Mixed convection and thermal plumes in solar domestic hot water systems

Mixed Convection and Thermal Plumes in Solar Domestic Hot Water Systems

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1. Introduction

Helically coiled tubes are widely used in heat exchange systems. A typical example can be found in the storage vessel of a Solar Domestic Hot Water System (SDHWS), sketched in figure (1). In a SDHWS the collector medium (water) is pumped through the collector where it is heated by solar radiation. To tide over the time gap between collection (maximal round noon) and utilization (mostly in the evening and morning) of solar energy, heat is tranferred from the collector medium to mains water in the storage vessel by means of the coiled heat exchanger. An effective way to operate a SDHWS is according to the low-flow principle: the amount of water pumped through the collector during a daytime is about equal to the storage capacity (typical 100 l). In combination with a stratified storage of the mains water this may lead to minimal exergy loss.

For the design of heat exchange systems Nusselt correlations are used. In case of the low-flow SDHWS, several problems are encountered. For the coiled heat exchanger, correlations are only available for two boundary conditions: a constant heat flux condition or a constant temperature condition.

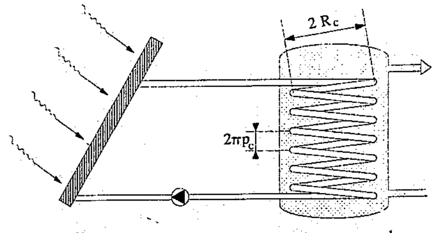

Figure 1. Solar Domestic Hot Water System with $R_c = \mathcal{O}(100\ mm)$ and $p_c = \mathcal{O}(10\ mm)$

The SDHWS-situation may be somewhere in between: the situation varies from a constant temperature at the coil when the storage vessel is unloaded, to a more or less constant heat flux boundary condition when the medium in the vessel is linearly stratified. Next, the Nusselt correlations are generally limited to fully developed flow, whereas in the SDHWS the flow is under development in a great part of the coil. Finally, even for a frequently used heat exchanger as the coiled heat exchanger, only a limited number of relevant studies dealing with mixed convection are available, and the Nusselt number predicted from these studies vary substantially.

Beside heat transfer in the coiled heat exchanger, the performance of a SDHWS is highly determined by the thermal mixing behaviour in the heat storage vessel. In this vessel heat transfer takes place by thermal plumes rising from the tube wall. The thermal mixing behaviour is highly influenced by transition phenomena of those thermal plumes and by stratification of the storage medium.

In the present paper tools used to study mixed convection in a helically coiled tube and thermal plumes rising from a heat source will be presented and some characteristic results will be discussed.

2. Governing equations

Most types of non-reacting flows can be described by a set of three equations, describing conservation of mass, momentum and energy [3]. In most cases it is valid to neglect the influence of pressure work, work by buoyant compression and viscous dissipation. Besides, the equations are often simplified using the Oberbeck-Boussinesq approximations [9]:

- the properties of the fluid are taken to be constant: $\rho = \rho_0$, $\mu = \mu_0$, $c_p = c_{p0}$ and $\lambda = \lambda_0$;

- the buoyancy term $(\rho - \rho_0)\vec{g}$ in the equation of motion is replaced by the term $\rho_0\beta_0(T_0 - T)\vec{g}$, where T_0 is the reference temperature of the medium;
- the medium is Newtonian and Fourier's law holds.

Taking these approximations into account the dimensionless equations read:
Continuity:

$$\vec{\nabla}\bullet\vec{u} = 0 \tag{1}$$

Momentum:

$$Sr\frac{\partial\vec{u}}{\partial t} + \vec{u}\bullet\vec{\nabla}\vec{u} + \vec{\nabla}p - \frac{1}{Re}\vec{\nabla}^2\vec{u} + \frac{Gr}{Re^2}T\vec{g} = \vec{0} \tag{2}$$

Energy:

$$Sr\frac{\partial T}{\partial t} + \vec{u}\bullet\vec{\nabla}T - \frac{1}{RePr}\vec{\nabla}^2T = 0 \tag{3}$$

with $\vec{u}$ the velocity, p the pressure, T the temperature, $\vec{g}$ the gravitational unit vector, Sr the Strouhal number, Re the Reynolds number ($Re = \frac{U\cdot d}{\nu}$ with U the mean velocity and d the diameter of the bend), Gr the Grashof number ($Gr = \frac{g\cdot\beta}{\nu^2}\cdot\Delta T\cdot d^3$ with ΔT the temperature difference between the fluid and the wall at the inlet) and Pr the Prandtl number. The Grashof number expresses the relative importance of the buoyancy forces. Instead of the Reynolds number in curved pipe flows often the Dean number Dn is used ($Dn = Re\sqrt{\delta}$; δ being the curvature ratio), expressing the relative importance of the centrifugal forces.

From a simplified force balance one may deduce that in the helical coiled tube laminar flow ($Re = O(10^2 - 10^3)$) is highly influenced by centrifugal ($Dn = O(10^2)$) and buoyancy effects ($Gr = O(10^6)$). Therefore, secondary vortices induced by centrifugal and buoyancy forces play an important role in the total heat transfer from the collector medium towards the mains water. In the storage vessel heat transfer takes place by thermal plumes rising from the tube wall. In a stratified storage these plumes are typically in the regime where transition takes place from a laminar state towards a turbulent one ($Ra = O(10^9 - 10^{11})$), leading to enhanced mixing.

3. Coiled heat exchanger with isothermally cooled wall

To study steady flow in the helical coiled tube the full elliptic equations are solved using the Finite Element Method (FEM) [5]. In combination with the direct solver used in the present study, it is only possible to solve the flow in the most proximate part of the heat exchanger [11]. The flow field in the complete coil may be resolved by using a segmented elliptic procedure. The coil is then covered by a number of axially partly overlapping domains (segments), in which a solution of the elliptic equations is computed, using the solution on the previous segment to prescribe the inflow condition for the present segment. Another way to calculate the flow in the complete coil is to solve the parabolized equations. When it is known that the flow has a dominant direction, the momentum and energy equations can be parabolized in this so-called streamwise direction. The major advantage of parabolizing the equations is the opportunity to use a marching procedure: each cross-section is treated separately, using information of the previous one [10].

The elliptic code is used to study mixed convection in the entrance section of a curved pipe. The domain consists of a $2d$ straight inflow section and a $11d$ (90^o) curved section with curvature ratio $\delta = 1/14$. At the entrance ($x = -2d$; x being the axial coordinate) the Hagen-Poiseuille velocity profile for fully developed pipe flow is assumed. At the tube wall no-slip conditions for the velocity are prescribed. At the outlet ($x = 11d$) the stress-free boundary conditions are used for the momentum equations and the zero heat flux condition for the energy equation. The dimensionless temperature of the inflowing medium is set to $T_{inlet} = 1$. The medium is cooled at the tube wall to $T_{wall} = 0$. At the wall a gradual decrease of the temperature from $T = 1$ to $T = 0$ is prescribed, a half cosine function from $x = -\frac{1}{2}d$ to $x = +\frac{1}{2}d$, to prevent a singularity at $x = 0d$. The domain was divided into 60×23 (cross-section $\times$ axial) tri-quadratic elements for the momentum equation and 124×40 tri-quadratic elements for the energy equation (due to the smaller thermal boundary layers).

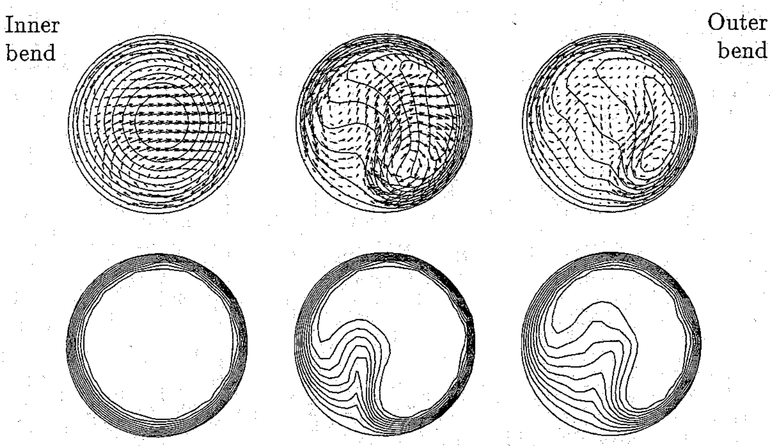

Figure 2. cross-section at $x = d$ (left), $x = 4d$ and $x = 8d$ (right) for $Re = 500$, $Dn = 134$, $Pr = 5$ and $Gr = 10^5$; elliptically computed isotachs and secondary velocity vectors (top) and isotherms (bottom)

In figure (2) the velocity field and the temperature field is shown as obtained from the elliptic computations for $Re = 500$, $Gr = 10^5$ and $Pr = 5$ at three axial positions: $x = d$ (at the left), $x = 4d$ and $x = 8d$ (at the right). The velocity field is visualized by isotachs for the axial component and by velocity vectors for the secondary components. The difference in velocity magnitude between the different isotachs equals 0.1 $\overline{u}_{ax}$. The scaling of the secondary flow is such that a vector length equal to the diameter of the pipe corresponds to twice the average axial velocity. The development of the temperature field is visualized by contours of equal temperature. The difference between the successive isotherms is 0.1, where the temperature at the wall is equal to 0 and at the inlet equal to 1.

At $x = 4d$ and $x = 8d$ it is clearly seen how buoyancy effects influence the secondary velocity field. Because the medium is cooled at the wall , resulting in a higher density there, the medium experiences a downward force near the wall. Consequently, in the core region the medium flows upwards. This phenomenon is not clearly visible at $x = d$ because here the centrifugal effects are still dominant. The highest secondary velocities are found near the upper and bottom wall of the tube. At $x = 8d$ the existence of a third vortex is seen near the center of the tube. This three-vortex solution is also found experimentally in air [4]. Besides, the magnitude of the secondary velocities at $x = 8d$ is considerably smaller than the magnitude at $x = 4d$. The influence of secondary flow on axial flow is clearly visible in a shift of the maximum of axial velocity towards the outer bend. The characteristic C-shape in the isotachs observed when only centrifugal effects are present, is severely deteriorated by buoyancy effects and by the third vortex in the core region. Also the isotherms clearly visualize the influence of the secondary flow field, resulting in large gradients at the upper-right regions and small gradients at the lower-left regions of the tube wall.

In figure (3) the dimensionless bulk temperature is displayed for the considered case of cooled water in a curved pipe at various Grashof numbers. Initially the dotted curve 0 for forced convection in a straight pipe is followed. At about $x = 2d$ the curved pipe lines $I - IV$ start to deviate from line 0, due to the manifestation of the secondary flow. The water loses its thermal energy more readily in a curved bend than in a straight tube, and also buoyancy effects on their turn show to encourage heat transfer.

The parabolic code is used to study mixed convection in a coiled heat exchanger with an isothermally cooled wall. The boundary conditions are the same as mentioned for the elliptic calculations. The domain with a length of about 200 tube diameters was divided into $40 \times 180 \times 7500$ nodes in radial, tangential and axial direction, respectively. In figure (4) the velocity field and the temperature field is presented as obtained from the parabolized equations for $Re = 500$, $Gr = 10^5$ and $Pr = 5$ at three axial

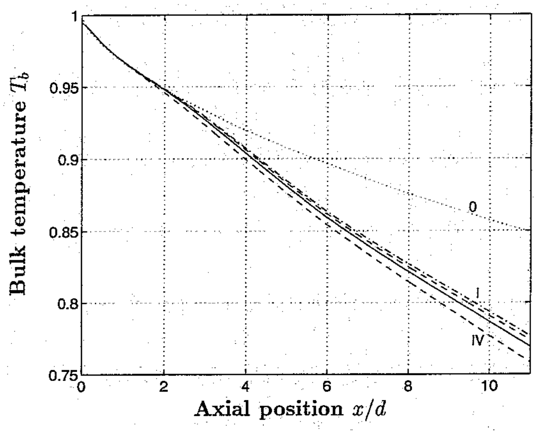

Figure 3: Dimensionless bulk temperature for $Re = 500$, $Dn = 134$, $Pr = 5$ and for various Grashof numbers: I: $Gr = 0$; II: $Gr = 5 \cdot 10^4$; III: $Gr = 10^5$; IV: $Gr = 2 \cdot 10^5$. As a reference also the bulk temperature number for forced convection in a straight pipe is plotted.

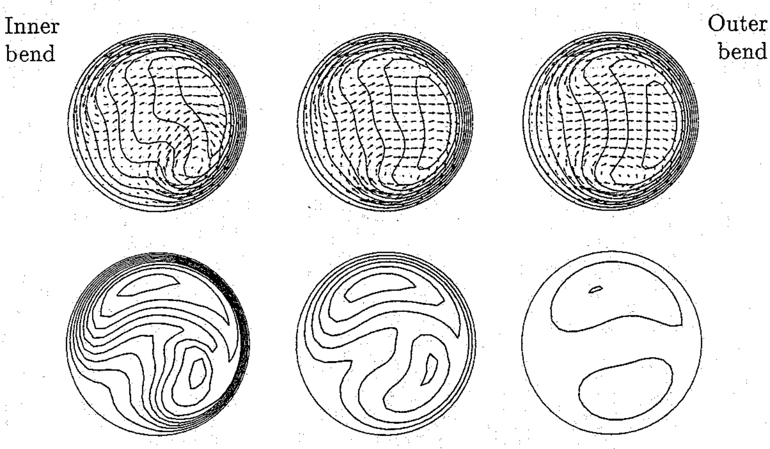

Figure 4: cross-section at $x = 20d$ (left), $x = 50d$ and $x = 100d$ (right) for $Re = 500$, $Dn = 134$, $Pr = 5$ and $Gr = 10^5$; parabolically computed isotachs and secondary velocity vectors (top) and isotherms (bottom)

positions: $x = 20d$ (at the left), $x = 50d$ and $x = 100d$ (at the right). From the secondary flow field it can be seen that the third vortex is not present anymore. Besides, the secondary velocities are much lower than in the entrance section due to the decreasing importance of buoyancy forces. At $x = 100d$ the flow configuration is almost symmetric and two Dean-type vortices can be observed due to centrifugal forces. At this position buoyancy effects play only a minor role because most of the heat is already transferred. This is also clearly visualized by the isotherms, showing that the bulk temperature is almost equal to the wall temperature.

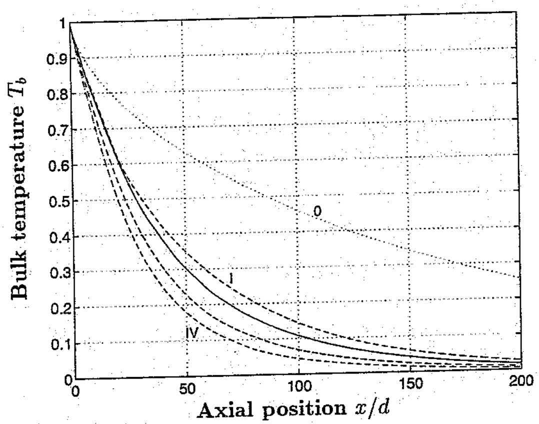

Figure 5. Dimensionless bulk temperature as a function of axial position in an isothermally cooled coil for $Re = 500$, $Dn = 134$ and $Pr = 5$: - - - (I): $Gr = 0$; —: $Gr = 10^5$; ---: $Gr = 5 \cdot 10^5$; - · - (IV): $Gr = 10^6$. As a reference also the bulk temperature number for developing forced convection is plotted as the dotted line 0.

In figure (5) the bulk temperature is given as function of axial position for various Grashof numbers. It can be concluded that higher Grashof numbers lead to enhanced heat transfer and that buoyancy effects play only a minor role for axial positions larger than $100d$. To transfer 90 % of the heat for $Gr = 10^5$ about 100 pipe diameters are needed, whereas for $Gr = 10^6$ about 70 pipe diameters are needed.

4. Large-Eddy Simulations of thermal plumes

Beside the heat transfer characteristics of the coiled heat exchanger, the performance of a SDHWS is also determined by the thermal mixing behaviour in the heat storage vessel. This thermal mixing is influenced by effects like stratification in the heat storage vessel and transition of the thermal plumes. To study those phenomena Direct Numerical Simulations (DNS) and Large-Eddy Simulations (LES) are aimed at. In DNS calculations all scales (both in space and time) are resolved using very fine grids and small time steps. For discretization of the equations the Spectral Element Method (SEM) is used, a high order Galerkin type of method [8]. To avoid the large calculation times involved with DNS, in LES calculations the governing equations are spatially filtered using a so-called SubGrid-Scale model [2]. With this method only the large scales (large eddies) are resolved, whereas the small scales are modelled. In the present study these LES calculations are performed using the Finite Volume Method (FVM).

Particle Tracking Velocimetry (PTV) is a method for measuring the velocity field in a fluid by means of keeping track of individual particles suspended in the fluid [1]. The advantage of the method over methods like Laser-Doppler Anemometry is that it is possible to measure $2D$ velocity components in a $2D$ cross section of a flow field. In this study the PT facility of the computer package *DigImage* is used. Details of the method as implemented in *DigImage* can be found in [6] and [7]. In PT experiments the flow configuration is intersected by a thin light sheet. The particles suspended in the fluid reflect the light. This reflection is recorded and the video system used is capable of grabbing each field of the recording separately. When each particle in the present field can be matched exactly to a particle in the next field, it is possible to determine the average velocity of that particle. For optimal matching it is required that the particles stay within the light sheet for several recordings.

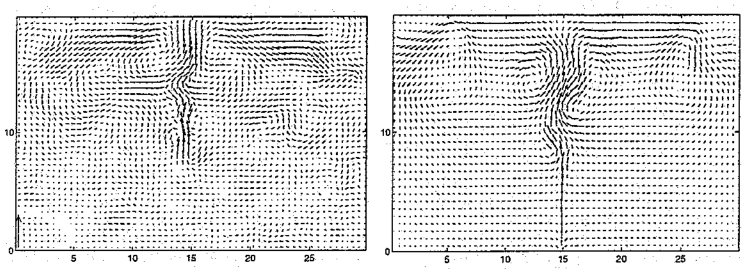

Figure 6. velocity field in a confined enclosure with a heat source at the bottom; experiments using PTV (left; $Ra \approx 10^{11}$) and calculations using LES (right; $Ra \approx 10^{12}$)

A typical result is given in figure (6) where a heat source (a vertically positioned strip) is placed near the bottom of a confined enclosure. At the left the velocity field is presented at $Ra \approx 10^{11}$ (Ra being the Rayleigh number) as obtained with PTV. At the right the result is shown of a LES calculation at $Ra \approx 10^{12}$. For both situations initial stratification was not present. Both methods give detailed insight into the flow phenomena occurring like the characteristic structures (for example the formation of a dipole at the initial phase of the experiment), like the characteristic frequencies (of the swaying motion), like the turbulent intensities, and so forth. At this moment efforts are undertaken to quantify the results as function of the Rayleigh number.

5. Conclusions

In the present paper tools used to study mixed convection and thermal plumes in a SDHWS are shown. Elliptic calculations with the Finite Element Method of the flow in the coiled heat exchanger show great detail in the flow phenomena occurring but are also very time consuming. For a certain range of Reynolds and Grashof numbers it is possible to analyze mixed convection flows using the parabolic solution procedure. Results are presented for an isothermally cooled wall but more complicated boundary conditions should be investigated. Especially the influence of stratification effects in the heat storage vessel are of interest. For the performance of a SDHWS also the thermal mixing behaviour in the heat storage vessel is of large importance, which on its turn is highly influenced by the transition behaviour of thermal plumes. Particle Tracking Velocimetry and Large-Eddy Simulations are powerful tools to study this phenomenon.

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