New research directions in flexural member failure at an interior support (Interaction of web crippling and bending moment)

By tests carried out by Hofmeyer [Hofm96b], it can be concluded that at least for members with small sections radii, only two mechanisms occur: web crippling failure and bending moment failure. Of course the web crippling failure is influenced by the bending moment and the bending moment failure is influenced by the concentrated load. For these failure modes, an interaction formula is not needed if the failure modes can be modeled directly.

To reduce the limitations of current design rules, the authors suggest that member failure at an interior support should be modeled by analytically describing each of the possible failure mechanisms and by leaving out the paradigm of the three failure modes: web crippling, bending moment and combined web crippling and bending moment .

For analytically describing each of the possible failure mechanisms, it is necessary to determine all possible mechanisms, by experimental research.

As already mentioned, Bakker has modeled one failure mechanism by describing the mechanism initiation load for the rolling mechanism for a concentrated load and a small bending moment.

In practice, as said before, the yield arc mechanism also can occur. Further, sections may be loaded by a concentrated load and a large bending moment, and for the rolling mechanism, it is preferred to know the ultimate strength.

A new research project has been launched at Eindhoven University of Technology. Sections and loading combinations as occurring in practice will be studied. Using this information a test series has been set up. During the research focus will be made on failure behaviour and load-web deformation behaviour. The experimental research, will be verified using advanced finite element programs. The experimental test data and finite element calculations should lead to the insight for developing an analytical model describing the member behaviour at an interior support for loading situations in practice. A part of the experimental research will be presented in a paper presented at the 13th International Specialty Conference on Cold-Formed Steel Structures in Missouri USA [Hofm96b].

LITERATURE

[Berg79a] Bergfelt, A. and Edlund, B. 1979, Effects of web buckling in light gauge steel beams, Rhodes, J. and Walker, A.C.: Thin-walled Structures, Recent Technical Advances and Trends in Design, Research and Construction.

[Bakk86a] Bakker, M. and Peköz, T., 1986, Comparison and evaluation of web crippling prediction formulas, EUT-report 86-B-01, Eindhoven University of Technology.

[Bakk92a] Bakker, M., 1992, Web crippling of cold-formed steel members, Dissertation Eindhoven University of Technology, The Netherlands.

[Gdin87a] Deutsche Norm DIN 18807. Stahltrapezprofile. Durchfürung und Auswertung von Tragfähigkeitswerte durch Berechnung, Juni 1987.

[Hetra78a] Hetrakul, N., 1978, Webs for cold-formed steel flexural members. Structural behaviour of beam webs subjected to web crippling and a combination of web crippling and bending. Civil engineering study 78-4, Structural series. Rolla: University of Missouri-Rolla, Department of Civil Engineering.

[Hofm96b] Hofmeyer, H., Kerstens, J.G.M., Snijder, H.H., Bakker, M.C.M. Research on the Behaviour of Combined Web Crippling and Bending of Steel Deck Sections, to be published, 13th International Speciality Conference on Cold-Formed Steel Structures, St. Louis, Missouri, U.S.A., October 18-19, 1996

[Pekö85a] Peköz, T. and Muzaffer, Y., Partial Stress Distribution in Cold-formed Steel. Journal of Structural Engineering, Vol. 111, No. 6, June, 1985.

[Rein83a] Reinsch, W., 1983, Das Kantenbeulen zur rechnerischen Ermittlung von Stahltrapezbleck-Tragern, Dissertation 17. Darmstadt: Technische Hochschule Darmstadt, Germany.

[Rsdr74a] 1974, Richtlijnen voor de berekening van Stalen Dakplaten-RSD 1974, Dumebo en Staalbouwkundig Genootschap, The Netherlands.

[Tsai86a] Tsai, Y.M./Crisinel, M., 1986, Moment redistribution in continuous profiled sheeting. Thin-walled metal structures in buildings. IABSE preceding 49, pp. 107-114.

[Wint47a] Winter, G. Strength of Thin Steel Compression Flanges, Transactions, American Society of Civil Engineers, Vol. 112, 1947, pp. 527-554.

[Vaes95a] Vaessen, M. On the elastic web crippling stiffness of thin-walled cold-formed steel members, Graduate Thesis, Eindhoven University of Technology, The Netherlands.

European Workshop
THIN-WALLED STEEL STRUCTURES
26-27 Sept., 1996, Krzyżowa (Kreisau), Poland

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NEW RESEARCH DIRECTIONS IN FLEXURAL MEMBER FAILURE AT AN INTERIOR SUPPORT (INTERACTION OF WEB CRIPPLING AND BENDING MOMENT)

SUMMARY

Design rules describing failure at an interior support of cold-formed steel flexural members are of an empirical nature. This is probably due to the complex character of the failure mechanisms, which makes an analytical approach difficult. An overview of research on this subject has been made. The Bakker model is reviewed. This is the first analytical model accurately describing one of the failure mechanisms occurring for a concentrated load and a small bending moment. A new research project is introduced for extending the Bakker model for a concentrated load and a medium or large bending moment.

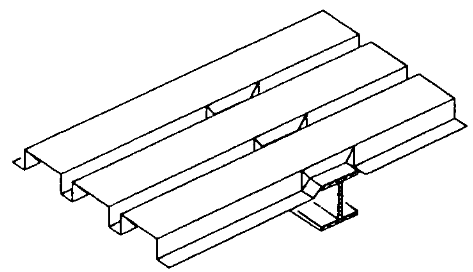

Figure 1, web crippling of cold-formed sheet at a interior support

1 INTRODUCTION

At an interior support, a cold-formed steel flexural member, for example a roof panel, floor panel, or a beam, is generally subjected to both a concentrated load and a bending moment. Due to these loads, the member may fail near the interior support.

If the member webs are strongly indented by the concentrated load and this is thought to be the cause of failure, the failure mechanism is called web crippling, as shown in figure 1.

A flexural member can also fail by bending moment. A bending moment causes compression stresses in one flange and tension stresses in the other flange. The member flange under tension can yield or the flange under compression can buckle and yield.

At this moment, it is believed that failure at an interior support can be described by web crippling, by bending moment, or by interaction of web crippling and bending moment. The failure interaction of web crippling and bending moment is predicted by a function of the web crippling strength and the bending moment strength.

2 EXISTING MODELS FOR FAILURE AT AN INTERIOR SUPPORT

EMPIRICAL MODELS

Empirical models for web crippling and for combined web crippling and bending moment are developed using experimental test data. The attention during the experimental research for empirical models has been focused on the registration and prediction of the ultimate load only.

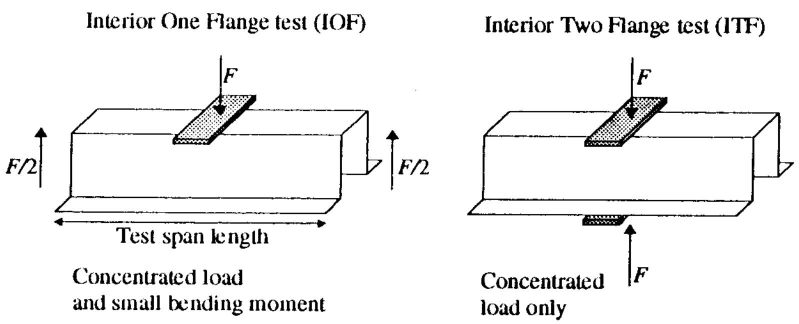

Figure 2, experimental web crippling tests

For web crippling, experiments are carried out as shown in figure 2. The members are loaded with a concentrated load only or with a concentrated load and a small bending moment. For the last load combination mentioned, it is assumed that the small bending moment has no influence on the ultimate concentrated load. The ultimate load is registrated. For both loading conditions as shown in figure 2, the ultimate load can be described as a function of the member properties, as shown below:

$$F_u = k * (t^2 * f_y) * k_{f_y} * k_{\theta_w} * k_{r_i} * k_{L_{1b}} * k_{b_w}$$

- k: This is a constant without dimension.
- t: Steel plate thickness.
- f_y: Steel yield strength.
- k_{fy}: This factor brings the steel yield strength into account. The steel yield strength is not linear correlated with the web crippling load and therefore this factor is needed.
- $k_{\theta w}$: This factor, without dimension, takes the influence of the angle between web and flange into account. If this angle decreases, the web crippling load decreases also.
- k_{ri}: This factor, without dimension, brings the corner radius influence into account. If the corner radius increases, the load eccentricity relatively to the webs increases. Therefore the web crippling load will be lower.
- k_{Lb}: This factor, without dimension, takes the influence of the support width into account. If the support width increases, the load concentration becomes less pronounced. Therefore the web crippling load will be higher.

- k_{bw}: This factor, without dimension, takes the influence of the web width into account. If the web width increases, there is a better opportunity for the web to bend out. This makes the web crippling load lower.

The k factors are determined by the interpretation of test results of many web crippling tests, as shown in figure 2. The factors can be different for members having one or two webs, and for members subjected to different load conditions.

A detailed overview of the research carried out on web crippling is presented in [Bakk92a].

For bending moment, an other approach is used. A section subjected to a bending moment only, has a flange under compression and a flange under tension. The flange under compression will buckle. A buckled plate's ultimate load is larger than the buckling load.

The stress distribution in a buckled plate will differ from the distribution before buckling, as shown in figure 3 (top half). Von Karmann introduced the effective width theory in which it is assumed that, after plate buckling, all compression stress is distributed over two effective zones at the plate edges. This is also shown in figure 3 (bottom half). The ineffective zones carry no stress. According to Von Karmann the effective width theory is only valid if the compression stress at the edges equal the yield stress. Winter modified the Von Karmann theorem which resulted in a formula useful for stresses lower than the yield stress [Wint47a].

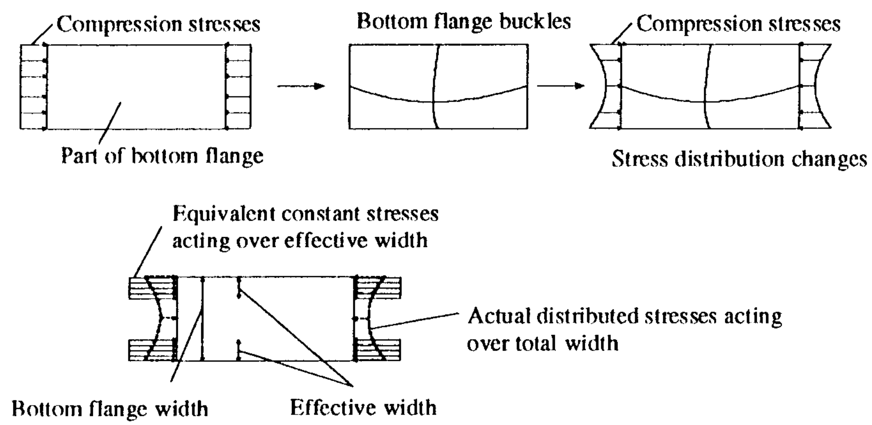

Figure 3

Using the effective width theory, a member cross section can be seen as a cross section built up out of the parts under tension and the effective parts under compression. For this cross section, the ultimate bending moment can be calculated as the bending moment for which the yield stress is reached in either the compression or the tension flange.

The member webs are subjected to a stress gradient containing compression and tension stresses. Due to the compression stresses the webs buckle. The influence of web buckling on the ultimate bending strength can also be taken into account by means of a effective width approach. However, since the web is subjected to a stress gradient, the effective width formulae for the web will differ from those for the flange under compression [Berg79a].

Later, it was believed that it is possible to permit the tension flange to yield until the compression flange fails. This implicates that the web zone near the tension flange yields, as shown in figure 4 at the middle. Pekōz carried out research to permit both the tension and compression flange to yield before failure [Pekö85a]. This implicates that web zones near the flanges will yield, as shown in figure 4 at the right.

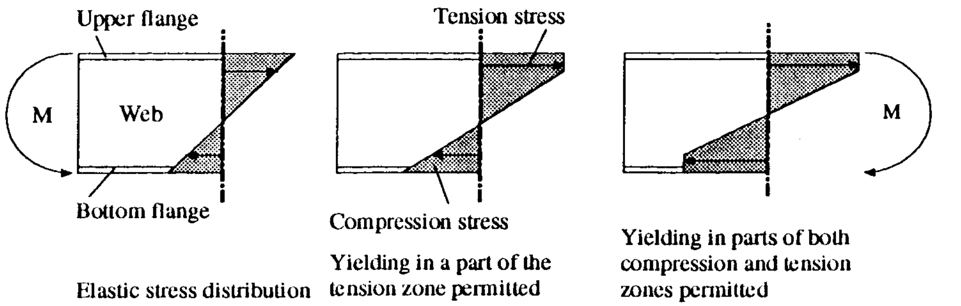

Figure 4

All design codes use the above presented methods for calculating F_u and M_u. However, in the different design codes, the k factors of the web crippling formula can differ. Furthermore, the calculation for bending moment can use different assumptions regarding the effective width and can be more or less conservative with regarding to the redistribution of stresses as shown in figure 4.

For interaction between web crippling and bending moment, an interaction formula has been designed, using the data of many experimental tests [Hetr78a]. It was found that the interaction can be described as follows:

$$\alpha \frac{M}{M_u} + \beta \frac{F}{F_u} \leq \gamma \text{ and } \frac{M}{M_u} < 1 \text{ and } \frac{F}{F_u} < 1$$

M and F are the actual bending moment and concentrated load acting on the member. F_u is the ultimate concentrated load without a bending moment acting on the member. M_u is the ultimate bending moment without concentrated load. They are predicted as already explained. The factors α, β, and γ are experimentally determined. This formula is graphically shown in figure 5.

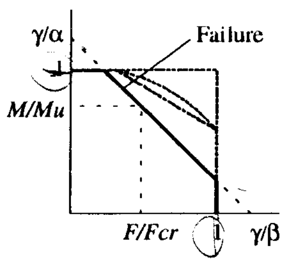

Figure 5, interaction diagram

This interaction formula can differ in different design codes. For example, the German DIN design code [Gdin87a] gives an interaction formula dependent on the member's ratio between F_u and M_u. This is shown in figure 5 by the dotted lines. However, in all design codes, the idea of the interaction formula is used.

ANALYTICAL MODELS

In 1986, Bakker stated [Bakk86a], that the concentrated load (web crippling) prediction formulae evaluated, did not give consistent satisfactory results. In most empirical formulae, it was assumed that the influences of the corner radius, the web slenderness, the load bearing plate length, the yield strength and the web angle were independent of each other. The predicted results differed up to about 40 %.

Before these results were presented, it was already thought that, because the physical behaviour is not taken into account, empirical models are not able to describe the web crippling behaviour accurately. Consequently, from 1974 on, several attempts have been made to develop an analytical model for web crippling.

In 1974, in the Dutch design code RSD, the first attempt of an analytical model for the description of web crippling was introduced. However, in the Dutch follow up RGSP design code [Rsdr74a], the model was replaced by the one according to the European design code, because the correctness of this model was questionable.

The second analytical model for failure at an interior support was developed by Reinsch [Rein83a]. This model was designed to improve the knowledge about moment redistribution in cold-formed multi span members. Traditional hot-rolled steel members can form a plastic hinge, which makes moment redistribution in many cases possible. Cold-formed, thin-walled steel members cannot form a plastic hinge, but can form, in some cases, a mechanism, which acts like a plastic hinge and makes moment redistribution possible.

In figure 6, bottom half, a multi-span member is shown. If the load q for this member increases, the difference between the moment at interior support B and the moment in the field stays equal: 1/8*q*L2. If the ultimate moment at B is reached because a plastic hinge has formed, the moment at B can not increase. However, the moment in the field may still increase as shown in figure 6, bottom half, at the right. The situation can also be reversed. If the ultimate bending moment in the field is reached first, only the support moment can increase.

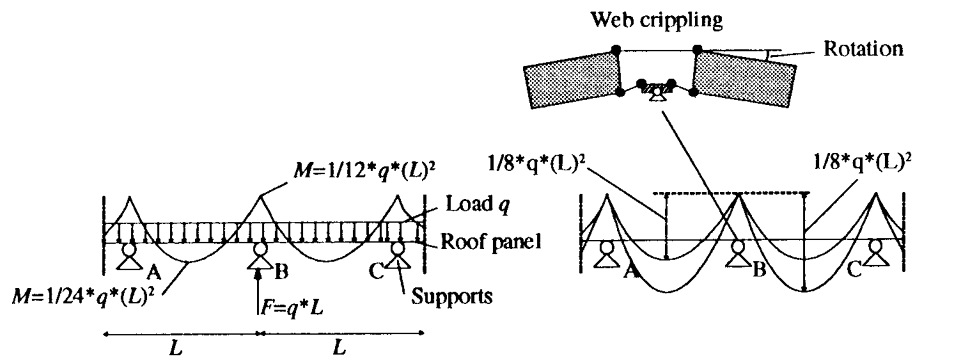

Figure 6

The member fails if both in the field and at the support the ultimate bending moment is reached. This approach is economical compared with the assumption that the member fails if either at the supports or in the field the ultimate moment is reached. To use the method of moment redistribution, information is needed about the moment curvature relation at the support. If, for example, during the curvature of the member near the support, the bending moment sharply decreases, moment redistribution will not be possible.

As shown in figure 6, top half, at the right, for web crippling a mechanism occurs, that causes the member to act like a beam with a plastic hinge. However, for moment redistribution, it cannot be assumed that the member has a simple plastic hinge, because the web crippling mechanism can have several forms of moment-rotation behaviour.

To improve knowledge on this subject, it is necessary to study the relations between the web crippling deformation, the rotation caused by the web crippling mechanism, and the bending moment at the support.

Reinsch wanted to describe the relation between the web crippling deformation and the rotation caused by the hinge mechanism. Therefore, web crippling experiments were carried out as already shown in figure 2 at the left, but in this case, the load deformation behaviour, the failure mechanisms, and the support rotations were carefully studied.

Reinsch made the following assumptions:

1. The deformation mode of a member subjected to combined bending and concentrated load is identical to the deformation mode of a member subjected to a concentrated load only as shown in figure 7 (top half).

2. The increase of internal energy dissipation due to an incrementally increased web crippling deformation of a member subjected to a concentrated load only, equals the increase of internal energy dissipation of a member subjected to combined bending and concentrated load, as seen in figure 7 (bottom half).

With these two assumptions and a spatial plastic mechanism with fixed yield lines, Reinsch reduced the ultimate concentrated load for bending moment by setting equal the external energy states of concentrated load only and concentrated load and bending moment:

$$\left.\begin{array}{l} W_{e;CB} = F_{CB} * \delta h_w + 2 * M * \delta\varphi = W_{i;CB} \\ W_{e;C} = F_C * \delta h_w = W_{i;C} \\ W_{i;CB} = W_{i;C} \end{array}\right\} \Rightarrow F_{CB} * \delta h_w + 2 * M * \delta\varphi = F_C * \delta h_w \Leftrightarrow$$

$$F_{CB} = F_C - 2 * M * \frac{\delta\varphi}{\delta h_w}$$

$$M = \frac{1}{2} * F_{CB} * \frac{L_{yb} - L_{test}}{2}$$

In this formulae, CB stands for combined concentrated load and bending moment and C stands for concentrated load only. $\delta\varphi/\delta h_w$ can be derived from the spatial yield line mechanism. This spatial yield line mechanism was chosen by Reinsch by observations of tests only. The yield line positions were not determined analytically. Reinsch developed an empirical formula for F_C. The calculation of the external energy caused by the bending moment is suggested by Bakker. Reinsch used an other approach.

Some remarks can be made on the Reinsch model:

- It was found by Bakker [Bakk92a] that the deformation mode, and thus the relation $\delta\varphi/\delta h_w$ varies with the span length.
- Reinsch made no comparison between his rigid plastic model and yield line models observed in tests.
- Assumption 2 of the model implies that the energy dissipated in the yield lines is not influenced by the stresses due to the load application. Conform yield line theory, the energy dissipation depends on the stresses and can therefore differ for the two cases of load application described above.
- The model is based on one failure mode observed in the tests. However, Bakker showed that several failure modes can occur.

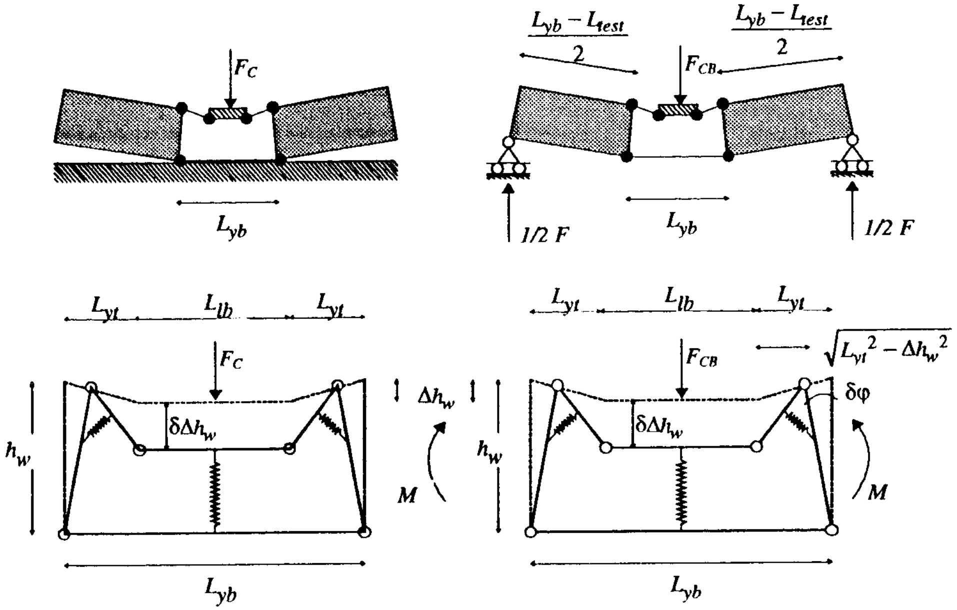

Figure 7, Reinsch model

In 1987, Tsai [Tsai87a] modified Reinsch's model. The Tsai model describes the load-deformation behaviour using load web crippling deformation curves. It can be seen that the research of Tsai was carried out for extending the knowledge for moment redistribution calculations, because the main results of the research are load deformation diagrams at the interior support

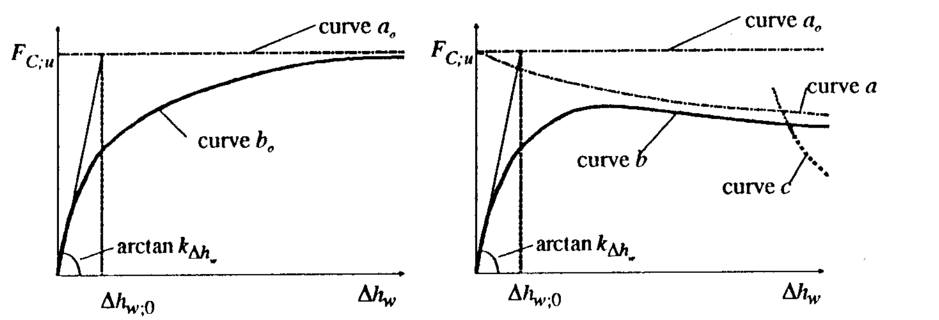

Figure 8, Tsai model

Tsai derived the ultimate web crippling load of a member subjected to combined bending and concentrated load from a member subjected to a concentrated load only. In figure 8, the load deformation diagram on the left gives the behaviour for a member subjected to a concentrated load only. Curve a_0 gives the rigid-plastic load-deformation behaviour. Curve b_0 is the realistic behaviour, derived form curve a_0 with an empirical formula. In figure 8 at the right, the load deformation curve is shown for a member subjected to combined bending and concentrated load. Here curve a is the rigid plastic behaviour, derived with the Reinsch formula. Curve b, the realistic behaviour, is derived from curve b_o with an empirical formula. Curve b has curve a as an asymptote.

The research of Bakker [Bakk92a] showed that for a certain failure mechanism the curve a_0 is an ascending one and not a horizontal line. For the yield arc mechanism, that will be presented in the next chapter, the curve a_0 was even descending.

THE BAKKER MODEL

In 1992 Bakker developed a more consistent model for web crippling, based on observed failure modes. This model was first meant to describe web crippling only, but it was found necessary to take the influence of the bending moment directly into account. Thus, the model described interaction between web crippling and bending moment fully analytically. The models of Reinsch and Tsai determined the web crippling load empirically. The Reinsch model made use of a yield line pattern with yield line positions determined by observations of tests results only.

Bakker carried out experimental tests, in which hat sections with several corner radii, web angles, and load bearing plate widths were subjected to a concentrated load only or to a concentrated load and a small bending moment. Bakker observed two failure modes near the load bearing plate: a yield line mechanism and a rolling mechanism, as shown in figure 8.

These mechanisms occurred for specific corner radii. For small corner radii the yield line mechanism occurred, for large corner radii the rolling mechanism occurred. During the tests, the web crippling deformation, the support rotations and the beam deflections were carefully measured. The two failure modes could be characterized by their concentrated load-web crippling deformation behaviour, as shown in figure 10 with bold drawn lines.

According to the model, this behaviour can be described by a linear curve until the formation of a yield line mechanism is initiated. Once this mechanism is formed, the behaviour can be described by a rigid-plastic curve.

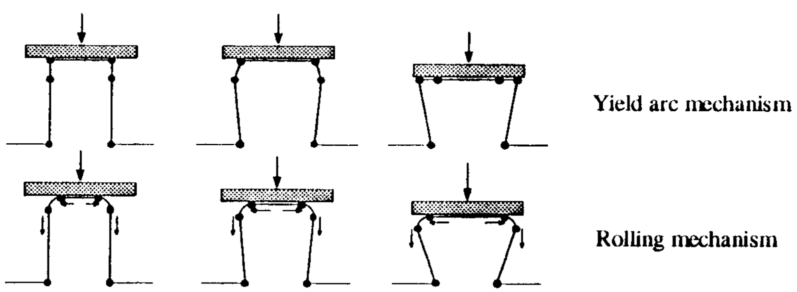

Figure 9, two failure modes for hat section cross-section

The mechanism initiation load is defined as the load at which the yield line mechanism is initiated. For the yield arc mechanism, the ultimate load is approximately the mechanism initiation load. For the rolling mechanism, the ultimate load is much higher than the mechanism initiation load. However, it is expected that for larger span lengths, the difference between mechanism initiation load and ultimate load will decrease.

Bakker developed a method to find the mechanism initiation load for the rolling mechanism. The load-web crippling deformation behaviour for the rolling mechanism is shown in figure 10 at the right. A

yield line mechanism was developed for described the rolling mechanism. For every point of elastic web deformation, Bakker calculated the load for which the yield line mechanism would initiate, giving a mechanism initiation curve. This curve is shown in figure 11. The intersection of the linear curve and mechanism initiation curve gives the mechanism initiation point.

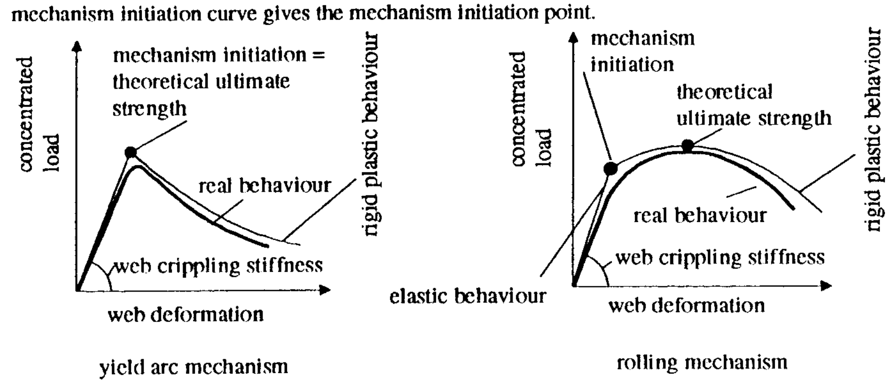

Figure 10, load deformation diagram for the yield arc and rolling mechanism

For finding the linear elastic curve, Vaessen developed an analytical model [Vaes95a]. It should be noted that the mechanism initiation curve does not describe the rigid plastic curve and has no physical meaning, except at the mechanism initiation point.

The Bakker model was verified for sections subjected to a concentrated load and a small bending moment. However, in practice large bending moments occur. Apart from this aspect, it is possible that the yield arc mechanism occurs also for sections with small corner radii. For the rolling mechanism, the Bakker model does not calculate the ultimate strength, but the theoretical mechanism initiation point.

During the development of the analytical yield line model, Bakker had to take the influence of the bending moment directly into account in order to get an accurate model. Bakker realized that it would be possible to release the paradigm of the three failure modes (web crippling, bending moment, and interaction of web crippling and bending moment).

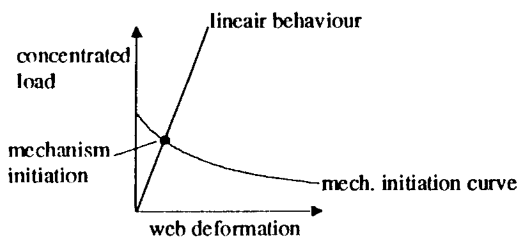

Figure 11, calculation of mechanism initiation point for rolling mechanism

3 NEW DIRECTIONS

During the research of Bakker, it was observed that the influence of the bending moment has to be taken into account, when predicting the web crippling load. That means that, at least for small bending moments, the interaction formula as presented in chapter 2 is not correct. After all, in the interaction formula it is assumed that the bending moment has no influence for small bending moments.