

Efficiency and time-optimal control of fuel cell - compressor electrical drive systems

Citation for published version (APA):

Boynov, K. O. (2008). *Efficiency and time-optimal control of fuel cell - compressor - electrical drive systems*. [Phd Thesis 1 (Research TU/e / Graduation TU/e), Electrical Engineering]. Technische Universiteit Eindhoven. https://doi.org/10.6100/IR633936

DOI: 10.6100/IR633936

Document status and date:

Published: 01/01/2008

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.

• The final author version and the galley proof are versions of the publication after peer review.

• The final published version features the final layout of the paper including the volume, issue and page numbers.

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Efficiency and Time-Optimal Control of Fuel Cell – Compressor – Electrical Drive Systems

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de Rector Magnificus, prof.dr.ir. C.J. van Duijn, voor een commissie aangewezen door het College voor Promoties in het openbaar te verdedigen op dinsdag 8 april 2008 om 16.00 uur

door

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geboren te Moskou, Rusland

Dit proefschrift is goedgekeurd door de promotoren:

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Tijdens de promotiezitting vervult prof.dr.ir. J.H. Blom namens wijlen prof.dr.ir. A.J.A. Vandenput de rol van eerste promotor.

This work was supported by TDO (Centrum Techologie voor Duurzame Ontwikkeling).

Cover design by Oranje Vormgevers.

Printed by Eindhoven University Press.

CIP-DATA LIBRARY TECHNISCHE UNIVERSITEIT EINDHOVEN

Boynov, Konstantin

Efficiency and time-optimal control of fuel cell – compressor – electrical drive systems / by Konstantin Boynov. – Eindhoven : Technische Universiteit Eindhoven, 2008. Proefschrift. – ISBN 978-90-386-1824-1 NUR 959 Trefw.: optimale regelsystemen / asynchrone machines ; regeling / compressoren / brandstofcellen. Subject headings: optimal control / induction motor drives / compressors / fuel cells.

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CHAPTER 1

Introduction

This thesis concerns with the analysis, control and design of a high-speed induction motor drive for use as a prime mover of a centrifugal air compressor in proton exchange membrane fuel cell systems.

A fuel cell (FC) power generation system is regarded as one of the perspective energy supply solutions for a wide variety of applications and power levels and could potentially be used to replace conventional power equipment. The mentioned range of applications includes distributed and central power plants, transport, auxiliary power and micro power supply.

The electrochemical conversion of chemical energy into electrical and thermal energy can be considered as a high-efficient and emission-free process, when pure hydrogen is used as fuel. The proper operation of a FC system is provided by an electrically driven auxiliary subsystem which is responsible for maintaining a key operation conditions, as well as the fuel and oxidant supply. Thus, the control of the auxiliary system may either significantly improve or deteriorate the performance characteristics and efficiency of the total system.

This chapter gives an overview of various FC technologies, their applications automotive and stationary power generation with a description of the basic structure of a FC system. It concludes with the problem description, the objectives and the layout of this thesis.

1.1 FC types

Fuel cells can be classified in several ways, depending on the combination of fuel and oxidant types, external or internal fuel processing (reforming), type of electrolyte, operation temperature, etc.

The most common classification of fuel cells uses the type of electrolyte used in the cells and includes [1], [2], [3]

- alkaline fuel cell (AFC);
- solid oxide fuel cell (SOFC);

- molten carbonate fuel cell (MCFC);
- phosphoric acid fuel cell (PAFC);
- proton exchange membrane fuel cell (PEMFC).

The AFC is one of the first modern fuel cells, developed in the beginning of 1960. It was used to provide on-board electrical power for the Apollo space vehicle. It is still used in the space shuttle, producing power for the on-board systems by combining the pure hydrogen and oxygen stored in the rocket-fuelling system, and producing potable water for the astronauts.

The concentrated (85 wt%) KOH is used as an electrolyte in the fuel cells operated at high temperature (250 °C), or less concentrated (35-50 wt%) KOH for lower temperature (< 120 °C) operation. The electrolyte is retained in a matrix (usually asbestos), and a wide range of electrocatalysts can be used (e.g. Ni, Ag, metal oxides, etc.).

The main drawback of AFCs for terrestrial applications consists of the carbon dioxide reaction with the alkaline electrolyte. It reduces the rate of the reaction at the anode, lowers the limiting current and increases the mass transport losses, activation and ohmic losses. The essential solution of the problem is removing the carbon dioxide from the air, which increases the system costs, complexity, mass and size.

The SOFC is a completely solid state device that uses an oxide ion-conducting ceramic material as the electrolyte. In the SOFC a negatively charged ion (O^-) is transferred from the cathode through the electrolyte to the anode. At the anode water is formed as a product. Solid oxide fuel cells operate at high temperatures, and therefore can operate in applications where high-temperature heat can be utilized - for heating processes in industry or at home (cogeneration), and also for integration with turbines for additional electricity production.

The SOFC operates in the range of 600 to 1000 $^{\circ}$ C. This means that high reaction rates can be achieved without expensive catalysts, and that gases such as natural gas can be used directly, or "internally reformed" within the fuel cell, without the need for a separate unit.

But the ceramic materials, these cells are made from, are difficult to manufacture, and quite a large amount of extra equipment is needed to produce a complete fuel cell system of this type. The extra elements include air and fuel pre-heaters and a relatively complex cooling system. In addition, this FC system type requires a complicated start-up procedure.

The MCFCs can occupy the same market segment as the SOFCs. The electrolyte in this fuel cell is usually a combination of alkali carbonates retained in a ceramic matrix of LiAlO₂. The fuel cell operates at 600 to 700 °C, where the alkali carbonates form a highly conductive molten salt, with carbonate ions providing ionic conduction. This temperature level results in several benefits: the cell can be made of commonly available sheet metals that can be stamped for less costly fabrication, the cell reactions occur with nickel catalysts rather than with expensive precious metal catalysts, reforming can take place within the cell provided a

reforming catalyst is added, CO is a directly usable fuel, and the rejected heat is at sufficiently high temperature to drive a gas turbine and/or produce a high-pressure steam for use in a steam turbine or for cogeneration. Another advantage of the MCFC is the efficient operation with $\rm CO_2$ -containing fuels such as bio-fuel derived gases. This benefit is due to the cathode performance enhancement resulting from $\rm CO_2$ enrichment.

However, the MCFC has some disadvantages: the electrolyte is very corrosive and mobile, and a source of CO_2 is required at the cathode (usually recycled from the anode exhaust) to form the carbonate ion. Operation requires the use of stainless steel as the cell hardware material. The higher temperatures promote material problems, particularly mechanical instability that impacts the life time.

The PAFC is one of the most advanced systems from the point of view of commercial development. It is mainly used in stationary power plants ranging from dispersed power to on-site generation plants. Phosphoric acid (H_3PO_4) concentrated to 100% is used for the electrolyte in this fuel cell type, which operates at 150 to 220 °C.

The advantages of the PAFC are its simple construction, its stability - thermal, chemical as well as electrochemical, and the low volatility of the electrolyte at the operating temperatures. These factors probably assisted the earlier deployment into commercial systems compared to the other fuel cell types.

The disadvantages are the cathode's slow oxygen reaction rate reducing the performance and the cell requirement for hydrocarbon fuels to be reformed into an H₂-rich gas.

The electrolyte in the *PEMFC* is an ion exchange membrane (fluorinated sulfonic acid polymer or an other similar polymer) being an excellent proton conductor. The PEMFC, like the SOFC, has a solid electrolyte. As a result, this cell exhibits excellent resistance to gas crossover. In contrast to the SOFC, the cell operates at low temperature (80-120 $^{\circ}$ C). It results in a capability to reach its operating temperature quickly, but the rejected heat cannot be effectively used for co-generation or additional power generation. Test results have shown that the cell can operate at very high current densities compared to other types. However, heat and water management issues may limit the operating power density of a practical system. The only liquid in this fuel cell is water; thus, corrosion problems are minimal.

Due to the set of PEMFC's advantages, like high power density, rapid startup and low temperature operation, they are ideal for use in transport, battery replacement, and domestic power production. Although PEMFCs operate at about 80 °C, this is sufficient for space heating and hot water production providing the opportunity to replace a domestic boiler. The ability to replace diesel stand-by generators is also engendering great interest.

The summary of the different FC types is presented in Table 1.1.

	SOFC	Ceramic				600-800 800-	1000	-0	No		Ceramic		Perovskites	Gaseous	product	Internal	reforming and	process gas		
FC types.	MCFC	Immobilized	liquid molten	$\operatorname{carbonate}$		650		CO_3^-	No		Stainless	based	Nickel	Gaseous	product	Internal	reforming and	process gas		
or differences of	PAFC	Immobilized	liquid phos-	phoric acid		205		$^{+}\mathrm{H}$	Yes		Graphite	based	Platinum	Evaporative		Process gas	and	independent	cooling	medium
ummary of majc	AFC	Mobilized or	immobilized	potassium	hydroxide	65-220		-HO	Yes		Carbon based		Platinum	Evaporative		Process gas	and	electrolyte	circulation	
Table 1.1: Su	PEMFC	Ion exchange	membranes			80-120		H^+	Yes		Carbon based		Platinum	Evaporative		Process gas	and	independent	cooling	medium
		Electrolyte				Operating tem-	perature [°C]	Charge carrier	External	reformer	Prime cell com-	ponents	Catalyst	Product water	management	Product heat	management			



Figure 1.1: Schematic drawing of a hydrogen/oxygen fuel cell based on the proton exchange membrane fuel cell.

The basic physical structure (Fig. 1.1) of a fuel cell consists of an electrolyte layer in contact with a porous anode on one side and a cathode on the other side [1]. In a typical fuel cell, gaseous fuel (e.g. hydrogen) is fed continuously to the anode (negative electrode) compartment and an oxidant (e.g. oxygen from air) is fed continuously to the cathode (positive electrode) compartment; the electrochemical reactions take place at the electrodes to produce an electrical current, water and heat.

Fuel cell technology allows to produce electricity at local sites from a wide range of fuels, and with high efficiency. Most types of fuel cells operate on hydrogen fuel, which can be produced from natural gas, liquid hydrocarbon fuels including biomass fuels, landfill gases, water and electricity (via the process of electrolysis), biological processes and from coal [2].

A fuel cell generation system becomes attractive due to a set of advantages over the conventional systems, such as:

- low pollution (or in some cases zero emission solutions);
- relatively high efficiency (55 65% stack);

- relatively high power density (> $1 \text{ MW/m}^3 \text{ stack}$);
- direct energy conversion (no combustion);
- absence of moving parts in the main energy converter;
- silent operation (stack);
- fuel flexibility;
- siting ability;
- ability of modular installations for load matching;
- high reliability;
- size flexibility;
- rapid load following capability (PEM).

1.2 FC systems applications

The major applications of fuel cells are stationary electrical power plants, including co-generation units, and electrically powered vehicles [1], [2], [3]. Derivative applications are summarized in the following sections.

1.2.1 Stationary

The most important attributes of fuel cells for stationary power generation are the high efficiency and the possibility for distributed power generation. Both lowtemperature and high-temperature fuel cells could, in principle, be utilized for stationary applications.

The power plants in the range above 250 kW can be operated by high-temperature fuel cell systems. The heat obtained from these systems can be used directly or for further electricity generation by steam engines. The start-up time of these systems is longer than for low-temperature systems, but the advantage of being able to operate the system without external reforming and the higher efficiencies of SOFCs and MCFCs make these systems more suitable for large-scale power plants.

For a small distributed power system, e.g. single-home or multiple-home power generation, a PEMFC (Fig. 1.2), or a PAFC combined with a heat cycle could be used to provide all energy needs. The start-up time of these types of FC is much lower than in high-temperature systems, which makes it more attractive for small-power generation. The heat generated by the fuel cell system can be employed for heating and hot water production.



Figure 1.2: Stationary Ballard PEMFC power system.

1.2.2 Transport

Since the late 1980s there has been a strong push to develop fuel cells for use in light-duty and heavy-duty vehicle propulsion. A major objective for this development is the need for clean, efficient cars, trucks, and buses that can operate on conventional fuels (gasoline, diesel), as well as renewable and alternative fuels (hydrogen, methanol, ethanol, natural gas, and other hydrocarbons). With hydrogen as the on-board fuel, such vehicles would be zero local emission vehicles. With on-board fuels other than hydrogen, the fuel cell systems would use an appropriate fuel processor to convert the fuel to hydrogen, yielding a vehicle with very low CO emissions and high efficiencies. Further, such vehicles offer the advantages of an electrical drive with low maintenance because of the few critical moving parts.

Transport applications require rapid start-up and instant dynamic response from fuel cell systems, thus a high-temperature fuel cell cannot be used in this case. The prime candidate for these vehicle propulsion systems is the PEMFC, which exhibits both characteristics while also having a very high power density. This is important as it must occupy a similar amount of space as an internal combustion engine.

In addition, the conventional car FC will also potentially be applied in other transport systems: locomotives, ships, scooters, etc. This development is being sponsored by various governments in North America, Europe, and Japan, as well as by major automobile manufacturers worldwide. Fig. 1.3 shows the 275 HP (205 kW) fuel cell engine that is used in buses in Canada and some North American states, and produced by Ballard Power Systems Inc. of Canada.



Figure 1.3: PEMFC for a bus (Ballard Power Systems Inc).

1.2.3 Automotive FC for distributed generation

In [4] it is proposed to use fuel cell vehicles (FCVs) as distributed electricity generating resources when parked at homes, offices, and shopping malls (Fig. 1.4). In addition to its primary function (transport), FCVs could help to meet local power needs, reducing demand for grid power as well as supplying power to the grid during times of peak demand. In principle, use of FCVs in this way could both reduce the need to construct new stationary "peak power" plants to supply peak electricity demands, as well as bring down the costs of FCV ownership.



Figure 1.4: Schematic of fuel cell vehicles parked at office building.

1.3 FC system engineering

1.3.1 General system structure

Generally, the total FC system comprises primary and secondary power system equipment (Fig. 1.5). The first group includes a reformer (fuel processor), fuel cell stack, electronic power converter and a recuperator. The reformer produces hydrogen gas from hydrocarbon fuel and then supplies it to the stack. The fuel cell stack output DC voltage is first corrected by a DC/DC converter and then transformed to AC through a DC/AC converter. Further, electrical energy is delivered to the primary and auxiliary electrical loads. In case of a domestic application of the FC, thermal energy can be used for space heating (through the recuperator).

The auxiliary subsystem or so-called "balance of plant" (BoP) includes the pumps, compressors, fans and blowers driven by electrical motors, and it is responsible for the continuous energy regeneration process of the fuel cells.

The system structure investigated in this thesis is limited to the PEMFC stack, the air compressor driven by an electrical drive and a simplified variable electrical load. A continuous stream of pure hydrogen is assumed to be available as compressed gas at the desired operating pressure and is used as the fuel. Ambient air, which is used as the cathode reactant, has to be pushed through the fuel cell stack using an air compressor.



Figure 1.5: Simplified diagram of a fuel cell system for residential applications.



Figure 1.6: Example of a PEMFC stack.

1.3.2 FC stack

The fuel cell stack is the key component of the system and performs the main function of energy conversion within the system. Due to the fact that the voltage of a single fuel cell is quite small, about 0.7 V when drawing rated current, the desired voltage can only be produced by a series connection of cells. Such a collection of cells in series is known as a "stack" (Fig. 1.6).

One of the commonly used methods of cell interconnection in practice is applying a "bipolar plate". This makes connections all over a cathode of one cell and an anode of the next cell (hence "bipolar"). At the same time the bipolar plate serves as a means of feeding oxygen to the cathode and fuel gas to the anode. This distribution of the reactant gases over the electrodes is done using "flow field" formed into the surface of the plate, usually a fairly complex serpentine pattern.

Platinum is usually used as catalyst material for both anode and cathode. The platinum catalyst is formed into very small particles on the surface of larger particles of finely divided carbon powders. The platinum is highly divided and spread out, so that a very high proportion of the surface area will be in contact with the reactants.

The operational principle of the fuel cell can be described as follows [2] (see also Fig. 1.1). Both reactant gases are supplied under pressure into the flow channels of the plate. At the anode side hydrogen reacts forming protons and electrons:

$$H_2 \to 2H^+ + 2e^-.$$
 (1.1)

Released protons and electrons are transferred through the membrane to the

cathode catalyst layer and the external electrical load, respectively. At the cathode side the oxygen is consumed along with the protons and electrons, and liquid water is produced as the product with heat:

$$2H^+ + 2e^- + 1/2O_2 \to H_2O + \text{Heat.}$$
 (1.2)

As a result, the overall chemical reaction of the fuel cell is represented by the reaction:

$$H_2 + 1/2O_2 \rightarrow H_2O + \text{Heat} + \text{Electrical energy.}$$
 (1.3)

1.3.3 Auxiliary system

The main functions of the auxiliary subsystem are [1], [2]:

- air management;
- water and thermal management;
- control systems.

The air management system is one of the most important BoP component of a fuel cell power system. The air management equipment for pressurized systems consists primarily of a compressor with an electrical drive and air valves. Operation at elevated pressure (e.g. 2-3 atmospheres) is favored by most of the fuel cell systems developers. Pressurization results in a higher specific stack performance (current and power density) which leads to an increased stack efficiency, reduced size and, respectively, cost. But at the same time it requires additional equipment for the pressure handling.

Water and thermal management are also key BoP functions. In a fuel cell power system, the temperatures of a number of critical components (particularly the stack and the different reactors of the fuel processor) have to be carefully controlled, and the flow and utilization of heat from several sources within the system have to be managed efficiently to achieve high overall efficiency. Water also plays a very important role in the operation of PEM fuel cell stacks and fuel processors. The pure water produced in cells from the electrochemical combination of hydrogen and oxygen must be removed continuously from the stack. This is accomplished either by condensing water in the cathode exhaust gas, or by using a combination of capillary action and pressure to "wick" water from the stack to a reservoir. Both approaches are now used. At the same time, the gases entering the stack must be humidified to prevent proton exchange membranes from drying out which could result in cross-over of hydrogen and/or oxygen and irreversible stack damage due to the heat generated by this "chemical short circuit." Pure water is also used as coolant to transfer waste heat from the stack to the fuel processing system and/or to a radiator.

Control systems are critically important components of an advanced fuel cell power system. Parameters, which have to be controlled, include various mass flows, temperature and pressure levels, compressor shaft speed and electrical values. These control functions need to be provided not only over a wide range of system power levels, but for substantially different system conditions that include cold start, rapid load transients, and system shutdown. The control subsystem includes a substantial number of sensors, compressors and valves for air and fuel flow, microprocessors to perform the numerous control functions, a computer-based master control, and the mechanical and electrical hardware required for connecting all these components with the appropriate subsystems.

1.4 Problem description

The major subject of this thesis is, primarily, to investigate the functions and characteristics of the FC stack together with the auxiliary electrically driven compression system and, secondly, to derive a system structure and a control strategy of the electrical drive and the compression system to achieve optimal steady-state and dynamic behavior of the FC system.

The electrically driven compressors play an important role in a FC system, supplying fuel and oxidant to the stack. The performance and efficiency of the whole system are directly influenced by their operation. While designing the FC generation system a number of aspects must be taken into account - from the performance characteristics of the system components to their availability on the market.

The well-known centrifugal compressor can be considered as one of the most attractive candidates. It has a simple and compact mechanical design, high reliability, high efficiency, relatively high pressure capabilities, oil-free output and continuous mass flow. Centrifugal compressors are commonly used in a wide range of compressor applications, from combustion engine superchargers to gas distribution systems. The FC system analysis presented in this thesis demonstrates that the main obstacle for the use of a centrifugal compressor in FC systems is its limited stable operation area.

The goal of this work is to achieve a trade-off between the fast response and efficiency of the compression system, using standard, reliable and commercially available components. The innovative control solution for the electrical drive with the centrifugal compressor is developed to fit their performance characteristics to the specifically demanding application.

1.5 Thesis objectives

The global goal of this thesis is to improve the steady-state and transient performance of a FC system exploiting the control functionality of the auxiliary electrical drives. To meet this global goal the following five objectives have been formulated:

1. To analyse and select appropriate system components: FC type, compressor and electrical drive. 2. To build comprehensive and coupled mathematical models of the system components in different physical domains (electrical, mechanical and thermodynamical).

Different energy conversion devices are usually designed for working at an optimal operating point. However, once assembled into one system, the designed operating points of different components may not always coincide with their own optimal one leading to the following objectives.

3. To expand the naturally imposed stability or efficiency boundaries of the system components by means of appropriate control approaches and verify them experimentally.

The design of a FC system may require system components with particular characteristics, which can not always be provided by standard or commercially available equipment. In this case, the characteristics of such components can be identified using a global design method. The fourth objective of this thesis is thus:

- 4. To propose a global design procedure of a high-speed electrical machine.
- 5. To develop an efficiency and time-optimal control of the system with the PEMFC, air compressor and electrical drive.

1.6 Thesis layout

According to the problem definition and the formulated objectives, this thesis is organized as follows.

In Chapter 2 the basic operating principles of a FC are described using fundamental electrochemical theory. It mainly focuses on a detailed mathematical model of the PEMFC. The presented static and transient models allow the simultaneous analysis of the electrochemical and thermodynamic processes for further coupling to the model of a compression system.

The compression system is the subject of Chapter 3. Starting from the general description of different compressor types, it focuses on the centrifugal compressor. Based on the detailed fundamental principles of its operation the control technique, called active surge control, is illustrated, which allows stable air flow in the compression system in the originally unstable area by means of a high-performance drive torque actuation.

Consequently, Chapters 4 and 5 are dedicated to electrical drives. Chapter 4 discusses mainly the control principles of major electrical motor types emphasizing the speed and efficiency control methods of induction motors. Chapter 5 presents design issues of a high-speed induction motor specific to the FC compression application. Additionally, the preliminary analysis of the induction motor selected for the experimental work is given.

The results of the experimental verification of the surge control method by means of the electrical drive are presented in Chapter 6.

Finally, in Chapter 7, the analysis of the complete PEMFC system is presented. Based on the developed and experimentally verified active surge control method of the centrifugal compressor by means of the high-performance control of the induction motor drive, as well as the described fundamental mathematical models of the FC system components, an optimal control method of the FC system is proposed. With this method a detailed study of the trade-offs between the efficiency of the system and its transient response on the load demands can be achieved.

The generalized scheme of the mathematical models and control strategies is shown in Fig. 1.7



Figure 1.7: Mathematical models and control strategies of the PEMFC system with the electrical drive and the air compressor.

CHAPTER 2

Proton exchange membrane fuel cells modeling

Fuel cell modeling is an important aspect in FC systems development since it facilitates a better understanding of the features and parameters affecting the performance of FCs and FC systems. Section 2.1 presents basic electrochemical notations describing hydrogen FC output characteristics. Based on the fundamental definitions, section 2.2 presents a more detailed FC mathematical model particulary focused on PEMFCs. Section 2.3 describes the PEMFC transient response to a change of the electrical load and a variation of the pressure and mass flow of the inlet gases. Section 2.4 gives an initial system description in terms of thermodynamic variables and the power flow inside a FC system.

2.1 Introduction into hydrogen FC modeling

The steady-state electrical performance of a FC is characterized by the relationship between the voltage across a FC and the current density. The generally accepted term for the plot of the cell voltage versus current density is *polarization curve*. The FC output operational voltage results from the sum of the *open-circuit voltage*¹ and several voltage drop components - *overvoltages*². The terminology for the FC description has been established in electrochemical science and is originally related to the electrochemical processes at the electrodes.

2.1.1 Open-circuit voltage

The electrical operation principle of the FC is based on the conversion of chemical energy stored in reactants into electrical energy. The energy released due to the reaction:

¹Also called *reversible voltage*, thermodynamic equilibrium potential or Nernst voltage.

 $^{^{2}}$ Also called *overpotentials*.

$$H_2 + \frac{1}{2}O_2 \to H_2O, \tag{2.1}$$

can be expressed in terms of change in Gibbs free energy (in molar form):

$$\Delta G = G_{\text{products}} - G_{\text{reactants}},\tag{2.2}$$

$$G = H - TS, \tag{2.3}$$

where

G - Gibbs free energy, [J], H - enthalpy, [J], S - entropy, [J/K], T - temperature, [K].

The Gibbs free energy can be defined as the "energy available to do external work, neglecting any work done by changes in pressure and/or volume" [2].

For (2.1) the change in Gibbs free energy is:

$$\Delta G = G_{\rm H_2O} - G_{\rm H_2} - G_{\rm O_2}. \tag{2.4}$$

In the idealized (lossless) case, the change in Gibbs energy results in electric energy corresponding to carrying a charge of one mole of electrons:

$$\Delta \text{Energy} = \text{Electrical work} = \text{Charge} \times \text{Voltage.}$$
(2.5)

For one mole of hydrogen the charge is defined as:

$$Charge = -nNe^{-} = -nF, \qquad (2.6)$$

where

- n number of electrons involved in a reaction (equals 2 for a hydrogen molecule),
- F Faraday's constant = 96487 [C/mol],
- N Avogadro's number = $6.02214199\times 10^{23},$
- e^- charge of one electron, [C].

Combining (2.4), (2.5) and (2.6) the fundamental equation for the electromotive force (EMF) or reversible voltage, E_{Nernst} , of the hydrogen fuel cell is derived:

$$E_{\text{Nernst}} = \frac{-\Delta G}{nF}.$$
(2.7)

The Gibbs free energy depends on the thermodynamic parameters: pressure and temperature. The influence of the pressure on the Gibbs free energy can be described using the concept of *activity*:

$$a = \frac{p_{\rm i}}{p_0},\tag{2.8}$$

where

a - activity, [-], p_i - partial pressure of the species, [Pa], p_0 - standard pressure = 1 atm (101.3 kPa), [Pa].

It modifies the Gibbs free energy as [2]:

$$\Delta G = \Delta G^{\circ} - RT \ln \left[\frac{a_{\rm H_2} a_{\rm O_2}^{1/2}}{a_{\rm H_2 \rm O}} \right],$$
(2.9)

where

 G° - reference value of Gibbs free energy, [J], R - universal gas constant = 8.3145 [J/(mol \cdot K)], a_{H_2} - hydrogen activity, [-], a_{O_2} - oxygen activity, [-], $a_{\text{H}_2\text{O}}$ - water vapor activity, [-].

Combination of (2.9) and (2.7) results in the expression for the reversible voltage (*Nernst equation*):

$$E_{\rm Nernst} = E^{\circ} - \frac{RT}{nF} \ln \left[\frac{a_{\rm H_2} a_{\rm O_2}^{1/2}}{a_{\rm H_2O}} \right], \qquad (2.10)$$

where

 E° - reference voltage at unit activity, [V], varying from the standard state³ reference voltage with temperature [5]:

$$E^{\circ} = E_0^{\circ} - (T - T_0) \left(\frac{\Delta S^{\circ}}{nF}\right), \qquad (2.11)$$

where

 S° - reference entropy, [J/K],

 E_0° - standard state reference voltage, [V],

 T_0 - standard state temperature = 298.15 [K].

The entropy change of the given reaction is approximately constant and can be set equal to the standard state reference value, S_0° [2]. Assuming that input gases

³Standard state: $T_0 = 298.15$ [K], $p_0 = 1$ [atm].

and produced vapor behave like "ideal gases", the following is applicable (for one mole of gas):

$$pV = RT, (2.12)$$

where

p - absolute pressure, [Pa], V - volume, $[\mathrm{m}^3].$

Finally, the Nernst equation (2.10) can be expressed as:

$$E_{\text{Nernst}} = E^{\circ} - \frac{RT}{nF} \ln \left[p_{\text{H}_2} \left(p_{\text{O}_2} \right)^{0.5} \right].$$
 (2.13)

2.1.2 Activation overvoltage

The activation overvoltage is caused by the finite reaction speed taking place at the surface of the electrodes and results in a rapid voltage drop when the current changes from zero. The kinetics of the electrochemical reaction is defined by the Butler-Volmer equation:

$$j = j_0 \left[e^{\frac{-\alpha n F \eta}{RT}} - e^{\frac{(1-\alpha) n F \eta}{RT}} \right], \qquad (2.14)$$

where

j - current density, [A/cm²] ⁴, j_0 - exchange current density (no load), [A/cm²], η - overvoltage, [V], α - transfer coefficient, [–].

In practice (for a large value of η), the second exponential in (2.14) can be neglected. Thus, the activation overvoltage can be expressed as (*Tafel equation*):

$$\eta_{\rm act} = -\frac{RT}{\alpha nF} \ln\left[\frac{j}{j_0}\right]. \tag{2.15}$$

2.1.3 Resistive overvoltage

The resistive overvoltage (or resistance loss) is caused by the resistance to the flow of protons through the electrolyte (main part) and, also by the resistance to the flow of electrons through electrodes and interconnections. In general form, the resistive overvoltage is:

$$\eta_{\rm ohm} = -jR_{\rm a},\tag{2.16}$$

 $^{^{4}}$ In fuel cell studies the unit [A/cm²] for current density is usually used rather than [A/m²] in order to get a more representative value.

where

 $\eta_{\rm ohm}$ - resistive overvoltage, [V], $R_{\rm a}$ - area-specific resistance, [Ωcm^2].

2.1.4 Concentration overvoltage

The concentration overvoltage (or mass-transport loss) occurs at high current density level and is caused by the reduction of the partial pressure (and, accordingly, concentration) of the reactant gases in the electrode region. The reduction depends on the current flowing through the FC and some physical characteristics of the system related to air supply and circulation properties. Using Nernst equation (2.10), the voltage dependence on the pressure is:

$$\Delta \eta = -\frac{RT}{nF} \ln\left(\frac{p_{\rm j}}{p_{\rm j=0}}\right),\tag{2.17}$$

where

 $p_{j=0}$ - pressure at zero current density, [Pa], p_j - pressure at instant current density, [Pa].

Assuming a linear decline in pressure to zero at the limiting current density, j_1 , i.e.:

$$p_{j} = p_{j=0} \left(1 - \frac{j}{j_{l}} \right),$$
 (2.18)

the concentration overvoltage, $\eta_{\rm con}$, can be expressed as:

$$\eta_{\rm con} = -\frac{RT}{nF} \ln\left(1 - \frac{j}{j_1}\right). \tag{2.19}$$

The FC output voltage, V_{cell} , combines the effect of the four components:

$$V_{\text{cell}} = E_{\text{Nernst}} + \eta_{\text{act}} + \eta_{\text{ohm}} + \eta_{\text{con}}.$$
 (2.20)

A typical FC output characteristic can be presented graphically as a polarization curve - FC output voltage versus current density (Fig. 2.1).

2.2 Steady-state electrochemical PEMFC models

2.2.1 Overview of PEMFC modeling approaches

There are different approaches to FC modeling, which can be classified using certain criteria (Table 2.1) [6]. The first criterion is related to the system boundary which defines the area of interest of the model:



Figure 2.1: FC polarization curve.

- very fundamental cell level (electrodes, membrane);
- middle level (FC stack);
- highest level (stack with auxiliary system).

The FC models can also be subdivided into empirical and theoretical ones. The theoretical, also called "mechanistic", FC models normally use the basic, phenomenological equations. For example, the Nernst-Planck equation describes the species transport, the Stefan-Maxwell equation is used for the gas-phase transport, the Butler-Volmer equation for the FC voltage.

Spatial dimensions are the key criteria for mechanistic models. For instance, to describe the FC phenomenon of mass transport limitation at least a one-dimensional model is required. For a proper treatment of the thermal and water management, except electrochemical relationships, the model should contain also thermodynamic and fluid dynamic equations. They are normally applied in two or three dimensions and can provide an appropriate representation of almost all processes in a FC and a FC system. Depending on its focus and complexity level, the model may provide details like FC flow pattern, current density distribution, voltage and pressure drops in the FC stack. Such a model is normally an appropriate tool for detailed system studies, since it allows a high flexibility in applications with a wide range of operating conditions. Usual drawbacks of such models are the time demand for

System	Model	Spatial	Complexity/	State
boundary	approach	dimension	details	
electrode	theoretical	zero to three	electrochemical,	steady-state,
(gas channels,	(mechanistic),	dimensions	thermodynamic,	quasi
catalyst layer),	empirical,		fluid dynamic,	steady-state,
fuel cell, stack,	semi-empirical		control	transient
system				

Table 2.1: Summary of major FC modeling features.

their development and validation, due to the difficulties in the achievement of the detailed FC stack features.

The other approach to model the FC is empirical. It is based on fitting experimental data by a set of mathematical functions. Usually, these models are related to the particular FC experimental data specific to each application and operating condition. They typically do not provide as many details as theoretical ones but may serve as a fast start into FC modeling and a simplified basis for engineering applications.

The next classification criterion is the state of the model - steady state or transient (or a special case, the quasi steady state). This criterion is especially useful for system engineers. Steady-state models describing one operating point in each step, are used mainly for parametric studies like sizing components in the system (stack and/or BoP size), calculating amounts of materials (e.g. catalyst), specification of the BoP equipment parameters. The transient models are useful for the design of the system interfaces, both electrical and thermal, and also the dynamic properties of the BoP equipment. The objective of the FC system modeling and simulation could be focused on both stationary and transportation FC applications.

2.2.2 Empirical electrochemical steady-state FC models

One of the well-known approaches related to empirical modeling of the PEMFC is described in [7]. In this model, the output cell voltage, V_{cell} , is expressed as a function of the current density, j, using the following notation:

$$V_{\text{cell}} = k_{\text{Nernst}} + k_{\text{act}} \log(j) - k_{\text{ohm}} j - k_{\text{con}} \exp(k_{\text{ij}} j), \qquad (2.21)$$

where k_{Nernst} , k_{act} , k_{ohm} , k_{con} , k_{j_1} are adjusted in a way to fit the experimental data. Some sets of values are presented in Table 2.2.

	10010 11	. i araineeer	o or one omp.		aon
Pr.,	$k_{\text{Nernst}},$	$k_{\rm act},$	$k_{ m ohm},$	$k_{\rm con},$	k_{j_1} ,
[bar]	[mV]	[mV/dec]	$\left[\Omega \mathrm{cm}^2\right]$	[mV]	$\left[\text{cm}^2/\text{mA} \right]$
1	943	62	0.238	3.75	6.38×10^{-3}
3	982	59	0.306	36	6.53×10^{-3}
5	1000	57	0.280	5.2	3.61×10^{-3}

Table 2.2: Parameters of the empirical FC model.

The simulation results corresponding to the mentioned values are presented in Fig. 2.2.



Figure 2.2: Simulation results of the PEMFC empirical model (at different pressures).

The four terms in the right part of (2.21) correspond to some extent to the wellknown overvoltages: Nernst, activation, resistive and concentration, respectively. This empirical model accurately predicts the PEMFC polarization curve but without complete consideration of thermodynamics and electrochemistry.

In [8] a similar model is considered with the incorporation of the logarithm of the pressure ratio that serves to model the oxygen pressure effect on the output voltage:

$$V_{\text{cell}} = k_{\text{Nernst}} + k_{\text{act}} \log(j) - k_{\text{ohm}} j - k_{\text{con}} \exp(k_{\text{lcd}} - k_{\text{p}}) \ln\left(\frac{p}{p_{\text{O}_2}}\right), \qquad (2.22)$$

where

 $k_{\rm lcd}$ and $k_{\rm p}$ - empirical coefficients.

These models are rather simple and give a good representation of the steady-state FC performance. One of the main drawbacks of such models is the limited operating and conditional range. Some terms are presented without a physicochemical interpretation and used just to fit experimental data. Also, the empirical parameters have to be redefined for different operating conditions.

2.2.3 Mechanistic and semi-empirical electrochemical steady-state FC models

The detailed mechanistic models [9], [10], [11], in opposite, provide considerable insight in the nature and the role of physical and chemical processes, like water transport, thermal and diffusion effect. Different operating conditions are presented in analytical form, allowing to model the FC performance in a wide range of current densities, temperature and pressure levels. Such mechanistic models are rather complex and in general require the knowledge of plenty of physical and chemical parameters to accurately predict the FC performance.

The combination of two common modeling approaches, namely the mechanistic and empirically based analysis, is applied in [5]. An approach combining both techniques inherits the mechanistic validity and the simplicity of the empirical approach. Originally, [5] has proposed a steady-state electrochemical model for a PEMFC which has been applied to particular fuel cells manufactured by Ballard Power Systems of Burnaby, BC, Canada. Its application was restricted to two particular cells: the Ballard Mark IV and Mark V. In [12] the original model has been modified in order to generalize it for a broader range of cell types, dimensions and operation regimes.

The fuel cell voltage is the result of the combined effect of thermodynamics, mass transport, kinetics and resistance:

$$V_{\text{cell}} = E_{\text{Nernst}} + \eta_{\text{act}} + \eta_{\text{ohm}}.$$
(2.23)

The open-circuit voltage, E_{Nernst} , is related to the maximum stored chemical energy. The last two components, η_{act} and η_{ohm} , (related to kinetics and resistance, respectively) represent the losses in the fuel cell. The concentration overvoltage is not modelled separately, but the mass transport effects are included in all three (right) terms of the equation.

Open-circuit voltage

For the open-circuit voltage, E_{Nernst} , the Nernst equation is written in the following form, using published values for the standard-state entropy:

$$E_{\text{Nernst}} = 1.229 - 0.85 \cdot 10^{-3} \cdot (T - 298.15) + + 4.3085 \cdot 10^{-5} \cdot T \cdot \left[\ln \left(p_{\text{H}_2}^{\text{int}} \right) + \frac{1}{2} \ln \left(p_{\text{O}_2}^{\text{int}} \right) \right], \qquad (2.24)$$

where

 $p_{\rm H_2}^{\rm int}$ - hydrogen partial pressure at the anode catalyst/gas interface, [Pa], $p_{\rm O_2}^{\rm int}$ - oxygen partial pressure at the cathode catalyst/gas interface, [Pa].

The evaluation of the hydrogen and oxygen partial pressures typically requires the mass transport calculation.

Activation overvoltage

Normally, the activation overvoltage, η_{act} , of the PEMFC is obtained as the sum of the anodic and the cathodic overvoltages:

$$\eta_{\rm act} = \eta_{\rm act\ anodic} + \eta_{\rm act\ cathodic}.$$
 (2.25)

The single expression for the total activation overvoltage can be presented as:

$$\eta_{\rm act} = \eta_{\rm act \ anodic} + \eta_{\rm act \ cathodic} = \xi_1 + \xi_2 T + \xi_3 T \left[\ln(c_{\rm O_2}^{\rm int}) \right] + \xi_4 T [\ln(i)], \quad (2.26)$$

where

$$\xi_1 = \left(\frac{-\Delta G_{\rm e}}{\alpha_{\rm c} nF}\right) + \left(\frac{-\Delta G_{\rm ec}}{2F}\right),\tag{2.27}$$

$$\xi_2 = \frac{R}{\alpha_{\rm c} n F} \ln \left[n F A k^0 (c_{\rm H^+}^{\rm int})^{(1-\alpha_{\rm c})} (c_{\rm H_2O}^{\rm int})^{(\alpha_{\rm c})} \right] + \frac{R}{2F} \ln \left(4 F A k^0 c_{\rm H_2}^{\rm int} \right), \qquad (2.28)$$

$$\xi_3 = \frac{R}{\alpha_{\rm c} n F} \left(1 - \alpha_{\rm c} \right), \tag{2.29}$$

$$\xi_4 = -\left(\frac{R}{\alpha_{\rm c} nF} + \frac{R}{2F}\right),\tag{2.30}$$

where

i - electrical current, [A],

 $\begin{array}{l} k^0 \mbox{ - intrinsic rate constant, [cm/s],} \\ n \mbox{ - number of equivalents involved in the reaction, [-],} \\ \alpha_{\rm c} \mbox{ - chemical activity parameter for the cathode, [-],} \\ A \mbox{ - active cell area, [cm^2],} \\ \Delta G_{\rm e} \mbox{ - change in standard-state free energy of the cathode reaction, [J/mol],} \\ \Delta G_{\rm ec} \mbox{ - change in standard-state free energy of activation for chemisorption, [J/mol],} \\ \Delta G_{\rm ec} \mbox{ - concentration of water, [mol/cm^3],} \\ c_{\rm D_2}^{\rm int} \mbox{ - concentration of hydrogen, [mol/cm^3],} \\ c_{\rm H_2}^{\rm int} \mbox{ - concentration of protons, [mol/cm^3].} \end{array}$

Resistance

The resistance results from the resistance to proton transfer in the solid polymer membrane and the resistance to electron transfer in the graphite collector plates and the electrodes.

The resistance to electron transfer is expected to be approximately constant over the operating temperature range (328 K < T < 358 K). The resultant voltage drop can be determined using Ohm's law:

$$\eta_{\rm ohm}^{\rm electrons} = -iR^{\rm electrons}.$$
(2.31)

The resistance to proton transfer through the membrane is not considered to be constant. The total drop in potential due to the resistance results from the combined resistance to electron and proton transport and can be defined using Ohm's law:

$$\eta_{\rm ohm}^{\rm total} = \eta_{\rm ohm}^{\rm electrons} + \eta_{\rm ohm}^{\rm protons} = -iR_{\rm internal}, \qquad (2.32)$$

where R_{internal} is the total resistance of the FC.

Due to the absence of an explicit mechanistic model, linear regression techniques are used to determine the "best fit" parametric model for the internal resistance:

$$R_{\rm internal} = \xi_5 + \xi_6 T + \xi_7 i. \tag{2.33}$$

Table 2.3 contains the coefficients values for the proposed model.

In [12] this model was more generalized with respect to its applicability to different FC configurations. A new empirical expression for the resistance is proposed, based on published recommendations and correlations of the set of FC performance curves including important membrane parameters:

$$R^{\rm protons} = \frac{r_{\rm M} l_{\rm M}}{A}, \qquad (2.34)$$

where

	1
ξ_1	$-0.948(\pm 0.004)$
ξ_2	$0.00286 + 0.0002 \ln A + 4.3 \times 10^{-5} \ln c_{\mathrm{H_2}}^{\mathrm{int}}$
ξ_3	$(7.6 \pm 0.2) \times 10^{-5}$
ξ_4	$-(1.93\pm0.05)\times10^{-4}$
ξ_5	3.30×10^{-3}
ξ_6	-7.55×10^{-6}
ξ_7	1.10×10^{-6}

Table 2.3: Parameters of the empirical FC model.

 $r_{\rm M}$ - membrane area-specific resistivity for the flow of hydrated protons, [Ω cm], $l_{\rm M}$ - thickness of the polymer membrane, [cm].

The membrane area-specific resistivity is defined as:

$$r_{\rm M} = \frac{181.6 \left[1 + 0.03 \left(\frac{i}{A} \right) + 0.062 \left(\frac{T}{303} \right)^2 \left(\frac{i}{A} \right)^{2.5} \right]}{\left[s - 0.634 - 3 \left(\frac{i}{A} \right) \right] \exp\left(4.18 \left[\frac{T - 303}{T} \right] \right)},\tag{2.35}$$

where

 \boldsymbol{s} - semi-empirical parameter representing the effective water content of the membrane.

2.2.4 One-dimensional steady-state diffusion

According to subsection 2.2.3, one of the main parameters of the electrochemical reaction influencing the electrical output performance of a FC is the concentration of the reactants at the membrane interface layer. The concentration defines the output voltage level, as well as the achievable current density [5], [13], [14].

Many publications could be found, presenting the analysis of the physical phenomena and based on the mass transport (diffusion), heat transfer, temperature effects, water balance, reactant gases flow, etc. The current work is not aimed at providing the analysis of all phenomena in details. The goal is limited to the description of the steady-state FC behavior with respect to thermodynamic processes (pressure and mass flow) and the estimate of the FC response time on the load and gas supply conditions. Thermal processes are considered much slower than electrical and pneumatic ones [15] and, thus, assumed constant.

The mass transport effects are the main issue in multi-domain models. Three distinct mass transport interfaces exist in both electrodes: gas flow channel - porous electrode, gas phase in the electrode - water film covering the catalyst site, water film - catalyst surface [5], [13]. In a mixed flow stream (O_2 , N_2 and H_2O for air), consumption of oxygen at the electrode would normally result in concentration gradients along the length of the gas flow channel. However, at the cathode side, the consumed oxygen represents a small fraction of the total flow. Thus, for simplification, the partial pressure can be assumed constant along the flow channel.

Diffusion in the electrodes

Gas diffusion through the porous carbon electrodes is generally described using the Stefan-Maxwell equation:

$$\nabla x_{i} = \sum_{j=1}^{n} \frac{RT}{pD_{ij}^{\text{eff}}} \left(x_{i}J_{j} - x_{j}J_{i} \right), \qquad (2.36)$$

where

x - mole fraction of species, [–], $J_{\rm i}$ - gas flux of species i, [mol/(cm²s)], $D_{\rm ij}^{\rm eff}$ - effective binary diffusivity coefficient of the i-j gas pair, [m²/s].

For simplification, the one-dimensional diffusion normal to the electrodes surface is considered. At the cathode side, the gas stream is represented as a mixture of oxygen and nitrogen saturated with water vapor:

$$x_{\rm O_2} + x_{\rm N_2} + x_{\rm H_20, \ vap} = 1. \tag{2.37}$$

The derivative along the gas diffusion path is:

$$\frac{\partial x_{O_2}}{\partial z} + \frac{\partial x_{N_2}}{\partial z} + \frac{\partial x_{H_20, \text{ vap}}}{\partial z} = 0.$$
(2.38)

For the case of constant flow channel pressure, and accounting for the fact that nitrogen is inert and can not be removed from the diffusion channel, zero fluxes of the water vapor and the nitrogen can be assumed [5]:

$$\frac{\partial x_{\rm H_20, \ vap}}{\partial z} = 0, \tag{2.39}$$

$$J_{\rm H_20, \ vap} = 0, \tag{2.40}$$

and

$$J_{N_2} = 0. (2.41)$$

Using (2.41) and (2.40), (2.36) for a one-dimensional flow of the nitrogen component can be simplified as:

$$\frac{\partial x_{\mathrm{N}_2}}{\partial z} = \frac{RT}{pD_{\mathrm{N}_2,\mathrm{O}_2}^{\mathrm{eff}}} \left(x_{\mathrm{N}_2} J_{\mathrm{O}_2} \right). \tag{2.42}$$

The effective value of the diffusivity coefficient is defined as [16], [17]:

$$D_{N_2,O_2}^{\text{eff}} = D_{N_2,O_2}^0 \frac{p_0}{p} \left(\frac{T}{T_0}\right)^{1.832} \epsilon f(s), \qquad (2.43)$$

where

 D_{N_2,O_2}^0 - standard-state binary diffusivity coefficient of the $N_2 - O_2$ gas pair, $[m^2/s]$, ϵ - porosity of the diffusion material, [-].

The empirical function f(s) related to liquid water saturation can be defined as [16]:

$$f(s) = (1-s)^2. (2.44)$$

The value of ϵ for the carbon fiber paper is chosen in the range between 0.25 and 0.35 [5], for s: 0.1 < s < 0.2 [16] and for D_{N_2,O_2}^0 : $0.2 \cdot 10^{-4} < D_{N_2,O_2}^0 < 0.25 \cdot 10^{-4} m^2/s$ [17].

Using Faraday's law, the oxygen flux is related to the current density:

$$J_{O_2} = \frac{j}{4F}.$$
 (2.45)

From (2.42) and (2.45) the oxygen mole fraction at the interface is derived:

$$x_{N_2}^{\text{int}} = x_{N_2}^{\text{ch}} \exp\left[\frac{R \cdot T \cdot j \cdot \delta}{4 \cdot F \cdot p \cdot D_{N_2,O_2}^{\text{eff}}}\right],$$
(2.46)

where

 δ - thickness of the gas diffusion layer (GDL), [m].

Combining (2.37), (2.46) and (2.43) the effective partial pressure of oxygen at the gas/liquid interface of the catalyst layer can be derived:

$$p_{O_2}^{\text{int}} = p \left[1 - x_{H_2O}^{\text{sat}} - x_{N_2}^{\text{ch}} \exp\left(\frac{0.291j}{T^{0.832}}\right) \right].$$
(2.47)

Using a similar procedure, the interface hydrogen partial pressure is determined:

$$p_{\rm H_2}^{\rm int} = p \left[1 - 0.5 x_{\rm H_2O}^{\rm sat} - x_{\rm CO_2}^{\rm ch} \exp\left(\frac{0.183j}{T^{0.832}}\right) \right].$$
 (2.48)

Diffusion through the water film

The oxygen flux through the water layer can be defined by the first Fick law:

$$J_{\mathcal{O}_2} = -D_{\mathcal{O}_2} \frac{\partial c_{\mathcal{O}_2}}{\partial z},\tag{2.49}$$

where

 D_{O_2} - diffusivity of the oxygen through the water film, $[m^2/s]$.

The concentration of oxygen at the gas/liquid interface is defined by Henry's law:

$$c_{\rm O_2}^{\rm int} = \frac{p_{\rm O_2}^{\rm int}}{5.08 \cdot 10^6 \cdot \exp\left(\frac{-498}{T}\right)}.$$
 (2.50)

A water film has a rather small thickness [5], thus the oxygen concentration at the catalyst layer can be approximated by the interface concentration (2.50).

For the anode side, the hydrogen concentration is:



 $c_{\rm H_2}^{\rm int} = \frac{p_{\rm H_2}^{\rm int}}{1.09 \cdot 10^6 \cdot \exp\left(\frac{77}{T}\right)}.$ (2.51)

Figure 2.3: Results of the steady-state simulation.

The presented model (2.23), (2.24), (2.26), (2.32), (2.33), (2.47), (2.48), (2.50), (2.51) describes the PEMFC performance over a rather expanded range of conditions in temperature and pressure. The simulation results of the model presented for the initial and the linear part of the polarization curve for three different pressure levels are shown in Fig. 2.3.

2.3 Transient PEMFC models

Section 2.2 describes a rather detailed multidomain steady-state FC model. The model presented reflects a steady-state response of the FC to the variation of the
electrical load and partial pressures of the inlet gases. This section provides a simplified description of several phenomena which influence the transient response of the FC. The presented transient models are not as detailed as the steady-state one. However they provide a proper estimate of the system time constants. The first subsection describes the FC transient response to an electrical load change, and the second one examines the time delay in the FC response to a chance in gas supply.

2.3.1 Zero-dimensional electrochemical model of the charge double-layer effect

The activation overvoltage (2.26) in a FC is mainly responsible for its dynamic electrical behavior [2]. It is related to the so-called "charge double-layer", an electrode phenomenon consisting of electrical charge arising at the surface of the contact between two different materials (Fig. 2.4).



Figure 2.4: Charge double-layer at the surface of a FC cathode.

It occurs due to diffusion effects and the reaction between electrons in the electrodes and ions in the electrolyte. As a result, the layer of charge on or near the electrode/electrolyte interface becomes a storage of electrical charge and energy, which behavior is similar to that of an electrical capacitor. This involves a certain time delay in the response of a FC voltage on the load change.

The common way of modeling this effect is by using an equivalent circuit (Fig. 2.5) with a charge double-layer represented by an electrical capacitance defined as:

$$C_{\rm cdl} = \varepsilon \frac{A_{\rm el}}{d}, \qquad (2.52)$$

where

 $C_{\rm cdl}$ - equivalent capacitance, [F], ε - electrical permittivity, [F/m], $A_{\rm el}$ - surface area, [m²], d - distance between the plates, [m].

In (2.52) $A_{\rm el}$ represents the real electrode surface area, which, due to its complex structure, has a very large value. The distance, d, is typically a few nanometers. Thus the value of the capacitance is in the order of a few Farads.



Figure 2.5: Simplified equivalent circuit of the FC, to model the phenomenon of the "charge double-layer" effect.

The experimental results describing the transient electrical behavior of the PEMFC can be found in [18, 19, 20, 21, 22]. In [18] measurements of the resistance are described. The measurements are performed applying square current pulses to the PEMFC. It is demonstrated, that with an applied current pulse the voltage response follows closely the current decay transition, corresponding to a very fast electrochemical reaction. The remaining voltage transient process is defined by the slowness of the charge recombination.

A similar experimental verification of the transient behavior of the PEMFC is described in [19, 20, 21]. A voltage transient due to a load step switching is shown in Fig. 2.6. It can be seen that the maximum response time of the FC stack voltage is around 0.7 s. Similar results are presented in [22], where the response time of the FC to a load change does not exceed 0.15 s. This approach gives a good representation of the transient electrical behavior of the PEMFC and may serve for a proper design of the electrical interface.

2.3.2 One-dimensional transient diffusion estimation

With the aim to complete the description of the FC characteristics, an analysis of the transient response of a FC on the gas (air) supply variation is desired. The distribution of the oxygen concentration along the cathode gas diffusion layer (GDL) for different levels of current, pressure and mass flow is illustrated in Fig. 2.7. In



Figure 2.6: Step response of the output voltage caused by a load current jump from 5 to 15 A. Courtesy of P.J.H Wingelaar, EPE, TUEindhoven.

general, it involves a very detailed analysis of the transients in all processes taking place in the FC, and such a research has not been found in literature. For simplifying the problem, the same assumptions are made as for the one-dimensional steady-state oxygen diffusion through a porous medium (section 2.2.4).

The transient diffusion is described by the second Fick law:

$$\frac{\partial c_{\mathrm{O}_2}}{\partial t} = D_{\mathrm{N}_2,\mathrm{O}_2}^{\mathrm{eff}} \frac{\partial^2 c_{\mathrm{O}_2}}{\partial z^2}.$$
(2.53)

Equation (2.53), also known as mass transfer equation, is solved by means of the separation of variables, followed by the use of Fourier series [23]. Applying the separation of variables, it can be split into time and space (along the coordinate z) dependent parts:

$$c_{O_2}(z,t) = c_1(z)c_2(t).$$
(2.54)

Substitution of (2.54) into (2.53) results in:

$$c_1(z)\frac{dc_2(t)}{dt} = D_{N_2,O_2}^{\text{eff}}c_2(t)\frac{\partial^2 c_1(z)}{\partial z^2}.$$
(2.55)

To separate variables, both parts are divided by $D_{N_2,O_2}^{eff}c_1(z)c_2(t)$ obtaining:

$$\frac{\frac{dc_2(t)}{dt}}{D_{N_2,O_2}^{\text{eff}}c_2(t)} = \frac{\frac{\partial^2 c_1(z)}{\partial z^2}}{c_1(z)},$$
(2.56)



Figure 2.7: Oxygen concentration distribution along the cathode GDL for different current, pressure and mass flow levels.

where each side must be equal to a constant. With the aim to obtain the exponential finite solution in time the separation constant is set to the negative value $-\frac{1}{\lambda^2}$, where λ has a unit of length. The time solution in this case is:

$$c_2(t) = c_{\rm A} e^{-\frac{D_{\rm N_2,O_2}^{\rm eff} \pi^2}{\lambda^2} t},$$
(2.57)

where

 $c_{\rm A}$ - constant,

and the spatial solution is:

$$c_1(z) = c_{\rm B} \cos\left(\frac{z}{\lambda}\right) + c_{\rm C} \sin\left(\frac{z}{\lambda}\right), \qquad (2.58)$$

where

 $c_{\rm B}$ and $c_{\rm C}$ - constants.

The general solution is:

$$c_{\rm O_2}(z,t) = c_1(z)c_2(t) = c_{\rm A}e^{-\frac{D_{\rm N_2,O_2}^{\rm eff}\pi^2}{\lambda^2}t} \left[c_{\rm B}\cos\left(\frac{z}{\lambda}\right) + c_{\rm C}\sin\left(\frac{z}{\lambda}\right)\right].$$
 (2.59)

To obtain the values of the coefficients in (2.59) the boundary and initial conditions should be specified. Setting the boundary conditions as:

$$c_{\mathcal{O}_2}(0,t) = 0, \tag{2.60}$$

and

$$c_{\mathcal{O}_2}(\delta, t) = 0,$$
 (2.61)

and applying (2.60) to (2.59) gives:

$$c_{\rm B}\cos\left(\frac{z}{\lambda}\right) = 0 \Rightarrow c_{\rm B} = 0.$$
 (2.62)

Then applying (2.61) to (2.59) gives:

$$c_{\rm C}\sin\left(\frac{\delta}{\lambda}\right) = 0 \Rightarrow \frac{\delta}{\lambda} = k\pi \Rightarrow \lambda = \frac{\delta}{k\pi},$$
 (2.63)

where k=1, 2, ...

Setting k=1, the general solution (2.59) becomes:

$$c_{\rm O_2}(z,t) = c_{\rm A} c_{\rm C} \sin\left(\frac{z\pi}{\delta}\right) e^{-\frac{D_{\rm N_2,O_2}^{\rm eff}\pi^2}{\delta^2}t}.$$
 (2.64)

From this solution it becomes possible to estimate the time constant of the diffusion delay:

$$\tau_{\rm dd} = \frac{\delta^2}{D_{\rm N_2,O_2}^{\rm eff} \pi^2}.$$
(2.65)

The diffusion delay is expressed as a function of the diffusivity coefficient and the width, δ , of the diffusion layer. The diffusivity coefficient is, in its turn, a function of pressure, temperature and water concentration (2.43). Setting the pressure level to the ambient value, and the values for δ and $D_{\rm N_2,O_2}^{\rm eff}$ to the same as in (2.46), the time of the diffusion delay is approximately $\tau_{\rm dd} \times 5 = 5 {\rm ms}$.

To find the values of the coefficients in the space solution, the proper initial conditions have to be specified. In practice, the spatial distribution of the oxygen concentration across the gas diffusion layer is rather difficult to obtain and this task lays beyond the scope of this research. Only the values of the concentration in the gas channel and at the membrane interface are applied, defined using (2.47), (2.50) and (2.64).

The analysis presented shows that the time delay of the FC transient response on the gas supply variation is rather low. The reactant gases are supplied and physically controlled by mechanical equipment. Since the time constant of the mechanical parts is expected to be significantly larger, the presented model for transient concentration (2.64) has not been included in the complete model.

2.4 Preliminary analysis of the PEMFC system

This section describes the first step in linking the mathematical models of the PEMFC and the compression system described in Chapter 3. In particular, the output variables of the compression system, air mass flow, m, and pressure, p, are coupled to the input variable of the FC: partial pressure of oxygen, $p_{O_2}^{int}$, related to the concentration of oxygen, $c_{O_2}^{int}$, according to (2.50).

As seen from (2.47), the oxygen partial pressure depends on the air pressure, p, and the mole fraction of nitrogen, $x_{N_2}^{ch}$. The nitrogen mole fraction, $x_{N_2}^{ch}$, in the channel can be expressed as an average value between inlet, $x_{N_2}^{in}$, and outlet, $x_{N_2}^{out}$, mole fractions:

$$x_{\rm N_2}^{\rm ch} = 0.5 \left(x_{\rm N_2}^{\rm in} + x_{\rm N_2}^{\rm out} \right). \tag{2.66}$$

The value of $x_{N_2}^{in}$ only depends on the air pressure and temperature. However, $x_{N_2}^{out}$ is defined by the amount of supplied air (mass flow, m) and the amount of oxygen consumed in the reaction.

Therefore, it is common practice to use a relation between supplied and consumed amount of oxygen, called the oxygen stoichiometry:

$$S_{\rm O_2} = \frac{m_{\rm O_2}^{\rm in}}{m_{\rm O_2}^{\rm in} - m_{\rm O_2}^{\rm out}},\tag{2.67}$$

where

 $m_{O_2}^{in}$ - inlet oxygen mass flow, [kg/s], $m_{O_2}^{out}$ - outlet oxygen mass flow, [kg/s].

Then, considering the oxygen and nitrogen molar proportions in dry air, which approximately equal 0.21 and 0.79, and assuming the inlet air and outlet gas mixture to be saturated, the nitrogen mole fraction of the outlet gas can be expressed as:

$$x_{N_2}^{\text{out}} = 0.79 \left(1 - x_{H_2O}^{\text{sat}}\right) \frac{S_{O_2}}{(S_{O_2} - 0.21)}.$$
 (2.68)

The combination of the FC model presented in section 2.2.3 with (2.66) - (2.68) allows to preliminary specify the performance characteristics of the air supply equipment in order to provide a proper system operation.

The amount of oxygen consumed can be derived from the following steps. First, using Faraday's law, the charge, Q delivered by one mole of hydrogen is defined as:

$$Q = nF \times 1 \text{ mole of } H_2. \tag{2.69}$$

Then, taking into account the basic chemical reactions (1.1) - (1.3) in the FC, the amount of oxygen consumed by one FC can be expressed as a function of the current drawn from the cell:

$$O_2$$
usage = $m_{O_2}^{in} - m_{O_2}^{out} = \frac{1}{2} \frac{i}{nF} = \frac{i}{4F}$ moles/s. (2.70)

Combining (2.70) and (2.67), and taking into account the air molecular weight (0.029 kg/mole) and the number of cells, N_{cell} , the air mass flow, m, [kg/s], is defined as:

$$m = N_{\text{cell}} \frac{S_{\text{O}_2}}{0.21} \frac{i}{4F} 0.029.$$
 (2.71)

The air mass flow which delivers only the amount of oxygen defined in (2.70) (being just enough for the reaction) leads to the deterioration of the FC performance due to the blocking of the oxygen supply by nitrogen, water vapor and lack of supplied oxygen. Therefore, in practice, the air is supplied with a stoichiometry value, $S_{O_2} = 2$, (2.67) [2], [24].

Equation (2.71) with the requirement of an oxygen stoichiometry, $S_{O_2} \geq 2$, preliminary defines the necessary mass flow, to be provided by an air compressor. The second variable, pressure, p, can be defined by considering a particular electrical load of the FC. As it can be seen from the FC model (section 2.2.3) or in Fig. 2.3, higher pressure results in a higher voltage of the FC.

Finally, it can be concluded, that the FC stack with a particular cell area and number of cells can deliver higher power at higher pressure and mass flow. On the other hand, an increased mass flow and pressure would result in extra power consumption by the compression system which supplies air into the stack. Figure 2.8 presents a power flow diagram of the system with a PEMFC, compressor and electrical drive, where

 $\begin{array}{l} P_{\rm d}^{\rm el} \mbox{ - electrical drive input power, [W],} \\ P_{\rm d} \mbox{ - output mechanical power of the electrical drive, [W],} \\ P_{\rm d}^{\rm loss} \mbox{ - electrical drive losses, [W],} \\ P_{\rm FC} \mbox{ - FC output electrical power, [W],} \\ P_{\rm FC,s} \mbox{ - FC system output electrical power (after the electrical drive), [W],} \\ P_{\rm thermal} \mbox{ - FC thermal output power, [W],} \\ P_{\rm c} \mbox{ - output power of the compressor, [W],} \\ P_{\rm c}^{\rm loss} \mbox{ - compressor losses, [W].} \end{array}$

In an autonomous FC system, the compression of air utilizes a portion of the FC output power. In high-temperature FC systems the thermal energy can be used to drive the compressor via a steam cycle. In a PEMFC the temperature of the exhaust gas/water does not exceed 80-90 °C and, thus, can not be used for energy recovering. Instead, a part (P_d^{el}) of the FC output electrical power, P_{FC} , is used by the compressor drive and is considered as power loss for the total system.



Figure 2.8: Power flow in a FC system.

2.5 Conclusions

From the steady-state one-dimensional model combining mechanistic and semiempirical approaches, the dependence of the electrical performance preliminary described by the polarization curve, on the air supply parameters (concentrations, partial pressures and temperature) is determined. It allows to define an initial specification of the compressor system with respect to steady-state characteristics.

The dynamic electrical properties of the PEMFC are described by simplified equations (section 2.3.1), illustrated by measurements. They impose dynamic performance requirements on the compression system.

The FC response time to the transient change in pressure or mass flow is estimated in section 2.3.2. Although the analysis of the transient diffusion is rather simplified and limited to one dimension, it gives an approximate value of the response time. As a result, it becomes clear that the FC thermodynamic transient performance is mainly defined by the air supply system (compressor or blower) as being a significantly slower system component (time limiting step).

Finally, when defining the static and dynamic requirements of the compression system, the power flow issues have to be taken into account. A detailed description of the static and dynamic characteristics of the compression system is treated in Chapter 3.

CHAPTER 3

Centrifugal air compressor modeling and control

3.1 Overview of air compressor technologies

An air compressor is employed in a fuel cell system to deliver the reactant oxygen to the fuel cell stack. The FC systems exist in two variants: pressurized and ambient pressure. Pressurization of a FC stack normally results in higher efficiency, improved response characteristics and higher power density of a FC system. However, pressurization uses part of the FC output power and, in some cases, may lead to a reduction in the system net efficiency. Hence, the compressor used to increase the pressure has a direct effect on the FC system performance.

This chapter describes the main properties of different types of compressors. The major attention is given to the centrifugal compressor, as it is considered as one of the best candidates for using in a FC compression system. Based on the fundamental operation principles, the performance characteristics of a centrifugal compressor are derived and modelled, as well as the methods for its active stabilization.

3.1.1 Positive displacement compressors

Compressors can be categorized by their operating principle. The compressors of the first group are of the "positive displacement" type. They operate on the principle of reduction of the gas volume, and, consequently, the pressure rise and further discharging of the compressed gas to a plant. Assuming the pressurization process adiabatic and reversible it can be described by the following relation, derived from the first law of thermodynamics:

$$dQ = pdV + c_{\rm v}dT = -Vdp + c_{\rm p}dT, \qquad (3.1)$$

$$pV^{\gamma} = \text{constant},$$
 (3.2)

where

 $\begin{array}{l} Q \text{ - internal energy, [J],} \\ p \text{ - pressure, [Pa],} \\ V \text{ - volume, [m^3],} \\ \gamma = \frac{C_{\mathrm{p}}}{C_{\mathrm{v}}} \text{ - specific heat ratio} = 1.4 \text{ for air, [-],} \\ c_{\mathrm{p}} \text{ - specific heat at constant pressure =1005 [J/(kg \cdot K)],} \\ c_{\mathrm{v}} \text{ - specific heat at constant volume =716 [J/(kg \cdot K)].} \end{array}$

This principle is applicable to reciprocating as well as rotary compressors. In reciprocating compressors, compression is realized by a moving piston in a cylinder with inlet and discharge valves (Fig. 3.1). This compressor type has quite good characteristics, but involve the use of complicated mechanics and expensive friction-less materials.

Rotary compressors, on the other hand, use rotating lobes and screws or sliding vanes. For example, the operation of the Roots compressor (Fig. 3.2) is based on the rotation of two lobes pushing the gas out of the chamber. This type of compressor has low production costs and usually operates over a wide range of flow rates. For example, one of the modern Roots compressors, manufactured by Eaton, has achieved valuable results in portable designs, reducing clearances between the air-pumping lobes and the camber walls, and an added twist - slightly helical rotors that smooth out the pulsating flow. But a reasonable efficiency is achieved only at low pressure rating.

The "twin-screw" Lysholm compressor (Fig. 3.3) looks similar to a helical Roots compressor, but with rotors that are far more twisted, and have a conical taper. As a result, these "screw-type" compressors have an internal compression ratio, making them more efficient in high-pressure applications. The outlet air in such a compressor can contain some amount of oil or can be oil-free, depending on the interconnection between the two rotors. The main advantages of the screw type compressors are the ability to provide a wide range of compression ratios (up to eight) and operate at high efficiency over a wide range of mass flow. However, the high-precision rotors make them very expensive to manufacture, and they usually operate with a high noise level.

The "vane-type" compressors (Fig. 3.4) work very similar to the Roots type: a chamber with a fixed volume is filled with air at atmospheric pressure, and then, this air is moved to the high pressure side. The characteristics are also similar to those of the Roots compressor.

The main drawback of positive displacement compressors is that the fill/discharge cycle is discrete: air comes in bursts, rather than smoothly and continuously. Additionally, in most positive displacement schemes (especially those which are not equipped with inlet and outlet valves) air can flow between a chamber with not-yet-pressurized air and a manifold with already-pressurized air. Thus, much of the air is compressed twice. The resulting turbulence heats the air, thereby reducing the compressor efficiency.



Figure 3.1: Reciprocating compressor.



Figure 3.2: Roots compressor.



Figure 3.3: Twin-screw compressor.

Figure 3.4: Vane compressor.

3.1.2 Centrifugal and axial compressors

The pressurization in the second group of compressors is done by the momentum transfer. This approach is realized in axial (Fig. 3.5) and centrifugal (Fig. 3.6) (or radial) compressors. In this type of compressors, the entering fluid is accelerated via the kinetic energy imparted in the rotor (often called impeller in centrifugal compressors), and then converting the kinetic into potential energy by decelerating the fluid in diverging channels. During the first step, the pressure of the fluid increases due to the momentum imparted to the flow by the rotor blades. The second pressurization step is the diffusion process in the diffuser or in the stator (in axial compressors). The deceleration of the fluid results in a static pressure rise. This process can partially be illustrated by Bernoulli's theorem (3.3), assuming incompressible flow and the process being reversible, however it can not be applied for description of the compressor operation:

$$\frac{1}{2}u_{\rm imp}^2 + \frac{p_{\rm imp}}{\rho} = \frac{1}{2}u_{\rm dif}^2 + \frac{p_{\rm dif}}{\rho},\tag{3.3}$$

where

 $u_{\rm imp}/u_{\rm dif}$ - gas velocity in an impeller/diffuser, [m/s], $p_{\rm imp}/p_{\rm dif}$ - static pressure in an impeller/diffuser, [Pa], ρ - gas density, [kg/m³].



Figure 3.5: Axial compressor.

Figure 3.6: Centrifugal compressor.

Many research and development efforts are aimed at different compressor technologies and their possible implementation in fuel cell systems [2], [25]. The analysis of the results shows that current compressor technologies do not always fit the fuel cell demands. In fact, they require the serious modification of an existing compressor type or the development of totally new technologies. Instead of mechanically redesigning the compressors, their adjustment to the FC demands could be achieved by means of the implementation of a new control strategy.

3.2 Steady-state and transient models of the system with a centrifugal air compressor

3.2.1 Overview

The current project deals with centrifugal compressors. Among other compressor types, centrifugal compressors are commonly used in a wide range of applications: from a combustion engine supercharger to the gas distribution systems. The appropriate characteristics of the centrifugal compressor, like simple and compact mechanical design, high reliability, high efficiency, no lubrication requirements, relatively high pressure capabilities and continuous mass flow make them very suitable.

It consists of two main parts: impeller and diffuser. The gas is drawn in at the center and flung out at high speed to the outer volute (Fig. 3.7). The pressure is increased due to the conversion of the kinetic energy of the gas. This type of compressor has a low cost, it is well developed and its efficiency is comparable to that of any other type [2].



Figure 3.7: Centrifugal compressor impeller and diffuser.

The operation area of centrifugal compressors is limited by the so-called "surge line" at the left side and the so-called "stone wall" at the right side. They define the stable operation area (Fig. 3.8). Surge is an unstable operation mode of centrifugal compressors which occurs when the operating point of the compressor is located to the left of the surge line. The "stone wall" limit is related to the "choking" of the air flow close to the sonic speed. The surge has been a major problem for designers and users of compressors since the invention of the turbocompressor. The surge phenomenon consists of oscillations of the mass flow, the pressure and the rotational speed. It significantly limits both static and dynamic behavior of the system and can cause severe damage to the machine. Section 3.3 discusses ways to extend the stable operation area beyond the surge limit by means of new control strategies.



Figure 3.8: Compressor map with the surge line.

3.2.2 General transient model of the compression system

With the aim to investigate the compressor behavior, to specify the requirements for its electrical drive (dynamic performance, type of applicable control strategy, etc) and to develop compressor/drive control algorithms, the dynamic model of the compressor has to be derived.

The dynamic model of a compression system, also known as a "Greitzer" model was first proposed in [26], where the mass flow and the pressure are states of the axial compressor system. This model has been further updated in [27], where the spool dynamic is included. In [28] and [29] this model has been applied to centrifugal compressors. The compression system used in the model consists of a centrifugal compressor, a compressor duct, a plenum volume and a valve (Fig. 3.9).

The dynamic model is described by:

$$\frac{dp_{\rm p}}{dt} = \frac{a_0^2}{V_{\rm p}} \left(m - m_{\rm v}\right),\tag{3.4}$$

$$\frac{dm}{dt} = \frac{A_{\rm c}}{L_{\rm c}} \left(p_2 - p_{\rm p} \right), \tag{3.5}$$

$$\frac{d\omega}{dt} = \frac{1}{J} \left(T_{\rm d} - T_{\rm c} \right), \qquad (3.6)$$

where





Figure 3.9: Compressor, duct, plenum and valve system.

- m compressor mass flow, [kg/s],
- $m_{\rm v}$ mass flow through the value, [kg/s],
- a_0 inlet stagnation sonic velocity, [m/s],
- $A_{\rm c}$ area of impeller eye, $[{\rm m}^2]$,
- $L_{\rm c}$ duct length, [m],
- J spool moment of inertia, [kgm²],
- ω compressor angular speed, [rad/s],
- p_0 ambient pressure, [Pa],
- p_2 pressure downstream of the compressor, [Pa],
- $p_{\rm p}$ plenum pressure, [Pa],
- $V_{\rm p}$ plenum volume, $[{\rm m}^3]$,
- $T_{\rm d}$ drive torque, [Nm],
- $T_{\rm c}$ compressor torque, [Nm].

Equation (3.4) for the pressure follows from the mass balance in the plenum, assuming the process is isentropic. Equation (3.5) for the mass flow follows from the impulse balance in the duct.

3.2.3 Steady-state performance characteristics of the centrifugal compressor

To obtain the compressor equations in a complete form, the expressions for the pressure downstream of the compressor, p_2 , the mass flow through the valve, m_v , and the compressor torque, T_c , should be obtained. There are a few approaches to model the compressor. In [30] the centrifugal compressor characteristics are approximated by a cubic polynomial function using stationary measurements, based on the approach used in [31] and [27]. In [29] the model of the compressor is derived based on the analysis of the energy transfer and losses in the different compressor parts.

Generally, the pressure rise in the compressor from the level p_0 to the level p_2 can be described as an isentropic process $1 \rightarrow 2$ in the enthalpy-entropy diagram (Fig. 3.10), which involves an increase in the stagnation enthalpy $\Delta h_c = h_c - h_0$. In practice, the pressure increase in the compressor is not isentropic as the entropy also increases due to different kinds of losses. Thus, the compression from p_0 to p_2 is modelled as an isentropic process in series with an isobaric one. For the isentropic process, the entropy differential is ds = 0 and the enthalpy differential dh = V dp, while for the isobaric process dp = 0 and dh = T ds. The isentropic process $1 \rightarrow 2$ takes place first, which ends in a state with pressure p_2 and stagnation enthalpy h_c . Then, the isobaric process $2 \rightarrow 3$ takes place, which corresponds to the entropy increase, and ends in a state with the same pressure level p_2 and total stagnation enthalpy $h_{c,t}$.

The process $1 \rightarrow 3$ can be described by:

$$\Delta h_{\rm c, t} = \Delta h_{\rm c} + T \Delta s = \Delta h_{\rm c} + \Delta h_{\rm loss}, \qquad (3.7)$$

where

 $\Delta h_{\rm c, t}$ - total specific enthalpy delivered to the fluid, [J/kg], $\Delta h_{\rm c}$ - change in stagnation enthalpy in the isentropic process, [J/kg], Δs - change in specific entropy, [J/(kg · K)], $\Delta h_{\rm loss}$ - change in stagnation enthalpy due to losses, [J/kg].

The compressor characteristics, $\Psi_{c}(\omega, m)$, are defined by the following pressure ratio:

$$\Psi_{\rm c}(\omega, \ m) = \frac{p_2}{p_0}.$$
(3.8)

From the standard isentropic relations it is valid that:

$$\frac{p_2}{p_0} = \left(\frac{T_2'}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\Delta h_c}{c_p T_0}\right)^{\frac{\gamma}{\gamma-1}},$$
(3.9)

where

 $T_2^{'}$ - is entropic temperature of the compressed air (point 2 in Fig. 3.10), [K], T_0 - ambient temperature, [K].

The applied torque is set equal to the change in the angular momentum of the fluid (Fig. 3.11):

$$T_{\rm c} = m \left(r_2 u_{\rm t,2} - r_1 u_{\rm t,1} \right), \tag{3.10}$$

where



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Figure 3.10: Enthalpy (h) - entropy (s) diagram for a centrifugal compressor.



impeller eye (inducer)

Figure 3.11: Centrifugal compressor geometry.

 $r_1 = \sqrt{\frac{1}{2} (r_{t1}^2 + r_{h1}^2)}$ - average impeller radius, [m], r_2 - external impeller radius, [m], $u_{t,1,2}$ - tangential gas velocity at the according radius, [m/s].

The power delivered to the fluid is defined as:

$$P_{\rm c} = \omega T_{\rm c} = \omega m \left(r_2 u_{\rm t,2} - r_1 u_{\rm t,1} \right) = m \left(U_2 u_{\rm t,2} - U_1 u_{\rm t,1} \right) = m \Delta h_{\rm c,t}, \tag{3.11}$$

where

U - tangential impeller velocity at the according radius, [m/s].

The relation between the tangential impeller and gas velocities (slip factor, σ) is defined using the so-called Stanitz's formula:

$$\sigma = \frac{u_{\mathrm{t},2}}{U_2} \approx 1 - \frac{2}{N_{\mathrm{bl}}},\tag{3.12}$$

where

 $N_{\rm bl}$ - number of compressor blades.

From (3.11) and (3.12) and using the fact [29], that $u_{t,1} = 0$, the total specific enthalpy, $\Delta h_{c,t}$, is derived:

$$\Delta h_{\rm c, t} = \frac{P_{\rm c}}{m} = \sigma \omega^2 r_2^2. \tag{3.13}$$

The compressor torque, T_c , is:

$$T_{\rm c} = mr_2 u_{\rm t,2} = mr_2 \sigma U_2 = m\sigma \omega r_2^2.$$
 (3.14)

In the ideal case the enthalpy is independent of the mass flow. In practice the energy transfer cannot be constant due to the compressor losses, $\Delta h_{\rm loss}$, - mainly incidence losses and fluid friction losses. They play the most important role in the determination of the compressor stable operation range.

The incidence losses, Δh_i , occur due to the instantaneous change of the gas velocity while it hits the inducer and can be written as [29]:

$$\Delta h_{\rm i} = \frac{r_1^2}{2} \left(\omega - k_{\rm ins} m \right)^2, \qquad (3.15)$$

where

 $k_{\rm ins}$ - constant, determined by the geometry of the compressor components.



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Figure 3.12: Specific enthalpies for $\omega = 5400$ rad/s.

The second type of compressor losses consists of friction losses, $\Delta h_{\rm f}$, due to the friction of a mass flow through a pipe [29]:

$$\Delta h_{\rm f} = k_{\rm f} m^2, \qquad (3.16)$$

where

 $k_{\rm f}$ - fluid friction constant.

They are independent on the rotational speed and are quadratic with respect to the mass flow. The specific energy transfer or specific enthalpies with friction and incidence losses are plotted in Fig. 3.12.

3.2.4 Complete transient model of the compression system

Finally, using (3.7), (3.8), (3.9), (3.15) and (3.16) the relation between the pressure rise and the energy transfer, where the losses are taken into account, is:

$$p_{2} = \left(1 + \frac{\sigma\omega^{2}r_{2}^{2} - \frac{r_{1}^{2}}{2}\left(\omega - k_{\rm ins}m\right)^{2} - k_{\rm f}m^{2}}{T_{0}c_{\rm p}}\right)^{\frac{\gamma}{\gamma-1}} p_{0} = \Psi_{\rm c}\left(m,\omega\right)p_{0}.$$
 (3.17)

The compressor characteristics calculated using (3.17) are shown in Fig. 3.13.



Figure 3.13: Calculated centrifugal compressor characteristics (for $\omega=2000$ - 4500 rad/s).

The surge line passes through the local maxima of the constant speed lines and divides the map into a stable and unstable (surge) operation area. The valve is regarded as a simplified model of a turbine, and the mass flow through the valve is modelled as:

$$m_{\rm v} = k_{\rm v} \sqrt{p_{\rm p} - p_0}.$$
 (3.18)

Combination of (3.4), (3.5), (3.6), (3.14), (3.17) and (3.18) results in the complete dynamic model of the compression system:

$$\frac{dp_{\rm p}}{dt} = \frac{a_0^2}{V_{\rm p}} \left(m - k_{\rm v} \sqrt{p_{\rm p} - p_0} \right), \qquad (3.19)$$

$$\frac{dm}{dt} = \frac{A_{\rm c}}{L_{\rm c}} \left(\left(1 + \frac{\sigma \omega^2 r_2^2 - \frac{r_1^2}{2} \left(\omega - k_{\rm ins} m\right)^2 - k_{\rm f} m^2}{T_0 c_{\rm p}} \right)^{\frac{\gamma}{\gamma - 1}} p_0 - p_{\rm p} \right), \qquad (3.20)$$

$$\frac{d\omega}{dt} = \frac{1}{J} \left(T_{\rm d} - m\sigma\omega r_2^2 \right). \tag{3.21}$$



3.2 Steady-state and transient models of the system with a centrifugal air compressor 51



Figure 3.14: Centrifugal compressor simulation results (stable operation area): (a) in m - p coordinates, (b) as a function of time.



Figure 3.15: Centrifugal compressor simulation results (surge): (a) in m - p coordinates, (b) as a function of time.

The presented model describes the dynamics of a centrifugal compressor, including the surge phenomenon. However, the compressor characteristics are derived using the expression for the steady-state losses. Therefore, in general, the model should be corrected for the case of transient mass flow and pressure during the surge. More detailed models of the centrifugal compressor including flow transients can be found in [32].

A MATLAB/Simulink model has been derived, based on the compressor equations (3.19) to (3.21). The initial values of the model parameters are taken from [30]. The inputs of the model are: $T_{\rm d}$ - drive torque, $k_{\rm v}$ - valve coefficient. The states are: $p_{\rm p}$ - plenum pressure, m - mass flow, ω - rotational speed.

In the first simulation the valve coefficient, $k_v = 0.0025$, which corresponds to a load curve I located to the right of the surge line. The compressor operates in the stable area (Fig. 3.14).

In the second simulation the load curve II is shifted to the left of the surge line, setting the valve coefficient, $k_{\rm v} = 0.001$, which initiates the surge in the compressor. The simulation results are presented in Fig. 3.15.

It can be seen, that specifying the load curve to the left of the surge line forces the compressor into surge with fluctuations of the mass flow, the pressure and the rotational speed of the compressor.

3.3 Surge control methods

3.3.1 Open-loop surge control

A few approaches are used with the aim to counteract the surge. Traditionally, surge has been avoided using the open-loop surge control or so-called "surge avoidance" schemes. Surge avoidance schemes use various techniques to keep the operating point of the compressor away from the surge line. The simplest arrangement is a fixed restriction orifice (or recycle valve) in a line leading back to the supply side of the compressor (Fig. 3.16).



Figure 3.16: Surge avoidance scheme with a recycle valve.

Typically, a surge control line is drawn at a specified distance from the surge line, and the surge avoidance scheme ensures that the operating point does not cross it. This method works properly, as proven by numerous installations. However, due to the presence of the surge margin, the method restricts the operating range of the machine, and the achievable efficiency is limited (Fig. 3.17).



Figure 3.17: Compressor map with the surge and control line.

3.3.2 Active surge control by means of a close coupled valve

Active surge control fundamentally differs from surge avoidance as unstable equilibria are sought to be stabilized instead of avoided. The motivation behind this is to overcome the mentioned limitations of surge avoidance. Active surge control was first introduced in [33]. In [34], [27] and [35] the active surge control method employs the so-called close coupled valve (CCV), installed in series with the compressor (Fig. 3.18). It is based on the following principle. Due to the friction losses, the maximum energy transfer and minimum incidence loss do not occur at the same mass flow. The friction shifts the point of maximum energy transfer, and consequently the pressure rise, to the left of the point of minimum incidence loss, Δh_i (Fig. 3.12). Therefore, the friction losses in fact have a stabilizing effect, and introducing additional fluid friction would move the point of maximum energy transfer to the left (Δh_c in Fig. 3.12). The result is that the surge line is shifted to the left, and the area of stable compressor operation is expanded (Fig. 3.19).



Figure 3.18: Compression system with the CCV.



Figure 3.19: Modification of the compressor characteristics due to the additional friction in the CCV.

Thus, the valve introduces a pressure drop in the system, and the characteristic of the valve has the same qualitative impact on the equivalent compressor characteristic as introducing more fluid friction. The pressure drop over this valve serves as control variable, and is used to introduce additional friction at low mass flows in order to avoid surge.

This method is applicable in compressor systems driven by prime movers of which the speed can not be controlled with high performance. This type of the surge control using a bleed valve has been successfully implemented in the laboratory of Mechanical Engineering Department of Eindhoven University of Technology [30]. The main drawback of this approach are the additional friction losses and the pressure drop in the valve. In addition, an extra actuator is required to obtain the mentioned control.

3.3.3 Active surge control by means of a controlled electrical drive

A distinctive active surge control method by means of an electrical drive powering the compressor is proposed in [36] and [37]. The method is based on the torque/speed control of the electrical drive (Fig. 3.20).

In this approach of active surge control the electrical drive itself is used for surge control, thus eliminating the need for additional actuators. Therefore, the compressor should be able to operate at a low mass flow without surge providing an extension of the compressor operation area. The rotational speed or drive torque can be chosen as a control variable, depending on the applied control scheme.

In this approach the dynamics around the equilibrium point are considered. The deviations are written as:

$$\Delta m = m - m^0, \tag{3.22}$$

$$\Delta p = p - p^0, \tag{3.23}$$

$$\Delta \omega = \omega - \omega^0, \tag{3.24}$$

$$\Delta \Psi_{\rm c} \left(\Delta \omega, \Delta m \right) = \Psi_{\rm c} \left(\Delta \omega + \omega^0, \Delta m + m^0 \right) - \Psi_{\rm c}^0, \tag{3.25}$$

where $(\cdot)^0$ - equilibrium values.

The key feature of this type of control is that the shaft speed becomes a function of the mass flow:

$$\Delta \omega = -K_{\Psi} \Delta m, \qquad (3.26)$$

where the value of the gain, K_{Ψ} , [36], [37]:

$$K_{\Psi} > \frac{\partial \Psi_{\rm c}/\partial m}{\partial \Psi_{\rm c}/\partial \omega}.$$
(3.27)

This control results in the modification of the compressor characteristics, shown in Fig. 3.21, making the operation stable [36].

From (3.24) and (3.26) the desired speed, ω^* , is defined as:

$$\omega^* = \omega^0 - K_{\Psi} \Delta m. \tag{3.28}$$

For the verification of the surge control approach, a proportional feedback speed control law is applied:

$$\Delta T_{\rm d} = K_{\omega} \left(\omega^* - \omega \right). \tag{3.29}$$

The corresponding electrical drive torque is defined as:

$$T_{\rm d} = \Delta T_{\rm d} + T_{\rm d}^0, \qquad (3.30)$$

where $T_{\rm d}^0$ is the torque required at the equilibrium point.

As a result, the control law becomes:

$$T_{\rm d} = -K_{\omega}\Delta\omega - K_{\rm m}\Delta m + T_{\rm d}^0, \qquad (3.31)$$

where the feedback gain for the mass flow is:

$$K_{\rm m} = K_{\Psi} K_{\omega}. \tag{3.32}$$

The electrical drive torque at the equilibrium point, $T_{\rm d}^0$, corresponding to the desired system performance, is defined as the result of the control of the complete system consisting of a FC, compressor, electrical drive and a FC electrical load, as discussed in Chapter 7.

Using a MATLAB/Simulink model of the compressor, the control law (3.31) has been implemented. The simulation results are presented in Fig. 3.22.

The presented results show that the active surge control by means of a torque actuation allows the compressor to operate in a stable mode, when the operating point is located to the left of the surge line. This would allow to extend the stable operation area of the compressor, resulting in a more flexible regime of air supply into a FC stack, retaining high pressure.



Figure 3.20: Diagram of performance and active surge control by means of a controlled electrical drive.



Figure 3.21: Modification of the compressor characteristics due to active surge control by means of a controlled electrical drive.



(a)



Figure 3.22: Simulation results with the active surge control by means of a controlled torque actuation: (a) in m - p coordinates, (b) as a function of time.

3.4 Conclusions

This chapter presents comprehensive transient mathematical models of the compression system together with the steady-state performance characteristics of the centrifugal compressor. The specific instability phenomenon for centrifugal compressors - surge, is highlighted. Two approaches of an active surge control are described. The first one uses a controlled valve and can be applied for compression systems, where the prime mover is not controlled with high performance. The special attention is given to the second active surge control method, which employs the torque/speed control of a high-performance electrical drive.

The models presented in this chapter allow to complete the coupling of the FC model with the model of the compressor in the thermodynamic domain, using the state variables - mass flow, m, and pressure, p. Moreover, the compression system model can also be linked to the model of the electrical drive in the mechanical domain via (3.21).

Additionally, the presented dynamic model serves as a basis for the analysis of the dynamic behavior of the compression system. On the one hand, the transients of the mass flow and pressure depend on the geometrical parameters used in the "Greitzer" model. On the other hand, they are also defined by the transient angular velocity of the impeller.

The described method of the surge control, in its turn, requires a very fast response of the electrical drive. Thus, the combined models of the FC and compression system impose a set of specifications on the electrical drive related to its steady-state and transient properties. The major types of electrical motors and their control methods are presented in Chapter 4.

Chapter 4

Drives and control methods

The function of the electrical drives in a FC system is to convert electrical energy produced by the FC into mechanical energy for the main BoP systems.

As discussed, the role of electrical drives can be extended with the aim to enhance the efficiency, stability and performance of the FC system. The electrical drives are part of the internal chain of energy conversion in a FC system. The drives have to satisfy requirements, resulting in an efficient and high-performance operation with very flexible regimes allowing to fulfill the main system demands.

Several known types of electrical machines can be used as a compressor drive: permanent magnet brushless DC (BLDC), permanent magnet AC synchronous (PMAC), synchronous reluctance (SynR), switched reluctance (SR) and induction motors (IM). The particular choice of the machine type is usually based on factors, such as reliability, efficiency and cost.

The permanent-magnet (PM) machines are more attractive from the point of view of efficiency and power density, while the strong points of induction motors are lower cost and higher reliability. The use of modern materials and the application of new technologies allow a substantial improvements in the efficiency of induction machines. Additional efficiency improvements, especially for the operation at non-rated regimes, can be achieved via energy-efficiency control.

The SynR and SR motors are also generally characterized as highly reliable and low-cost machines due to their simple structure. On another hand, they usually require a very small airgap to achieve proper performance characteristics, which can compromise their cost advantage. Additionally, a set of precautions must be taken in SR motors to prevent their high-level noise and vibrations.

This chapter, first, summarizes the basic principles of energy conversion in electrical machines. Further, the mathematical models and control principles of PMAC and induction machines, as the best candidates for using as a FC compressor drive are reviewed. Special attention is paid to the energy-efficiency control strategies for an induction motor fed by a power electronic converter. The high-performance indirect rotor flux oriented control of the induction motor, which has been implemented in this work, is described in detail.

4.1 Energy conversion in electrical drives

4.1.1 Electrical equations

Stationary coil actuators

The electrical equations for an electromechanical device follow from circuit theory, which in turn is based on quasi-static field theory. In this section a generalized version of Kirchhoff's voltage law is derived which also takes into account the effects of electromechanical coupling.



Figure 4.1: Electromechanical actuator circuit.

At the beginning an electromechanical device with a stationary electrical circuit is studied (Fig. 4.1). It consists of a voltage source, v_s , a resistor, R, and a magnetic circuit. The magnetic circuit comprises a fixed coil, wounded on the stator soft magnetic core, and a soft magnetic rotor. At any point of $C_{1-2-3-1}$ the electrical field strength is given by [38]:

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \varphi, \qquad (4.1)$$

where

E - electrical field strength, [V/m], A - vector magnetic potential, [Wb/m], φ - scalar potential, [V].

The component, $\nabla \varphi$, corresponds to the accumulation of charge along the circuit

and $\frac{\partial A}{\partial t}$ is related to the time-varying current. Evaluating the closed line integral of (4.1) gives:

$$\oint_{\mathcal{C}} \boldsymbol{E} d\boldsymbol{l} = -\oint_{\mathcal{C}} \frac{\partial \boldsymbol{A}}{\partial t} d\boldsymbol{l} - \oint_{\mathcal{C}} \nabla \varphi d\boldsymbol{l}.$$
(4.2)

The first term is decomposed into integrals over individual strokes:

$$\int_{1}^{2} \boldsymbol{E} d\boldsymbol{l} + \int_{2}^{3} \boldsymbol{E} d\boldsymbol{l} + \int_{3}^{1} \boldsymbol{E} d\boldsymbol{l} = -\oint_{C} \frac{\partial \boldsymbol{A}}{\partial t} d\boldsymbol{l} - \oint_{C} \nabla \varphi d\boldsymbol{l}.$$
(4.3)

The integration of the conservative vector, $\nabla \varphi,$ is performed along a closed path, thus:

$$\oint_{\mathcal{C}} \nabla \varphi d\boldsymbol{l} = 0. \tag{4.4}$$

Assuming that the voltage source is ideal without internal resistance its voltage is defined as:

$$v_{\rm s} \equiv -\int_1^2 \boldsymbol{E} d\boldsymbol{l}.\tag{4.5}$$

Thus, (4.3) becomes:

$$v_{\rm s} = \int_2^3 \boldsymbol{E} d\boldsymbol{l} + \int_3^1 \boldsymbol{E} d\boldsymbol{l} + \oint_{\rm C} \frac{\partial \boldsymbol{A}}{\partial t} d\boldsymbol{l}.$$
(4.6)

The voltage drop across the resistor is described by Ohm's law:

$$\boldsymbol{J} = \sigma \boldsymbol{E},\tag{4.7}$$

where

- J current density, $[A/m^2]$,
- σ conductivity, [S/m].

Therefore:

$$\int_{2}^{3} \boldsymbol{E} d\boldsymbol{l} = \int_{2}^{3} \frac{\boldsymbol{J}}{\sigma} d\boldsymbol{l} = i_{\rm s} R.$$
(4.8)

Similarly, Ohm's law is applied to the coil:

$$\int_{3}^{1} \boldsymbol{E} d\boldsymbol{l} = \int_{3}^{1} \frac{\boldsymbol{J}}{\sigma} d\boldsymbol{l} = i_{\rm s} R_{\rm coil}.$$
(4.9)

The resistances, R, and $R_{\rm coil}$ are connected in series, and can be replaced by the common resistance, $R_{\rm s}$:

$$R_{\rm s} = R + R_{\rm coil}.\tag{4.10}$$

Substituting (4.8) - (4.10) into (4.6) yields:

$$v_{\rm s} = i_{\rm s} R_{\rm s} + \oint_{\rm C} \frac{\partial \boldsymbol{A}}{\partial t} d\boldsymbol{l}.$$

$$\tag{4.11}$$

The last term in (4.11) is rewritten as:

$$\oint_{\mathcal{C}} \frac{\partial \boldsymbol{A}}{\partial t} d\boldsymbol{l} = \int_{\mathcal{S}} \left(\nabla \times \frac{\partial \boldsymbol{A}}{\partial t} \right) d\boldsymbol{s} = \int_{\mathcal{S}} \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{s} = \frac{d}{dt} \int_{\mathcal{S}} \boldsymbol{B} d\boldsymbol{s} = \frac{d\lambda_{s}}{dt}, \quad (4.12)$$

where

 $\lambda_{\rm s}$ - flux linkage, [Wb], S - area bounded by the circuit path, [m²].

The flux linkage is defined as:

$$\lambda_{\rm s} \equiv \int_{\rm S} \boldsymbol{B} d\boldsymbol{s}. \tag{4.13}$$

The complete evaluation of the integral (4.13) can be very complicated depending on the circuit topology. Instead, the flux linkage is expressed in the simplified form:

$$\lambda_{\rm s} = N_{\rm coil}\Phi,\tag{4.14}$$

where

 Φ - flux through each coil, [Wb], $N_{\rm coil}$ - number of coils.

Substituting (4.12) into (4.11) the Kirchhoff's voltage law is obtained:

$$v_{\rm s} = i_{\rm s} R_{\rm s} + \frac{d\lambda_{\rm s}}{dt}.\tag{4.15}$$

The last term in (4.15) represents the voltage induced in the circuit by the rate of change of magnetic flux linked by the circuit. The flux linkage is a function of both current (i_s) in the coil and position (θ) of the rotor, thus the full differential is written as:

$$\frac{d\lambda_{\rm s}}{dt} = \frac{\partial\lambda_{\rm s}(i_{\rm s},\theta)}{\partial i_{\rm s}}\frac{di_{\rm s}}{dt} + \frac{\partial\lambda_{\rm s}(i_{\rm s},\theta)}{\partial\theta}\frac{d\theta}{dt}.$$
(4.16)

Moving coil actuators

In this section Kirchhoff's voltage law is derived for circuits with a coil moving through an external magnetic field (Fig. 4.2). This figure shows a linear actuator consisting of a conductive bar in sliding contact with a pair of stationary conducting rails. The rails are connected to a voltage source, v_s , and a resistor, R, that limits the current. The bar moves with velocity \boldsymbol{u} through $\boldsymbol{B}_{\text{ext}}$, which is constant with respect to the origin O.



Figure 4.2: Moving conductor actuator.

Kirchhoff's voltage law for a circuit with a moving bar is derived by applying Faraday's law to a stationary contour, C, that passes through the circuit and coincides with the bar for a given position (dashed line in Fig. 4.2). Thus, C coincides with the entire circuit at a single instant in time. Evaluating the integral:

$$\oint_{\mathcal{C}} \boldsymbol{E} d\boldsymbol{l} = -\frac{d}{dt} \int_{\mathcal{S}} \boldsymbol{B} d\boldsymbol{s}$$
(4.17)

around C yields:

$$\int_{1}^{2} \boldsymbol{E} d\boldsymbol{l} + \int_{2}^{3} \boldsymbol{E} d\boldsymbol{l} + \int_{3}^{1} \boldsymbol{E} d\boldsymbol{l} = -\frac{d}{dt} \int_{S} \boldsymbol{B} d\boldsymbol{s}.$$
(4.18)

Using (4.5) - (4.9):

$$v_{\rm s} = i_{\rm s}R + \int_3^1 \boldsymbol{E}d\boldsymbol{l} + \frac{d}{dt}\int_{\rm S}\boldsymbol{B}d\boldsymbol{s}.$$
(4.19)

The velocity of the bar is \boldsymbol{u} . The electrical field strength, \boldsymbol{E} , corresponding to the stationary reference frame with the origin O is expressed as:

$$\boldsymbol{E} = \frac{\boldsymbol{J}}{\sigma} - \boldsymbol{u} \times \boldsymbol{B}_{\text{ext}}.$$
(4.20)

Therefore,
$$\int_{3}^{1} \boldsymbol{E} d\boldsymbol{l} = \int_{3}^{1} \left(\frac{\boldsymbol{J}}{\sigma} - \boldsymbol{u} \times \boldsymbol{B}_{\text{ext}} \right) d\boldsymbol{l} = i_{\text{s}} R_{\text{coil}} - \int_{3}^{1} (\boldsymbol{u} \times \boldsymbol{B}_{\text{ext}}) d\boldsymbol{l}.$$
(4.21)

Substituting (4.21) into (4.19) and using (4.10) results in:

$$v_{\rm s} = i_{\rm s} R_{\rm s} + \int_3^1 (\boldsymbol{u} \times \boldsymbol{B}_{\rm ext}) d\boldsymbol{l} + \frac{d}{dt} \int_{\rm S} \boldsymbol{B} d\boldsymbol{s}.$$
(4.22)

In case of a magnetically linear system, \boldsymbol{B} in the last term of (4.22) can be expressed as a superposition of two field components: \boldsymbol{B}_{i} - due to the current i_{s} and \boldsymbol{B}_{ext} due to the external field:

$$\boldsymbol{B} = \boldsymbol{B}_{i} + \boldsymbol{B}_{ext}.$$
 (4.23)

Using the definition of the inductance:

$$L_{\rm s} = \frac{\partial \lambda_{\rm coil}(i_{\rm s})}{\partial i_{\rm s}},\tag{4.24}$$

and expressing the coil flux linkage as:

$$\lambda_{\rm coil}(i_{\rm s}) = \int_{\rm S} \boldsymbol{B}_{\rm i} d\boldsymbol{s}, \qquad (4.25)$$

the last term in (4.22) is rewritten as:

$$\frac{d}{dt} \int_{\mathcal{S}} \boldsymbol{B} d\boldsymbol{s} = \frac{d}{dt} \int_{\mathcal{S}} \boldsymbol{B}_{i} d\boldsymbol{s} + \frac{d}{dt} \int_{\mathcal{S}} \boldsymbol{B}_{ext} d\boldsymbol{s} = L_{s} \frac{di_{s}}{dt} + \frac{d}{dt} \int_{\mathcal{S}} \boldsymbol{B}_{ext} d\boldsymbol{s}.$$
(4.26)

Finally, substituting (4.26) into (4.22) the generalized Kirchhoff's voltage law is derived which takes into account the voltage induced in a coil moving through B_{ext} :

$$v_{\rm s} = i_{\rm s} R_{\rm s} + L_{\rm s} \frac{di_{\rm s}}{dt} + \int_3^1 (\boldsymbol{u} \times \boldsymbol{B}_{\rm ext}) d\boldsymbol{l} + \frac{d}{dt} \int_{\rm S} \boldsymbol{B}_{\rm ext} d\boldsymbol{s}.$$
 (4.27)

If B_{ext} is static, (4.27) is reduced to:

$$v_{\rm s} = i_{\rm s} R_{\rm s} + L_{\rm s} \frac{di_{\rm s}}{dt} + \int_3^1 (\boldsymbol{u} \times \boldsymbol{B}_{\rm ext}) d\boldsymbol{l}.$$
(4.28)

This voltage equation can also be applied to the case of a rotating coil. If the segment dl of the coil is rotating with an angular velocity, ω , at a distance, r, with respect to a fixed reference frame, the linear velocity is:

$$\boldsymbol{u} = \boldsymbol{\omega} \times \boldsymbol{r}.\tag{4.29}$$

Substituting (4.29) into (4.28) the circuit equation for the rotational motion is obtained:

$$v_{\rm s} = i_{\rm s} R_{\rm s} + L_{\rm s} \frac{di_{\rm s}}{dt} + \int_3^1 [(\boldsymbol{\omega} \times \boldsymbol{r}) \times \boldsymbol{B}_{\rm ext}] d\boldsymbol{l}.$$
(4.30)

4.1.2Electromechanical energy conversion

The mechanism of electromechanical energy conversion can also be clearly represented considering a singly excited actuator (Fig. 4.1) and applying the virtual work principle. Electrical energy, supplied to the actuator, is transformed into mechanical energy and losses [39] (Fig. 4.3). The magnetic field acts as a transfer medium in the conversion process.

Losses occur in all components of the electromechanical system. Ohmic losses occur in the conductors. In the mechanical system losses are produced due to friction and windage. In the ferromagnetic parts, hysteresis and eddy currents cause losses.



Figure 4.3: Energy transfer in electromechanical systems operation in a motor mode.

The energy conservation law can be expressed as:

$$W_{\rm e} = W_{\rm f} + W_{\rm m} + W_{\rm l}^{\rm e} + W_{\rm l}^{\rm f} + W_{\rm l}^{\rm m}, \qquad (4.31)$$

where

 $W_{\rm e}$ - electrical energy input, [J],

 $W_{\rm f}$ - stored magnetic field energy, [J],

 $W_{\rm m}$ - mechanical energy, [J],

- W_1^e electrical energy loss, [J], W_1^f magnetic energy loss, [J],

 $W_1^{\rm m}$ - mechanical energy loss, [J].

The losses yield irreversible heat dissipation and are not contributing to the actual process of the conversion of electrical energy into mechanical. For the conservative or lossless energy conversion (4.31) is simplified as:

$$W_{\rm e} = W_{\rm f} + W_{\rm m}.$$
 (4.32)

For an infinitely short time interval, dt, the relation between the three forms of energy (4.32) can be expressed as:

$$dW_{\rm e} = dW_{\rm f} + dW_{\rm m},\tag{4.33}$$

where

 $dW_{\rm e}$ - differential change in electrical energy input, [J], $dW_{\rm f}$ - differential change in stored magnetic field energy, [J], $dW_{\rm m}$ - differential change in mechanical energy, [J].

During this time interval, the rotor is moved over an angle, $d\theta$, under the action of the electromagnetic torque, $T_{\rm d}$. The mechanical work, $dW_{\rm m}$, done by the torque acting on the rotor during this time interval is:

$$dW_{\rm m} = T_{\rm d}d\theta. \tag{4.34}$$

The amount of electrical energy, $dW_{\rm e}$, transferred into magnetic field energy, $dW_{\rm f}$, and converted into mechanical work, $dW_{\rm m}$, during this time interval can be calculated using (4.15) by subtracting the energy loss dissipated in the winding resistance from the total electrical energy fed into the excitation winding:

$$dW_{\rm e} = ei_{\rm s}dt = v_{\rm s}i_{\rm s}dt - R_{\rm s}i_{\rm s}^2dt.$$

$$\tag{4.35}$$

Combining (4.15), (4.33) and (4.34) to (4.35) yields:

$$i_{\rm s}d\lambda_{\rm s} = dW_{\rm f} + T_{\rm d}d\theta. \tag{4.36}$$

According to Fig. 4.1, the magnetic field energy is expressed as a function of the flux linkage, λ_s , and angular position, θ , of the rotor. Thus, the differential of the magnetic field energy is:

$$dW_{\rm f} = \frac{\partial W_{\rm f}(\lambda_{\rm s}, \ \theta)}{\partial \lambda_{\rm s}} d\lambda_{\rm s} + \frac{\partial W_{\rm f}(\lambda_{\rm s}, \ \theta)}{\partial \theta} d\theta.$$
(4.37)

Treating λ_s and θ as independent variables an expression for the electromagnetic torque can be derived from (4.36) and (4.37):

$$T_{\rm d} = -\frac{\partial W_{\rm f}(\lambda_{\rm s}, \ \theta)}{\partial \theta}.$$
(4.38)

In a per-unit volume the energy, $w_{\rm f}$, stored in the magnetic field is expressed as:

$$w_{\rm f} = \int_0^{\rm B_{\rm max}} \boldsymbol{H} d\boldsymbol{B},\tag{4.39}$$

where

B - magnetic flux density, [T], H - magnetic field strength, [A/m].

Magnetic flux density and field strength are related as:

$$\boldsymbol{B} = \boldsymbol{\mu} \boldsymbol{H},\tag{4.40}$$

where

 μ - permeability, [H/m].



Figure 4.4: Graphical representation of the magnetic field energy, $W_{\rm f}$, and co-energy, $W_{\rm c}$, (nonlinear case).



In electrical machines the values of current and flux linkage are used rather than magnetic field strength and magnetic flux density. The relation between the current and the flux linkage is usually expressed by the magnetization curve (Figs. 4.4 and 4.5). Therefore, the energy stored in the magnetic field is written as (upper part of the shaded area):

$$W_{\rm f} = \int_0^{\lambda_{\rm max}} i_{\rm s} d\lambda_{\rm s}, \quad \text{with} \quad i_{\rm s} = f(\lambda_{\rm s}, \ \theta). \tag{4.41}$$

With the aim to express the electromagnetic torque as a function of current and position, the so-called co-energy, W_c , is used, which corresponds to the area below the magnetization curve (Fig. 4.4 and 4.5) and is expressed as:

$$W_{\rm c} = \int_0^{i_{\rm max}} \lambda_{\rm s} di_{\rm s}, \quad \text{with} \quad \lambda_{\rm s} = f(i_{\rm s}, \ \theta). \tag{4.42}$$

Treating i_s and θ as independent variables another expression for the electromagnetic torque can be derived as a function of the co-energy:

$$T_{\rm d} = \frac{\partial W_{\rm c}(i_{\rm s},\theta)}{\partial \theta}.$$
(4.43)

For a magnetically linear system (Fig. 4.5), the inductance, $L_s(\theta)$ - the ratio between the electrical current and flux linkage, does not depend on the current level and is expressed as:

$$L_{\rm s}(\theta) = \frac{\lambda_{\rm s}(i_{\rm s},\theta)}{i_{\rm s}}.\tag{4.44}$$

In this case the general expression for the magnetic co-energy (4.42) becomes:

$$W_{\rm c} = \frac{1}{2} \lambda_{\rm s}(i_{\rm s}, \theta) i_{\rm s}. \tag{4.45}$$

Finally, the electromagnetic torque for the singly exited actuator can be expressed as:

$$T_{\rm d} = \frac{1}{2} \frac{\partial \lambda_{\rm s}(i_{\rm s}, \theta)}{\partial \theta} i_{\rm s}, \qquad (4.46)$$

or

$$T_{\rm d} = \frac{1}{2} \frac{\partial L_{\rm s}(\theta)}{\partial \theta} i_{\rm s}^2 = \frac{1}{2} \frac{dL_{\rm s}(\theta)}{d\theta} i_{\rm s}^2.$$
(4.47)

The general principle for torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in Fig. 4.6. The differential electrical energy can be derived as follows:

$$dW_{\rm e} = e_{\rm s} i_{\rm s} dt + e_{\rm r} i_{\rm r} dt, \qquad (4.48)$$

where values $(\cdot)_r$ correspond to the rotor winding.

The differential magnetic energy and co-energy functions are:

$$dW_{\rm f}\left(\lambda_{\rm s},\lambda_{\rm r},\theta\right) = \frac{\partial W_{\rm f}\left(\lambda_{\rm s},\lambda_{\rm r},\theta\right)}{\partial\lambda_{\rm s}}d\lambda_{\rm s} + \frac{\partial W_{\rm f}\left(\lambda_{\rm s},\lambda_{\rm r},\theta\right)}{\partial\lambda_{\rm r}}d\lambda_{\rm r} + \frac{\partial W_{\rm f}\left(\lambda_{\rm s},\lambda_{\rm r},\theta\right)}{\partial\theta}d\theta \quad (4.49)$$



Figure 4.6: A doubly excited rotating actuator.

and

$$dW_{\rm c}\left(i_{\rm s}, i_{\rm r}, \theta\right) = \frac{\partial W_{\rm c}\left(i_{\rm s}, i_{\rm r}, \theta\right)}{\partial i_{\rm s}} di_{\rm s} + \frac{\partial W_{\rm c}\left(i_{\rm s}, i_{\rm r}, \theta\right)}{\partial i_{\rm r}} di_{\rm r} + \frac{\partial W_{\rm c}\left(i_{\rm s}, i_{\rm r}, \theta\right)}{\partial \theta} d\theta.$$
(4.50)

Following the procedures described above, the electromagnetic torque is:

$$T_{\rm d} = -\frac{\partial W_{\rm f}\left(\lambda_{\rm s}, \lambda_{\rm r}, \theta\right)}{\partial \theta},\tag{4.51}$$

or

$$T_{\rm d} = \frac{\partial W_{\rm c}\left(i_{\rm s}, i_{\rm r}, \theta\right)}{\partial \theta}.$$
(4.52)

For magnetically linear systems, the currents and flux linkages are related by only position dependent inductances:

$$\begin{bmatrix} \lambda_{\rm s} \\ \lambda_{\rm r} \end{bmatrix} = \begin{bmatrix} L_{\rm ss} & L_{\rm m} \\ L_{\rm m} & L_{\rm rr} \end{bmatrix} \begin{bmatrix} i_{\rm s} \\ i_{\rm r} \end{bmatrix}, \qquad (4.53)$$

where

 $L_{\rm ss}$ - stator self-inductance, [H], $L_{\rm rr}$ - rotor self-inductance, [H], $L_{\rm m}$ - mutual inductance, [H].

The magnetic co-energy then is:

$$W_{\rm c}(i_{\rm s}, i_{\rm r}, \theta) = \frac{1}{2} L_{\rm ss} i_{\rm s}^2 + \frac{1}{2} L_{\rm rr} i_{\rm r}^2 + L_{\rm m} i_{\rm s} i_{\rm r}.$$
(4.54)

From (4.54), the expression for the electromagnetic torque for the configuration of a doubly exited rotating electrical machine can be derived:

$$T_{\rm d} = \frac{1}{2}i_{\rm s}^2\frac{dL_{\rm ss}}{d\theta} + \frac{1}{2}i_{\rm r}^2\frac{dL_{\rm rr}}{d\theta} + i_{\rm s}i_{\rm r}\frac{dL_{\rm m}}{d\theta}.$$
(4.55)

From (4.55) two torque types can be distinguished. The first two terms of (4.55) correspond to the so-called reluctance torque. It is caused by the tendency of the rotor to align itself with the stator such that minimum reluctance is produced. Generally, a reluctance torque can arise in any kind of electrical machine. However, a useful reluctance torque is produced only if the stator or rotor yoke or both have a salient shape. This is typical for several types of permanent-magnet synchronous machines and especially for reluctance machines. Otherwise, this torque component is regarded as a disturbance.

The so-called electrodynamic torque - the third term of (4.55) is caused by the tendency of the excited rotor to align itself with the excited stator. The mutual inductance depends on the rotor position regardless of the saliency of both stator and rotor. Cylindrical synchronous and induction machines are typical examples.

4.2 Permanent magnet AC synchronous motor drives

The permanent magnet AC synchronous motor mainly distinguished by their rotor configuration: surface permanent-magnet (SPMSM) and interior permanent-magnet (IPMSM) synchronous motors (Figs. 4.7 - 4.8).



Figure 4.7: Basic topology of the SPMSM.

Figure 4.8: Basic topology of the IPMSM.

SPMSMs have arc-shaped magnets mounted on the surface of a cylindrical rotor. In relatively high-speed applications, they are normally covered by a thin nonmagnetic steel to prevent them from detaching due to centrifugal forces. The rotor of a SPMSM does not have a magnetic saliency, thus only the electrodynamic torque is produced. A similar structure of the rotor is used in the blushless DC motors. The BLDC motors are designed to have a trapezoidal back-EMF waveform and a rectangular current waveforms. In PMAC synchronous motors a conventional threephase stator is normally used. They are designed to have a sinusoidal back-EMF waveform and the current waveform is also controlled to be sinusoidal, which allows a smooth torque control.

In IPMSMs the permanent magnets are embedded into the rotor core. These synchronous motors are designed to produce both a reluctance and an electrodynamic torque. Interior PM synchronous machines offer some significant advantages, compared to conventional surface PM synchronous machines because of the hybrid nature of their torque production (magnet and reluctance) and a stiff rotor construction. These advantages include high efficiency and power density, as well as their ability to achieve a wide speed range of constant power operation, making IPMSMs in one of the best candidates to drive a FC compression system. The rest of the section presents a mathematical description together with control principles of the IPMSMs

4.2.1 Modeling of interior permanent-magnet synchronous motors

One of the most fundamental principles of AC machines is the creation of a rotating, sinusoidally distributed magnetic field in the airgap. Neglecting the effect of slotting and space harmonics due to the non-ideal winding distribution, it can be shown that a sinusoidal three-phase balanced power supply in the three-phase stator winding creates a rotating magnetic field.

The three-phase sinusoidal currents flowing in the three-phase stator windings, and are given as:

$$i_{\rm as} = I_{\rm s} \cos\left(\omega_{\rm e} t\right),$$

$$i_{\rm bs} = I_{\rm s} \cos\left(\omega_{\rm e} t - \frac{2\pi}{3}\right),$$

$$i_{\rm cs} = I_{\rm s} \cos\left(\omega_{\rm e} t + \frac{2\pi}{3}\right),$$

(4.56)

where

 $\omega_{\rm e}$ - electrical angular velocity, [rad/s].

Each phase winding independently produces a sinusoidally distributed magnetomotive force (MMF) wave (F), pulsating along the respective axis. At spatial angle, θ , the instantaneous MMF waves can be expressed as:

$$F_{\rm as}(\theta, t) = w_{\rm s}i_{\rm as}\cos(\theta),$$

$$F_{\rm bs}(\theta, t) = w_{\rm s}i_{\rm bs}\cos\left(\theta - \frac{2\pi}{3}\right),$$

$$F_{\rm cs}(\theta, t) = w_{\rm s}i_{\rm cs}\cos\left(\theta + \frac{2\pi}{3}\right),$$

(4.57)

where

 $w_{\rm s}$ - number of turns in a phase winding.

The resultant MMF wave at angle θ is:

$$F_{\rm s}(\theta, t) = F_{\rm as}(\theta, t) + F_{\rm bs}(\theta, t) + F_{\rm cs}(\theta, t) =$$
$$= w_{\rm s}i_{\rm as}\cos(\theta) + w_{\rm s}i_{\rm bs}\cos(\theta - \frac{2\pi}{3}) + w_{\rm s}i_{\rm cs}\cos(\theta + \frac{2\pi}{3}).$$
(4.58)

Substituting (4.56) in (4.58) gives:

$$F_{\rm s}(\theta,t) = w_{\rm s}I_{\rm s}\left[\cos(\omega_{\rm e}t)\cos(\theta) + \cos\left(\omega_{\rm e}t - \frac{2\pi}{3}\right)\cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\omega_{\rm e}t + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{2\pi}{3}\right)\right]. \tag{4.59}$$

After simplification, (4.59) can be written as:

$$F_{\rm s}(\theta, t) = \frac{3}{2} w_{\rm s} I_{\rm s} \cos(\omega_{\rm e} t - \theta), \qquad (4.60)$$

which is a MMF wave rotating at the speed, $\omega_{\rm e}$.

In order to avoid the complicated trigonometric transformations, the directquadrature (dq)-coordinate system synchronously rotating with an electrical angular velocity, $\omega_{\rm e}$, is normally used for modeling synchronous machines, where the d - axis coincides with the direction of the permanent-magnet flux linkage $\lambda_{\rm PM}$ [40, 41]. The flux linkages and the voltage equations for the stator of an IPMSM are expressed in the dq-coordinate system:

$$\begin{bmatrix} \lambda_{\rm d} \\ \lambda_{\rm q} \end{bmatrix} = \begin{bmatrix} L_{\rm d} & 0 \\ 0 & L_{\rm q} \end{bmatrix} \begin{bmatrix} i_{\rm d} \\ i_{\rm q} \end{bmatrix} + \begin{bmatrix} \lambda_{\rm PM} \\ 0 \end{bmatrix}, \qquad (4.61)$$

$$\begin{bmatrix} v_{\rm d} \\ v_{\rm q} \end{bmatrix} = R_{\rm s} \begin{bmatrix} i_{\rm d} \\ i_{\rm q} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{\rm d} & 0 \\ 0 & \lambda_{\rm q} \end{bmatrix} + \omega_{\rm e} \begin{bmatrix} 0 & -\lambda_{\rm q} \\ \lambda_{\rm d} & 0 \end{bmatrix}, \qquad (4.62)$$

$$\begin{bmatrix} v_{\rm d} \\ v_{\rm q} \end{bmatrix} = R_{\rm s} \begin{bmatrix} i_{\rm d} \\ i_{\rm q} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_{\rm d} & 0 \\ 0 & L_{\rm q} \end{bmatrix} \begin{bmatrix} i_{\rm d} \\ i_{\rm q} \end{bmatrix} + \begin{bmatrix} 0 & -\omega_{\rm e}L_{\rm q} \\ \omega_{\rm e}L_{\rm d} & 0 \end{bmatrix} \begin{bmatrix} i_{\rm d} \\ i_{\rm q} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{\rm e}\lambda_{\rm PM} \end{bmatrix},$$
(4.63)

where

 $\begin{array}{l} v_{\rm d}, v_{\rm q} - dq\text{-components of stator voltage, [V],} \\ i_{\rm d}, i_{\rm q} - dq\text{-components of stator current, [A],} \\ R_{\rm s} - \text{stator phase resistance, [}\Omega\text{],} \\ L_{\rm d}, L_{\rm q} - dq\text{-components of stator phase inductance, [H],} \\ \lambda_{\rm PM} - \text{permanent-magnet flux linkage, [Wb],} \\ \lambda_{\rm d}, \lambda_{\rm q} - dq\text{-components of flux linkage, [Wb].} \end{array}$

The dq-components of the state variables - voltages, currents and flux linkages - are obtained via the transformation from the stationary three-phase coordinate system, (abc), into the synchronously rotating dq-system using the consequent power-invariant transformations [42]. The rotor equations are omitted, because of the absence of an electrical circuit in the rotor. From (4.63), the steady-state vector diagram is built as shown in Fig. 4.9.



Figure 4.9: Steady-state vector diagram for an IPMSM in the dq - reference frame.

The electromagnetic torque is described by:

$$T_{\rm d} = p \left(\lambda_{\rm PM} i_{\rm q} + \left(L_{\rm d} - L_{\rm q}\right) i_{\rm d} i_{\rm q}\right). \tag{4.64}$$

Another way to express the torque is via the amplitude of the stator current, I_s ,

and the current phase angle β - electrical angle between the current vector and the EMF, directly related to the rotor position:

$$T_{\rm d} = p\left(\lambda_{\rm PM}I_{\rm s}\cos(\beta) + \frac{1}{2}\left(L_{\rm d} - L_{\rm q}\right)I_{\rm s}^2\sin(2\beta)\right),\tag{4.65}$$

where

p - pole-pair number, [-], β - current phase angle, [rad].

The first term in (4.64) or (4.65) corresponds to the electrodynamic torque, and the second term - to the reluctance torque component.

The model of an IPMSM can be regarded as a general-type synchronous motor model. The equivalent circuits and the equations that describe the behavior of a SPMSM or a SynRM can be derived from those of an IPMSM if equal inductances in *d*- and *q*-axes are obtained $(L_d = L_q)$ or the rotor excitation is eliminated $(\lambda_{\rm PM})$, respectively. Therefore, based on the model and control schemes of the IPMSM, as described in this section, the corresponding control techniques for the SPMSM and the SynRM can be derived.

4.2.2 Current vector control for PM synchronous machines

With a PWM controlled power electronic converter the d and q current components or the magnitude and the phase angle of the stator current can be separately controlled. Such a current vector control has a strong influence on the motor performance. Several control schemes derived from the dq-model can be used.

Maximum torque-per-current control

The maximum torque-per-current (MTpC) control strategy allows to minimize the current level keeping a constant value of torque or to achieve the maximum torque under a constant current level. From the torque expression (4.65) the optimal current phase angle corresponding to the maximum torque-per-current is derived [43, 44, 45]:

$$\beta = \arcsin\left(\frac{-\lambda_{\rm PM} + \sqrt{\lambda_{\rm PM}^2 + 8\left(L_{\rm q} - L_{\rm d}\right)^2 I_{\rm s}^2}}{4\left(L_{\rm q} - L_{\rm d}\right)^2 I_{\rm s}}\right).$$
(4.66)

The relation between d- and q-axis current components is defined as:

$$i_{\rm d} = \frac{\lambda_{\rm PM}}{2\left(L_{\rm q} - L_{\rm d}\right)} - \sqrt{\frac{\lambda_{\rm PM}^2}{4\left(L_{\rm q} - L_{\rm d}\right)^2}} + i_{\rm q}^2.$$
(4.67)

The trajectory which corresponds to this control strategy is shown in Fig. 4.11.

Maximum torque-per-flux control

The optimal operating point can also be achieved by the maximum torque-per-flux linkage (MTpF). The optimal current vector in this case is given as [44]:

$$i_{\rm d} = \frac{\lambda_{\rm PM} + \Delta \lambda_{\rm d}}{L_{\rm d}},\tag{4.68}$$

$$i_{\rm q} = \frac{\sqrt{\lambda_{\rm s}^2 - \Delta \lambda_{\rm d}^2}}{L_{\rm q}},\tag{4.69}$$

$$\Delta \lambda_{\rm d} = \frac{-L_{\rm q} \lambda_{\rm PM} + \sqrt{(L_{\rm q} \lambda_{\rm PM})^2 + 8 (L_{\rm q} - L_{\rm d})^2 \lambda_{\rm s}^2}}{4 (L_{\rm q} - L_{\rm d})}.$$
 (4.70)

This control is equivalent to the maximum torque per EMF, thus it can also be named as the maximum torque-per-voltage control. The corresponding trajectory is given in Fig. 4.11.

Field-weakening control

The speed of a synchronous machine can be controlled beyond the rated speed by the so-called "field-weakening (FW) control" [46, 47, 48, 49]. This method is based on the suppression of the EMF in the high-speed region. The flux linkage, $\lambda_{\rm PM}$, due to the permanent magnets is in fact constant. However, the *d*-axis flux linkage can be adjusted by utilizing the *d*-axis armature reaction.

The flux linkage, λ_s , is:

$$\lambda_{\rm s} = \sqrt{\lambda_{\rm d}^2 + \lambda_{\rm q}^2} = \sqrt{\left(L_{\rm d}i_{\rm d} + \lambda_{\rm PM}\right)^2 + \left(L_{\rm q}i_{\rm q}\right)^2}.$$
(4.71)

From the condition by which the EMF becomes equal to its limited value, E_{0m} , the following equation is obtained:

$$(L_{\rm d}i_{\rm d} + \lambda_{\rm PM})^2 + (L_{\rm q}i_{\rm q})^2 = \left(\frac{E_{\rm 0m}}{\omega_{\rm e}}\right)^2,$$
 (4.72)

$$i_{\rm d} = -\frac{\lambda_{\rm PM}}{L_{\rm d}} \pm \frac{1}{L_{\rm d}} \sqrt{\left(\frac{E_{\rm 0m}}{\omega_{\rm e}}\right)^2 - \left(L_{\rm q}i_{\rm q}\right)^2}.$$
 (4.73)

By controlling the current vector along the voltage limit ellipse (see FW trajectory in Fig. 4.11), the torque can be controlled.

Loss minimization control

The optimal current vector for minimizing motor losses at any operating condition can be derived from the equivalent circuit shown in Fig. 4.10 [50, 51]. This circuit

considers the iron losses in addition to the copper losses. The optimal current vector trajectory which corresponds to the minimum losses at a particular speed is shown in Fig. 4.11 as "the loss minimization (LM) trajectory". The optimal current vector is a function of both variables - torque and speed. At zero speed there is no core loss (if harmonic losses due to PWM are ignored), thus the optimal LM trajectory at standstill corresponds to the MTpC trajectory where the copper losses are minimum. With increasing speed the LM trajectory moves to the left towards the MTpF trajectory.



Figure 4.10: Equivalent circuits of an IPMSM in the dq-reference frame including the iron loss resistance $R_{\rm m}$: (a) d-axis, (b) q-axis.

Field-oriented control

The field-oriented control is used to achieve a linear torque control of the motor. In this control method, the *d*-axis current component is set to zero resulting in the current space angle, β being zero as well. The torque expression (4.64) is simplified:

$$T_{\rm d} = p\left(\lambda_{\rm PM} i_{\rm q}\right). \tag{4.74}$$

Summary of IPMSM current vector control methods

The performance of the IPMSM strongly depends on the control scheme. Fig. 4.11 shows the current vector trajectories of five kinds of current vector control.

The current vector trajectories of the FW control and the LM control move depending on the operating speed.

The points A to E represent the operating points of MTpF, FW, LM, MTpC, and field-oriented control respectively for generating the same torque.

At point A, the flux linkage and the EMF voltage are minimum. At point B, the EMF voltage is kept at the limited voltage. At point C, the total loss becomes minimal and, thus, the highest drive efficiency is achieved in case of constant output



Figure 4.11: Current vector trajectories for different vector controls.

power. At point D, the current value becomes minimum. At point E, the d-axis current component is not used.

In general, using the dq-motor model, an appropriate current vector control scheme for a particular operating condition can be derived necessary to obtain a high-performance IPMSM drive.

4.3 Induction motor drives and motion control methods

4.3.1 Mathematical modeling of the induction motor

Induction motors (Fig. 4.12) are the most widely used type of electrical machines mainly because of their simple construction, low cost, high reliability and high efficiency [52, 53, 54, 42, 55]. Except for standard applications, they are also widely employed in high-speed applications like compressors, spindle machines, etc.

The three-phase stator windings are formed by interconnecting conductors in slots around the circumferential periphery of the machine, and produce a pure traveling MMF, like in the case of synchronous machines. As a consequence, a constant-amplitude traveling flux wave is present in the airgap. The most common configuration of the rotor consists of short-circuited bars – so-called "squirrel cage". For convenience of modeling, it is also represented as a three-phase winding rotat-



Figure 4.12: General view of an induction motor.

ing at the mechanical speed, ω . Thus, in the rotor reference frame, the sinusoidally distributed stator flux wave is traveling with the so-called "slip" speed, which is expressed as:

$$\omega_{\rm slip} = \omega_{\rm e} - \omega_{\rm r}, \tag{4.75}$$

where

 $\omega_{\rm r} = p\omega$ - electrical rotor speed, [rad/s].

According to Faraday's law, the voltages, induced in each rotor phase winding are described as:

$$v_{\rm ar} = V_{\rm r} \cos\left(\omega_{\rm slip} t\right),$$

$$v_{\rm br} = V_{\rm r} \cos\left(\omega_{\rm slip} t - \frac{2\pi}{3}\right),$$

$$v_{\rm cr} = V_{\rm r} \cos\left(\omega_{\rm slip} t + \frac{2\pi}{3}\right),$$

(4.76)

where

 $V_{\rm r}$ - amplitude of the induced rotor voltage, [V].

The rotor phase currents are:

$$i_{\rm ar} = I_{\rm r} \cos\left(\omega_{\rm slip} t - \phi\right),$$

$$i_{\rm br} = I_{\rm r} \cos\left(\omega_{\rm slip} t - \phi - \frac{2\pi}{3}\right),$$

$$i_{\rm cr} = I_{\rm r} \cos\left(\omega_{\rm slip} t - \phi + \frac{2\pi}{3}\right),$$

(4.77)

where

 ϕ - phase shift angle between voltage and current, [rad].



Figure 4.13: Induction motor in the rotating dq-reference frame.

Like the stator phase windings, the rotor phase windings being displaced by the angle $\frac{2\pi}{3}$ in space and conducting a balanced set of three-phase currents, will establish a rotor MMF wave in the airgap that is constant in amplitude and travels with respect to the rotor at the slip speed. Consequently, the airgap of the induction motor has two constant-amplitude MMF waves rotating with synchronous angular speed, $\omega_{\rm e}$. The action of these two fields to align produces the electromagnetic torque, $T_{\rm d}$.

Under the assumption that the stator and rotor phase windings are electrically and magnetically symmetrical, the airgap is uniform and the field distribution is sinusoidal, a three-phase induction motor can be modelled by an equivalent twophase motor model $\alpha\beta$ (Fig. 4.13). With the additional assumptions that the effects of magnetic saturation, eddy-currents and hysteresis are negligible, the model equations for an induction motor can be expressed in the synchronously rotating dq-frame:

$$\begin{aligned} v_{qs}^{e} &= R_{s}i_{qs}^{e} + \frac{d\lambda_{qs}^{e}}{dt} + \omega_{e}\lambda_{ds}^{e}, \\ v_{ds}^{e} &= R_{s}i_{ds}^{e} + \frac{d\lambda_{ds}^{e}}{dt} - \omega_{e}\lambda_{qs}^{e}, \\ 0 &= R_{r}'i_{qr}^{'e} + \frac{d\lambda_{qr}^{'e}}{dt} + (\omega_{e} - \omega_{r})\lambda_{dr}^{'e}, \\ 0 &= R_{r}'i_{dr}^{'e} + \frac{d\lambda_{dr}^{'e}}{dt} - (\omega_{e} - \omega_{r})\lambda_{qr}^{'e}, \\ \lambda_{qs}^{e} &= L_{ls}i_{qs}^{e} + L_{m}\left(i_{qs}^{e} + i_{qr}^{'e}\right), \\ \lambda_{ds}^{e} &= L_{ls}i_{ds}^{e} + L_{m}\left(i_{qs}^{e} + i_{dr}^{'e}\right), \\ \lambda_{qr}^{'e} &= L_{lr}'i_{qr}^{'e} + L_{m}\left(i_{qs}^{e} + i_{dr}^{'e}\right), \\ \lambda_{dr}^{'e} &= L_{lr}'i_{qr}^{'e} + L_{m}\left(i_{qs}^{e} + i_{dr}^{'e}\right), \\ \lambda_{dr}^{'e} &= L_{lr}'i_{dr}^{'e} + L_{m}\left(i_{ds}^{e} + i_{dr}^{'e}\right), \end{aligned}$$

$$(4.79)$$

where

 $\begin{array}{l} R_{\rm r}' \mbox{ - rotor resistance, } [\Omega], \\ L_{\rm ls} \mbox{ - stator leakage inductance, } [{\rm H}], \\ L_{\rm lr}' \mbox{ - rotor leakage inductance, } [{\rm H}], \\ L_{\rm m} \mbox{ - magnetizing or mutual inductance, } [{\rm H}], \\ \lambda_{\rm ds}^{\rm e}, \lambda_{\rm qs}^{\rm e} \mbox{ - } dq\mbox{-components of stator flux linkage, } [{\rm Wb}], \\ \lambda_{\rm dr}^{\rm e}, \lambda_{\rm qr}^{\rm e} \mbox{ - } dq\mbox{-components of rotor flux linkage, } [{\rm Wb}], \\ \nu_{\rm ds}^{\rm e}, \nu_{\rm qs}^{\rm e} \mbox{ - } dq\mbox{-components of stator voltage, } [{\rm V}], \\ i_{\rm ds}^{\rm e}, v_{\rm qs}^{\rm e} \mbox{ - } dq\mbox{-components of stator current, } [{\rm A}], \\ i_{\rm dr}^{\rm 'e}, i_{\rm qr}^{\rm 'e} \mbox{ - } dq\mbox{-components of rotor current, } [{\rm A}], \\ ()_{\rm r}' \mbox{ - rotor values referred to the stator.} \end{array}$

The equations (4.78)- (4.79) can be combined to give the model of the electrical dynamics of an induction motor in matrix form (for one pole-pair):

$$\begin{bmatrix} v_{\rm qs}^{\rm e} \\ v_{\rm ds}^{\rm e} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{\rm s} + sL_{\rm s} & \omega_{\rm e}L_{\rm s} & sL_{\rm m} & \omega_{\rm e}L_{\rm m} \\ -\omega_{\rm e}L_{\rm s} & R_{\rm s} + sL_{\rm s} & -\omega_{\rm e}L_{\rm m} & sL_{\rm m} \\ sL_{\rm m} & (\omega_{\rm e} - \omega_{\rm r})L_{\rm m} & R'_{\rm r} + sL'_{\rm r} & (\omega_{\rm e} - \omega_{\rm r})L'_{\rm r} \\ -(\omega_{\rm e} - \omega_{\rm r})L_{\rm m} & sL_{\rm m} & -(\omega_{\rm e} - \omega_{\rm r})L'_{\rm r} & R'_{\rm r} + sL'_{\rm r} \end{bmatrix} \begin{bmatrix} i_{\rm qs}^{\rm e} \\ i_{\rm ds}^{\rm e} \\ i_{\rm dr}^{\rm e} \\ i_{\rm dr}^{\rm e} \end{bmatrix},$$

$$(4.80)$$

where

s - Laplace operator (d/dt), $L_{\rm s} = L_{\rm ls} + L_{\rm m}$ - stator self-inductance, [H], $L'_{\rm r} = L'_{\rm lr} + L_{\rm m}$ - rotor self-inductance, [H].

In the assumption that the induction motor is supplied from an ideal current source, the stator currents, $i_{\rm ds}^{\rm e}$, and $i_{\rm qs}^{\rm e}$ are impressed to the stator. The stator equations can then be omitted.

The dq-components of the rotor current can be expressed in terms of the rotor flux linkage and the stator current components. From (4.79) it can be written:

$$i_{\rm dr}^{'\rm e} = \frac{\lambda_{\rm dr}^{'\rm e} - L_{\rm m} i_{\rm ds}^{\rm e}}{L_{\rm r}^{\prime}},$$

$$i_{\rm qr}^{'\rm e} = \frac{\lambda_{\rm qr}^{'\rm e} - L_{\rm m} i_{\rm qs}^{\rm e}}{L_{\rm r}^{\prime}}.$$
(4.81)

After substitution of (4.81) into (4.78) the rotor equations becomes:

$$\lambda_{\rm qr}^{'\rm e} = \frac{1}{s} \begin{bmatrix} \frac{L_{\rm m}}{\tau_{\rm r}} i_{\rm qs}^{\rm e} - \frac{\lambda_{\rm qr}^{'\rm e}}{\tau_{\rm r}} - \omega_{\rm slip} \lambda_{\rm dr}^{'\rm e} \\ \lambda_{\rm dr}^{'\rm e} = \frac{1}{s} \begin{bmatrix} \frac{L_{\rm m}}{\tau_{\rm r}} i_{\rm ds}^{\rm e} - \frac{\lambda_{\rm dr}^{'\rm e}}{\tau_{\rm r}} + \omega_{\rm slip} \lambda_{\rm qr}^{'\rm e} \end{bmatrix},$$

$$(4.82)$$

where

 $\tau_{\rm r} = \frac{L_{\rm r}'}{R_{\rm r}'}$ - the rotor time constant, [s].

The electromagnetic torque can be expressed as:

$$T_{\rm d} = p \frac{3}{2} \frac{L_{\rm m}}{L'_{\rm r}} \left(i_{\rm qs}^{\rm e} \lambda'_{\rm dr}^{\rm e} - i_{\rm ds}^{\rm e} \lambda'_{\rm qr}^{\rm e} \right).$$
(4.83)

From (4.81 - 4.83) it is possible to construct a block diagram of an induction motor in the synchronous reference frame.

From the block diagram in Fig. 4.14 the problems of the control of an induction motor can be seen.

The two direct couplings of the stator current with the rotor flux make this setup to be non-linear, thus, very poorly controllable. If the high performance is not required, the so-called scalar speed control can be implemented. In case of a very precise speed or position adjustment, the direct torque control (DTC) [56] or direct self control (DSC) [57] can be used, as well as the field oriented control (FOC), or vector control [42, 58]. Taking into account a personal experience and the group expertise, the FOC approach is analyzed and implemented.

With the aid of vector control a decoupling can be achieved. This decoupling consists of controlling the selected variable (e.g. flux linkage along a particular axis)



Figure 4.14: Block diagram of an induction motor in the synchronous reference frame.

in such a way that they are always equal to zero. This results in a simplification of the mathematical model. The following sections present the most commonly used motion control algorithms for induction motors (IM).

4.3.2 Scalar control

The simplest motion control type of the IM is the scalar control. It consists of the simultaneous control of the amplitude and the frequency of the supplied voltage so that the stator flux linkage, $(\lambda_{\rm s} = V_{\rm s}/\omega_{\rm e})$, remains constant. The block diagram of this control method is shown in Fig. 4.15.



Figure 4.15: Open-loop V/Hz speed control with PWM voltage source inverter.

In general, no feedback signals are required for the control. The synchronous frequency, $\omega_{\rm e}$, is the primary control variable. In addition, in the scalar control method the motor model is only considered in steady state. Therefore, the scalar controller can not achieve the adequate dynamic performance. This can be considered as the basic drawback of the scalar control method.

4.3.3 Field oriented control

The field oriented control (FOC) strategy is used for achieving fast dynamic speed and torque responses [42, 58]. From the control point of view, the use of the FOC approach allows to treat an AC machine as a DC machine, in which the electromagnetic torque is produced by a flux and a current that are decoupled.

Therefore, the control objective is to decouple the stator current vector into two components: a flux producing component, i_{ds}^{e} , and a torque producing component, i_{gs}^{e} . Fig. 4.16 shows a schematic diagram of the applied field orientation scheme.



Figure 4.16: General setup of a field oriented scheme.

The diagram in Fig. 4.16 is a general scheme that can be aligned with the rotor, the airgap or the stator flux linkage. The implementation is achieved by employing an explicit angular relation, $\theta_{\rm e}$, between the stator current vector and the flux vector by either direct or indirect methods. With the direct field oriented control the position of the flux vector is measured directly by sensors (Hall probes or flux coils) or estimated by a flux observer. In the indirect methods the rotor speed or position is measured using, for example, an encoder, and these measurements are used for the estimation of the magnitude and the angle of the flux linkage. In order to apply the FOC, a suitable model of the IM is needed. Here, the mathematical description in the form of a space vector description is used.

Rotor flux oriented model

The requirement for the rotor flux orientation is that the stator current components, $i_{\rm ds}^{\rm e}$, and $i_{\rm qs}^{\rm e}$, have to be in phase and quadrature with respect to the rotor flux. This can be realized by selecting a synchronously rotating dq-frame at speed, $\omega_{\rm e}$, whose d-axis is aligned with the rotor flux. This implies that the q-component of the rotor flux linkage, $\lambda_{\rm qr}^{\rm 'e}$, is equal to zero:

$$\lambda_{\rm qr}^{\rm 'e} = 0. \tag{4.84}$$

The machine is said to be rotor flux oriented.

With this method the independent control of flux and torque is possible. By keeping the *d*-component of the rotor flux linkage, $\lambda_{dr}^{'e}$, constant it is possible to achieve a fast torque response, resulting in a fast speed or position response.

From (4.83) the developed electromagnetic torque can be written as:

$$T_{\rm d} = p \frac{3}{2} \frac{L_{\rm m}}{L_{\rm r}'} \lambda_{\rm dr}^{\rm 'e} i_{\rm qs}^{\rm e}.$$
(4.85)

From this expression it can be seen that if the rotor flux linkage is not disturbed, adjusting the q-component of the stator current can independently control the electromagnetic torque. The reduced block diagram is given in Fig. 4.17.



Figure 4.17: Rotor flux oriented IM.

Direct rotor flux oriented controller

In the direct rotor flux oriented control scheme the rotor flux angle is directly obtained from the machine's electrical quantities, eliminating the need for a speed or position sensor.

The scheme, which uses the direct measurement of the airgap flux with flux sensing coils or Hall elements, is illustrated in Fig. 4.18.

The rotor flux equation in the stationary reference frame is:

$$\lambda_{\rm qdr}^{\prime \rm s} = L_{\rm m} i_{\rm qds}^{\rm s} + L_{\rm r}^{\prime} i_{\rm qdr}^{\prime \rm s}, \qquad (4.86)$$

while, the measured airgap flux is:



Figure 4.18: Direct rotor flux oriented control (airgap flux measuring).

$$\lambda_{\rm qdm}^{\rm s} = L_{\rm m} \left(i_{\rm qds}^{\rm s} + i_{\rm qdr}^{'\rm s} \right). \tag{4.87}$$

Combination of (4.86) and (4.87) yields:

$$\lambda_{\rm qdr}^{\prime \rm s} = \frac{L_{\rm r}^{\prime}}{L_{\rm m}} \lambda_{\rm qdm}^{\rm s} - L_{\rm lr}^{\prime} i_{\rm qds}^{\rm s}, \qquad (4.88)$$

from which the values of the rotor flux magnitude and angle can be obtained. Only two motor parameters have to be known: the rotor leakage inductance, $L'_{\rm lr}$, which is usually a constant and the ratio $\frac{L'_{\rm r}}{L_{\rm m}}$ which is mainly affected only by saturation of the main flux path of the machine. Consequently, the main disadvantage of this method is the need for flux sensing elements, which may affect the cost and reliability of the drive.

The stator flux linkage can also be estimated using an observer based on the measurements of the stator terminal voltage and current (Fig. 4.19), thus avoiding the use of special sensors. In the stator reference frame the voltage equation is:

$$v_{\rm qds}^{\rm s} = R_{\rm s} i_{\rm qds}^{\rm s} + s \lambda_{\rm qds}^{\rm s}, \tag{4.89}$$

from which:

$$\lambda_{\rm qds}^{\rm s} = \frac{1}{s} \left(v_{\rm qds}^{\rm s} - r_{\rm s} i_{\rm qds}^{\rm s} \right). \tag{4.90}$$

The stator flux linkage is expressed as:

$$\lambda_{\rm qds}^{\rm s} = L_{\rm s} i_{\rm qds}^{\rm s} + L_{\rm m} i_{\rm qdr}^{\rm \prime s}, \tag{4.91}$$



Figure 4.19: Direct rotor flux oriented control (stator voltage and current measuring).

from which the rotor current is expressed:

$$i_{\rm qdr}^{\prime \rm s} = \frac{\lambda_{\rm qds}^{\rm s} - L_{\rm s} i_{\rm qds}^{\rm s}}{L_{\rm m}}.$$
(4.92)

The rotor flux linkage can then be written as:

$$\lambda_{\rm qdr}^{\rm 's} = L_{\rm m} i_{\rm qds}^{\rm s} + \frac{L_{\rm r}^{\prime}}{L_{\rm m}} \left(\lambda_{\rm qds}^{\rm s} - L_{\rm s} i_{\rm qds}^{\rm s} \right) = \frac{L_{\rm r}^{\prime}}{L_{\rm m}} \left(\lambda_{\rm qds}^{\rm s} - L_{\sigma \rm s} i_{\rm qds}^{\rm s} \right), \tag{4.93}$$

where

 $L_{\sigma s} = L_s - \frac{L_m^2}{L_r'}$ - so-called transient inductance. In this case, the values for three motor parameters, the stator phase resistance, R_s , the stator phase inductance, L_s , and the ratio, $\frac{L_r'}{L_m}$, are necessary. The stator phase resistance might cause significant difficulties to the control accuracy at low speed due to its temperature dependence and skin effects, when the voltage drop, $i_{\rm s}R_{\rm s}$, becomes dominant. Extra measures to overcome this problem at very low speed are given in [58].

Indirect rotor flux oriented controller

The indirect rotor flux oriented controller has to maintain the correct orientation during flux changes, thus the slip has to be calculated continuously. To derive the controller, the relationship between the rotor flux linkage and the stator current as well as the slip equation has to be used.

Using (4.84), the rotor equations (4.82) can be rewritten as:

$$\lambda_{\rm qr}^{'\rm e} = 0 = \frac{1}{s} \left[\frac{L_{\rm m}}{\tau_{\rm r}} i_{\rm qs}^{\rm e} - \omega_{\rm slip} \lambda_{\rm dr}^{'\rm e} \right],$$

$$\lambda_{\rm dr}^{'\rm e} = \frac{1}{s} \left[\frac{L_{\rm m}}{\tau_{\rm r}} i_{\rm ds}^{\rm e} - \frac{\lambda_{\rm dr}^{'\rm e}}{\tau_{\rm r}} \right].$$
(4.94)

From the first equation of (4.94) the angular slip speed can be expressed:

$$\omega_{\rm slip} = \frac{L_{\rm m} i_{\rm qs}^{\rm e}}{\lambda_{\rm dr}^{\rm 'e} \tau_{\rm r}}.$$
(4.95)

The relation between the rotor flux linkage, λ'_{dr}^{e} , and the stator current component, i_{ds}^{e} , is given by the second equation of (4.94):

$$\lambda_{\rm dr}^{\rm 'e} = \frac{L_{\rm m}}{s\tau_{\rm r} + 1} i_{\rm ds}^{\rm e},\tag{4.96}$$

from which a reference value of, i_{ds}^{e} , is computed:

$$i_{\rm ds}^{\rm e*} = \lambda_{\rm dr}^{'\rm e*} \frac{s\tau_{\rm r} + 1}{L_{\rm m}}.$$
 (4.97)

In order to maintain the correct field orientation the time lag in the flux response has also to be taken into account. This is obtained by substituting (4.96) in (4.95) and is:

$$\omega_{\rm slip} = \frac{\frac{1}{\tau_{\rm r}} i_{\rm qs}^{\rm e*}}{\frac{1}{1+s\tau_{\rm r}} i_{\rm ds}^{\rm e*}}.$$
(4.98)

The slip equation (4.98) and the flux-current relation (4.97) are used in the controller (Fig. 4.20). By adding the angular slip speed, $\omega_{\rm slip}^*$, to the rotor speed, $\omega_{\rm r}$, the reference angular velocity, $\omega_{\rm e}$, is obtained (Fig. 4.21). Integrating $\omega_{\rm e}$ gives information on the angle, $\theta_{\rm e}$, of the rotor flux. This can be used for the dynamic transformation of the stator current components in the stator reference frame to the stator current components in the synchronous reference frame.

The feed-forward mechanism of the indirect rotor flux oriented controller assumes no feedback loop for the rotor flux. In this case, the drive performance becomes dependent on the knowledge of the motor parameters. It is visible from the model in Fig. 4.20 that the rotor time constant, τ_r , has an important impact on the proper field orientation. In practice, this constant varies during operation because of the changing of the rotor phase resistance due to the temperature and the frequency dependent skin effect, and the variable magnetic saturation. This fact can have a negative influence on the drive performance. On the other hand, there is just one external sensor needed to measure the speed of the rotor.



Figure 4.20: IM current model.



Figure 4.21: Indirect rotor flux oriented control (speed measuring).

4.3.4 Current control

PWM voltage source inverters (VSI) with a switching frequency in the range of tens of kHz today are the most commonly used power electronic converters in the low to medium power range of high-performance drives. The current regulated PWM inverter uses closed-loop control with feedback of the AC currents and can function as a regulated current supply having good dynamic response and low harmonic content (Fig. 4.22).

The current controllers are classified into three groups: hysteresis controller, ramp comparison and predictive controllers. The hysteresis controller is the simplest one. It applies the current error signal to a hysteresis element, and its output supplies the logic signal to the gate of a positive or negative inverter switching element. This system provides good current amplitude control. Its main disadvantage is the highly variable PWM switching rate. Low frequency components appear in the current spectrum regardless of the switching frequency. The variation of the switching rate is opposite to the needs for a good current control with the highest switching rates associated with the lowest reference frequencies.

The more advanced type of current controller is the ramp comparison one, which is the most widely used nowadays. Its operation principle is based on the comparison of the processed current error with a triangle waveform, which then defines the polarity of the inverter leg. The current error signal is usually conditioned using a PI controller, which provides a very satisfactory static and dynamic performance. The switching frequency is constant or, if necessary, can be variable, but well controllable. The predictive current controllers are beyond the framework of this study.

The ramp comparison controller can be realized in a stationary or rotating reference frame as shown in Fig. 4.23 and Fig. 4.24, respectively.

The current controller in a rotating reference frame requires more complex hardware for implementation because of the requirements to transform the measured currents into a rotating frame and also to transform the calculated reference voltage components back into the stationary frame. On the other hand, the "synchronous" controller provides a better performance due to the fact that the steady-state currents are represented as DC signals resulting in a zero steady-state error of the PI controller. With the aim to simplify the structure of the controller, the synchronous controller can be transformed into the stationary frame [42]. The corresponding controller is shown in Fig. 4.25. The performance of the controller is identical to



Figure 4.22: PWM-VSI system with current regulation.

that of the synchronous frame regulator shown in Fig. 4.24, but requires simpler hardware for implementation.



Figure 4.23: System diagram of a PI current controller in a stationary reference frame.



Figure 4.24: System diagram of a PI current controller in a rotating reference frame.



Figure 4.25: Synchronous PI current controller in a stationary reference frame.

4.4 Overview of control strategies for the induction motor efficiency

The control methods for induction motors (both scalar and vector) normally assume the motor operation at the rated flux level which results in a good dynamic performance i.e. fast torque response. However, in many industrial applications, induction motors often operate at a reduced load level. In this case the remained rated flux level may lead to excessive core losses. A number of control approaches has been developed aiming at the improvement of the IM efficiency at reduced load, usually indicated as: "energy optimal control", "loss minimization control", "part load optimization" or "efficiency optimized control". This section briefly summarizes the control principles related to the efficiency improvement of induction motors.

4.4.1 Induction motor losses

Besides mechanical and windage losses, losses in induction machines also occur in the stator and rotor windings and in the stator and rotor magnetic cores. They determine the efficiency of the energy conversion in the electrical machine [59].

A classification of electromagnetic losses by origin includes fundamental losses and losses due to space harmonics and time harmonics. Time harmonics are to be considered when the induction motor is fed by a static converter, and thus the voltage time harmonics content depends on the type of the converter and the pulse width modulation method used. Space harmonics result from a non-sinusoidal space distribution of the rotating flux due to the non-ideal winding distribution and the periodic variation of magnetic circuit reluctance, caused by rotor and stator slots. Losses due to harmonics are not investigated, and the main attention is given to the minimization of the fundamental core and winding losses.

4.4.2 Principle of efficiency control

In induction motor applications the energy saving criteria must be taken into account from the design of the machine, using advanced magnetic materials and process technologies up to the concept of control algorithms. The current investigation deals with energy saving by means of control approaches.

The electromagnetic losses are represented in the model shown in Fig. 4.26 by the stator winding, rotor winding and core resistances.



Figure 4.26: Single-phase equivalent circuit of an induction motor in the stationary reference frame.



Figure 4.27: Loss variation in an induction motor with varying flux.

4.4 Overview of control strategies for the induction motor efficiency 95

The motor winding and core losses and the converter losses depend on the motor flux level and, thus, can be influenced by the efficiency control [60, 61]. The main issue in the efficiency optimized control is the flux level. At constant torque, the reduction of the flux level results in the rise of the rotor current (and respectively rotor copper losses). The core losses are reduced significantly. Further reduction of the flux results in rising of the rotor and stator copper losses. Thus, at a certain load level there is an optimal flux level which corresponds to the minimum motor loss (Fig. 4.27).

Many efficiency control approaches dealing with flux optimization are reported in literature. They are mainly distinguished by the definition of the optimal flux level and the way of reaching the point of optimal flux (and optimal efficiency, respectively). They can be classified as follows:

- Simple state control,
 - displacement power factor control,
 - rotor slip frequency control,
- Model based control (scalar and vector drives),
- Search control (scalar and vector drives).

4.4.3 Simple state control

In the simple state control a single motor parameter is controlled. This is typically the displacement power factor, $cos(\phi)$, or rotor slip frequency, ω_{slip} . They can be controlled either according to a constant value or to a predefined scheme. The advantages of the $cos(\phi)$ control [62, 63, 64] (Fig. 4.28) are the simplicity of realization, the fast adaptation and the fact that no speed or load information is required. The disadvantage is that the $cos(\phi)$ reference can not be unified for different motors.



Figure 4.28: Simple state control $(cos(\phi))$.

In [65] the efficiency control strategy where the optimal rotor slip frequencies, ω_{slip}^* , are put in a look-up table (Fig. 4.29), is proposed. The values are based on off-line measurements on the motor drive, and for that given motor drive, the optimal slip frequency values depend on the speed only.



Figure 4.29: Simple state control (slip frequency).

4.4.4 Model based control

The flux optimization based on a machine loss model can be realized in different ways depending on the motor control applied (e.g. vector control or scalar control). For example, the simple control scheme for a scalar controlled drive is proposed in [66] and [67] (Fig. 4.30). The steady-state motor model is used to calculate an optimal stator frequency for further V/Hz control:

$$\omega_{\rm e} = p\omega + \sqrt{\frac{R_{\rm s}R_{\rm r}^{'2}}{R_{\rm s}\left(L_{\rm m} + L_{\rm r}^{'}\right)^2 + R_{\rm r}^{'}L_{\rm m}^2}}.$$
(4.99)



Figure 4.30: Model based control (scalar drive).

The efficiency optimization for a vector controlled drive can be best illustrated by the approach used in [68, 69, 70, 71]. The optimal operating point can be determined using the simplified equivalent circuit of an induction motor (Fig. 4.31). The motor losses are represented by the separated power dq-components, depending on the corresponding current components, using a concept of "negative" power:



Figure 4.31: Steady-state rotor flux oriented model of an induction motor including core losses.

$$P_{\rm loss,d} = \left(\frac{\left(\omega_{\rm e} L_{\rm m}\right)^2}{R_{\rm m}} + R_{\rm s} + \left(\omega_{\rm e} L_{\rm m}\right)^2 \frac{R_{\rm s}}{R_{\rm m}^2}\right) i_{\rm ds}^{\rm s2},\tag{4.100}$$

$$P_{\rm loss,q} = (R_{\rm s} + R_{\rm r}') \, i_{\rm qs}^{\rm e2}, \tag{4.101}$$

$$P_{\rm loss,dq} = -2\omega_{\rm e}L_{\rm m}\frac{R_{\rm s}}{R_{\rm m}}i^{\rm e}_{\rm ds}i^{\rm e}_{\rm qs}.$$
(4.102)

According to [68], the electromagnetic torque of the rotor flux oriented drive is:

$$T_{\rm d} = pL_{\rm m} i^{\rm e}_{\rm ds} i^{\rm e}_{\rm qs}. \tag{4.103}$$

The current components ratio is defined as:

$$A = \frac{i_{\rm qs}^{\rm e}}{i_{\rm ds}^{\rm e}}.\tag{4.104}$$

Using (4.103) the following relations are obtained:

$$i_{\rm qs}^{\rm e2} = A \frac{T_{\rm d}}{pL_{\rm m}}, \quad i_{\rm ds}^{\rm e2} = \frac{1}{A} \frac{T_{\rm d}}{pL_{\rm m}}, \quad i_{\rm ds}^{\rm e} i_{\rm qs}^{\rm e} = \frac{T_{\rm d}}{pL_{\rm m}}.$$
 (4.105)

The total losses are expressed by combination of (4.100) - (4.102) and (4.105):

$$P_{\text{loss}} = P_{\text{loss},\text{d}} + P_{\text{loss},\text{q}} + P_{\text{loss},\text{dq}} = \frac{T_{\text{d}}}{pL_{\text{m}}} \left[\left(\frac{(\omega_{\text{e}}L_{\text{m}})^2}{R_{\text{m}}} + R_{\text{s}} + (\omega_{\text{e}}L_{\text{m}})^2 \frac{R_{\text{s}}}{R_{\text{m}}^2} \right) \frac{1}{A} + (R_{\text{s}} + R_{\text{r}}') A - 2\omega_{\text{e}}L_{\text{m}} \frac{R_{\text{s}}}{R_{\text{m}}} \right].$$
(4.106)

Neglecting saturation effects which result in a dependence of the model parameters (in particular $R_{\rm m}$ and $L_{\rm m}$) on the flux level and assuming the torque to be constant, the minimal loss is found by differentiating (4.106) with respect to A:

$$\begin{aligned} \frac{\partial P_{\text{loss}}}{\partial A} &\Rightarrow \\ &\Rightarrow -\left(\frac{\left(\omega_{\text{e}}L_{\text{m}}\right)^{2}}{R_{\text{m}}} + R_{\text{s}} + \left(\omega_{\text{e}}L_{\text{m}}\right)^{2}\frac{R_{\text{s}}}{R_{\text{m}}^{2}}\right)\frac{1}{A^{2}} + \left(R_{\text{s}} + R_{\text{r}}'\right) = 0 \Rightarrow \\ &\Rightarrow P_{\text{loss,d}} = P_{\text{loss,g}}. \end{aligned}$$
(4.107)

As an example, (4.107) is implemented using a simple PI-controller in Fig. 4.32.



Figure 4.32: Model based control (vector drive).

4.4.5 Search control

In the search control approach the output power of the motor is kept constant while the motor flux level is iteratively adapted to find a minimum of the input power [72]. Theoretically, this method offers the optimal solution without knowledge of the motor parameters. But some disadvantages appear in practice, such as continuous disturbances in the torque, slow adaptation, difficulties to tune the algorithms for a given application and the need for precise load information. For the scalar controlled drive the DC-link input power can be used as the controlled variable which is minimized by adjusting the V/Hz ratio (Fig. 4.33) [73].



Figure 4.33: Search control: input power minimization for a scalar controlled IM.

A similar approach for vector controlled drives can be found in [74] and [75] where the search controller is combined with a rotor-flux oriented vector control scheme, and minimizes the drive input power, keeping the motor output power constant by controlling the speed and assuming the load characteristic to be constant.

In [76] the loss minimization in scalar controlled induction motor drives with search controllers is investigated comparing different controlled variables. It is shown, that the method of measuring the DC-link power usually suffers from the low sensitivity of the DC power in the vicinity of the minimum and, additionally, its speed dependence. Thus, instead of measuring the power or current of the inverter DC-link the stator current is proposed for minimization. It is demonstrated, that the sensitivity of the DC input power or current. Additionally, the stator current sensitivity with respect to the airgap flux is much larger than the sensitivity function is independent of the speed and does not significantly depend on the motor efficiency. Experimental results showed the advantage of the stator current minimization over the input power minimization. Stator current minimization control (Fig. 4.34) leads to even more loss reduction, due to the absence of oscillations in the airgap flux and the minimization of the inverter losses since the inverter losses are proportional to the stator current.

In [77] a fuzzy logic based on-line efficiency optimization control of an induction motor drive is described which uses an indirect vector control in the inner loop. At steady-state light load condition, a fuzzy controller adaptively decrements the excitation current on the basis of the measured input power such that, for a given load torque and speed, the drive settles to the minimum input power, i.e. operates at maximum efficiency. The low-frequency pulsating torque due to the decrease of the rotor flux is compensated in a feed-forward way. If the load torque or speed command changes, the efficiency search algorithm is abandoned and the rated flux is established to get the best transient response.

In [78] the authors present a method of efficiency maximization that utilizes the sensing of the third-harmonic content of the airgap flux. This signal is used



Figure 4.34: Search control: stator current minimization for a vector controlled IM.

to determine the resulting instantaneous position of the fundamental component of the airgap flux and, consequently, the torque- and flux-producing components of the stator current. In addition, the third harmonic signal is also used to determine the rotor speed. Hence, the output power of the machine can be calculated with only one signal sensor wire attached to the neutral point of the machine. The fluxproducing component can now be readily adjusted to produce the minimum input power for a fixed amount of output power at constant speed.

In [79] dynamic programming is used to minimize the losses of a field-oriented induction motor drive during a closed-cycle operation. Constraints on the input currents, input voltages, maximum speed, and rotor flux are taken into account in the problem formulation. Iron saturation effects are modelled to provide a state-varying constraint on the magnitude of the rotor flux, and the load is modelled as a function of motor position, as can be found in many mechanical processes. The dynamic optimization results correlate well with experimental results. The efficiencies of the dynamically optimized (varying flux) trajectories for the whole cycle are better than those with an optimal constant flux by 10 to 20 %.

The comparison of different energy-efficiency control strategies and experimental data [61], [80] are presented in the Tables 4.1 and 4.2.

Control method	Simple state	Model based	Search
Motor drive specific	yes	yes	no
Parameter sensible	yes	yes	no
Efficiency precision	medium	medium	high
Efficiency convergence	fast	fast	slow
Complexity	simple	medium/high	medium
Speed measurement req.	ω_{slip} :yes; $\cos(\phi)$:no	sometimes	yes

Table 4.1: Comparison of energy-efficiency control methods.

(published experimental data).		
Energy-optimal control strategy	Scalar drive	Vector drive
Simple state - $cos(\phi)$	7 s	1 s
Model based	2-3 s	0.5-2 s
Model based (off-line calculated airgap flux)	1 s	2 s
Stator current minimizing search control	7-8 s	$3 \mathrm{s}$
DC-link power minimizing search control	9 s	4 s

Table 4.2: Comparison of convergence time in energy-efficiency control methods (published experimental data).

4.5 Conclusions

This chapter discussed the operational principles and control methods of two types of electrical machines (IPMSM and IM) which are the main candidates for using in the fuel cell compression system.

The advantage of synchronous motors compared to induction motors is mainly due to the synchronous rotation. It significantly affects the IPMSM power factor and efficiency. The main losses in the IPMSM consist of stator ohmic losses and iron losses caused by the airgap flux harmonics.

In addition to the mentioned losses the IM also contains rotor copper losses and extra stator copper losses due to the high magnetizing current. Thus, the efficiency of the IPMSM is noticeably higher than that of the IM. Another advantage of the IPMSM is related to the mechanical construction of the rotor. It is lighter compared to that of an equivalent IM's rotor, which makes it suitable for servo motor drives where a high power to weight ratio is required. This is an important remark for the application studied in this thesis - active surge suppression by a speed controlled electrical drive.

The main disadvantage of the IPMSM is the use of PM materials which results in increased motor cost and, additionally, in a limited operational temperature range. The induction machine has a very simple rotor construction, which makes the machine more reliable and cheaper. In practice, for low-power applications PM machines are preferable, but in application of a few tens of kW induction motors are usually employed.

The advantage of IMs over PMSMs with respect to the efficiency mainly appears in applications where the speed is relatively high at low load torque [68]. In this case, the flux reduction technique for the induction motor results in a valuable efficiency gain due to reduced core losses. The choice of the efficiency control is defined by the requirements to the performance of the system and the applied motion control. For example, in low-performance applications, like HVAC systems, the simple state or search control is the most applicable. The fast model-based control could be applied in high-performance systems where powerful computers and accurate measurement equipment are already in use.

From the variety of motion control methods the indirect field-oriented control has been chosen for the active surge control of the compressor due to the following
reasons. As it is seen from section 3.3.3, the surge control method requires the precise speed information and, thus the use of a speed sensor. In principle, speed could be estimated via the measurements of the electrical quantities of the machine [81, 82]. This approach seems to be very attractive from the point of view of simplification of the system as well as cost reduction, and it can be recommended for future research. But for the initial study of the active surge control, the precise speed measurement is preferable.

Chapter 5 deals with the design aspects of high-speed induction motors. A design is proposed, which can be recommended for compressor applications and surge control. Additionally, the analysis of the electromagnetic behavior of a high-speed induction machine which will be used for the experiments, is described.

Chapter 5

Design and numerical modeling of high-speed induction motors

As discussed in section 3.3.3, the active surge control of the centrifugal compressor can be performed by the precise torque regulation of the motor. From the system of dynamic equations of the compressor it can be seen, that the primary process variables - mass flow and pressure - directly depend on the rotational speed of the impeller (or driving motor). The compressor and motor dynamic interaction is described by the mechanical dynamic equation, where the inertia is presented as a parameter. Hence, the drive speed response would be defined by two factors: the electromagnetic torque and the motor inertia.

The high-performance torque control of the motor can be implemented using, for example, a FOC scheme, which has been described in Chapter 4. This chapter mainly focusses on the design aspects of the induction motor. The relations between the performance characteristics and the geometrical parameters are discussed. A simplified derivation of the efficiency and a thermal analysis are also presented. In addition to the torque production and the motor inertia, which define the process controllability, the thermal and efficiency aspects of the induction motor operation must also be taken into account, since they are directly related to the system efficiency.

The design procedure outlined in Fig. 5.1 has been applied for a high-speed induction motor. The initially specified parameters and analytically obtained performance characteristics have been verified by high-precision numerical modeling, using the Finite Elements Method. The analytical and numerical methods have also been used for the preliminary analysis and design verification of the induction motor selected for the full-scale laboratory experiment [83].



Figure 5.1: IM iterative design flow-chart.

5.1 Design of the high-speed induction motor

5.1.1 Performance specification

The initial design specification data for the high-speed induction motor has been derived from the analysis of the compression system operation regimes (Chapter 3), [84]. First, the desired compression system performance variables (pressure and mass flow) and the compressor parameters are defined (Table 5.1).

Table 5.1: Compressor rated performance parametersPressure, $p_{\rm p}$, [Bar]1.5Maximum mass flow, $m_{\rm max}$, [kg/s]0.5Minimum mass flow, $m_{\rm max}$, [kg/s]0.2

Maximum mass flow, $m_{\rm max}$, [kg/s]	0.5
Minimum mass flow, m_{\min} , $[kg/s]$	0.2
Impeller radius, r_2 , [m]	0.12
Impeller inertia, J , $[kgm^2]$	0.001

Next, using the compressor dimensions and the compressor torque, T_c , (3.14), the required motor output mechanical power, P_d , is defined:

$$P_{\rm d} = \omega T_{\rm c},\tag{5.1}$$

where the speed, ω , is defined from the compressor performance map (Fig. 3.22), which equals approximately 4000 rad/s or 600 Hz.

Taking into account a high-speed operation, a two-pole induction motor is proposed. Additionally, with the aim to reduce losses in the power electronic inverter operating at high frequency, an increased voltage level with respect to a classical application has been selected.

The input apparent power, S, of the motor is a function of the efficiency, η_{IM} , and the power factor, $\cos \phi$:

$$S = P_{\rm d} \frac{k_{\rm e}}{\eta_{\rm IM} \cos \phi},\tag{5.2}$$

where

 $P_{\rm d}$ - mechanical power, [W], $k_{\rm e}$ - ratio between back EMF and terminal voltage (preliminary set equal to 0.98), $\eta_{\rm IM}$ - motor efficiency, $\cos\phi$ - power factor.

The efficiency and the power factor can be preliminary estimated using the following empirical equations [85]:

$$\eta_{\rm IM} \sim 0.856 P_{\rm d}^{0.025} p^{-0.02} f_{\rm s}^{-0.05},$$
(5.3)

$$\cos\phi \sim 1.08 P_{\rm d}^{0.015} p^{-0.08} f_{\rm s}^{-0.07}.$$
 (5.4)

 $f_{\rm s}$ - stator frequency, [Hz], p - number of pole pairs, [-].

Finally, the initial motor specifications have been be formulated (Table 5.2).

Table 5.2: Design data of the direct-drive high-speed 2-pole, 30 kW IM.

Output power, $P_{\rm d}$, [kW]	30
Phase rms voltage, $V_{\rm s}$, [V]	380
Stator frequency, $f_{\rm s}$, [Hz]	600
Number of pole pairs, p , $[-]$	1
Power factor, $\cos\phi$, $[-]$	0.76
Motor efficiency, $\eta_{\rm IM}$, [%]	95.0

5.1.2 Selection of the main dimensions

At the next stage of an electrical machine design the main machine dimensions are defined. It is performed using the general equation of the electromechanical power conversion (5.5) which links the main motor dimensions with the electrical and magnetic loading [86, 87]:

$$\frac{D^2 l_{\delta} \omega}{S} = \frac{2}{\pi \alpha_{\delta} k_{\rm B} k_{\rm w} A B_{\delta}},\tag{5.5}$$

where

D - stator internal diameter, [m],

- l_{δ} active length of the magnetic core, [m],
- ω angular speed, [rad/s],
- S input apparent power, [VA],
- B_{δ} airgap flux density (magnetic loading), [T],
- A electrical loading, [A/m],
- α_{δ} pole overlap coefficient, [-],
- $k_{\rm B}$ coefficient of flux density distribution, [-],
- $k_{\rm w}$ winding coefficient, [-].

The left part of (5.5) is related to the output dynamic performance of the machine, and the right part corresponds to the particular limitations caused by the thermal and magnetic properties of the materials. The required dynamic performance of the machine with respect to the application can be characterized by the electromechanical time-constant, $\tau_{\rm em}$:

$$\tau_{\rm em} = \frac{J_{\rm m}\omega}{T_{\rm d}},\tag{5.6}$$

where the inertia, $J_{\rm m}$, of the rotor is:

$$J_{\rm m} = \frac{k_\gamma \gamma_{\rm st} l_\delta \pi D_{\rm r}^4}{32},\tag{5.7}$$

where

 $D_{\rm r}$ - rotor diameter, [m],

the product, $k_{\gamma} \cdot \gamma_{\text{st}}$, is the average rotor density expressed via the steel density, γ_{st} , $[\text{kg/m}^3]$, and the density coefficient, k_{γ} .

Assuming that $D \approx D_r$ from (5.5), (5.2) and (5.7) it is visible that $T_d \propto D^2 l_\delta$ and $J_m \propto D_r^4 l_\delta$, which finally shows:

$$\tau_{\rm em} \propto D_{\rm r}^2.$$
 (5.8)

From this function, it is visible that a reduction of $\tau_{\rm em}$ is achieved by means of a reduced rotor diameter $D_{\rm r}$. The required value of torque or power is kept by proportionally increasing the active motor length l_{δ} . On the other hand, the aspect ratio, $\lambda_{\rm main}$, defined as:

$$\lambda_{\rm main} = \frac{l_{\delta}}{D_{\rm r}},\tag{5.9}$$

is limited by the mechanical strength properties of the rotor assembly. Within the present work, a comprehensive mechanical analysis is not performed. Using the common experience of electrical machine design and published reports related to high-speed induction motors [88, 89], the maximum limit for λ_{main} is set to two.

The main machine dimensions also depend on the product, AB_{δ} , which on its turn defines the machine characteristics. The values of electrical loading and airgap flux density can originally be selected from a set of their combinations corresponding to $AB_{\delta} = const$.

5.1.3 Stator sizing and winding parameters

The next step is the stator dimensioning and the stator winding design. At first, the number of stator slots, Z_s , and the number of turns per phase, w_s , are defined. The number of turns per phase, w_s , should provide the values of the electrical loading and airgap flux density close to the initially specified ones. The number of stator slots, Z_s , should guarantee an equal distribution of the winding.

To fulfill those conditions, the tooth pitch is initially specified, depending on the type of the winding, the rated voltage and the pole pitch. For a more uniformly distributed winding, a high number of teeth should be selected. However, the slot width should not be very small, as it influences the filling factor, the mechanical strength of the stator teeth, as well as the machine production cost.

The number of stator slots, Z_s , must be a multiple of the number of stator phases, m, and thus the corresponding number of stator slots per pole and per phase, q_s , must be integer:

$$q_{\rm s} = \frac{Z_{\rm s}}{2pm}.\tag{5.10}$$

The number of effective conductors in a slot, $u_{\rm sl}$, is calculated in the following way. First, a preliminary number, $u'_{\rm sl}$, is defined as:

$$u_{\rm sl}' = \frac{\pi DA}{I_{\rm s} Z_{\rm s}},\tag{5.11}$$

where the stator rms current, $I_{\rm s}$, is:

$$I_{\rm s} = \frac{P_d}{m V_{\rm s} \eta_{\rm IM} \cos \phi},\tag{5.12}$$

where

 $V_{\rm s}$ - rms phase voltage, [V].

Further, this number is corrected with respect to the number of parallel branches in a phase winding, a:

$$u_{\rm sl} = a u_{\rm sl}^{\prime}.\tag{5.13}$$

Finally, the effective number of turns per phase, $w_{\rm s}$, is:

$$w_{\rm s} = \frac{u_{\rm sl} Z_{\rm s}}{2am}.\tag{5.14}$$

After the calculation of the number of stator slots and the winding parameters the values of A and B_{δ} are verified. The final value of the electrical loading, A, is:

$$A = \frac{2I_{\rm s}w_{\rm s}m}{\pi D}.\tag{5.15}$$

To calculate the airgap magnetic flux density, first, the flux, Φ , is expressed based on the obtained effective number of turns per phase, w_s :

$$\Phi = \frac{k_{\rm e} V_{\rm s}}{4k_{\rm B} w_{\rm s} k_{\rm w} f_{\rm s}}.\tag{5.16}$$

In the airgap of the induction motor, a flux density with a nearly sinusoidal shape is supposed within this derivation. The magnetic flux corresponding to one pole pitch, $\tau_{\rm p}$, is defined as:

$$\Phi = l_{\delta} \int_{0}^{\tau_{\rm p}} B_{\delta {\rm x}} dx, \qquad (5.17)$$

where

 $B_{\delta x}$ - flux density in the airgap at point, x, [T].

In practice, the exact function of the flux density distribution is difficult to calculate by analytical methods. Thus, an equivalent pole pitch, $\tau'_{\rm p}$, is introduced, along which the flux density is assumed constant. The meaning of $\tau'_{\rm p}$ is defined from the condition of equal fluxes per unit length of the magnetic core:

$$B_{\delta} \cdot \tau_{\rm p}' = \int_{0}^{\tau_{\rm p}} B_{\delta {\rm x}} dx.$$
 (5.18)

The value of $\tau'_{\rm p}$ is defined as:

$$\tau_{\rm p}' = \alpha_{\delta} \tau_{\rm p}, \tag{5.19}$$

where the pole overlap coefficient for the sinusoidal flux density, $\alpha_{\delta} = \frac{2}{\pi}$.

Taking into account (5.19), the flux density in the airgap is defined as:

$$B_{\delta} = \frac{\Phi}{a_{\delta}\tau_{\rm p}l_{\delta}} = \frac{p\Phi}{Dl_{\delta}}.$$
(5.20)

The obtained value of B_{δ} should not differ from the initially recommended one by more than 5 %. Otherwise the calculations are repeated iteratively selecting a different number of effective conductors in a slot, $u_{\rm sl}$.

If the electrical loading and the airgap flux density do not exceed the appropriate limits, the next step is the determination of the cross-section area of the conductor. The cross-section of the conductor is defined from the stator current in one branch and the allowed current density in the winding, J_s :

$$S_{\rm eff,s} = \frac{I_{\rm s}}{aJ_{\rm s}}.$$
(5.21)

Normally, the value of the current density is selected with respect to the type of cooling, insulation material and the motor size. From the point of view of utilizing the active material, the maximum value of the current density should be selected. On the other hand, the current density is directly related to the losses, the temperature of the winding and the efficiency of the machine. In some cases the conductor is divided into several elementary conductors, $n_{\rm el}$, thus:

$$S_{\rm el}n_{\rm el} = S_{\rm eff,s}.\tag{5.22}$$

After selection of $S_{\rm el}$ and $n_{\rm el}$ the stator winding current density is corrected:

$$J_{\rm s} = \frac{I_{\rm s}}{aS_{\rm el}n_{\rm el}}.\tag{5.23}$$

The dimensions of the stator slot in an induction motor should be defined according to several criteria. First, the cross-section area of the slot must correspond

to the sizes of the winding including the insulation. Secondly, the flux density in the stator teeth and yoke should stay within certain limits depending on the power level and the used material of the magnetic core. Slots and teeth configuration is defined by the winding type which, on its turn, depends on electrical quantities, such as power and rated voltage.

5.1.4 Selection of the airgap length

A correct selection of the airgap length significantly influences the performance characteristics of an induction motor. A small length of the airgap leads to a lower value of the MMF and, consequently, a small magnetizing current. This results in an increased power factor and reduced copper losses. On the other hand, a very small airgap would lead to higher stray losses (section 5.1.8). Thus, the efficiency of a motor with a very small airgap could be less. The airgap length is usually selected in order to obtain the minimum of the total losses, using an optimization routine. Preliminary, the airgap length, δ , can be defined as [90]:

$$\delta \approx (0.3 + 1.5D) \cdot 10^{-3}. \tag{5.24}$$

As described in section 5.1.8, the surface and pulsation losses depend on both the amplitude and the frequency of the pulsation of the airgap flux density. Thus, it is recommended to make a somewhat larger airgap in high-speed machines to reduce this kind of losses. Finally, the airgap chosen according to the mentioned recommendations should be larger than minimal possible due to the mechanical properties of the shaft.

5.1.5 Rotor design

A squirrel cage rotor has been selected as one of the most appropriate solutions for high-speed induction motors. The short-circuited winding of the rotor does not have a defined number of phases or number of poles. It is generally accepted that one rotor bar corresponds to one phase of the short-circuited winding. Then, the number of rotor phases, m_r , is equal to the number of slots ($m_r = Z_r$), and every phase contains one half of a turn.

The winding coefficient in this case is equal to unity, and the number of slots per pole and per phase is:

$$q_{\rm r} = \frac{Z_{\rm r}}{2pm_{\rm r}} = \frac{1}{2p}.$$
 (5.25)

Special attention should be paid to the selection of the number of rotor slots during the motor design procedure. It is important due to the fact that in the airgap the magnetic flux density contains a set of higher harmonics in addition to the fundamental. Every harmonic induces an EMF in the rotor winding. As a result of the interaction of the rotor currents and the high field harmonics additional components of the electromagnetic torque appear. Thus, depending on the relation between the slot numbers of stator and rotor, different synchronous and asynchronous torque components can seriously influence the mechanical characteristics of the machine.

Additionally, higher field harmonics appear due to certain relations between the pole and teeth number of the rotor, [86]. They normally lead to extra noise and vibrations of the machine at the rated regime. Especially, this effect becomes important in machines with a very small airgap.

In low-power motors the usual case is $Z_{\rm r} < Z_{\rm s}$. However, in high-power motors, $Z_{\rm r} > Z_{\rm s}$ is usually selected with the aim to reduce the current in the rotor bars and improve the distribution of the conductors.

The rotor current, $I_{\rm r}$, is:

$$I_{\rm r} = k_{\rm i} I_{\rm s} \frac{2mw_{\rm s}k_{\rm w}}{Z_{\rm r}k_{\rm skew}},\tag{5.26}$$

where

 $k_{\rm i}$ - coefficient describing the influence of the magnetizing current, and the skew coefficient, $k_{\rm skew},$ is defined as:

$$k_{\rm skew} = \frac{2\sin(\gamma_{\rm skew}/2)}{\gamma_{\rm skew}},\tag{5.27}$$

and

$$\gamma_{\rm skew} = \frac{b_{\rm skew} 2p}{\tau_{\rm z,r} Z_{\rm r}},\tag{5.28}$$

where

 $\tau_{z,r}$ - rotor slot pitch, [m], b_{skew} - rotor slot skew, [m].

The cross-section of the bar, S_{bar} , is:

$$S_{\rm bar} = \frac{I_{\rm r}}{J_{\rm r}},\tag{5.29}$$

where

 $J_{\rm r}$ - rotor bar current density, [A/m²].

The current density in the rotor bar, J_r , is defined depending on the conductor material and power of the machine. For aluminium bars the value of the current density usually equals $(2.5 - 3.5) \cdot 10^6 [A/m^2]$, and for copper bars it can reach $(4 - 8) \cdot 10^6 [A/m^2]$. Higher values correspond to low-power machines.

The current in the rotor end ring, $I_{\rm er}$, is defined as:

$$I_{\rm er} = \frac{I_{\rm r}}{\Delta_{\rm er}},\tag{5.30}$$

$$\Delta_{\rm er} = 2\sin\frac{\pi p}{Z_{\rm r}}.\tag{5.31}$$

The current density in the rotor end rings is usually assumed 15-20 % lower than that in the rotor bars based on the cooling conditions.

The cross-section area of the rotor rings, $S_{\rm er}$, is:

$$S_{\rm er} = \frac{I_{\rm er}}{J_{\rm er}},\tag{5.32}$$

where

 $J_{\rm er}$ - end ring current density, [A/m²].

The size of the rotor slots is primarily defined by the required cross-section area of the rotor bar and the maximum allowable flux density in the teeth. The shape of the slot also influences the starting characteristics of the machine. Usually, long or double-cage slots would provide better starting characteristics due to the skin effect at high rotor frequency. However, since in the present work the IM is assumed to be well-controlled using a power electronic converter, the direct-on-line starting characteristics are not relevant.

To reduce the surface and pulsation losses, a closed-slot rotor has been selected. Additionally, the closed-slot rotor structure provides sufficient mechanical strength of the rotor assembly, which is an important factor in a high-speed motor. However, the closed rotor slots lead to an increased rotor leakage flux. Normally, the height of the iron bridge, $h_{\rm br}$, over the rotor bar should be minimized to achieve a high saturation level and, as a consequence, reduce the rotor leakage permeance. In the case of a high-speed machine, the minimal height of the bridge is limited by its mechanical strength. In the present design, the height of the iron bridge is set to 1 mm. The cross-section of the proposed IM is shown in Fig. 5.2.

5.1.6 Magnetizing current

The magnetizing current is the main parameter that reflects the magnetic loading, since it provides the magnetomotive force (MMF) necessary to drive the flux through the magnetic flux path.

For the magnetic circuit analysis Ampere's law is used [85, 91]:

$$\oint \boldsymbol{H} d\boldsymbol{l} = i_{\text{enclosed}},\tag{5.33}$$



Figure 5.2: Cross-section of the proposed IM with closed rotor slots.

 i_{enclosed} - enclosed current, [A], **H** - magnetic field strength vector, [A/m], dl - unit vector along the closed magnetic path, [m].

The magnetomotive force, F, is defined to be:

$$F \triangleq \oint \boldsymbol{H} d\boldsymbol{l} = i_{\text{enclosed}}.$$
 (5.34)

The appropriate value of \boldsymbol{B} is obtained from Gauss's law:

$$\oint_{S} \boldsymbol{B} d\boldsymbol{s} = 0, \tag{5.35}$$

where

S - closed surface, $[m^2]$.

As applied to an electrical machine, (5.35) expresses the principle of continuity of the flux, Φ , along the magnetic flux path.

For simplifying the analysis, the magnetic flux path of the machine is split in separated segments along the magnetic path. Each segment has a comparatively simple geometrical configuration and a particular magnetic characteristic. As sketched in Fig. 5.3, for each pole pair this flux path consists of two airgap lengths (c - d and g - h), two tooth heights in the stator (b - c and h - a), two tooth heights in the rotor (d - e and f - g), one core (yoke) length for the stator (a - b) and one for the

rotor (f - e). The material of the shaft is selected to be non-magnetic to exclude the losses in the bearings due to induced currents. The direction of the magnetic field is assumed to be known. In such a representation, the vectors **B** and **H** can be replaced by scalars **B** and **H**, correspondingly. The values of the magnetic field strength, *H*, are taken from the magnetization curve provided by a manufacturer of electrical steels. In the present work, the data for the electrical steel 3.5%SiFe NO20 is used (Fig. 5.4).



Figure 5.3: Simplified magnetic path in an IM.

Starting from the airgaps, and assuming that H does not vary along the length of the gap, the MMF in the airgaps, F_{δ} , is:

$$F_{\delta} = \frac{2}{\mu_0} B_{\delta} \delta k_{\delta}, \qquad (5.36)$$

where

 $\mu_0 = 4\pi \cdot 10^{-7}$ permeability of vacuum, [H/m], δ - airgap length, [m],



Figure 5.4: B-H curve of the 3.5%SiFe NO20 laminated material.

 k_{δ} - Carter's coefficient.

Carter's coefficient takes into account the influence of the stator and rotor teeth on the MMF in comparison to flat surfaces. In general, it is defined as the product of the stator, $k_{\delta,s}$, and the rotor, $k_{\delta,r}$, Carter's coefficients:

$$k_{\delta} = k_{\delta,\rm s} k_{\delta,\rm r}.\tag{5.37}$$

For the closed-slot rotor structure, $k_{\delta,r} = 1$, thus

$$k_{\delta,\mathrm{s}} = \frac{\tau_{\mathrm{z,s}}}{\tau_{\mathrm{z,s}} - \gamma_{\mathrm{s}}\delta},\tag{5.38}$$

$$\gamma_{\rm s} = \frac{\left(b_{\rm o,s}/\delta\right)^2}{5 + b_{\rm o,s}/\delta},\tag{5.39}$$

where

 $b_{\rm o,s}$ - width of the stator slot opening, [m],

 $\tau_{\rm z,s}$ - stator slot pitch, [m],

 $b_{z,s}$ - width of the stator tooth, [m].

The MMFs in the stator and rotor yoke, $F_{\rm y,s}$, and $F_{\rm y,r}$, are defined as:

$$F_{y,s} = l_{y,s} H_{y,s},$$
 (5.40a)

$$F_{\mathbf{y},\mathbf{r}} = l_{\mathbf{y},\mathbf{r}} H_{\mathbf{y},\mathbf{r}},\tag{5.40b}$$

where

 $l_{\rm y,s}$ - average path length of the magnetic flux in the stator yoke, [m], $l_{\rm y,r}$ - average path length of the magnetic flux in the rotor yoke, [m], $H_{\rm y,s}$ - magnetic field strength in the stator yoke, [A/m], $H_{\rm y,r}$ - magnetic field strength in the rotor yoke, [A/m].

The magnetic flux densities in the stator yoke, $B_{y,s}$, and in the rotor yoke, $B_{y,r}$, are:

$$B_{\rm y,s} = \frac{\Phi}{2h_{\rm y,s}l_{\delta}k_{\rm mc}},\tag{5.41a}$$

$$B_{\rm y,r} = \frac{\Phi}{2h_{\rm y,r}l_{\delta}k_{\rm mc}},\tag{5.41b}$$

where

 Φ - magnetic flux per pole, [Wb],

 $h_{\rm y,s}$ - stator yoke height, [m],

 $h_{\rm y,r}$ - rotor yoke height, [m],

 $k_{\rm mc}$ - filling factor of the magnetic core.

From the geometry of the machine, the stator yoke height, $h_{\rm y,s}$, is:

$$h_{\rm y,s} = \frac{D_{\rm s} - D}{2} - h_{\rm sl,s},\tag{5.42}$$

where

 $D_{\rm s}$ - stator external diameter, [m], $h_{\rm sl,s}$ - stator slot height, [m].

The average path length of the stator yoke, $l_{y,s}$, is:

$$l_{\rm y,s} = \frac{\pi \left(D_{\rm s} - h_{\rm y,s} \right)}{2p}.$$
 (5.43)

The rotor yoke height, $h_{y,r}$, is defined as:

$$h_{\rm y,r} = \frac{D_{\rm r} - D_{\rm sh}}{2} - h_{\rm sl,r},$$
 (5.44)

where

 $D_{\rm sh}$ - shaft diameter, [m], $h_{\rm sl,r}$ - rotor slot height, [m].

The average path length of the rotor yoke, $l_{y,r}$, is:

$$l_{\rm y,r} = \frac{\pi \left(D_{\rm sh} + h_{\rm y,r} \right)}{2p}.$$
 (5.45)

Stator and rotor teeth MMF

In general, the MMFs of the stator and rotor teeth zone, $F_{z,s}$ and $F_{z,r}$, are:

$$F_{z,s} = \int_{0}^{h_{z,s}} H_{z,s} dx, \qquad (5.46a)$$
$$F_{z,r} = \int_{0}^{h_{z,r}} H_{z,r} dx, \qquad (5.46b)$$

where

 $H_{z,s}$ - magnetic field strength in the stator tooth, [A/m], $H_{z,r}$ - magnetic field strength in the rotor tooth, [A/m], $h_{z,s}$ - height of the stator tooth, [m], $h_{z,r}$ - height of the rotor tooth, [m].

For a constant cross-section of the tooth, $H_{\rm z,s}$ and $H_{\rm z,s}$ can be assumed constant, then:

$$F_{z,s} = 2h_{z,s}H_{z,s},$$
 (5.47a)

$$F_{\rm z,r} = 2h_{\rm z,r}H_{\rm z,r}.$$
 (5.47b)

The flux densities in the stator and rotor teeth, $B_{z,s}$, and $B_{z,r}$, can be defined as:

$$B_{\rm z,s} = \frac{B_{\delta} \tau_{\rm z,s}}{b_{\rm z,s} k_{\rm mc}},\tag{5.48a}$$

$$B_{\rm z,r} = \frac{B_{\delta} \tau_{\rm z,r}}{b_{\rm z,r} k_{\rm mc}}.$$
(5.48b)

Total MMF

The total MMF, F_{Σ} , is equal to the sum of the MMFs of the individual segments:

$$F_{\Sigma} = F_{\delta} + F_{z,s} + F_{z,r} + F_{y,s} + F_{y,r}.$$
 (5.49)

The saturation coefficient of the magnetic circuit is:

$$k_{\mu} = \frac{F_{\Sigma}}{F_{\delta}}.$$
(5.50)

A full-pitch concentrated winding having w_s/p turns per pole-pair and carrying an alternating current, I_{μ} , generates an MMF-wave with a rectangular distribution in space and pulsating in time. The amplitude of its fundamental, F_1 , is [85]:

$$F_1 = \sqrt{2} \frac{4}{\pi} \frac{w_{\rm s} k_{\rm ws}}{p} I_{\mu}.$$
 (5.51)

Each of the m phases contributes one-half of its pulsating wave to the forward rotating field. It follows that the magnetizing current is:

$$I_{\mu} = \frac{F_{\Sigma}}{\frac{m}{2}\sqrt{2}\frac{4}{\pi}\frac{w_{\rm s}k_{\rm ws}}{p}} = \frac{pF_{\Sigma}}{0.9mw_{\rm s}k_{\rm ws}},\tag{5.52}$$

where

 $k_{\rm ws}$ - stator winding coefficient (preliminary set equal to 0.95).

The magnetizing current can also be expressed as a portion of the rated stator current:

$$I_{\mu}^{*} = \frac{I_{\mu}}{I_{\rm s}}.$$
 (5.53)

The value of I^*_{μ} is a measure to represent the correctness of the performed calculations. The usual value of I^*_{μ} should be in the range between 0.2-0.4.

5.1.7 Rated parameters

An induction machine is usually described by the well-known T-equivalent circuit (Fig. 5.5(a)). The main parameters of the induction machine are the resistances and reactances of the stator, the rotor and the magnetizing branch. The rotor values are usually referred to the number of turns of the stator winding.

The equivalent circuit shown in Fig. 5.5(a) is not convenient to use for predicting the performance of the induction machine. As a result, several simplified versions have been proposed in various textbooks on electrical machines. There is no general agreement on how to treat the shunt branch i.e. $(R_{\rm m} \text{ and } X_{\rm m})$, particulary the resistance, $R_{\rm m}$, representing the core loss in the motor.

In practice, the magnetizing branch with parallel connected $R_{\rm m}$ and $X_{\rm m}$ can be replaced by series connected R_{μ} and X_{μ} , based on:

$$\frac{jR_{\rm m}X_{\rm m}}{R_{\rm m}+jX_{\rm m}} = R_{\mu} + jX_{\mu}.$$
(5.54)

For further convenience of the machine analysis, the magnetizing branch can be moved to the machine terminals forming the Γ - equivalent circuit as shown in



Figure 5.5: T - equivalent circuit: (a) Classical, (b) Simplified.

Fig. 5.6 This approximation of the equivalent circuit considerably simplifies the computation, because the magnetizing current, \bar{I}''_{μ} , and the load component, \bar{I}''_{r} , of the machine current can be directly computed from the terminal voltage, \bar{v}_s , by dividing it by the corresponding impedances, corrected with the leakage factor, $\sigma_{\rm s}$.

$$\begin{split} \bar{Z}_{\rm s} &= R_{\rm s} + j X_{\rm ls}, \quad (5.55a) \\ \bar{Z}'_{\rm r} &= R'_{\rm r} + j X'_{\rm lr}, \quad (5.55b) \\ \bar{Z}_{\mu} &= R_{\mu} + j X_{\mu}, \quad (5.55c) \end{split}$$

$$Z'_{\rm r} = R'_{\rm r} + j X'_{\rm lr},$$
 (5.55b)

$$Z_{\mu} = R_{\mu} + j X_{\mu},$$
 (5.55c)

$$\sigma_{\rm s} = \sqrt{\frac{\left(R_{\rm s} + R_{\mu}\right)^2 + \left(X_{\rm ls} + X_{\mu}\right)^2}{R_{\mu}^2 + X_{\mu}^2}}.$$
 (5.55d)

Stator and rotor resistances

The stator phase resistance, $R_{\rm s}$, is defined by the general equation:

$$R_{\rm s} = \rho_{\rm w} \frac{l_{\rm w}}{S_{\rm eff} a},\tag{5.56}$$



Figure 5.6: Γ - equivalent circuit.

where

 $\rho_{\rm w}$ - resistivity of the winding material, $[\Omega {\rm m}],$ $S_{\rm eff}$ - cross-section area of the effective conductor, $[{\rm m}^2],$ $l_{\rm w}$ - total length of the effective conductors of a phase winding, [m].

The resistance is usually defined for a particular operating temperature and correspondingly corrected using the well-known equation:

$$R_{\rm s}(T_{\rm w}) = R_{\rm s}(T_0) \left[1 + \alpha_{\rm t}(T_{\rm w} - T_0) \right], \qquad (5.57)$$

where

 $\begin{array}{l} T_0 \mbox{ - ambient temperature, [K],} \\ \alpha_t \mbox{ - temperature coefficient, [1/K],} \\ T_w \mbox{ - winding temperature, [K].} \end{array}$

The rotor winding resistance, R_r , is defined from the condition that the power loss in R_r due to the bar current, I_r , is equal to the sum of the losses in the bar and end ring due to the currents, I_r and I_{er} , respectively:

$$R_{\rm r}I_{\rm r}^2 = R_{\rm bar}I_{\rm r}^2 + R_{\rm er}I_{\rm er}^2.$$
(5.58)

The end ring resistance, $R_{\rm er}$:

$$R_{\rm er} = \rho_{\rm er} \frac{\pi D_{\rm er,av}}{Z_{\rm r} S_{\rm er}},\tag{5.59}$$

where

 $\begin{array}{l} \rho_{\rm er} \ \text{-resistivity of the end ring material, } [\Omega m], \\ D_{\rm er,av} \ \text{-average end ring diameter, } [m], \\ S_{\rm er} \ \text{-end ring cross-section area, } [m^2]. \end{array}$

The bar resistance, R_{bar} , is:

$$R_{\rm bar} = \rho_{\rm bar} \frac{l_{\rm bar}}{S_{\rm bar}},\tag{5.60}$$

where

$$\begin{split} \rho_{\rm bar} & \text{-} \text{ resistivity of the bar material, } [\Omega m], \\ l_{\rm bar} & \text{-} \text{ bar length, } [m], \\ S_{\rm bar} & \text{-} \text{ bar cross-section area, } [m^2]. \end{split}$$

The rotor resistance, $R_{\rm r}$, is finally obtained:

$$R_{\rm r} = R_{\rm bar} + 2\frac{R_{\rm er}}{\Delta_{\rm er}^2}.$$
(5.61)

The value of $R_{\rm r}$ should be referred to the stator winding. It is performed via the coefficient, $w_{\rm sr}$, expressed for the short-circuit rotor as:

$$w_{\rm sr} = 4m \frac{(w_{\rm s}k_{\rm ws})^2}{Z_{\rm r}k_{\rm skew}^2}.$$
 (5.62)

As a result, the rotor resistance referred to the number of turns of the stator winding, $R'_{\rm r}$, is:

$$R_{\rm r}' = R_{\rm r} w_{\rm sr}.\tag{5.63}$$

Stator and rotor leakage reactances

Leakage fields are linking only the stator or rotor. They contribute to the winding self inductance but not to the energy transfer. The general expression for the leakage inductance can be derived from the expression for the stored magnetic field energy (or coenergy), $W_{\rm f}$, [86]:

$$W_{\rm f} = \frac{1}{2} \int\limits_{V} (\boldsymbol{B}\boldsymbol{H}) dV. \tag{5.64}$$

Assuming that, in a particular region i, the magnetic field is produced by a single current source, I_i , an inductance, L_i , links the magnetic field with a circuit element:

$$W_{\rm f} = L_{\rm i} \frac{I_{\rm i}^2}{2}.$$
 (5.65)

Applying Ampere's law (5.33) to that region, and using the known relation $\boldsymbol{B} = \mu_0 \boldsymbol{H}$, saturation being neglected, the leakage inductance, $L_{\rm li}$, can be expressed in a simplified way as:

$$L_{\rm li} = \mu_0 l_\delta \lambda_{\rm i},\tag{5.66}$$

where

 λ_i - geometrical specific permeance, [-], depending on the geometry of the particular region *i*.

For the stator slot presented in Fig. 5.7 the geometrical specific permeance, $\lambda_{sl,s}$, is [85]:

$$\lambda_{\rm sl,s} = \frac{2h_{\rm 1,s}}{3\left(b_{\rm 1,s} + b_{\rm 2,s}\right)} + \frac{h_{\rm 2,s}}{b_{\rm 1,s}} + \frac{2h_{\rm 3,s}}{b_{\rm 1,s} + b_{\rm 0,s}} + \frac{h_{\rm 0,s}}{b_{\rm 0,s}}.$$
(5.67)

Finally, the stator phase leakage inductance is:



Figure 5.7: Stator slot.

$$L_{\rm ls} = 2\mu_0 w_{\rm s}^2 \frac{l_\delta}{pq_{\rm s}} \left(\lambda_{\rm sl,s} + \lambda_{\rm ew} + \lambda_{\rm dl,s}\right),\tag{5.68}$$

 $\lambda_{\rm sl,s}$ - stator slot specific permeance, [-], $\lambda_{\rm ew}$ - stator end winding specific permeance, [-], $\lambda_{\rm dl,s}$ - stator differential specific permeance, [-].

The expressions or values for the end winding and differential specific permeance, $\lambda_{\rm ew}$, and $\lambda_{\rm dl,s}$, can be taken from literature [86]. The stator leakage reactance, $X_{\rm ls}$, is:

$$X_{\rm ls} = 2\pi f_{\rm s} L_{\rm ls}.\tag{5.69}$$

An expression for the rotor slot (Fig. 5.8) specific leakage permeance can be derived in a similar way as for the stator slot. Neglecting saturation of the tooth



Figure 5.8: Rotor slot.

top, Ampere's law for the bridge zone yields:

$$H_{\rm br} = \frac{I_{\rm r}}{b_{1,\rm r}},$$
 (5.70)

where

 $H_{\rm br}$ - magnetic field strength in the iron bridge, [A/m], $b_{1,\rm r}$ - upper width of the rotor slot, [m],

and for the slot zone:

$$H_{\rm sl} = \frac{I_{\rm r}}{b_{\rm av,r}},\tag{5.71}$$

where

 $H_{\rm sl}$ - magnetic field strength in the rotor slot, [A/m],

and average slot width:

$$b_{\rm av,r} = \frac{b_{1,r} + b_{2,r}}{2},\tag{5.72}$$

where

 $b_{2,r}$ - lower width of the rotor slot, [m].

The rotor slot leakage inductance obtained, $L_{\rm l,sl,r}$, per slot is:

$$L_{\rm l,sl,r} = \frac{2}{I_{\rm r}^2} W_{\rm f} = \frac{2}{I_{\rm r}^2} \frac{1}{2} l_{\delta} \left(b_{\rm av,r} \int_{0}^{h_{\rm 1,r}} \mu_0 H(x)^2 dx + b_{\rm 1,r} \int_{h_{\rm 1,r}}^{h_{\rm 1,r}+h_{\rm br}} \mu_0 \mu_{\rm r} H(x)^2 dx \right),$$
(5.73)

 $\mu_{\rm r}=\frac{B_{\rm br}}{H_{\rm br}}$ - permeability of the rotor iron bridge, obtained from the B-H characteristic of the lamination material, [H/m].

Substituting (5.70), (5.71) and (5.72) into (5.73) yields:

$$L_{\rm l,sl} = \mu_0 l_\delta \left(\frac{2h_{\rm 1,r}}{3(b_{\rm 1,r} + b_{\rm 2,r})} + \frac{h_{\rm br}}{b'_{\rm o,r}} \right), \tag{5.74}$$

where the term in parenthesis corresponds to the specific permeance, $\lambda_{sl,r}$:

$$\lambda_{\rm sl,r} = \frac{2h_{\rm 1,r}}{3(b_{\rm 1,r} + b_{\rm 2,r})} + \frac{h_{\rm br}}{b'_{\rm o,r}},\tag{5.75}$$

where the equivalent rotor slot opening, $b'_{o,r}$, is:

$$b'_{\rm o,r} = \frac{b_{\rm 1,r}\mu_0}{\mu_{\rm r}}.$$
(5.76)

Taking into account that the number of rotor slots per pole and per phase, $q_r = \frac{1}{2p}$, and adding the skew specific permeance, λ_{skew} , the rotor leakage inductance, L_{lr} , is:

$$L_{\rm lr} = \mu_0 l_\delta \left(\lambda_{\rm sl,r} + \lambda_{\rm er} + \lambda_{\rm dl,r} + \lambda_{\rm skew} \right), \qquad (5.77)$$

where

 $\lambda_{\rm er}$ - rotor end ring specific permeance, [-], $\lambda_{\rm dl,r}$ - rotor differential specific permeance, [-], $\lambda_{\rm skew}$ - skew specific permeance, [-].

The rotor leakage reactance, $X_{\rm lr}$, is:

$$X_{\rm lr} = 2\pi f_{\rm s} L_{\rm lr}.\tag{5.78}$$

The rotor leakage reactance should also be referred to the number of turns of the stator winding:

$$X_{\rm lr}' = X_{\rm lr} w_{\rm sr}.\tag{5.79}$$

The magnetizing reactance, X_{μ} , is defined as [85]:

$$X_{\mu} = 2\pi f_{\rm s} \frac{\mu_0 m w_{\rm s}^2 D l_{\delta}}{\pi p^2 \delta}.$$
(5.80)

The resistance, R_{μ} , which corresponds to the core loss is calculated as:

$$R_{\mu} = \frac{P_{\text{core}}}{mI_{\mu}^2},\tag{5.81}$$

where

 $P_{\rm core}$ - core loss, [W], is defined in the next section.

5.1.8 Losses and efficiency

The losses is high-speed induction machines consist of copper losses $(P_{\rm cu})$, main core losses $(P_{\rm core})$, stray core losses, consisting of surface $(P_{\rm surf})$ and pulsation $(P_{\rm pul})$ losses, windage $(P_{\delta,{\rm wind}})$ and acceleration $(P_{\delta,{\rm acc}})$ losses in the airgap, and mechanical losses $(P_{\rm mech})$. The copper losses take place in both the stator and rotor windings mainly due to the resistances. The stator copper losses, $P_{\rm cu,s}$, are calculated as follows:

$$P_{\rm cu,s} = m I_{\rm s}^2 R_{\rm s},\tag{5.82}$$

and the rotor copper losses are:

$$P_{\rm cu,r} = Z_{\rm r} I_{\rm r}^2 R_{\rm r}.\tag{5.83}$$

The main core losses in electrical machines are caused by the eddy currents and the hysteresis effects in the magnetic core. For an induction machine, the main core losses are usually calculated only for the stator core. In the rotor, the frequency, $f_{\rm r}$, is usually very low:

$$f_{\rm r} = s f_{\rm s},\tag{5.84}$$

where

s - slip, [-].

Thus, in the operation regimes close to the rated one the core losses in the rotor are very low even at a high flux density level. In non-controlled induction motor drives, the rotor core losses can be significant during starting due to the high slip and, consequently, the high magnetic field frequency in the rotor. However, in the case of a controlled drive with a power electronic converter, where the slip is wellregulated and kept small, the rotor core losses can be neglected. One of the general equations for the core losses, $P_{\rm core}$, is [92]:

$$P_{\rm core} = k_{\rm h} B_{\rm m}^{\alpha+\beta B_{\rm m}} f_{\rm s} + \frac{\sqrt{\sigma}}{\gamma_{\rm st}} k_{\rm exc} \frac{1}{T} \int_{\rm T} \left| \frac{dB}{dt} \right|^{1.5} dt + \frac{\sigma d^2}{12\gamma_{\rm st}} \frac{1}{T} \int_{\rm T} \left| \frac{dB}{dt} \right|^2 dt, \qquad (5.85)$$

 $\begin{array}{l} B_{\rm m} \mbox{ - maximum flux density, [T],} \\ B \mbox{ - instantaneous flux density, [T],} \\ T \mbox{ - period, [s],} \\ \sigma \mbox{ - electrical conductivity, [S/m],} \\ \gamma_{\rm st} \mbox{ - steel density, [kg/m^3],} \\ k_{\rm h}, \alpha, \beta \mbox{ - hysteresis loss constants,} \\ k_{\rm exc} \mbox{ - excess loss constant,} \\ d \mbox{ - lamination thickness, [m].} \end{array}$

In practice, a simplified equation can be applied for the core loss calculation based on the specific loss data supplied by a steel manufacturer:

$$P_{\text{core}} = P_0 \left(\frac{f_{\text{s}}}{f_0}\right)^{\beta_{\text{f}}} \left[\left(\frac{B_{\text{z,s}}}{B_0}\right)^2 m_{\text{z,s}} K_{\text{tz,s}} + \left(\frac{B_{\text{y,s}}}{B_0}\right)^2 m_{\text{y,s}} K_{\text{ty,s}} \right], \tag{5.86}$$

where

 P_0 - specific losses at the rated frequency (f_0) and the rated flux density (B_0) , [W/kg],

 $\beta_{\rm f}$ - coefficient related to the frequency dependence of the losses,

 $m_{\rm z,s}$ - mass of the stator teeth, [kg],

 $m_{\rm y,s}$ - mass of the stator yoke, [kg],

 $B_{z,s}$ - flux density in the stator tooth, [T],

 $B_{\rm y,s}$ - flux density in the stator yoke, [T],

 $K_{\rm tz,s}$ - technological coefficient for the stator tooth,

 $K_{\rm ty,s}$ - technological coefficient for the stator yoke.

Due to the stator and rotor slotted structure and, correspondingly, high-frequency airgap harmonics, stray core losses occur: surface core losses (rotor and stator) and tooth flux pulsation core losses (rotor and stator).

The induced EMF due to the slot harmonics causes eddy currents in a thin surface layer of the teeth. The stator, $P'_{\text{surf},s}$, and rotor, $P'_{\text{surf},r}$, specific surface losses can be defined as [90]:

$$P_{\rm surf,s}' = k_{\rm lt} \left(\frac{Z_{\rm r}\omega}{2\pi \cdot 10000}\right)^{1.5} \left(10^3 B_{\delta 0} \tau_{\rm z,r}\right)^2, \qquad (5.87a)$$

$$P_{\rm surf,r}' = k_{\rm lt} \left(\frac{Z_{\rm s}\omega}{2\pi \cdot 10000}\right)^{1.5} \left(10^3 B_{\delta 0} \tau_{\rm z,s}\right)^2, \qquad (5.87b)$$

 k_{lt} - coefficient related to the lamination thickness, $B_{\delta 0}$ - amplitude of the flux density pulsations [T] in the airgap, defined as:

$$B_{\delta 0} = B_{\delta} k_{\delta} \beta_0, \tag{5.88}$$

where β_0 is a function of the ratio between the stator slot opening, $b_{o,s}$, and the airgap length, δ .

Another type of stray core losses is flux pulsation losses. They are caused by the pulsations of the flux in the teeth, and corresponding eddy currents induced in the teeth. The frequency of the pulsations on a particular side of the airgap corresponds to the mechanical angular speed and the slot number on the opposite side. The stator and rotor pulsation losses can be calculated as [90]:

$$P_{\rm pul,s} = 0.1 \left(\frac{Z_{\rm r}\omega}{2\pi \cdot 1000}\right)^2 B_{\rm pul,s}^2 m_{\rm z,s}, \tag{5.89a}$$

$$P_{\rm pul,r} = 0.1 \left(\frac{Z_{\rm s}\omega}{2\pi \cdot 1000}\right)^2 B_{\rm pul,r}^2 m_{\rm z,r},$$
 (5.89b)

(5.89c)

where $B_{\text{pul,s}}$ and $B_{\text{pul,r}}$ - amplitude of the flux density pulsations [T] in the middle cross-section of the stator and rotor tooth correspondingly:

$$B_{\rm pul,s} = \frac{\gamma_{\rm s}\delta}{2\tau_{\rm z,s}} B_{\rm z,s},\tag{5.90a}$$

$$B_{\rm pul,r} = \frac{\gamma_{\rm r}\delta}{2\tau_{\rm z,r}} B_{\rm z,r}.$$
(5.90b)

The windage losses, $P_{\delta,\text{wind}}$, take place mainly in the airgap resulting from the high peripheral speed of the rotor being around 130 m/s. They are roughly proportional to the cube of the rotor peripheral speed. In contrast to conventional machines, the windage losses in high-speed machines contribute a rather large part to the total losses.

The windage losses are defined by the gas (air) properties and its velocity field. The nature of a gas flow is determined by the ratio between the inertia and viscous forces, called the Couette Reynolds number, Re_{δ} , [93]:

$$Re_{\delta} = \frac{\rho_{\rm air} u_{\rm r} \delta}{\mu_{\rm air}},\tag{5.91}$$

where

 $\begin{array}{l} \nu_{\rm air} \mbox{ - dynamic viscosity of air, [Pa \cdot s],} \\ \rho_{\rm air} \mbox{ - air density, [kg/m^3],} \\ u_{\rm r} \mbox{ - peripheral speed of the rotor, [m/s].} \end{array}$

The windage loss in the airgap, $P_{\delta,\text{wind}}$, follows the equation:

$$P_{\delta,\text{wind}} = k_{\text{s}} C_{\text{f}} \rho_{\text{air}} \pi \omega^3 \left(\frac{D_{\text{r}}}{2}\right)^4 l_{\delta}, \qquad (5.92)$$

where

 $k_{\rm s}$ - surface coefficient, [-],

and the friction coefficient, $C_{\rm f}$, for turbulent flow is:

$$C_{\rm f} = 0.515 \frac{\left(\frac{2\delta}{D_{\rm r}}\right)^{0.3}}{Re_{\delta}^{0.5}}.$$
 (5.93)

In case of open-circuit cooling, when the cooling gas (air) passes through the airgap, additional acceleration losses take place in the airgap. The rotating rotor forces the cooling gas into a tangential movement. Assuming that the initial tangential velocity of the air is zero, the power needed for the acceleration is [93]:

$$P_{\delta,\mathrm{acc}} = \frac{2}{3}\pi\rho_{\mathrm{air}}u_{\mathrm{a}}u_{\mathrm{t}}\left(\left(\frac{D}{2}\right)^{3} - \left(\frac{D_{\mathrm{r}}}{2}\right)^{3}\right)\omega,\tag{5.94}$$

where

 $u_{\rm a}$ - axial velocity, [m/s],

 $u_{\rm t}$ - tangential velocity, [m/s].

The tangential velocity, u_t , is determined by the final velocity distribution in the airgap and proportional to the peripheral speed of the rotor:

$$u_{\rm t} = C_{\rm a}\omega \frac{D_{\rm r}}{2},\tag{5.95}$$

 $C_{\rm a} = 0.48$ - acceleration coefficient, [-].

It can be seen, that the acceleration losses, $P_{\delta,\text{acc}}$, are proportional to the air axial velocity, u_a , which is:

$$u_{\rm a} = \frac{m_{\rm c}}{\rho_{\rm air} S_{\delta}},\tag{5.96}$$

where

 $m_{\rm c}$ - cooling air mass flow, [kg/s].

The cross-section area of the airgap, S_{δ} , is:

$$S_{\delta} = \frac{\pi (D^2 - D_{\rm r}^2)}{4}.$$
(5.97)

From (5.94)-(5.97), it follows that the acceleration losses are mainly defined by the mass flow of the cooling air, m_c , and the airgap length, δ . It would introduce a set of contradictory criteria: a larger airgap length would result in reduced acceleration losses, but increased magnetizing current. A larger mass flow of the cooling fluid, which could be necessary for the heat removal, increases the losses. The thermal analysis, which takes into account the mentioned aspects, is presented in section 5.1.9.

The mechanical losses, P_{mech} , are caused by the friction in the bearings. For the preliminary design, the mechanical losses can be assumed 2% of the rated output power.

The efficiency of the motor is preliminary defined as a function of the output mechanical power, $P_{\rm d}$, and the sum of all losses in the machine, $\Sigma P_{\rm loss}$:

$$\eta_{\rm IM} = 100 \frac{P_{\rm d}}{P_{\rm d} + \Sigma P_{\rm loss}},\tag{5.98}$$

where

$$\Sigma P_{\text{loss}} = P_{\text{cu,s}} + P_{\text{cu,r}} + P_{\text{core}} + P_{\text{surf,s}} + P_{\text{surf,r}} + P_{\text{pul,s}} + P_{\text{pul,r}} + P_{\delta,\text{wind}} + P_{\delta,\text{acc}} + P_{\text{mech}}.$$
(5.99)

5.1.9 Thermal analysis

The goal of the thermal analysis is to estimate the temperature of the motor components, estimate the necessary mass flow of the cooling air and also investigate the influence of different design parameters on the thermal behavior. Due to the high loss density in high-speed electrical machines they usually require very effective cooling. The cooling requirements are dictated by the temperature rise of the stator winding and the rotor. The maximum allowable temperature is determined by the insulation class of the stator winding and the thermal rotor expansion.

Heat transfer mechanism

Heat is transferred to the environment by means of convection, conduction and radiation. Convection occurs at the interface between a solid and a fluid, which in electrical machines represents the cooling medium. The heat removal takes place due to the relative motion of the cooling medium and the surface. If the motion of the fluid is induced naturally by the density variation of the fluid as the result of temperature gradients, it is referred to as free convection. In case when the motion of the fluid, arising from externally imposed pressure differences, it is referred to as forced convection. The rate of heat transfer, $Q'_{t,conv}$, [W], by convection between the fluid and the boundary surface is evaluated by Newton's equation of cooling [17]:

$$Q'_{\rm t,conv} = \lambda_{\rm conv} S_{\rm surf} \Delta T_{\rm s,fl}, \qquad (5.100)$$

where

 $\lambda_{\rm conv}$ - convection heat transfer coefficient, [W/(m² · K)], $\Delta T_{\rm s,fl}$ - temperature difference between surface and fluid, [K], $S_{\rm surf}$ - surface area, [m²].

Consequently, the heat flux, $Q_{t,conv}$, $[W/m^2]$, is:

$$Q_{\rm t,conv} = \frac{Q'_{\rm t,conv}}{S_{\rm surf}}.$$
(5.101)

For the airgap, the λ_{conv} can be approximately derived as following [93]: Nusselt number, Nu, for the airgap flow is:

$$Nu = \frac{\lambda_{\rm conv}\delta}{\lambda_{\rm cond}},\tag{5.102}$$

where

 λ_{cond} - thermal conductivity of air, [W/(m · K)].

Due to rotation of the rotor, the tangential flow of the air takes place, which is characterized by the Taylor number, Ta:

$$Ta = \frac{\rho_{\rm air}^2 \omega^2 r_{\rm av} \delta^3}{\nu_{\rm air}^2},\tag{5.103}$$

where

 $r_{\rm av}$ - average stator and rotor radii, [m].

The rotor radius and the airgap length are taken into account by the modified Taylor number:

$$Ta_{\rm m} = \frac{Ta}{Fg},\tag{5.104}$$

where

Fg - geometrical factor, [-].

Since the airgap in electrical machines is very small, making $Fg \approx 1$ [93], $Ta \approx Ta_{\rm m}$.

The dependance of the Nusselt number on the modified Taylor number is a very complicated function of the fluid flow, the thermo-physical properties of the fluid and the geometrical arrangements of the system. Usually, it is quite impossible to obtain exact analytical solutions and approximations become necessary. In practice the dependance is evaluated from empirical equations obtained by correlating experimental results with methods of dimensional analysis.

For the designed induction motor, the modified Taylor number lies in the range $10^4 < Ta_m < 10^7$, corresponding to the turbulent flow, and the Nusselt number is [93]:

$$Nu = 0.409Ta_{\rm m}^{0.241}. (5.105)$$

Conduction is the dominant mechanism for transfer of heat through metallic and solid insulation materials. Heat conduction is described by a second order differential equation of the diffusion type and in many respects parallels the phenomenon of conduction of electricity. Materials with high electrical conductivity usually have also high thermal conductivity. The conduction heat transfer rate, $Q'_{t,cond}$, in one dimensional coordinate system, along the coordinate x, is described by Fourier's law [17]:

$$Q'_{\rm t,cond} = -\lambda_{\rm cond} S_{\rm m} \frac{dT}{dx}, \qquad (5.106)$$

where

 λ_{cond} - thermal conductivity of the material, [W/(m · K)], S_m - cross-sectional area, [m²],

or for the heat flux, $Q_{t,cond}$,

$$Q_{\rm t,cond} = -\lambda_{\rm cond} \frac{dT}{dx}.$$
(5.107)

The minus sign is necessary as heat is always transferred in the direction of decreasing temperature.

For steady state conduction, with no heat generation and constant thermal conductivity, the temperature varies linearly with x. Thus (5.106) can be rewritten in a simplified way:

$$Q_{\rm t,cond}' = \frac{\lambda_{\rm cond} S_{\rm m}}{l_{\rm m}} \Delta T_{\rm m}, \qquad (5.108)$$

where

 $\Delta T_{\rm m}$ - temperature difference across the block, [K], $l_{\rm m}$ - block length, [m].

Radiation is the process of heat transferring by means of electromagnetic waves. The power radiated per unit surface is proportional to the difference between the fourth powers of the absolute temperatures of the hot body and of the ambient:

$$Q_{\rm t,rad} = \sigma \varepsilon f_{\rm w} \left(T_1^4 - T_2^4 \right), \qquad (5.109)$$

where

$$\begin{split} \sigma &= 5.67 \cdot 10^{-8} \, \left[\mathrm{Wm}^{-2} \mathrm{K}^{-4} \right] \text{-} \mathrm{Stefan-Boltzmann \ constant}, \\ \varepsilon &- \mathrm{emissivity \ of \ the \ surface, \ [-],} \\ f_{\mathrm{w}} &- \mathrm{view \ factor, \ [-],} \\ T_1, \ T_2 &- \mathrm{absolute \ temperatures \ of \ the \ hot \ body \ and \ the \ ambient, \ [K].} \end{split}$$

In electrical machines the temperature rise over the ambient is rather low, about 70-100K. At such temperature difference, the energy dissipation due to the radiation are comparable with the free convection. However, in the investigated system the motor would normally be surrounded by surfaces having an equivalent temperature level. This leads to the mutual exchange of energy. Consequently, the radiation heat removal would not play a significant role and, thus, is neglected.

Thermal equivalent circuit and cooling scheme

Using (5.100) and (5.108), the analogy between thermal and electrical parameters can be evaluated. Defining thermal resistance as the ratio of a driving potential, i.e. the temperature difference to the corresponding transfer rate, i.e. heat transfer rate, it follows from (5.100) and (5.108):

$$R_{\rm conv} \equiv \frac{\Delta T_{\rm s,fl}}{Q'_{\rm t,conv}} = \frac{1}{\lambda_{\rm conv} S_{\rm surf}},\tag{5.110}$$

where

 $R_{\rm conv}$ - thermal resistance to convection, [K/W], and

$$R_{\rm cond} \equiv \frac{\Delta T_{\rm m}}{Q'_{\rm t,cond}} = \frac{l_{\rm m}}{\lambda_{\rm cond} S_{\rm m}},\tag{5.111}$$

 $R_{\rm cond}$ - thermal resistance to conduction, [K/W].

Thus, the heat transfer rate representing the power loss, P_{loss} , corresponds to an electrical current, the temperature difference to a voltage and the thermal resistance to an electrical resistance. The thermal analysis presented in this section is based on the thermal equivalent circuit (Fig. 5.9) taking into account the following assumptions:

- thermal flows in radial and axial directions are independent;
- for each machine component, only one average temperature defines a radial and axial heat flow;
- circumferential heat flow is not considered;
- heat generation is uniformly distributed.

The calculations are performed using the values of losses for the rated operating regime and the values of coefficients of heat conduction and heat convection defined for the particular part and surface of the machine. To incorporate multi-dimensional effects, each component of the machine is modelled by means of a network consisting of six resistors corresponding to either a cylindrical or a cuboidal block (Fig. 5.10). The corresponding thermal resistances for the cylindrical (5.112) and cuboidal (5.113) blocks are [17, 93]:



Figure 5.9: Thermal equivalent circuits for thermal flows in radial and axial directions



Figure 5.10: Component models and their equivalent circuits : (a) cylindrical, (b) cuboidal.

$$R_{1a} = R_{2a} = \frac{L}{2\pi\lambda_{\text{cond,a}} \left(r_1^2 - r_2^2\right)},$$
(5.112a)

$$R_{3a} = \frac{L}{6\pi\lambda_{\text{cond},a} \left(r_1^2 - r_2^2\right)},$$
(5.112b)

$$R_{1r} = \frac{1}{4\pi\lambda_{\text{cond,r}}L} \left[1 - \frac{2r_2^2 \ln\left(\frac{r_1}{r_2}\right)}{r_1^2 - r_2^2} \right], \qquad (5.112c)$$

$$R_{2\rm r} = \frac{1}{4\pi\lambda_{\rm cond,r}L} \left[\frac{2r_2^2 \ln\left(\frac{r_1}{r_2}\right)}{r_1^2 - r_2^2} - 1 \right], \qquad (5.112\rm d)$$

$$R_{3r} = \frac{-1}{8\pi \left(r_1^2 - r_2^2\right) \lambda_{\text{cond},r} L} \left[r_1^2 + r_2^2 - \frac{4r_1^2 r_2^2 \ln\left(\frac{r_1}{r_2}\right)}{r_1^2 - r_2^2} \right],$$
(5.112e)

$$R_{1i} = R_{2i} = \frac{L_i}{2\lambda_{\text{cond},i}S_{\text{m},i}},$$
 (5.113a)

$$R_{3i} = -\frac{1}{3}R_{1i},\tag{5.113b}$$

 $\begin{array}{l} L \mbox{ - block length, [m],} \\ r_1 \mbox{ - external radius, [m],} \\ r_2 \mbox{ - internal radius, [m],} \\ \lambda_{\rm cond,a,r} \mbox{ - thermal conductivity in axial or radial direction, [W/(m \cdot K)],} \\ \lambda_{\rm cond,i} \mbox{ - thermal conductivity along } x, y \mbox{ or } z \mbox{ coordinate, [W/(m \cdot K)],} \\ S_{\rm m,i} \mbox{ - block cross-sectional area normal to } x, y \mbox{ or } z \mbox{ coordinate, [m^2].} \end{array}$

The values of the temperatures in the equivalent thermal network are obtained using the node potentials method. These temperatures correspond to the temperature rise of the nodes with respect to that of the reference node e.g. ambient air. In this method, Kirchhoff's current law is used to write the power losses in every node. Then, the power losses are rewritten with the aid of the node temperature and the thermal resistances of the components.

For high-speed electrical machines the open-circuit scheme (Fig. 5.11) is considered as one of the most effective cooling methods. The cooling fluid flows through the machine e.g. airgap, significantly reducing the temperature of the inside air. It also easily removes the heat produced by windage losses in the airgap. Additionally, the rotor losses are effectively removed due to the high peripheral speed of the rotor.

The cooling air enters the machine at the temperature, T_{in} , which corresponds to the reference temperature. Inside the electrical machine the temperature rise of



Figure 5.11: Temperature of the cooling fluid.

the cooling fluid along an axial direction is assumed to be linear and the slope for each flow section ($T_{\rm ew,in}$ and $T_{\rm ew,out}$ correspond to the temperatures in the endwinding regions at the inlet and outlet sides, T_{δ} corresponds to the temperature in the airgap and $T_{\rm out}$ corresponds to the temperature of the outlet air) depends on the absorbed losses and mass flow of the cooling fluid. The air leaves the machine at the temperature, $T_{\rm out}$. The mean temperature rise, $\Delta T_{\rm i}$, for every section is defined as [93]:

$$\Delta T_{\rm i} = \frac{1}{2} \frac{P_{\rm i}}{m_{\rm c} c_{\rm p}},\tag{5.114}$$

where

 $c_{\rm p}$ - air specific heat at constant pressure, [J/(kg · K)], $P_{\rm i}$ - sum of the power loss in the node i, [W].

The resulting value of the temperature of a particular part of the machine must be below the defined limit. For example, the winding temperature, $T_{\rm w}$, should not exceed its maximum value, specified for a particular insulation class.

The necessary cooling air mass flow, m_c , can be approximately determined using the following simplified expression:

$$m_{\rm c} = \frac{\Sigma P_{\rm i,air}}{c_{\rm p} \Delta T_{\rm i,amb}},\tag{5.115}$$

where
$\Sigma P_{\rm i,air}$ - sum of losses removed by the internal air flow, [W],

 $\Delta T_{i,amb}$ - difference between internal and ambient temperatures, [K].

5.1.10 Critical speed

A critical speed is the rotational frequency of the rotor at which it coincides with the natural frequency of the rotor leading to excessive vibrations, including bending. The first bending critical speed is regarded is the most important. In electrical machines, the value of the critical speed must lie in a range far remote from the rated speed. In classical electrical machines that value is usually significantly higher than rated. However, in high-speed machines it can be difficult to achieve, since a high-value of a critical speed would require a thicker rotor or using a solid rotor magnetic core instead of laminated one.

A precise calculation of the critical speed would require rather detailed data related to a mechanical design and materials properties, which lies beyond a scope of the thesis. Nevertheless, the critical speed can be approximately estimated using simplified methods, widely used in engineering. The expressions for the critical speed are usually obtained from a lumped-parameter model of the shaft with a disc, and adapted for a typical construction of electrical machines using either detailed finite element methods or, more often, derived from experiments. For example, for a laminated rotor the following expression can be used [94, 95]:

$$\omega_{\rm c} = \sqrt{\left(\frac{D_{\rm sh}}{D_{\rm r}}\right)^4 \frac{3ED_{\rm r}^2}{\rho_{\rm st}l_\delta}},\tag{5.116}$$

where

 $\omega_{\rm c}$ - first bending critical speed, [rad/s], $E = 210 \cdot 10^9$ - Youngs modulus of the steel, [Pa].

However, a laminated rotor has a lower stiffness, than a solid rotor, consequently, in (5.116) the lower value of the Youngs modulus should be taken, e.g. 30 GPa [95].

The approximate estimation of the first bending critical speed can also be performed using the simplified empirical expression given in [96]:

$$\omega_{\rm c} = 8.85 \cdot 10^4 \frac{D_{\rm sh}'}{\sqrt{m_{\rm rot} l_{\rm rot}^3}},\tag{5.117}$$

where

 $D'_{\rm sh}$ - equivalent (average) shaft diameter, [m], $m_{\rm rot}$ - total rotor mass, [kg], $l_{\rm rot}$ - distance between bearings, [m]. This section presented the analytical procedure of the electromagnetic design and the thermal analysis of a high-speed induction motor. The next section describes the numerical Finite Element Analysis of the induction motor. The results of both analytical and FEM methods together with the results of the thermal analysis are presented in section 5.3.

5.2 Finite Element Analysis of the induction motor

For further motor investigation and its performance prediction including the transient electromagnetic process associated with the rotational movement, a detailed electromechanical model of the induction motor is implemented in the Maxwell 2D transient solver. This software module provides the possibility to solve coupled field, circuit and motion equations in the time-domain using the finite element method (FEM).

5.2.1 Field equations

The electromagnetic field calculations are usually based on the fundamental Maxwell equations (in differential form):

$$\nabla \cdot \boldsymbol{D} = \rho_{\rm ch} \quad (\text{Gauss's law}), \tag{5.118}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{5.119}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
 (Faraday – Lenz's law), (5.120)

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$
 (extended Ampere's law), (5.121)

where

- D electrical flux density vector, $[C/m^2]$,
- $\rho_{\rm ch}$ charge density, [C/m³],
- B magnetic flux density vector, [T],
- E electrical field strength vector, [V/m],
- H magnetic field strength vector, [A/m],
- J current density vector, $[A/m^2]$.

The relationship between the different field vectors is defined using the material properties:

$$\boldsymbol{D} = \boldsymbol{\varepsilon} \boldsymbol{E} = \boldsymbol{\varepsilon}_0 \boldsymbol{\varepsilon}_{\mathrm{r}} \boldsymbol{E}, \qquad (5.122)$$

$$\boldsymbol{B} = \boldsymbol{\mu} \boldsymbol{H} = \boldsymbol{\mu}_0 \boldsymbol{\mu}_r \boldsymbol{H},\tag{5.123}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E} = \frac{1}{\rho_{\rm E}} \boldsymbol{E},\tag{5.124}$$

where

Usually the Maxwell equations are not solved in terms of the field quantities. They are transformed into ODEs or PDEs using potential formulations [97, 98]. The $\varphi - A$ potential combination is used as the most appropriate one for a two dimensional problem.

The magnetic vector potential, \boldsymbol{A} , corresponds to magnetic field:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}.\tag{5.125}$$

Using the expression for \boldsymbol{B} in (5.120) gives:

$$\nabla \times \boldsymbol{E} + \frac{\partial}{\partial t} \nabla \times \boldsymbol{A} = 0.$$
 (5.126)

Assuming sufficient continuity of the fields to interchange the spatial and temporal differentiations, (5.126) can be written:

$$\nabla \times \left[\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} \right] = 0.$$
 (5.127)

The vector, $\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t}$, has zero curl and thus can be written as the gradient of a scalar potential, φ :

$$\boldsymbol{E} = -\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t}.$$
(5.128)

Equations (5.125) and (5.128) give the magnetic and electrical fields in terms of the vector potential, \boldsymbol{A} and the scalar potential, φ . Substituting the obtained expressions into (5.121) results in:

$$\frac{1}{\mu}\nabla \times \nabla \times \boldsymbol{A} = \boldsymbol{J}_{\text{source}} - \varepsilon \nabla \left(\frac{\partial \varphi}{\partial t}\right).$$
(5.129)

The last term in (5.129) is the displacement current density, which can often be neglected at low frequencies.

In the case of a two dimensional problem, the magnetic vector potential, A, and the current density vector, J, contain only a z-component. Thus, the magnetic field equations become scalar PDEs. Finally, the electromagnetic problem with

motion involved is described by the following time-dependent magnetic diffusion equation [99]:

$$\frac{1}{\mu}\nabla\times\nabla\times A = J_{\text{source}} - \sigma \frac{\partial A}{\partial t} + \sigma u \times \nabla \times A, \qquad (5.130)$$

where

 J_{source} - source current density, [A/m²], u - relative velocity of moving parts, [m/s].

Further simplification is achieved using the approach, when the reference frame is fixed with respect to the investigated components, thus eliminating the component containing the relative velocity, u. Finally, (5.130) is reduced to:

$$\frac{1}{\mu}\nabla \times \nabla \times A = J_{\text{source}} - \sigma \frac{\partial A}{\partial t}, \qquad (5.131)$$

The source and induced (or eddy) currents contribute to the total current density, which is important for the calculation of the Joule loss:

$$J_{\text{tot}} = J_{\text{source}} + \sigma \frac{\partial A}{\partial t} = \frac{\sigma}{l_{\delta}} V_{\text{b}} + \sigma \frac{\partial A}{\partial t}.$$
 (5.132)

where

 $V_{\rm b}$ - voltage across the conductor, [V].

Equation (5.131) is applied for the main machine regions: the airgap and the magnetic core, the stator winding and solid conductors or rotor bars [100]. For the region of the airgap and the magnetic core (5.131) becomes:

$$\frac{1}{\mu}\nabla \times \nabla \times A + \sigma \frac{\partial A}{\partial t} = 0.$$
(5.133)

In the stranded windings the skin effect is neglected together with a contribution of the eddy currents to the current density over the area of the winding [100]. Thus, in the region of the stranded windings (5.131) can be expressed as:

$$\frac{1}{\mu}\nabla \times \nabla \times A - \frac{d_{\rm f}n_{\rm el}}{S_{\rm eff}a}i_{\rm s} = 0, \qquad (5.134)$$

where

 $d_{\rm f}$ - polarity (+1 or -1), which represents forward or return path,

and in the solid conductors (for one conductor):

$$\frac{1}{\mu}\nabla \times \nabla \times A + \sigma \frac{\partial A}{\partial t} - \frac{d_{\rm f}\sigma}{l_{\delta}}V_{\rm b} = 0.$$
(5.135)

The current density in the solid conductor is:

$$J_{\rm bar} = -d_{\rm f}\sigma \frac{\partial A}{\partial t} + \frac{\sigma}{l_{\delta}}V_{\rm b}.$$
 (5.136)

The resistances and inductances of the end parts of the stator windings and rotor conductors are included in the external circuits.

5.2.2 Circuit equations

The field equations are coupled with the circuit equations and solved simultaneously. For simplicity, the circuit equations of the stranded windings and the solid conductors include only the parts located in the iron stack. In practice, each winding and solid conductor might be connected to a voltage source and have a resistance and an inductance in series, which are expressed as equivalent values derived from external circuits. The stranded windings and solid conductors can be represented by a back EMF, e, and the ohmic voltage drop. For stranded windings, the effect of the ohmic voltage drop corresponds to the external resistance. Thus, the electrical components associated with a voltage source constitute a simple electrical circuit [101]:

$$[R]\{i_{\rm w}\} + [L]\frac{d\{i_{\rm w}\}}{dt} + \{e\} = \{v_{\rm source}\},$$
(5.137)

where

 $\begin{array}{l} [R] \mbox{-} resistance matrix, $[\Omega]$, $$ [L] - inductance matrix, $[H]$, $$ {$v_{\rm source}$}$ - voltage sources column matrix, $[V]$, $$ {e}$ - induced back EMF column matrix, $[V]$. } \end{array}$

The induced back EMF, e, provides the magnetic coupling with the electrical circuit:

$$e = d_{\rm f} \frac{n_{\rm el} l_{\rm w}}{S_{\rm eff} a} \int\limits_{S_{\rm w}} \frac{dA}{dt} dS_{\rm w}.$$
(5.138)

For the squirrel-cage rotor the solid conductor circuit equations are applied:

$$[R_{\rm er}]\{i_{\rm r}\} + [L_{\rm er}]\left\{\frac{di_{\rm r}}{dt}\right\} = -[M][M]^{\rm T}\{V_{\rm b}\}, \qquad (5.139a)$$

$$\{J_{\rm r}\} = -\sigma \left\{\frac{dA}{dt}\right\} + \frac{\sigma}{l_{\rm bar}}\{V_{\rm b}\},\tag{5.139b}$$

where

 $\{J_r\}$ - column matrix of the rotor bar current density, $[A/m^2]$, $[L_{er}]$ - matrix of the end ring inductance, [H], [M] - connection bar matrix, [-].

The field equations are further discretized in the space domain by applying the Galerkin method. Next, the field and circuit equations are discretized in the time domain. It results in a global equation system with the following unknowns: nodal values of the vector potential, the currents in the winding, the values of the electrical scalar potential between two ends of the solid conductor. Further, the obtained system of equations is solved by the Newton-Raphson method [101].

5.2.3 Motion equations

The motion equation is written as:

$$J_{\rm m}\frac{d\omega}{dt} = T_{\rm d} - T_{\rm load} - K_{\rm D}\omega, \qquad (5.140)$$

where

 $K_{\rm D}$ - damping coefficient, [Nm · s/rad], $T_{\rm load}$ - load torque, [Nm].

At each time step, the electromagnetic torque is computed employing the method of virtual work. Solving the equation of motion allows to obtain the rotor angular acceleration and, consequently angular displacement of the rotor. The motion of the rotor is introduced by means of independent meshing of two parts of the motor - stationary (stator with slots and windings) and movable (rotor with bars and a shaft) and the coupling of them at the interface. Such a technique allows to avoid re-meshing due to the movement [101].

5.3 Results of the analytical design and numerical modeling

Using the procedure described in section 5.1, the global design of the high-speed 30 kW induction motor, specified in Table 5.2 has been performed. The principal motor design parameters, obtained from the calculations, are given in Table. 5.3. Major electromagnetic and thermal design results are presented in Table 5.4.

The results of the analytical method show the expected motor performance characteristics. According to the thermal analysis, the average temperature of the stator winding does not exceed the limit of the insulation of class F. The first bending critical speed of the machine is much below the rated speed. However, if in a practical application, the critical speed is required to be outside of the operating speed range, the machine can be redesigned with respect to the rotor construction. As it seen from (5.116) and (5.117), the critical speed is significantly influenced by the ratio

Rotor diameter, $D_{\rm r}$, [mm]	70
Stator outside diameter, $D_{\rm s}$, [mm]	150
Shaft diameter, $D_{\rm sh}$, [mm]	25
Airgap length, δ , [mm]	0.5
Number of stator slots, Z_s , $[-]$	24
Number of rotor slots, $Z_{\rm r}$, $[-]$	20
Skew coefficient (rotor), k_{skew} , [-]	0.8
Core length, l_{δ} , [mm]	140
Rotor inertia, $J_{\rm m}$, $[\rm kgm^2]$	0.0024

Table 5.3: Principal motor design parameters of the high-speed 2-pole, 30 kW IM.

Table 5.4: Calculated electromagnetic, mechanical and thermal parameters of the high-speed 2-pole, 30 kW IM.

Parameter	Value
Stator rms current, $I_{\rm s}$, [A]	38
Stator copper loss, $P_{cu,s}$, [W]	201
Rotor copper loss, $P_{cu,r}$, [W]	43
Main core loss, $P_{\rm core}$, [W]	723.5
Stray core loss, $P_{\text{surf}} + P_{\text{pul}}$, [W]	238
Mechanical loss, P_{mech} , [W]	600
First bending critical speed, $\omega_{\rm c}$, [Hz]	170
Stator winding temperature rise over $T_0, T_w, [^{\circ}C]$	75.5
Motor efficiency, η_{IM} , [%]	94.0
Power factor, $\cos(\phi)$, [-]	0.74

between the shaft and the rotor diameters. With respect to that, the shaft diameter can be increased. Additionally, the shaft can be made of magnetic steel, which would allow to make a narrow rotor yoke and, consequently, keep the rotor diameter low. In this case, a special attention should be paid to the losses calculation in the rotor and bearings.

In addition to the design parameters, the performance characteristics of the IM have been derived based on the Γ - equivalent circuit (Fig. 5.6). The performance characteristics of the designed motor have also been verified using the Ansoft Maxwell 2D FEM software package. Figs. 5.12 - 5.13 present the characteristics of the designed IM obtained from the analytical calculations (solid lines) and FEM (dashed lines).



Figure 5.12: Performance characteristics of the high-speed 2-pole, 30 kW IM (solid line - analytical results, dashed line - FEM results): (a) Torque vs. slip, (b) Stator current vs. slip.



Figure 5.13: Performance characteristics of the high-speed 2-pole, 30 kW IM (solid line - analytical results, dashed line - FEM results): (a) Efficiency vs. slip, (b) $cos(\phi)$ vs. slip.

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Additionally, as FEM results the magnetic flux lines and magnetic flux density distribution are presented in Figs. 5.14(a) and 5.14(b), respectively. It can be seen that the maximum flux density (higher 2 T) occurs in the steel bridges on top of the rotor bars as expected.



Figure 5.14: FEM results for the high-speed 2-pole, 30 kW IM: (a) Magnetic flux lines, (b) Magnetic flux density distribution.

In Table 5.5 the obtained FEM results for the main performance variables are compared with the analytical ones. It can be seen that both design results match quite well. However, there is a noticeable difference in the obtained results for the power factor and the stator phase current. These differences appear due to several reasons. First, the calculations related to the nonlinear magnetic circuit analysis are normally performed more accurately by a numerical method. In the FEM eddycurrents and skin effects are also easier taken into account. On the other hand, in the analytical method the influence of technological inaccuracy can be incorporated. In general, for an optimal design of a high-speed IM, a combination of analytical design methods, FEM tools and a proper optimization technique should be applied.

Parameter	Analytical	FEM
Stator rms current, $I_{\rm s}$, [A]	38	35.4
Motor efficiency, η_{IM} , [%]	94	95
Power factor, $cos\phi$, [-]	0.74	0.78

Table 5.5: Comparison of main analytical and FEM results.

5.4 Analysis of the high-speed induction motor selected for the experiments

A high-speed direct drive would be a favorite solution for the active surge control experiment due to its good dynamic properties. Due to time and resources limitations within the project the complete design and manufacturing of the direct-drive system have not been performed. Moreover, a standard automotive centrifugal turbocharger has been used in the experiment. This compressor is equipped with a gearbox with the ratio 3.45 coupled to the impeller shaft. Thus, the required motor speed becomes $36000/3.45 \approx 10000 \ rpm$. Finally, the available high-speed 2-pole, 30 kW induction motor shown in Fig. 5.15 has been chosen as the compressor drive for the full-scale laboratory experiment. Its design parameters are specified in Table 5.6.



Figure 5.15: High-speed 2-pole, 30 kW induction motor selected for the experiments.

Prior to the experiments the performance of the induction motor has been analyzed using the straightforward analysis flow-chart, shown in Fig. 5.16, which is based on the design method, described in sections 5.1 and 5.2.

The geometric data, the stator winding and rotor bar arrangements, the material properties and the main mechanical and electrical data are entered as input parameters. The motor has also been modelled by means of the Maxwell 2D FEM software.

Output power, $P_{\rm d}$, [kW]	30
Terminal line-to-line rms voltage, $V_{\rm s,ll}$, [V]	500
Frequency, $f_{\rm s}$, [Hz]	300
Stator rms current, $I_{\rm s}$, [A]	47
Over-current ratio, $[-]$	6.88
Rated speed, ω , [rpm]	17900
Power factor, $\cos\phi$, [-]	0.82
Motor efficiency, η_{IM} , [%]	89.9
Rotor diameter, $D_{\rm r}$, [mm]	110
Stator outside diameter, $D_{\rm s}$, [mm]	200
Airgap length, δ , [mm]	0.5
Number of stator slots, $Z_{\rm s}$, [-]	36
Number of rotor slots, $Z_{\rm r}$, $[-]$	30
Skew coefficient, k_{skew} , [-]	0.8
Core length, l_{δ} , [mm]	120
Rotor inertia, $J_{\rm m}$, [kgm ²]	0.020

Table 5.6: Design data of the high-speed 2-pole, 30 kW IM selected for the experiments.

Two operating modes have been investigated. First, the rated operating mode at 30 kW output power and about 18000 rpm rotor speed. Secondly, the operating mode required by the experimental set-up, i.e. 30 kW output power and about 9000 rpm rotor speed. The respective performance characteristics are shown in Figs. 5.17 - 5.18 and 5.19 - 5.20, wherein the solid lines represent the analytical results and the dashed lines the FEM results.

As can be seen in the presented figures, the value of the rated slip obtained using FEM is located to the left of the analytical value. A possible reason is that the rotor bar behaves as a bar in a double cage. In that case considerable skin effect appears in the rotor bar, resulting in an increased rotor resistance that moves the break-down torque to the left. It should be emphasized that the skin effect has not comprehensively been studied in the analytical design method.

Finally, the thermal analysis showed the following average stator winding temperatures: 114 °C at rated operating mode and 138 °C at reduced speed. These values are at the edge and, respectively beyond the maximum allowable temperature of the stator winding insulation (class F). As a result, water cooling and forced ventilation of the motor are recommended, especially at the operating mode with reduced speed and high torque.



Figure 5.16: Analysis flow-chart for the high-speed 2-pole, 30 kW IM selected for the experiments.



Figure 5.17: Performance characteristics of the high-speed 2-pole, 30 kW IM selected for the experiments ($f_s = 300 \text{ Hz}$)(solid line - analytical results, dashed line - FEM results): (a) Torque versus slip, (b) Stator current versus slip.



Figure 5.18: Performance characteristics of the high-speed 2-pole, 30 kW IM selected for the experiments ($f_s = 300 \text{ Hz}$)(solid line - analytical results, dashed line - FEM results): (a) Efficiency versus slip, (b) $\cos(\phi)$ versus slip.



Figure 5.19: Performance characteristics of the high-speed 2-pole, 30 kW IM selected for the experiments ($f_s = 150 \text{ Hz}$)(solid line - analytical results, dashed line - FEM results): (a) Torque versus slip, (b) Stator current versus slip.



Figure 5.20: Performance characteristics of the high-speed 2-pole, 30 kW IM selected for the experiments ($f_s = 150 \text{ Hz}$)(solid line - analytical results, dashed line - FEM results): (a) Efficiency versus slip, (b) $cos(\phi)$ versus slip.

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5.5 Conclusions

This chapter presents the analytical electromagnetic design procedure and thermal analysis of high-speed induction motors for compressor applications. A rather good agreement between the results of the analytical calculations and those of the FEM confirms that the presented method is suitable for the preliminary design and analysis of high-speed induction motors.

The results obtained satisfy the initially specified motor requirements which have been derived from the compressor system analysis. According to the thermal analysis the temperatures of the main components of the machine stay within appropriate limits. Furthermore, for the given rated motor power and speed (30 kW, 36000 rpm), the rotor inertia, $J_{\rm m} = 0.0021$ kgm², which corresponds to a rather low value of the electromechanical time constant, as required for high controllability.

As far as the high-speed induction motor selected for the experiments is concerned, its performance has also been analyzed analytically and by means of the FEM. The main attention has been given to the operating mode at reduced speed and correspondingly higher torque. From the thermal analysis it follows that the combination of forced ventilation and water cooling is recommended.

CHAPTER 6

Experimental results

The primary goal of the experiments is the verification of the compressor active surge control by means of torque/speed control of the electrical drive. Prior to the final experiment on the surge control, a set of system identification tests has been performed. The first experiment is aimed at the identification of the parameters of the equivalent circuit of the high-speed induction motor with further implementation of the field-oriented control. Next, the static compressor performance map and the equivalent parameters of the transient model of the compression system are determined. Finally, the compressor active surge control in combination with the field-oriented control of the IM has been implemented and tested.

6.1 Experimental set-up and measurement equipment

The experiments have been carried out on the laboratory test set-up shown in Fig. 6.1. The main components of the set-up are: the high-speed IM drive, the power electronic converter (Semikron), the centrifugal compressor (Vortech) with an embedded gearbox of ratio 1:3.45 and the PVC piping system. The data acquisition (DAQ) and control algorithms have been realized on the dSpace DS1104 controller board. The "turbo-machinery" part of the installation shown in Fig. 6.2 has been built in collaboration with the Energy Technology Group of the TU/e-Mechanical Engineering Department.



Figure 6.1: Laboratory experimental test set-up.



Figure 6.2: Basic topology of the piping system with the compressor.

6.2 Standard IM parameter identification tests

The initial set of tests aimes at the experimental determination of the major parameters of the induction motor. The knowledge of the parameters of the IM equivalent circuit (Fig. 5.5(a)) is necessary for the implementation of the highperformance field-oriented control, as well as for the system simulation and analysis. In the present work, the most widely used methods have been executed: measurement of the DC resistance of the stator winding, no-load and locked-rotor tests in correspondence with the IEEE standard 112-1996 [102].



Figure 6.3: DC test scheme.

6.2.1 DC test

The purpose of the DC test is to determine the value of the stator winding resistance. This test is accomplished by connecting two 'Y' - connected stator phases to a variable voltage DC supply (Fig. 6.3). The value of the resistance, R_s , is determined from the measured voltage, $V_{s,DC}$, and current, $I_{s,DC}$:

$$R_{\rm s} = \frac{V_{\rm s,DC}}{2I_{\rm s,DC}}.\tag{6.1}$$

Due to the limited value of the maximum current level of the DC supply (4.5 A), the rated motor current value (47A) could not be achieved. During the experi-



Figure 6.4: Measurement results in the DC test: (a) $V_{\rm s,DC}$ versus $I_{\rm s,DC}$, (b) $R_{\rm s}$ versus $I_{\rm s,DC}$.

ments, the supply voltage is gradually increased till the maximum current level was achieved. Further, the recorded data have been fitted with a linear function:

$$V_{\rm s,DC}^{\rm fitt} = 0.11798 * I_{\rm s,DC}^{\rm fitt} + 0.088577.$$
 (6.2)

Substituting the rated current value $(I_{\rm s}^{\rm rat} = 47 \text{ A})$ into (6.2) and (6.1), the value of the stator phase resistance is obtained:

$$R_{\rm s} = \frac{V_{\rm s, DC}^{\rm fitt} \left(I_{\rm s}^{\rm rat}\right)}{2I_{\rm s}^{\rm rat}}.$$
(6.3)

The value of the stator phase resistance equals $0.06 \ \Omega$. The measurement results are presented in Figs. 6.3 and 6.4, respectively and in Appendix B.1.

6.2.2 No-load test

The no-load test is aimed at the estimation of the values of the magnetizing reactance, $X_{\rm m}$. It also gives information on the magnetizing current and rotational losses. The no-load test is performed with the motor shaft decoupled from any mechanical load with rated three-phase excitation applied to the motor input terminals. The rotational speed is very close to the synchronous one ($\omega \approx \frac{2\pi f_s}{p}$, $s \approx 0$), thus the secondary circuit parameters are neglected. The equivalent circuit for the no-load test is shown in Fig. 6.5.

From the measured complex values of stator voltage and current the no-load impedance, \bar{Z}_{nl} , is determined:



Figure 6.5: IM equivalent circuit for no-load test.

$$\bar{Z}_{\rm nl} = \frac{\bar{V}_{\rm s}}{\bar{I}_{\rm s}}.\tag{6.4}$$

Assuming $R_{\rm m} >> X_{\rm m}$, the no-load impedance, $\bar{Z}_{\rm nl}$, is:

$$\bar{Z}_{nl} = R_s + j(X_{nl}).$$
 (6.5)

The no-load reactance, X_{nl} , is:

$$X_{\rm nl} = X_{\rm m} + X_{\rm ls} = \sqrt{\left(\frac{V_{\rm s}}{I_{\rm s}}\right)^2 - R_{\rm s}^2},$$
 (6.6)

where

 $X_{\rm ls}$ - leakage reactance of a stator phase winding, $[\Omega]$, $X_{\rm m}$ - magnetizing reactance, $[\Omega]$.

The no-load reactance, $X_{\rm nl}$, is obtained from the curve of $X_{\rm nl}$ versus $V_{\rm s}$. The values located at the right-hand side of the maximum, i.e. at higher voltage values, correspond to high saturation of the magnetic core. And, at low voltage values, the influence of the friction and windage loss becomes significant. Thus, the highest point on this curve is used as the value of $X_{\rm nl}$.

The small power loss in the machine at no load arises due to core, friction and windage losses. Separation of the core loss from the friction and windage loss is done using the standard procedure [102]. The motor is supplied at 50 Hz and a rms voltage value ranging from approximately 125 percent of the corresponding rated value (constant V/Hz curve) down to the point where further voltage reduction increases the current. The curve extended to zero voltage, which is obtained by substraction of $I_s^2 R_s$ from the total input power, $P_{\rm in}$, indicates the windage and friction loss.

The no-load measurement results are summarized in Table 6.1 and Figs. 6.8 - 6.9, and in Appendix B.2.



Figure 6.6: Measurement circuit for no-load and locked-rotor tests.

Table 6.1: Summary of the high-speed IM no-load test results at 50 Hz (per phase).

$V_{\rm s},$	$I_{\rm s},$	$P_{\rm in},$	$P_{\rm in} - I_{\rm s}^2 R_{\rm s},$	$X_{\rm ls} + X_{\rm m},$	$L_{\rm ls} + L_{\rm m},$
[V]	[A]	[W]	[W]	$[\Omega]$	[mH]
18.551	4.359	25.792	24.654	4.255	13.544



Figure 6.7: Measurement results of the no-load test (50 Hz): (a) $P_{\rm in}$ versus $V_{\rm s},$ (b) $I_{\rm s}$ versus $V_{\rm s}.$



Figure 6.8: Measurement results of the no-load test (50 Hz): (a) $P_{\rm in} - I_{\rm s}^2 R_{\rm s}$ versus $V_{\rm s}$, (b) $P_{\rm in} - I_{\rm s}^2 R_{\rm s}$ versus $V_{\rm s}$ (zoomed).



Figure 6.9: Measurement results of the no-load test (50 Hz): (a) $X_{\rm s}+X_{\rm m}$ versus $V_{\rm s},$ (b) $L_{\rm ls}+L_{\rm m}$ versus $V_{\rm s}.$

6.2.3 Locked-rotor test

The locked-rotor test on an induction machine gives information on the leakage reactances and the rotor resistance, referred to the stator. In this test, the rotor is locked, and reduced balanced polyphase voltages are applied to the stator terminals. The locked-rotor test should be performed under the same conditions of the rotor current and frequency that will prevail in the normal operation. For example, if the performance characteristics in the normal condition (i.e. low-slip) are required, the locked-rotor test should be performed at a reduced voltage and rated current. The frequency should also be reduced because the rotor effective resistance and leakage reactance at the reduced frequency (corresponding to lower values of slip) may differ appreciably from their values at the rated frequency. This will be particularly true for double-cage or deep-bar rotors. The IEEE recommends a frequency of 25 % of the rated frequency for the locked-rotor test [102], which makes it useful to use the 50 Hz grid supply for the high-speed IM. The leakage reactances at the rated frequency can then be obtained by considering that the reactance is proportional to the frequency.

Due to the condition ($\omega = 0, s = 1$) the magnetizing branch of the equivalent circuit is neglected. The equivalent circuit for the locked-rotor test is shown in Fig. 6.10.



Figure 6.10: IM equivalent circuit for locked-rotor test.

From the measurements the locked-rotor impedance, related to the values of the total leakage reactance and the resistances of the stator and the rotor, is determined:

$$\bar{Z}_{\rm br} = \frac{\bar{V}_{\rm s}}{\bar{I}_{\rm s}},\tag{6.7}$$

where

$$\bar{Z}_{\rm br} = (R_{\rm s} + R'_{\rm r}) + j (X_{\rm ls} + X'_{\rm lr}).$$
 (6.8)

The total leakage reactance is:

$$X_{\rm ls} + X'_{\rm lr} = \sqrt{\left(\frac{V_{\rm s}}{I_{\rm s}}\right)^2 - \left(R_{\rm s} + R'_{\rm r}\right)^2}.$$
 (6.9)

The rotor resistance, $R'_{\rm r}$ is:

$$\frac{P_{\rm in}}{I_{\rm s}^2} - R_{\rm s},$$
 (6.10)

where $P_{\rm in}$ - active input power of the induction motor, [W].

The locked-rotor measurement results are summarized in Table 6.2, Fig. 6.11 and in Appendix B.3.

Table 6.2: Summary of the high-speed IM blocked-rotor test results.

$V_{\rm s}, \ [V]$	$I_{\rm s}, \ [A]$	$X_{\rm ls} + X'_{\rm lr}, \ [\Omega]$	$L_{\rm ls} + L'_{\rm lr}, \ [mH]$	$R'_{ m r},~[\Omega]$
11.481	47.717	0.130	0.415	0.142



Figure 6.11: Measurement results of the locked-rotor test (50 Hz): (a) $I_{\rm s}$ versus $V_{\rm s}$, (b) $L_{\rm ls} + L'_{\rm lr}$ versus $V_{\rm s}$.

Combining the results of all three tests and assuming that $X_{\rm ls} = X'_{\rm lr}$ all equivalent circuit parameters can be derived, as summarized in Table 6.3.

Table 6.3: Equivalent circuit parameters of the high-speed IM at 50 Hz.

$R_{\rm s}, \ [\Omega]$	$L_{\rm ls}, [mH]$	$L'_{\rm lr}, [mH]$	$L_{\rm m}, [mH]$	$R'_{\rm r}, \ [\Omega]$
0.06	0.207	0.207	13.479	0.142

6.3 Identification of the compressor performance map

The next phase of the experimental work is dedicated to measurements on the compression system. First, measurements have been performed to determine the overall performance and efficiency of the compressor and to detect the surge limit. Five values have been measured (Fig. 6.12): pressure, p_p , mass flow, m, temperature of the pressurized air, T_2 , ambient temperature, T_0 , and speed of the motor, ω . The compression system has been equipped with a pressure transducer, a Pitot tube for steady-state mass flow measurement and temperature probes. The angular speed of the motor is measured with a laser speed transducer.

With the aim to perform accurate measurements of the mass flow, the piping system has been configured as illustrated in Fig. 6.12. The valve is placed in a flange, located next to the compressor, and the Pitot tube has been placed beyond the valve, where the air flow is less turbulent. The details of the operation principle of the Pitot tube can be found in Appendix C.

The compressor performance characteristics are measured starting at the maximum distance from the surge line, corresponding to an opened valve. Then, the compressor mass flow is reduced by gradually closing the valve to move the operating point of the system towards the surge line. The surge initiation point corresponds to the point where the amplitude of the pressure oscillations starts to grow. The steady-state compressor characteristics have been obtained for seven speed levels and six valve positions for each speed. The measurement results are summarized in Appendix D.



Figure 6.12: Measurement scheme for the steady-state performance characteristics of the compressor.

Based on the obtained measurement results, the mathematical model (3.17) of the compressor has been corrected. The coefficients in (3.17) corresponding to the friction, $(k_{\rm f})$, and incidence, $(k_{\rm ins})$, loss components have been adjusted to fit the calculated performance characteristics to the measured ones. It allows to approximate the performance characteristics of the compressor in the unstable area, for which the experimental measurements are not available. The measured and calculated compressor performance map is presented in Fig. 6.13, wherein the curve corresponding to $\omega_{\rm imp} = 25000$ rpm is omitted



Figure 6.13: Compressor performance map with indicated values of the isentropic efficiency (solid lines - measured; dashed lines - calculated).

From the obtained data, the efficiency of the compressor is defined in the following way. The compressor operation is based on the expression of constant entropy:

$$\frac{T_2'}{T_0} = \left(\frac{p_{\rm p}}{p_0}\right)^{\frac{\gamma-1}{\gamma}},\tag{6.11}$$

where

 T'_2 - outlet temperature (isentropic), [K].

The work delivered to the gas is equal to the change in enthalpy of the gas:

$$W = c_{\rm p} \left(T_2 - T_0 \right) m, \tag{6.12}$$

However, the isentropic work is:

$$W' = c_{\rm p} \left(T_2' - T_0 \right) m. \tag{6.13}$$

In practice, the temperature rise $(T_2 - T_0)$ is higher than $(T'_2 - T_0)$ due to various losses in the compressor. Consequently, the isentropic efficiency of the compressor, η_c is expressed as the ratio of the real and isentropic work:

$$\eta_{\rm c} = \frac{\text{isentropic work}}{\text{real work}} = \frac{W'}{W} = \frac{T'_2 - T_0}{T_2 - T_0}.$$
(6.14)

Combination of (6.11) and (6.14) results in:

$$\eta_{\rm c} = \frac{T_0}{T_2 - T_0} \left[\left(\frac{p_{\rm p}}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right].$$
(6.15)

The values of the isentropic efficiency are also indicated in the map of Fig. 6.13.

6.4 Identification of the lumped-parameter Greitzer model

From the previous tests, the steady-state compressor performance map has been obtained. The next step is to determine the transient characteristics of the compression system, mathematically described by the lumped-parameter Greitzer model (Fig. 3.9). The major parameters of the Greitzer model correspond to the geometry of the system: plenum volume, $V_{\rm p}$, equivalent compressor duct length, $L_{\rm c}$, and area of the impeller eye, $A_{\rm c}$.

For transient measurements and further surge experiments, the compression system has been modified: the position of the back-pressure valve has been moved to the outlet of the system (Fig. 6.14). This increases the plenum volume, $V_{\rm p}$, and consequently results in a reduced surge frequency.

The initial values of the Greitzer lumped parameters are defined from the geometrical sizes of the compressor and the piping system. In practice, the area of the impeller eye, A_c , the duct length, L_c , and the plenum volume, V_p , are not strictly defined by the system geometry and their values should be corrected according to the dynamic test results. The parameters are tuned in such a way, that the amplitude and the frequency of the pressure oscillation fairly coincide. Figure 6.15 presents the simulated and measured results for a pressure oscillation during deep surge. The original and recalculated parameters of the Greitzer model are collected in Table 6.4. It can be seen, that the duct length and, consequently the plenum volume parameters increased by approximately a factor of two, compared to their physical values. A suggested reason for the mismatch can be the shape of the real compression system, where the duct and the plenum are not strictly distinguished, like it is assumed in the Greitzer model.

The obtained steady-state and transient models of the complete compression system allow to use them as a mass flow observer for the surge control.



Figure 6.14: Measurement scheme for the surge control.



Figure 6.15: Pressure plots for Greitzer model identification pressure plots: upper - original model values; lower - adjusted model values (solid lines measured results, dashed lines - simulated results).

 Parameter
 Original
 Recalculated

 $V_{\rm p}$, $[{\rm m}^3]$ 0.0140
 0.0319

 $A_{\rm c}$, $[{\rm m}^2]$ 0.0064
 0.0064

 $L_{\rm c}$, $[{\rm m}]$ 2.20
 5.0160

Table 6.4: Lumped parameters of the Greitzer model.

6.5 Implementation and verification of the active surge control

The last test is dedicated to the implementation and verification of the active surge control of the compressor by means of the field-oriented controlled induction motor. The experiment has been performed at a rather low pressure level to reduce the harmful effects of the surge on the compressor.

As described in section 3.3.3, the key aspect of the surge control is to control the angular speed of the impeller proportionally to the deviation of the mass flow from the equilibrium point. In its turn, the torque control of the motor should provide an appropriate speed response. The indirect field-oriented controller described in section 4.3.3 has been implemented using the values of the equivalent circuit parameters obtained from the motor tests, described in section 6.2.

The feedback gain for the mass flow, $K_{\rm m}$, (3.32) is defined according to the procedure described in section 3.3.3.

A band-pass filter has been used to reduce the impact of the high-frequency noise in the measured pressure signal:

$$\frac{Ks^2}{(s+\omega_1)^2(s+\omega_2)^2},\tag{6.16}$$

where

s - Laplace operator, $K = 4 \cdot 10^4$ - gain, [-], $\omega_1 = 2\pi 10, \omega_2 = 2\pi 22$ - circular frequencies, [rad/s].

The Bode diagram of the filter is presented in Fig. 6.16. A high-level differential action has been added to the controller to compensate the phase lag caused by the inertia of the rotor of the motor, shaft coupling, gearbox and compressor impeller. At the surge frequency ($\approx 12Hz$) the phase lead of the filter is approximately 40°.

The test is done in the following steps. The speed of the compressor is set to the value corresponding to the pressure of 1.3 bar. Thus, the first operating point (an intersection of the compressor characteristic and the load curve I) corresponds to the opened valve, and it is located within the stable area (Fig. 6.17). Next, the valve is gradually closed towards the position where the surge is initialized; the load curve is moved towards the unstable area.





Figure 6.16: Bode diagram of the band-pass filter.

When the operating point is moved beyond the surge limit, i.e. being in the unstable area on load curve II, the surge phenomenon appears (Fig. 6.17). Then, the feedback control gain, $K_{\rm m}$, is gradually increased initializing the controller. As can be seen in Fig. 6.17 and Fig. 6.19 the low-frequency, large-amplitude deep surge oscillations are successfully damped. The effect of the surge suppression can also be clearly demonstrated using the Fourier analysis of the pressure, $p_{\rm p}$, (Fig. 6.20).



Figure 6.17: Simulation and experimental results: Surge.



Figure 6.18: Simulation and experimental results: Surge control.



Figure 6.19: Experimental results showing the effect of the active surge control.



Figure 6.20: Fourier spectrum of the pressure magnitude: (a) Surge, (b) Surge control.

6.6 Conclusions

This chapter presents the experimental results of one of the key contributions of the research project - realization of the active surge control of the centrifugal compressor by means of the field-oriented controlled high-speed induction motor. Initially, the equivalent circuit parameters of the induction motor have been determined from the standard experimental tests. They were further incorporated into the indirect rotor flux oriented vector controller, which has been successfully implemented.

Next to that, the steady-state tests of the compressor have been used to determine the performance characteristics of the centrifugal compressor. Furthermore, the Greitzer lumped parameters of the compression system have been identified from the geometrical sizes and further tuned by means of dynamic tests, in which the deep surge occurred. Finally, the data obtained has been used for the surge control implementation. The presented experimental results clearly demonstrate the effectiveness of the active surge control and, consequently, the extension of the stable operational area of the centrifugal compressor.

CHAPTER 7

System efficiency and time-optimal control

This chapter is dedicated to the description of the complete system consisting of the FC stack, the high-speed induction motor with field-oriented control and the centrifugal air compressor [103, 104, 105]. Several possible control schemes dedicated to an improvement of the efficiency and performance of the FC system are described and simulated using the mathematical models of the separated system components presented in the previous chapters.

7.1 Energy saving approaches in compression systems

7.1.1 Process control schemes

A typical compression system serves to provide and control one or several process parameters, such as pressure, mass flow or temperature. An appropriate control method is determined by the requirements for its performance, cost or efficiency. In practice, process control systems can be characterized as *discrete* or *continuous*.

Bang-bang control is a typical representative of discrete control and one of the most simplest control methods. The prime mover (i.e. electrical drive) is turned on or off depending on the controlled parameter value. The duty-cycle normally depends on a buffer size. This method usually shows rather high efficiency, especially at rated load, since the electrical drive is then operating at the point of maximum efficiency.

In a stepwise control, the comparatively large electrical drive can be split in a few smaller ones, and the bang-bang control may be applied to each of them separately. In this case, it results in a more flexible operation and improved efficiency, since most of the motors operate near rated load. However, the efficiency of a smaller drive is usually lower, and it requires extra installation cost.

The more demanding applications would require continuous control where the process variables are smooth and uninterrupted in time. In compression applications


Figure 7.1: System structure with a constant compressor characteristic.



Figure 7.2: State map of the system with a constant compressor characteristic.

mass flow and pressure can be controlled continuously by means of a regulating valve, installed next to the compressor (Fig. 7.1), by using a compressor with a variable geometry (Fig. 7.3(a)), or by using a variable-speed electrical drive (Fig. 7.3(b)).

The first approach, also referred to as a mechanical process control, can be illustrated by the following example. Consider a typical pumping or compression system consisting of a prime mover, a compressor and a piping system with a plant (e.g. pneumatic tool or ventilation chamber) which serves as a load of the compressor. For simplicity the plant characteristic is assumed constant. The operating point of the system is defined by a certain value of the mass flow through the plant and the pressure drop over the plant. It corresponds to the intersection of the compressor characteristic (CC) (one curve corresponding to a particular speed) and the system characteristic (SC) in the mass flow - pressure map. Two operating points are considered: o.p.1 at the rated value of the mass flow and o.p.2 at 50 % of the rated

mass flow.

If a constant-speed electrical drive (CSD) is used, the system process parameters are controlled by the mechanical valve placed next to the compressor (Fig. 7.1). In this case, the flow/pressure regulation is possible by opening/closing the valve, i.e. changing the system characteristic, while the compressor characteristic remains constant (Fig. 7.2). This method is rather simple, but leads to extra energy loss in the regulating valve at the reduced mass flow.

The second possibility of continuous process control is the control of the compressor characteristic, keeping the system characteristic unchanged (Fig. 7.4). It could be achieved by using a compressor with a variable geometry which allows the control of the mass flow at constant speed (Fig. 7.3(a)). This method results in a good efficiency and flexibility, but requires a complicated mechanical construction of the compressor.

The third, more appropriate continuous process control method is using a compressor with solid geometry together with a variable-speed electrical drive (VSD) (Fig. 7.3(b)). This method is more energy efficient compared to the first approach because of the absence of the unwanted energy loss in the valve over the total operational area. In some cases it can be combined with other control principles, which are referred to as energy-efficiency control methods of the electrical drive discussed in section 4.4.



Figure 7.3: System structure with a variable compressor characteristic: (a) variable compressor geometry, (b) variable-speed electrical drive.

The aforementioned process control methods can normally be applied to systems where only one of the process states is controlled - either pressure or mass flow. This is typical for systems like heating, ventilation and air conditioning (HVAC). For such type of applications the fast response of the compressor is not critical.

For fuel cell applications the fast response of the compression system is required



Figure 7.4: State map of the system with a variable compressor characteristic.

in addition to the energy efficiency. Moreover, the performance of the FC depends on both process parameters - pressure and mass flow. Thus, the control of the compression system needs the combined action of a variable-speed electrical drive and a valve control. The configuration of the FC compression system and different control strategies are described in the following sections.

7.1.2 Optimal compression system configuration

In general, the choice of an appropriate compression system configuration is not trivial. The choice is defined by the characteristics, efficiency, weight, cost, etc. Three main types of a compression system arrangement can be considered. The most common is a compressor/expander combination (Fig. 7.5). This is regarded as one of the simplest and most efficient configurations. In the compression system of a PEMFC, the electrical drive is used as a prime mover, and the expander can be used for the power recovering from the exhaust flow. The arrangement shown in Fig.7.6 is typical for this application. The compressor and expander should be matched properly to provide an acceptable energy balance over a wide range of operating conditions. In such case, the expander may act as the back-pressure device. However, when using a fixed geometry compressor and expander, a good match over the entire operating range is difficult to realize. Hence, only a limited area match between the FC, compressor and expander may be achieved, and any deviation from this area is subject to a degradation of the performance of the overall system.

A similar system can also be used, where the compressor and the expander are decoupled, and the expander is connected to an electrical generator (Fig. 7.7). Such





Figure 7.5: Compressor-expander.

Figure 7.6: Drive-compressor-expander configuration.

a connection may add more flexibility in the operating regimes and provide better matching. However, the generator with the power electronic converter may double the price of the total compression system.



Figure 7.7: Drive-compressor-expander-generator configuration.

Here, the simplest compression configuration is investigated. It consists of a variable-speed electrical drive, a compressor and a back-pressure valve (Fig 7.8).

In addition to the aforementioned matching considerations, the transient response of the compression system has to be considered. The electrical current in the FC can change rapidly. However, the centrifugal compressor has a much larger time constant and, thus, the time delay in the change of the air mass flow and pressure to the appropriate levels may be critical.



Figure 7.8: Drive-compressor-valve configuration.

7.2 Operating regimes of the BoP compression system

As seen from the dynamic model of the compression system (3.19) - (3.21), there are two sets of primary physical parameters that may result in a transient delay in the change of the state of the air (mass flow and pressure) in the entire physical volume. The geometrical parameters are the duct length, L_c , the area of the impeller eye, A_c , and the plenum volume, V_p , and they define the first time constant. A second time delay is caused by the combined inertia of the compressor impeller and the rotor of the motor. In general, the dynamics of the back-pressure valve should also be included in the analysis. In practice, the valve inertia is considerably lower than the primary motor inertia and, thus, neglected in this study.

Three operating regimes are further investigated: constant-speed (Fig. 7.9), variable-speed (Fig. 7.10) and load-following-mass flow (L-F-MF) regime at approximately constant pressure (Fig. 7.11). In the first regime, the motor speed and the valve position are kept unchanged for all FC load levels. In the second one, the speed of the motor is adjusted in a way to provide the necessary mass flow level. The last regime provides the same mass flow regulation but keeps a nearly constant pressure and angular speed. In the presented plots, the control line shows an additional restriction of the operational area in case of application of the surge avoidance control, which was discussed in section 3.3.1. The behavior in these regimes are further discussed, simulated and compared in section 7.4.



Figure 7.9: FC-compressor operating points in the constant-speed operating regime.



Figure 7.10: FC-compressor operating points in the variable-speed operating regime.



Figure 7.11: FC-compressor operating points in the load-following-mass flow operating regime at approximately constant pressure.

7.3 System efficiency at steady state

The useful output energy of a FC is the electrical energy. The amount of fuel is normally considered as the input to a FC. Since this research mainly deals with the efficiency of the BoP considering also the FC performance, the system efficiency, $\eta_{\rm FC,s}$, is defined, according to Fig. 2.8, as the ratio of the FC system output electrical power, $P_{\rm FC,s}$,

$$P_{\rm FC,s} = V_{\rm FC,s} \cdot I_{\rm FC,s},\tag{7.1}$$

where

 $V_{\rm FC,s}$ - FC system output voltage, [V], $I_{\rm FC,s}$ - FC system output current, [A],

to the sum of $P_{\rm FC,s}$ and the active power consumed by the electrical drive, $P_{\rm d}^{\rm el}$, which actuates the compressor:

$$\eta_{\rm FC,s} = \frac{P_{\rm FC,s}}{P_{\rm FC,s} + P_{\rm d}^{\rm el}},\tag{7.2}$$

where

 $P_{\rm d}^{\rm el}$ - active power consumed by the electrical drive, [W].

The produced thermal power is not investigated. The performance of the FC stack depends on the pressure and mass flow, which define the output power of the compressor, $P_{\rm c}$ (3.11). The relation between $P_{\rm c}$ and $P_{\rm d}^{\rm el}$ can be formulated via the efficiencies of the electrical drive, $\eta_{\rm d}$, and the compressor, $\eta_{\rm c}$:

$$P_{\rm c} = P_{\rm d}^{\rm el} \eta_{\rm d} \eta_{\rm c}. \tag{7.3}$$

The efficiency of the electrical drive with an IM, η_d , normally changes with the operating point. However, applying the control methods described in section 4.4, the electrical drive efficiency can be kept nearly constant.

The efficiency of the compressor, η_c , can be predicted from its basic operational principle presented in section 3.2.3:

$$\eta_{\rm c}\left(m,\omega\right) = \frac{\Delta h_{\rm c,t}}{\Delta h_{\rm c,t} + \Delta h_{\rm loss}},\tag{7.4}$$

where

 $\Delta h_{\rm loss} = \Delta h_{\rm i} + \Delta h_{\rm f}$ -sum of the major compressor losses (incidence and friction).

Then, collecting the various other losses (back flow, clearance, volute and diffusion losses) [106] mainly related to the mechanical construction of the compressor, (7.4) can be corrected as follows:

$$\eta_{\rm c}(m,\omega) = \frac{\Delta h_{\rm c,t}}{\Delta h_{\rm c,t} + \Delta h_{\rm loss}} - \Delta \eta_{\rm add}, \tag{7.5}$$

where

 $\Delta \eta_{\rm add}$ - efficiency drop due to additional losses.

The operational area of the compression equipment is aimed at covering the demanded air mass flow levels. But in practice, the operational area of the compressor is bounded by the surge limit.

Thus, the efficiency of the system not only depends on the efficiency of the separated components (compressor and IM drive), but also on the ability of the

compression equipment to provide an optimal operating condition for the FC stack with the minimal energy consumption.

The ability to extend the stable operational area of the compressor can be accounted for in (7.2) by introducing the term *utilization*, κ_c , of the compressor mass flow (and, respectively, its output power in case of constant pressure). In other words, the utilization, κ_c , corresponds to the deviation from the constant stoichiometry. This denotes the ratio of the desired compressor output power at the operating point, P_c^{des} , where the stoichiometry approaches 2, to the real power, P_c :

$$\kappa_{\rm c} = \frac{P_{\rm c}^{\rm des}}{P_{\rm c}}, \ m^{\rm des} \le m \ (\text{or } S_{\rm O_2} \ge 2), \ p = \text{const},$$
(7.6)

where

$\kappa_{\rm c}$ - utilization of the compressor power at constant pressure, [-].

Substituting (7.6) into (7.3) and combining with (7.2) the efficiency, $\eta_{\text{FC},s}$, of the system with the FC stack, the centrifugal air compressor and the induction motor drive can be expressed as:

$$\eta_{\rm FC,s} = \frac{P_{\rm FC,s}}{P_{\rm FC,s} + \frac{P_{\rm c}^{\rm des}}{\kappa_c \eta_d \eta_c}}.$$
(7.7)

The derived relation allows to include the utilization, κ_c , serving as a measure of the surge control performance, into the expression of the system efficiency, $\eta_{FC,s}$, alongside with the efficiencies of the compressor, η_c , and the electrical drive, η_d . The graphical representation of the obtained expression is shown in Fig. 7.12. It can be seen, that the maximum system efficiency, $\eta_{FC,s}$, corresponds to the utilization, κ_c , equal to unity, which means no exceeding mass flow and, consequently, no exceeding energy consumption by the induction motor driving the compressor. As a comparison, Fig. 7.13 demonstrates the efficiency of the compressor, η_c , as a function of κ_c . In this case, the efficiency curves directly correspond to the performance characteristics, where the maximum is located in the vicinity of the surge line (Fig. 6.13).

The following section presents the dynamic simulation results of the complete FC system with the centrifugal compressor driven by the high-speed IM.



Figure 7.12: System efficiency, $\eta_{\rm FC,s}$: (a) in $m - m^{\rm des}$ coordinates, (b) in $\kappa_{\rm c} - m^{\rm des}$ coordinates.



Figure 7.13: Compressor efficiency, $\eta_{\rm c}:$ (a) in $m-m^{\rm des}$ coordinates, (b) in $\kappa_{\rm c}-m^{\rm des}$ coordinates.

7.4 System simulation results

With the aim to investigate the correlation between the system efficiency and the dynamic performance, a transient mathematical model of the complete system has been derived (Fig. 7.14).

The FC stack is modelled as an equivalent electrical circuit with the *charge* double-layer capacitance (Fig. 2.5). The value of each overvoltage is defined by the description of the FC steady-state model given in section 2.2.3. The high-speed induction motor with the field-oriented control is modelled as described in sections 4.3.1 and 4.3.3. The model of the compression system with the centrifugal air compressor corresponds to that presented in section 3.2.4. The parameters of the compressor and the compression system are taken the same the experimental ones (sections 6.3 - 6.5). The variable load of the FC stack is modelled as an impedance consisting of $R_{1 \text{ load}} = 0.001\Omega$, $R_{2 \text{ load}} = 1.35\Omega$, $R_{3 \text{ load}} = 0.6\Omega$, $L_{\text{load}} = 0.04H$, and an ideal switch, S. The equivalent auxiliary load caused by the converter fed electrical motor driving the compressor is represented by a controlled current source, $I_{\rm D}$:

$$I_{\rm D} = \frac{P_{\rm d}^{\rm el}}{V_{\rm FC}}.\tag{7.8}$$

In this scheme, three major controllers provide an appropriate system operation. First, the mass flow controller calculates the desired mass flow, m^{des} , from the FC current, I_{FC} , based on (2.71). Its value is used in the performance controller to define the torque component $T_{\rm d}^0$, which corresponds to the compressor torque, $T_{\rm c}$, in a particular operating point according to (3.14). It also defines the valve position, $k_{\rm v}$, using (3.18):

$$k_{\rm v} = \frac{m^{\rm des}}{\sqrt{p_{\rm p}^* - p_0}},\tag{7.9}$$

where

$p_{\rm p}^*$ - reference pressure, [Pa].

Therefore, the performance controller is used to provide a proper operating point of the compression system depending on the FC load and a selected operating regime mentioned in section 7.2: constant-speed operating regime, variable-speed operating regime or load-following-mass flow operating regime. However, the load-followingmass flow operating regime requires stabilization of the compressor in the area beyond the surge line. This function is performed by the surge controller, presented in section 3.3.3. It defines the torque component ΔT_d based on (3.29). Additionally, due to the fact, that the transient mass flow can not be measured directly, the estimated value of the mass flow \hat{m} is used in the surge control. The mass flow observer is based on the dynamic model of the compression system (3.19) - (3.20).



Figure 7.14: Transient system model.

All three aforementioned operating regimes of the system have been simulated and compared. Initially, the electrical load of the FC has a low value i.e. the switch, S, (Fig. 7.14) is open. Then, at t = 5 sec. the switch is closed, resulting in a higher electrical load.

Figures 7.15 - 7.17 represent the major FC and compressor state trajectories: cell voltage, V_{cell} , versus current density, j, and pressure, p_{p} , versus mass flow, \hat{m} , respectively. In all three operating regimes, the steady-state electrical operating points of the FC are similar: o.p.1 corresponds to the low electrical load, and o.p.2 corresponds to the high electrical load. However, the compression system behaves differently in every operating regime, thus resulting in different system performance and efficiency.

The results are better understood and compared based on the plots of the main system performance characteristics as a function of time, corresponding to the constant-speed, variable-speed and L-F-MF operating regimes and illustrated in the top, middle and bottom plots of Figs. 7.18 - 7.25, respectively.

In the constant-speed operating regime, the mass flow controller is not activated, thus the operating point of the compressor (lower plot in Fig. 7.15 and Figs. 7.21 and 7.22) is set to that corresponding to the maximum load of the FC. The valve remains opened, and the electrical drive delivers the maximum torque. The operating points of the FC stack are located at the polarization curve corresponding to the maximum pressure (Fig. 7.15). The mass flow at o.p.1 is double of the desired one (Fig. 7.22), also indicated by the low value of utilization (Fig. 7.23).

As a result, the electrical drive always consumes the maximum amount of electrical power (Fig. 7.24) during both, low and high FC electrical load (Figs. 7.18, 7.19 and 7.20). Consequently, the system efficiency is reduced at low FC electrical load (Fig. 7.25). On the other hand, the FC electrical power transition (Figs. 7.18 - 7.20) is not disturbed by mechanical and thermodynamic transients (Figs. 7.21 and 7.22).

In the variable-speed operating regime, the mass flow controller is active and sets the operating point of the compressor according to the condition of constant stoichiometry, $SO_2 = 2$ (lower plot in Fig. 7.16). The valve in this operating regime is always opened, thus the system operates at different pressures (lower plot in Fig. 7.16 and Fig. 7.21). The operating points of the FC stack are now located on different polarization curves (upper plot in Fig. 7.16). The mass flow is regulated according the FC load and settles at the desired value at o.p.2 (Figs. 7.22). The utilization equals unity at both operating points (Fig. 7.23).

As a result, the variable-speed operating regime provides a higher system efficiency (Fig. 7.25) at low FC load, due to less energy consumption by the compressor drive (Fig. 7.24). On the other hand, the inertia of the motor rotor and the compressor impeller slows down the speed transient, and, consequently, the pressure and mass flow response (Figs. 7.21 and 7.22). A sudden drop in the FC voltage (Fig. 7.16 and 7.18) and, consequently, in the FC output power (Fig. 7.20) is visible.

Finally, in the load-following-mass flow operating regime, the mass flow controller operates in the same way as in the variable-speed operating regime. In addition, the valve is controlled in a way to keep a constant pressure, i.e. it is closed at reduced mass flow (o.p.1) and opened at maximum mass flow (o.p.2) (lower plot in Fig. 7.17). Both operating points of the FC stack are located at the same polarization curve, as in the constant-speed regime (upper plot in Fig. 7.17). The mass flow and, consequently, utilization follow closely the desired values also during transients (Figs. 7.22 and 7.23).

It can be seen, that in the L-F-MF operating regime, the value of the drive input power and, consequently, system efficiency at low FC load is in between those at the constant-speed and variable-speed regimes (Figs.7.24 and 7.25) due to the fact that only one process parameter, namely the mass flow, is regulated corresponding to the FC electrical load (lower plot in Fig. 7.17). The FC output power transition is rather smooth (Figs. 7.18, 7.19 and 7.20) due to a fast response of the compression system on the increased FC load, since there are nearly no mechanical transients. The applied active surge control keeps the process stable in the compressor unstable area (o.p.1 in lower plot in Fig. 7.17).

The values of the system efficiency at low FC load as well as the transient time interval of the mass flow in the three operating regimes are summarized in Table 7.1.

Operating regime	Transient time interval, $\tau_{\rm c}$	System efficiency, $\eta_{\rm FC,s}$
	of the (mass flow)	at low FC load
Constant speed	0 s	0.88
Variable speed	$\approx 1.2s$	0.99
L-F-MF	$\approx 0.2s$	0.95

Table 7.1: Major comparative results of the BoP operating regimes.



Figure 7.15: Fuel cell - compressor simulation results in state coordinates (constant-speed regime).



Figure 7.16: Fuel cell - compressor simulation results in state coordinates (variable-speed regime).



Figure 7.17: Fuel cell - compressor simulation results in state coordinates (load-following-mass flow regime).



Figure 7.18: Cell voltage.



Figure 7.19: Current density.



Figure 7.20: FC output power.



Figure 7.21: Pressure.



Figure 7.22: Mass flow (dashed line - $m^{\rm des},$ solid line - $\hat{m}).$



Figure 7.23: Utilization κ_c .



Figure 7.24: Drive input power.



Figure 7.25: System efficiency.

7.5 Dynamic time-optimal control

The previous section described and compared three independent operating regimes of the compression system. It is demonstrated, that the system efficiency can be improved by regulating the performance parameters (mass flow and pressure) of the compression system according to the FC electrical load. In the variable-speed operating regime, both parameters are regulated, and in the load-following-mass flow operating regime, only the mass flow is varied with the FC electrical load in order to eliminate the mechanical transients associated with the speed and, consequently, the pressure transient. The load-following-mass flow operating regime requires the application of the active surge control, the effectiveness of which has been confirmed by simulation and experimental results in section 6.5.

However, despite of preventing the deep surge of the compression system in the unstable area, continuous operation in this area might still be accompanied with high-frequency oscillations or the so-called "mild surge", as it can be seen from the experimental results in Fig. 6.19.

Therefore, in this section, another system control method is proposed, which provides the load-following regulation of both compression system parameters - mass flow and pressure. The m-p trajectory passes through the unstable area only during the transients. However, the steady-state operating points are located in the stable area. This control is realized by means of "time-optimal control" technique.

In contrast to the system control described in section 7.4 (Fig. 7.14), the new

time-optimal controller combines both, performance and surge control. Consequently, both system input variables: the drive torque, $T_{\rm d}^*$ and the valve position, $k_{\rm v}$, are used for the performance as well as for the surge control.

It is expected, that the efficiency of the system at low FC load would be equivalent to that in the variable-speed operating regime, due to similar locations of the steady-state operating points, but the transient time interval would be reduced by adjusting the m - p trajectory between the operating points.



Figure 7.26: LQG controller scheme.

For simplification, the linear quadratic Gaussian (LQG) approach is used in the control design. LQ stands for linear systems with quadratic performance criteria. The system may be affected by disturbances and measurement noise represented as stochastic processes, in particular, by Gaussian white noise. The optimal control design method converts a control design problem into an optimization problem with time-domain performance criteria. With the LQG approach, the coefficients of the feedback controller in Fig. 7.26 are designed in such a way that the cost functional, J_c , is minimized under the "constraint" of the system dynamics, \dot{x} :

$$\min_{\boldsymbol{u}(t)} \quad J_{c} = \frac{1}{2} \int_{0}^{\infty} \left(\boldsymbol{y}(t)^{\mathrm{T}} Q \ \boldsymbol{y}(t) + \boldsymbol{u}(t)^{\mathrm{T}} R \ \boldsymbol{u}(t) \right) dt,$$

subject to $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}), \ \boldsymbol{x}(t_{0}) = \boldsymbol{x}_{0},$ (7.10)

where

$$\boldsymbol{x}(t) = \begin{bmatrix} p_{\mathrm{p}} \\ m \\ \omega \end{bmatrix}$$
 - state vector,

$$\boldsymbol{u}(t) = \begin{bmatrix} k_{\mathrm{v}} \\ 0 \\ T_{\mathrm{d}}^{*} \end{bmatrix} \text{ - input vector,}$$
$$\boldsymbol{y}(t) = \begin{bmatrix} p_{\mathrm{p}} \\ m \\ \omega \end{bmatrix} \text{ - output vector,}$$

Q and R weighing matrices.

The deviations of the output vector, \boldsymbol{y} , and the control input, \boldsymbol{u} , from their reference values are penalized quadratically with nonnegative symmetric weighing matrices, Q, and R, correspondingly, in order to reflect different weights attached to different state and input components. With the choice of the weighing matrices, Q and R, a trade-off between control performance (increasing Q) and low input energy (increasing R) can be achieved. The Q and R parameters have been tuned in order to obtain a fast-response and stable behavior of the compression system.

The linearized system around the equilibrium point is presented in the following way:

$$\Delta \boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{x}(t)^{0},$$

$$\Delta \boldsymbol{u}(t) = \boldsymbol{u}(t) - \boldsymbol{u}(t)^{0},$$
(7.11)

where

 $(\cdot)^0$ - equilibrium point, $\Delta(\cdot)$ - deviation from the equilibrium point.

The state-space equations of the linearized system are:

$$\begin{aligned} \Delta \dot{\boldsymbol{x}}(t) &= A \Delta \boldsymbol{x}(t) + B \Delta \boldsymbol{u}(t) + G \Delta \boldsymbol{y}(t) \boldsymbol{\nu}(t), \\ \Delta \boldsymbol{z}(t) &= C_{\boldsymbol{z}} \Delta \boldsymbol{x}(t), \\ \Delta \boldsymbol{y}(t) &= C \Delta \boldsymbol{x}(t) + \boldsymbol{\varpi}(t), \end{aligned} \tag{7.12}$$

where

 $\begin{array}{l} \Delta \boldsymbol{z}(t) \text{ - controllable output,} \\ \Delta \boldsymbol{y}(t) \text{ - measured output, available for feedback,} \\ \nu(t) \text{ - plant disturbances,} \\ \overline{\boldsymbol{\omega}}(t) \text{ - measurement noise,} \\ C_{\mathrm{z}} \text{ and } G \text{ - matrices defining controllable output and plant disturbances,} \\ A, B, C \text{ - state-space matrices defined from the compression system model (3.19) - (3.21):} \end{array}$

$$A = \begin{bmatrix} -\frac{a_{01}^2 k_{\rm v}}{2V_{\rm p} \sqrt{p_{\rm p} - p_0}} & -\frac{A_{\rm c}}{L_{\rm c}} & 0\\ \frac{a_{01}^2}{V_{\rm p}} & \frac{A_{\rm c} \Xi^{\frac{\gamma}{\gamma-1}} \gamma((\omega r_1 - k_{\rm ins} m) k_{\rm ins} - 2k_f m) p_0}{L_{\rm c}(\gamma - 1) C_{\rm p} T_0 \Xi} & -\frac{\sigma D_2^2 \omega}{4J}\\ 0 & \frac{A_{\rm c} \Xi^{\frac{\gamma}{\gamma-1}} \gamma(\frac{1}{2} \sigma D_2^2 \omega - (\omega r_1 - k_{\rm ins} m) r_1) p_0}{L_{\rm c}(\gamma - 1) C_{\rm p} T_0 \Xi} & -\frac{\sigma D_2^2 m}{4J} \end{bmatrix},$$
(7.13)

where

$$\Xi = \left(1 + \frac{\frac{1}{4}\sigma D_2^2 \omega^2 - \frac{1}{2} \left(\omega r_1 - k_{\rm ins} m\right)^2 - k_{\rm f} m^2}{C_{\rm p} T_0}\right),\tag{7.14}$$

$$B = \begin{bmatrix} -\frac{a_{01}^2 \sqrt{p_{\rm p} - p_0}}{V_{\rm p}} & 0\\ 0 & 0\\ 0 & \frac{1}{J} \end{bmatrix},$$
(7.15)

$$C = \begin{bmatrix} 1\\0\\1 \end{bmatrix}. \tag{7.16}$$

This control scheme allows to handle MIMO system problems almost as easy as SISO system problems. An optimal trajectory is generated by choosing the input vector for $t \ge 0$ as:

$$\boldsymbol{u}(t) = -F_{c}\Delta\boldsymbol{x}(t),$$

$$F_{c} = R^{-1}B^{T}X,$$
(7.17)

where

 $F_{\rm c}$ - state feedback gain matrix,

X - the symmetric matrix - the non-negative-definite solution of the algebraic Riccati matrix equation (ARE):

$$A^{\mathrm{T}}X + XA + C_{\mathrm{z}}^{\mathrm{T}}BC_{\mathrm{z}} - XBR^{-1}B^{\mathrm{T}}X = 0.$$
(7.18)

This solution requires that the state, $\boldsymbol{x}(t)$, is fully accessible for measurement. However, the state, m, of the system is not directly accessible and only the outputs, ω , and $p_{\rm p}$, can be measured. Thus, the reconstruction of the state, m, is performed using an observer of the form:

$$\Delta \hat{\boldsymbol{x}}(t) = A \Delta \hat{\boldsymbol{x}}(t) + B \Delta \boldsymbol{u}(t) + K \left(\Delta \boldsymbol{y}(t) - C \Delta \hat{\boldsymbol{x}}(t) \right),$$

$$\Delta \boldsymbol{u}(t) = -F_{\rm c} \Delta \hat{\boldsymbol{x}}(t),$$
(7.19)

where

 $\Delta \hat{\boldsymbol{x}}(t)$ - estimate $\Delta \boldsymbol{x}(t)$, K - observer gain matrix.

The observation error, $\Delta \mathbf{y}(t) - C\Delta \hat{\mathbf{x}}(t)$, is the difference between the actual measured output, $\Delta \mathbf{y}(t)$, and the output, $\Delta \hat{\mathbf{y}}(t) = C\Delta \hat{\mathbf{x}}(t)$, as reconstructed from the estimated state, $\hat{\mathbf{x}}(t)$. The extra input term, $K[\Delta \mathbf{y}(t) - C\Delta \hat{\mathbf{x}}(t)]$, on the right-hand side of (7.19) provides a correction that becomes active as soon as the observation error is nonzero.

The Riccati equations for the observer are written as follows:

$$AY + YA^{\rm T} + GVG^{\rm T} - YC^{\rm T}W^{-1}CY = 0, K = YC^{\rm T}W^{-1},$$
(7.20)

where

Y - the symmetric matrix,

V, W - intensity matrices of the white noise processes.

The state estimate error is defined as:

$$e(t) = \Delta \hat{\boldsymbol{x}}(t) - \Delta \boldsymbol{x}(t). \tag{7.21}$$

Combination of (7.21), (7.19) and (7.12) results in the linearized closed-loop system:

$$\begin{bmatrix} \Delta \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix} = \begin{bmatrix} A - BF & -BF \\ 0 & A - KC \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x}(t) \\ \boldsymbol{e}(t) \end{bmatrix} + \begin{bmatrix} Gv(t) \\ Gv(t) + K\varpi(t) \end{bmatrix}.$$
(7.22)

The eigenvalues of the open-loop system, the feedback controller with the observer and the closed-loop system at the point, located to the left from the surge line (corresponding to 0.p.1 in the lower plot of Fig. 7.11), are presented in Fig. 7.27.



Figure 7.27: System eigenvalues.

An operating regime, similar to the variable-speed operating regime in Figs. 7.10 and 7.16, but with applied LQG time-optimal control has been simulated. The results are presented in state coordinates in Fig. 7.28, and as a function of time in Figs. 7.29 - 7.31.

It can be seen, that the steady-state operating points of the FC and the compression system in both operating regimes are the same when comparing Fig. 7.28 with Fig. 7.16. As a result, the system efficiency at low FC load remains high, as indicated in the lower plot of Fig. 7.31.

The transient characteristics are estimated from the curves (Figs. 7.29 - 7.31) and compared with those corresponding to the constant-speed, variable-speed and L-F-MF operating regimes (Figs. 7.18 - 7.25).

The most important result of the operating regime with applied time-optimal control is the smaller transient time interval of 0.9 s (first and second plots in Figs. 7.30) of the mass flow (and also pressure), compared to 1.2 s in the variable-speed operating regime (second plots in Fig.7.22 and 7.21), which finally results in a smoother power transient (second plot in Fig. 7.20 compared to third plot in Fig. 7.29).



Figure 7.28: Fuel cell - compressor simulation results with LQG control in m-p coordinates.



Figure 7.29: Cell voltage, current density and FC power as a function of time (LQG control).



Figure 7.30: Pressure, mass flow and utilization as a function of time (LQG control),(dashed line - m^{des} , solid line - \hat{m}).



Figure 7.31: Drive input power and FC system efficiency as a function of time (LQG control).

7.6 Conclusions

This chapter summarizes the analysis of BoP system in a FC, resulting in a formulation of the criteria for a combined efficient and fast operation. Using the mathematical models of the separated components of the FC system, as well as the described and tested active surge control of the compressor, the static and dynamic behavior of the FC system at different operation regimes have been investigated and compared.

In the constant-speed operating regime there is no mechanical transient in the compression system. Thus, it results in the fastest response, corresponding only to the FC electrical time constant, which lies in the range of 0.1 - 0.2 s. However, the system efficiency is reduced due to the extra power consumption by the compressor.

In the variable-speed operating regime, the compressor works entirely in the stable area, and it results in a high efficiency at reduced FC load. However, it causes a significant delay in the response due to the mechanical transient. The electromechanical time constant of the induction motor $\tau_{\rm em}$ according to (5.6) equals 1.2 s. However, the total mechanical time constant is higher due to the extra inertia of the impeller and the load. In practical applications, this regime would require an additional electrical energy buffer installed next to the FC stack to compensate for that delay.

A tradeoff between efficiency and response time has been achieved in the loadfollowing-mass flow operating regime, wherein active surge control is applied. The transient response time is reduced by approximately 83 % in comparison to that in the variable-speed operating regime, but at the cost of 4 % efficiency. However, this operating regime still exhibits 7 % higher efficiency at low FC load than the constant-speed operating regime. The transient time constant here is defined mainly by the geometry of the compression system, and its value is close to the FC electrical time constant. The FC diffusion delay, $\tau_{\rm dd}$, (2.65) equals 1 ms, and is neglected.

With the aim to improve the transient behavior of the most efficient variablespeed operating regime, a time-optimal LQG control has been applied. The resulting operating regime provides a faster transition between the steady-state operating points along the time-optimal trajectory. As demonstrated, a 25 % reduction of the transient response time can be achieved in comparison with the variable-speed operating regime while keeping the same system efficiency at low FC load. In practical applications, it would still require an electrical energy buffer [107], but of a significantly lower size.

CHAPTER 8

Conclusions and recommendations

8.1 Conclusions

The conclusions of the thesis are organized in accordance with the main objectives listed in the introduction:

- 1. To analyse and select appropriate system components: FC type, compressor and electrical drive.
- 2. To build comprehensive and coupled mathematical models of the system components in different physical domains (electrical, mechanical and thermodynamical).
- 3. To expand the naturally imposed stability or efficiency boundaries of the system components by means of appropriate control approaches and verify them experimentally.
- 4. To propose a global design procedure of a high-speed electrical machine.
- 5. To develop an efficiency and time-optimal control of the system with the PEMFC, air compressor and electrical drive.

8.1.1 Components selection

Based on the overview of existing FC systems and their present and potential applications, as well as a preliminary analysis of the system components in Chapter 1, the research has focused on the PEMFC system (a few hundred kilowatt), the centrifugal air compressor and the induction motor drive.

The applications require a high energy efficiency and a fast transient response, which are directly affected by the BoP mainly controlled by electrical drives.

8.1.2 Mathematical models

The mathematical models of the selected system components are presented in Chapters 2, 3, and 4. The combined model is described in Chapter 7.

Fuel cells

After a comprehensive literature study, the PEMFC model, which combines fundamental electrochemical theory and empirical equations, has been derived in Chapter 2. This model allows the analysis of the FC performance depending on both the electrical load and input variables related to the fuel and oxidant supply. The results of the PEMFC analysis can be summarized as follows.

The FC output characteristics are mainly defined by the concentration of the reactants at the catalyst layer of the cell. The reactants concentration, in turn, depends on the balance between the supply and the consumption of fuel and oxidant. The consumption is primarily related to the FC electrical load. On the supply side, (considering only a cathode) the concentration is defined by the pressure and mass flow (oxygen stoichiometry) of the air.

The steady-state model demonstrates that at higher pressure and, accordingly, concentration, the FC generates a higher voltage at the same current density. Moreover, at the elevated pressure a higher maximum current density is achievable. These effects can be interpreted as an improvement of the efficiency of the FC stack (ratio between the output power and the amount of fuel) and higher power density of the stack.

As far as the transient behavior is concerned, the PEMFC has a rather fast transient response on both, output electrical load variation (double charge layer effect) and input change of pressure and mass flow, which corresponds to the transient diffusion. From the presented analysis it follows that the compression system is the slowest component, the so-called time limiting component.

Centrifugal air compressor

The model of the centrifugal air compressor, presented in Chapter 3, involves the thermodynamical and mechanical physical domains, therefore it can be linked to the FC model using the mass flow and pressure variables, and to the electrical drive via the torque and speed variables.

From the steady-state model of the compressor, the dependence of the compressor efficiency on the operating point can be defined. The input and output power can also be determined. This makes it convenient for the analysis of the efficiency of the complete system. The model also clearly identifies the natural stability limits of centrifugal compressors.

The analysis of the dynamic properties shows that the transient time interval between two operating points is determined by two factors: the geometrical sizes of the compression system and the mechanical time constant. The expected low volume of the internal channels of the FC system makes the "geometrical" component of the transient time interval relatively small. Thus, in the case of a variable-speed regime, the transient performance of the compression system primarily depends on the activation by the electrical drive.

Induction motor drive

Electrical drives are presented in Chapter 4. Their operational principles logically suggest their modeling in two domains - electromagnetic and mechanical. Consequently, the electrical drive model is linked to the compressor using a common mechanical equation, and to the FC by electrical ones. In general, a model of the power electronic converter would be required to link the electrical drive to the output of the FC. In the present work, the models are linked in a simplified way using an equivalent current source.

The major focus is given to an induction motor drive. Originally, induction motors were intended to be used in constant-speed applications. However, a number of well-developed control methods, such as field-oriented control or direct torque control, can be used to provide a high-performance electrical drive. As far as the efficiency is concerned, it is not constant and depends on the operating point.

8.1.3 Control methods of the BoP components

The presented description and mathematical models of the major auxiliary system components - centrifugal air compressor and induction motor drive - illustrate naturally imposed operational limitations.

Active surge control of the centrifugal air compressor

The stable operational area of the centrifugal compressor is limited by the surge line on one side and the "stone wall" on the opposite side. The surge limit is regarded as of prime importance. It restricts the compressor operation in the area of reduced mass flow at high pressure, which can be highly demanded by many compressor applications. Moreover, the surge line passes through the area of maximum efficiency of the compressor, forcing, in practice, the operating point into the less efficient area. During the last two decades a number of active surge control methods with experimental verifications have been reported. However, they are still not reliable enough for a wide range of practical applications. The active surge control by means of the highly dynamic torque/speed control of the electrical drive is the most recent development in this area. The experimental results, presented in section 6.5, confirm the effectiveness of this method: the operation of the compressor remained stable beyond the surge line.

Energy-efficiency control of the induction motor drive

A number of methods to control the energy efficiency of an IM have been reported in section 4.4. Most of the techniques are based on the reduction of the magnetic flux and the corresponding losses at low torque and high speed. However, the reduction of the magnetic flux can seriously affect the motion controllability. This matter is particulary important regarding the discussed application. The area of low torque and high speed directly corresponds to the unstable area of the compressor, where the high-performance drive motion control is demanded for the active surge stabilization. Due to time limitation, the energy-efficiency control of the induction motor in combination with the active surge control of the compressor has not been implemented.

8.1.4 Design of the high-speed induction motor

The centrifugal compressor operates at high-speed. Additionally, the transient requirements call for a low-inertia prime mover. These factors, in combination with the required compactness and high reliability of the auxiliary components in a FC system, exclude the use of a standard electrical motor with a gearbox in favor of a high-speed motor. Chapter 5 presents the developed simplified design and analysis procedures of a high-speed induction motor. Its initial specifications have been derived using the previously described models of the FC and the centrifugal compressor. As a result, an electromagnetic design of a low-inertia and rather high-efficiency induction motor has been described in Chapter 5.

8.1.5 Total system model and time-optimal control

Finally, Chapter 7 discusses the system control strategies, using the combined mathematical models of the FC stack, the centrifugal compressor and the field-oriented controlled induction motor drive. Three major system operating regimes can be applied, depending on the predominant control goal.

The constant-speed operating regime can be used, when the FC electrical load changes frequently and fast. In the case of a slow variation of the FC electrical load, the variable-speed operating regime is advisable, providing a high energy efficiency at low FC load. In intermediate cases the load-following-mass flow operating regime with the application of the active surge control of the compressor becomes preferable. This operating regime eliminates the relatively long mechanical transient response, keeping the energy consumption of the BoP approximately linearly proportional to the main load.

The operating regime with applied LQG time-optimal control has been proposed as an alternative to the load-following-mass flow operating regime and the variablespeed operating regime. The transition between two steady-state operating points, where the system efficiency is maximum, follows the time-optimal trajectory, keeping the transient response time small.

8.2 Thesis contributions

The major scientific contributions of this thesis can be summarized as follows:

The multidomain mathematical model and the energy-efficiency and time-optimal control of the system consisting of the FC stack, the centrifugal air compressor and the induction motor. The modeling and control of the separated components are not new, and system models can also be found in the literature. However, they are composed of rather simplified models of the components and are limited to steadystate system performance. A comprehensive modeling and analysis of the static and dynamic system performance have not been found in literature.

The experimental verification of the active surge control by means of a fieldoriented controlled induction motor drive. The idea of active surge control is not new, neither the high-performance field-oriented control of an IM. However, the experimental verification of the active surge control by means of the high-performance motion control of an induction motor drive is novel.

The double purpose of the surge control: energy efficiency and fast system response. The original purpose of the published surge control methods was the energyefficiency improvement of compression systems. In this thesis, in addition to the energy-efficiency improvement, a fast response of the system to load changes is also achieved by the surge control, which is new.

The global design method of high-speed induction motors. The presented design and analysis allow a good estimation of the performance characteristics of a highspeed induction motor. A lot of publications on this subject can be found, but they are mainly focussed on one particular discipline. The electromagnetic design together with the thermal analysis and the FEM verification is a contribution.

8.3 Recommendations for further research

8.3.1 Fuel cells

The mathematical models presented in this thesis are mainly based on the published theoretical work supplemented by experimental results. However, the dynamic response of a FC system to the transient operation of a compression system has not been thoroughly investigated. Further theoretical and experimental research on this topic is recommended.

8.3.2 Active surge control by means of an electrical direct drive

Despite of the fact that the effectiveness of the surge control has been proven experimentally, a further improvement of its performance is a subject of scientific and engineering interest. The main obstacle in the present experimental work was the partial mismatch of the rated torque/speed parameters of the motor and the compressor. Moreover, the gearbox introduced additional losses and control difficulties. The inertia of the compressor impeller is approximately ten times less than that of the motor rotor. However, the use of the gearbox doubles the total system inertia, reducing the controllability of the system. Thus, the use of a direct drive is highly recommended.

8.3.3 Combined energy-efficiency and high-performance control of an induction motor drive

The majority of energy-efficiency control methods described in section 4.4 are applied for low-performance applications, like HVAC systems. On the other hand, in high-performance applications, transient response characteristics of an induction motor drives are primarily concerned, rather than energy-efficiency. The situation is different in the system operating regimes described in this thesis, like the loadfollowing-mass flow operating regime of the compression system in a FC. In such applications, the induction motor drive operates *continuously* in a *transient* regime with active surge control. However, the application itself together with the main idea of the surge control suggests an energy-efficient operation of the electrical drive.

Additionally, depending on motor parameters and control performance, the surge control may require operation of the induction motor drive in both motoring and generating modes. Therefore, the operating regime of the drive would require a number of modifications in energy-efficiency control methods, taking into account the bi-directional energy flow in an autonomous system.

8.3.4 Time-optimal control together with a surge control by means of an electrical drive and valve actuation

During the last few decades a number of experimental results on active surge control methods employing a valve control have been reported. In the surge control method presented and verified in this research, the high-performance drive has been used. However, the time-optimal control of the compression system requires the high-performance control of both, an electrical drive and a back-pressure valve. Finally, a combined action of a high-performance electrical drive and a valve has to be experimentally verified, since it could lead to very promising results in a fast transient behavior of the compression system, as well as more effective surge stabilization.

Appendix A

List of symbols

Latin symbols

	J	
A	electrical loading	[A/m]
A	active cell area	$\left[\mathrm{cm}^{2}\right]$
$oldsymbol{A}$	vector magnetic potential	[Wb/m]
$A_{ m c}$	area of impeller eye	$[m^2]$
$A_{ m el}$	surface area	$[m^2]$
A, B, C	state-space matrices defined from the com-	[-]
	pressor model	
a	activity	[—]
a	number of parallel branches in a phase wind-	[—]
	ing	
a_0	inlet stagnation sonic velocity $= 340$	[m/s]
$a_{\rm sl,r}$	lower width of the rotor slot	[m]
B	magnetic flux density vector	[T]
B_{δ}	flux density in the airgap	[T]
$B_{ m y,s}$	flux density in the stator yoke	[T]
$B_{\rm z,s}$	flux density in the stator tooth	[T]
$B_{\delta 0}$	amplitude of the flux density pulsations	[T]
$B_{ m pul,s}$	amplitude of the flux density pulsations in the	[T]
	middle cross-section of the stator tooth	
$B_{ m pul,r}$	amplitude of the flux density pulsations in the	[T]
	middle cross-section of the rotor tooth	
$B_{\delta \mathbf{x}}$	flux density in the airgap at point x	[T]
$B_{\delta,\mathrm{av}}$	average value of the flux density in the airgap	[T]
$b_{\rm av,r}$	average slot width	[m]
$b_{ m o,s}$	stator slot-opening	[m]
$b_{ m o,r}$	rotor slot-opening	[m]
$b_{\rm sl,ins}$	slot insulation thickness	[m]
$b_{\rm ew,ins}$	end winding insulation thickness	[m]
$b_{\rm skew}$	rotor slot skew	[m]
$b_{\rm sl}$ r	upper width of the rotor slot	[m]
-------------------------	--	--
$b'_{a,a}$	equivalent rotor slot-opening	[m]
C_{π} and G	matrices defining controllable output and	[_]
- 2	plant disturbances	
$C_{\rm f}$	friction coefficient	[—]
$C_{\rm v}$	specific heat at constant volume	$[J/(kg \cdot K)]$
$c_{\rm H}^{\rm int}$	concentration of water	$[mol/cm^3]$
$c_{\rm int}^{\rm int}$	concentration of oxygen	$[mol/cm^3]$
$c_{\rm out}^{\rm int}$	concentration of hydrogen	mol/cm^3
$C_{\rm H_2}^{\rm int}$	concentration of protons	$[mol/cm^3]$
$C_{\rm H^+}$	equivalent capacitance	[F]
CA	constant	[_]
CP.	constant	[_]
С _Б	constant	[_]
C.	specific heat at constant pressure	$[J/(k\sigma \cdot K)]$
$c_{\rm p}$	power factor	[0/(ng n)] [_]
D	electrical flux density vector	$\left[C/m^{2} \right]$
$D_{\rm eff}^{\rm eff}$	effective binary diffusivity coefficient of the $i-$	$\left[\frac{m^2}{s}\right]$
Dij	<i>i</i> gas pair	
$D_{\rm W}^0$	standard-state binary diffusivity coefficient of	$\left[\mathrm{m}^2/\mathrm{s}\right]$
P_{N_2,O_2}	the $N_2 - \Omega_2$ gas pair	
D_{\odot}	diffusivity of ovvgen through a water film	$\left[\mathrm{m}^{2}/\mathrm{s}\right]$
D_{0_2}	stator internal diameter	[m]
D.	stator external diameter	[m]
$D_{\rm s}$	rotor diameter	[m]
$D_{\rm r}$	average end ring diameter	[m]
d er,av	distance between the plates	[m]
dW_{\cdot}	differential change in electrical energy input	[T]
dW_{c}	differential change in stored magnetic field en-	[J]
<i>ww</i>	erov	[0]
dW_{m}	differential change in mechanical energy	[1]
d d	lamination thickness	[b]
d dc	polarity	[]
E	Youngs modulus	[Pa]
E	reversible voltage (open-circuit)	[V]
E_{Nernst}	electrical field strength vector	[V/m]
e ⁻	charge of one electron	[C]
e(t)	state estimate error	[-]
F	Faraday's constant = 96487	[C/mol]
F	magnetomotive force	[A]
$\overline{F_c}$	state feedback gain matrix	[_]
Fa	geometrical factor	[_]
- 9 f	surge frequency	[Hz]
j fa	stator frequency	[Hz]
JS	stator moquonoj	[***]

f	rotor field frequency	[H ₂]
Jr f	view factor	[112] []
J w F	MME in the stater volce	
$F_{\rm y,s}$	MMF in the rotor voko	
$\Gamma_{\rm y,r}$	MMF in the airgan	
Γ_{δ} E_	total MME	
$\Gamma \Sigma$ F	MME in the stater teeth zone	
$\Gamma_{z,s}$	MMF in the stator teeth zone	
$\Gamma_{z,r}$	Cibbs free energy	
G H	GIDDS filee energy	[J] [T]
11 H	magnetic field strongth voctor	[J]
	magnetic field strength in the stater value	[A/III]
$m_{\rm y,s}$	magnetic field strength in the reter value	[A/III]
$m_{y,r}$	magnetic field strength in the states testh	[A/III]
$\Pi_{z,s}$	magnetic field strength in the actor tooth	[A/III]
$\Pi_{z,r}$	magnetic field strength in the iner bridge	[A/III]
$n_{\rm br}$	magnetic field strength in the notar slat	[A/III]
$\Pi_{\rm Sl}$	magnetic neid strength in the rotor slot	[A/III]
$n_{\rm y,s}$	stator yoke neight	[m] []
$n_{y,r}$	rotor yoke neight	[m]
$n_{\rm y,r}$	equivalent rotor yoke neight	
$n_{\rm sl,r}$	rotor slot neight	
$n_{\rm z,s}$	neight of the stator tooth	
$n_{z,r}$	neight of the rotor tooth	
$h_{\rm sl,s}$	stator slot height	
$h_{\rm y,s}$	stator yoke height	
$h'_{y,r}$	equivalent rotor yoke height	
$h_{ m br}$	height of the iron bridge	
$I_{ m FC}$	FC current	
$I_{\rm s}$	stator current	
I_{μ}	magnetizing current	
$I_{\rm r}$	bar current	
$I_{\rm er}$	end-ring current	
I^*_μ	relative magnetizing current	[-]
$I_{\rm er}$	current in the rotor end ring	
i	number of compressor blades	[-]
i	electrical current	
$i_{ m d},i_{ m q}$	dq-components of stator current	
$i_{ m ds}^{ m e},i_{ m qs}^{ m e}$	dq-components of stator current in the rotat-	[A]
, ,	ing dq -reference frame	
$i_{ m dr}^{ m e}, i_{ m qr}^{ m e}$	dq-components of rotor current in the rotating	[A]
	dq-reference frame	F 93
J	moment of inertia	$\left[\text{kg} \cdot \text{m}^2 \right]$
J_{i}	gas flux of species i	$\left[mol/(cm^2s) \right]$
J	current density	$\left[A/m^{2} \right]$

J	current density vector	$\left[A/m^2\right]$
$J_{ m r}$	rotor bar current density	$\left[A'/m^2 \right]$
$J_{\rm er}$	end-ring current density	$\left[A'/m^{2} \right]$
$J_{\rm s}$	current density in the stator winding	A/m^2
$J_{ m m}$	rotor inertia	$\left[\text{kg} \cdot \text{m}^2 \right]$
$J_{\rm c}$	cost functional	[-]
j j	current density (in FC)	$\left[A/cm^{2} \right]$
<i>i</i> 1	limiting current density	A/cm^2
$\frac{j_0}{j_0}$	exchange current density	A/cm^2
$\overset{j\circ}{K}$	controller coefficient	[_]
K	observer gain matrix	[_]
$K_{tz s}$	technological coefficient for the stator tooth	[_]
Kty s	technological coefficient for the stator voke	[_]
K_{D}	damping coefficient	$[Nm \cdot s/rad]$
$K_{\rm m}^{\rm E}$	feedback gain for the mass flow	[_]
K_{Ψ}	feedback gain for characteristic slope	[_]
K.,	feedback gain for the speed	[_]
K_{μ}	saturation coefficient	[_]
k^{μ}	general for coefficient (locally defined)	[_]
$k_{\rm v}$	valve gain	[_]
k^0	intrinsic rate constant	[cm/s]
k_{δ}	Carter's coefficient	[-]
k_{ws}	stator winding coefficient	[_]
$k_{\rm B}$	ratio between the AC and DC phase resistance	[_]
$k_{\rm mc}$	filling factor of the magnetic core	[_]
$k_{\rm h}, \alpha, \beta$	hysteresis loss constants	[_]
keyc	excess loss constant	[_]
kskew	skew coefficient	[_]
k_{1t}	coefficient related to the lamination thickness	[_]
$k_{\rm B}$	coefficient of flux density distribution	[_]
k_{e}	ratio between back EMF and voltage	[_]
k_{w}	winding coefficient	[_]
k_{i}	coefficient describing the influence of the mag-	[_]
1	netizing current	
k_{s}	surface coefficient	[_]
$k_{\delta s}$	stator Carter's coefficient	[_]
$k_{\delta r}$	rotor Carter's coefficient	[_]
$k_{\rm B}$	coefficient of flux density distribution	[_]
$\vec{k_{\gamma}}$	density coefficient	[_]
$k_{ m f}$	fluid friction constant	[_]
$k_{\rm ins}$	constant, determined by the geometry of the	[_]
1113	compressor components	
L_{ss}	stator self-inductance	[H]
$L_{\rm rr}$	rotor self-inductance	[H]
		L J

L	mutual inductance	[H]
$L_{\rm m}$	magnetizing inductance	[11]
L_{μ}	magnetizing branch inductance (ser. connec-	[H]
$-\mu$	tion)	[**]
$L_{\sigma s}$	transient inductance	[H]
Ld. La	dq-components of stator phase inductance	[] [H]
L_{dc}	stator leakage inductance	[+-] [H]
L_{1s}'	rotor leakage inductance referred to the stator	[] [H]
$L_{\rm s}$	stator inductance	[] [H]
L'_n	rotor inductance referred to the stator	[H]
L_{c}^{1}	duct length	[m]
$L_{\rm er}$	end-ring inductance	[H]
L_{eq}	equivalent inductance	[H]
l_{M}	thickness of the polymer membrane	[cm]
l	length of the closed magnetic path	[m]
$l_{\rm v.s}$	average path length of the magnetic flux in the	[m]
5,-	stator yoke	
$l_{\rm v,r}$	average path length of the magnetic flux in the	[m]
5.7-	rotor yoke	
l_{δ}	active length of the magnetic core	[m]
$l_{\rm w}$	total length of the effective conductors of a	[m]
	phase winding	
$l_{\rm bar}$	bar length	[m]
$l_{\rm p,sl}$	perimeter of the stator slot	[m]
$l_{\rm p,ew}$	perimeter of the end winding	[m]
$l_{\rm ew,b}$	length of the end-winding block	[m]
$l_{\rm av}$	average turn length	[m]
$l_{\rm y,s}$	average path length of the stator yoke	[m]
$l_{\rm y,r}$	average path length of the rotor yoke	[m]
[M]	connection bar matrix	[—]
m	compressor mass flow	[kg/s]
$m_{ m v}$	mass flow through the valve	[kg/s]
m	number of stator phases	[—]
$m_{\rm z,s}$	mass of the stator teeth	[kg]
$m_{\rm y,s}$	mass of the stator yoke	[kg]
$m_{ m c}$	cooling air mass flow	[kg/s]
$m_{ m r}$	rotor phase number	[—]
N	Avogadro's number = $6.02214199 \times 10^{23}$	[—]
$N_{ m bl}$	number of compressor blades	[-]
$N_{ m coil}$	number of coils	[-]
Nu	Nusselt number	[—]
n	number of electrons involved in a reaction $(2$	[—]
	for hydrogen)	

n	amount of gas	[mol]
$n_{\rm el}$	number of elementary conductors	[_]
$P_{\rm FC}$	FC power (internal)	[W]
$P_{\rm FC,s}$	FC system power (terminal)	W
P_0	specific losses	[W/kg]
$P_{\rm core}$	core loss	[W]
$P_{\rm cu,ew}$	end-winding losses	[W]
$P_{\rm cu,sl}$	copper loss in the stator slots	[W]
$P_{\rm cu,s}$	stator copper losses	[W]
$P_{\rm cu,r}$	rotor copper losses	[W]
P_{δ}	airgap friction losses	[W]
$P_{\delta,\mathrm{acc}}$	air acceleration losses in the airgap	[W]
$P_{\rm d}$	drive output mechanical power	[W]
$P_{\rm d}^{\rm el}$	active power consumed by the electrical drive	[W]
$P_{\rm in}^{\rm d}$	power consumed by the induction motor	[W]
$P_{\rm pul,r}$	rotor pulsation losses	[W]
$P_{\rm pul,s}$	stator pulsation losses	W
$P_{\rm surf,r}$	rotor surface losses	W
$P_{\rm surf,s}$	stator surface losses	W
$P_{\delta, \text{wind}}$	windage loss in the airgap	[W]
p	pole pair number	[-]
p	pressure	[Pa]
$p_{\rm i}$	partial pressure of the species	[Pa]
p_0	standard pressure (ambient pressure)	[Pa]
p_2	pressure downstream of the compressor	[Pa]
p_{P}	plenum pressure	[Pa]
$p_{\mathrm{H}_2}^{\mathrm{int}}$	hydrogen partial pressure at the anode cata-	[Pa]
2	lyst/gas interface	
$p_{\Omega_2}^{\text{int}}$	oxygen partial pressure at the cathode cata-	[Pa]
~ 2	lyst/gas interface	
Q	internal energy	[J]
Q, R	weighting matrices	[-]
$q_{ m s}$	number of stator slots per pole and per phase	[-]
$q_{ m r}$	number of rotor slots per pole and per phase	[-]
$Q'_{\rm t,conv}$	heat transfer rate by convection	[W]
$Q'_{\rm t,cond}$	heat transfer rate by conduction	[W]
$R^{'}$	universal gas constant $= 8.3145$	$[J/(mol \cdot K)]$
$R_{\rm a}$	area specific resistance	$[\Omega \ { m cm}^2]$
$R_{\rm internal}$	total resistance of the FC	[Ω]
$R^{\rm electrons}$	resistance to electrons transfer	$[\Omega]$
R^{protons}	resistance to protons transfer	$[\Omega]$
$R_{ m m}$	magnetizing branch resistance (par. connec-	$[\Omega]$
	tion)	

D		[0]
R_{μ}	magnetizing branch resistance (ser. connec-	[22]
_	tion)	
$R_{ m er}$	end ring resistance	$[\Omega]$
$R_{ m eq}$	equivalent resistance	$[\Omega]$
$R_{ m r}$	rotor winding resistance	$[\Omega]$
$R_{\rm s}$	stator phase resistance	$[\Omega]$
$R_{\rm bar}$	bar resistance	$[\Omega]$
$R'_{\rm r}$	rotor resistance referred to the stator winding	$[\Omega]$
Re_{δ}	Couette Reynolds number	[_]
$R_{\rm cond}$	thermal resistance to conduction	[K/W]
$R_{\rm conv}$	thermal resistance to convection	[K/W]
r_1	average impeller radius	[m]
r_2	external impeller radius	[m]
$r_{\rm M}$	membrane area-specific resistivity for the flow	$\left[\Omega \mathrm{cm}\right]$
101	of hydrated protons	
S	entropy	[J/K]
\tilde{S}	coil cross-section area	$[m^2]$
$\tilde{\overline{S}}$	input apparent power	[VA]
\tilde{S}	closed surface	$[m^2]$
\tilde{S}_{s}	cross-section of the airgan	m^2
S. T	cross-section area of the effective conductor	m^2
S_{eff}	elementary conductor cross-section area	m^2
$S_{\rm el}$	bar cross-section area	m^2
$S_{\rm bar}$	oquivalent surface area	m^2
S_{eqv}	bar gross sostion	¹¹¹ ²
$S_{\rm bar}$	bar cross-section	[111]
$S_{\rm er}$	cross-section area of the sines	^{III-}
S_{δ}	cross-section area of the airgap	[m²]
s	semi-empirical parameter representing the ef-	[—]
	fective water content of the membrane	г 1
s	slip	[-]
<u>s</u>	Laplace operator	[-]
T	temperature	[K]
T	period	$[\mathbf{s}]$
Ta	Taylor number	[-]
$Ta_{\rm m}$	modified Taylor number	[-]
$T_{\rm d}$	drive torque	[Nm]
$T_{ m g}$	generator torque	[Nm]
$T_{\rm c}$	compressor torque	[Nm]
T_2	outlet temperature	[K]
T'_2	outlet temperature (isentropic)	[K]
T_0	ambient temperature	[K]
$T_{\rm w}$	winding temperature	[K]
$T_{\rm load}$	load torque	[Nm]
$T_{\rm d}$	electromagnetic torque	[Nm]

$T_{\rm in}$	temperature of the inlet cooling air	[K]
$T_{\rm out}$	temperature of the outlet cooling air	[K]
$T_{\rm ew,in}$	air temperature in the end-winding region at	[K]
	the inlet side	
$T_{\rm ew,out}$	air temperature in the end-winding region at	[K]
,	the outlet side	
T_{δ}	temperature in the airgap	[K]
t	time	$[\mathbf{s}]$
u	velocity of moving parts	[m/s]
U	tangential impeller velocity	[m/s]
$u_{\rm sl}$	number of effective conductors in a slot	[_]
u'_{cl}	preliminary number of effective conductors in	[_]
51	a slot	
$u_{ m r}$	peripheral speed of the rotor	[m/s]
u_{a}	axial velocity	[m/s]
u _{t.}	tangential gas velocity	[m/s]
u	input vector	[_]
$u_{\rm imp}$ dif	gas velocity in an impeller/diffuser	[m/s]
V	volume	$[m^3]$
$V_{\rm P}$	plenum volume	[m ³]
Vcell	FC output voltage	[V]
VEC	FC stack terminal voltage	[V]
V, W	intensity matrices of the white noise processe	[_]
v	gas velocity	[m/s]
$v_{\rm d}, v_{\rm q}$	dq-components of stator voltage	[V]
v_{1}^{e}, v_{re}^{e}	dq-components of stator voltage in the rotat-	ĪVĪ
as qs	ing dq -reference frame	[.]
V_{c}	rms phase voltage	[V]
$V_{\rm b}$	voltage difference across a conductor	[V]
v_{a}	voltage drop across a capacitor	[V]
Ŵ	work	[J]
W'	isentropic work	[J]
We	electrical energy input	[J]
$W_{\rm f}$	stored magnetic field energy	[J]
Wmech	mechanical energy	[J]
W_1^{e}	electrical energy loss	$[\mathbf{J}]$
W_1^{f}	magnetic energy loss	$[\mathbf{J}]$
W_1^{im}	mechanical energy loss	$[\mathbf{J}]$
$W_{\rm m}$	magnetic energy	[J]
Wf	stored magnetic field energy per unit volume	$[J/m^3]$
wt%	weight percent	L-,]
$w_{ m sr}$	coefficient referring to the stator winding	[-]
$w_{ m s}$	number of turns per phase	[_]
$X_{\rm ls}$	leakage reactance of the stator phase winding	ĪΩĪ

V	ma maticing has stands	[0]
$\Lambda_{\rm m}$	magnetizing reactance	
X	symmetric matrix	[-]
$X'_{\rm r}$	rotor leakage reactance referred to the stator	$[\Omega]$
-	winding	
$X_{\rm nl}$	no-load reactance	$[\Omega]$
x	mole fraction of species	[-]
\boldsymbol{x}	state vector	[-]
Y	symmetric matrix	[-]
\boldsymbol{y}	measured output, available for feedback	[-]
$Z_{ m r}$	number of rotor slots	[-]
$Z_{\rm s}$	number of stator slots	[-]
$ar{Z}_{ m nl}$	no-load impedance	$[\Omega]$
$ar{Z}_{ m br}$	blocked-rotor impedance	$[\Omega]$
$\Delta z(t)$	controllable output	[-]

Greek symbols

α	transfer coefficient	[-]
$lpha_{ m c}$	chemical activity parameter for the cathode	[-]
$lpha_{ m t}$	temperature coefficient	[1/K]
$lpha_{\delta}$	pole overlap coefficient	[-]
β	current phase angle	[rad]
$eta_{ m f}$	coefficient related to the frequency depen-	[-]
	dence of core losses	
β_0	function of the ratio between the stator slot	[-]
	opening $b_{\rm so}$ and the airgap length δ	
γ	specific heat ratio $= 1.4$ for air	[-]
$\gamma_{ m st}$	steel density	$[kg/m^3]$
$\Delta G_{\rm e}$	change in standard - state free energy of the	[J/mol]
	cathode reaction	
$\Delta G_{\rm ec}$	change in standard - state free energy of acti-	[J/mol]
	vation for chemisorption	
$\Delta h_{ m c, t}$	total specific enthalpy delivered to the fluid	$[\mathrm{J/kg}]$
$\Delta h_{\rm c}$	change in stagnation enthalpy in the isentropic	$[\mathrm{J/kg}]$
	process	
Δs	change in specific entropy	$[{ m J}/({ m kg}\cdot{ m K})]$
$\Delta h_{\rm loss}$	change in stagnation enthalpy due to losses	$[\mathrm{J/kg}]$
$\Delta T_{\rm s,fl}$	temperature difference between surface and	[K]
	fluid	
δ	thickness of the gas diffusion layer (GDL)	[m]
δ	airgap length	[m]
ϵ	porosity of the diffusion material	[-]
ε	electrical permittivity	[F/m]
ε	emissivity of the surface	[-]
ε_0	electrical permittivity of vacuum $(8.85 \cdot 10^{-12})$	[F/m]

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$\varepsilon_{ m r}$	relative permittivity	[-]
η	overvoltage	[V]
η_{IM}	efficiency of the induction motor	[-]
η_{c}	compressor efficiency	[-]
$\eta_{\rm FC,s}$	fuel cell system efficiency	[-]
$\theta_{\rm e}$	angular relation between the stator current	[rad]
	vector and the flux vector	
$\kappa_{ m c}$	utilization of the compressor power at con-	[-]
0	stant pressure	
$\lambda_{\rm sl,s}$	stator slot specific permeance	[-]
$\lambda_{ m ew}$	stator end winding specific permeance	[_]
$\lambda_{\rm dl,s}$	stator differential specific permeance	[_]
$\lambda_{\rm dl,r}$	rotor differential specific permeance	[_]
$\lambda_{ m er}$	rotor end-ring specific permeance	[_]
λ_{surf}	convection heat coefficient	$[W/(m^2 \cdot K)]$
$\lambda_{ m m}$	thermal conductivity of the material	$[W/(m \cdot K)]$
$\lambda_{ m main}$	geometrical main machine dimensional coeffi-	[-]
	cient	
$\lambda_{ m sl\ r}$	rotor slot specific permeance	[-]
$\lambda_{ m skew}$	skew specific permeance	[_]
$\lambda_{ m PM}$	permanent-magnet flux linkage	[Wb]
$\lambda_{\rm d}, \lambda_{\rm q}$	dq-components of flux linkage	[Wb]
$\lambda_{\rm ds}^{\rm e}, \lambda_{\rm qs}^{\rm e}$	dq-components of stator flux linkage	[Wb]
$\lambda_{\rm dr}^{'e}, \lambda_{\rm or}^{'e}$	dq-components of rotor flux linkage	[Wb]
$\lambda_{ m s}$	stator flux linkage	[Wb]
μ	permeability	[H/m]
μ_0	permeability of vacuum = $4\pi \cdot 10^{-7}$	[H/m]
$\mu_{ m r}$	relative permeability	[-]
$ u_{ m air}$	dynamic viscosity of air	$[Pa \cdot s]$
u(t)	plant disturbances	[-]
ν	fluid velocity	[m/s]
ξ	empirical coefficient	[—]
$\varpi(t)$	measurement noise	[—]
ho	gas density	$\left[\text{kg/m}^3 \right]$
$ ho_{ m bar}$	resistivity of the bar material	[Ωm]
$ ho_{ m ch}$	charge density	$\left[\mathrm{C/m^{3}}\right]$
$ ho_{ m er}$	resistivity of the end-ring material	[Ωm]
$ ho_{ m w}$	resistivity of the winding material	$[\Omega m]$
σ	slip factor	[-]
σ	electrical conductivity	[S/m]
σ	Stefan-Boltzmann constant = $5.67 \cdot 10^{-8}$	$\left[\mathrm{Wm^{-2}K^{-4}}\right]$
$\sigma_{ m s}$	leakage factor	[—]
$\Sigma P_{\rm i,air}$	sum of losses removed by the internal air flow	[W]
$\Sigma P_{\rm loss}$	sum of all losses in the machine	[W]

_	1: Construction of the second second	[_]
$ au_{ m dd}$	diffusion time constant	[S]
$ au_{ m r}$	rotor time constant	$[\mathbf{s}]$
$ au_{\mathrm{z,s}}$	stator slot pitch	[m]
$ au_{ m z,r}$	rotor slot pitch	[m]
$ au_{ m p}$	pole pitch	[m]
$ au_{ m p}^{\prime}$	equivalent pole pitch	[m]
$ au_{ m em}$	electromechanical time-constant	[s]
ϕ	phase shift angle	[rad]
Φ	magnetic flux	[Wb]
φ	scalar potential	[V]
$\Psi_{ m c}$	compressor characteristics	[-]
ω	angular velocity	[rad/s]
$\omega_{ m c}$	first bending critical speed	[rad/s]
$\omega_{ m imp}$	angular velocity of the impeller (used in Chap-	[Hz]
	ter 6 and Appendix D)	
$\omega_{ m e}$	electrical angular velocity	[rad/s]
$\omega_{ m r}$	electrical rotor speed	[rad/s]
$\omega_{ m rated}$	rated angular speed	[rad/s]
$\omega_{ m slip}$	slip speed	[rad/s]

sub- end superindexes (if not separately defined)

	defined)
$(\cdot)_0$	standard state (ambient) value (at 298.15 K
	and 1 atm)
$(\cdot)^{\circ}$	reference value (at unit activity)
$(\cdot)_0^{\circ}$	standard state reference value
$(\cdot)_{\mathrm{H}_2}$	hydrogen
$(\cdot)_{O_2}$	oxygen
$(\cdot)_{\mathrm{H_2O}}$	water vapor
$(\cdot)_{\mathrm{act}}$	activation
$(\cdot)_{\rm con}$	concentration
$(\cdot)_{ m ohm}$	ohmic
$(\cdot)_{s}$	stator
$(\cdot)_{\mathrm{r}}$	rotor
$(\cdot)_{\delta}$	airgap
$(\cdot)_{\mathbf{v}}$	valve
$(\cdot)_{\mathrm{t}}$	thermal
$(\cdot)_{\mathbf{p}}$	plenum
$(\cdot)^{s}$	stationary reference frame
$(\cdot)^{\mathrm{e}}$	rotating reference frame
$()'_r$	rotor values referred to the stator
$(\cdot)^0$	equilibrium point
× /	· ·

	Abbreviations
AFC	alkaline fuel cell
BoP	balance of plant
CCV	close coupled valve
CSD	constant-speed drive
CHP	combined heat and power
\mathbf{DAQ}	data acquisition
DSC	direct self control
DTC	direct torque control
EMF	electromotive force
\mathbf{FC}	fuel cell
FCVs	fuel cell vehicles
FEM	finite element method
FOC	field-oriented control
\mathbf{FW}	field weakening
GDL	gas diffusion layer
HVAC	heating, ventilation and air conditioning
IM	induction motor
IPMSM	interior permanent-magnet synchronous motors
LM	loss minimization
LQG	linear quadratic Gaussian
MCFC	molten carbonate fuel cell
L-F-MF	load-following-mass flow
MIMO	multiple-input multiple-output
MMF	magnetomotive force
MTpC	maximum torque-per-current
MTpF	maximum torque-per-flux linkage
PAFC	phosphoric acid fuel cell
PEMFC	proton exchange membrane fuel cell
\mathbf{PM}	permanent magnet
PVC	polyvinyl chloride
PWM	pulse-width modulation
SISO	single-input multiple-output
SOFC	solid oxide fuel cell
SPMSM	surface permanent-magnet synchronous motors
SynRM	synchronous reluctance motor
VSD	variable speed drive
VSI	voltage source inverter

Appendix B

Parameters identification of the high-speed induction motor selected for the experiments

B.1 DC test

Diff. Summary of the light speed in De test ie						
$I_{\rm s,DC},$ [A]	$R_{\rm s}, [\Omega]$					
0.633	0.128					
0.989	0.103					
1.294	0.093					
1.619	0.086					
2.103	0.080					
2.420	0.077					
2.717	0.076					
3.121	0.073					
3.538	0.071					
	$\begin{array}{c} I_{\rm s,DC}, [{\rm A}] \\ \hline 0.633 \\ \hline 0.989 \\ \hline 1.294 \\ \hline 1.619 \\ \hline 2.103 \\ \hline 2.420 \\ \hline 2.717 \\ \hline 3.121 \\ \hline 3.538 \end{array}$					

Table B.1: Summary of the high-speed IM DC test results.

Appendix B. Parameters identification of the high-speed induction 222 motor selected for the experiments

B.2 No-load test

					· · ·
$V_{\rm s},$	$I_{\rm s},$	$P_{\rm in},$	$P_{\rm in} - I_{\rm s}^2 R_{\rm s},$	$X_{\rm ls} + X_{\rm m},$	$L_{\rm ls} + L_{\rm m},$
[V]	[A]	[W]	[W]	$[\Omega]$	[mH]
59.545	24.506	159.989	124.016	2.429	7.732
56.631	20.643	127.250	101.724	2.743	8.730
53.744	17.628	104.177	85.564	3.048	9.703
50.718	15.285	88.749	74.754	3.318	10.560
47.410	13.420	77.328	66.540	3.532	11.243
42.643	11.433	66.541	58.711	3.729	11.871
38.238	9.787	56.982	51.245	3.907	12.436
33.577	8.277	48.949	44.846	4.056	12.911
27.970	6.736	38.267	35.549	4.152	13.216
24.030	5.681	32.693	30.760	4.230	13.463
18.551	4.359	25.792	24.654	4.255	13.544
11.923	3.043	19.143	18.588	3.917	12.470
10.275	2.790	17.752	17.286	3.683	11.722
4.040	3.330	12.504	11.840	1.212	3.857
3.444	3.947	12.578	11.645	0.871	2.771

Table B.2: Summary of the high-speed IM no-load test results at 50 Hz (per phase).

B.3 Blocked-rotor test

phase).					
$V_{\rm s}, [{\rm V}]$	$I_{\rm s}, [A]$	$X_{\rm ls} + X'_{\rm lr}, [\Omega]$	$L_{\rm ls} + L'_{\rm lr}, [\rm mH]$	$R_{\rm r}^{\prime}, [\Omega]$	
13.254	55.627	0.134	0.428	0.137	
12.382	51.686	0.132	0.421	0.140	
11.481	47.717	0.130	0.415	0.142	
11.210	46.453	0.129	0.411	0.144	
10.680	44.392	0.131	0.417	0.142	
9.260	38.392	0.130	0.415	0.143	
7.741	32.083	0.131	0.417	0.143	
7.163	29.625	0.130	0.415	0.144	
6.833	28.115	0.126	0.402	0.148	
5.676	23.344	0.126	0.402	0.148	
4.608	18.953	0.127	0.403	0.148	
4.006	16.472	0.126	0.402	0.148	
2.826	11.707	0.128	0.408	0.145	
2.048	8.527	0.128	0.406	0.144	

Table B.3: Summary of the high-speed IM locked-rotor test results at 50 Hz (per phase).

AppendixB. Parameters identification of the high-speed induction224motor selected for the experiments

Appendix C

Mass flow measurement method (Pitot tube)

The Pitot tube (named after Henri Pitot in 1732) measures a fluid velocity by converting the kinetic energy of the flow into potential energy. The conversion takes place at the stagnation point, located at the Pitot tube entrance (Fig. C.1). A pressure higher than the free-stream, i.e. dynamic pressure results from the kinetic to potential conversion. This "static" pressure is measured by comparing it with the flow's dynamic pressure with a differential manometer (water column).

The conversion of the resulting differential pressure measurement into a fluid velocity measurement, ν [m/s], is:

$$\frac{\nu^2}{2} + \frac{p_{\text{static}}}{\rho} = \frac{p_{\text{stagnation}}}{\rho}.$$
 (C.1)

The velocity of the flow can be obtained as (valid for laminar flow):

$$\nu = \sqrt{\frac{2\left(p_{\text{stagnation}} - p_{\text{static}}\right)}{\rho}}.$$
 (C.2)

The pressure difference $(p_{\text{stagnation}} - p_{\text{static}})$ is equal to the hydrostatic pressure:

$$(p_{\text{stagnation}} - p_{\text{static}}) = \rho_{\text{H}_2\text{O}}gh_{\text{m}}, \qquad (C.3)$$

where

$$\begin{split} \rho_{\rm H_2O} &= 1000 \text{ - water density, } [\rm kg/m^3], \\ h_{\rm m} &= \rm height \ of \ the \ water \ column, \ [m], \\ g &= 9.81 \text{ - gravitational acceleration, } [\rm m/s^2]. \end{split}$$

The air density ρ at the atmospheric pressure and normal temperature (298 K) is equal to 1.186 [kg/m³]. For conditions different from the normal one, the value of the velocity is corrected using the ideal gas equation:

$$p_{\text{static}}V_{\text{p}} = nRT,\tag{C.4}$$

where

 $V_{\rm p}$ - volume, [m³], n - amount of gas, [mol], R - gas constant, [J/(mol \cdot K)], T - temperature, [K].

Finally, the mass flow, m,[kg/s], is obtained as:

$$m = \nu \rho A_{\rm c},\tag{C.5}$$

where

 $A_{\rm c}$ - area of impeller eye, [m²].



Figure C.1: Pitot tube: longitudinal cross-section.

Appendix D

Compressor performance map identification

	-	-	- (F	
$T_0,$	<i>p</i> _p ,	$T_2,$	$\Delta p,$	$\omega_{\rm imp},$	m,
[°C]	$\left[\text{Pa} \cdot 10^{-5} \right]$	[°C]	$[mmH_2O]$	[Hz]	[kg/s]
22.7	0.04	25.4	20	170	0.24
22.7	0.04	26.2	20	170	0.24
22.7	0.04	26.6	15	170	0.21
22.7	0.05	27.9	5	170	0.12
22.7	0.06	30.5	1	170	0.05
22.7	0.04	29.1	0	170	0.00
22.7	0.06	31.1	45	260	0.36
22.7	0.06	31.6	41	260	0.35
22.7	0.08	32.1	32	260	0.31
22.7	0.10	34.4	9	260	0.16
22.7	0.10	40.8	2	260	0.08
22.7	0.09	39.8	0	260	0.00

Table D.1: Compressor performance map ($\omega_{imp} = 170 - 260 \text{ Hz}$).

$T_0,$	$p_{\rm p},$	$T_2,$	$\Delta p,$	$\omega_{\rm imp},$	m,
[°C]	$\left[\mathrm{Pa} \cdot 10^{-5} \right]$	$[^{\circ}C]$	$[mmH_2O]$	[Hz]	[kg/s]
22.7	0.10	38.7	82	360	0.49
22.7	0.10	38.8	77	360	0.48
22.7	0.13	39.7	59	360	0.42
22.7	0.18	42.7	20	360	0.24
22.7	0.20	47.3	6	360	0.13
22.7	0.19	49.8	2	360	0.08
22.7	0.13	44.1	110	420	0.57
22.7	0.14	43.9	101	420	0.54
22.7	0.18	44.9	80	420	0.48
22.7	0.22	46.8	50	420	0.38
22.7	0.24	49.0	28	420	0.29
22.7	0.26	53.3	9	420	0.16

Table D.2: Compressor performance map ($\omega_{imp} = 360 - 420$ Hz).

Table D.3: Compressor performance map ($\omega_{imp} = 470-535$ Hz).

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$T_0,$	$p_{\mathrm{p}},$	$T_2,$	$\Delta p,$	$\omega_{\mathrm{imp}},$	m,			
[°C]	$\left[\operatorname{Pa} \cdot 10^{-5} \right]$	$[^{\circ}C]$	$[mmH_2O]$	[Hz]	[kg/s]			
21.0	0.16	44.9	146	470	0.66			
21.0	0.16	46.3	108	470	0.56			
21.0	0.22	47.6	92	470	0.52			
21.0	0.27	49.7	53	470	0.39			
21.0	0.30	51.7	34	470	0.32			
21.0	0.32	58.6	12	470	0.19			
21.0	0.22	53.3	151	535	0.67			
21.0	0.24	54.5	146	535	0.66			
21.0	0.28	55.3	114	535	0.58			
21.0	0.35	58.1	68	535	0.45			
21.0	0.39	60.7	33	535	0.31			
21.0	0.42	67.2	21	535	0.25			

Table D.4: Compressor performance map ($\omega_{imp} = 570 \text{ Hz}$).

$T_0,$	$p_{\rm p},$	$T_2,$	$\Delta p,$	$\omega_{\rm imp},$	m,
[°C]	$\left[\mathrm{Pa} \cdot 10^{-5} \right]$	$[^{\circ}C]$	$[mmH_2O]$	[Hz]	[kg/s]
21.0	0.24	57.2	165	570	0.70
21.0	0.24	58.1	152	570	0.67
21.0	0.31	59.3	131	570	0.62
21.0	0.40	62.6	64	570	0.43
21.0	0.46	66.3	43	570	0.36
21.0	0.48	72.6	26	570	0.28

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Acknowledgements

This thesis presents a description of research and design performed on the subject of "Efficiency and Time-Optimal Control of Fuel Cell – Compressor – Electrical Drive Systems". It was carried at the group Electromechanics and Power Electronics at Eindhoven University of Technology.

First of all, I owe my sincere thanks to Professor André Vandenput for his invaluable guidance, encouragement and support throughout the duration of my PhD research and the writing of this thesis.

I wish to express my genuine gratitude and appreciation to Elena Lomonova for being my every-day supervisor. I am sincerely thankful for her most supportive attitude, attention and assistance.

I would also like to thank my colleagues from the Energy Technology Group of the TU/e-Mechanical Engineering Department for the design and construction of the compressor equipment, and in particular Rick de Lange for his support and valuable advices.

Many thanks to the company "EN Motoren Nijmegen BV" for providing a special high-speed induction motor for the experiments.

The experimental work conducted during the PhD research would not be possible without a great help of our brilliant technicians, Marijn Uyt de Willigen and Wim Thirion. They have not only managed to build a complex test set-up, but also to provide fast solutions to a huge amount of problems concerning the mechanical installation, EMC problems, measurements, power electronics, etc.

I am also thankful to Jorge Duarte for his good ideas, valuable advices and fruitful discussions.

Thanks are also extended to my colleague from Control Systems group, Andrey Tyagunov, for his helpful assistance and advices relating to control engineering.

And I am also very obliged to all my colleagues, students and PhD students of EPE group, with whom I could exchange the thoughts on technical, non-technical, relevant and non-relevant matters.

Efficiency and Time-Optimal Control of

Fuel Cell – Compressor – Electrical Drive Systems

Summary

The proton exchange membrane fuel cell (PEMFC) based power generation system is regarded as one of the perspective energy supply solutions for a wide variety of applications including distributed power plants and transport. The main component of the FC system is the FC stack, where the process of electrochemical energy conversion takes place. Additionally, such systems usually contain an auxiliary compression subsystem which supplies the reactant gases to the FC stack as well as maintains certain operation conditions: pressure, temperature, humidity, etc. The proper operation of the compression system significantly improves the performance characteristics of the total system. On the other hand, it consumes a portion of the electrical energy produced, thus reducing the net efficiency of the total system.

This thesis focuses on an innovative way to improve both the energy efficiency and the response characteristics of a power generation system with a PEMFC. The approach principally consists of the control of the air compressor powered by the electrical drive. This method could be considered as an alternative to a redesign of the complete system (changing the power level, using an extra energy buffer, etc).

The modern high-speed centrifugal compressor has been regarded as one of the best candidates for the FC system. It has appropriate characteristics with respect to efficiency, reliability, compact design, etc. However, the presence of a stability margin or so-called "surge line" limits its operation area. With the aim to overcome this constraint, a novel active surge suppression approach has been proposed for application in the system. This control method relies on the high-performance speed control of the electrical drive and accurate measurement and estimation of the thermodynamic quantities, such as air pressure and mass flow.

The choice of an induction motor drive has been justified by its commonly known advantages: low cost, simple construction, high reliability, etc. These features become especially important in high-speed applications. For the detailed investigation and performance prediction of the prime mover, a global electromagnetic design procedure with thermal analysis of a high-speed induction motor has been performed. The obtained analytical results have been verified numerically by a high-precision Finite Elements Method. A good agreement between the analytical and FEM simulation results has been achieved. The mentioned active surge control in combination with the high-performance field-oriented control of the induction motor has been implemented and tested. The test bench comprises the centrifugal compressor with the PVC piping system, the high-speed induction motor drive, the real-time data acquisition and the control system. The experimental results proved the effectiveness of the active surge suppression by means of the drive torque actuation: the operation point of the compressor can be moved beyond the surge line while the process remains stable.

Using the combined mathematical models of the FC stack, the centrifugal compressor and the field-oriented controlled induction motor drive, the static and dynamic behavior of the total system have been simulated, allowing to clarify the interaction between the electrochemical processes in the FC stack, the thermodynamic processes in the compression system and the electromechanical performance of the drive.

Various system operating regimes have been proposed and analyzed. When the FC electrical load changes frequently and fast, the constant-speed operating regime can be used. In case of a slow variation of the FC electrical load, the variable-speed operating regime is advisable, providing a high energy efficiency at low FC load. In intermediate cases, the load-following-mass flow operating regime with the application of the active surge control of the compressor becomes preferable. This operating regime eliminates the relatively long mechanical transient process, keeping the energy consumption of the balance of plant (BoP) approximately linearly proportional to the main load. The operating regime with applied linear quadratic Gaussian (LQG) time-optimal control has been proposed as an alternative to the load-following-mass flow operating regime. The transition between two steady-state operating points, where the system efficiency is maximum, follows the time-optimal trajectory, keeping the transient response time small.

Finally, recommendations for further research have been formulated concerning the dynamic response and energy-efficiency of a fuel cell system. Mainly, the recommendations concern further improvements of presented control strategies and their more comprehensive experimental verification using a complete FC system.

First of all, the use of a *direct* induction motor drive for the compressor stabilization would significantly improve the effectiveness of the surge control. It would allow to control the surge of higher frequency, or to stabilize the compressor operation at larger distance from the surge line.

Second, a combination of the electrical drive torque control with a valve position control would result probably in a more effective surge control, together with fast transients of the system operating point.

Third, the application of the electrical drive for the compressor active surge control in a FC system would require new control algorithms for energy-efficiency improvement of the induction motor, not compromising its high-performance capabilities.

Curriculum Vitae

Konstantin Boynov was born in Moscow, Russia, in 1975.

He received the M.Sc. degree in electrical engineering from the Moscow State Aviation Institute (Technical University), Moscow, Russia, in 1999. He performed the Master thesis during a NUFFIC scholarship (1998 - 1999) at the Control Laboratory of the Delft University of Technology in the Netherlands.

From 1999 - 2001 he worked in the Russian State Scientific Designing Institute "ATOMENERGOPROECT" as a scientific programmer.

From 2001 till 2005 he was a PhD student at Eindhoven University of Technology, the Netherlands. This thesis is the result of the research, done in the group Electromechanics and Power Electronics of the Electrical Engineering Department.

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