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by

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Design Parameters for a Siphon System

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1 Introduction

DHI are interested in understanding a rather unusual water extraction system that is operated by a water supply company. Typically when water is extracted from the ground a well is dug and a pump is installed in the well to push the water to the surface where it enters a distribution system of pipes. Such a system may consist of a dozen or so wells each connected to a single collection pipe. The system that DHI wish to more fully understand consists of a series of ten wells connected to a single collection pipe. The difference in the mode of operation is that the system contains no pumps in the wells. The force to collect the water comes from placing the end of the collection pipe in a tank that is continuously pumped to keep it at approximately 0.5 bar below atmospheric pressure. In this way the water is drawn out of the wells by a siphon mechanism. Such a system appears cheaper to install with fewer pumps and water supplied in this manner costs roughly half the price of water from a standard pump system. How this multiple siphon system works and how it might be controlled were the general problems of interest to the study group.

A plan view of the the well system is shown in figure 1 where the wells to the north and west are conventional pumped wells and the ten wells to the south-east are the siphon system. The ten siphon wells are approximately 50 meters apart horizontally and sunk to a depth of around eighty meters into a limestone aquifer that is confined by an overlying clay layer.

The siphon system has been operated for about twenty years but there is very little data to indicate how it is operating. There has been one set of measurements taken of the pressure at the top of ten wells along with the flow up each well. From this one set of data, given in table 1, measurements on the collection tank indicate it was operating at about 5.20 meters of head below atmospheric with a total flow rate of 340 m^3/hr. The data show that the drop in pressure from the first well is 3.20 m (which is around 60% of
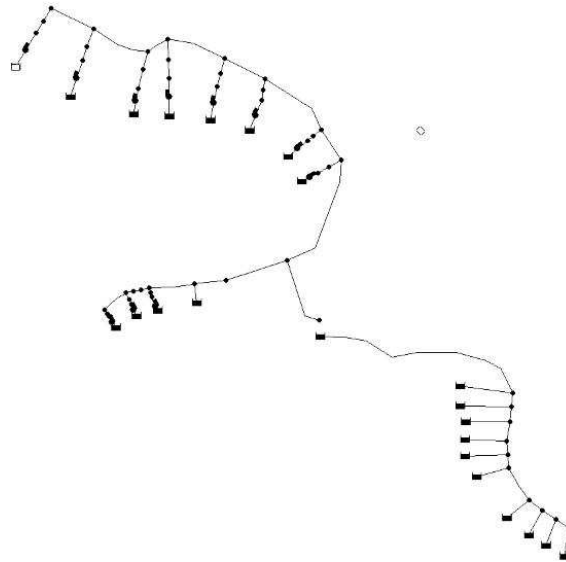
Figure 1: Plan view of pumped and siphoned wells

the total) and that the first five of the ten wells contributed about 90% of the flow into the collection system. The remaining wells were operating at pressures up to 1 meter of head below atmospheric and contributed significantly less to the overall flow from the system. There is no other data from the siphon system but this demonstrates that the amount of water flowing from each well seems to be strongly influenced by its distance from the collection tank. Given these observations and the desire to understand how the system might be controlled the following questions were asked of the study group:

1. What parameters in the system will determine the flow distribution from the wells?

2. How will the water table in the wells influence the flow?

3. How will the pipe dimensions determine the flow?

4. Can water go from one well to another?

5. Where in the system is the greatest risk of cavitation?

Well	7a	8a	9a	10a	11	12a	14a	15a	16a	17a
Diameter d (mm)	203.4	203.3	203.5	202.8	203.3	202.6	203.1	203.2	200.9	194.9
Velocity v (m/s)	0.67	0.47	0.50	0.48	0.43	0.06	0.06	0.22	0.20	0.18
Pressure head P (m)	2.00	2.10	2.12	1.84	1.73	1.53	1.40	1.44	1.07	1.23
Flow Q (m^3/h)	69.38	46.41	49.26	46.88	41.87	4.57	4.08	19.04	17.07	14.00
fraction of total flow	22%	15%	16%	15%	13%	1%	1%	6%	5%	4%

2   Basic flow model

We consider a model for a system of circular pipes each of uniform crossection connected at various junction points. At each point along the pipe we have pressure p, velocity v and height above a reference vertical position, h. As the flows typically have large Reynolds number we shall consider the total stagnation pressure

$$P = p + (1/2)\rho v^2 + \rho g h \tag{1}$$

For a single pipe in the system a feasible model is to consider fully developed flow in each pipe and hence to use a relationship between the flow down the pipe and the total stagnation pressure drop along the pipe. Such flow laws are well known in the literature and are summarized in a Moody Diagram [4] and are typically written as

$$\Delta P = \frac{fL}{2D}\rho v|v| \tag{2}$$

where ΔP is the difference in the stagnation pressure from one end of the pipe to the other, v is the average flow velocity, D is the diameter of the pipe, L is the pipe length and f is a friction factor determined by the Reynolds number of the flow and the roughness of the pipe surface. For the pipe flows here we can determine that a typical Reynolds number in the system is

$$Re = \frac{\rho v D}{\nu} \approx 10^6 \tag{3}$$

and the plastic pipes are very smooth.

To simulate the flow in the system we consider the situation set out in figure 2. Here the variables $P_{0,i}$ indicate the total stagnation pressure of water down in well i (all heights are measured relative to the same reference such as the top of the water in the tank to simplify the analysis) while P_i is the total stagnation pressure head at the point where the well pipe enters the collection pipe. We use v_i as the average velocity of water up well i and u_i is the average velocity in the collection pipe between well i and $i+1$. We adopt a convention where $i = 1$ is the well furthest from the tank, $i = N$ the well closest to the tank and P_{N+1} is the tank. Conservation of mass in the system then requires

$$A_{i+1}u_{i+1} = A_i u_i + a_i v_i \qquad 1 \le i \le N-1 \tag{4}$$

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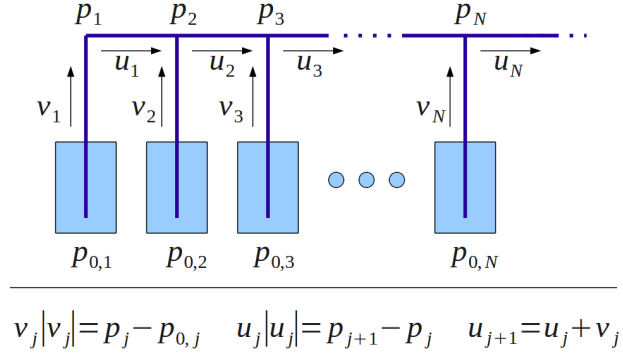$$v_j|v_j| = p_j - p_{0,j} \qquad u_j|u_j| = p_{j+1} - p_j \qquad u_{j+1} = u_j + v_j$$

Figure 2: Nomenclature used for several wells

and $u_1 = v_1$. Here $A_i = \pi D_i^2/2$ is the crossectional area of each collection pipe and $a_i = \pi d_i^2/2$ is the crossectional area of the pipes in the wells. The difference in pressure head along each pipe then gives, for the collection pipes

$$P_i - P_{i+1} = \frac{fL_i}{2D_i}\rho|u_i|u_i \qquad 1 \le i \le N \tag{5}$$

and for the well pipes

$$P_{0,i} - P_i = \frac{f\ell_i}{2d_i}\rho|v_i|v_i \qquad 1 \le i \le N \tag{6}$$

where we know that $P_{N+1} = -H$ where H is the vacuum pressure exerted within the tank. (We have used A_i for the crossectional area of the collection pipes and a_i for the crossectional area of the well bores pipes. With L_i being the distance between pipes and ℓ_i being the riser pipe length.)

The model given above is that currently used by DHI to consider flow within the system. We now use this to look at two different situations with the intention of demonstrating behavior of the equations in different ways. We shall first consider the case where there are many wells and so a continuum model can be exploited while subsequently we consider just two wells, which is the smallest number of wells that gives nontrivial behavior.

2.1   Continuous model

We consider a distributed system of wells with x measuring the distance along the collection pipe from the tank $x = 0$ to the furthest well $x = L$. Taking the limit $N \to \infty$ of the previous discrete model (4) we get

$$\frac{d(A(x)u)}{dx} = B(x)v \qquad \text{for } 0 \le x \le L \ . \tag{7}$$

where $A(x)$ is the crossectional area of the collection pipe along its length and $B(x)$ is the crossectional area of well pipe per length of the collection pipe (ie. a continuous version

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of a_i/L_i). To this equation we add the boundary condition $u(0) = 0$, to indicate there is no external flow into the collection pipe before the well furthest from the tank, and the difference in pressure head along each pipe then gives, for the collection pipes

$$\frac{dP}{dx} = -\frac{f}{2D(x)}\rho|u|u \tag{8}$$

and for the well pipes

$$P_0(x) - P = \frac{f\,l(x)}{2d(x)}\rho|v|v \tag{9}$$

and finally we know that $P(L) = -H$ with H the vacuum pressure exerted within the tank. Here we have used the notation that in the continuum limit $A(x) = \pi D(x)^2/4$ and $l(x) = $ (riser pipe length$/L_i$).

This problem can be solved by taking (9) to give

$$v = (P_0(x) - P)\sqrt{\frac{2d(x)}{fl(x)\rho|P_0(x) - P|}} \tag{10}$$

and using this to give the system for u and P

$$\frac{d(A(x)u)}{dx} = B(x)(P_0(x) - P)\sqrt{\frac{2d(x)}{fl(x)\rho|P_0(x) - P|}} \tag{11}$$

$$\frac{dP}{dx} = -\frac{f}{2D(x)}\rho|u|u \tag{12}$$

with the boundary conditions

$$u(0) = 0, \qquad P(L) = -H\ . \tag{13}$$

This problem was solved numerically for the case where all the pipes are identical $D(x) = d(x) = 1$ and all the physical parameters are unity. The pressure head in the wells, $P_0(x)$ was taken to be linear ($P_0(x) = x$). The results for the total stagnation head and the velocity in the collection pipe are given in figure 3 and 4. These show that the pressure drops steeply near the tank, due primarily to the square law dependence of the turbulent friction, and that this caused the wells nearest the tank to contribute a large proportion of the water to the system as shown by the solution for v as given in figure 5.

This continuous model gives the possibility of considering how to design $A(x)$ and $B(x)$ so that all wells contribute the same flow to the system. This could be done by imposing the fact that v is known and then finding the conditions needed for $A(x)$ and $B(x)$. One case that is easily considered is where all the risers are the same length and diameter and the head in the well increases linearly with distance so that $P_0 = b(L - x)$. If we take the
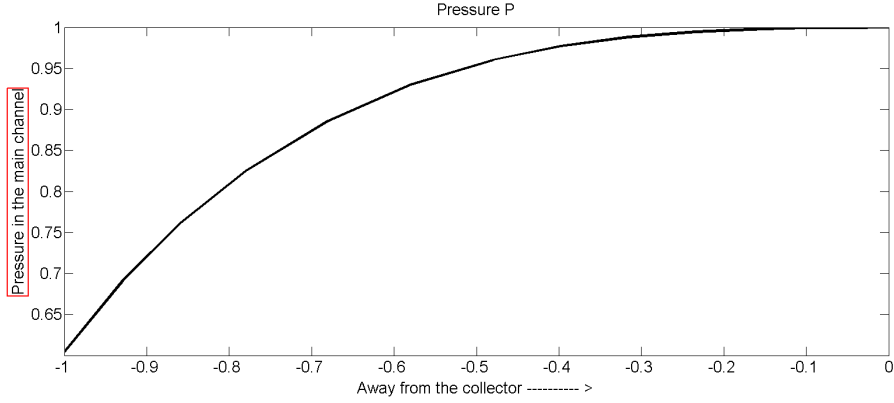
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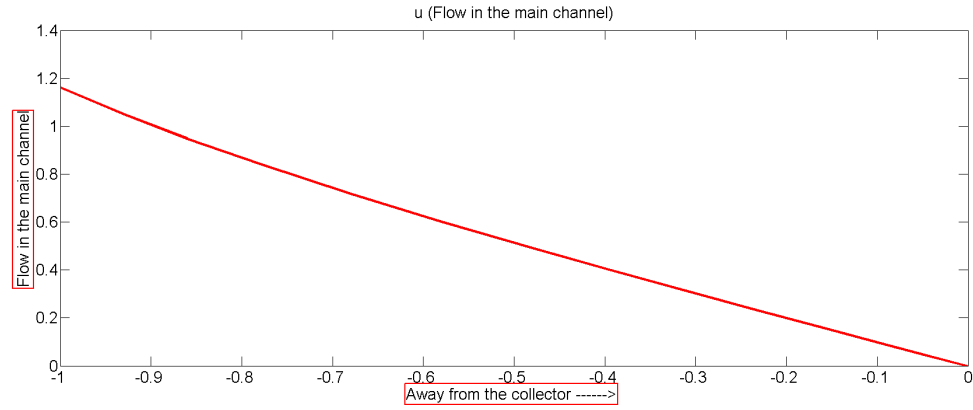Figure 3: The total stagnation pressure, P, predicted from the continuous model

Figure 4: The velocity along the collection pipe, u, predicted from the continuous model

outflow from each well to be the same so that v is a constant, from equation (9) we find that $P = bx - B$ (where B is a constant). Furthermore, from equation (8)

$$b \propto -\frac{1}{\sqrt{A}} u^2. \tag{14}$$

We can also integrate equation (7) to give

$$A(x)u = Bv(x - L) \tag{15}$$

and so combining these results, we find that

$$A \propto (L - x)^{\frac{4}{5}}. \tag{16}$$

Hence we conclude that the pipe network must be constructed so that the area increases significantly as we get near to the reservoir tank.

One other interesting case where analytical progress can be made is to consider the case where all the wells are at the same height with identical risers ($P_0(x)$, $\ell(x)$, $d(x)$ constant).
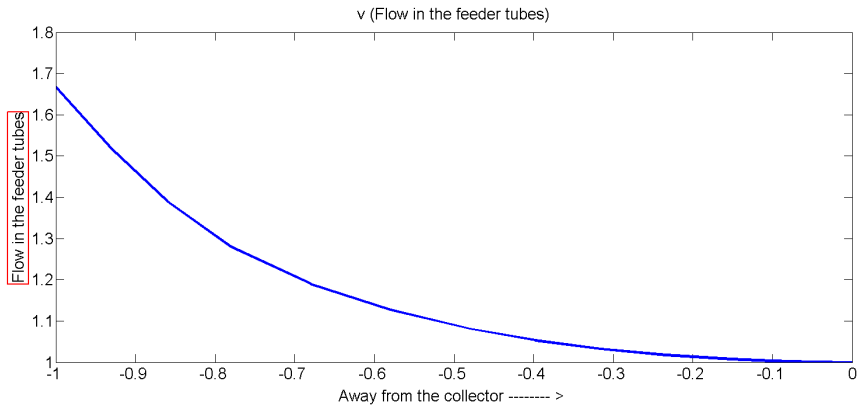
Figure 5: The velocity up the wells, v, predicted from the continuous model

In this case it is not possible to make all wells contribute equally and we find the more distance well contributing a small fraction of the flow. We do not present the details here. We note that the continuous model may allow methods to be applied to understand the stability of the solution with respect to the small variations in the pressure changes in individual wells. A formal derivation of the continuous model from the discrete equations remains to be performed.

2.2 Two well model

In order to make analytical progress and to understand some of the properties of the model it is worth considering the case of two wells, which is the least number that is not the trivial case usually considered in a simple siphon. As the aim of this simple model is to gain insight a highly simplified situation is considered as shown in figure 6 where we just consider three wells and no reservoir tank, we take simple pipes from each well (no distinction of riser and connection pipes), the pipes to the first and last are assumed identical and L is the ratio of the length of the middle pipe to the others. The equations governing this system are

$$P_2 = v_1|v_1| \, , \tag{17}$$

$$h - P_2 = Lv_2|v_2| \, , \tag{18}$$

$$1 - P_2 = v_3|v_3| \, , \tag{19}$$

$$v_1 + v_2 + v_3 = 0 \, . \tag{20}$$

The pressures have been scaled so that the pressure in the reservoir is zero ($P_1 = 0$), the pressure in the furthest well is unity ($P_3 = 1$) and the pressure in the middle well is h.

Figure 7 shows the total contribution from each well to the flow arriving at the reservoir as a function of the height h of well 1. This is shown for three different cases: $L = 0.01$
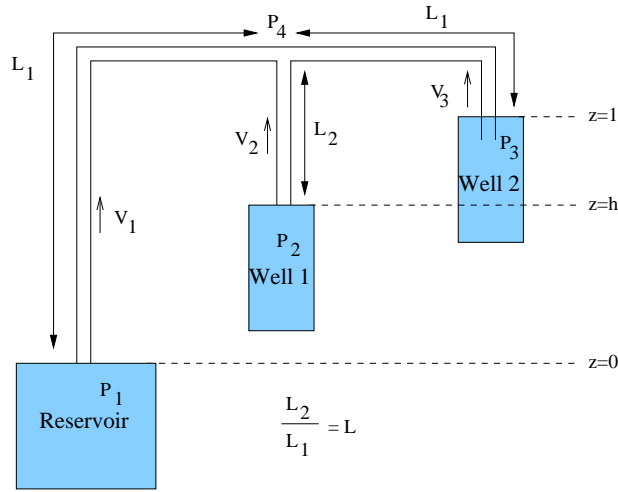
Figure 6: Notation adopted for the simple two-wells problem

(a short riser pipe relative to the inter-well spacing), $L = 0.5$ and $L = 1$ (the riser pipe and inter-well spacing are the same). The light line shows the contribution from the well 2 while the dark line shows the contribution from well 1. It can be seen that for all three cases, when $h < 0.5$ there is no flow contribution from well 1. In fact here water enters well 1 from well 2.

The most interesting difference comes when $h \to 1$. For $L = 1$, the contribution from both wells is equal when they are both at the same height. This is because both wells are the same distance away from the reservoir and so the system is symmetrical. However when $L < 1$, well 2 is further away from the reservoir than well 1. This means that there is more pipe friction between well 2 and the reservoir than there is between well 1 and the reservoir and so there is a greater headloss from well 2. Thus the flow from well 1 can contribute a significantly larger proportion of the total flow than well 2, even when $h < 1$. In the DHI system, the inter-well spacing is approximately 50m, while the riser pipes are around 10m in length. Thus $L \approx 0.2$, and so it is to be expected that there will be a significantly larger proportion from a closer wells than a farther away wells when the water tables in the wells are at similar heights.

This simple example nicely illustrates why the majority of flow in DHI's data comes from the nearest wells to the reservoir. This is because pipe friction causes significant stagnation pressure head loss from more distant wells, so flow will be reduced relative to the closer wells.
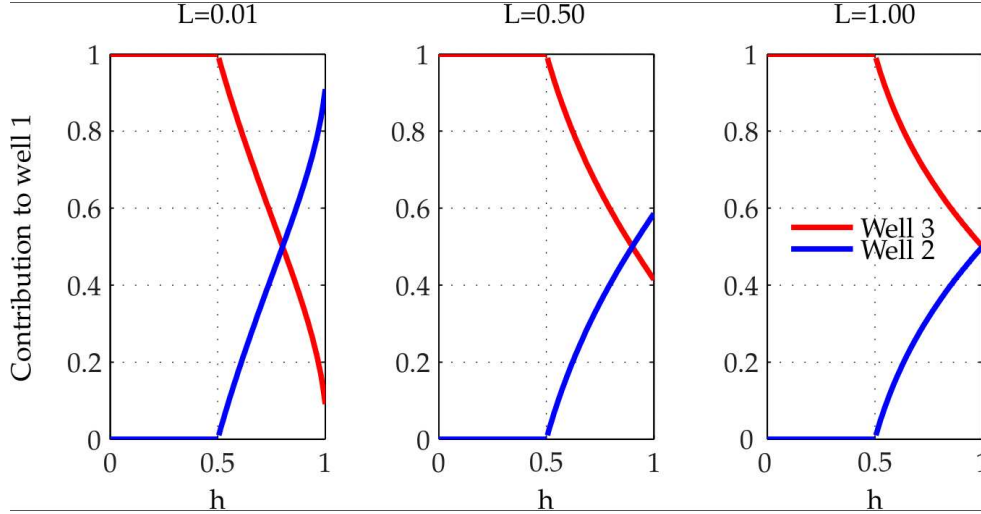
Figure 7: Flow from the wells as the relative height of the wells is changed in the two-well problem

3 Electrical analogies

One approach to studying the flow problem is to draw on the extensive ideas available from the area of electrical circuit behavior. For example the two-well problem can be replaced by three resistors and three batteries as shown in figure 8 Here the volume flow rate Q is
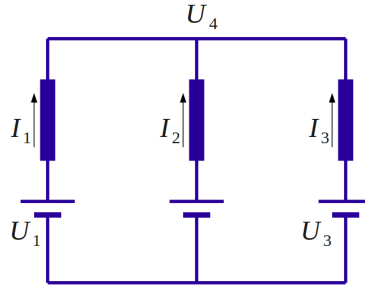
Figure 8: Notation for the two-well problem as an electrical circuit

replaced by the current I and the head loss P by the voltage V with the unit for current is m^3/s and for voltage $\mathrm{m}^3/\mathrm{s}^2$ so that the unit for resistance is $\mathrm{m}^{-1}\mathrm{s}^{-1}$. Also the current for a crossectional area we have $I = Q = Av$ and the applied voltages due to batteries are E_i.

For the two well problems the electrical equivalent is:

$$V_4 = R_1 \tag{21}$$

$$V_2 - V_4 = R_2 Q_2 + E_2 \tag{22}$$

$$V_3 - V_4 = R_3 Q_3 + E_3 \tag{23}$$

$$Q_1 = Q_2 + Q_3 . \tag{24}$$

An interesting result is that it is possible to analyze this problem to find when there is no flow from well 1 while well 2 is working, by taking $Q_2 = 0$. This shows that this occurs when

$$P_2 = \frac{\frac{L_1}{A_1 d_1}}{\frac{L_1}{A_1 d_1} + \frac{L_3}{A_3 d_3}} P_3 . \tag{25}$$

Similarly it can be found that well 2 stops flowing when

$$P_3 = \frac{\frac{L_1}{A_1 d_1}}{\frac{L_1}{A_1 d_1} + \frac{L_2}{A_2 d_2}} P_2 . \tag{26}$$

4   Including junction effects

Experiments show that the basic model outlined above does not predict the observations made on a simple two-well problem examined by taking three buckets of water at different heights and connecting them with piping and a T-junction connector. In particular it was observed that the junction seemed to have a significant effect on the behavior. A model of junction behavior was found in [3] which considers the pressure head loss across different shapes of junction.

Note in extending the basic model we are relaxing the very strong condition imposed in the previous model that the pressure at the junction was the same in all pipes at the junction. This condition does not account for the boundary layer effects near and in the junction. For very low Reynolds number flow the pressure will be almost uniform within the junction. However as the Reynolds number grows boundary layer effects will be become important. There will be significant entry regions before the flow becomes fully developed and there will be growing shear layers separating one flow from another We have not studied these effects in detail but observe, for example, that at high Reynolds number fully developed flow may require 20 pipe diameters to become valid.

The extension of the model is to consider the pressures in the pipes entering a junction to be different and to use approximations and empirical laws in order to understand how these pressures are related. Noe that all this analysis is done in the high Reynolds number limit and hence we are looking at jumps in the stagnation pressure of the fluid namely $p + (1/2)\rho v^2$

In the paper by Bassett et.al. [3] six different possible flow configurations are considered for each junction. These are shown in figure 9. The expressions for the various pressure drops are rather complicated but we note that they do significantly simplify in the case of a T-junction.
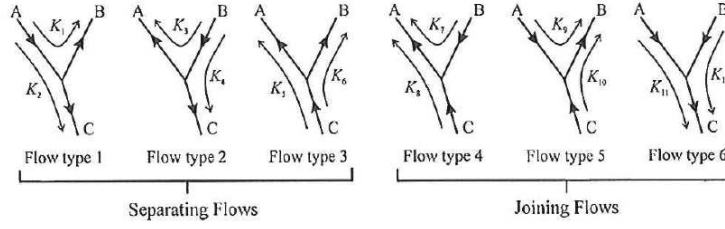
Fig. 2 Possible flow types and losses for a three-pipe junction

Figure 9: Flow configuration for determining pressure losses (reproduced from [3])

The two-well problem described earlier was extended to account for a single T-junction where the pipes connect. To accommodate this the single pressure P_4 at the junction is replaced by three pressures being the pressure on the legs of the junction. See figure 10 for the notation used and note that again we scale pressure so that $P_1 = 0$ and $P_3 = 1$. The
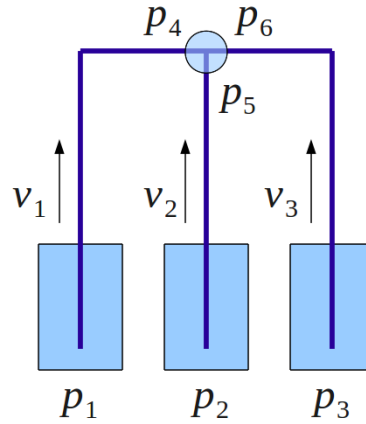

Figure 10: Pressure notation used in the legs of the T-junction

model equations are therefore:

$$P_4 = v_1 |v_1| \,, \tag{27}$$

$$P_2 - P_5 = L v_2 |v_2| \,, \tag{28}$$

$$1 - P_6 = v_3 |v_3| \,, \tag{29}$$

$$v_1 + v_2 + v_3 = 0 \,, \tag{30}$$

$$P_5 - P_4 = F_1(v_1, v_2) \,, \tag{31}$$

$$P_6 - P_4 = F_1(v_1, v_3) \tag{32}$$

where the functions F_1 and F_2 are given in [3]. we note that these functions are not continuous functions of the two velocities particularly because of the jump in behavior

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when the flows reverse. To this problem a simple model of the behaviors of each of the wells was added. This model represents the wells as buckets so that the pressure in each well depends on the flow of water into the bucket. Hence the model is

$$\frac{dP_1}{dt} = -v_1, \qquad \frac{dP_2}{dt} = -v_2, \qquad \frac{dP_3}{dt} = -v_3 \ . \tag{33}$$

where the initial values of P_1 P_2 and P_3 are given. This model was then solved numerically in order to determine the possible behavior that can occur. One difficulty in finding such a solution is the solution of the nonlinear equations representing the junction. It was found that there may be multiple solutions to the system and hence the current numerical method simply finds one of these and then attempts to follow the resulting behavior assuming that the pressure remains continuous.

In figure 11 the behavior of the three different wells is shown for the case when the junction is not considered to be special but has a single pressure, as in the earlier section. Here there are no issues of non-uniqueness of problems with numerical solution and the behavior is smooth and monotone.
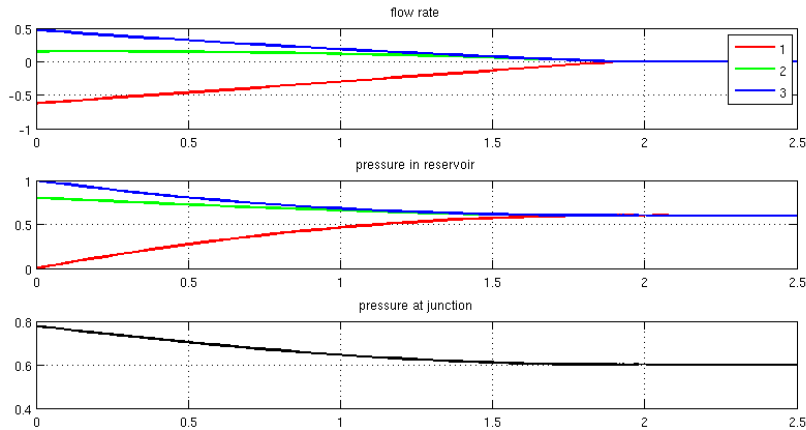

Figure 11: The flow and pressure in each of the three legs of the two-well problem and the pressure at the junction (no special junction behavior accounted for)

In figure 12 the behavior of the three different wells is shown for the case when the junction is not considered to be at a single pressure. Here the flow and pressure behavior shows some rather unexpected behavior. In particular the problem initially starts with the main flow from the first bucket to the last. This continues well past the time when the pressure of the middle bucket gets higher than both the others and might therefore be expected to contribute significantly to the flow. This is caused because the flow across the horizontal legs of the T-junction prevent the vertical leg from contributing until the flow across slows to a halt.
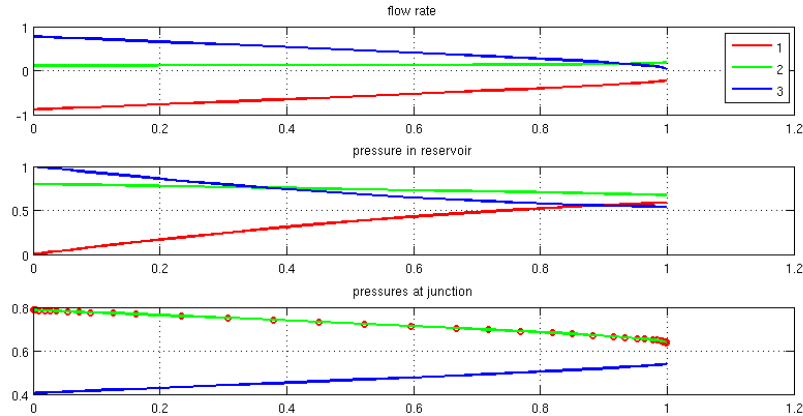
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Figure 12: The flow and pressure in each of the three legs of the two-well problem with junction behavior as given by [3]

Thus, when junction effects are included the solution can become non-unique. This means that the flow from a well may be augmented or retarded depending on the initial conditions and time-course of the problem. For example, in the case described above the flow from the middle well cannot force its way into the the collection pipe because of the force generated by the strong flow between the other two wells.

5 Including groundwater effects

There is data from the pumped wells close to the siphon system that indicates that there are significant interactions between pumped wells through the groundwater flow. DHI use a complex computational groundwater model to understand this behavior but it was deemed as appropriate to see how these effects might be incorporated into the model of the siphon. DHI have connected a computational model of the siphon system (similar to the basic model above) to the computational model of the groundwater and found interesting behavior and some computational difficulties. The study group therefore briefly examined how such integration might make numerical implementation difficult.

For flow in a confined aquifer with a horizontal top confining layer and of thickness d the pressure head in the system can be modelled using a simple equation

$$\frac{\partial(SP)}{\partial t} = \nabla\left(T\nabla P\right) \tag{34}$$

where S is the specific storativity, T is the transmissivity of the limestone and the spatial derivatives are in the horizontal plane only. To this model we would like to include the ten wells and these can be represented by ten point sources. As the system is linear we can

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add the sources together and so it is simple to consider a single source and seek the radially symmetric solution around it. The radially symmetric solution is outlined in [2](page 216) or [1] and for the problem

$$\frac{\partial(SP)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rT\frac{\partial P}{\partial r}\right) \tag{35}$$

with and initial pressure of $P = 0$ and a sink such that

$$2\pi rT\frac{\partial P}{\partial r} \to -Q \qquad \text{as } r \to 0 \tag{36}$$

is given as

$$P = \frac{-QS}{4\pi}\int_{(r^2 S/4Tt)}^{\infty}\frac{\exp -\eta}{\eta}\,d\eta\ . \tag{37}$$

The important part of this behavior is that the response at a distance of $r \sim l$ occurs of a time scale of order $t \sim l^2 S/T$. If the data from the submerged pumps is accurate then it is noted that this response appears to be within an hour. This requires very small value of S and large values of T. The change in the pressure at the different wells when all pumps have been off for some time and the first pump is turned on is shown in figure 13.
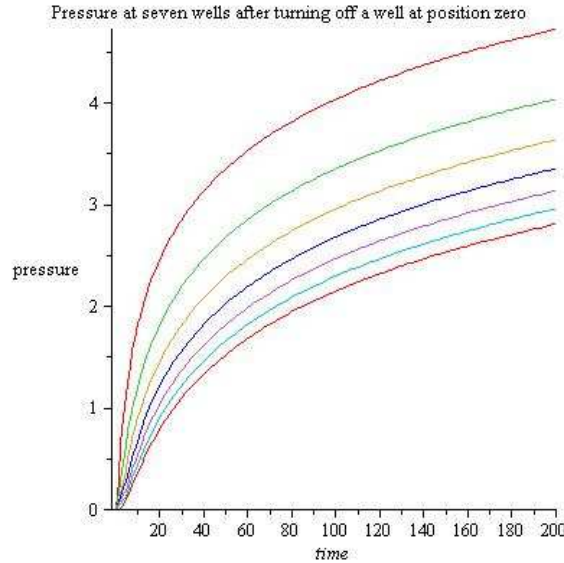

Figure 13: The pressure in the groundwater at eight different equally spaced wells when the first pump is suddenly started.

One of the consequences of having such small S and high T values is that this connects the groundwater levels between wells on a very short time scale, as expected to fit with data. However, this implies that the numerical procedure of coupling the groundwater to the pipe system must account for this. Currently the connection is made by assuming that

at each timestep the water levels in the groundwater model are constant and to introduce a small model to connect these given levels to the flow in the well accounting for local coning and losses in the screen on a single well. To get better convergence it may be necessary to ensure such local interaction is done at all wells simultaneously and even this may be inadequate.

6   Conclusions

In this report we have considered the flow of water from a number of interconnected wells into a single reservoir under the action of a siphon. We have found that the following behavior can occur.

- Wells further away tend to contribute less to the total due to pipe friction causing a loss in stagnation pressure head.

- Depending on the height of the water in the wells, and assuming it increases away from the reservoir the collector pipe can be made to make the flow uniform from all wells and this requires the collector pipe to increase in area significantly near to the reservoir tank.

- The existing model does not include the effects due to flow at the junctions. These make a significant difference to the flow in the pipes. In particular the behavior can be non-unique with the flow depending on the initial conditions and the history of the well heights. A well that is not producing but whose head is increasing may remain a small contributer if the junction is such that the cross-flow through the junction due to the other wells makes it hard to break into the stream.

- Careful design of the junctions may allow more uniform flows from the wells to occur.

- An electrical analogue has identified conditions in a small system where a well may cease to contribute to the flow (and hence might actually reverse its flow).

- The dependency of the flow in the collection pipe to the ground water heights is such that all the wells interact strongly through the groundwater. Hence numerical methods that link models of each part must account from this strong linking.

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