

## Near-optimal heuristics to set base stock levels in a two-echelon distribution network

**Citation for published version (APA):**

Basten, R. J. I., & Houtum, van, G. J. J. A. N. (2010). *Near-optimal heuristics to set base stock levels in a two-echelon distribution network*. (BETA publicatie : working papers; Vol. 324). Technische Universiteit Eindhoven.

**Document status and date:**

Published: 01/01/2010

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

**General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.tue.nl/taverne](http://www.tue.nl/taverne)

**Take down policy**

If you believe that this document breaches copyright please contact us at:

[openaccess@tue.nl](mailto:openaccess@tue.nl)

providing details and we will investigate your claim.

## **Near-optimal heuristics to set base stock levels in a two-echelon distribution network**

R.J.I. Basten, G.J. van Houtum

Beta Working Paper series 324

BETA publicatie	WP 324 (working paper)
ISBN	978-90-386-2357-3
ISSN	
NUR	804
Eindhoven	September 2010

# Near-optimal heuristics to set base stock levels in a two-echelon distribution network

R.J.I. Basten\* • G.J. van Houtum

*Eindhoven University of Technology, School of Industrial Engineering*

*P.O. Box 513, 5600 MB, Eindhoven, The Netherlands*

## **Abstract**

We consider a continuous-review two-echelon distribution network with one central warehouse and multiple local stock points, each facing independent Poisson demand for one item. Demands are fulfilled from stock if possible and backordered otherwise. We assume base stock control with one-for-one replenishments and the goal is to minimize the inventory holding and backordering costs. Although this problem is widely studied, only enumerative procedures are known for the exact optimization. A number of heuristics exist, but they find solutions that are far from optimal in some cases (over 20% error on realistic problem instances). We propose a heuristic that is computationally efficient and finds solutions that are close to optimal: 0.1% error on average and less than 3.0% error at maximum on realistic problem instances in our computational experiment.

**Keywords: Service logistics, spare parts inventories, heuristic, two-echelon, distribution network**

---

\*Corresponding author. Tel.: +31 40 247 2601; fax: +31 40 247 4531  
*E-mail addresses:* r.j.i.basten@tue.nl, g.j.v.houtum@tue.nl

# 1 Introduction

Capital goods are expensive, technologically advanced products or systems that are critical for the companies that are using them. Examples are MRI-scanners in hospitals, baggage handling systems at airports, or radar systems on board naval vessels. Since the availability of these systems is so important, spare parts are generally located close to the installed base. However, since spare parts are often expensive, also more central stocking locations at higher echelon levels are used to use pooling effects. Although in principle any number of echelon levels is possible, spare parts networks for capital goods often have a two-echelon structure (Cohen et al., 1997). In such networks, many different items are stocked: Gallego et al. (2007) give the example of General Motors' spare parts organization, which manages over four million stock-keeping units serving eight thousand dealers and is resetting stock allocations daily. Because of the number of components that is involved and the number of stock points in the network, efficient heuristics are required to set stock levels. In this paper, we develop two such heuristics. Although this work is motivated by problems in the area of spare parts networks for capital goods, this work may be applied in any situation in which there is a distribution network in which large numbers of low-demand items are stocked. If replenishment intervals are short, it may even be applied in higher-demand settings.

Because of the practical relevance, there has been a lot of interest in distribution networks, starting with the seminal papers by Clark and Scarf (1960) for *serial* ( $N$ -echelon) *networks*, which are networks in which each stock point has at most one successor and at most one predecessor, and Sherbrooke (1968) for general *distribution networks*, which are networks in which each stock point has (at most) one predecessor and which are the networks in which we are interested.

The paper by Clark and Scarf (1960) started the research on periodic-review models. The optimal policy is not known for these models, but Eppen and Schrage (1981) introduced the *balance assumption* which effectively means that the inventory position of a

local stock point immediately after ordering is allowed to be lower than the inventory position just before ordering. In other words, after ordering, the inventory is balanced over the various stock points again, even if it was unbalanced before ordering. Using this assumption, the optimal policy can be characterized under various circumstances, see, e.g., Diks and de Kok (1998, 1999) or van der Heijden et al. (1997).

We, however, consider continuous-review and we assume base stock control and FCFS allocation at the central warehouse, which means that no central information is required. This stream of research started with the paper by Sherbrooke (1968). An overview can be found in Sherbrooke (2004) or Muckstadt (2005). For a more general overview of literature on multi-echelon inventory systems, we refer to van Houtum (2006).

Here, we only discuss the most relevant literature in more detail: the literature on continuous-review distribution networks with Poisson demand processes, base stock control, and backordering, in which a penalty is paid for items that are on backorder (as opposed to models in which there is a service constraint). First, we discuss two papers in which the model that we use is evaluated exactly and second, we discuss a number of papers in which heuristics are developed to optimize the base stock levels.

The model that we consider is identical to the model studied by Axsäter (1990). He gives recursive formulas to evaluate the inventory holding and penalty costs. Furthermore, he finds that the costs are convex in each of the local base stock levels, but not in the central base stock level. Axsäter gives a lower and upper bound on the central stock level, between which he enumerates all possible values to set optimal base stock levels in the network. Because of his recursive formulas, this enumeration is still relatively fast, e.g., in comparison with the exact evaluation that Graves (1985) proposes.

Graves (1985) develops both an exact and approximate procedure to determine the distribution of the items in inventory and on backorder at the local stock points. If items are on backorder at the central warehouse, they delay the replenishments at the local stock points. In the exact evaluation, Graves (1985) has to calculate a convolution which is computationally intensive. In the approximate procedure, he fits a negative

binomial distribution on the first two moments of the distribution of the number of items on backorder at the central warehouse. Optimization of the base stock levels is not considered.

Graves' approximate evaluation procedure can be seen as an extension of Sherbrooke's (1968) METRIC method, since Sherbrooke approximates the number of components on backorder at the central warehouse using a Poisson distribution, so using one moment only (the Poisson distribution has the same mean and variance). The one-moment approximation by Sherbrooke and the two-moment approximation by Graves can both be extended to the general multi-echelon (and multi-indenture) case.

Heuristics for this problem exist as well. They are important because exact optimization is often not possible in practice due to the number of components and the size of the networks involved. Gallego et al. (2007) compare FCFS allocation with other forms of allocation in which central information is required. Although this is very relevant in certain business situations, we are interested in FCFS allocation only here. Gallego et al. (2007) develop the *restriction-decomposition* heuristic (*RD*), in which they restrict the central base stock level to have one of three possible values: zero ('cross-docking'), the expected demand at the central warehouse ('zero safety stock'), or the maximum value at the central warehouse if each local stock point would hold no inventory ('stock-pooling'). Using each of these three values, the base stock levels at each of the local stock points can be optimized independently using simple newsvendor equations.

Rong et al. (2010) propose two different heuristics for distribution systems with a general number of echelon levels. In the *recursive optimization* (*RO*) heuristic, first the base stock levels of all local stock points (lowest echelon level) are optimized assuming that there are no delays at higher echelon levels. These problems reduce to simple newsvendor equations. Then the next higher echelon's base stock levels are optimized using the base stock level at the lowest echelon level as an input. This cost function is convex and can be solved easily. Then the next higher echelon level can be solved (if existent), etc.

In the second heuristic of Rong et al., the *decomposition-aggregation* (*DA*) heuristic,

the problem is decomposed into one serial system per local stock point. Each of these serial systems consists of the path from the most central location to (and including) the local stock point. Such serial systems can be solved exactly, but for sake of optimization time, Rong et al. (2010) use newsvendor approximations of Shang and Song (2003). Each non-local stock point is part of a number of serial systems. The backorders at such a stock point that result from the optimization of each of the serial systems are summed (aggregated) and it is determined which base stock level is required at that stock point to achieve the summation of the backorders. This is the final base stock level. Rong et al. (2010) compare their heuristics with the heuristic by Gallego et al. (2007); we show this comparison for the two-echelon case in Section 5, where we also compare our results with the results of Rong et al..

Our main contribution is that we develop a heuristic that finds base stock levels that lead to costs that are much closer to optimal and is still faster than most existing heuristics. Recall that such a heuristic is useful in practice where one has to control large numbers of components. A second contribution is that we derive new properties of the total cost function.

The remainder of this paper is structured as follows. In Section 2, we give the mathematical problem formulation and in Section 3, we show some analytical properties of the cost function. In Section 4, we explain the heuristics that we develop and we show the results of our computational experiment in Section 5. We conclude in Section 6 with conclusions and recommendations for further research.

## 2 Problem description and notation

We consider a two-echelon distribution network consisting of one central warehouse and  $N$  local stock points. The central warehouse has index 0 and the local stock points are indexed  $1, \dots, N$ .

The demand process at local stock point  $i$  is a Poisson process with rate  $\lambda_i (> 0)$ . A

demand is backordered if it cannot be fulfilled immediately. Local stock point  $i$  uses a base stock policy with basestock level  $S_i(\geq 0)$ . Hence, each demand at a local stock point is immediately followed by a replenishment order at the central warehouse. As a result, the demand process at the central warehouse is also a Poisson process, with rate  $\lambda_0 = \sum_{i=1}^N \lambda_i$ . Demands that cannot be fulfilled immediately at the central warehouse are backordered and fulfilled first come first served (FCFS). The central warehouse uses a base stock policy with basestock level  $S_0(\geq 0)$ .

The central warehouse orders components at an external supplier with infinite supply or repairs components using an uncapacitated repair facility (this is equivalent). The total lead time from the occurrence of a demand until the central warehouse has a new or repaired component is an i.i.d. random variable with mean  $L_0(> 0)$ . Each shipment from the warehouse to local stock point  $i$  takes  $L_i(> 0)$  time units (deterministic).

We consider holding costs  $h_0(> 0)$  at the central warehouse and  $h_i(> 0)$  at each of the local stock points  $i$  for the time that a spare part is in stock (considering holding costs for spare parts in transit is straightforward since the average number of spare parts in transit is a constant given the fact that we use backordering). Furthermore, we consider penalty costs  $\beta_i(> 0)$  per unit per time unit for backordered demand at local stock point  $i$ .

We need the following additional notation for the cost function that we want to minimize and the analysis in Section 3. The number of items on order at the central warehouse is  $X_0$ . This number is Poisson-distributed with parameter  $\lambda_0 L_0$ . The corresponding number of backorders is  $B_0(S_0) = (X_0 - S_0)^+$  and the number of items on hand is  $I_0(S_0) = (S_0 - X_0)^+$ .  $B_0^{(i)}(S_0)$  is the number of backorders at the central warehouse that is due to local stock point  $i$  ( $\sum_{i=1}^N B_0^{(i)}(S_0) = B_0(S_0)$ ). The number of items on order at local stock point  $i$ ,  $X_i(S_0)$ , is equal to the number of demands over the leadtime,  $Y_i$ , which is Poisson-distributed with parameter  $\lambda_i L_i$ , plus the number of backorders at the central warehouse that are due to stock point  $i$ ,  $B_0^{(i)}(S_0)$  ( $Y_i$  and  $B_0^{(i)}(S_0)$  are mutually independent). The corresponding number of backorders is  $B_i(S_0, S_i) = (X_i(S_0) - S_i)^+$  and the number of items on hand is  $I_i(S_0, S_i) = (S_i - X_i(S_0))^+$ . Finally,  $Z_i$  is a Bernoulli-



distributed random variable for all  $i \in \{1, \dots, N\}$ :  $Z_i = 1$  with probability  $\frac{\lambda_i}{\lambda_0}$  and 0 otherwise. The interpretation is that  $Z_i = 1$  if the first backorder in the queue at the central warehouse is due to local stock point  $i$ . However, it is also defined if there is no queue at the central warehouse (although the interpretation is not clear then).

The goal of the optimization is to set the base stock levels  $S_i$  for  $i \in \{0, \dots, N\}$  such that total average costs per time unit in steady state are minimized:

$$C(S_0, S_1, \dots, S_N) = h_0 \mathbb{E}I_0(S_0) + \sum_{i=1}^N [h_i \mathbb{E}I_i(S_0, S_i) + \beta_i \mathbb{E}B_i(S_0, S_i)] \quad (1)$$

### 3 Analytical properties

In this section, we give some properties of the cost function. However, these properties are not strong enough to give an easy way to optimize this function. For example, the cost function is not convex in  $S_0$  nor multi-modular in general. What we can show is as follows:

**Lemma 1.** *The cost function  $C(S_0, S_1, \dots, S_N)$  is supermodular in  $(S_0, S_i)$  for all  $i \in \{1, \dots, N\}$ , which means that:*

$$\begin{aligned} & C(S_0, S_1, \dots, S_i - 1, \dots, S_N) - C(S_0 + 1, S_1, \dots, S_i - 1, \dots, S_N) \\ & \geq C(S_0, S_1, \dots, S_i, \dots, S_N) - C(S_0 + 1, S_1, \dots, S_i, \dots, S_N) \end{aligned}$$

*Proof.* See Axsäter (1990, p. 68, Equations (29) and (31); notice that he does not use the term supermodularity). □

This supermodularity property implies that if costs can be reduced by adding one spare at the central warehouse for a given  $S_i$  (right hand side), costs can certainly be reduced by adding one spare at the central warehouse if  $S_i$  is one lower (left hand side).

Before we can show that the cost function is convex in  $S_i$  for all  $i \in \{1, \dots, N\}$ , we need an additional lemma:

**Lemma 2.** For  $i \in \{1, \dots, N\}$ , it holds that:

$$\mathbb{E}I_i(S_0, S_i + 1) - \mathbb{E}I_i(S_0, S_i) = \mathbf{P}\{B_i(S_0, S_i) = 0\} \quad (2)$$

$$\mathbb{E}B_i(S_0, S_i + 1) - \mathbb{E}B_i(S_0, S_i) = -\mathbf{P}\{B_i(S_0, S_i) > 0\} \quad (3)$$

*Proof.* Equation (2) can be proven as follows:

$$\begin{aligned} & \mathbb{E}I_i(S_0, S_i + 1) - \mathbb{E}I_i(S_0, S_i) \\ &= \sum_{x=0}^{S_i} (S_i + 1 - x) \mathbf{P}\{X_i(S_0) = x\} - \sum_{x=0}^{S_i-1} (S_i - x) \mathbf{P}\{X_i(S_0) = x\} \\ &= \sum_{x=0}^{S_i} \mathbf{P}\{X_i(S_0) = x\} = \mathbf{P}\{X_i(S_0) \leq S_i\} = \mathbf{P}\{B_i(S_0, S_i) = 0\} \end{aligned}$$

The proof of Equation (3) is analogous. □

**Lemma 3.** The cost function  $C(S_0, S_1, \dots, S_N)$  is convex in  $S_i$  for all  $i \in \{1, \dots, N\}$ .

*Proof.* This can be seen by looking at the difference function:

$$\begin{aligned} & \Delta_i C(S_0, S_1, \dots, S_N) \\ &= C(S_0, S_1, \dots, S_i + 1, \dots, S_N) - C(S_0, S_1, \dots, S_i, \dots, S_N) \\ &= h_i [\mathbb{E}I_i(S_0, S_i + 1) - \mathbb{E}I_i(S_0, S_i)] + \beta_i [\mathbb{E}B_i(S_0, S_i + 1) - \mathbb{E}B_i(S_0, S_i)] \\ &= h_i \mathbf{P}\{B_i(S_0, S_i) = 0\} - \beta_i \mathbf{P}\{B_i(S_0, S_i) > 0\} \\ &= -\beta_i + (\beta_i + h_i) \mathbf{P}\{B_i(S_0, S_i) = 0\} \end{aligned}$$

Since  $\mathbf{P}\{B_i(S_0, S_i) = 0\}$  is strictly increasing in  $S_i$ ,  $\Delta_i C(S_0, S_1, \dots, S_N)$  is strictly increasing, so the cost function is convex in  $S_i$ . □

Axsäter (1990) mentions this result, without the deduction.

**Corollary 4.** *The optimal  $S_i$  for a given  $S_0$ ,  $S_i^*(S_0)$ , is the smallest  $S_i$  such that:*

$$\mathbf{P}\{B_i(S_0, S_i) = 0\} \geq \frac{\beta_i}{\beta_i + h_i} \quad (4)$$

*Proof.* This follows directly from Lemma 3.  $\square$

**Lemma 5.** *If  $S_0$  is increased by one, then for all  $i \in \{1, \dots, N\}$  the optimal  $S_i$ ,  $S_i^*(S_0)$ , stays the same or decreases by one:*

$$S_i^*(S_0) - 1 \leq S_i^*(S_0 + 1) \leq S_i^*(S_0)$$

*Proof.* The first inequality can be proven as follows. It is easily seen that  $B_0(S_0 + 1) \geq_{\text{st}} B_0(S_0) - 1$ , where  $A \geq_{\text{st}} \tilde{A}$  denotes that a random variable  $A$  is stochastically larger than another random variable  $\tilde{A}$ . This implies that  $B_0^{(i)}(S_0 + 1) \geq_{\text{st}} B_0^{(i)}(S_0) - 1$ , and thus  $Y_i + B_0^{(i)}(S_0 + 1) \geq_{\text{st}} Y_i + B_0^{(i)}(S_0) - 1$ , which is equivalent to  $X_i(S_0 + 1) \geq_{\text{st}} X_i(S_0) - 1$ . Hence, in particular  $\mathbf{P}\{X_i(S_0 + 1) \leq S_i^*(S_0) - 2\} \leq \mathbf{P}\{X_i(S_0) - 1 \leq S_i^*(S_0) - 2\}$ , which is equivalent to  $\mathbf{P}\{X_i(S_0 + 1) \leq S_i^*(S_0) - 2\} \leq \mathbf{P}\{X_i(S_0) \leq S_i^*(S_0) - 1\}$ . Since  $S_i^*(S_0)$  is the smallest  $S_i$  for which Equation (4) holds, we know that Equation (4) does not hold for  $S_i^*(S_0) - 1$ . In other words,  $\mathbf{P}\{X_i(S_0) \leq S_i^*(S_0) - 1\} < \frac{\beta_i}{\beta_i + h_i}$ , and therefore  $\mathbf{P}\{X_i(S_0 + 1) \leq S_i^*(S_0) - 2\} < \frac{\beta_i}{\beta_i + h_i}$ . This means that  $S_i^*(S_0 + 1) > S_i^*(S_0) - 2$  or, equivalently,  $S_i^*(S_0 + 1) \geq S_i^*(S_0) - 1$ .

The second inequality can be proven analogously, but follows also directly from the supermodularity of the cost function.  $\square$

The cost function  $C(S_0, S_1, \dots, S_N)$  is not convex in  $S_0$  in general. For given values of  $S_i$  for all  $i \in \{1, \dots, N\}$ , the cost function generally decreases with an increasing  $S_0$  until a minimum is reached. It then increases such that the derivative becomes close to  $\sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i$  after which the derivative converges to  $h_0$ . Therefore, if  $h_0 < \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i$ , the cost function is generally first convex in  $S_0$  and then concave in  $S_0$ . This suggests that the function  $C(S_0, S_1, \dots, S_N)$  is unimodal in  $S_0$ , but we are not able to prove this. However,

lemmas 6 and 8 give two cases in which the cost function is convex in  $S_0$ .

**Lemma 6.** *If  $S_i = 0$  for all  $i \in \{1, \dots, N\}$ , then the cost function  $C(S_0, S_1, \dots, S_N)$  is convex in  $S_0$  and its corresponding difference function is as follows:*

$$\Delta_0 C(S_0, 0, \dots, 0) = h_0 \mathbf{P}\{B_0(S_0) = 0\} + \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} [-\beta_i + \beta_i \mathbf{P}\{B_0(S_0) = 0\}] \quad (5)$$

*Proof.* See Appendix A. □

**Corollary 7.** *The  $S_0$  that minimizes Equation (5) in Lemma 6,  $S_0^u$ , is the smallest  $S_0$  such that:*

$$\mathbf{P}\{B_0(S_0) = 0\} \geq \frac{\sum_{i=1}^N \frac{\lambda_i}{\lambda_0} \beta_i}{\sum_{i=1}^N \frac{\lambda_i}{\lambda_0} \beta_i + h_0}$$

$S_0^u$  is an upper bound for  $S_0$ .

*Proof.* The first part follows directly from Lemma 6. Because of the supermodularity of the cost function,  $S_0^u$  is an upper bound for  $S_0$ . □

The upper bound that we have thus derived is at least as high as the upper bound that Axsäter (1990, p. 68) derives. However, the computational costs are lower and it turns out that the upper bounds are not too far apart (a few percent on larger problem instances).

**Lemma 8.** *The cost function  $C(S_0, S_1, \dots, S_N)$  is convex in  $S_0$  if  $h_0 \geq \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i$ .*

*Proof.* See Appendix B. □

Notice that this includes the important case that  $h_0 = h_i$  for all  $i \in \{1, \dots, N\}$ .

## 4 Heuristics

We propose two heuristics. The first heuristic, called *Smart Enumeration (SE)* is not very fast, but in a computational experiment (see Section 5), it always finds the optimal solution. The second heuristic, called *Step and Check (SC)* is very fast and will be shown to be near-optimal via a computational experiment.

## 4.1 Smart Enumeration

The basic idea of the SE heuristic is to start at an easily found upper bound for  $S_0$ , use the fact that  $S_i^*(S_0) \geq S_i^*(S_0 + 1) \geq S_i^*(S_0) - 1$ , decrease  $S_0$  by one at a time, and assume that the function  $C(S_0, S_1^*(S_0), \dots, S_N^*(S_0))$  is almost unimodal (although it is not) so that we know when to stop.

In pseudo-code:

1. best  $S_0 := S_0 := S_0^u$  (using Corollary 7)
2.  $S_i := S_i^*(S_0)$  (for all  $i \in \{1, \dots, N\}$ , using Corollary 4)
3. best costs  $:= C(S_0, S_1, \dots, S_N)$  (using Equation (1))
4. counter  $:= 0$
5.  $S_0 := S_0 - 1$
6.  $S_i := S_i^*(S_0)$  (for all  $i \in \{1, \dots, N\}$ , using Lemma 5 and Corollary 4)
7. new costs  $:= C(S_0, S_1, \dots, S_N)$  (using Equation (1))
8. Compare new costs with best costs:
  - If new costs  $\leq$  best costs:
    - counter  $:= 0$
    - best costs  $:=$  new costs
    - best  $S_0 := S_0$
    - Go to step 5
  - Otherwise:
    - If counter  $\leq N + 1$ :
      - \* counter  $:=$  counter + 1
      - \* Go to step 5
    - Otherwise:
      - \* Stop

The stopping criterion is the critical part of this heuristic. Stopping already when counter  $\leq N$  leads to a suboptimal solution in a few cases.

## 4.2 Step and Check

The basic idea of the SC heuristic is to start at an upper bound for  $S_0$ , decrease  $S_0$  in steps of size  $N$  and assume that the function  $C(S_0, S_1^*(S_0), \dots, S_N^*(S_0))$  is almost unimodal (although it is not) so that we know when to stop. When we have found a presumed local optimum, we perform a local search around this local optimum.

At each step we check the costs using the two-moments approximation of Graves (1985). We could use another approximating distribution than the negative binomial distribution, but our results are so satisfactory that we do not need to consider that. We could also calculate the costs exactly using Graves (1985), but this is too time-consuming.

In pseudo-code:

1. step size :=  $N$
2. best costs :=  $bigM$
3.  $S_0 := S_0^u$  (using Corollary 7)
4. Fit a negative binomial distribution to the first two moments of  $X_i(S_0)$  (for all  $i \in \{1, \dots, N\}$ , using Graves, 1985)
5.  $S_i := S_i^*(S_0)$  (for all  $i \in \{1, \dots, N\}$ , using Corollary 4)
6. new costs :=  $C(S_0, S_1, \dots, S_N)$  (using Equation (1))
7. Compare new costs with best costs:
  - If new costs  $\leq$  best costs:
    - best costs := new costs
  - Otherwise:
    - $S_0 := S_0 + \text{step size}$
    - Go to step 10
8.  $S_0 := S_0 - \text{step size}$
9. Go to step 4
10. Check step-size:
  - If step size  $> 1$ :
    - step size :=  $\lceil \frac{\text{step size}}{2} \rceil$ :

- Otherwise:
- Stop
11.  $S_0 := S_0 + \text{step size}$
  12. Perform Steps 4, 5, and 6
  13. Compare new costs with best costs:
    - If new costs  $\leq$  best costs:
      - best costs := new costs
      - Go to step 10
    - Otherwise:
      - Go to step 14
  14.  $S_0 := S_0 - 2 \cdot \text{step size}$
  15. Perform Steps 4, 5, and 6
  16. Compare new costs with best costs:
    - If new costs  $\leq$  best costs:
      - best costs := new costs
      - Go to step 10
    - Otherwise:
      - $S_0 := S_0 + \text{step size}$
      - Go to step 10

Steps 1 to 9 are used to find a local optimum. The ‘otherwise’ clause in Step 7 will not be used in the first iteration of the algorithm (when  $S_0 = S_0^u$ ). When the clause is used, we have found a solution that is not as good as the local optimum. Therefore, we add ‘step size’ to  $S_0$  so that it is the local optimum again.

In step 10, we divide the step size in half. We then check whether using  $S_0 + \text{step size}$  would lead to a better result than using  $S_0$  (our local optimum), and if so, we use  $S_0 + \text{step size}$  as the new local optimum and go back to step 10. Otherwise, we check whether  $S_0 - \text{step size}$  would lead to a better result. If so, this is the new local optimum. In both cases, we go back to step 10.

## 5 Computational experiment

We perform an extensive computational experiment to examine the performance of our heuristics, both in terms of optimization time and quality of the solution. For the quality of the solution, we use two indicators. The *average error* is defined as (using  $P$  problem instances, numbered  $1, \dots, P$ ):  $\frac{\sum_{p=1}^P \text{Costs}_{\text{heuristic}}(p) - \sum_{p=1}^P \text{Costs}_{\text{optimal}}(p)}{\sum_{p=1}^P \text{Costs}_{\text{optimal}}(p)}$ . The *maximum error* is defined as:  $\arg \max_p \frac{\text{Costs}_{\text{heuristic}}(p) - \text{Costs}_{\text{optimal}}(p)}{\text{Costs}_{\text{optimal}}(p)}$ . First, we use problem instances from the literature to compare our heuristics with the optimal solution and existing heuristics by Gallego et al. (2007) and Rong et al. (2010). Second, we perform more extensive experiments in which we compare our heuristics with the optimal solution only. We use these experiments to get more insights into the performance of our heuristics.

Rong et al. (2010) give a set of problem instances on which they compare their two heuristics with the optimal solution and the RD heuristic of Gallego et al. (2007). To get the optimal solution, Rong et al. enumerate all possible values for  $S_0$  between a lower and an upper bound, determine the optimal  $S_i$  that belong to that  $S_0$  value and use the exact evaluation of Graves (1985) to determine the costs. The settings for the problem instances that are used can be found in Table 1. Since for each local stock point  $i$ ,  $L_i$  and  $\beta_i$  are drawn from a uniform distribution, 200 problem instances are generated for each combination of  $\lambda_i$ ,  $L_0$ , and  $h_0$ , leading to 72,000 problem instances in total (notice that in each problem instance it holds for all  $i, j \in \{1, \dots, N\}$  that  $\lambda_i = \lambda_j$ ). The results that Rong et al. (2010) report can be found in Table 2. Notice that the heuristics by Rong et al. find base stock levels only; the corresponding costs are determined afterwards and are not included in the computation time.

Since the RO heuristic is quite slow and the RD heuristic is relatively slow and leads to a relatively high average error compared with the DA heuristic, we may see the DA heuristic as the best heuristic. Hence, we compare our heuristics with the DA heuristic. We implement the DA heuristic, our heuristics, and the algorithm of Axsäter (1990) in Python and perform the experiments on a laptop computer running Windows XP on an



Parameter	Value(s)
$N$	4
$L_i$	$U [0.1, 0.25]$
$L_0$	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1
$\lambda_i$	4, 8, 16, 32
$h_i$	1
$h_0$	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
$\beta_i$	$U [9, 39]$

Table 1: Problem instances from Rong et al. (2010, page 11)

	Rong et al.		Gallego et al.	Graves
	RO	DA	RD	Exact
Average error (%)	1.4	1.9	3.4	—
Maximum error (%)	22.2	23.6	12.7	—
Average time (s.)	0.548	0.015	0.176	4.012

Table 2: Results from Rong et al. (2010, Table 1, page 11)

Intel Core Duo P8600 2.40 GHz, 3.45 GB RAM. The results for the problem instances of Rong et al. (2010) (see Table 1), solved using our implementation can be found in Table 3. The most striking observation is that the error that we find for the DA heuristic is lower than the error that Rong et al. report. The difference may be due to the randomness in the problem instances. Notice further that our implementation is somewhat faster than the original implementation (0.006 seconds on average compared with 0.015 seconds). The key result is that the SC heuristic performs much better than the DA heuristic in terms of solution quality, whereas the optimization time is still very low. Another important result is that the SE heuristic always finds the optimal solution. Lastly, our results suggest that the algorithm by Axsäter (1990) is faster than the algorithm of Graves (1985), although we cannot conclude this with certainty from these results.

	Rong et al.	Our heuristics		Axsäter
	DA	SE	SC	Exact
Average error (%)	1.60	0.00	0.11	—
Maximum error (%)	14.13	0.00	2.70	—
Average time (s.)	0.006	0.145	0.020	0.279
Maximum time (s.)	0.025	0.782	0.056	1.768

Table 3: Our results on the tests by Rong et al. (2010)

Parameter	Value(s)
$N$	2, 8, 32
$L_0$	1, 2, 4
$h_0$	1
$L_i$	0.25, 1
$\lambda_i$	0.25, 1, 4
$h_i$	1, 2, 4
$\beta_i$	16, 64

Table 4: Additional problem instances

We perform a more extensive experiment which is quite different from the one by Rong et al. (2010). We are interested in a broader range of local stock points and a long lead time at the central warehouse relative to the lead time at the local stock points. This better reflects the typical situation in the area of capital goods, and especially in the military context. Furthermore, we are interested both in the setting in which the holding costs at the local stock points are higher than those at the central warehouse and in the setting in which holding costs are equal at all locations. Because the DA heuristic cannot deal with the situation of equal holding costs at both echelon levels, we cannot compare our results with those of that algorithm.

For the exact values that we use, see Table 4. The local stock points are divided into two sets of equal size and we vary the relevant values ( $L_i$ ,  $\lambda_i$ ,  $h_i$ , and  $\beta_i$ ) in the two groups independently from each other. So, there are three values for  $\lambda_i$ , which means that there are nine combinations possible, three of which lead to ‘balanced’ networks in which all local stock points have the same demand rate. In total, there are 9 settings at the central warehouse. For each of these settings, there are 36 settings at the local stock points. Since each of these settings in one half of the local stock points is combined with each of these settings in the other half of the local stock points, there are 1,296 settings in total at the local stock points, and 11,664 problem instances in total.

The results can be found in Table 5. The SE heuristic still always finds the optimal solution. For the SC heuristic, the maximum error is 2.9% and only 270 (out of 11,664) problem instances have an error of more than 1.0%. Notice that our heuristic does find a

	Our heuristics		Axsäter
	SE	SC	Exact
Average error (%)	0.00	0.11	0.00
Maximum error (%)	0.00	2.92	0.00
Average time (s.)	0.846	0.054	1.536
Maximum time (s.)	17.406	0.305	36.118

Table 5: Additional test results

	$N$			$L_0$		
	2	8	32	1	2	4
Average time SE (s.)	0.014	0.168	2.356	0.396	0.737	1.405
Maximum time SE (s.)	0.085	1.224	17.406	4.554	8.598	17.406
Average error SC (%)	0.08	0.14	0.11	0.08	0.11	0.15
Maximum error SC (%)	1.66	2.92	2.60	2.71	2.11	2.92
Average time SC (s.)	0.006	0.027	0.130	0.051	0.054	0.059
Maximum time SC (s.)	0.020	0.070	0.305	0.290	0.274	0.305

Table 6: Detailed test results for our heuristics (1/3)

cost estimate, but we do not know the exact costs corresponding with the base stock levels that we find; these costs are calculated after execution of our heuristic. The difference between the cost estimate and the true costs is  $-0.5\%$  on average and varies between  $-7.6\%$  and  $2.3\%$ .

We show more detailed results in Table 6, in which we vary  $N$  and  $L_0$ , Table 7, in which we vary  $\lambda_i$  and  $\beta_i$ , and Table 8, in which we vary  $h_i$  and  $L_i$ . In the latter two tables, we only show the results of the problem instances for which the relevant parameter ( $\lambda_i$ ,  $\beta_i$ ,  $h_i$ , or  $L_i$ ) is equal for all local stock points. The most interesting things to notice are:

- The optimization time of the SC heuristic increases approximately linear with the

	$\lambda_i(\forall i \in \{1, \dots, N\})$			$\beta_i(\forall i \in \{1, \dots, N\})$	
	0.25	1	4	16	64
Average time SE (s.)	0.084	0.365	2.166	0.759	0.929
Maximum time SE (s.)	0.454	2.129	14.954	14.954	17.406
Average error SC (%)	0.04	0.06	0.06	0.08	0.13
Maximum error SC (%)	1.27	0.85	1.34	1.69	2.92
Average time SC (s.)	0.029	0.042	0.075	0.050	0.058
Maximum time SC (s.)	0.098	0.131	0.272	0.272	0.305

Table 7: Detailed test results for our heuristics (2/3)

	$h_i(\forall i \in \{1, \dots, N\})$			$L_i(\forall i \in \{1, \dots, N\})$	
	1	2	4	0.25	1
Average time SE (s.)	0.954	0.838	0.751	0.643	1.053
Maximum time SE (s.)	17.406	15.959	14.418	9.561	17.406
Average error SC (%)	0.24	0.13	0.05	0.13	0.11
Maximum error SC (%)	2.71	2.21	1.22	2.71	1.59
Average time SC (s.)	0.061	0.053	0.049	0.045	0.064
Maximum time SC (s.)	0.305	0.279	0.277	0.195	0.305

Table 8: Detailed test results for our heuristics (3/3)

number of local stock points ( $N$ ), whereas the optimization time of the SE heuristic increases much harder.

- The optimization time of the SC heuristic is almost independent of the lead time at the central warehouse ( $L_0$ ), whereas the optimization time of the SE heuristic increases approximately linear with  $L_0$ . It is also interesting to see that the optimization time of the exact algorithm by Axsäter (1990) increases even harder (for  $L_0 = 4$ , SE is on average 53% faster than the exact algorithm, whereas for  $L_0 = 1$ , it is only 18%).
- The optimization time of the SC heuristic increases less than linear with the demand rate ( $\lambda_i$ ), whereas the optimization time of the SE heuristic increases more than linear with the demand rate. Again, the optimization time of the exact algorithm increases even harder (for  $\lambda_i = 4$ , SE is on average 53% faster, whereas it is 1.5% slower for  $\lambda_i = 0.25$ ).
- Notice that the problem instances in which half of the local stock points have one value for  $\lambda_i$ , whereas the other half of the problem instances have another value for  $\lambda_i$  are excluded from the comparison in the previous bullet. Since the average errors that we report in Table 7 are on average (over the three values for  $\lambda_i$ ) lower than those that we report in Table 5, we can conclude that in the ‘unbalanced’ problem instances the error is higher on average than in the ‘balanced’ problem instances (equal demand rate at all local stock points).

- The quality of the SC solution depends heavily on the value of  $h_i$ ; the quality increases with an increasing  $h_i$  (notice that the problem instances in which half of the local stock points have one value for  $h_i$ , whereas the other half of the problem instances have another value for  $h_i$  are excluded from this comparison).

## 6 Conclusions and recommendations for further research

We have studied two-echelon distribution networks. We have developed a fast heuristic (Step and Check) that finds base stock levels that are close to optimal, and a heuristic (Smart Enumeration) that always finds the optimal solution in our computational experiment. In that experiment, we have shown that the former heuristic is faster than most existing heuristics and gives a better solution than all existing heuristics.

It may be possible to extend the SC heuristic in a couple of ways. First, it is relatively straightforward to extend our algorithm to the general multi-echelon case. This would be useful since only two heuristics exist for this case so far (those of Rong et al., 2010).

Second, it may be possible to consider compound Poisson demand. The approximation by Graves (1985) that we use in our heuristic can also be used as an approximation for the compound Poisson demand (with some changes), although we do not know the approximation quality. A comparison could be made with the exact algorithm that exists for this case (with two echelon levels) by Forsberg (1995).

Third, it may be interesting to extend the  $(S - 1, S)$ -policies to  $(R, Q)$ -policies, especially at the central warehouse. The extension to  $(R, Q)$ -policies at all stock points may be difficult. A comparison can be made with the exact algorithm by Forsberg (1996). Combining  $(R, Q)$ -policies and compound Poisson demand may be even harder, but it would lead to a model that fits many practical situations. Comparison in this case could be made with the exact algorithm by Axsäter (2000).

## Appendix A: Proof of Lemma 6

Before we can go to the actual proof, we need two additional lemmas.

**Lemma 9.** *It holds that:*

$$\mathbb{E}I_0(S_0 + 1) - \mathbb{E}I_0(S_0) = \mathbf{P}\{B_0(S_0) = 0\} \quad (6)$$

and, for all  $i \in \{1, \dots, N\}$  it holds that:

$$\mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0 + 1) \right) - \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0) \right) = \frac{\lambda_i}{\lambda_0} \mathbf{P}\{B_0(S_0) > 0\} \quad (7)$$

$$\mathbb{E}B_i(S_0 + 1, S_i) - \mathbb{E}B_i(S_0, S_i) = -\frac{\lambda_i}{\lambda_0} \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\} \quad (8)$$

*Proof.* The proof of Equation (6) is analogous to the proof of Lemma 2.

Equation (7) can be proven as follows:

$$\begin{aligned} \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0 + 1) \right) - \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0) \right) &= \mathbb{E}B_0^{(i)}(S_0 + 1) - \mathbb{E}B_0^{(i)}(S_0) \\ &= \mathbf{P}\{B_0(S_0) > 0 \cap Z_i = 1\} \\ &= \frac{\lambda_i}{\lambda_0} \mathbf{P}\{B_0(S_0) > 0\} \end{aligned}$$

The second equality is the crucial step in this derivation. Notice that there is a difference between  $\left( S_i - Y_i - B_0^{(i)}(S_0 + 1) \right)$  and  $\left( S_i - Y_i - B_0^{(i)}(S_0) \right)$  if an increment of  $S_0$  by one leads to a decrease in  $B_0^{(i)}(S_0)$ . This occurs only if the following two statements hold:

- the number of backorders at the central warehouse decreases by adding a spare part, so  $B_0(S_0) > 0$ , and
- the first backorder in the pipeline at the central warehouse is due to local stock point  $i$ , so  $Z_i = 1$ .

The proof of Equation (8) is as follows:

$$\begin{aligned}
\mathbb{E}B_i(S_0 + 1, S_i) - \mathbb{E}B_i(S_0, S_i) &= \mathbb{E} \left( Y_i + B_0^{(i)}(S_0 + 1) - S_i \right)^+ - \mathbb{E} \left( Y_i + B_0^{(i)}(S_0) - S_i \right)^+ \\
&= -\mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \cap Z_i = 1\} \\
&= -\frac{\lambda_i}{\lambda_0} \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\}
\end{aligned}$$

Again, the second equality is the crucial step in this derivation. Notice that there is a difference between  $\left( Y_i + B_0^{(i)}(S_0 + 1) - S_i \right)^+$  and  $\left( Y_i + B_0^{(i)}(S_0) - S_i \right)^+$  if an increment of  $S_0$  by one leads to a decrease in  $B_0^{(i)}(S_0)$ , which in turn leads to a decrease in  $\left( Y_i + B_0^{(i)}(S_0) - S_i \right)^+$ . This is a stronger requirement than we had in the proof of Equation (7). Therefore, now three statements need to hold simultaneously:

- the number of backorders at the central warehouse decreases by adding a spare part, so  $B_0(S_0) > 0$ ,
- the first backorder in the pipeline at the central warehouse is due to local stock point  $i$ , so  $Z_i = 1$ , and
- the number of backorders at local stock point  $i$  decreases if the number of backorders that it is due to stock point  $i$  at the central warehouse decreases, so  $B_0^{(i)}(S_0) + Y_i > S_i$ .

□

**Lemma 10.** *The difference function  $\Delta_0 C(S_0, S_1, \dots, S_N)$  is as follows:*

$$\begin{aligned}
&\Delta_0 C(S_0, S_1, \dots, S_N) \\
&= C(S_0 + 1, S_1, \dots, S_N) - C(S_0, S_1, \dots, S_N) \\
&= h_0 \mathbf{P}\{B_0(S_0) = 0\} \\
&+ \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} \left[ h_i \mathbf{P}\{B_0(S_0) > 0\} - (\beta_i + h_i) \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\} \right]
\end{aligned}$$

*Proof.* The cost function, Equation (1), can be rewritten as follows:

$$\begin{aligned}
C(S_0, S_1, \dots, S_N) &= h_0 \mathbb{E}I_0(S_0) + \sum_{i=1}^N [h_i \mathbb{E}I_i(S_0, S_i) + \beta_i \mathbb{E}B_i(S_0, S_i)] \\
&= h_0 \mathbb{E}I_0(S_0) + \sum_{i=1}^N [h_i (\mathbb{E}I_i(S_0, S_i) - \mathbb{E}B_i(S_0, S_i)) + (\beta_i + h_i) \mathbb{E}B_i(S_0, S_i)] \\
&= h_0 \mathbb{E}I_0(S_0) + \sum_{i=1}^N \left[ h_i \left( \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0) \right)^+ - \mathbb{E} \left( Y_i + B_0^{(i)}(S_0) - S_i \right)^+ \right) \right. \\
&\quad \left. - (\beta_i + h_i) \mathbb{E}B_i(S_0, S_i) \right] \\
&= h_0 \mathbb{E}I_0(S_0) + \sum_{i=1}^N \left[ h_i \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0) \right) - (\beta_i + h_i) \mathbb{E}B_i(S_0, S_i) \right]
\end{aligned}$$

Next, we can derive the difference function, using Lemma 9:

$$\begin{aligned}
\Delta_0 C(S_0, S_1, \dots, S_N) &= h_0 (\mathbb{E}I_0(S_0 + 1) - \mathbb{E}I_0(S_0)) \\
&\quad + \sum_{i=1}^N \left[ h_i \left[ \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0 + 1) \right) - \mathbb{E} \left( S_i - Y_i - B_0^{(i)}(S_0) \right) \right] \right. \\
&\quad \left. + (\beta_i + h_i) [\mathbb{E}B_i(S_0 + 1, S_i) - \mathbb{E}B_i(S_0, S_i)] \right] \\
&= h_0 \mathbf{P}\{B_0(S_0) = 0\} \\
&\quad + \sum_{i=1}^N \left[ h_i \frac{\lambda_i}{\lambda_0} \mathbf{P}\{B_0(S_0) > 0\} - (\beta_i + h_i) \frac{\lambda_i}{\lambda_0} \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\} \right] \\
&= h_0 \mathbf{P}\{B_0(S_0) = 0\} \\
&\quad + \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} \left[ h_i \mathbf{P}\{B_0(S_0) > 0\} - (\beta_i + h_i) \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\} \right]
\end{aligned}$$

□

Now we are ready to proof Lemma 6:

*Proof.* If  $B_0(S_0) > 0$  and  $Z_i = 1$  for some  $i \in \{1, \dots, N\}$ , then  $B_0^{(i)}(S_0) > 0$ . If  $S_i = 0$ , then



$B_0^{(i)}(S_0) > 0$  implies that  $B_0^{(i)}(S_0) + Y_i > S_i$ . Therefore, if  $S_i = 0$  for all  $i \in \{1, \dots, N\}$ , the difference function in Lemma 10 reduces to:

$$\begin{aligned} \Delta_0 C(S_0, 0, \dots, 0) &= h_0 \mathbf{P}\{B_0(S_0) = 0\} + \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} (-\beta_i) \mathbf{P}\{B_0(S_0) > 0\} \\ &= h_0 \mathbf{P}\{B_0(S_0) = 0\} + \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} [-\beta_i + \beta_i \mathbf{P}\{B_0(S_0) = 0\}] \end{aligned}$$

Since  $\mathbf{P}\{B_0(S_0) = 0\}$  is increasing in  $S_0$ , the difference function is increasing in  $S_0$  as well, so the cost function is convex in  $S_0$  (under the given conditions).  $\square$

## Appendix B: Proof of Lemma 8

*Proof.* To proof Lemma 8, we rearrange the terms of the difference function in Lemma 10 as follows:

$$\begin{aligned} \Delta_0 C(S_0, S_1, \dots, S_N) &= h_0 (1 - \mathbf{P}\{B_0(S_0) > 0\}) \\ &+ \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} \left[ h_i \mathbf{P}\{B_0(S_0) > 0\} - (\beta_i + h_i) \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\} \right] \\ &= h_0 - \left( h_0 - \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i \right) \mathbf{P}\{B_0(S_0) > 0\} \\ &- \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} (\beta_i + h_i) \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\} \end{aligned}$$

If  $h_0 \geq \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i$ , then  $(h_0 - \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i) \mathbf{P}\{B_0(S_0) > 0\}$  is decreasing in  $S_0$  (it is 0 if  $h_0 = \sum_{i=1}^N \frac{\lambda_i}{\lambda_0} h_i$ ). Furthermore,  $\sum_{i=1}^N \frac{\lambda_i}{\lambda_0} (\beta_i + h_i) \mathbf{P}\{B_0(S_0) > 0 \cap B_0^{(i)}(S_0) + Y_i > S_i \mid Z_i = 1\}$  is also decreasing in  $S_0$ , so the difference function is increasing in  $S_0$  and the cost function is convex in  $S_0$  (under the given conditions).  $\square$

## References

- Axsäter, S. (1990). Simple solution procedures for a class of two-echelon inventory problems. *Operations Research*, 38(1):64–69.
- Axsäter, S. (2000). Exact analysis of continuous review  $(R, Q)$  policies in two-echelon inventory systems with compound poisson demand. *Operations Research*, 48(5):686–696.
- Clark, A. J. and Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Management Science*, 6(4):475–490.
- Cohen, M. A., Zheng, Y.-S., and Agrawal, V. (1997). Service parts logistics: a benchmark analysis. *IIE Transactions*, 29:627–639.
- Diks, E. B. and de Kok, A. T. (1998). Optimal control of a divergent multi-echelon inventory system. *European Journal of Operational Research*, 111:75–97.
- Diks, E. B. and de Kok, A. T. (1999). Computational results for the control of a divergent  $N$ -echelon inventory system. *International Journal of Production Economics*, 59:327–336.
- Eppen, G. and Schrage, L. (1981). Centralized ordering policies in a multi-warehouse system with lead times and random demand. In Schwarz, L. B., editor, *Multi-Level Production/Inventory Control Systems, Theory and Practice*, volume 16 of *TIMS Studies in the Management Sciences*, pages 51–67. North-Holland Publishing Company.
- Forsberg, R. (1995). Optimization of order-up-to- $S$  policies for two-level inventory systems with compound poisson demand. *European Journal of Operational Research*, 81:143–153.
- Forsberg, R. (1996). Exact evaluation of  $(R, Q)$ -policies for two-level inventory systems with poisson demand. *European Journal of Operational Research*, 96:130–138.

- Gallego, G., Özer, Ö., and Zipkin, P. H. (2007). Bounds, heuristics, and approximations for distribution systems. *Operations Research*, 55(3):503–517.
- Graves, S. C. (1985). A multi-echelon inventory model for a repairable item with one-for-one replenishment. *Management Science*, 31(10):1247–1256.
- Muckstadt, J. A. (2005). *Analysis and Algorithms for Service Parts Supply Chains*. Springer, New York (NY).
- Rong, Y., Atan, Z., and Snyder, L. V. (2010). Heuristics for base-stock levels in multi-echelon distribution networks. *Working Paper*. Available at SSRN: <http://ssrn.com/abstract=1475469>, last checked on September 1, 2010.
- Shang, K. H. and Song, J.-S. (2003). Newsvendor bounds and heuristic for optimal policies in serial supply chains. *Management Science*, 49(5):618–638.
- Sherbrooke, C. C. (1968). METRIC: A multi-echelon technique for recoverable item control. *Operations Research*, 16(1):122–141.
- Sherbrooke, C. C. (2004). *Optimal inventory modelling of systems. Multi-echelon techniques*. Kluwer, Dordrecht (The Netherlands), second edition.
- van der Heijden, M. C., Diks, E. B., and de Kok, A. T. (1997). Stock allocation in general multi-echelon distribution systems with  $(R, S)$  order-up-to-policies. *International Journal of Production Economics*, 49:157–174.
- van Houtum, G. J. (2006). Multi-echelon production/inventory systems: Optimal policies, heuristics, and algorithms. In Johnson, M. P., Norman, B., and Secomandi, N., editors, *Tutorials in operations research: models, methods, and applications for innovative decision making*, chapter 8, pages 163–199. INFORMS, Hanover (MD).

Working Papers Beta 2009 - 2010

nr.	Year	Title	Author(s)
327	2010	<a href="#">A combinatorial approach to multi-skill workforce scheduling</a>	Murat Firat, Cor Hurkens
326	2010	<a href="#">Stability in multi-skill workforce scheduling</a>	Murat Firat, Cor Hurkens, Alexandre Laugier
325	2010	<a href="#">Maintenance spare parts planning and control: A framework for control and agenda for future research</a>	M.A. Driessen, J.J. Arts, G.J. v. Houtum, W.D. Rustenburg, B. Huisman
324	2010	<a href="#">Near-optimal heuristics to set base stock levels in a two-echelon distribution network</a>	R.J.I. Basten, G.J. van Houtum
323	2010	<a href="#">Inventory reduction in spare part networks by selective throughput time reduction</a>	M.C. van der Heijden, E.M. Alvarez, J.M.J. Schutten
322	2010	<a href="#">The selective use of emergency shipments for service-contract differentiation</a>	E.M. Alvarez, M.C. van der Heijden, W.H. Zijm
321	2010	<a href="#">Heuristics for Multi-Item Two-Echelon Spare Parts Inventory Control Problem with Batch Ordering in the Central Warehouse</a>	B. Walrave, K. v. Oorschot, A.G.L. Romme
320	2010	<a href="#">Preventing or escaping the suppression mechanism: intervention conditions</a>	Nico Dellaert, Jully Jeunet.
319	2010	<a href="#">Hospital admission planning to optimize major resources utilization under uncertainty</a>	R. Seguel, R. Eshuis, P. Grefen.
318	2010	<a href="#">Minimal Protocol Adaptors for Interacting Services</a> <a href="#">Teaching Retail Operations in Business and Engineering Schools</a>	Tom Van Woensel, Marshall L. Fisher, Jan C. Fransoo.
317	2010	<a href="#">Design for Availability: Creating Value for Manufacturers and Customers</a>	Lydie P.M. Smets, Geert-Jan van Houtum, Fred Langerak.
316	2010	<a href="#">Transforming Process Models: executable rewrite rules versus a formalized Java program</a>	Pieter van Gorp, Rik Eshuis.
315	2010		Bob Walrave, Kim E. van Oorschot, A. Georges L. Romme
314	2010	<a href="#">Ambidexterity and getting trapped in the suppression of exploration: a simulation model</a>	

313	2010	<a href="#">A Dynamic Programming Approach to Multi-Objective Time-Dependent Capacitated Single Vehicle Routing Problems with Time Windows</a>	S. Dabia, T. van Woensel, A.G. de Kok
312	2010	<a href="#">Tales of a So(u)rcerer: Optimal Sourcing Decisions Under Alternative Capacitated Suppliers and General Cost Structures</a>	Osman Alp, Tarkan Tan
311	2010	<a href="#">In-store replenishment procedures for perishable inventory in a retail environment with handling costs and storage constraints</a>	R.A.C.M. Broekmeulen, C.H.M. Bakx
310	2010	<a href="#">The state of the art of innovation-driven business models in the financial services industry</a>	E. Lüftenegger, S. Angelov, E. van der Linden, P. Grefen
309	2010	<a href="#">Design of Complex Architectures Using a Three Dimension Approach: the CrossWork Case</a>	R. Seguel, P. Grefen, R. Eshuis
308	2010	<a href="#">Effect of carbon emission regulations on transport mode selection in supply chains</a>	K.M.R. Hoen, T. Tan, J.C. Fransoo, G.J. van Houtum
307	2010	<a href="#">Interaction between intelligent agent strategies for real-time transportation planning</a>	Martijn Mes, Matthieu van der Heijden, Peter Schuur
306	2010	<a href="#">Internal Slackening Scoring Methods</a>	Marco Slikker, Peter Borm, René van den Brink
305	2010	<a href="#">Vehicle Routing with Traffic Congestion and Drivers' Driving and Working Rules</a>	A.L. Kok, E.W. Hans, J.M.J. Schutten, W.H.M. Zijm
304	2010	<a href="#">Practical extensions to the level of repair analysis</a>	R.J.I. Basten, M.C. van der Heijden, J.M.J. Schutten
303	2010	<a href="#">Ocean Container Transport: An Underestimated and Critical Link in Global Supply Chain Performance</a>	Jan C. Fransoo, Chung-Yee Lee
302	2010	<a href="#">Capacity reservation and utilization for a manufacturer with uncertain capacity and demand</a>	Y. Boulaksil; J.C. Fransoo; T. Tan
300	2009	<a href="#">Spare parts inventory pooling games</a>	F.J.P. Karsten; M. Slikker; G.J. van Houtum
299	2009	<a href="#">Capacity flexibility allocation in an outsourced supply chain with reservation</a>	Y. Boulaksil, M. Grunow, J.C. Fransoo
298	2010	<a href="#">An optimal approach for the joint problem of level of repair analysis and spare parts stocking</a>	R.J.I. Basten, M.C. van der Heijden, J.M.J. Schutten
297	2009	<a href="#">Responding to the Lehman Wave: Sales Forecasting and Supply Management during the Credit Crisis</a>	Robert Peels, Maximiliano Udenio, Jan C. Fransoo, Marcel Wolfs, Tom Hendrikx
296	2009	<a href="#">An exact approach for relating recovering surgical patient workload to the master surgical schedule</a>	Peter T. Vanberkel, Richard J. Boucherie, Erwin W. Hans, Johann L. Hurink, Wineke A.M. van Lent, Wim H. van Harten
295	2009	<a href="#">An iterative method for the simultaneous optimization of repair decisions and spare parts stocks</a>	R.J.I. Basten, M.C. van der Heijden, J.M.J. Schutten

294	2009	<a href="#">Fujaba hits the Wall(-e)</a>	Pieter van Gorp, Ruben Jubeh, Bernhard Grusie, Anne Keller
293	2009	<a href="#">Implementation of a Healthcare Process in Four Different Workflow Systems</a>	R.S. Mans, W.M.P. van der Aalst, N.C. Russell, P.J.M. Bakker
292	2009	<a href="#">Business Process Model Repositories - Framework and Survey</a>	Zhiqiang Yan, Remco Dijkman, Paul Grefen
291	2009	<a href="#">Efficient Optimization of the Dual-Index Policy Using Markov Chains</a>	Joachim Arts, Marcel van Vuuren, Gudrun Kiesmuller
290	2009	<a href="#">Hierarchical Knowledge-Gradient for Sequential Sampling</a>	Martijn R.K. Mes; Warren B. Powell; Peter I. Frazier
289	2009	<a href="#">Analyzing combined vehicle routing and break scheduling from a distributed decision making perspective</a>	C.M. Meyer; A.L. Kok; H. Kopfer; J.M.J. Schutten
288	2009	<a href="#">Anticipation of lead time performance in Supply Chain Operations Planning</a>	Michiel Jansen; Ton G. de Kok; Jan C. Fransoo
287	2009	<a href="#">Inventory Models with Lateral Transshipments: A Review</a>	Colin Paterson; Gudrun Kiesmuller; Ruud Teunter; Kevin Glazebrook
286	2009	<a href="#">Efficiency evaluation for pooling resources in health care</a>	P.T. Vanberkel; R.J. Boucherie; E.W. Hans; J.L. Hurink; N. Litvak
285	2009	<a href="#">A Survey of Health Care Models that Encompass Multiple Departments</a>	P.T. Vanberkel; R.J. Boucherie; E.W. Hans; J.L. Hurink; N. Litvak
284	2009	<a href="#">Supporting Process Control in Business Collaborations</a>	S. Angelov; K. Vidyasankar; J. Vonk; P. Grefen
283	2009	<a href="#">Inventory Control with Partial Batch Ordering</a>	O. Alp; W.T. Huh; T. Tan
282	2009	<a href="#">Translating Safe Petri Nets to Statecharts in a Structure-Preserving Way</a>	R. Eshuis
281	2009	<a href="#">The link between product data model and process model</a>	J.J.C.L. Vogelaar; H.A. Reijers
280	2009	<a href="#">Inventory planning for spare parts networks with delivery time requirements</a>	I.C. Reijnen; T. Tan; G.J. van Houtum
279	2009	<a href="#">Co-Evolution of Demand and Supply under Competition</a>	B. Vermeulen; A.G. de Kok
278	2010	Toward Meso-level Product-Market Network Indices for Strategic Product Selection and (Re)Design Guidelines over the Product Life-Cycle	B. Vermeulen, A.G. de Kok
277	2009	<a href="#">An Efficient Method to Construct Minimal Protocol Adaptors</a>	R. Seguel, R. Eshuis, P. Grefen
276	2009	<a href="#">Coordinating Supply Chains: a Bilevel Programming Approach</a>	Ton G. de Kok, Gabriella Muratore
275	2009	<a href="#">Inventory redistribution for fashion products under demand parameter update</a>	G.P. Kiesmuller, S. Minner
274	2009	<a href="#">Comparing Markov chains: Combining aggregation and precedence relations applied to sets of states</a>	A. Busic, I.M.H. Vliegen, A. Scheller-Wolf

273	2009	<a href="#">Separate tools or tool kits: an exploratory study of engineers' preferences</a>	I.M.H. Vliegen, P.A.M. Kleingeld, G.J. van Houtum
272	2009	<a href="#">An Exact Solution Procedure for Multi-Item Two-Echelon Spare Parts Inventory Control Problem with Batch Ordering</a>	Engin Topan, Z. Pelin Bayindir, Tarkan Tan
271	2009	<a href="#">Distributed Decision Making in Combined Vehicle Routing and Break Scheduling</a>	C.M. Meyer, H. Kopfer, A.L. Kok, M. Schutten
270	2009	<a href="#">Dynamic Programming Algorithm for the Vehicle Routing Problem with Time Windows and EC Social Legislation</a>	A.L. Kok, C.M. Meyer, H. Kopfer, J.M.J. Schutten
269	2009	<a href="#">Similarity of Business Process Models: Metics and Evaluation</a>	Remco Dijkman, Marlon Dumas, Boudewijn van Dongen, Reina Kaarik, Jan Mendling
267	2009	<a href="#">Vehicle routing under time-dependent travel times: the impact of congestion avoidance</a>	A.L. Kok, E.W. Hans, J.M.J. Schutten
266	2009	<a href="#">Restricted dynamic programming: a flexible framework for solving realistic VRPs</a>	J. Gromicho; J.J. van Hoorn; A.L. Kok; J.M.J. Schutten;

Working Papers published before 2009 see: <http://beta.ieis.tue.nl>