Strain gradient plasticity : a tensorial gradient approach

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Strain Gradient Plasticity — A Tensorial Gradient Approach

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1. Introduction

Size-dependent behaviors are observed in many experiments except when the deformation field is uniform [1]. Classical constitutive models are not capable of predicting such behaviors. One approach to capture these size effects is to enhance the constitutive models with plastic strain gradients. However, models incorporating gradients of the (scalar) effective plastic strain are problematic when there is a change in direction of the plastic strain field. A gradient formulation based on the full plastic strain tensor is proposed to address this issue.

2. Scalar gradient model

The scalar gradient model [2] is summarized as:

$$\begin{aligned} \sigma_{eq} &= \sqrt{\tfrac{3}{2}\sigma'_{ij}\sigma'_{ij}} = \sigma_0 + \bar{h}(p - \overline{p}) + hp^n \\ p &= \overline{p} - l^2\nabla^2\overline{p} \end{aligned}$$

where p is the plastic strain accumulation and $()'$ implies the deviatoric part of a tensor. For a beam in pure bending, the equivalent stress evolution is shown in Fig 1. It is observed that a negative (non-physical) result is obtained.

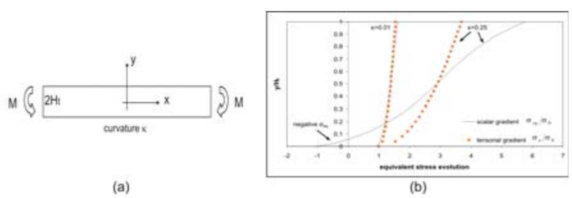

Figure 1: (a) beam in pure bending (b) equivalent stress evolution

3. Tensorial gradient model

The proposed tensorial gradient model is summarized as:

$$\begin{aligned} \sigma_e &= \sqrt{\tfrac{3}{2}(\sigma'_{ij} - x_{ij})(\sigma'_{ij} - x_{ij})} = \sigma_0 + hp^n \\ x_{ij} &= \hat{h}(\varepsilon^p_{ij} - \bar{\varepsilon}^p_{ij}) \\ \varepsilon^p_{ij} &= \bar{\varepsilon}^p_{ij} - l^2\nabla^2\bar{\varepsilon}^p_{ij} \end{aligned}$$

For the same example in Fig 1, non-physical responses are now circumvented with the tensorial gradient model.

4. Smooth plastic strain field

When the plastic strain field is smooth, the scalar gradient model does not suffer from any non-physical responses. In such cases, similar numerical results are obtained for both scalar and tensorial gradient models. This is demonstrated in the flat punch indentation example (Figs 2 and 3).

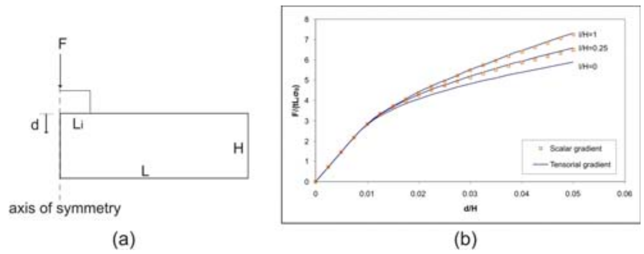

Figure 2: (a) flat punch indentation (b) size effect predictions

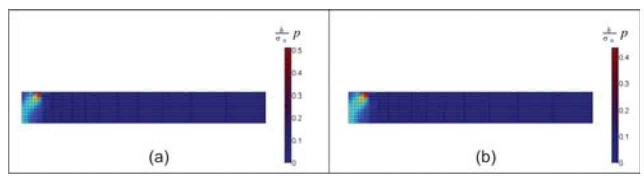

Figure 3: Plastic strain accumulation in the (a) scalar gradient model (b) tensorial gradient model

5. Conclusion

The tensorial gradient model is able to capture size effects without suffering from non-physical responses. For deformations where the plastic strain field is smooth, numerical results from the scalar gradient model are similar to those of the proposed model.

6. Future work

- Incorporating damage mechanisms to model strain softening behaviors
- Utilizing gradient enhancements to avoid mesh dependency issues during softening

References

[1] Fleck, N. A., Hutchinson, J. W. (1997). Strain Gradient Plasticity. *Advances in Applied Mechanics 33*. 295-361.

[2] Peerlings, R.H.J. (2007). On the role of moving elastic-plastic boundaries in strain gradient plasticity. *Model Simul Mater Sci Eng 15*. 109-20.