transition is given by

$$\ln p[\underline{z}_l - \underline{y}(s_l,s_{l+1})] = -\ln(\sqrt{2\pi N_0})^M +$$

$$-\frac{1}{2N_0}\sum_{i=1}^{M}\{z_{il} - y_i(s_l,s_{l+1})\}^2 \tag{3.35}$$

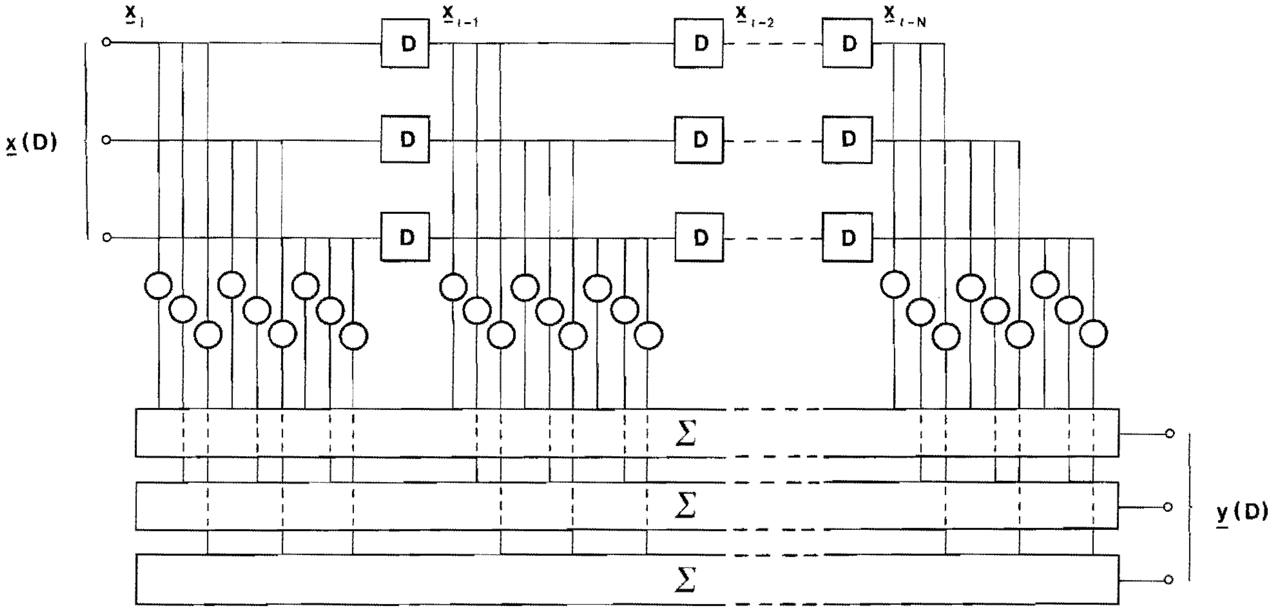

Fig. 3.1 Model of a multiple finite state machine

where z_{il} and $y_i(s_l,s_{l+1})$ are the $i^{\underline{th}}$ elements of respectively $\underline{z}_l$ and $\underline{y}(s_l,s_{l+1})$. In ML sequence estimation the first term of the right-hand member of (3.35), being independent of l, can be omitted and the same applies to the factor $\frac{1}{2N_0}$ in the second term. Given the received sequence $\underline{y}(D)$ one can associate a distance with each allowable state transition

$$D^2(s_l,s_{l+1}) \triangleq \sum_{i=1}^{M} \{z_{il} - y_i(s_l,s_{l+1})\}^2. \tag{3.36}$$

The Viterbi algorithm now recursively finds that state sequence for which the metric

$$J \triangleq \sum_{l} D^2(s_l,s_{l+1}) \tag{3.37}$$

is minimal, i.e. the maximum likelihood estimation of the state (input) sequence [1, 2, 8 and 9]. At this point the vector Viterbi algorithm is in fact reduced to the scalar version and we refer to [1], [2], [8] and [9] for further details. In its implementation the Viterbi algorithm requires one metric and one path register for each state. Hence, the complexity of implementation grows exponentially not only with the channel memory N but also with the number of channels M. This fact severely limits the practical applicability of the Viterbi algorithm for MDI correction.

3.4 The vector Ungerboeck algorithm

Ungerboeck describes an alternative algorithm for making ML sequence estimations on data disturbed by ISI and white Gaussian noise [3].

Using the algorithm, the tapped delay line is omitted and the sampled output of the matched filter is used directly as input for the algorithm. We shall generalize below the Ungerboeck algorithm for ML vector sequence estimation of data that are disturbed by MDI and white Gaussian noise.

If a vector sequence $\underline{x}(D)$ is transmitted, the corresponding received vector signal is defined as

$$\underline{u}(t) \triangleq \sum_{l} R(t-lT)\, \underline{x}_l + \underline{n}(t) \qquad l = \ldots,-1,0,+1,\ldots \tag{3.38}$$

Among all possible input sequences $\underline{\xi}(D)$ we choose as the estimate $\hat{\underline{x}}(D)$ for $\underline{x}(D)$ that vector sequence which maximizes $ln\ p[\underline{u}(t)|\underline{\xi}(D)]$, which is equivalent to minimizing

$$J = \left|\left|\underline{u}(t) - \sum_{l} R(t-lT)\underline{\xi}_l\right|\right|_2^{\,2} =$$

$$= <[\underline{u}(t) - \sum_{l} R(t-lT)\underline{\xi}_l]^T\ ,\ [\underline{u}(t) - \sum_{k} R(t-kT)\underline{\xi}_k]> \tag{3.39}$$

over all allowable $\underline{\xi}(D)$. Rewriting (3.39) we obtain

$$J = <\underline{u}^T(t),\underline{u}(t)> - <\underline{u}^T(t),\ \sum_{k} R(t-kT)\underline{\xi}_k> - <\sum_{l} \underline{\xi}_l^T R^T(t-lT),\underline{u}(t)> +$$

$$+ <\sum_{l} \underline{\xi}_l^T R^T(t-lT),\ \sum_{k} R(t-kT)\underline{\xi}_k>. \tag{3.40}$$

Define

$$\underline{v}_l \triangleq <R^T(t-lT),\underline{u}(t)> \qquad l = \ldots,-1,0,1,\ldots \tag{3.41}$$

This vector is interpreted as the sampled output of the MMF. By means of definition (3.41) J is written as

$$J = \langle \underline{u}^T(t), \underline{u}(t) \rangle - 2 \sum_l \underline{\xi}_l^T \underline{v}_l + \sum_l \sum_k \underline{\xi}_l^T V_{l-k} \underline{\xi}_k \quad . \tag{3.42}$$

The first term of (3.42) is independent of $\underline{\xi}_l$ and thus may be ignored in the minimization process. The metric $J\{\underline{\xi}(D)\}$ can be calculated in a recursive manner

$$\begin{aligned} J_l(\ldots, \underline{\xi}_{l-1}, \underline{\xi}_l) &\triangleq -2 \sum_{n=-\infty}^{l} \underline{\xi}_n^T \underline{v}_n + \sum_{n=-\infty}^{l} \sum_{k=-\infty}^{l} \underline{\xi}_n^T V_{n-k} \underline{\xi}_k \\ &= J_{l-1}(\ldots, \underline{\xi}_{l-1}) + F(\underline{v}_l; \underline{\xi}_{l-N}, \ldots, \underline{\xi}_l) \end{aligned} \tag{3.43}$$

where

$$F(\underline{v}_l; \underline{\xi}_{l-N}, \ldots, \underline{\xi}_l) = \underline{\xi}_l^T \, (V_0 \, \underline{\xi}_l + 2 \sum_{k=1}^{N} V_k \, \underline{\xi}_{l-k} - 2 \, \underline{v}_l) \tag{3.44}$$

and N is the degree of the matrix polynomial $H(D)$ (see (3.23) and (3.24)). We define the survivor metric $\tilde{J}_l$ as follows

$$\tilde{J}_l(s_l) \triangleq \tilde{J}_l \; (\underline{\xi}_{l+1-N}, \ldots, \underline{\xi}_l) \triangleq \min_{\{\ldots, \underline{\xi}_{l-N}\}} \{J_l(\ldots, \underline{\xi}_{l-N}, \underline{\xi}_{l-N+1}, \ldots, \underline{\xi}_l)\} \, . \tag{3.45}$$

The sequence $(\ldots, \underline{\xi}_{l-N})$, which results in a minimum of (3.45) is called the path history of the state

$$s_l \triangleq (\underline{\xi}_{l+1-N}, \ldots, \underline{\xi}_l) \tag{3.46}$$

at time lT $(l = \ldots, -1, 0, 1, \ldots)$.

It is easy to see that there are again L^{NM} different states. One can imagine that these states correspond to the states of a finite state machine. Like in the Viterbi algorithm one now uses dynamic programming to find recursively the ML state sequence [3]. Although at first glance the metric calculation of the Ungerboeck algorithm seems more complicated than that of the Viterbi algorithm, a closer inspection of (3.44) shows that the metric up-dating is a rather simple operation from a programming point of view. Namely, the quantity $\underline{\xi}_l^T[V_0\,\underline{\xi}_l + 2\sum_{k=1}^{N} V_k\,\underline{\xi}_{l-k}]$ depends only on the channel response, assumed to be fixed, and on the particular transition. Hence, this value can be stored in a memory and need not be calculated in real time.

3.5 The error performance of the ML receiver

The analysis in this section closely resembles that given in [1] and [3]. Assume, without loss of generality, that the error event ε, associated with the vector error sequence

$$\underline{e}(D) \triangleq \underline{\hat{x}}(D) - \underline{x}(D) \tag{3.47}$$

starts at $t=0$, i.e. $\underline{e}(D)$ can be represented by

$$\underline{e}(D) = \underline{e}_0 + \underline{e}_1 D + \ldots + \underline{e}_H D^H \qquad \text{with } ||\underline{e}_0||_2, ||\underline{e}_H||_2 \geq \delta_0 \tag{3.48}$$

where δ_0 denotes the minimum nonzero value of the Euclidean norm of the error vector $\underline{e}_i$ $(i=1,\ldots,H)$ and the length of the error event ε is $H+N$. The value of δ_0 equals

$$\delta_0 = \min_{i\neq j} \{|a_{jl} - a_{il}|\} \tag{3.49}$$

which equals unity in our case. From [3] we know that the probability of an error event ε can be written as

$$Pr(\varepsilon) = Pr(\varepsilon_1)Pr(\varepsilon_2|\varepsilon_1) \leq Pr(\varepsilon_1)Pr(\varepsilon_2'|\varepsilon_1) \tag{3.50}$$

where the sub-events $\varepsilon_1, \varepsilon_2$ and ε_2' are defined as follows:

ε_1 : $\underline{x}(D)$ is such that $\underline{x}(D)+\underline{e}(D)$ is an allowable data vector sequence,

ε_2 : the noise vector sequence is such that $\underline{x}(D)+\underline{e}(D)$ is ML (within the observation interval),

ε_2': the noise vector sequence is such that $\underline{x}(D)+\underline{e}(D)$ has greater likelihood than $\underline{x}(D)$, but not necessarily ML.

From the preceding section it is concluded that $Pr(\varepsilon_2'|\varepsilon_1)$ is the probability that

$$J\{\underline{x}(D)\} > J\{\underline{x}(D) + \underline{e}(D)\}. \tag{3.51}$$

It can be shown that inequality (3.51) is identical to

$$\delta^2(\varepsilon) \triangleq ||V_0^{-1}||_2 \sum_{l=0}^{H} \sum_{k=0}^{H} \underline{e}_l^T V_{l-k}\, \underline{e}_k < 2||V_0^{-1}||_2 \sum_{l=0}^{H} \underline{e}_l^T \underline{n}_l' \tag{3.52}$$

where $\underline{n}_l'$ are the sample values of the noise at the output of the MMF. The quantity $\delta(\varepsilon)$ is called the magnitude of the error event ε. Consider the random variable α given by the right-hand member of (3.52)

$$\alpha \triangleq 2||V_0^{-1}||_2 \sum_{l=0}^{H} \underline{e}_l^T \underline{n}_l'. \tag{3.53}$$

This random variable is Gaussian distributed with zero-mean and variance

$$E[\alpha^2] = 4N_0||V_0^{-1}||_2\ \delta^2(\varepsilon). \tag{3.54}$$

From this it follows that

$$Pr(\varepsilon_2'|\varepsilon_1) = Pr\{\alpha > \delta^2(\varepsilon)\} = Q\left(\frac{\delta(\varepsilon)}{2\{N_0||V_0^{-1}||_2\}^{\frac{1}{2}}}\right) \tag{3.55}$$

where the $Q(.)$-function is defined in [5] and (2.21). Let E be the set of all possible error events ε. Then the probability that any error event occurs becomes

$$Pr(E) = \sum_{\varepsilon\epsilon E} Pr(\varepsilon). \tag{3.56}$$

Let Δ be the set of all possible $\delta(\varepsilon)$ and E_δ the subset of error events for which $\delta(\varepsilon) = \delta$. Then from (3.50) the event error probability is bounded by

$$Pr(E) \leq \sum_{\delta\epsilon\Delta} Q\left(\frac{\delta}{2\{N_0||V_0^{-1}||_2\}^{\frac{1}{2}}}\right) \sum_{\varepsilon\epsilon E_\delta} Pr(\varepsilon_1). \tag{3.57}$$

Because of the exponential behavior of the $Q(.)$-function for large values of the argument, this expression will at moderate SNR values already be dominated by the term involving the minimum value δ_{min} out of the set Δ. At moderate and large signal-to-noise ratios ε_2' implies ε_2 with a probability almost equal to one. For these SNR values $Pr(E)$ is approximated by

$$Pr(E) \simeq Q\left(\frac{\delta_{min}}{2\{N_0||V_0^{-1}||_2\}^{\frac{1}{2}}}\right) \sum_{\varepsilon\in E_{\delta_{min}}} Pr(\varepsilon_l). \tag{3.58}$$

Assuming the input symbols to be independent and equiprobable, the probability of ε_l can be written as

$$Pr(\varepsilon_l) = \prod_{i=1}^{M} \prod_{l=0}^{H} \frac{L-|e_{il}|}{L} \tag{3.59}$$

with e_{il} the i^{th} component of $\underline{e}_l$. In the Appendix (Section 3.8) it is shown that under the constraint

$$||V_0^{-1}||_2 \sum_{l=-\infty}^{\infty}{}' ||V_l||_2 \leq 1 \tag{3.60}$$

no error event whatever has a smaller magnitude than the single error events with magnitude δ_0. By a single error event we mean an error event with an error sequence that consists of one error vector $\{\underline{e}(D)=\underline{e}_0\}$ and of this vector only one component differs from zero. In this situation the single error events with magnitude δ_0 dominate the expression for the event error probability and moreover, the event error probability approximates the symbol error probability, i.e.

$$Pr(e) \simeq Q\left(\frac{\delta_0}{2\{N_0||V_0^{-1}||_2\}^{\frac{1}{2}}}\right) \sum_{\varepsilon\in E_{\delta_0}} \frac{L-1}{L}. \tag{3.61}$$

Since $\dfrac{\delta_0^{\,2}}{||V_0^{-1}||_2}$ is the total amount of energy that is measured at the receiving end on transmission of a single symbol out of the set E_{δ_0}, the symbol error probability is not increased by MDI.

3.6 Maximum a posteriori receivers

In this section we shall extend the algorithms of Sections 3.3 and 3.4 to provide maximum a posteriori (MAP) detection of signals disturbed by noise and MDI. We can start with the finite state machine models developed in those sections. As is shown in [9] the contribution of a certain transition in the trellis to the probability of a certain path is

$$\lambda(\underline{\xi}_l) \triangleq \ln p(\underline{z}_l|\underline{\xi}_l) + \ln Pr(s_{l+1}|s_l). \tag{3.62}$$

As far as the Viterbi algorithm is concerned, this results in a change of the distance in the following way,

$$D_{MAP}{}^2(s_l,s_{l+1}) \triangleq \sum_{i=1}^{M} \{z_{il} - y_i(s_l,s_{l+1})\}^2 - 2N_0 \ln Pr(s_{l+1}|s_l). \tag{3.63}$$

In the case of the Ungerboeck algorithm we obtain as a modified metric contribution

$$F_{MAP}(\underline{v}_l;\underline{\xi}_{l-N},\ldots,\underline{\xi}_l) \triangleq \underline{\xi}_l{}^T \, [V_0\,\underline{\xi}_l + 2\sum_{k=1}^{N} V_k\,\underline{\xi}_{l-k} - 2\,\underline{v}_l] + \\ - 2N_0 \ln Pr(s_l|s_{l-1}). \tag{3.64}$$

From (3.63) it follows that for large SNR values the MAP algorithm will give the same performance as the ML algorithm. Only for small SNRs can the MAP rule offer significant improvement, depending on the several transition probabilities. This will be demonstrated in the next section.

3.7 Examples

Example 3.7.1

As a first example we take a multiple channel with $M=2$. The elements of the transmission matrix $R(t)$ are given in Fig. 3.2. We take $T=1$ and for this system the $V(D)$ matrix polynomial is as follows:

$$V(D) = \begin{bmatrix} 37 & 12 \\ 12 & 37 \end{bmatrix} \left(\frac{1}{72} D^{-1} + \frac{5}{144} + \frac{1}{72} D\right). \tag{3.65}$$

One can easily verify that this $V(D)$ satisfies condition (3.60). Decomposition of $V(D)$ according to (3.24) yields

$$H^T(D) = \frac{1}{12} \begin{bmatrix} 6 & 1 \\ 1 & 6 \end{bmatrix} (2+D). \tag{3.66}$$

The trellis diagram for this system is depicted in Fig. 3.3, whereas the values of $\underline{y}(s_l, s_{l+1})$ are given in Table 3.1. The system described by (3.65) together with a ML receiver designed for this system is simulated on a minicomputer. In Fig. 3.4 the bit error probability for a binary alphabet $\{+1,-1\}$ and independent, equiprobable input symbols is plotted as a function of the SNR, together with the $Pr(e)$ for isolated pulses. The two curves merge at a $Pr(e)$ of about 10^{-4}. Thus for bit error probabilities smaller than 10^{-4} the performance of the ML receiver on signals disturbed by MDI is as good as the performance of a ML receiver designed for signals without MDI. In the case of larger bit error probabilities the difference between the two curves is maximal by 1.2 dB.

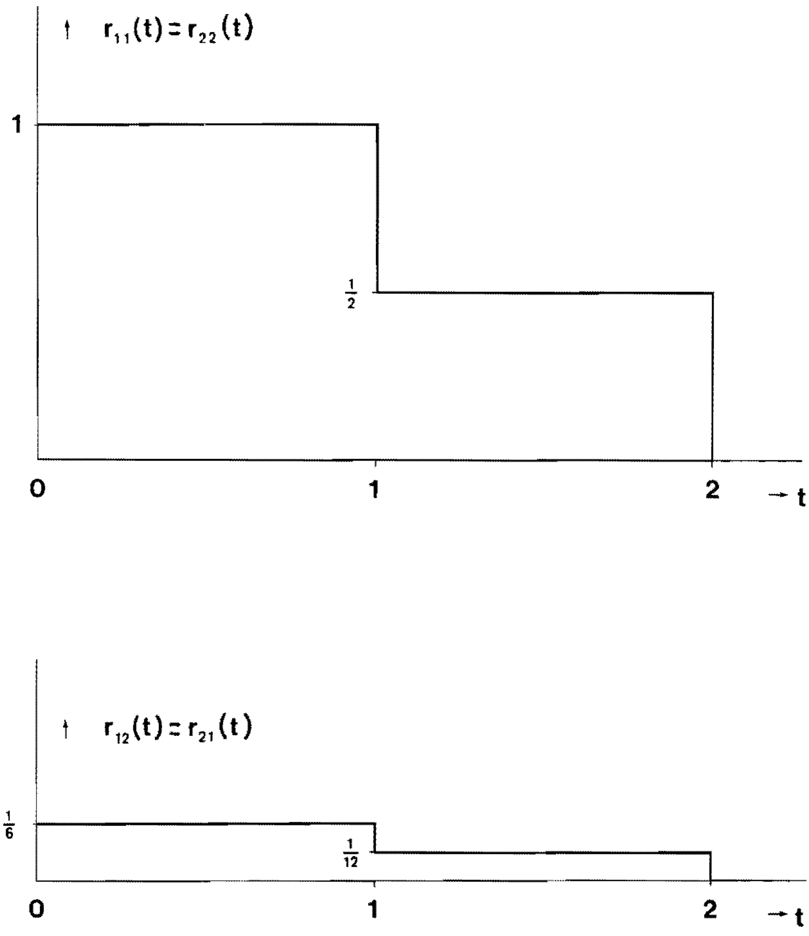

Fig. 3.2 Received signal set for the Examples 3.7.1 and 3.7.2

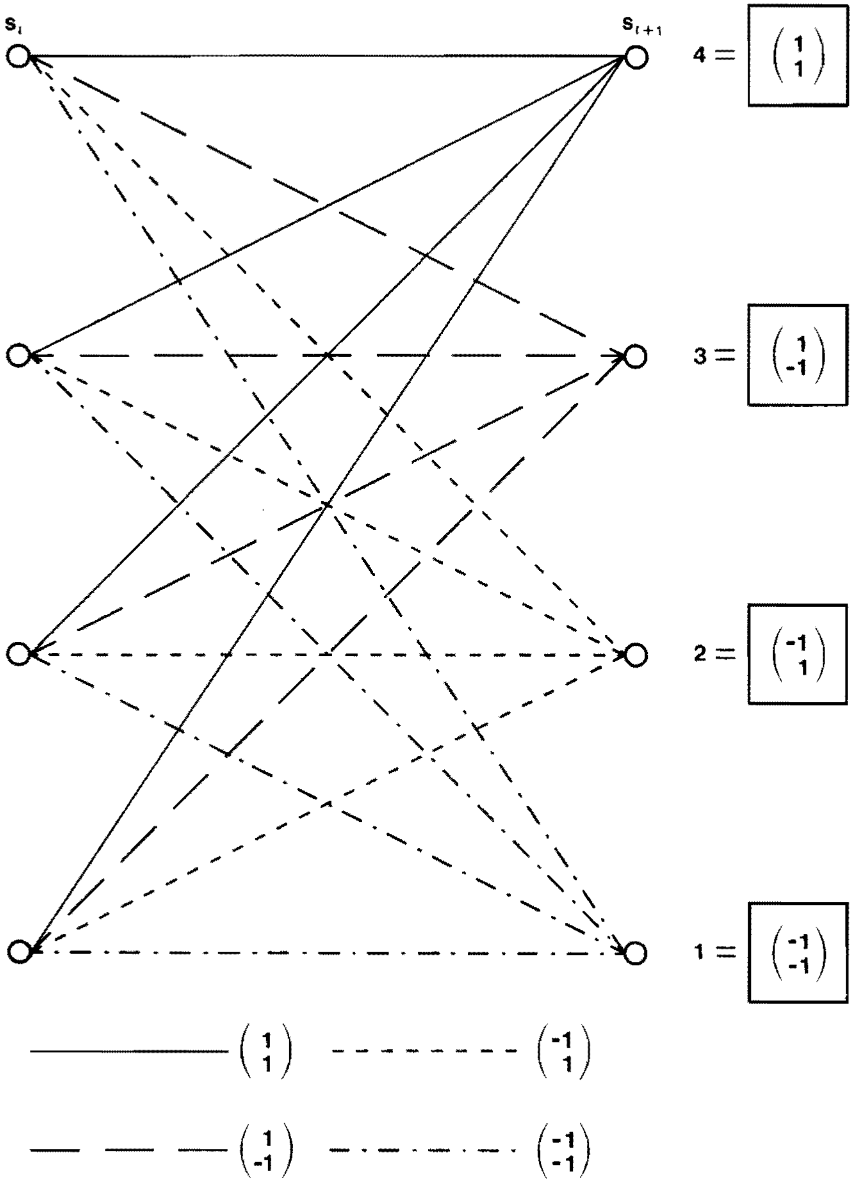

Fig. 3.3 The trellis diagram of the Examples 3.7.1 and 3.7.2

Table 3.1

old state s_l	new state s_{l+1}	output $\underline{y}(s_l, s_{l+1})$
1	1	$\frac{1}{12}\begin{pmatrix}-21\\-21\end{pmatrix}$
2	1	$\frac{1}{12}\begin{pmatrix}-19\\-9\end{pmatrix}$
3	1	$\frac{1}{12}\begin{pmatrix}-9\\-19\end{pmatrix}$
4	1	$\frac{1}{12}\begin{pmatrix}-7\\-7\end{pmatrix}$
1	2	$\frac{1}{12}\begin{pmatrix}-17\\3\end{pmatrix}$
2	2	$\frac{1}{12}\begin{pmatrix}-15\\15\end{pmatrix}$
3	2	$\frac{1}{12}\begin{pmatrix}-5\\5\end{pmatrix}$
4	2	$\frac{1}{12}\begin{pmatrix}-3\\17\end{pmatrix}$

Table 3.1 (continued)

old state s_l	new state s_{l+1}	output $\underline{y}(s_l,s_{l+1})$
1	3	$\frac{1}{12}\begin{pmatrix} 3 \\ -17 \end{pmatrix}$
2	3	$\frac{1}{12}\begin{pmatrix} 5 \\ -5 \end{pmatrix}$
3	3	$\frac{1}{12}\begin{pmatrix} 15 \\ -15 \end{pmatrix}$
4	3	$\frac{1}{12}\begin{pmatrix} 17 \\ -3 \end{pmatrix}$
1	4	$\frac{1}{12}\begin{pmatrix} 7 \\ 7 \end{pmatrix}$
2	4	$\frac{1}{12}\begin{pmatrix} 9 \\ 19 \end{pmatrix}$
3	4	$\frac{1}{12}\begin{pmatrix} 19 \\ 9 \end{pmatrix}$
4	4	$\frac{1}{12}\begin{pmatrix} 21 \\ 21 \end{pmatrix}$

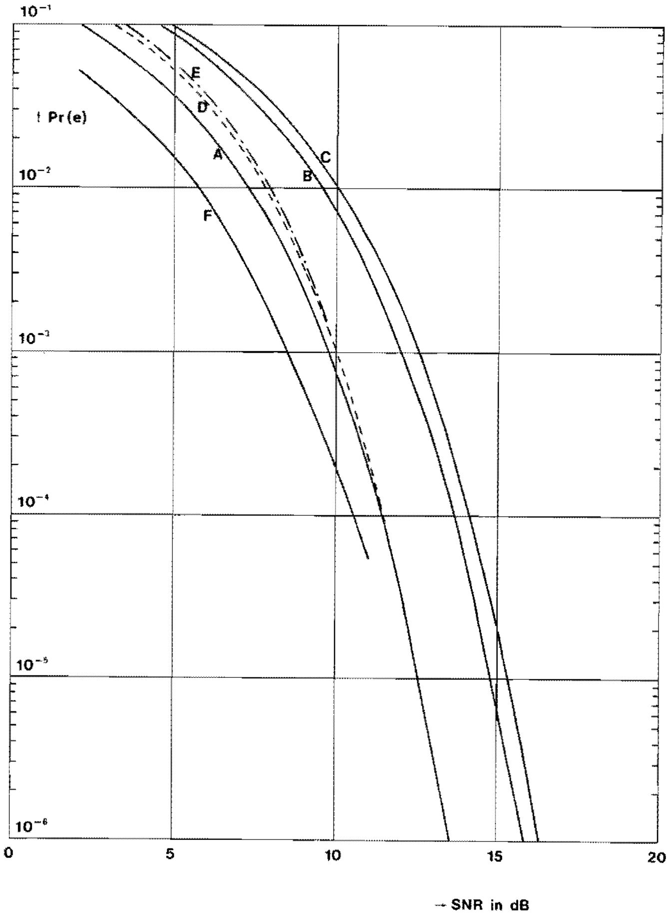

Fig. 3.4 Symbol error probability versus signal-to-noise ratio

Curve A single pulse

Curve B monochannel with linear correction and bit-by-bit detection

Curve C multiple channel with $M = 2$, linear correction and bit-by-bit detection

Curve D monochannel with ML sequence estimation

Curve E multiple channel with $M = 2$ and ML vector sequence estimation

Curve F monochannel and multiple channel with $M = 2$; correlated sources and MAP vector sequence estimation

These results are compared with those of an optimum constrained linear receiver (see Chapter 2 and [4]). The difference between the linear receiver and the single-pulse performance is 2.7 dB, showing the superiority of the vector ML receiver. We also simulated a ML receiver for a monochannel with impulse response $r_{11}(t)$. Now the maximum difference from the single-pulse performance is found to be 1 dB, but these two curves also merge at a $Pr(e)$ value of about 10^{-4}. Linear correction with bit-by-bit detection gives an increase of 2.2 dB in this case.

Example 3.7.2

As a second example we consider the channel of Example 3.7.1, with the same parameters, but correlated input symbols. It is assumed that the two symbol sources are independent, but that each source is first-order Markoff with transient probabilities as given in Fig. 3.5.

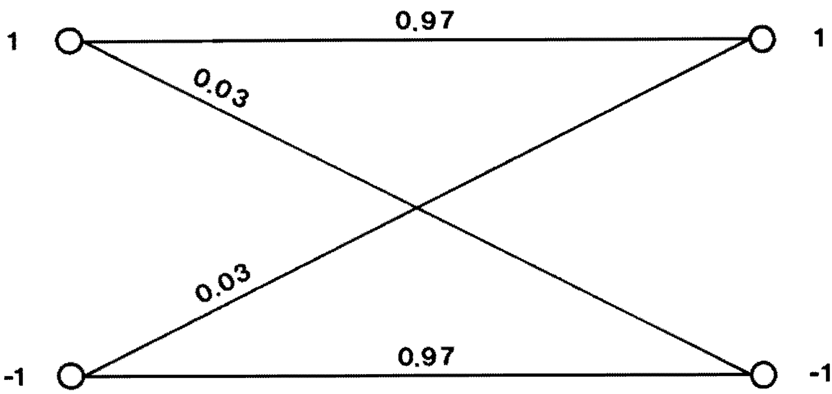

Fig. 3.5 The transient probabilities of the sources in Example 3.7.2

By means of this it is easy to see that the transient probabilities in the trellis diagram (see Fig. 3.3) are

$$P = \begin{bmatrix} 0.0009 & 0.0291 & 0.0291 & 0.9409 \\ 0.0291 & 0.0009 & 0.9409 & 0.0291 \\ 0.0291 & 0.9409 & 0.0009 & 0.0291 \\ 0.9409 & 0.0291 & 0.0291 & 0.0009 \end{bmatrix}. \tag{3.67}$$

The symbol error probability as a function of SNR for this system is also given in Fig. 3.4 (curve F). The error curve for the monochannel case with transient probabilities according to Fig. 3.5 coïncides practically with this curve. As can be seen, the error performance at low SNR values is substantially better than that of the single-pulse response. If the probabilities in Fig. 3.5 are changed to 0.05 and 0.95 the error curve coïncides practically with the single-pulse curve. Probabilities of 0.1 and 0.9 show no difference from the uncorrelated case. From this it follows that MAP estimation only makes sense where there are considerable differences between the probabilities of the transitions in the trellis and the SNR is low.

N.B. - In the simulations the path-register length was 16 bits in all cases.

- The number of transmissions was chosen such that the 90% confidence interval extends from $0.95\ Pr(e)$ to $1.05\ Pr(e)$.

3.8 Appendix

Let

$$||\underline{e}_0||_2, ||\underline{e}_H||_2 \geqslant \delta_0 \tag{3.68}$$

where δ_0 is the minimum nonzero value of the Euclidean norm of $\underline{e}_i$ $(i=1,\dots,H)$.

Let $\{V_l\}_{l=-\infty}^{\infty}$ be given and assume

$$||V_0^{-1}||_2 \sum_{l=-\infty}^{\infty}{}' ||V_l||_2 \leq 1 \ . \tag{3.69}$$

The matrix V_0 equals $< R^T(t), R(t) >$ and it is easy to show that this matrix is positive definite, under the condition derived in Section 3.1.

$$\begin{aligned}\delta^2(\varepsilon) &= ||V_0^{-1}||_2 \sum_{l=-H}^{H} \sum_{k=0}^{H} \underline{e}_{l+k}^T \ V_l \ \underline{e}_k \\ &= ||V_0^{-1}||_2 \sum_{k=0}^{H} \underline{e}_k^T \ V_0 \ \underline{e}_k + ||V_0^{-1}||_2 \sum_{l=-H}^{H}{}' \sum_{k=0}^{H} \underline{e}_{l+k}^T \ V_l \ \underline{e}_k \ .\end{aligned} \tag{3.70}$$

Consider the first term of (3.70). Because V_0 is positive definite we have the inequality

$$\underline{e}_k^T \ V_0 \ \underline{e}_k \geq \lambda_{min}(V_0) \ \underline{e}_k^T \ \underline{e}_k \tag{3.71}$$

where $\lambda_{min}(V_0)$ is the smallest eigenvalue of V_0. Moreover,

$$||V_0^{-1}||_2 = \frac{1}{\lambda_{min}(V_0)} . \tag{3.72}$$

From (3.71) and (3.27) it follows that

$$||V_0^{-1}||_2 \sum_{k=0}^{H} \underline{e}_k^T \ V_0 \ \underline{e}_k \geq \sum_{k=0}^{H} \underline{e}_k^T \ \underline{e}_k = \sum_{k=0}^{H} ||\underline{e}_k||_2^2 \ . \tag{3.73}$$

Now consider the second term of (3.70). Due to the Schwarz inequality and from (3.68) and (3.69) we have

$$\begin{aligned}&\left| ||V_0^{-1}||_2 \sum_{l=-H}^{H}{}' \sum_{k=0}^{H} \underline{e}_{l+k}^T \ V_l \ \underline{e}_k \right| \leq \\ &\leq ||V_0^{-1}||_2 \sum_{l=-H}^{H}{}' ||V_l||_2 \sum_{k=0}^{H} ||\underline{e}_{l+k}^T||_2 \ ||\underline{e}_k||_2 \\ &\leq \{||V_0^{-1}||_2 \sum_{l=-H}^{H}{}' ||V_l||_2\}\{ \sum_{k=0}^{H} ||\underline{e}_k||_2^2 - \delta_0^2\}.\end{aligned} \tag{3.74}$$

From (3.70), (3.73) and (3.74) it follows that

$$\delta^2(\varepsilon) \geq \left(\sum_{k=0}^{H} ||\underline{e}_k||_2^2 - \delta_0^2 \right) - \{ ||V_0^{-1}||_2 \sum_{l=-H}^{H}{}' ||V_l||_2 \} \cdot$$

$$\cdot \{ \sum_{k=0}^{H} ||\underline{e}_k||_2^2 - \delta_0^2 \} + \delta_0^2 \geq \delta_0^2. \quad (3.75)$$

This last inequality holds good if (3.69) is satisfied.

3.9 References

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CHAPTER 4

CONCLUSIONS AND FINAL REMARKS

In this thesis it is shown that for a multiple channel transmission system both the optimum linear receiver (minimum symbol error probability) and the optimum linear constraint receiver (minimum symbol error probability under the zero-forcing condition) have the same structure as the optimum linear receiver found by Kaye and George who used the minimum mean square error criterion. Moreover, it is found that by means of the multidimensional Nyquist criterion and the generalization to MDI of a theorem by Lucky for ISI, it is fairly easy in systems for which this latter theorem is valid, to find the optimum tap settings for a finite-length MTDL. The algorithm to calculate the tap settings is further simplified in cases where the noise in unimportant and the sampling instant is smaller than the bit time. If the system to be considered does not satisfy the requirements of the generalization of Lucky's theorem, the calculation of the optimum tap settings requires more complicated optimization methods, such as linear programming or computer search.

Furthermore, it is shown that the outputs of the multiple matched filter as defined in the Section 2.1 on the structure of the optimum linear receiver, form a set of sufficient statistics for estimating the transmitted vector sequence. A multiple whitened matched filter is derived, the output of which is used for ML vector sequence estimation by means of a vector version of the Viterbi algorithm. A modified algorithm, derived by Ungerboeck, is also generalized to combat noise and MDI. If this algorithm is used, the MTDL is omitted and the algorithm is applied directly to the sampled output of the MMF. From analysis of the error performance of the ML receiver it follows that, under a certain constraint, for moderate and large SNRs, the symbol error probability is not substantially affected by MDI, i.e. the symbol error probability is approximated by the error probability for a single pulse. The example of the MAP receiver shows that this kind of receiver only makes sense where there are considerable differences between the a priori probabilities of the transitions in the trellis and the SNR is low.

From a practical point of view the linear receiver is easier to implement than the ML receiver. The latter will require equipment, which is nearly as complicated as a micro-computer. Moreover, the operations with this equipment will be very time-consuming. Hence, the linear receiver is eminently suited for incorporation in systems where a high bit rate at a prescribed low symbol error probability and low equipment costs are first requirements, as is the case in digital cable networks. The application of ML receivers will lie in the field of high quality systems, which operate at a relative low speed, such as communication systems for space vehicles. In cases where the MTDL is unstable or the overall worst-case MDI at the receiver output is greater than unity, the linear receiver is of no use. In those cases the ML receiver can offer a way out.

Comparing the results of this thesis with the theory on monochannel systems it is concluded that ICI plays the same role as ISI. If these two disturbances are considered simultaneously, then MDI can be treated as a generalization of ISI. However, in applying the concepts developed in this thesis, it should be ensured that the conditions of the relevant theorems are satisfied.

ACKNOWLEDGMENT

The author wishes to acknowledge all who contributed to the realization of this thesis. The 200 Mb/s system is implemented by T.Lammers, who also delivered an important contribution to the simulation algorithms. J.Swijghuisen Reigersberg has developed the 20 Mb/s system and E.de Jong programmed the optimization algorithms. Further the author is grateful to L.de Jong of the mathematical department, who was very helpful with the proofs of several theorems. Finally, Marion Garos typed the first draft and Gwenny van Hulsen the final version.

SAMENVATTING

Dit proefschrift behandelt de detektie van synchrone datasignalen, die verzonden zijn over meervoudige kanalen en gestoord worden door ruis, intersymbool- en interkanaalinterferentie.

In hoofdstuk 1 geven we definities van de begrippen intersymbool- en interkanaalinterferentie. Om het gezamenlijk effekt van deze twee storingen aan te geven voeren we de term meerdimensionale interferentie in. Na een korte historische inleiding wordt vervolgens het model van het meervoudige kanaal beschreven, zoals dat in dit proefschrift wordt gehanteerd.

Hoofdstuk 2 is geheel gewijd aan lineaire ontvangers. Eerst leiden we de struktuur van het optimale, lineaire ontvangfilter af. Dit filter bestaat uit twee delen, resp. het meervoudig "matched" filter en de meervoudige, afgetakte vertragingslijn genoemd. Bij de afleiding hanteren we als kriteria minimale symboolfoutenkans of minimale symboolfoutenkans onder de bijvoorwaarde dat de meerdimensionale interferentie nul is. De gevonden struktuur blijkt dan dezelfde te zijn, als de struktuur die gevonden wordt, indien men het kleinste-kwadraten-kriterium gebruikt, zoals Kaye en George hebben gedaan. Verder formuleren we het meerdimensionale Nyquist kriterium, dat overeenkomt met het gegeneraliseerde Nyquist kriterium, zoals Shnidman dat definieert. Er blijkt een eenvoudige uitdrukking te bestaan voor de symboolfoutenkans van systemen die aan dit meerdimensionale Nyquist kriterium voldoen. Daarna worden optimale, realiseerbare, afgetakte vertragingslijnen (d.w.z. afgetakte vertragingslijnen met eindige lengte) beschouwd en algorithmen worden gegeven om de weegfaktoren in verschillende praktische situaties te berekenen. Aan het eind van dit hoofdstuk beschrijven

we twee experimenten, waarop de theorie wordt toegepast, die in dit hoofdstuk is ontwikkeld. Deze voorbeelden hebben betrekking op de transmissie van vier binaire datastromen over een vieraderige kabel. De experimenten zijn uitgevoerd met resp. 5 Mb/s per kanaal en 50 Mb/s per kanaal.

In hoofdstuk 3 onderzoeken we "maximum likelihood"-ontvangers. Teneinde "maximum likelihood sequence estimation" toe te kunnen passen op de ontvangen signalen, tonen we eerst aan, dat de verzameling samplewaarden van de meervoudig "matched" filter uitgangen een "sufficient statistic" vormen voor de gezonden datareeks. Vervolgens worden twee algorithmen voor "maximum likelihood sequence estimation" veralgemeend voor "maximum likelihood vector sequence estimation". Voor het vektor-Viterbi-algorithme definieren we een "whitened matched" filter. Het vektor-Ungerboeck-algorithme maakt rechtstreeks gebruik van de samplewaarden van de meervoudig "matched" filter uitgangen. Bij gebruik van dit algorithme kan de meervoudige, afgetakte vertragingslijn achterwege blijven, terwijl dit algorithme in feite niet ingewikkelder is dan het Viterbi-algorithme. Uit een onderzoek naar de kwaliteit van dit soort ontvangers volgt, dat, onder een bepaalde voorwaarde, voor gemiddelde en grote signaal-ruisverhoudingen de meerdimensionale interferentie de symboolfoutenkans niet noemenswaardig beïnvloedt. Tenslotte wordt nog enige aandacht besteed aan "maximum a posteriori"-ontvangers.

De belangrijkste conclusie van dit onderzoek is, dat meerdimensionale interferentie opgevat kan worden als een veralgemening van intersymboolinterferentie. Diverse belangrijke resultaten uit de theorie omtrent intersymboolinterferentie lenen zich voor generalisatie voor meerdimensionale interferentie.

CURRICULUM VITAE

De auteur werd geboren op 1 maart 1942 te Zevenbergen, waar hij het LO en ULO doorliep. In 1957 behaalde hij het MULO-A-diploma en in 1958 het MULO-B-diploma.
Van 1958 tot 1962 studeerde hij aan de Hogere Technische School "Sint Virgilius" te Breda en legde met goed gevolg het eindexamen af in de afdeling der Elektrotechniek.
Gedurende zijn militaire diensttijd van 1962 tot 1964 verzorgde hij technische dokumentatie voor verbindingsapparatuur.
In 1964 liet hij zich inschrijven in de afdeling Elektrotechniek van de Technische Hogeschool Eindhoven.
Van 1966 tot 1969 was hij als technisch ambtenaar verbonden aan deze Technische Hogeschool en was belast met het ontwerpen van nauwkeurige gelijkspannings- en verschilversterkers.
Het ingenieursdiploma werd behaald in 1969.
Van 1969 tot 1970 was hij werkzaam bij NV Philips' Gloeilampenfabrieken in een ontwerpgroep oscilloscopen.
Sinds 1970 is hij als wetenschappelijk medewerker verbonden aan de Technische Hogeschool Eindhoven en als zodanig werkzaam in de vakgroep Telekommunikatie op het gebied van de digitale lijntransmissie.

STELLINGEN

1

Bij korte meeraderige kabels, bestaande uit evenwijdige geleiders, kan overspraak voor sprongvormige signalen worden vermeden door een geschikt gekozen afsluitnetwerk.

W.van Etten,

"Crosstalkless termination of multiwire cables",

Electronics Letters, 16th October 1975, Vol.11.No.21, pp.505-506.

2

In een meeraderige verbinding voor digitale transmissie kan bestrijding van intersymbool- en interkanaalinterferentie zowel in de eindapparatuur als door de kabelkonstruktie geschieden. Bij de realisering van zo'n verbinding is het daarom aan te bevelen, dat kabel en eindapparatuur door één instantie worden ontworpen, opdat ekonomisch een optimale oplossing gevonden wordt.

W.van Etten and J.van der Plaats,

"Alternatives in multiwire cables for digital transmission",

Electronics Letters, 14th November 1974, Vol.10.No.23, pp.477-478.

3

Het lijkt de moeite waard een onderzoek in te stellen naar de ekonomische en technische aspekten van een meeraderige striplijnkabel voor digitale transmissie.

W.van Etten and J.van der Plaats,

"Alternatives in multiwire cables for digital transmission",

Electronics Letters, 14th November 1974, Vol.10.No.23, pp.477-478.

4

Door korrektiemethoden voor meerdimensionale interferentie toe te passen, zoals in dit proefschrift is beschreven, kan de transmissiekapaciteit van meeraderige kabels beter benut worden dan nu het geval is.

CCITT Joint working party CNC , GM/CNC- No.34-E.

5

In de toekomst kunnen charge-coupled-devices een belangrijke rol gaan spelen bij de realisering van afgetakte vertragingslijnen en matched filters in de ontvangers van digitale transmissiesystemen.

6

Indien men besluit matched filters te gebruiken in een digitaal lijntransmissiesysteem, bestaande uit een aantal identieke repeatersekties, kan bij de realisering van zo'n filter met vrucht gebruik worden gemaakt van de kabel van de volgende sektie. Hierdoor kan het aantal repeaters in een verbinding gehalveerd worden, wat behalve een besparing van elektronika ook een reduktie van de cumulatieve jitter tot gevolg heeft.

7

Bij de schatting van de foutenkans van een "maximum likelihood"-ontvanger gaat Forney er van uit, dat bij een "error event" het gekozen pad een grotere waarschijnlijkheid heeft dan het gezonden pad. Hij ziet echter over het hoofd, dat het gekozen pad ook een grotere waarschijnlijkheid moet hebben dan alle andere mogelijke paden.

G.D.Forney, Jr.,

"Maximum likelihood sequence estimation of digital sequences in the presence of intersymbol interference",

IEEE Trans. on Inf. Th., Vol.IT-18, May 1972, pp.363-378.

8

Gezien de grote overeenkomst tussen "maximum likelihood" detektiealgorithmen en dekodeeralgorithmen voor konvolutiekodes, lijkt het zinvol een onderzoek in te stellen naar de vraag of deze algorithmen met voordeel geïntegreerd kunnen worden.

9

Wat betreft meeraderige kabels voor digitale transmissie zijn er binnen de CCITT tendensen om aanbevelingen te doen met betrekking tot interkanaalinterferentie. Deze tendensen worden echter niet bespeurd met betrekking tot intersymboolinterferentie. Gezien in het licht van de artikelen van Shnidman, Kaye en George en dit proefschrift, komt dat enigszins vreemd voor. Uit deze studies blijkt namelijk, dat intersymbool- en interkanaalinterferentie gelijksoortige verschijnselen zijn.

1) *CCITT Joint working party CNC , GM/CNC - No.29-E,*
GM/CNC - No.34-E,
GM/CNC - No.58-E.

2) *D.A.Shnidman,*
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Bell System Technical Journal, November 1967, pp.2163-2177.

3) *A.R.Kaye and D.A.George,*
"Transmission of multiplexed PAM signals over multiple channel and diversity systems",
IEEE Trans. on Comm. Tech., Vol.COM-18, October 1970, pp.520-525.

10

Het nut van stellingen bij een proefschrift wordt steeds meer in twijfel getrokken. Zij zijn dan ook vaak meer een afspiegeling van de spitsvondigheid van de promovendus dan van diens algemeen wetenschappelijk inzicht. Het zou daarom beter zijn dit algemeen wetenschappelijk inzicht te toetsen door middel van een examen.

W.van Etten, Eindhoven, 18 mei 1976.