Periodic structures in 3D mixing flows

Periodic Structures in 3D Mixing Flows

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Introduction

The laminar mixing process can be improved by introducing chaos in the flow. Time-periodic flows produce chaotic trajectories and the chaos is determined by periodic points. Figure 1 represents the flow geometry and the front and back wall induce the time-periodic motion.

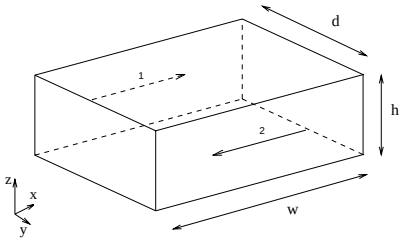

Fig. 1 *3D flow geometry*

Objectives

- study chaotic mixing using periodic point analysis
- develop numerical tools to analyze mixing

Chaotic Mixing

Chaos in the flow is determined by the periodic points. Periodic points return to their original position after one period T and are classified into two groups :

Periodic Points

- elliptic : center of non-mixing regions (islands)
- hyperbolic : center of stretching and folding

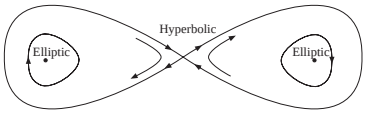

Fig.2 *Periodic points*

Governing Equations

The flow is described by the Stokes equations :

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nu \nabla^2 u - \nabla p + f & \text{in } \Omega, \\ \nabla \cdot u = 0 & \text{in } \overline{\Omega}, \\ u = g & \text{on } \Gamma = \partial\Omega, \end{cases}$$

Numerical Techniques

- Time discretization : Pressure correction method
- Space discretization : Spectral element method

Mixing Analysis

In symmetrical flows, periodic lines cross the plane of symmetry $x = 0$ at the times $t_1 = T/4$ and $t_2 = 3T/4$. This plane is tracked from t_1 and t_2.

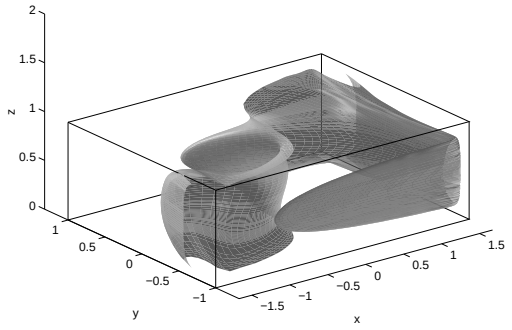

Fig.3 *Tracking of surface $x = 0$ for one period*

The intersection with $x = 0$ is tracked to time $t = T$, and the true position of the periodic lines is found, see Figure 4.

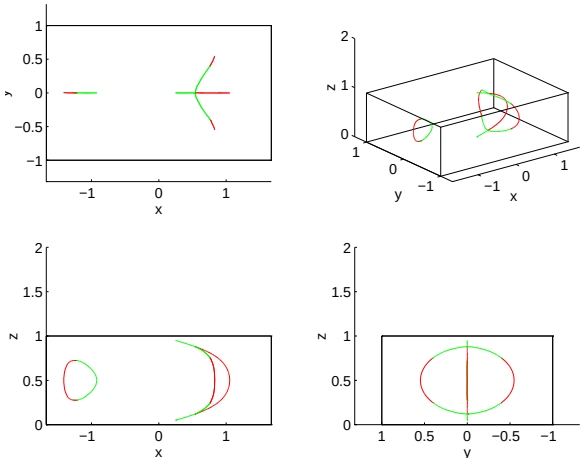

Fig.4 *Red are hyperbolic lines; green are elliptic lines*

Conclusions

Periodic structures, which consist of lines, are found and classified in 3D cavity flows.