

# Collision warning system based on probability density functions

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# Collision Warning System based on Probability Density Functions

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**Abstract**—In this paper, a collision warning method between the host vehicle and target object(s) is studied. A probabilistic collision warning method is proposed, which is, in particular, useful for objects, e.g. vulnerable road users, which trajectories can rapidly change heading and/or velocity with respect to time. A vehicle is equipped with the probabilistic collision warning system and its functionality is validated in experiments with a bicyclist as target object.

**Index Terms**—Advance driver assistant system, collision warning system, vulnerable road user, probability density function.

## I. INTRODUCTION

In this paper, a collision warning system is designed, which is able to handle the unpredictable behavior of vulnerable road users (e.g. bicyclists and pedestrians). Today's cars are more and more equipped with advance driver assistant (ADA) systems. One of the most popular ADA systems in the research field are active safety systems, i.e. collision warning and avoidance (CW/A) systems. For decades CW systems are only used in air traffic control [1], but recently the focus is also on automotive applications. The current CW/A systems are strongly focused on car-to-car applications, i.e. the host vehicle with CW/A system prevents an accident with another vehicle. Most of these car-to-car CW/A applications only focus on head-on collision [2],[3],[4]. Regrettably, not all road users are protected with car-to-car collision warning systems. For instance, 750 road fatalities are recorded in the Netherlands in 2008, of which 181 are bicyclist, 75 are moped driver and 62 pedestrians [5]. Over 42 % of the total road fatalities are vulnerable road users (VRUs).

Before customer cars are equipped with collision warning systems, which are able to handle all road

users in all possible collision scenarios, many technical and scientific challenges must be faced such as robust classification of all road users, accurate acceleration measurements for a better future prediction, handling the unpredictable behavior of vulnerable road users, etc. The focus of the current paper is handling the unpredictable behavior of vulnerable road users. The main focus is on bicyclists and/or moped drivers with their nonholonomic constraints, but the general theory is applicable for pedestrians as well. The general theory is based on probability density functions (PDFs).

In recent literature a few methods are described based on probability density functions. In [2],[3] an ADA system is designed, which is based on an algorithm that calculates the risk factor. The risk factor is based on the PDF of steering maneuvers. The model used in the algorithm is based on a single track model, with car state variables. Since the model is based on car-to-car scenarios it considers state variable of a car, based on Kamm's circle of frictional forces. Therefore, the proposed method is not robust to different road users, e.g. bicyclists, pedestrians. Secondly, the focus of the algorithm are steering maneuvers, which results that a change of velocity or acceleration in the longitudinal direction is not taken into account.

A general method for computing the risk of a collision, which is based on PDFs, is described in [4] for a collision mitigation by braking (CMbB) application. The model predicts trajectories of objects that are supposed to follow straight line segments and circle segments. The probability density function is approximated with the use of particle filtering. The PDFs of the acceleration behaviour of a driver is an empirical distribution and the measurement noise is based on bimodal Gaussian probability density functions. Regrettably, the research of [4] is limited to head-on car-to-car collisions, which is described as forward collision

avoidance systems.

A Monte Carlo method is used for computing the probability of an impact of airplanes in [1]. The advantage of the Monte Carlo method is that complex PDFs, e.g. non-Gaussian, multi-modal, can be used in complicated state space models. The disadvantage is the increase in calculation time. The authors of [1] propose a small adaptation to the usual Monte Carlo approach. The future trajectories of the objects are approximated by individual path segments. With the use of linear path segments in combination with change points to change the heading of the object, more complicated and time-consuming calculations are no longer necessary. This approach is feasible, since the position error of airplanes is relative small compared to absolute positions. However, the position error of VRUs is relative large compared to absolute positions, since the measured positions are much smaller compared to airplanes.

In this paper a probabilistic method of collision warning is proposed, which is, in particular, useful for objects, e.g. VRUs, which trajectories can rapidly change heading and/or velocity with respect to time. The complexity is limited to decrease computation time. The collision warning system is validated during experiments in a Citroën C4.

This paper is organized as follows. In Section II, definitions and theorems that are used in the remainder of the paper are presented. The collision warning system based on probability density functions is explained in detail in Section III. In Section IV, the proposed collision warning system is validated in experiments. The conclusions are presented in Section V.

## II. PRELIMINARIES

Throughout this paper numerous theoretical results will be used. In this section these theoretical results and definitions are briefly recalled.

*Theorem 1:* [6] A function  $f(x)$  is a probability density function for some continuous random variable  $X$  if and only if it satisfies the properties

$$f(x) \geq 0; \quad (1)$$

for all real  $x$ , and

$$\int_{-\infty}^{\infty} f_X(x) dx = 1. \quad (2)$$

*Proof:* See page 65 of [6]. ■

*Definition 1:* [6] Random variables  $X_1, \dots, X_k$  are independent if for every  $a_i < b_i$ ,

$$P(a_1 \leq X_1 \leq b_1, \dots, a_k \leq X_k \leq b_k) = \prod_{i=1}^k P(a_i \leq X_i \leq b_i). \quad (3)$$

*Definition 2:* [7] A joint probability density function for the continuous random variables  $X$  and  $Y$ , denoted as  $f_{XY}(x, y)$ , satisfies the following properties:

- $f_{XY}(x, y) \geq 0$  for all  $x, y$ ;
- $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x, y) dx dy = 1$ ;
- for any region  $R$  of two-dimensional space,

$$P([X, Y] \in R) = \iint_R f_{XY}(x, y) dx dy. \quad (4)$$

*Theorem 2:* [6] Suppose that  $X$  is a continuous random variable with probability density function  $f_X(x)$  and assume that  $Y = h(X)$  defines a one-to-one transformation from  $A = \{x | f_X(x) > 0\}$  on to  $B = \{y | f_Y(y) > 0\}$  with inverse transformation  $x = w(y)$ . If the derivative  $(d/dy)w(y)$  is continuous and nonzero on  $B$ , then the probability density function of  $Y$  is

$$f_Y(y) = f_X[w(y)] \left| \frac{d}{dy} w(y) \right| \quad y \in B. \quad (5)$$

*Proof:* See page 198 of [6]. ■

*Theorem 3:* [6] Suppose that  $X = (X_1, X_2, \dots, X_k)$  is a vector of continuous random variables with joint probability density function  $f_X(x_1, x_2, \dots, x_k) > 0$  on  $A$ , and  $Y = (Y_1, Y_2, \dots, Y_k)$  is defined by the one-to-one transformation

$$Y_i = u_i(X_1, X_2, \dots, X_k) \quad i = 1, 2, \dots, k. \quad (6)$$

The inverse transformation is defined as  $x = w(y)$ . If the Jacobian is continuous and nonzero over the range of the transformation, then the joint probability density function of  $Y$  is

$$f_Y(y_1, \dots, y_k) = f_X(w_1(y_1, \dots, y_k), \dots, w_k(y_1, \dots, y_k)) |J|, \quad (7)$$

where  $J$  is the Jacobian, which is given by:

$$J = \begin{vmatrix} \partial x_1 / \partial y_1 & \partial x_1 / \partial y_2 & \dots & \partial x_1 / \partial y_k \\ \partial x_2 / \partial y_1 & & & \vdots \\ \vdots & & & \\ \partial x_k / \partial y_1 & \dots & & \partial x_k / \partial y_k \end{vmatrix}. \quad (8)$$

*Proof:* See page 206 of [6]. ■

*Definition 3:* [6] If the pair  $(X_1, X_2)$  of continuous random variables has the joint probability density function  $f(x_1, x_2)$ , then the marginal probability density functions of  $X_1$  and  $X_2$  are

$$f_1(x_1) = \int_{-\infty}^{\infty} f(x_1, x_2) dx_2, \quad (9)$$

and

$$f_2(x_2) = \int_{-\infty}^{\infty} f(x_1, x_2) dx_1. \quad (10)$$

### III. PROBABILISTIC RISK ESTIMATION

In this section, the approach of the probabilistic collision warning system is explained in detail. First, in Section III-A, the inputs are defined for the probabilistic risk estimation module. Then, in Section III-B, the trajectory is predicted for two cases, namely constant velocity with constant heading and constant acceleration with constant curvature. The probabilistic trajectory prediction is explained in Section III-C. This approach is particularly interesting to predict trajectories of objects, of which the predicted trajectory is uncertain in time due to sudden unexpected movements of the object. The objects in this document are currently only bicyclists. In Section III-D, the collision probability is determined between object and host. And finally, in Section III-E, the overall probabilistic risk estimation algorithm is given.

#### A. Inputs

There are two types of input data necessary for an accurate trajectory prediction of both host and object. The required input data consists of probability density functions and the measured states of the object and host vehicle. Individual PDFs are determined for each object and host vehicle. These distributions represent the predicted behaviour of the VRU and host vehicle. The measured states are defined as the generalized position, velocity and acceleration and are defined as a vector *SVA* according to:

$$SVA = (x \ y \ \phi \ \dot{x} \ \dot{y} \ \dot{\phi} \ \ddot{x} \ \ddot{y} \ \ddot{\phi}), \quad (11)$$

where the states of the host vehicle and objects are indicated by subscripts  $h$  and  $o$ , respectively. Not all states of (11) are necessary which will become clear in Section III-B. Currently, the probabilistic risk estimation module consists of 1 host vehicle and  $i$  objects, with  $i = 1, \dots, N$ .

The *SVA* of the host and objects are absolute quantities, since the local coordinate frame of the host is globally fixed at the time of calculation. The coordinate frame is located in front of the host vehicle with the  $x$ -axis pointing in the longitudinal direction and the  $y$ -axis pointing in the leftwards lateral direction. The orientation is the angle between the heading of the object or host and the  $x$ -axis, see Figure 1.

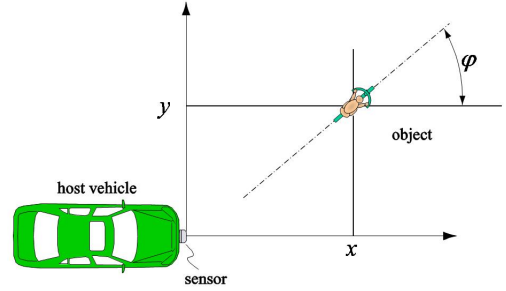


Fig. 1. Host vehicle coordinate frame.

Note that the position of the object is given at the center of gravity. Common sensors are not able to detect the center of gravity, but detect the object boundaries. Since the model of the object and host is based on a point mass, this effectively means that the point mass is located at the boundary of the object.

#### B. Trajectory Prediction

In this section the prediction of the objects and host trajectories are explained. First, the trajectory prediction is explained for objects and host which are driving with a constant velocity and no curvature. The second trajectory prediction is based on objects and host with a constant acceleration and a constant curvature, which is not equal to zero. This division is made, since the object trajectory prediction is experimentally validated in Section IV, and the sensors that are used in the experiments are not able to determine acceleration and curvature.

1) *Constant Velocity & Constant Heading:* The trajectory of object and host, with a constant velocity and constant heading, is predicted using the following continuous time kinematic model:

$$\begin{aligned} \dot{x} &= v \cos(\phi) \\ \dot{y} &= v \sin(\phi) \\ \dot{\phi} &= 0 \end{aligned}, \quad (12)$$

where the longitudinal velocity  $v$  at  $t = t_0$  is determined as

$$v(t_0) = \sqrt{\dot{x}^2(t_0) + \dot{y}^2(t_0)}. \quad (13)$$

If the orientation  $\phi$  is not measured by the sensor(s), it is assumed that the heading, i.e. yaw angle, is aligned with the velocity direction of the object, which is defined as

$$\phi(t_0) = \arctan\left(\frac{\dot{y}(t_0)}{\dot{x}(t_0)}\right). \quad (14)$$

The analytical solution of the kinematic model of (12) is easily derived and given as

$$\begin{aligned} x(t) &= \left(v(t_0) \cos(\phi(t_0))\right)t + x(t_0) \\ y(t) &= \left(v(t_0) \sin(\phi(t_0))\right)t + y(t_0) \end{aligned} \quad (15)$$

This model is used in the experimental validation in Section IV.

### 2) Constant Acceleration & Constant Curvature:

The trajectory of object and host, with a constant acceleration and constant curvature, is predicted using the following continuous time kinematic model:

$$\begin{aligned} \dot{x} &= v \cos(\phi) \\ \dot{y} &= v \sin(\phi) \\ \dot{\phi} &= \kappa v \\ \dot{v} &= a_{long} \\ \dot{a}_{long} &= 0 \\ \dot{\kappa} &= 0 \end{aligned}, \quad (16)$$

where the longitudinal acceleration  $a_{long}$  and curvature  $\kappa$  at  $t = t_0$  are determined according to

$$a_{long} = \ddot{x}(t_0) \cos(\phi(t_0)) + \ddot{y}(t_0) \sin(\phi(t_0)) \quad (17)$$

and

$$\kappa(t_0) = \frac{\dot{\phi}(t_0)}{v(t_0)}, \quad (18)$$

respectively. If the yaw rate  $\dot{\phi}$  is not directly measured by the sensor(s), the curvature is determined according to

$$\kappa(t_0) = \frac{a_{lat}(t_0)}{v(t_0)^2}, \quad (19)$$

with

$$a_{lat} = -\ddot{x}(t_0) \sin(\phi(t_0)) + \ddot{y}(t_0) \cos(\phi(t_0)). \quad (20)$$

Again, the kinematic model (16) is solved analytical, resulting in the following two equations

$$\begin{aligned} x(t) &= \frac{1}{\kappa(t_0)} \sin\left(\kappa(t_0)\left(\frac{1}{2}a(t_0)t^2 + v(t_0)t\right) + \phi(t_0)\right) \\ &\quad + x(t_0) - \frac{1}{\kappa(t_0)} \sin(\phi(t_0)) \\ y(t) &= -\frac{1}{\kappa(t_0)} \cos\left(\kappa(t_0)\left(\frac{1}{2}a(t_0)t^2 + v(t_0)t\right) + \phi(t_0)\right) \\ &\quad + y(t_0) - \frac{1}{\kappa(t_0)} \cos(\phi(t_0)) \end{aligned} \quad (21)$$

where  $a$  is equal to the longitudinal acceleration  $a_{long}$ . The model (21) is currently developed for future research application, since the model can not be experimentally validated. The current host vehicle, which we use during experiments, is not equipped with sensors which are able to determine real-time the accelerations and/or yaw rate.

In the next section we determine the likelihood that the host and object are following their trajectories. This estimation is performed with the use of probability density function.

### C. Probabilistic Trajectory Prediction

Let us determine the probability of the future trajectory of the object, e.g. VRU, and the host with a constant velocity and a constant heading, which is explained in Section III-B.1. Note that the subscripts  $o$  and  $h$  are still not used, since the probabilistic trajectory prediction theory holds for both object and host. Bare in mind that the starting position of the host  $(x_h(t_0), y_h(t_0)) = (0, 0)$ , see Figure 1. We assume a certain predefined distribution for the probability of the velocity and the heading of both object and host. The distribution of the velocity and heading is not defined in this section, since this theory is applicable for all distributions that satisfy the properties of Theorem 1, which is defined in Section II. Therefore, the distribution of the velocity and yaw angle is defined as  $f_v$  and  $f_\phi$ , respectively.

The joint probability density function of the position  $f_{x(t),y(t)}(x(t), y(t))$  is dependent on the PDF of the velocity  $f_v$  and yaw angle  $f_\phi$  as follows:

$$\begin{aligned} f_{x(t),y(t)}(x(t), y(t)) &= \\ f_{v(t_0)}\left(v(x(t), y(t))\right) f_{\phi(t_0)}\left(\phi_0(x(t), y(t))\right) |J|, \end{aligned} \quad (22)$$

where  $J$  is the Jacobian and defined as

$$J = \begin{vmatrix} \frac{\partial v(t_0)}{\partial x(t)} & \frac{\partial v(t_0)}{\partial y(t)} \\ \frac{\partial \phi(t_0)}{\partial x(t)} & \frac{\partial \phi(t_0)}{\partial y(t)} \end{vmatrix}, \quad (23)$$

as explained in Theorem 3, assuming  $v(t_0)$  and  $\phi(t_0)$  are independent. It becomes clear in (22) that (15)

should be inverted as follows

$$\begin{aligned} v(t_0) &= \sqrt{\left(\frac{x(t)-x(t_0)}{t}\right)^2 + \left(\frac{y(t)-y(t_0)}{t}\right)^2} \\ \phi(t_0) &= \arctan\left(\frac{y(t)-y(t_0)}{x(t)-x(t_0)}\right) \end{aligned} \quad (24)$$

The Jacobian is given as

$$J = \frac{1}{t\sqrt{(x(t)-x(t_0))^2 + (y(t)-y(t_0))^2}}. \quad (25)$$

Define a grid, which is in front of the host vehicle, with numerous  $x$ - and  $y$ -values. For each future timestep  $t$ , the joint probability density function is determined according to (22), i.e. the likelihood of the objects and host future trajectory is determined. In Figure 2, the grid, which is in front of the host vehicle, is shown.

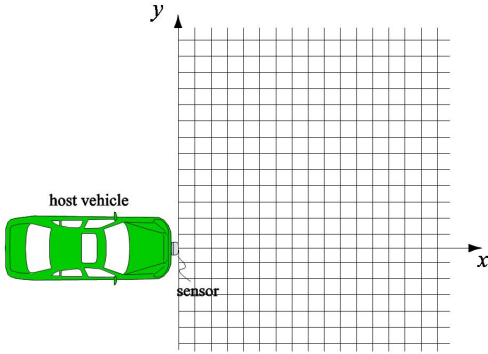


Fig. 2. Grid in front of host vehicle.

Due to calculation time limitations, we are limited to 2 distributions, namely  $f_v$  and  $f_\phi$ . If we choose more than 2 distributions, the one-to-one transformation of (24), as explained in Theorem 3, results in more outputs instead of only the position  $(x, y)$ . Then, the marginal PDF has to be calculated, see Definition 3, to limit the result of the joint probability density function to the position  $(x, y)$ . Determining the marginal PDF is very calculation time consuming, due to its integral.

Note that the probabilistic path prediction for more complicated trajectories, e.g. (21), is solved in a similar fashion. In the next section, we determine the collision probability.

#### D. Collision Probability

In Section III-C the future trajectories of object and host are determined stochastically. Based on these

future trajectories, the collision probability is determined in this section. The area of overlapping predictions of object and host is defined as the collision area. The volume of probability density function in the collision area is equal to the probability that the object and host are located in the collision area at the same time. The volume of the probability density function is equal to (4) in Definition 2. Let us assume the probabilities of object  $i$  and host, respectively  $P_{oi}$  and  $P_h$ . Then, the collision probability is defined as

$$P_i = P_{oi}([X, Y] \in R) \cdot P_h([X, Y] \in R), \quad (26)$$

where  $X$  and  $Y$  define the collision area  $R$  and where  $P_{oi}$  and  $P_h$  are independent. The continuous integral of (4) is solved with the use of Monte Carlo integration [8], where the ranges  $X$  and  $Y$  are uniformly distributed over the collision area.

#### E. Probabilistic Risk Estimation Algorithm

The probabilistic risk estimation algorithm is subdivided into four steps to limit the calculation time for real time applications. First, the minimum deterministic distance between each object  $i$  and host is determined. Second, the collision probability is determined for each object  $i$  and host for the time interval around the corresponding time of the minimum distance of object  $i$  and host. Then, the most important object (MIO) is determined. We have chosen to combine the collision probability between object  $i$  and host and the collision time, since the collision warning system should only activate when the collision probability is high and the time-to-collision is low. Finally, if the MIO-value exceeds a threshold, the collision warning system is activated, see Algorithm 1.

## IV. EXPERIMENTAL RESULTS

In this section an experiment is performed to validate the probabilistic collision warning system, which is proposed in Section III. In Section IV-A the experimental setup is presented and the experimental result is discussed in Section IV-B.

#### A. Experimental Setup

The probabilistic risk estimation algorithm is integrated into a Citroën C4. The object's position and velocity is determined with lidar (OMRON Laser Radar). The collision warning algorithm runs on a dSpace Autobox with a sample rate of 10 Hz. Both signal processing and algorithm implementation are executed in Matlab/Simulink and are real-time monitored and tuned in dSpace ControlDesk.

1. Calculate deterministic the minimum distance ( $\Delta d_i$ ) between the host vehicle and object  $i$  and corresponding time  $t_i$  for the time interval  $T_{RD} = [0, t_{max}]$ . Here,  $t_{max}$  is defined as the maximum time horizon to determine collision probabilities.

$$\Delta d_i = \min \left( \sqrt{(x_h(T_{RD}) - x_i(T_{RD}))^2 + (y_h(T_{RD}) - y_i(T_{RD}))^2} \right) \quad (27)$$

2. Calculate the collision probability, based on probability density functions, for the time interval  $T_i = \left[ t_i - \frac{t_{range}}{2}, t_i + \frac{t_{range}}{2} \right]$ , where  $t_{range}$  is a constant that determines the time range around the time  $t_i$  that is determined in step 1, for each object  $i$ ,

$$P_{T_i} = P_h([X, Y] \in R) \cdot P_{oi}([X, Y] \in R), \quad (28)$$

where  $P_h$  and  $P_{oi}$  are probabilities of the collision area  $R$  of the host vehicle and object  $i$ , respectively. The probability is defined as (4).

3. Calculate most important object (MIO) for all objects  $i$  and time intervals  $T_i$

$$\text{MIO} = \max_i \left( \frac{P_i}{T_i} \right). \quad (29)$$

4. Threshold the MIO to determine if the system should activate.

$$\text{If } (\text{MIO} > \text{threshold}) \rightarrow \text{Activate Warning}$$

#### Alg. 1. Probabilistic Risk Estimation Algorithm

### B. Experimental Results

Let us assume a scenario where a bicyclist is coming from the right and is passing in front of the vehicle. Since the coordinate frame is located in front of the host vehicle, the position of the lidar is always  $(0, 0)$ , see Figure 1. In this example we are only interested in the position  $(x, y)$ , orientation  $\phi$  and velocity  $v$  of (12), since the lidar is not able to determine the accelerations and yaw rate real-time of the bicyclist, see Figures 3 and 4.

The joint probability density function of both vehicle and bicyclist are solved with the trajectory prediction of Section III-B.1 and the probabilistic trajectory prediction of Section III-C. A Gaussian distribution is chosen for the random behaviour of the bicyclist and vehicle. Standard deviations of  $(0.05, 0.01)$  and  $(0.1, 0.1)$  are chosen for both 'uncertain' parameters, i.e. velocity  $v$  and heading  $\phi$  of host vehicle and bicyclist, respectively. Let us take a closer look at the time intervals of 0, 1, 2 and 3 seconds. In Figures 5 and 6, the joint PDFs of the vehicle and bicyclist are shown for the time intervals of 0 and 1 seconds and 2 and 3 seconds in a three dimensional and two dimensional

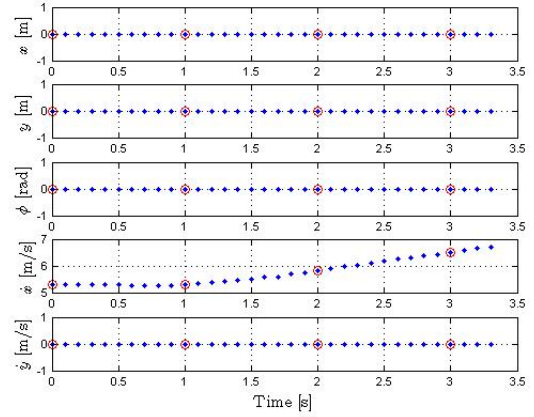


Fig. 3. The position, orientation and velocity of the host vehicle.

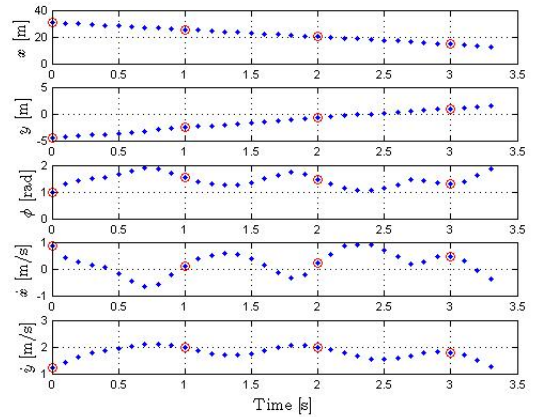


Fig. 4. The position, orientation and velocity of the bicyclist.

view, respectively. The largest MIO, see Algorithm 1, is shown for both time intervals with the corresponding collision probability and time-to-collision.

Figures 5 and 6 show that the probabilities of both vehicle and bicyclist are different due to the chosen standard deviation of the Gaussian distribution. The joint PDF of the bicyclist is lower in height and spread over a larger area compared to the joint PDF of the host. This means that the future position of bicyclist is more unpredictable than the host vehicle's position. Both joint PDFs are overlapping each other, i.e. there is a possibility that a collision can occur. The height of both joint PDFs of bicyclist and host vehicle are increasing in time, while the time-to-collision is decreasing, i.e. the future position becomes more reliable.

In Figure 7 the collision probability, time-to-collision and MIO-value are shown for the entire scenario.

Figure 4 shows that the forward velocity of the bi-

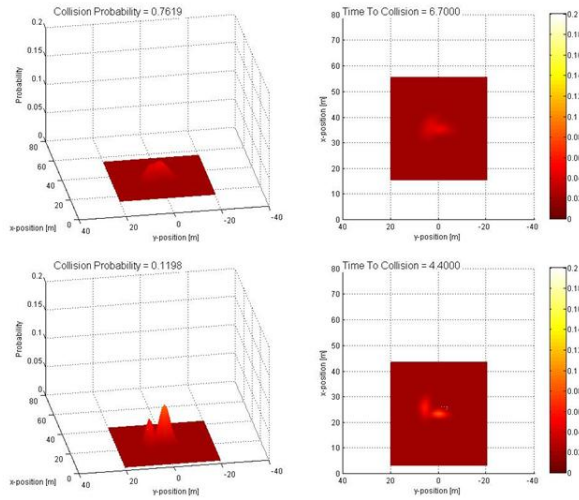


Fig. 5. The collision probability and time-to-collision for  $t = 0, 1$  [s].

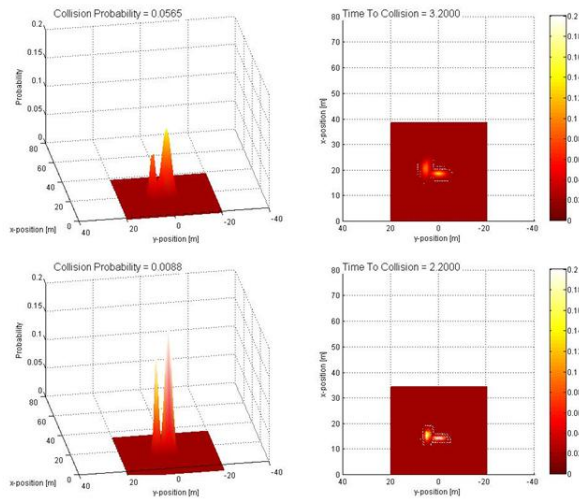


Fig. 6. The collision probability and time-to-collision for  $t = 2, 3$  [s].

cyclist varies a little. The measurement uncertainties influence the collision probability and time-to-collision, see Figure 7. The TTC varies with the same frequency as the measured velocity of the bicyclist. Therefore, it is important that the inputs are measured as good as possible.

## V. CONCLUSION

In this paper a probabilistic risk estimation algorithm is given. The risk estimation algorithm is based on probability density functions. The stochastic approach corresponds to the nature of vulnerable road users and makes the approach feasible for vulnerable road users in general.

Although PDFs require numerical intensive calculations, it is shown in an experimental environment

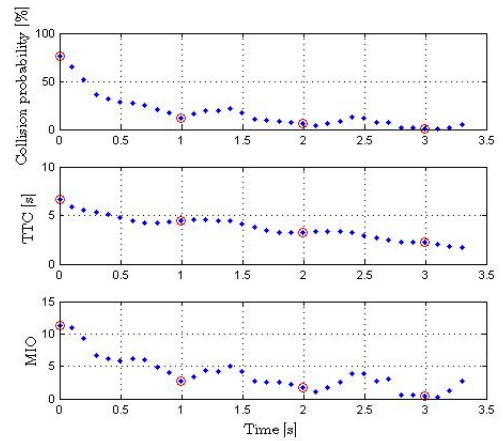


Fig. 7. The collision probability, TTC and MIO value for the entire scenario.

that due to efficient calculation a real-time implementation is feasible.

Currently, the behaviour of the bicyclist and host vehicle are estimated and simplified with the use of Gaussian distributions. Different distributions result in different estimated behaviours of the bicyclist and host vehicle. First, it is recommended to determine distributions that represent realistic behaviour of bicyclists. Second, the distributions should be expanded for all road users.

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